ROCK BED
HEAT ACCUMULATORS

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FOREWORD

This report on Rock Bed Heat Accumulators was prepared by the University of Minnesota, Minneapolis, Minnesota under U.S. Energy Research and Development Administration Contract EY-76-S-02-4009-M001.

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ABSTRACT

The principal objectives of the research program on rock bed heat accumulators (or RBHA) are, a) to investigate the technical and economic feasibility of storing large amounts of thermal energy (in the tens of Mwt range) at high temperature (up to 500°C) over extended periods of time (up to 6 months) using native earth or rock materials; b) to conduct studies to establish the performance characteristics of large rock bed heat accumulators at various power and temperature levels compatible with thermal conversion systems; c) to assess the materials and environmental problems associated with the operation of such large heat accumulators.

In the generic configuration of the RBHA, a large bed of crushed or naturally formed porous rock is heated by hot air flowing from a central manifold toward two outer cold manifolds; on discharge, the flow is reversed with cold air introduced at the top and bottom manifolds being heated in its passage through the porous media so that hot air is harvested from the central manifold.

The results of the study indicate that rock bed heat accumulators for seasonal storage are both technically and economically feasible, and hence could be exploited in various applications in which storage plays an essential role such as solar power and total energy systems, district and cogeneration heating systems. The economic advantage

of RBHA stems from the low cost inherent in the preparation of on-site native materials using a variety of construction techniques that can be adapted to local geological conditions and siting constraints with minimum environmental impact.

Section 1

EXECUTIVE SUMMARY

1.1 Description of RBHA Concept

Thermal energy storage on a large scale in terms of both time and volume of rock material can play an important role in energy conservation and management. In electric utility systems, load leveling provides for improved utilization of capital equipment resulting in substantial cost savings; in district heating systems, the use of waste or process heat in cogeneration modes can reduce premium fuel requirements; in solar power generation and/or heating systems, the inherent cyclic and intermittent nature of solar energy requires the use of efficient storage systems to achieve optimum energy utilization. While most of these applications have been concerned with short-term storage (hourly, daily, or weekly), there is a need also to consider long-term storage (seasonal or yearly). Among the various options such as ground or aquifer storage, the use of native earth or rock materials to form a large rock bed heat accumulator with air as heat transfer medium for charge and discharge offers a promising approach to the long-term thermal storage issue. The present study on rock bed heat accumulators RBHA is therefore aimed at determining the technical and economic feasibility of large-scale thermal energy storage materials with minimum environmental impact.

The concept of the rock bed heat accumulator, as originally presented [1]*, involves heating crushed or naturally formed porous rock by forcing hot air to flow from a central manifold to collector manifolds located at the top and bottom of the bed; on discharge, the air flow is reversed: cold air is introduced through the top and bottom manifolds and hot air is harvested through the central manifold. Input and output temperatures of up to 500°C and charging and discharging times of four to six months are possible with storage capacity limited only by the size of the bed. Various configurations of rock bed heat accumulators can be envisioned with local geological conditions, environmental constraints, and cost dictating the appropriate selection. Figure 1.1 illustrates two particular configurations applicable to surface and underground construction techniques.

1.2 Summary of Results

(a) System Performance

- Performance Characteristics: The design and performance of rock bed thermal storage systems are governed by these principal requirements: (a) heat leaks to surrounding (uninsulated) earth or groundwater must be minimized, thereby also insuring high first-law efficiency; (b) volume and second-law

* The references listed in Section 1 appear as appendices to this report.

efficiencies must be optimized by proper selection of rock materials and of bed dimensioning; (c) pumping losses (pressure drop) associated with the charging and discharging of the bed must be minimized relative to the input energy; (d) the stability of the flow through the porous bed heated from below must be maintained by inhibiting the onset of free convection.

- Dynamic Containment: This requirement is achieved in the generic triple manifold system by stopping the charging of the bed when the temperature begins to rise at the outer cold manifolds. Furthermore, the follow-up discharge process is allowed to continue until the temperature of the central hot manifold falls substantially. These conditions imposed on the cyclic operation of the bed are necessary in order to inhibit the build-up, after two or three years, of a temperature gradient that would otherwise fill the entire bed regardless of its length [3].

- Mathematical Models: Analytical and computer models of one- and two-dimensional configurations of earth storage beds have been developed [2,3,5]. The simple single-phase one-dimensional model in which air and rock are at the same temperature and which uses an effective thermal conductivity of the bed $k_e = k_{bed} + (v_{air} \rho_{air} c_{air})^2 / h_v$ is particularly suited to estimate the long-term performance. The single-blow solutions provide a good approximation to the dynamic containment operation that can be visualized

in terms of moving thermal wavefronts having simple mathematical closed forms. Such analyses serve to identify the basic parameters which affect the design and performance of earth thermal storage systems in terms of such criteria as volume, first- and second-law efficiencies. In particular, volume efficiency η_v is a function of $H/\sqrt{\alpha\tau}$ as illustrated in Fig. 1.2 for two dimensionless outlet temperatures θ_f [H is the bed height, τ the charging time, and α the effective bed diffusivity]. Clearly with bed length and charging time prescribed, volume efficiency can be improved only with a reduction in diffusivity of the bed.

• Pumping and Stability Requirements: The work done in pumping the air through the rock bed must be minimized in relation to the energy input. This ratio (which must be less than a few percent) is proportional to the square of the bed length and inversely proportional to the permeability of the porous material. The flow stability requirement that no free convection occurs in the portion of the bed heated from below is satisfied when the Rayleigh number is less than 40; this criterion constraints the product of bed height and permeability to be less than some critical value which depends on the temperature. The satisfaction of the constraints of pressure drop and stability establishes acceptable domains of operation from which the permeability and bed length can be selected as illustrated in Fig. 1.3 for the case of a 300 MWt

storage system with a charging time of 4 months, 8 hours per day, assuming a 40% volume efficiency. In this instance, bed height cannot exceed 35 to 45 m and the optimum permeability is about 5000 darcys ($5 \times 10^{-9} \text{ m}^2$) [6].

(b) Construction and Material Evaluation

- Construction Techniques: Various geometric arrangements and excavation methods have been identified for the construction of earth heat storage units in a variety of soil/rock types [6]. Surface construction methods such as trenching, rock quarrying, and strip mining, as well as underground techniques in competent rock involving mass blasting or large excavation are examined and compared (see Table I). Of particular promise is the use of naturally occurring porous rock such as St. Peter sandstone or reconstructed aggregates of fine particles as the storage medium.

- Material Evaluation: A literature survey [4] of the thermo-mechanical properties of rocks over the temperature range of 0°C to 500°C shows that for most rocks the thermal properties are temperature dependent and indicates that very little data is available on the mechanical properties of rock in this particular range. A series of laboratory experiments were conducted to evaluate the possibility of thermo-mechanical degradation of seven rock types subjected to up to 100 thermal cycles at 100° to 500°C. All samples showed dimensional changes and weight loss. The granites and sandstone

experienced a significant strength loss after thermal cycling whereas the other rock types tested (limestone, basalt, and quartzite) had no strength reduction. Even after the 100 thermal cycles, all rock samples, still had significant strength, and thus would be suitable for a RBHA.

(c) Environmental Considerations

The mode and degree of interaction of the accumulator with the hydrogeologic environment strongly influences its technological and ecological feasibility. A digital model based on finite-element techniques has been used to simulate two-dimensional heat transport from a RBHA unit into and with the groundwater flow system. It investigates the influence of different hydrologic parameters on the spatial evolution of dimensionless isothermal lines with time. The controlling parameters are groundwater flow velocity, aquifer dispersivity, thermal diffusivity, and position of the unit in the unsaturated soil zone, with the first two being the most important. The main results are that isotherms move very slowly under average flow and operating conditions, and that the most critical situation is one where intermediate velocities and dispersivities allow temperature buildup over a relatively large aquifer volume. However, if the temperature at the bottom of the RBHA is only 10 to 20°C higher than ambient soil temperature, this temperature rise will damp out quickly, and,

within a distance of 2 or 3 times the lateral extent of the RBHA, it will not be distinguishable from normal soil temperature fluctuations [8].

(d) Economic Assessment

The physical configurations and excavation methods considered for the various heat storage schemes are the primary cost determinants. For comparison and estimation purposes, it is convenient to express the total construction cost in dollars per ton of in-situ material removed and backfilled. Construction cost estimates were compiled from comparisons with similar construction projects, such as crushed rock quarrying and pipe laying, and do not include input/output heat exchangers, air pumping equipment, nor cost of energy input to the system. The construction cost indices for the various heat accumulator schemes studied ranged from a low of $\$1.77/\text{m}^3$ ($\$1.00/\text{ton}$) to a high of $\$10.60/\text{m}^3$ ($\$6.00/\text{ton}$) of rock material. An economic analysis computer program was developed that considered interest during construction, cost of capital, interest on debt, debt ratio, operation and maintenance with escalation, overhead, federal tax on profits, federal investment credit, other taxes, and depreciation. For an annual storage heat accumulator with a 300 MWt rating operating at 500°C , and assuming construction costs of $\$1/\text{ton}$ to $\$3/\text{ton}$, energy costs and capital costs are in the range of 5 to 15 mills/kWht and

150 to 400 \$/kWt, respectively. Such costs for long-term thermal energy storage in rock materials appear economically attractive.

1.3 Conclusions

The principal conclusions to be drawn from this study of rock bed heat accumulators can be summarized as follows:

- Large-scale seasonal TES in native rock is technically feasible, potentially economical, and environmentally acceptable.
- Performance criteria can be met by a dynamic containment design with a penalty in volume and second-law efficiencies, allowing:
- One-dimensional single-blow conductivity models to predict long-term performance with reasonable accuracy.
- Pumping and stability requirements can be satisfied by selection of material and bed dimensions.
- Several construction techniques have been identified to meet local site conditions.
- Laboratory temperature cycling of various rock types has indicated their suitability as storage medium.
- Naturally occurring porous materials provide for low-cost large-scale TES units.
- RBHA interaction with groundwater meets acceptable environmental constraints.

1.4 Recommendations

Based on these conclusions, it is recommended that further research be carried out on RBHA along the following main lines:

- Conduct a small-scale experiment on a long-term storage module constructed in St. Peter sandstone to validate the concept feasibility and assess environmental impact.
- Perform laboratory tests and measurements on natural and reprocessed rocks and sandstone with improved thermal and mechanical properties.
- Develop analyses for beds with non-homogeneous porous materials, and for hybrid bed configurations suitable for combining both long- and short-term storage.
- Establish the preliminary design of a rock bed accumulator unit for seasonal storage appropriate to a cogeneration heating system operating at moderate to low source temperatures.

1.5 Publications

(The research work reported in the following publications was performed by the University of Minnesota with the support of ERDA under the present contract.)

1. M. Riaz, P. L. Blackshear, and H. O. Pfannkuch, "High Temperature Energy Storage in Native Rocks", Proc. of ISES Joint Conference: Sharing the Sun, Winnipeg, Canada, Vol. 8, pp. 123-137, Aug. 1976 (Permagon Press).
2. M. Riaz, "Transient Analysis of Packed-Bed Thermal Storage Systems", Proc. of 1977 ISES Meeting in Orlando, FL, Vol. 1, pp. 17.11-17.15, June 1977.
3. P. L. Blackshear, P. Emerson, B. R. Baliga, and M. Riaz, "Gravel-Filled Trenches in Earth for Annual Thermal Energy Storage", Ibid., pp. 16.16-16.19.
4. H. O. Pfannkuch and M. H. Edens, "Rock Properties for Thermal Energy Storage Systems in the 0° to 500° Range", Ibid., pp. 18.5-18.9.
5. M. Riaz, "Analytical Solutions for Single- and Two-Phase Models of Packed-Bed Thermal Storage Systems". J. of Heat Transfer, ASME Trans., Vol. 99, pp. 489-492, Aug. 1977.

6. M. P. Hardy, V. D. Albertson, T. P. Bligh, M. Riaz, and P. L. Blackshear, "Large-Scale Thermal Storage in Rock: Construction, Utilization, and Economics", Proc. of 12th Intersociety Energy Conversion Engineering Conference in Washington, D.C., Vol. 1, paper No. 779093, pp. 583-590, Sept. 1977.
7. T. P. Bligh, "Thermal Energy Storage in Large Underground Systems and Buildings", Proc. of Rockstore 1977/Conference, Stockholm, Sweden, Sept. 1977.
8. D. W. Pollock, "Numerical Simulation of Energy Transport in Shallow Aquifers Subjected to a Thermal Stress from High-Temperature Energy Storage in the Unsaturated Zone", MS thesis, July 1977.

