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Rotation and Shape
of High Altitude Reflectors
Controlled from the Ground

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In this note we will be concerned with passive reflector mirrors of electromagnetic radiation, located above (most of) the atmosphere. We assume that the reflector has generally the shape of a dish facing down, and that it remains stationary above a chosen point on the surface of the earth. In this sense one may say that the reflector is in "stationary orbit" around the earth. Its altitude, however, can be varied. In particular, the reflector may be located fairly close to the earth, much closer than what is often referred to as the "radius of a stationary orbit around the earth"; in fact, it may be located below the ionosphere. The mirror is prevented from falling on the earth by the pressure of the electromagnetic radiation which it reflects. The dish is constructed as a mesh of fine wires to make the reflector weigh as little as possible.

The shape of such a mirror will be determined by the two dominant forces which act on it: radiation pressure and gravitation. As a result of the interplay of these two, the mirror will be "blown up" to the desired face-down dish shape, just as a parachute assumes such a shape as a result of the interplay between air pressure and gravitation. Furthermore, it can be shown that the reflector will be stable against tilting, if a weight is suspended from its rim in the manner in which a parachutist is attached to his parachute. It will also be stable against small horizontal displacements away from the center of the radiation beam which impinges on it.

It was pointed out recently⁽¹⁾ that such passive reflectors located at stationary points above the atmosphere may replace communications satellites for intermediate range (a few hundred or thousand miles) communications. The main advantages of passive reflectors are: lower cost, broad bandwidth, relatively unsophisticated equipment required, and local relay stations are not needed. The latter fact, in turn, not only allows more flexibility in reception but is also advantageous from the environmentalist point of view, because it eliminates the undesirably high fields around relay stations.

There are several reasons why one may wish to adjust and control from the ground the orientation, rotation and shape of a passive reflector dish, supported above the atmosphere by radiation pressure:

First of all, if one orients the reflector correctly, and the supporting radiation emitted from the ground is plane polarized, then less radiation power is needed to prevent the reflector from falling. Indeed, if the wires which form the reflecting mesh are oriented perpendicularly to the plane of radiation polarization, then the wire mesh will be (almost) transparent to the radiation. On the other hand, if the wires are parallel to the polarization plane, then the reflexivity will be maximal. Therefore, to obtain highest reflexivity for a given number of wires, it would be advantageous to use plane polarized radiation, and construct the mesh of a large number of mutually parallel "longitudinal" wires oriented parallel to the radiation polarization plane, plus a smaller number of cross wires of negligible weight to hold the mesh together. (See Fig. 1.) If it were impossible to adjust the orientation of the wires from the ground, then the face-down dish of such a reflector could rotate around its vertical axis of symmetry in an uncontrollable manner. (As shown by the arrows in Fig. 1.) In fact, the mirror would tend to orient the bulk of its wires perpendicularly to the polarization plane of the incident radiation, thus becoming essentially transparent. In such a case the mesh would have to be constructed from an equal number of longitudinal and cross wires to insure equally good reflexivity for all orientations. The weight of the cross wires would then no longer be negligible, in fact, it would equal the weight of the longitudinal wires, thus effectively doubling the weight of the mirror. That, in turn, would require twice as much reflected radiation to support the mirror above the atmosphere.

The second reason for controlling reflector orientation, is that by continuously changing the orientation of the wires it is possible to keep the dish rotating around its vertical axis of symmetry (shown by the arrows in Fig. 1.). As seen from a coordinate frame rotating with the dish, centrifugal forces will then act on all non central points in the wire mesh, pushing them outward. This will change the curvature of the mirror. In this manner one moment of the mirror shape can be adjusted from the ground by adjusting the angular velocity of rotation.

Third, it is possible to suspend flaps from the reflector dish. When the dish is set in rotation, the flaps will open up, changing the shape of the reflector. Furthermore, if the distance between neighboring wires in a flap differs from that in the dish, then that flap will have different reflexivity. It can be so adjusted, for example, as to be transparent to radiation which supports the dish, but to reflect radiation carrying signals of some chosen longer wavelength.

