THE INFLUENCE OF CENTERING FORCES
ON IFR HOSE INSTABILITY

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H. Lee Buchanan

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Introduction:

The beam physics memo¹ by E. P. Lee addressed the magnitude of the centering force which arises from the image of a displaced ion channel on a relativistic electron beam propagating in the ion focused mode within a circular pipe. While this force is not large, it is always in a direction that tends to center the beam and may be therefore important in certain situations. A separate report by H. Lee Buchanan² discussed the development of a transverse (hose) instability which develops between mobile channel ions and a particle beam assuming that all forces were negligible except the electrostatic force between the beam and the channel ions. A result of that calculation was that for parameters characteristic of the ETA injected into one micron of Benzene, the instability should be readily discernible over propagation lengths of several tens of centimeters. There is therefore some concern that the IFR hose could affect the operation of IFR cells used for damping of the beam break-up (BBU) instability or emittance tailoring.

It is of interest to determine the effect of the centering force upon the IFR hose instability. In this note we add the appropriate centering force

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terms to the IFR hose equations and calculate their effect on hose development. It is found that while hose at very high frequencies is stabilized, the overall suppression is only moderate and other stabilization processes need to be explored.

The Model:

As in previous work we take the beam and the channel to have the same radius (a) and profile (gaussian). The beam and channel are oppositely charged and the channel charge density is taken to be a fraction, $f_n < 1$, of the beam charge density. If the beam and channel are separated transversely by a distance Δ , the force per unit length between them is attractive and given by

$$\text{force per length} = - \frac{2f_n I_b^2}{\Delta c^2} \left(1 - e^{-\Delta^2/2a^2} \right), \quad (1)$$

where I_b is the beam current. Beam particles are assumed to be relativistic ($v_b = c$) and ions are assumed to be longitudinally fixed.

We now assume that the beam is propagating down a smooth pipe of radius b parallel to the pipe axis but at a distance y from the pipe center. The ion channel is also parallel to the pipe axis at a distance d along the same radius. If the pipe is grounded, the ion channel induces a charge density on the pipe wall equivalent in effect to an image channel at a distance b^2/d from the axis and having a charge density $-f_n I_b/c$. The equation of motion for the beam is then

$$c^2 m_e N_b \frac{\partial^2 y}{\partial z^2} = - f_n \frac{I_b^2}{ac^2} F\left(\frac{y-d}{a}\right) + f_n \frac{I_b^2}{ac^2} F\left(\frac{y}{a} - \frac{b^2}{da}\right), \quad (2)$$

where we have defined

$$F(u) = \frac{2}{u} (1 - e^{-u^2/2}) \quad (3)$$

Note that $F(u)$ is the shape of the interaction potential between two beams separated by u and depends on beam and channel dimension and profile only.

The first term on the right hand side represents the attractive force of the ion channel on the beam. The second term represents the centering force on the beam from the channel image. There is also a force on the beam due to the image of the beam charge but this is cancelled to a fraction $1/\gamma^2$ by the image force of the beam current so both are neglected. N_b and m_e denote beam number density per unit length and beam particle mass respectively.

Similarly, we may find an equation of motion for the ion channel

$$c^2 f_n m_i N_b \frac{\partial^2 d}{\partial x^2} = f_n \frac{I_b^2}{ac^2} F[(y-d)/a] - f_n^2 \frac{I_b^2}{ac^2} F[(d - \frac{b^2}{d})/a] \\ + f_n \frac{I_b^2}{ac^2} F[(d - \frac{b^2}{y})/a] \quad (4)$$

The first term on the right hand side gives the direct attractive force of the beam on the ion channel. The second and third terms give the forces on the channel from the images of the channel and the beam respectively.

We can simplify both equations of motion first by definition of the new variables

$$\begin{aligned} Y &= y/a; & D &= d/a; & B &= b/a \\ Z^2 &= \frac{z^2}{a^2} \frac{I_b}{I_A} f_n; & X^2 &= \frac{x^2}{a^2} \frac{I_b}{I_A} \frac{m_e}{m_i} \gamma; & I_A &= \frac{\beta_{ymc}^3}{q} \end{aligned} \quad (5)$$

Note that we have absorbed the parameter f_n into the independent variable Z but could make no such scaling in X . Although this procedure risks some confusion, it is convenient for the numerical calculations that follow since the amplitude of the saturated growth is what is of ultimate interest and the normalization affects only the real part of the oscillation frequency. Second we note the following limiting expressions

$$\begin{aligned} F(u \rightarrow \infty) &= 2/u \\ F(u \rightarrow 0) &= u \end{aligned} \quad (6)$$

If we then assume $y - d \ll a$, $d - b^2/y \gg a$ and $y - b^2/d \gg a$ the equations become

$$\begin{aligned} \frac{\partial^2}{\partial Z^2} Y &= -(Y-D) - \frac{2}{B^2} \left[1 + \left(\frac{Y}{D} \right) \left(\frac{D^2}{B^2} \right) \right] D \\ \frac{\partial^2}{\partial X^2} D &= (Y-D) + \left[f_n D - \left(\frac{2}{B^2} \right) Y \right] \left(1 + \frac{D^2}{B^2} \right) \end{aligned} \quad (7)$$