Table I: Comparison of construction techniques

CONSTRUCTION SCHEME	CRUSHED ROCK SUPPLY	ROCK QUALITY	TRANSPORTATION	SITING	ENVIRONMENTAL IMPACT	MAJOR COST	COST INDEX	BEST SUITED	CONSTRUCTION PERIOD (YRS.)
TRENCHING	EXTERNAL	GOOD AS SPECIFIED	OF CRUSHED ROCK	DEEP SOILS MUST BE AVAILABLE	PUMPING	TRANSPORTING CRUSHED ROCK TRENCHING	\$6/TON	SMALL STORAGE	1
STRIP MINING	SORTED ON SITE	MIXED	ON SITE OF CRUSHED ROCK AND FINES	SANDY SOIL MOST APPROPRIATE	LAY IMPERVIOUS BED	?	\$2 to \$3/TON	LARGE STORAGE	2
ROCK QUARRY	ADJACENT	GOOD BY SELECTION	ON SITE OF CRUSHED ROCK ONLY	OUTCROPS OF SUITABLE MATERIAL	LAY IMPERVIOUS BED	DRILL AND BLAST IN QUARRY	\$2 to \$3/TON	ALL SIZES	2
NATURAL POROUS ROCK	NATURAL	NATURAL	REMOVAL OF ACCESS	LOCALLY FAVORABLE CONDITIONS	PUMPING IF NECESSARY	ACCESS HORIZONTAL DRILL HOLES	\$1 to \$2/TON	WHERE NATURAL CONDITIONS APPLY	3
UNDERGROUND WITH MASS BLASTINGS	MASS BLASTING	GOOD	20% to 40% EXTRACTION DISPOSED ON SURFACE	ROCK AND GROUNDWATER CONDITIONS DICTATE	IN TIGHT ROCK IN FLOW LOW	ACCESS AND SHAFT IF AT DEPTH	\$3/TON	LARGE STORAGE	2-3
UNDERGROUND, CRUSHED ROCK BACKFILL	CRUSHED EXCAVATION MATERIAL	GOOD	ON SITE ALL MATERIAL MOVED	ROCK AND GROUNDWATER CONDITIONS DICTATE	IN TIGHT ROCK IN FLOW LOW	EXCAVATION	\$4 to \$6/TON	LARGE STORAGE	3-6

Figure 1.1 Examples of surface and underground RHBA.

Figure 1.2 Volume efficiency.

Figure 1.3 Pumping and stability constraints.

Section 2

RBHA MODELS

2.1 RBHA Sizing

Because the heat accumulator must be capable of storing large amounts of thermal energy (in terms of megawatt-years) at high temperatures (ranging from 200 to 500°C) over long duration (up to six months in the case of seasonal storage) and, furthermore, must be inexpensive (less than 50 \$/GJ or 0.18 \$/kWh) and constructed with abundant materials that tolerate high temperature cycling, all these considerations have led to a generic configuration of a rock bed module as illustrated in Fig. 2.1. In this arrangement, the accumulator consists of crushed or naturally formed porous rock placed in large caverns, trenches or holes surrounded by undisturbed earth and fed by hot or cold air from above or below. In the charge mode, hot air is forced to flow from a central manifold to collector manifolds located at the top and bottom of the bed. Upon discharge, the air flow is reversed with cold air introduced at the top and bottom manifolds while hot air is harvested from the central manifold.

To help establish order of magnitude estimates of size and cost for such an RBHA, a simple energy balance is applied to the accumulator. The thermal energy stored can be expressed as the volumetric capacity of the rock bed and its earth surroundings times the

Fig. 2.1 Generic configuration of RBHA.

temperature rise and total volume; this energy in turn is equal to mass rate of flow of air that must be pumped through an effective area A_a over a period of time τ_c times the specific heat of air and the temperature potential ΔT . Since all the rock volume will not be heated to the input air temperature, a volume efficiency η_v must be included in the overall heat balance which can then be expressed mathematically as

$$W = \eta_v \rho_b c_b V_b \Delta T = \eta_v \rho_a c_a v_a A_a \Delta T \tau_c \quad (2.1)$$

As estimating guide, Table II indicates the capacity and cost of RBHA units derived on the basis of assumed unitary constants for the material properties. The air flow is chosen here to correspond to a charge rate of about 12 days per meter, appropriate to seasonal storage. The average temperature potential ΔT is chosen for the convenience of round numbers as 100°C and so is the cost of 1 \$/ton. A volume efficiency η_v of 0.4 is considered to be realistic. Despite the somewhat artificiality of the numbers used in Table II, the potential economic advantages of RBHA for thermal energy storage systems are clearly evidenced. A major aspect of the work to be reported here is to determine realistic figures for such materials properties (see specifically Appendix A.3) and costs based on proposed construction techniques (see specifically Appendix A.2). In order to assess the performance characteristics of RBHA units, it is necessary to develop suitable analytical and computer models. Such models are next discussed.

ORDER OF MAGNITUDE ESTIMATES

PROPERTIES		Base $\Delta T = 100^\circ\text{C}$	
Bed	Air	SIZE	
$k_b = 1 \text{ W/m}^\circ\text{C}$ $\rho_b = 1 \text{ ton/m}^3$ $c_b = 1 \text{ kJ/kg}^\circ\text{C}$	$\rho_a = 1 \text{ kg/m}^3$ $c_a = 1 \text{ kJ/kg}^\circ\text{C}$	Energy density: $\rho_b c_b \Delta T = 0.1 \text{ GJ/m}^3$	
$\alpha_b = 10^{-6} \text{ m}^2/\text{s}$		Volume density: $25 \text{ m}^3/\text{GJ}$	
$v_b = 10^{-6} \text{ m/s}$ charge rate: $10^6 \text{ s/m} \cong 12 \text{ d/m}$	$v_a = 10^{-3} \text{ m/s}$ flowrate: 10^{-3} kg/s m^2	$1/\eta_v \rho_b c_b \Delta T \cong 0.1 \text{ m}^3/\text{kWh}$ $(\eta_v = 0.4) \quad 2000 \text{ m}^3/\text{MWd}$	
ENERGY BALANCE		COST	
$W = \eta_v \rho_b c_b V_b \Delta T$ $\downarrow$ volume efficiency	$\Delta T = \eta_v \rho_a c_a v_a A_a \Delta T \tau_c$ $\downarrow$ charge time	$1 \$ / \text{ton}$ $\downarrow$ $25 \$ / \text{GJ}$	$10\% \text{ interest}$ $\downarrow$ 10 mills/kWh

Table II

2.2 Analytical Models

In the usual formulation of a one-dimensional model of a packed bed, the following assumptions are first introduced:

- the fluid (air) is in plug flow
- the physical parameters are taken to be constant
- thermal gradient within solid particle is neglected
- radiation effects are negligible
- losses to environment are ignored
- internal heat generation is absent.

An energy balance is then performed on each of the fluid and solid phases yielding these two coupled partial differential equations:

$$\epsilon \rho_a c_a \left(\frac{\partial T_a}{\partial \tau} + v_i \frac{\partial T_a}{\partial \xi} \right) = h_v (T_b - T_a) + \epsilon k_a \frac{\partial^2 T_a}{\partial \xi^2} \quad (2.2a)$$

$$(1-\epsilon) \rho_b c_b \frac{\partial T_b}{\partial \tau} = h_v (T_a - T_b) + (1-\epsilon) k_b \frac{\partial^2 T_b}{\partial \xi^2} \quad (2.2b)$$

with appropriate initial and boundary conditions. The simplest of such conditions are those associated with the so-called single-blow operation in which a semi-infinite bed initially at a uniform temperature is suddenly subjected to a step change of inlet air temperature. Closed-form solutions of the single-blow conditions have been obtained for only two limiting situations corresponding to the so-called Schumann model in which axial conduction is ignored ($k_a = k_b = 0$), and to the single-phase conductivity model in which rock-air heat transfer is ignored ($h_v = \infty$).

With both dispersive effects present and/or arbitrary initial conditions, computer simulations must be resorted to obtain the transient solutions. However, despite their inherent restrictions, the closed-form analytical solutions of the Schumann and conductivity models for the single-blow case of a packed bed are important because they provide insight into the dynamic operation of packed beds, allow for the verification of computer solutions, and, as shown later, can be used effectively as design tools for predicting the behavior of the long-term cyclical performance of packed beds used for seasonal thermal energy storage. Thus, the principal analytical solutions are presented here with details given in Appendices A-5, A-6, and A-7.

Schumann Model

When the conductivity terms are dropped and the thermal capacity of the fluid phase is ignored (a warranted assumption in air systems), Eqs. (2.2) reduce to:

$$\rho_a c_a v_a \frac{\partial T_a}{\partial \xi} = h_v (T_a - T_b) \quad (2.3a)$$

$$(1-\epsilon) \rho_b c_b \frac{\partial T_b}{\partial \tau} = h_v (T_b - T_a) \quad (2.3b)$$

where v_a , the superficial air velocity is related to the interstitial velocity v_i through $v_a = \epsilon v_i$. These equations can be written in nondimensional form by introducing the following parameters

$$\lambda_2 = \frac{\rho_a^c v_a}{h_v} , \text{ a characteristic distance and}$$

$$\theta_2 = \frac{(1-\epsilon)\rho_b^c}{h_v} , \text{ a characteristic time with the result}$$

that

$$\frac{\partial T_a}{\partial x} = T_b - T_a = -\frac{\partial T_b}{\partial t} \quad (2.4)$$

in which $x = \xi/\lambda_2$ and $t = \tau/\theta_2$ are the dimensionless space-time variables. Finally, the unit step response for the single-blow case can be expressed by

$$\begin{aligned} T_a &= J(x, t) \\ T_b &= 1 - J(t, x) \end{aligned} \quad (2.5)$$

where the function J can be written in various equivalent forms as

$$\begin{aligned} J(\alpha, \beta) &= 1 - e^{-\beta} \int_0^{\alpha} e^{-u} I_0(2\sqrt{\beta u}) du \\ &= e^{-\alpha-\beta} \sum_{n=0}^{\infty} \beta^n M_n(\alpha\beta) = 1 - e^{-\alpha-\beta} \sum_{n=0}^{\infty} \frac{\alpha^n}{n!} N_{n-1}(\beta) \\ &= 1 - e^{-\alpha-\beta} \sum_{n=1}^{\infty} \alpha^n M_n(\alpha\beta) = e^{-\alpha-\beta} \sum_{n=0}^{\infty} \frac{\beta^n}{n!} N_n(\alpha) \end{aligned}$$

in which

$$M_0(\alpha\beta) = I_0(2\sqrt{\alpha\beta}) = \sum_{n=0}^{\infty} \frac{(\alpha\beta)^n}{(n!)^2}$$

$$M_n(\alpha\beta) = \frac{d^n M_0}{d(\alpha\beta)^n} = \sum_{k=n}^{\infty} \frac{(\alpha\beta)^{k-n}}{k!(k-n)!}$$

$$N_n(r) = \sum_{k=0}^n \frac{r^k}{k!} .$$

Furthermore, Eqs. (4) also yield

$$T_a - T_b = e^{-x-t} I_0(2\sqrt{\alpha t}) \quad (2.6)$$

The function I_0 denotes a modified Bessel function of the first kind of zero order; M_n is the n^{th} derivative of this function (first introduced by Schumann); $N_n(r)$ represents the first n terms in the series expansion of the exponential function e^r . Plots of the step response based on Eqs. (2.5) were given originally by Schumann [1] and, for an extended range of the variables x and t , by Furnas [2]. A convenient plot of the J function over a wide range of α and β values is shown in Fig. 2.2, reproduced from Vermeulen, Klein, and Hiester [4] (also given in reference [3] which includes, in addition, a table of values for this function).