Fourth, if the rotating dish is not cylindrically symmetric around its axis of rotation, or if flaps are attached to the dish, then the "footprint" on the ground of the radiation reflected from a rotating mirror will move around an orbit. The mirror will reflect radiation to different points on that orbit, at least once during each revolution. For example, if the emitter is located directly below the reflector, then the signals will be reflected by the flaps to points (or areas) which are located on the ground on the circumference of circles around the emitter. The radius of the circles will generally decrease if the angular velocity of the rotating dish increases, because in that case the flaps orientation will be closer to horizontal. By synchronizing signal emission with mirror rotation, one can insure that signals will be sent only to prechosen points (or areas), and no others. Using several flaps (and perhaps several wavelengths) and correctly intertwining signals, one can simultaneously send different messages to different

prechosen points (areas).

Calculation of the Torque Acting on a Wire

We calculate the torque which an electromagnetic wave of frequency ω exerts on an electron moving along a straight line inside a wire.

The x and y components of the electric field $\bar{E}$ are respectively

$$\begin{aligned} E_x &= \text{Re}[A e^{-i\omega t}] & ; & \quad A = |A| e^{i\alpha_0} \\ E_y &= \text{Re}[B e^{-i\omega t}] & ; & \quad B = |B| e^{i(\alpha_0 + \alpha)} \end{aligned} \quad (1)$$

where A and B are the complex amplitudes along the x and y directions respectively, and α is their relative phase.

A straight wire makes an angle χ with the x axis (see Fig. 2.). The component of E along the wire is $E_{\parallel}$, and its component perpendicular to the wire is $E_{\perp}$.

$$\begin{aligned} E_{\parallel} &= \text{Re}[A' e^{-i\omega t}] & ; & \quad A' = |A| e^{i\alpha_0} \cos \chi + |B| e^{i(\alpha_0 + \alpha)} \sin \chi \\ E_{\perp} &= \text{Re}[B' e^{-i\omega t}] & ; & \quad B' = -|A| e^{i\alpha_0} \sin \chi + |B| e^{i(\alpha_0 + \alpha)} \cos \chi \end{aligned} \quad (2)$$

An electron located at point ξ located inside a wire illuminated by an electromagnetic wave will experience a longitudinal force along the wire, due to $E_{\parallel}$. It will also experience a "frictional force" due to collisions with other particles in the wire. The frictional force will be proportional to $\frac{d\xi}{dt}$, the velocity of the electron and to the damping constant g . Therefore, the equation of motion of the electron will be

$$m \frac{d^2 \xi}{dt^2} = e E_{\parallel}(\xi) - m g \frac{d\xi}{dt} \quad (3)$$

In addition, the electron will experience a force perpendicular to the wire caused by $E_{\perp}$, but we assume the wire to be sufficiently thin so that any transverse motion of the electron can be neglected. There are also magnetic forces acting on the electron, but we assume that the electrons are suf-

ficiently nonrelativistic for these forces to be negligible. The solution of Eq. (5) is

$$\xi(t) = \frac{i}{\omega} \frac{1}{g - i\omega} \frac{e}{m} A e^{-i\omega t} \equiv \frac{1}{\omega} \frac{e^{i\phi}}{\sqrt{g^2 + \omega^2}} \frac{e}{m} A e^{-i\omega t} \quad (4)$$

The phase angle ϕ is defined by Eq. (4). The velocity of the electron is

$$v(t) = \frac{-ie^{i\phi}}{\sqrt{g^2 + \omega^2}} \frac{e}{m} A e^{-i\omega t} \quad (5)$$