Now we can linearize for a small displacement assuming $D, Y \ll B$ to get (for $2/B^2 = \alpha$)

$$\begin{aligned} \frac{\partial^2}{\partial Z^2} Y &= (1 - \alpha) D - Y \\ \frac{\partial^2}{\partial X^2} D &= (1 - \alpha) Y - (1 - \alpha f_n) D \end{aligned} \quad (8)$$

As expected, the previous equations of motion are recovered by letting $\alpha \rightarrow 0$ ($b/a \rightarrow \infty$).

Analysis:

Much can be learned about the consequences of $\alpha = 0$ dispersion relation.

If we apply a perturbation to the form

$$Y \sim D \sim \exp(-iKX - i\Omega Z), \quad (9)$$

and assume K to be real, then the growth rate, Γ , is found to be:

$$\Gamma = \text{Im} \left[1 - \frac{(1 - \alpha)^2}{(1 - \alpha f_n - K^2)} \right]^{1/2}. \quad (10)$$

From this expression we see that instability arises for

$$\alpha (2 - f_n - \alpha) < K^2 < (1 - \alpha f_n). \quad (11)$$

Increasing α from zero (i.e. reducing the ratio of pipe radius to beam radius) both lowers the upper limit and raises the lower limit, reducing from both ends the range of unstable mode numbers. In order to close off the range, thereby stabilizing all modes, we require $\alpha = 1$ (i.e. $b/a = \sqrt{2}$). This shrinks the pipe to within the beam, however, and the linear analysis breaks down. It is interesting to note that the ions play a mixed role in the development. Increasing f_n from zero with a finite α aids in the stabilization of the low frequency modes because of the inertia of the massive ions. The channel inhibits the stabilization of high frequency modes, however, because the force of the ion channel's image on itself is attractive and therefore de-centering. For very low wave numbers this works against the centering force from the beam.

Numerical Results:

The beam and channel equations of motion, Eq. (8), have been solved numerically to obtain linearized saturated growths as a function of α and f_n . As in reference 2, the spread mass model,³ was used to simulate phase mixing of particles in anharmonic potentials. The character of the numerical results are similar to those obtained previously.² For a fixed X , a perturbation shows exponential initial growth but reaches a saturation due to phase mixing. For fixed X the instability appears to damp away; actually the instability continues to develop by growing convectively in Z .

A measure of unstable growth may be obtained by determining the maximum amplitude of growth for a fixed X without regard to the location in Z where that saturated maximum occurs. When this saturated amplitude $Y_{\max}$ is plotted versus X on a semilog scale, the resulting curve is a straight line to excellent approximation. Figure 1 shows $Y_{\max}(X)$ for various values of α with $f_n = 1$. A qualitatively similar plot results with $f_n = 0$. This seems paradoxical in light of the fact that with f_n identically zero the channel no longer exists. One recalls, however, the normalization of $Z^2 \propto f_n$ and the fact that as $f_n \rightarrow 0$, the real part of the wavelength goes to infinity while the imaginary part remains finite.

The data of Figs. 1 and 2 indicates the functional form

$$Y_{\max} = \exp \left[p(\alpha, f_n) X \right] , \quad (12)$$

where p is a growth rate and depends on $\alpha = 2a^2/b^2$, indicative of wall effects, and $f_n = N_c/N_b$, indicative of channel effects. This growth rate is plotted as a function of α and f_n in Fig. 3. Dependence of the growth on channel strength is weak since it is due primarily to the second order effect of the force of the channel upon itself via its image. The dependence on α is quite linear and a modest extrapolation would suggest complete stabilization for α somewhat less than unity. Recall however that our linearization is strictly valid for $\alpha \ll 1$ indicating some caution is necessary.

One may conclude from Fig. 3 that wall effects do tend to stabilize ion hose but the effect is not great. A reduction in growth rate of a factor of two requires a wall to beam radius ratio of $b/a \approx \sqrt{8}$ under the most favorable conditions of $f_n \ll 1$. In this configuration one must then be concerned with scrape-off effects that have not been included here.

References

1. Edward P. Lee, "Possible Centering Force in the IFR," Beam Physics Memo #34, June 30, 1982.
2. H. Lee Buchanan, "Linear and Non-Linear Calculations of the Hose Instability in the Ion Focused Regime," LLNL report UCID-19495.
3. E. P. Lee, Phys. Fluids 21, 1327, (1978).

Figure Captions

Figure 1 Plot of $Y_{\max}$ versus X for $f_n = 1$.

Figure 2 Plot of $Y_{\max}$ versus X for $f_n = 0$.

Figure 3 Plot of growth rate p as a function of α for several values of f_n .

Figure 1

Figure 2

Figure 3