Single-Phase Conductivity Model

The single-phase conductivity model of a packed-bed can be derived formally from Eqs. (2.2) by letting h_v tend to infinity. In this limit, the air and bed temperatures coalesce to the same value T and Eqs. (2.2) reduce to the single equation

$$\begin{aligned} [\epsilon \rho_a c_a + (1-\epsilon) \rho_b c_b] \frac{\partial T}{\partial \tau} + \epsilon \rho_a c_a v_i \frac{\partial T}{\partial \xi} = \\ [\epsilon k_a + (1-\epsilon) k_b] \frac{\partial^2 T}{\partial \xi^2} \end{aligned} \quad (2.7a)$$

which can be written in the simpler form

$$\frac{\partial T}{\partial \tau} + v \frac{\partial T}{\partial \xi} = \alpha \frac{\partial^2 T}{\partial \xi^2} \quad (2.7b)$$

Fig. 2.2 Plot of the J-function

$$J(\alpha, \beta) = 1 - e^{-\beta} \int_0^{\alpha} e^{-u} I_0(2\sqrt{\beta u}) du$$

by introducing the parameters:

$$v = \frac{\epsilon \rho_a c_a}{\epsilon \rho_a c_a + (1-\epsilon) \rho_b c_b} v_i$$

representing a thermal wave velocity in the absence of conduction effects ($\alpha=0$), and

$$\alpha = \frac{\epsilon k_a + (1-\epsilon) k_b}{\epsilon \rho_a c_a + (1-\epsilon) \rho_b c_b} v_i$$

representing a combined axial diffusivity. For air systems, the air capacity terms can justifiably be neglected compared to the solid-phase capacity so that

$$v \approx \frac{\rho_a c_a}{(1-\epsilon) \rho_b c_b} v_a$$

and

$$\alpha \approx \frac{k_b}{\rho_b c_b}.$$

To Eqs. (2.7) must be added a boundary condition constraining $\frac{\partial T}{\partial \xi}$ at the inlet of the bed: such a condition is the Danckwert type

$$\frac{\partial T}{\partial \xi} = \frac{v}{\alpha} (T - T_{in}) \quad (2.7c)$$

With the introduction of $\lambda_1 = \alpha/v$ as characteristic distance and $\theta_1 = \alpha/v^2$ as characteristic time, Eqs. (2.7) can be written in terms of the dimensionless x and t variables as

$$\frac{\partial T}{\partial x} + \frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} \quad \text{with} \quad \frac{\partial T}{\partial x} = T - T_{in} \quad \text{at} \quad x=0 \quad (2.8)$$

In Appendix A.5, it is shown with the application of Laplace transform methods that the unit step response of the single-phase model of a semi-infinite packed bed (initially relaxed) can be written in the closed form

$$T = \frac{1}{2} \operatorname{erfc} \left(\frac{x-t}{2\sqrt{t}} \right) + \frac{\sqrt{t}}{\pi} e^{-\frac{(x-t)^2}{4t}} - \frac{1}{2} (1+x+t) e^x \operatorname{erfc} \left(\frac{x+t}{2\sqrt{t}} \right) \quad (2.9)$$

in which erfc denotes the complementary error function. Plots of Eq. (2.9) as shown in Figs. 2.3 and 2.4 illustrate the space-time distribution of temperature inside the packed bed for the single blow case.

By comparing these plots of Eq. (2.9) with those of Furnas [2], it can be verified that they are nearly identical. For values of time greater than 10, the curves practically coincide; for small values of t , the agreement between T of Eq. (2.9) and T_b of Eq. (2.5) is within ten percent; clearly, for small t , T_a differs appreciably from T or T_b as expressed by Eq. (2.6).

For large values of t , the following approximations hold

$$T(x,t) \cong 0.5 \operatorname{erfc} \left(\frac{x-t}{2\sqrt{t}} \right) \quad (t > 10) \quad (2.10)$$

and, at $x = t$,

$$\left. \begin{aligned} T &\cong 0.5 [1 - (\pi x^3)^{-1/2}] \cong T_b \\ \frac{\partial T}{\partial x} &\cong -0.5 (\pi x)^{-1/2} \\ T_a - T_b &\cong 0.5 (\pi x)^{-1/2} \end{aligned} \right\} \quad (2.11)$$

Fig. 2.3 Single-blow response of packed bed (as a function of time).

Fig. 2.4a Single-blow response of packed bed (as a function of logarithmic distance).

Fig. 2.4b Single-blow of packed bed (as a function of linear distance).

Fig. 2.5 Piecewise linear approximation of moving thermal wavefront.

Equation (2.10) for the temperature profile may be visualized as constituting one half a complementary error function which moves in the axial direction at unity dimensionless velocity (or v actual) and which disperses with increasing time. Equation (2.10) provides simple measures of the location and sharpness of the moving wavefront. These approximations lead to a piecewise linear representation of the moving wave as shown in Fig. 2.5. In this approximation, the length of the wavefront is given by

$$\Delta x = 2\sqrt{\pi x} = 2\sqrt{\pi t} \quad (\text{dimensionless})$$

or

$$\Delta \xi = 2\sqrt{\pi \alpha \tau} \quad (\text{dimensional})$$

Similarly, the length of the bed corresponding to an exit temperature of T_f at time τ (or t) is given by

$$x_b = t + \sqrt{\pi t} (1 - 2T_f)$$

or

$$H = H_m + \sqrt{\pi \alpha \tau} (1 - 2T_f) \text{ with } H_m = v\tau$$

The corresponding wave velocity is

$$\frac{dx_b}{dt} = 1 - \frac{1}{2} \sqrt{\frac{\pi}{t}} (1 - 2T_f)$$

or

$$v_f = v - \frac{1}{2} \sqrt{\frac{\pi \alpha}{\tau}} (1 - 2T_f)$$

where the midpoint velocity ($T_f = 0.5$) is the previously defined velocity $v \cong \frac{\rho_a c_a}{(1-\epsilon)\rho_b c_b} v_a$.

Finally, the volume efficiency can be expressed as the ratio of the area under the wave to the length of the

bed with the result

$$\eta_v = 1 - \frac{\sqrt{\pi\alpha\tau}}{H} (1 - T_f)^2$$

Since the above expressions are derived from a piecewise linear wave representation, they are sufficiently accurate only for the case of a developed wave (i.e. $t > 5$) and for values of T_f in the range $0.1 < T_f < 0.9$. Figure 1.2 gives a plot of the volume efficiency as a function of the dimensionless parameter $H/\sqrt{\alpha\tau} = x/\sqrt{t}$, as derived by numerical integration applied to the actual curves of Fig. 2.4.

Clearly the single-phase model of a packed bed is simpler than the two-phase Schumann model. As elaborated in Appendix A.5, the closed-form solution for the transient step response given by Eq. (2.9) makes it readily adapted for obtaining the response to arbitrary time-varying inlet temperatures through the use of convolution techniques.

Equivalent and Combined Models (Appendix A.6)

The close agreement between the transient responses of the single- and two-phase models of a packed bed suggests that they are equivalent and can therefore be effectively combined. This equivalence is predicated upon a physical interpretation of the heat transfer occurring in a packed bed in which the various individual dispersive mechanisms involved in stagnant bed conduction, air-rock heat transfer, and internal particle conduction act separately (i.e., they are statistically independent) and can be combined by superposition. In accordance with this view, the

characteristic distance and time for a combined heat-transfer, conductivity model of a packed bed may be defined as the sum of the individual characteristic quantities associated with each model so that

$$\lambda_{eq} = \lambda_1 + \lambda_2 = \frac{v_a \rho_a c_a}{h_{eq}} = \frac{k_{eq}}{v_a \rho_a c_a} \quad (2.12)$$

$$\theta_{eq} = \theta_1 + \theta_2 = \frac{\rho_b c_b}{h_{eq}} = \frac{k_{eq} \rho_b c_b}{(v_a \rho_a c_a)^2}$$

where

$$\frac{1}{h_{eq}} = \frac{1}{h_v} + \frac{k_b}{(v_a \rho_a c_a)^2}$$

$$k_{eq} = k_b + \frac{(v_a \rho_a c_a)^2}{h_v} \quad (2.13)$$

These relations indicate that the effect of conductivity of the bed can be included in the Schumann model (this is especially relevant at low Reynolds numbers) by introducing the modified heat transfer coefficient h_{eq} . Similarly, in the single-phase model, the effect of air-rock heat transfer can be accounted for by introducing an effective thermal conductivity k_{eq} . To account for temperature gradients within the rock particles, the value of h_v may be reduced by a factor which incorporates the Biot number. It is of interest to point out that the ratio $\lambda_{eq}/\theta_{eq} = v$ and is independent of both h_v and k_b .

2.3 Computer Models

To account for the wide variety of possible operations of rock bed heat accumulators of different geometries (which

obviously cannot be treated analytically), a number of computer models have been developed based on a general program that solves the energy and Navier-Stokes equations in two spatial dimensions. Specifically, the program solves the following equation (an extension of Eq. 2.7 to two dimensions):

$$\frac{\partial T}{\partial \tau} + rv_{ax} \frac{\partial T}{\partial \xi} + rv_{ay} \frac{\partial T}{\partial y} = \alpha \left(\frac{\partial^2 T}{\partial \xi^2} + \frac{\partial^2 T}{\partial y^2} \right) + s \quad (2.14)$$

where

$$r = \frac{(\rho c)_a}{(\rho c)_b} \quad \text{and} \quad \alpha = \frac{k_b}{(\rho c)_b}$$

This program employs a fully implicit finite difference scheme to formulate the difference equations by integration over the control volume surrounding a grid point. A hybrid approach which combines the central difference scheme with the upwind difference scheme is used to approximate the temperature profile over space [5]. This technique resolves the exact but computationally more expensive exponential solution into three flow regions that are a function of Peclet number and diffusivity. Thus the profile will change in proportion to the relative magnitudes of the convection and diffusion terms in the general energy equation. The solution of the resulting nodal equations is done by an alternating direction line-by-line method using the tridiagonal matrix algorithm. The iterative nature of this procedure allows for temperature dependence of the conductivities and densities which cause nonlinearities in the system. The use of a line-by-line approach over a point-

by-point iteration scheme facilitates convergence and allows for greater flexibility in setting up the equations.

The various computer models are evaluated on the basis of their volume, first- and second-law efficiencies which are defined as follows:

- Volume efficiency: it represents the amount of energy recovered from the bed during discharge in relation to the total energy capacity of the bed and is defined as

$$\eta_v = \frac{(\rho c)_a \times v_a \times \text{inlet area} \times \int_{\text{discharge}} T d\tau}{\text{length} \times \text{width} \times \text{depth} \times (\rho c)_b \times T_{\text{hot}}}$$

- First-law efficiency: the first-law efficiency is the amount of energy recovered during the discharge cycle divided by the amount of energy injected into the bed during the charge cycle. Assuming constant air properties, velocities, and inlet areas during charge and discharge, the first-law efficiency can be expressed as

$$\eta_1 = \frac{\int_{\text{discharge}} T d\tau}{\int_{\text{charge}} T d\tau}$$

- Second-law efficiency: the second-law efficiency is the amount of available energy divided by the total energy divided by the efficiency of a fully stratified bed (the Carnot efficiency). It is a measure of how much energy cannot be returned because the system must be operated at an ambient temperature other than absolute zero. Generally, it increases as the bed is operated at elevated temperatures. Mathematically, this efficiency can be expressed as:

$$\eta_2 = \left(\frac{T_{\text{hot}}}{T_{\text{hot}} - T_{\text{cold}}} \right) \times \left(1 - \frac{T_{\text{cold}}}{T_{\text{c}} - T_{\text{d}}} \ln \frac{T_{\text{c}}}{T_{\text{d}}} \right)$$

(Carnot Effic.⁻¹ x available energy efficiency)

where:

T_{hot} = charging temperature

T_{cold} = discharging temperature

T_c, T_d = reference temperatures near inlet after
charge and discharge, respectively.

In Appendix A.1, computer simulations of several storage configurations with various boundary conditions are presented with typical results and possible applications indicated. While other configurations with slightly different boundary conditions were also studied, it was determined that they were not significantly different to warrant inclusion in this report.

2.4 References

1. Schumann, T. E. W., "Heat Transfer: Liquid Flowing through a Porous Prism", J. of The Franklin Institute, 208, pp. 405-416, 1929.
2. Furnas, C. C., "Heat Transfer from a Gas Stream to a Bed of Broken Solids", Trans. A. Inst. Chem. E., 24, pp. 142-169, 1929.
3. Sherwood, T. K., Pigford, R. L. and Wilke, C. R., Mass Transfer, chapter 10, McGraw-Hill, NY, 1975.
4. Vermeulen, T. G., Klein, G. and Hiester, N. K., Sec. 16 in J. H. Perry (ed.) Chemical Engineers Handbook, McGraw-Hill, NY, 1973.
5. Spaulding, D. B., Int. J. of Numerical Methods in Eng., 4, p. 551, 1972.