When there is not just one electron, but n electrons per unit volume of the wire, all of which satisfy Eq. (5), then the conductivity of the wire is clearly

$$\sigma = \frac{e^2}{m} \frac{-in}{\sqrt{g^2 + \omega^2}} e^{i\phi} \quad (6)$$

At time t the instantaneous torque around the z axis exerted on an electron in the wire can be calculated particularly simply whenever $\bar{E}$ is independent of ξ . In cases of interest to us this condition is satisfied to a good approximation, so that we can write for the torque

$$\begin{aligned} T_z &= e \xi(t) E_{\perp} [\xi(t)] = e \xi(t) E_{\perp}(t) = \\ &= \frac{e^2}{m} |A| |B| \frac{1}{\omega} \frac{1}{\sqrt{g^2 + \omega^2}} [\cos(-\omega t + \alpha_0 + \phi)] \cos(-\omega t + \alpha'_0 + \alpha') \end{aligned} \quad (7)$$

When there are n electrons per unit volume of the wire, all of which experience the same torque, then the torque experienced by a unit volume of the wire will be n times the right hand side of Eq. (7). Substituting A' , B' and α' from Eq.s (2) and averaging time over one period of oscillation of the electromagnetic field, we obtain the time average torque around the z axis exerted by the electromagnetic field on a section of wire which has length l and crosssection s :

$$\begin{aligned} \langle T_z \rangle &= l s \frac{1}{\omega} |\sigma| \frac{1}{2} \left\{ [|A| |B| (\cos \alpha) \cos 2\mathcal{X} + \frac{1}{2} (|A|^2 - |B|^2) \sin 2\mathcal{X}] \right. \\ &\quad \left. \cdot \cos \phi + |A| |B| (\sin \alpha) \sin \phi \right\} \end{aligned} \quad (8)$$

When the electromagnetic wave is circularly polarized, one has $|A| = |B|$ and $|\alpha| = \frac{\pi}{2}$. Then $\langle T_z \rangle$ is independent of $\mathcal{X}$, as expected, and

$$\langle T_z \rangle_o = \ell s \frac{1}{\omega} |\sigma| \frac{1}{2} |A|^2 \sin \phi. \quad (8a)$$

If $g \gg \omega$, then $\sin \phi \approx 1 - \frac{1}{2}(\omega/g)^2$.

When the electromagnetic wave is linearly polarized along the x axis, one has $|B| = 0$, and $\alpha = 0$. Then $\langle T_z \rangle$ is

$$\langle T_z \rangle_x = -\ell s \frac{1}{\omega} |\sigma| \frac{1}{4} |A|^2 (\sin 2\mathcal{X}) \cos \phi. \quad (8b)$$

This torque vanishes for $\mathcal{X} = 0, \pm \frac{\pi}{2}$, and assumes its largest absolute value for $\mathcal{X} = \pm \frac{\pi}{4}, \pm \frac{3\pi}{4}$. If $g \gg \omega$, then $\cos \phi \approx -\omega/g$.

Angular Acceleration

A torque acting around the z axis, exerted on the wires of a reflector will cause the mirror to slow down or accelerate its rotation around the z axis. In addition, various torques acting on various wire segments may induce tensions, distortions, and may even set up elastic waves propagating along the wires, but here we are concerned only with the rotation of the mirror as a whole, around the z axis, going through the center of the dish, perpendicular to the surface.

The resultant torque acting on the mirror will be the sum of all torques acting on all the wires. The time averaged torque around the z axis exerted on any segment of a wire is given in Eq. (8). In that equation it was assumed that there are n conduction electrons per unit volume in the wire, and all satisfy Eq. (5). If the wire diameter, $2r$, is negligible compared to the skin depth of the radiation in the wire under consideration, then n is simply n_0 , the number of conduction electrons per unit volume. On the other hand, if $2r$ is comparable to, or much larger than the skin

depth, the effective n is only a fraction of n_0 , because the conduction electrons located deeper in the wire, experience only a reduced electric field strength. In cases of most interest to us, the effective wire radius will be comparable to the skin depth, so that $n = f n_0$, where f is less than one, typically $f \approx \frac{1}{2}$. For purposes of the following estimates, we shall assume $f = \frac{1}{2}$.

$$n = f n_0 = \frac{1}{2} n_0. \quad (9)$$

The moment of inertia around the z axis of the reflector will be the sum of the moments of inertia of each wire. If the total number of wires in the mesh is N , then the total moment of inertia M , will be

$$M = F N M_W \quad (10)$$

where M_W is the moment of inertia of a typical wire, and F is a correction factor which may differ from unity to take into account the fact that various wires may have different moments of inertia. In cases of interest to us the dish cross section in the (x,y) plane will not deviate much from a circle, most wires will have comparable length and uniform cross section, so that if we choose as the typical wire one whose length is a diameter of the dish: $2R$, the $F \approx 1$. We will assume this for purposes of the following estimate.