NOMENCLATURE

c	specific heat
h_v	volumetric heat transfer coefficient
k	thermal conductivity
t	dimensionless time, τ/θ
x	dimensionless distance, ξ/λ
H	length of bed
T	temperature
T_{in}	temperature at inlet of bed
v_a	superficial air velocity
v_i	interstitial air velocity
v	velocity of thermal wavefront
V	volume of bed
W	thermal energy stored
ΔT	temperature potential
α	diffusivity, $k/\rho c$
ϵ	porosity or void fraction
θ	characteristic time
λ	characteristic distance
ρ	density
ξ	distance
τ	time
η	efficiency

Subscripts

a	<u>a</u> ir (fluid)
b	<u>b</u> ed

Section 3

PERFORMANCE ANALYSIS

The object of this section is to examine the effects of employing RBHA composed of alternate gravel-filled trenches (or tunnels) and undisturbed earth. In particular, it is shown that average storage parameters can be obtained by introducing an effective thermal diffusivity (that is related to the thermal properties and the bed dimensions) into the one-dimensional theory discussed in Section 2, and solving the appropriate homogeneous one-dimensional problem. The use of packed beds for long-term storage implies small Reynolds number flow (small particles, low velocities) which in turn suggests that a theory based on the equilibrium between fluid and particle can be assumed. Hence, concentration distribution developed in equilibrium chromatography theory not only can suggest fruitful avenues of investigation, but in many instances can give good approximations of real system performance.

3.1 Constraints Imposed by Continuous Use and Environmental Impact

Consider a homogeneous system of constant properties initially at a uniform temperature T_0 . When fed with a high-temperature fluid at T_1 , a thermal wave or thermocline is produced which, as shown earlier, travels at its midpoint or $(T_1 - T_0)/2$ with a velocity $v \approx v_a \rho_a c_a / (1 - \epsilon) \rho_b c_b$ and which develops a front width increasing with time by

approximately $2\sqrt{\pi\alpha\tau}$. With the use of this piecewise linear approximation, the temperature of the bed at the exit corresponding to a length H will begin to rise when $v\tau + \sqrt{\pi\alpha\tau} \geq H$.

If it is desirable that the storage bed not produce an excessive net heating effect upon the environment, then the less time the extremity of the bed is at elevated temperature the better. On the other hand, if there is no environmental constraint, then the surrounding earth material can play an additional role in energy storage and, in this case, the bed can be completely filled. However, if environmental constraints limit the exit temperature to some fraction of $T_1 - T_0$, then the flow into the bed must be reversed when this temperature rise is reached at the exit. Furthermore, unless the end of the bed into which the hot gas is introduced and withdrawn is allowed to cool down to virtually T_0 , the bed cannot operate for an indefinite number of cycles. A typical cyclical or transient response is shown in Fig. 3 of Appendix A.7. Parenthetically, if the duration of the withdrawal cycle and storage cycle are the same, the flow rate must be greater during the withdrawal period. A consequence of these considerations is that the time-dependent temperature distribution within the bed rapidly approaches a steady state. A further consequence is that single transient solutions that lend themselves to analysis without computer can be employed to estimate the effects of different

parameters more readily than case-by-case studies. For example, when the volume efficiency is estimated from the fraction of the energy storage that can be used in the single transient solutions with the temperature rise at the exit limited to say, $0.1 (T_1 - T_0)$, i.e. $T_f = 0.1$, the result is as shown in Fig. 2.3. The shape of the volume efficiency versus dimensionless parameter $H/\sqrt{\alpha\tau}$ shows that with $T_f > 0.1$, $\eta_0 > 0.5$ only when $H/\sqrt{\alpha\tau} > 3$.

The cost per unit depth is frequently influenced by geological limitations. For example, we have considered the dry sandstone layer under the University of Minnesota campus as a readily accessible thermal energy storage medium. A storage depth of 6 m can be obtained at low cost; beyond 6 m, the water table must be manipulated at environmental and dollar cost. Thus, with a length limitation of 6 m total and for a double bed configuration, the effective bed length is then 3 m. If we used the property values reported by Pfannkuch and Edens in Appendix A.4, $\alpha = 10^{-6} \text{ m}^2/\text{s}$, the volume efficiency at $H = 3 \text{ m}$ is approximately five percent for $T = 15.8 \cdot 10^6 \text{ s}$ (6 months).

Other than by reducing water level, insulating, or accepting an environmental influence, three methods have been explored for achieving high volume efficiencies with six-month storage in 3 m of earth (or thereabouts): (a) the use of a heat pump, (b) the exploitation of local material property variations, and (c) the use of disturbed and rebonded particles with a reduced diffusivity and increased permeability.

- Heat Pump and the Environmental Limit

The environmental concern that no (or negligible) heat flow be permitted at the boundary of an uninsulated dynamically contained TES system permits transient heat fluxes providing the annual average heat flux is appropriately small. Our group has collaborated in the preliminary design of a system in which waste heat from a power plant on the University of Minnesota campus is stored during the summer and withdrawn during the winter to help cope with peaks in the heating load. A heat pump is employed in these designs to ensure that variations in the temperature of the heat recovered from stores can be compensated for. A potential fringe benefit of this pump system is that the cold return from the heat pump could be used to counteract the heating effect resulting from storage. It was found that at best the reduction was only a few percent and could not offer a significant reduction in environmental heating.

- Local Property Variations

The inexorable spreading of the thermal front as it is slowly translated into the storage bed is the cause of the low volume efficiency in short beds filled and emptied in an annual cycle. Chromatographic analogs show that if the dependence of the heat capacity of sandstone upon temperature reported by Pfannkuch and Eden (see Appendix A.3) were allowed for the spreading of the wave would be even worse than we predict for constant properties during the

charging cycle and would tend toward a constant wave front thickness during discharge (see Amundson et al (1)). One of us (Perry L. Blackshear, Jr.) who is working on a University-sponsored chromatographic separation process called the "adsorptivity reversal" process has explored the possibility of adapting the process to minimize the spreading of the wave front during the charging cycle. The proposal is to inject a bolus of condensing vapor, say water, into the bed at the onset of each charging cycle. Then, if the thermal and absorptive properties have the required values, the bolus will follow along with the thermal wave, restricting its spread. One of the thermal diffusivity modulating sandstone alterations described below (the use of sodium silicate as binder) holds promise for securing the desired absorptive properties. An estimate of the consequences show that the shortest bed that could permit this asymptotic wave width to be reached is one for which $H/\sqrt{\alpha\tau} \approx 2$; all beds for which $H/\sqrt{\alpha\tau} > 2$ thus treated will have higher volume efficiencies than those without treatment. Although this helps performance, it does not alter the requirement that for adequate volume efficiency, $\eta_v > 0.5$, $H/\sqrt{\alpha\tau}$ must be approximately 5 or better. One important consequence would result from the possibility of achieving an asymptotic wave front in both filling and emptying. A steady-state cyclical operation would be possible without the necessity of nearly complete filling or emptying as is the case otherwise.

• Modification of Permeability and Thermal Properties

In order to raise $H/\sqrt{\alpha\tau}$ to a level where good volume efficiencies may be expected within an upper bound placed on H leaves us with the possibility of altering $\alpha\tau$.

Reduction in α have been achieved by disrupting the sandstone and also by reassembling the sand into spherical pellets of order 1 cm diameter using sodium silicate as the bonding agent. The thermal changes accompanying these modifications have been examined experimentally by Abdel-Wahed and Emerson; a first draft of their paper is in preparation. Briefly, the results are summarized in the following table:

	Sandstone	Sand	3/4" Sand Pebbles	1/4" Sand Pebbles
Density	2152 kg/m ³	1652	885.7	975.6
Porosity	30.1%	32.1	-----	-----
Conductivity	1.64 W/m°K	.23	.1805	.2098
Specific Heat	1141 J/kg°K	1235	1188	1188
Diffusivity	6.67E-7 m ² /s	1.12E-7	1.71E-7	1.81E-7

It can be seen that simply disrupting the partial-particle bonds reduces the thermal conductivity six-fold in St. Peter sandstone. Aggregating the particles into pellets appears to reduce this gain somewhat. However, an approximately twofold increase in $H/\sqrt{\alpha\tau}$ can be achieved in sandstone with fixed H and τ by relatively simple maneuvers in which the sand is modified in situ.

If τ is reduced, i.e., the bed is used for short-term storage, there is an accompanying increase in air velocity,

all things being equal, and thus a demand for increased permeability. These demands can be readily met through pelletizing the sandstone.

3.2 Single-blow and Asymptotic Results.

Repeated storage without some sort of wavefront confining mechanism as described above and constrained by environmental limitations to low (or brief) temperature rises at the exhaust requires that the heat removal from the bed be nearly complete each cycle. Thus, the single-blow models such as the one discussed in Section 2 offer a convenient way of assessing the influence of system design variables on the performance of repeatedly used beds. Should wave confining mechanisms as described above be employed, asymptotic results as reported by Amundson and Rhee [1] become useful means of estimating performance. For example, it should be, in principle, possible to move such waves back and forth within a storage chamber indefinitely.

On the other hand, when interaction with the environment is to be studied, the effects can involve transients that persist for 20 years or more depending on the size of the structure. Computer modeling under these circumstances is of great convenience. Thus, the influence of the heat pump on the environmental impact of long-term thermal energy storage, or the use of the surrounding earth as storage media, are more conveniently explored via computer modeling. Similarly, the use of a series of uninsulated beds or tanks to be employed for daily storage embedded in earth such that

the heat leakage constitutes a form of long-term storage can better be analyzed by the computer model as well.

- Heterogeneous beds.

The convenience of the assumption that the solid and fluid are in thermal equilibrium is self-evident. A number of authors have shown that these equilibrium solutions also represent two-phase solutions under many circumstances of practical importance. We have discussed above that the single-blow homogeneous equilibrium system represents a useful approximation of an environmentally limited annual cycle because each cycle must start from a virtually cold bed in order to achieve a steady-state operation with an adequate volume use efficiency. In Appendix A.7, we show that the computer model solutions for a single year as the solutions approach a steady state can be modeled very closely by the single blow model in respect to volume efficiency.

It is also shown in Appendix A.7, that with air confined to flow through gravel-filled trenches or tunnels, separated by undisturbed earth or rock, that up to spacings of 6 m the annual cyclical variation in temperature across one module of the array appears to be a perturbation of the temperature field a single homogeneous equilibrium bed can afford. Thus the estimate of the performance can be made from the relatively simple solutions of the single-blow homogeneous bed providing the effective thermal diffusivity is taken to be

$$\alpha_e = \alpha \left(1 + x^2 \frac{W^2}{L^2} \right)$$

where W is the module width, L is the width of the bed, and x , the dimensionless length. Then, when this value of thermal diffusivity, α_e , is employed in calculating the behavior of the modules by treating them as single-blow homogeneous beds, the volume efficiencies obtained are remarkably close to those obtained from the computer analysis up to a spacing of 6 m. These comparisons are made in Table 2 of Appendix A.7.

Once again the system is constrained to operate such that at the start of each cycle the bed was nearly cold so that the single-blow model is expected to apply. In those cases where property values are grossly different between trench and undisturbed earth, where properties vary with temperature such that wave front confinement occurs in the trench but not in the surroundings, or where significant interplay with the surroundings are permitted, the simple model loses its utility.

3.3 Stability

With a thermally stratified bed, the second-law efficiency is at a maximum. However, if the bed is heated from below to minimize surface losses, buoyant forces can disrupt the stratification through convection. Thus it is desirable to predict the onset of convection under the various flow conditions that might be encountered in thermal storage.

Two situations have already been discussed both theoretically and experimentally in the literature [2,3,4]: These are the cases of heating from below with no flow or horizontal flow. The case of no flow would correspond to charging the bed solely by conduction. Natural convection sets in when a Rayleigh number of about 40 has been reached. The case of heating from below with horizontal flow would rarely be encountered in packed bed thermal storage but is mentioned here for completeness. In this situation between the Rayleigh numbers of 40 and 260 a series of convective rolls forms which may or may not travel in the direction of the flow. Above $Ra=260$, there is no pattern to the flow and a perfect instability has been reached.

A third situation which is far more typical to packed bed storage is the case of heating from below with vertical throughflow. Only one paper [5] treats the subject to any length; in it, the authors show that a convective instability will occur due to either buoyancy forces or a viscosity difference between two parts of the fluid phase which causes the hotter and less viscous to "channel" through the colder and more viscous part. The flow will either favor the instability if it runs counter to the gravity vector, or hinder it if it runs in the opposite direction. The analysis is not, however, complete with respect to the present application. Since the authors have sought a closed-form solution they have neglected non-Darcy effects, non-Boussinesq effects, and the somewhat controversial inertial term of the basic momentum equation. The former

two could enter at the very high flow speeds (~ 10 cm/s for air) which might be encountered in daily storage or at large temperature differentials. The validity of the inclusion of some form of the inertial term would have to be determined by experiment.

3.4 Pumping and Stability Requirements on RBHA Materials and Size

For the efficient and economic operation of an RBHA, the pumping work that must be done to move the air through the packed bed must be minimized in relation to the energy carried by the air. Furthermore, for the portion of the bed heated from below, conditions of flow stability must be insured so that no free convection can take place. Such pumping and stability requirements impose definite constraints upon the design and operation of an RBHA. A simplified analysis to establish those constraints is presented.

• Pumping Requirements

The pumping work to move air at velocity v_a through a bed of height H can be expressed as a pressure drop ΔP in terms of the air viscosity μ_a and bed permeability K as

$$\Delta P = \frac{\mu_a}{K} v_a H \quad (3.1)$$

or, since

$$v_a \approx \frac{(\rho c)_b}{(\rho c)_a} v \approx \frac{(\rho c)_b}{(\rho c)_a} \frac{H}{\tau_c}, \quad \Delta P = \frac{\mu_a}{K} \frac{(\rho c)_b}{(\rho c)_a} \frac{H^2}{\tau_c} \quad (3.2)$$

If this pumping energy associated with charging and discharging the bed is to be kept below a prescribed fraction r of the energy carried by the air, the following condition must be satisfied

$$\frac{2\Delta P}{(\rho c)_a \Delta T} = \frac{2\mu_a}{K} \frac{(\rho c)_b}{(\rho c)_a^2} \frac{H^2}{\tau_c} \frac{1}{\Delta T} \leq r \quad (3.3)$$

so that

$$\frac{H^2}{K} \leq \frac{(\rho c)_a^2}{2\mu_a (\rho c)_b} r \Delta T \tau_c \quad (3.4)$$

where the factor 2 is introduced to account for the pumping energy being expended during both charge and discharge cycles.

• Stability Requirements

As discussed previously, the stability of porous systems heated from below depends upon the Rayleigh number. For Rayleigh numbers less than 40, no free convection columns are established so that bed stratification can be maintained: The stability condition can be expressed by the Darcy-Rayleigh relation

$$Ra: \frac{KHg\beta}{\alpha_m v_a} \Delta T \leq 4\pi^2 \quad (3.5)$$

so that

$$KH \leq \frac{4\pi^2 \alpha_m v_a}{g\beta \Delta T} \quad (3.6)$$

where

α_m is a modified bed diffusivity $k_b/(\rho c)_a$,

ν_a is the kinematic viscosity μ_a/ρ_a ,

β is the volumetric coefficient of expansion of air ($1/T$ for an ideal gas), and

g is the gravitational constant.

Interrelationship of Bed Permeability and Height

To estimate the effects of pumping and stability constraints upon bed design, the following material constants are assumed

$$\mu_a = 2.5 \times 10^{-5} \text{ Pa.s} \quad k_b = 1 \text{ W/m}^\circ\text{C}$$

$$(\rho c)_a = 10^3 \text{ J/m}^3\text{ }^\circ\text{C} \quad (\rho c)_b = 1.4 \times 10^6 \text{ J/m}^3\text{ }^\circ\text{C}$$

$$\nu_a = 2.5 \times 10^{-5} \text{ m}^2/\text{s} \quad g = 9.8 \text{ m/s}^2$$

$$\beta \approx \frac{2}{T_h + T_c} = \frac{2}{\Delta T + 2T_c} \quad (T_c = 300^\circ\text{K})$$

and the following parameters are taken as base:

$$r = 0.02$$

$$\Delta T = 500^\circ\text{C}$$

$$\tau_c = 3.456 \times 10^6 \text{ s (40 days)}$$

Substituting these values in Eqs. (3.4) and (3.6) gives the following two (extreme) conditions:

$$\frac{H^2}{K} = 4.94 \times 10^{11} \text{ (dimensionless)}$$

$$HK = 1.11 \times 10^{-7} \text{ m}^3$$

which are shown in log-log plots in Fig. 3.1. The figure indicates the acceptable domains of operation for which both constraints of pressure drop and flow stability are

satisfied and from which the permeability and height of beds can be selected. For the assumed conditions, it is seen that the bed height cannot exceed 40m. At this maximum height, which also maximizes volume efficiency, the optimum permeability is about $3 \times 10^{-9} \text{ m}^2$ (3000 darcys).

The effect of modifying the operating parameters can be visualized simply by translating the boundary limits of Fig. 3.1 either vertically or horizontally maintaining the slopes of the thermal stability line at -1 and the pumping requirement line at +2, respectively. For instance, changing one of the parameters r , ΔT , or τ_c by the multiplying factor m means displacing the pumping boundary line downwards by the distance m if $m > 1$ (increase in parameter value), and upwards if $m^{-1} > 1$ (decrease in parameter value). The stability boundary, on the other hand, is only a function of ΔT (but not time) and moves upward with a decrease in ΔT in accordance with Eq. (3.6).

3.5 Concluding Remarks

The most demanding storage system, namely the annual cycle system with minimal environmental impact and low-cost construction, can be achieved in those sites where the depth (5 - 30 m per bed in a double bed) can be realized. Thus, in regions where the water table is deep, in sandstone layers where water is no problem and in bed rock, this most demanding system could be within reason. The calculations and cost estimates of this report can be employed to

Fig. 3.1 Interrelationship of bed permeability and height.

estimate the technical and economic (energy and dollar) feasibility of such an installation.

The next most demanding system the annual cycle at minimum cost where surrounding earth or rock can be part of the system is attractive where environmental restrictions can be relaxed. This system has not yet been evaluated in the search for an optimum configuration, but it appears that many of the calculations reported herein can be useful in designing prototype systems. For example, if a series of beds in parallel can each in turn be filled, the first-order effect is to have a long bed (with its corresponding high efficiencies) that includes surrounding earth as part of the storage system.

A single fast-charge-fast discharge bed fed from above but located under a layer of rock that can be heated could serve as a daily storage system embedded in an annual cycle system.

The possibility of using the double bed as a short-term storage system is attractive from a number of points of view. First, the dimensionless time and length can be sufficiently large to assure good efficiencies within a wide range of water table limitations. Second, because the system is used more often than once a year, the fierce cost restriction can be limited. Although the instability problem will be greater (these fast-blowing pumping limitations require large particles and are more subject to thermal instability since the Rayleigh number is proportional to particle diameter squared), the possibility exists of using an aspirated small particle

short bed as an insulator on top of a larger particle long-bed heated from below.

During the course of preparing this final report, one of its coauthors, Perry L. Blackshear, found that there exists the possibility of modifying the bed properties in such a way that an asymptotic thermal wave thickness could be achieved during both charge and discharge cycles. Although the concept has not been explored beyond some first-order estimates as to the limiting asymptotic wave thickness, it seems clear that all the efficiencies would benefit from the correctly varying heat capacities and that the length restriction eased somewhat. It is believed that this application of chromatography theory to thermal energy storage offers a potentially promising way to achieve stratification in a cycling system without the demand that the system be purged each cycle or that the exit temperature be permitted to rise.

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Section 4

MATERIALS EVALUATION

4.1 Introduction

During the life of a Rock Bed Heat Accumulator the rock will be thermally cycled at least once per year. Depending upon operating conditions and positions of the rock within the bed, the thermal cycles could range from ambient temperatures to 500°C. How does rock respond to such cycling and are some rock types more stable than other, requiring the selection of specific rock types in the construction of a Rock Bed Heat Accumulator?

4.2 Material Properties Influence on Bed Performance

Before discussing the response of various rock types to thermal loading, the consequences of mechanical breakdown and degradation of the bed will be presented so that the importance of selecting a stable rock type can be assessed. The two most important parameters in the rock bed design are its effective thermal conductivity and permeability (see Appendix A-2). Both these parameters are relatively poorly understood for the application and material here studied, but two things are known: both the porosity and particle size distribution will influence both parameters. A reduction in porosity will cause an increase in effective thermal conductivity (see, for example, the discussion by Combarnous and Baries), and a decrease in bed permeability. Both are detrimental to the performance of the Rock Bed Heat Accumulator. A change in particle size and size distribution by degradation

will cause first an overall reduction in particle size and secondly, clogging of air paths by the smaller particles, both effects contributing to a reduction in permeability. An approximate relationship between permeability, porosity and particle size given by Carmen, 1937, illustrates the dependence of these parameters, namely

$$k = \frac{d^2}{150} \frac{\phi^3}{(1-\phi)^2}$$

An increase in the number of small particles relative to larger particles will influence the effective thermal conductivity as the particle-to-particle contact area will be increased, thereby increasing the flow paths for heat conduction within the rock bed.

In addition to the mechanical aspects of particle size changes affecting the effective thermal conductivity and permeability, chemical changes within the bed could result from thermal cycling. These changes could result in a change in thermal properties of the particles in the bed and hence the effective thermal conductivity of the bed. Such phenomena as dehydration could leave the rock particle unchanged in overall dimension but with a high porosity and lower conductivity. This would be beneficial to the effective thermal conductivity of the bed. Chemical changes could result, however, in the rock particles losing sufficient strength so that particle breakdown and compaction could occur under the self weight of the bed. Such collapse would reduce the porosity and permeability and would increase the effective thermal conductivity of the bed and possibly render the bed inoperable.

Hence a study was conducted to evaluate the changes that might occur both to the mechanical structure and chemical composition of a rock bed, of naturally formed rock material, subject to thermal cycling over a 40-year life. A review of the literature revealed that very little information is available on this subject particularly because of the severe thermal loading and the relatively short time span. Most papers concerning chemical and mechanical changes recorded in the laboratory are for temperatures near the melting point of some minerals within the rock or at high confining pressure (see, for example, Murrell and Chakrovanty, 1973, and Heard, 1967).

Somewhat similar heat cycling conditions are applied to rocks in a sauna. During operation of the sauna the rocks are heated constantly and intermittently cooled by water, the steam generated heating the sauna room. As the sauna is used only occasionally the rocks cool to ambient temperature when not in use. The temperature of the rocks probably reaches 200°C and cools to maybe 100°C during the application of water. The temperature inside a sauna is typically 45 to 72°C. Often locally available alluvial gravels are used in saunas with no particular selection process used. These gravels usually consist of hard crystalline rocks such as quartzites and granites. A local sauna dealer told us that such rocks "last forever" with no cracking, spalling or degradation. The conditions in a sauna, though at a lower

temperature range than for the Rock Bed Heat Accumulator, illustrate that probably many rock types will meet the needs of the accumulator. It is difficult, however, to show conclusively at this time that a particular rock type is acceptable for an accumulator. The chemical and mechanical changes that might occur in 40 years cannot be compressed into a short-term laboratory study.

4.3 Laboratory Study

To illustrate the influence of thermal cycling on some typical rocks that might be considered suitable for a Rock Bed Heat Accumulator, a laboratory study was undertaken. This study consisted of evaluating the mechanical changes that occurred during thermal cycling of seven rock types from 100 to 500°C. One hundred cycles were performed over a four day period. The heating medium was hot air from the combustion of propane gas.

The seven rock types selected for testing were:

- Dresser basalt
- St. Cloud gray granodiorite
- Sioux quartzite
- Berea sandstone
- Holston limestone
- Plattville limestone
- St. Peter sandstone

The first five rock types were chosen as they have been intensively studied at the United States Bureau of Mines and their subcontractors, Kreck et al., 1974. The latter two rock types represent weaker rocks found beneath the University of Minnesota campus. Table 4.1 summarizes the properties of these seven rock types.