Table I shows the angular acceleration of the mirror around the z axis which can be achieved in a typical case.

Adjusting Mirror Orientation from the Ground

It is clear from Eq. (8) (or Table I) that larger angular acceleration can be imparted to the mirror, by radiation which is circularly polarized. Indeed, for reasonable radiation intensities and frequencies the mirror orientation angle (e.g. χ) of a mirror which is originally at rest, can

be changed by one radian within a time which is typically a fraction of a second. On the other hand, when the radiation incident on the mirror has the same intensity but is linearly polarized, then the corresponding time is of the order of minutes, even if the wires are so oriented as to allow the field to exert maximum torque. Indeed, denoting by $\langle T_2 \rangle_0$ and $\langle T_2 \rangle_x$ the time average torque around the z axis exerted by a circularly polarized field, and by a linearly polarized field respectively, their ratio, α can be found from Eq. (8):

$$\alpha \equiv \frac{\langle T_z \rangle_x}{\langle T_z \rangle_0} = \frac{1}{2} \frac{1}{\tan \phi} \sin 2\bar{\chi}. \quad (9)$$

Here $\bar{\chi}$ is some effective orientation angle, obtained by averaging χ over all wires. If the mirror is as shown in Fig. 1, then $\bar{\chi}$ is essentially χ_L , the angle between the polarization direction and the longitudinal wires. When $g \gg \omega$,

$$\alpha \approx -\frac{1}{2} \frac{\omega}{g} \sin 2\bar{\chi}. \quad (9a)$$

Assuming that the frequency of the radiation is $\leq 10^{10} \text{ sec}^{-1}$, $g = 3.10^{13} \text{ sec}^{-1}$ (see Table I.), and that the mirror is as shown in Fig. 1, one has

$$|\alpha| \lesssim 10^{-3} \sin 2\chi_L. \quad (10)$$

This last inequality demonstrates that it is possible to use a mirror of the type depicted in Fig. 1 to reflect radiation polarized linearly parallel to the bulk of the wires, i.e. parallel to the longitudinal wires ($\chi_L=0$), and, furthermore, that it is possible to control the orientation of the mirror from the ground.

To show this, assume first of all that only linearly polarized radiation impinges on the mirror and that $\chi_L=0$, i.e. that the longitudinal wires are parallel to the polarization of the incident radiation, and the mirror is at rest. Then the mirror will have maximal reflexivity, and its weight (as well as the supporting radiation intensity) need be only half as much as it would have to be if χ_L could not be controlled from the ground. Furthermore, Eq. (8) shows that in this case $\langle T_z \rangle = 0$: the mirror will be in equilibrium as far as rotations around its vertical axis of symmetry (the z axis) are concerned. However, this equilibrium is unstable. If for some reason χ_L were to assume a value different from zero, a torque $\langle T_z \rangle \neq 0$ would tend to further increase $|\chi_L|$. The increase would be exponential, with a typical time constant of minutes. Within a time interval of some minutes this torque would have to be counteracted in order to keep χ_L near zero. But this could easily be done by adding a small admixture of circularly polarized component (of the correct polarity) to the radiation to be reflected from the mirror. The intensity of this component would have to be only $|\alpha|$ times that of the linearly polarized component. According to Eq. (10), $|\alpha| \lesssim 10^{-3}$, and if the unwanted $|\chi_L|$ is small, $|\alpha|$ is even less.

In practice even if the angular velocity of the mirror is zero ($\dot{\chi}_L=0$), one would probably not want to maintain the direction of linear polarization parallel to x , but vary it periodically, allowing it to sweep a range of angles θ between $+\theta_0$ and $-\theta_0$, where $|\theta_0| \ll 1$, and has some suitably chosen value. As a result, on the one hand, the reflexivity of the mirror will vary as $\cos^2 \theta$, but for sufficiently small θ this variation is relatively small, so that only a small fractional increase in radiation intensity will be needed to compensate for it and keep the mirror at the desired altitude. On the other hand, the variation of reflexivity as a function of θ can be observed on the ground as an increase of reflected radiation, peaking when

θ goes through zero. In this manner, the orientation of the mirror can be monitored. Should any unwanted change in orientation occur, an appropriate small admixture of circularly polarized radiation can correct for it. If one wishes to change the angular velocity of the mirror, an admixture of circular polarization can be applied, and thereafter the direction of linear polarization will have to rotate with the new angular velocity, to maintain synchronization between the two, apart from the sweeping of an angular range $(\theta_0, -\theta_0)$ near the instantaneous orientation of the longitudinal wires to monitor their position. Again, should any undesirable change occur either in angular position or angular velocity, that can be corrected as described above.

References

- (1) Paul L. Csonka, Microwave Journal, October, 1979.

TABLE 1.

$n_o(\text{cm}^{-3})$	$8 \cdot 10^{22}$	
$g(\text{sec})$	$3 \cdot 10^{13}$	
$ A (\text{V/cm})$	30	
$\ell(\text{cm})$	$2 \cdot 10^2$	
f	0.5	
F	1	
M_W	$1.5 \cdot 10^{-1}$	
$\langle T_z \rangle (\frac{\text{g cm}}{\text{sec}^2})$	$\frac{1.3}{\gamma(\text{MH}_2)}$	if circular polarization
$\langle T_z \rangle (\frac{\text{g cm}}{\text{sec}^2})$	$1.3 \cdot 10^{-4} \sin 2\chi$	if linear polarization along x
$\ddot{\chi}(\text{sec}^{-2})$	$\frac{9}{\gamma(\text{MH}_2)}$	if circular polarization
$\ddot{\chi}(\text{sec}^{-2})$	$9 \cdot 10^{-4} \sin 2\chi$	if linear polarization along x

The n_o and g which appear in Eq. 3, are chosen to have values which characterize Cu wires. The values assumed for the correction factors f and F appearing in Eqs. 9 and 10 respectively, are explained in the text. The M_W is the moment of inertia of a typical wire of length ℓ , diameter $2.5\mu\text{m}$ and density 8.9 g/cm^3 (as for Cu). The torque around the z axis averaged over one oscillation of the electromagnetic radiation is $\langle T_z \rangle$, and the angular acceleration of the wire produced by that torque is $\ddot{\chi}$. The linear polarization is assumed to be along x , the wire makes an angle χ with x .

Figure Captions

Fig. 1   A flat dish shaped passive reflector facing downwards, is made of a mesh of a large number of "longitudinal" (i.e. parallel to the plane of polarization of the reflected electromagnetic radiation) wires and a few "cross" (perpendicular to the polarization plane) wires. If the dish is allowed to rotate around its center around its axis of vertical symmetry (as indicated by the arrows), it will reflect a smaller fraction of the radiation impinging on it, the reflexivity becomes (almost) zero when the "longitudinal" wires become perpendicular to the polarization plane as a result of such rotation. To prevent a decrease of reflexivity as the mirror rotates, the number of cross wires would have to be equal to the number of longitudinal wires. (The dashed lines indicate possible additional wires needed for structural reasons.)

Fig. 2   The components of electric field $\bar{E}$ along the x and y axes are E_x and E_y respectively. A straight wire is rotated with respect to the x axis by an angle χ . The component of $\bar{E}$ along the wire is $E_{\parallel}$ and perpendicular to the wire the component is $E_{\perp}$. The x, y and z axes form a right handed triad.

Fig. 1

Fig. 2