TABLE 4.1 SOME MECHANICAL PROPERTIES OF ROCK

	Compressive Strength MN/m ²	Tensile Strength MN/m ²	Porosity %	Density gm/cc	Permeability μm/sec	Diffusivity x 10 ⁻⁶ m ² /sec	Conductivity Watts/m °C
Dresser Basalt	479	22.9	0.16	3.02	>10 ⁻⁴	5.0	2.2
St. Cloud Grey Granodiorite	280	6.90	0.07	2.72	>10 ⁻⁴	5.5	2.0
Sioux Quartzite	512	10.3	0.15	2.64	>10 ⁻⁴	12.0	4.0
Berea Sandstone	44.5	0.984	19.1	2.11	0.56	0.55	0.81
St. Peter Sandstone	3.45	0.7	28.0	2.00	23.6	0.4	0.52
Halston Limestone	119	10.1	0.24	2.7	>10 ⁻⁴	NA	NA
Platteville Limestone	83	NA	NA		NA	NA	NA

Tests were designed to measure uniaxial compressive strength, tensile strength, weight loss and dimension changes as a function of number of heat cycles. To accomplish this, one-inch diameter cores, approximately two inches long, were cored from a single block of rock cut and the ends surface ground so that the ends were flat, smooth and parallel. The samples were measured and weighed. Forty samples were cut and prepared for each rock type.

The samples were heated in a batch roaster at the Mineral Resources Research Center at the University of Minnesota. This batch roaster had been designed and used for the study of pelletizing of taconite for the steel industry. The basket of the batch roaster is approximately 20 inches in diameter and 20 inches high, large enough to accommodate all the rock samples in one run. Thermocouples were placed at the top and bottom of the bed so that the heating cycle could be stopped when all the rock had reached 500°C.

After 3, 10, 30 and 100 heating cycles, the rock samples were allowed to cool and seven samples of each rock removed for later testing. These samples were measured, weighed and five samples tested for uniaxial compressive strength, one sample cut into two short cylinders and each tested for tensile strength and one sample retained for later inspection.

4.4 Test Results

The strength test results are summarized; however, as the project is not yet complete (the subject of a Master's

thesis) a full explanation of the mechanisms for strength reduction is not given.

4.4.1 Weight Loss

All samples showed some weight loss after three cycles but little additional loss with subsequent heat cycles. The loss was probably due to drying out of the samples as some color change was apparent in most cases and the loss was less than 3 percent. The St. Peter sandstone samples were damaged during the process of the experiments; this was probably due to abrasion during handling as the samples were very weak and friable even before heating. No dimension change measurements were made, and no strength tests performed, on the St. Peter sandstone samples.

4.4.2 Dimension Changes

All samples showed some increase in length after heat cycling. However, after the initial cycles no additional dimension changes occurred.

4.4.3 Uniaxial Compressive Strength

The uniaxial compressive strength is simply the force required to cause rock fracture divided by the initial rock sample cross sectional area. The force must be evenly distributed to the rock sample to avoid localized stress concentrations. In these tests the force was transferred to the rock via a cylindrical steel end platen of the same diameter as the rock.

The results of these tests are summarized in Fig. 4.1 which shows the normalized strength for each rock type as a function of the number of heating cycles. The normalized strength is the strength divided by the rock strength before heating. Only average strengths are shown. St. Peter sandstone is not included because of breakdown in sample integrity. All rock type, except the granodiorite, maintained at least 80 percent of their initial strength.

All samples, except the granodiorite, showed no difference in fracture mode or brittleness as a function of heat cycling. The granodiorite, however, showed a dramatic change in fracture mode with number of heat cycles. After heating the granodiorite was less brittle and broke into more fragments, more like granulated sugar, than the unheated samples, which formed fewer, larger angular pieces.

It appears then that in Rock Bed Heat Accumulator design where rock particles will be loaded by point contacts similar to that imposed by the Brazilian test, some splitting could occur. This could impose a height limitation on the bed, although this is not considered serious except for St. Peter sandstone, Halston limestone, Plattville limestone and Berea sandstone. The other rocks have sufficient strength even after thermal weakening to resist this type of failure.

From the weight loss tests and visual inspection of the samples it was apparent that no significant dust problem would occur. By extrapolating this finding to a RBHA design, it appears that most of the dust or fine particles would be

generated from crushing or splitting of rock particles following thermal weakening.

The test results indicate that during the life of the Rock Bed Heat Accumulator structural changes must be assumed. Over the extended life of the accumulator other time dependent changes, similar to weathering processes, will occur and hence the strength reductions and structural changes seen in the short-term laboratory experiments represent the minimum strength loss. It follows that unless extraordinary precautions are taken in material selection some bed degradation will occur during the life of the accumulator. More extensive investigation of the consequences of these changes are required before any one rock type can be excluded from consideration. However these laboratory tests indicate that a strong rock such as basalt, quartzite and even granite would be best suited for the accumulator material than a weaker sedimentary rock such as the limestones and sandstones.

4.4.4 Tensile Strength

The tensile strength was measured using the Brazilian test, sometimes known as the diametral compression test. The compressive force, W , required to diametrically split a cylinder of diameter D and length L was recorded. The tensile strength is calculated from

$$\sigma_t = \frac{2W}{\pi DL}$$

Fig. 4.2 shows the normalized tensile strength plotted as a function of number of heating cycles. As only two tests

were performed per sample per number of cycles, the data is more scattered than for the compression tests. However, a more pronounced strength reduction is evident here than in the uniaxial compression tests. Only the basalt appears relatively unaffected; the test result for zero cycles for basalt is probably statistically low, thereby giving the other results the appearance of strengthening with thermal cycling. All other rock types lost as least 40 percent of their original strength after 100 cycles with the majority of the loss appearing after the first three cycles. The Berea sandstone appeared to maintain its strength until 10 cycles of heating, but lost significant strength after 100 cycles.

4.5 Discussion of Results

The test results indicate that some form of cracking occurred in most of the rock types tested and that the majority of this cracking appears in the initial cycles. The compressive strength was relatively insensitive to this cracking except for the granodiorite. The tensile strength reduction, however, was significant for all rock types except the Dresser basalt.

4.6 References

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Figure 4.1 Change in compressive strength of six rock types with thermal cycling.

Figure 4.2 Change in tensile strength of five rock types with thermal cycling.

Section 5

ENVIRONMENTAL EFFECTS

The central question concerning environmental effects of an RBHA Storage site deals with the inherent heat loss from the storage unit, the temperature rise and distribution, in the surrounding geologic and hydrogeologic environment, and its absolute (especially maximal) values in space and time. From such considerations critical effects on the biological and geological environment can be assessed. In order to describe temperature distribution around the RBHA the governing mechanisms and equations for heat transport in the hydrogeologic environment have to be identified, material parameters assigned and boundary configurations and conditions described.

5.1 Mechanisms of Heat Transport

The main mechanisms of heat transport in the hydrogeologic environment are heat conduction and convection. The complex constitution of the container and distribution of the mobile phases make analytical treatment extremely difficult. In order to simplify the situation it is best to look at two zones in which different transfer mechanisms dominate, and two regimes where two different modes of transfer occur.

The first zone of heat transfer outside the RBHA is the partially water-saturated zone. There, the principal mechanism of heat transfer is by conduction through the

multiphase aggregate of rock-mineral-water and air combinations. The degree of saturation may vary from totally dry to totally saturated. The first condition is not likely to be found at the beginning of a storage project, but it is reasonable that after some time of application of a temperature gradient total expulsion of capillary water from the pore space will result. Of all other possible degrees of saturation, the so-called field capacity is the most likely to occur, this being the amount of water that can be held against gravitational forces by capillarity. Finally, under high absolute storage temperatures the possibility exists that banks of continuous water form at 100% saturation as a result of vaporization, migration, and condensation. These do not necessarily have to be connected with the groundwater flow field.

The second zone of importance is the totally saturated groundwater flow field where a hydraulic potential gradient generally sets up circulation patterns which give rise to forced convection in the temperature field. Heat transport by convection is further influenced by mechanical dispersion of heat due to the flowing fluid carrier.

Two temperature regions have to be considered. The first is where temperatures are always lower than the boiling point of the saturating fluid, in this case water. Mass transport is then in only one phase, but one still has to consider whether actual temperatures deviate in a critical degree from reference or undisturbed temperatures. This

is of importance when considering the groundwater flow field. According to Hubbert (1940), the groundwater flow field can be treated as a (velocity) potential field only when the temperature dependent parameters density and viscosity can either be considered constant or when it is expressed as a unique function of temperature, and properly incorporated into the governing equations. The studies of heat transport in shallow aquifers to be discussed below were carried out under the assumption that density and viscosity differences were small enough to be neglected. If this is not the case, especially when a high temperature RBHA is situated relatively near the watertable and temperatures are close to the boiling point empirical and engineering methods will have to give some information. They can be used to at least establish a configuration of the first non-critical isotherm which then can be used as a boundary for the rest of the field.

If temperatures at a given point or region are above the boiling point of the saturating liquid, an increase of heat transfer will occur due to a vaporization-condensation capillary effect. The saturating liquid will evaporate at the high temperature level, thus latent heat will migrate with the vapor along a vapor pressure gradient and subsequently will be released on condensation. Depending on whether the system can be considered open or closed, some or all of the condensed water will return by capillary action to the high-temperature end. The overall effect is that the effective

heat transfer coefficient can be several times greater than the one due to conduction alone. It will depend on the absolute permeability of the medium, the mobility ratios of liquid and vapor, liquid phase saturation, capillary pressure gradient, heat of vaporization, and temperature gradient. Again the analytical treatment of this problem and experimental verification are far from perfected (Somerton 1975) and would necessitate experimental assessment of orders of magnitude for local conditions of interest. For a complete assessment of a high temperature RBHA installation this factor has to be included.

In summary the basic transport equations can be expressed by the following relation

$$J_i = L_{ij} X_j \quad (5.1)$$

where J are the generalized fluxes, x the generalized driving forces and L the material transport parameters, and the indices indicate that these equations have to be written for each quantity transported. Using the proper conservation statement and thermodynamic equation of state the proper field equations can be generated and solved in principle when initial and boundary conditions are specified.

5.2 Material Properties

One general objective of this study is to determine the order of magnitude of material parameters in heat transfer as exemplified by Eq. (5.1), their dependence on mineralogic and petrophysical parameters and on the temperature

conditions imposed by the RBHA operations. One important consideration is to present the results in a form useful to engineering applications. This has been done by compilation and critical review of scattered literature data. The special difficulty was to find and to extend results into the temperature range of interest, namely 100°C to 500°C. Expression for conductivity for minerals and dense rocks, for dry porous rocks, partially and totally saturated are given.

For minerals and dense monomineral aggregates the general form of the temperature dependency in the temperature range of interest is

$$K = BT^m \quad (5.2)$$

where K is the thermal conductivity ($\text{Wm}^{-1}\cdot\text{K}^{-1}$), T the temperature ($^{\circ}\text{K}$), and B and m denote a coefficient and an exponent. The exponent m is negative for most crystalline materials, approaching -1 for pure quartz, but larger than -1 for most mixed crystals and more disordered crystals. In the cases of some minerals it may actually become positive, which it definitively is for amorphous substances. Numerical values for the coefficients have been compiled and are prescribed in Appendix A-4.

Aggregates of polymineral composition will exhibit conductivities that depend on the relative proportions of the mineral constituents and the composite conductivity model. The most reasonable and simple model is that of conduction in series yielding

$$\frac{1}{K(A)} = \sum \frac{1}{P(I)K(I)} \quad (5.3)$$

where $K(A)$ is the conductivity of the aggregate $P(I)$ the volume fraction of mineral constituent I with conductivity $K(I)$.

More elaborate expressions exist such as the one by Hashin and Strickman (1962) involving variational principles to establish upper and lower bounds of aggregate conductivity. The particular adaptation to thermal conductivity is due to Horai (1971).

Let K_U and K_L be the upper and lower bounds of a macroscopically homogeneous and isotropic multiphase material. They can be expressed in terms of the phase conductivities $K(I)$, ($I=0, 1 \dots n$) and the volume fractions $P(I)$ ($I=0, 1 \dots n$) as in following

$$K_U = K_{\max} + A_{\max} / (1 - \alpha_{\max} A_{\max}) \quad (5.4a)$$

$$K_L = K_{\min} + \frac{1}{1 - \gamma_{\min}} \frac{A_{\min}}{A_{\min}} \quad (5.4b)$$

where

$$K_{\max} = \text{Max}(K_0, K_1, K_2 \dots K_n)$$

$$K_{\min} = \text{Min}(K_0, K_1, K_2 \dots K_n)$$

$$\gamma_{\max} = (3K_{\max})^{-1}$$

$$\gamma_{\min} = (3K_{\min})^{-1}$$

$$A_{\max} = \sum_{i(K_i \neq K_{\max})} \frac{P(I)}{(K(I) - K_{\max})^{-1} + \gamma_{\max}}$$

$$A_{\min} = \sum_{i(K_i \neq K_{\min})} \frac{P(I)}{(K(I) - K_{\min})^{-1} + \gamma_{\min}}$$

The representative value for the aggregate can be found by appropriate averaging techniques between K_u and K_L , which becomes arithmetic averages when the difference between K_u and K_L and the range of $K(I)$ values is narrow.

But the question has to be asked whether the more sophisticated forms are warranted in view of the overall accuracy of the input data.

Substitution of Eq. (5.2) in (5.3) gives

$$\frac{1}{K(A,T)} = \sum \frac{1}{P(I)B(I)T^m(I)} \quad (5.5)$$

Calculated values have been checked against experimental results and give good agreement for basalts, but much less so for granites. The question has been discussed in Pfannkuck and Edens (1977 b) in Appendix (A.4).

The same approach of proportional transmission through different mineral phases can also be extended to the case where one of the phases is a void or pore space filled with air, the conductivity of which can be neglected as a first approximation. Such equations have been given by Woodside and Messmer (1961). Based on a three element resistor model, it is

$$K(A) = \frac{aK_D \cdot K_F}{K_D(1-d)+dK_F} + cK_F \quad (5.6)$$

where

$$a+c=1, \quad d+(1-d)=1$$

are the sides of the hypothetical resistor in cubic form. They also express bulk phase distribution or porosity. Other approaches have been used to correlate petrophysical

parameters to conductivity; Somerton (1975) gives the following expression for dry sandstones:

$$K = 0.59\rho - 5.54\phi + 0.92k^{0.10} + 0.022F - 0.054 \quad (7)$$

with K: conductivity [W/m °K], ρ bulk density [g/m³], ϕ ; fractional porosity, k: permeability in millidarcies; F: dimensionless formation resistivity factor.

For totally saturated materials a number of expressions have been derived. Again on the basis of petrophysical parameters Somerton (1975) derived a correlation on the bases of regression analysis, namely

$$\frac{K_s}{K_D} = 1.0 + 0.3 \left[\frac{K_f}{K_a} - 1 \right]^{0.33} + 4.57 \left[\frac{\phi}{1-\phi} \frac{K_f}{K_D} \right] \left[\frac{\rho_s}{\rho_a} \right]^{-4.3} \quad (5.8)$$

with K representing conductivities, ρ the bulk densities, and the subscripts indicating: s liquid saturated rock, d dry rock, f saturating liquid, a air, and C_m the cementation factor from Archies' equation, a petrophysical parameter describing tortuosities of pore paths and degree of consolidation.

For unconsolidated totally saturated sands we have

$$K = 0.38 - 1.19 \phi + 0.02k + 7.23K_f + 0.088K_D - 0.0025 \frac{D_{90}}{D_{10}} + 0.305D_{50} \quad (5.9)$$

where D_{90}/D_{10} is a grain size distribution parameter and D_{50} is the median grade diameter in mm.

Based on a relation developed experimentally by Asaad (1955) Edens (1977) has combined equ. (2) with that of Asaad for well cemented massive, quartz rich sandstones.

$$K = K_D \left(\frac{K_f}{K_D} \right)^\phi \quad (5.10)$$

and obtaining K as a function of temperature

$$K(T) = \frac{2000}{T} (-0.28 \times 10^{-3} T + 3.1 \times 10^{-6} T^2 - 3.9 \times 10^{-6} T^3)^\phi \quad (5.11)$$

for a fully saturated quartzite sandstone.

For the case of partial saturation this expression has been extended by a factor $b S_w^n$ which describes the fractional decrease of conductivity from total saturation to very low partial saturation ($\sim 1\%$). The saturation S_w is given in percent of total pore space, b and n are empirical constants that have the values $b = 0.48$, $n = 0.08$ for $1 < S_w \leq 20$, and $n = .245$ and $n = 0.308$ for $20 < S_w \leq 100$. This gives:

$$K(T, S_w) = \frac{2000}{T} (-0.28 \times 10^{-3} T + 3.1 \times 10^{-6} T^2 - 3.9 \times 10^{-6} T^3)^\phi [b(S_w)^n] \quad (5.12)$$

Somerton (1975) gives the following expression for sandstone with varying brine saturations as

$$\frac{K_{S_w}}{K_{S_w=0}} = 1 + \left[\frac{K_{S_w=1}}{K_{S_w=0}} - 1 \right] S_w^{1/3} \quad (5.13)$$

where the K 's are again the conductivities at different saturations, and the subscripts S_w designate liquid saturation in samples, $S_w=0$ for zero saturation dry sample, $S_w=1$ for total saturation, and S_w is the fractional brine saturation.

It needs to be repeated that the empirical and semi-empirical reductions given above are based on limited laboratory

experiments and often only over limited ranges of mineral composition, porosities and saturation as well as limited temperatures. No adequately generalized expression, based either on composition or petrophysical parameters exists. It is clear, however, that a strong dependency on temperature exists which is further amplified when the transport parameter for solution of the field equation is used, namely thermal diffusivity D

$$D(T) = \frac{K(T)}{\rho(T)C_p(T)} \quad (5.14)$$

where ρ is the mass density, and C_p the specific heat at constant pressure. Since ρ varies very little with temperature, but $C_p(T)$ varies generally with T^n , where n is a positive exponent. Since from Eqn. (5.2) $K(T)$ varies as T^m where m is a negative exponent for minerals and crystalline aggregates, it can be seen that the effects are additive and thereby increases the temperature dependence of diffusivity.

In a realistic field situation other factors of scale, inhomogeneity and anisotropy, as well as the presence of preferential channels of heat or mass conduction are possible. In this respect the relevant representative elemental volume (REV) has to be defined over which material properties including thermal diffusivity and hydrologic transmissivity can be averaged (Bear 1970 and Pollock 1977).

In this context a combined diffusion parameter has to be specified which contains those terms describing molecular heat diffusivity and the vaporization-condensation-capillary effects. This has not been done in the literature, yet all

vapor-heat transfer models have been developed for normal soil temperatures, and it was not within the scope of the present investigation.

On the other hand Pollock (1977) developed an expression for the dispersion parameter for heat transport in a flowing groundwater field which is as follows

$$D_{dij} = \left[\frac{\phi K_f + (1-\phi) K_r}{(\rho C)_{\text{bulk}}} \right] \delta_{ij} + D_{vij} \quad (5.15)$$

with D_{dij} = thermal dispersion coefficient for the porous medium
[L²/T]

D_{vij} = mechanical dispersion coefficient

$D_c = \left[\frac{\phi K_f + (1-\phi) K_r}{(\rho C)_{\text{bulk}}} \right] \delta_{ij}$ - thermo molecular heat diffusion
coefficient for a saturated porous
medium

δ_{ij} = Kronecker delta

The mechanical dispersion coefficient is a tensor which contains in its explicit form longitudinal and transverse mechanical dispersion coefficients as well as average pore velocity components. On an empirical basis the coefficients depend on dispersivities which are geometric characteristics of the porous medium (dimensions such as length) and the average flow velocity within the pore space.

Under most situations representative for groundwater flow the mechanical dispersion component is much larger than the molecular diffusion component. Furthermore whatever few field verifications are present, they seem to indicate that the

dispersivity coefficients in the field are one or two orders of magnitude larger than the estimates based on laboratory experiments or theoretical predictions. This is an important factor to keep in mind when assessing real situations.

In this discussion, idealizations and simplifications have been carried out to show how the ideal system would behave and to investigate what the important and most sensitive parameter in a heat transport process in shallow aquifer would be.

5.3 Heat Transport from RBHA

In order to assess and evaluate the simplifications introduced in modeling heat flow from a hypothetical RBHA, it seems necessary to give a short overview of actual operating and storage conditions. The emplacement of the RBHA will most definitely be in the unsaturated zone at a certain distance above the water table, therefore involving conduction in one and dispersion in the other as principal heat transfer mechanisms. The storage unit will operate on a cyclic basis, ideally two to four cycles per year for large units, with higher frequencies for smaller or auxiliary units. Only at a certain distance from the RBHA will a constant isotherm be established. Its position will depend on the thermal operating characteristics, that is range of temperatures, cycling frequencies, thermal properties and geometries of unit and geologic units surrounding it.

The purpose of the present study is to model heat transport principally in a shallow groundwater flow system which was intuitively assumed to be the most effective heat dispersion system. The

objective is to study the influence of different hydrologic and geologic parameters on heat flow for a number of boundary geometries and conditions.

This concentration on the saturated zone has the practical advantage of not having to treat heat transport in the unsaturated zone explicitly, although it is possible to obtain a fairly detailed physical description of the heat transport processes in the unsaturated zone, this description far exceeds the ability to measure or predict the value of many of the variables. Because of this, the most useful approach is to develop an approximate description of heat transport in the unsaturated zone that is more compatible with the ability to observe the processes that take place there. With this simplified model it is possible to estimate heat transfer in the unsaturated zone based on a simple conduction model, and then use this estimate as input into the saturated zone where heat transport can be treated more explicitly.

This study consists of two main stages (Pollock, 1977). In the first, the general question of energy transport in groundwater flow is addressed. In the second, several hypothetical cases are investigated and the energy transport characteristics observed.

In the first part of this study simplifying assumptions are discussed, specifically that the energy transfer in the groundwater flow can be treated as a continuum process by defining an appropriate averaging process over a representative volume of the aquifer. This assumption is coupled with several others: constant density of the groundwater, negligible heat generation

due to viscous dissipation and chemical reaction, and local thermal equilibrium between solid and liquid. Together they lead to the energy transfer equation for groundwater flow, which in its complete form is written

$$\frac{\partial T}{\partial t} + \frac{\phi (\rho C)_f}{(\rho C)_{\text{bulk}}} \vec{q} \cdot \nabla T = \nabla \cdot (D \nabla T) + Q_{\text{bulk}} \quad (5.16)$$

consider T = temperature

$(\rho C)_f$ = volumetric heat capacity of water ($J/M^3\text{-deg C}$)

$(\rho C)_{\text{bulk}}$ = volumetric heat capacity of (J/ $M^3\text{-deg C}$)
water-solid system

$\vec{q}$ = groundwater flux ($M^3/M^2\text{-sec}$)

D = dispersion coefficient tensor (M^2/sec)

Q_{bulk} = internal heat generation (deg C/sec)

ϕ = porosity of the rock

t = time

and where

$$\vec{q} = -K \nabla h \quad (5.17)$$

with

h = hydraulic head (M)

K = hydraulic conductivity tensor (M/sec)

The continuum approach to energy transfer in a porous medium introduces the process of mechanical dispersion. Mechanical dispersion is due to small scale mixing in the porous medium. On the macroscopic scale it is necessary to employ an appropriate empirical correlation for mechanical dispersion. This is complicated by the fact that the magnitude of mechanical

dispersion in natural aquifer systems tends to be several orders of magnitude larger than mechanical dispersion observed in laboratory tests. In any case, mechanical dispersion can be expressed as a function of groundwater velocity and the dispersion tensor is expressed (Eqn. 5.15) in the form developed by Bear and Bachmat (1967).

The convective term in the energy equation requires an accurate representation of the groundwater velocity. The steady state groundwater flow equation is employed. Several assumptions are necessary in order to treat groundwater flow in this way. One of these assumptions is that the density remains relatively constant and the flow of groundwater can be described by the distribution of the hydraulic head. The assumption of steady state flow is based on the idea that water table fluctuations occur about some average position over the period of time considered. These assumptions seem to be well justified for the problem at hand.

A two-dimensional Galerkin Finite Element computer model has been developed to solve Eqn. (5.16) for any specified RBHA-groundwater flow systems.

The computer routine actually solves two problems. First, the groundwater flow equation is solved to obtain the value of the hydraulic head at all node points. From the hydraulic heads q can be calculated using Eqn. (5.17). These values of q can in turn be used as input for Eqn. (5.16) which can then be solved by an analogous procedure to produce nodal temperature values at a given point in time. The process can be repeated to yield the nodal temperatures at some later time. Both areal and cross-

sectional models have been studied for a number of boundary configurations and conditions, and energy transport parameters. The results are studied in view of the following: general pattern of energy transport, trends of energy transport with various parameters, and magnitude of temperature effects for various transport conditions.

The boundary conditions for the cross-sectional model are considered to consist of an impermeable layer at the bottom, the water table at the top, both considered no heat flux boundaries. The two side boundaries are far enough removed from the storage unit to be considered as no heat flux boundaries. In the areal model all lateral boundaries are far enough away to not be reached by the temperature disturbance for the time period considered.

A second set of two layered-unsaturated-saturated-model runs has been performed and detailed description of results again can be found in Pollock (1977). The most important result from the point of view of modeling technique is that the simplified single layer model with a heat source in the form of a line segment on top of the water table gives comparable results to the physically more realistic but mathematically more complicated two-layer case. General trends established for the first can be interpreted and discussed as representative for the prototype situations.

Examples of cross-sectional profiles are shown in figures 5-1, 5-2, and 5-3. The different conditions of energy transport are given in the legend. The isotherms are given as dimensionless temperatures θ , since none of the parameters

PROGRESS OF A TEMPERATURE FRONT WITH TIME

Model Parameters : $\bar{v}_w = 40 \text{ m/y}$, $d_L = 25 \text{ m}$, $d_L/d_T = 5$, $D_c = 50 \text{ m}^2/\text{y}$

Fig. 5-1

INFLUENCE OF GROUNDWATER VELOCITY ON THE TEMPERATURE DISTRIBUTION AFTER 10 YEARS

Model Parameters : $d_L = 25 \text{ m}$, $d_L / d_T = 5$, $D_c = 50 \text{ m}^2/\text{y}$

Fig. 5-2

EFFECT OF THERMAL DISPERSIVITY ON THE TEMPERATURE DISTRIBUTION AFTER 10 YEARS

Model Parameters: $\bar{v}_w = 40 \text{ m/y}$, $d_L/d_T = 5$, $D_c = 50 \text{ m}^2/\text{y}$

investigated are considered to be functions of temperature. We have

$$\theta = \frac{T - T_o}{T_{wt} - T_o} \times 100 \quad (5.18)$$

with T , temperature at some point in the aquifer; T_o , the ambient groundwater temperature; T_{wt} , the elevated temperature at the water table below the storage unit. The dimensionless temperature has the effect of normalizing the temperature in the aquifer to the temperature change as the water table below the accumulator unit so that any arbitrary temperature applied can easily be converted throughout the aquifer by multiplication of the value of T_{wt} .

The parameters and conditions studied dealt with the influence of the following: magnitude of longitudinal and transversal dispersivities, d_L and d_T ; the change of ratio of d_L/d_T ; groundwater velocity (v_c); thermal diffusivity (D_c); and finally effect on interconnected surface waters.

In a plan view temperature evolution around the RBHA is quite similar. It has to be remembered that the solution is for a 2-D case, and that therefore the position of the isotherms is more extreme than will be expected if some of the heat is conducted in a direction perpendicular to the plane. The form of the temperature field around the RBHA depends on the geometry of the unit and its orientation with respect to the groundwater flow field but otherwise corroborates the findings based on the cross sectional models.

The models studied confirm a number of intuitive guesses

about energy transport in groundwater: 1. Aquifer dispersivity and groundwater velocity are by far the most important factors governing energy transport in the aquifer. 2. The thermal properties of the unsaturated zone, especially thermal diffusivity, are instrumental in determining the maximum temperature change in an aquifer. All of these factors are highly variable in natural systems and indicate a need to investigate the hydrogeologic properties of shallow aquifers in any test project.

In addition, the models yield information that could not have been easily anticipated: 1. An understanding about the temperature effects at the water table for various thermal and hydrologic conditions. 2. The existence of a particularly bad temperature effect in the aquifer for a middle range of velocities and aquifer dispersivities is predicted. At very high velocities or very high dispersivities the energy transport in the groundwater flow is very efficient and the temperature increase is relatively small. At very low velocities or dispersivities the temperature disturbance does not propagate far from the storage unit and the overall temperature disturbance is also small. The middle range of velocities produce significant temperature increases over a sizable volume of the aquifer. 3. Comparison of single layer and two layer models gives some information about how well single layer models really work and what sort of assumptions are necessary to describe the temperature boundaries of the single layer models.

In the end the single layer models will probably be the most useful since the computational restrictions on the two layer

models are more severe.

The results presented here are not exactly representative of real operating conditions since the temperature cycling procedure for the storage unit will significantly affect the temperature pattern in the underlying aquifer. However, this study merely looks at the general effect of an arbitrary high temperature storage unit on the temperature of the groundwater. The specific operational procedure for a given storage unit must certainly be looked at in detail for any field test. Particular modes of operation however are not considered to be critical for the general questions addressed in this study. They most certainly can be incorporated in the future as it becomes necessary to obtain more specific results. The real accomplishment of this study, however, is to present a well constructed model of energy transport in groundwater flow. The model may be applied to real aquifer systems in the future. The computer programs are constructed in such a way that changes can be readily incorporated as the transport processes become better understood, as more reliable values for field parameter becomes available, and as local conditions, boundary configurations and types as well as operating rules are specified for a prototype system. (Pollock, 1977b)

5.4 Feasibility of Sites

It will be of interest to note where shallow aquifer systems exist, that would correspond to the model outlined above. Furthermore construction techniques for RBHA's have been proposed

where the undisturbed porous and permeable native rock units can be used for heat storage by hot gas circulation in the unsaturated zone above the water table. Figure 5-4 gives a generalized overview of potential sites in the U.S. It is based on information dealing with sedimentary bedrock or unconsolidated materials, altitude position of the units with respect to topography, and hydrogeologic characteristics and conditions. The two main classes of sites are those found in connection with the permeable sedimentary rock units close to the surface which are connected especially with the flanks of the Appalachian mountain systems and the sedimentary basins of the interior; the other deals with more localized unconsolidated deposits in connection with alluvial or colluvial deposits in the western mountain and in tramontane basins, and those contained in the glacial drift of the continental ice sheets which covers the northern midwest regions, the great Lakes and New-England regions.

Figure 5-4 is only meant to show areas of potential locations and the areal extent of possible sites. In each individual case the hydrologic parameters porosity and permeability as well as distance of proposed accumulator to water table have to be established and assessed with the other feasibility criteria.

Fig. 5- 4 – Potential R BHA Site Locations

Sedimentary bedrock sites

Unconsolidated deposits

Southern extent of continental glaciation deposits

5.5 Environmental Implication

Temperature leakage and transport from an RBHA site may have potential effects on the mineralogic-petrologic environment and on the biosphere.

The mineralogic effects deal with increased mobilization of mineral constituents which would tend to dissolve faster at increased temperatures. The magnitude of change at the relatively low increases of groundwater temperatures will be small. The overall effect also small, since the same minerals will tend to redeposit when they reach regions of normal groundwater temperatures. The hydrologic effect would be to increase porosities and permeabilities in the vicinity of the site by dissolution, and possibly decreasing these parameters in regions of redeposition. Since both porosity and permeability in heterogeneous materials can be treated as a series or arithmetic averaging problem, the overall effect will be low.

Biologic impact of high temperatures will extend directly to the region around and near the site, where high temperature gradients are expected. In the unsaturated zone and on the surface these effects will be negligibly small after a distance of approximately three RBHA diameters.

The more important locations to watch are areas of surface water-groundwater interface where heated groundwater would enter into open surface water bodies. Possibilities of increased temperature effects that need to be investigated are reduced oxygen, increased metabolism and respiration, disruption of

producer-consumer food chains, and internal function aberrations. Furthermore there may be a decrease in the resistance of the organisms to toxic substances. Increased temperatures may interfere with spawning and other critical activities.

Temperature rise in open water bodies will be very small since heated groundwater seepage rates will be very low by comparison with effluent discharge rates from thermal plants for example. If the interface is about 10 RBHA length dimensions away from the storage site temperatures will have decreased to such a degree that heat will be rapidly dispersed by the natural dissipation process of the open water body.

The question of temperature effects on the biotic environment can best be answered by design characteristics in that critical distances can be calculated on the basis of initial temperatures, cycling operations, conductivities in the unsaturated and saturated zone, groundwater flow velocities and seepage amount compared to reservoir volume and turnover time.

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Section 6

ECONOMIC ANALYSIS

6.1 Introduction

The method of economic comparison used is revenue requirements analysis. Revenue requirements are defined as the revenue that would have to be obtained to cover all expenses associated with a project, including the minimum acceptable return on investors' committed capital in the enterprise. The revenue requirements method is used in economic analyses by many utilities, and in particular has formed the basis for economic comparisons of various energy storage systems. (1,2,3,4,5,6) Thus the method of economic analysis used herein is consistent with that used in other recognized energy storage studies. Appendix A.9 describes the computer program written for the revenue requirements method of economic analysis.

In general, the cost of a storage unit may be broken down into a cost associated with maximum power transfer of the storage unit C_p , and a cost associated with the time for discharge at maximum power, C_s . Equation (6.1) then gives the total cost, C_t , where t is the time for discharge at the maximum power rating:

$$C_t = C_p + C_s t \quad (6.1)$$

6.2 Cost Indices of Various Construction Methods

The RBHA storage unit can be constructed by a multitude of techniques, some of which are site-dependent but of lower cost than the more general techniques. The first proposed technique was in situ blasting of rock with the resulting rubble forming the rockbed. An analysis of this system showed that there were potential problems with stability of the air flow in the rock bed for rock particles over a minimum size. Since the size of rock particles is not controlled precisely in blasting, a number of other techniques are available which offer excellent particle size control. The second technique known as "trenching" consists of a series of trenches dug in the ground which are then backfilled with crushed rock after the manifolds are put in position. The main problem with this technique is that the cost will be at least \$6/ton for the removal and backfill which is uneconomical compared to the alternative methods. For large thermal storage units, it is possible to build a rock bed that is similar to the trenched system by removing all of the original material from the storage bed and replacing half of it with crushed rock and the other half with the original material. This technique would be used on large systems so that all of the material would not be removed at once, i.e., one section can be backfilled while another is still being excavated. The total operation would resemble a strip mining operation and cost about \$3/ton, which is half the previous cost. Another alternative is the use of an old rock quarry to construct the storage unit. The assumption is made that the entire volume of the unit is

to be filled with crushed rock from an adjacent quarry at about \$3/ton. Since no excavation is necessary, the cost will then be the latter figure. Another alternative is to construct a large underground cavern in rock such as basalt and fill it with crushed rock from the excavation. This technique would also cost around \$3/ton for excavation and backfill processes. The last proposed technique is to use an underground layer of sandstone and excavate only as much material as necessary to place the manifolds into position, using the sandstone to store the heat. The sandstone must be under an impervious layer of rock to avoid groundwater contamination. Obviously, the last three techniques depend heavily upon the geology of the region in which the storage unit is being constructed. Figure 6.1 gives the two basic layouts of the finished unit indicating the dimensions. Note that there are three sets of manifolds. The upper and lower sets carry the cold air, and the center set carries the hot air. This means that the air flow must reverse direction for a change from charge to discharge mode or vice versa. The manifold must be capable of withstanding high temperatures, on the order of 500°C (1000°F) for long periods of time. Therefore a ceramic pipe will probably be used rather than metal.

For the configuration shown in Figure 6.1, the cost of the unit for storage capacity, C_s , in dollars per gigajoule is given by Eqns. (6.2) and (6.3) where the terms are defined in Figure 6.1

A. Cavity type construction.

H = Total height of the unit

S = Manifold spacing dimension

m = number of manifold groups (usually 2 or 3)

B. Trench type construction.

FIGURE 6.1 Construction of the RBHA unit.

$$C_s = \frac{m \cdot C_m + H \cdot S \cdot C_e}{w_{\text{eff}} \cdot (H_s - \pi r_m^2)} \quad (6.2)$$

$$w_{\text{eff}} = w \cdot (\text{volumetric efficiency}) \quad (6.3)$$

Equation (6.3) defines an effective energy density which is a volumetric efficiency times the actual value for the energy efficiency if the whole bed between the manifolds were brought up to the hot side temperature. Thus, the effective energy density represents an average value over the whole rock bed that accounts for the fact that the cold manifold is not allowed to rise in temperature more than a few degrees above ambient to prevent excessive thermal pollution of the local soil and groundwater. For the construction schemes that require location near the surface of the ground, i.e., trench method, "strip mine" method, or use of an abandoned rock quarry, it is necessary to compute the land area required for the storage unit. Eqn. (6.4) gives the energy per unit area in gigajoules per square meter, w_L , which can be divided into the required amount of energy storage in gigajoules to give the land area needed in square meters as in Eqn. (6.5).

$$w_L = (H - \pi r^2 \frac{m}{s}) w_{\text{eff}} \quad (6.4)$$

$$\begin{aligned} A_s &= \frac{(\text{rating of storage in gigajoules})}{w_L} \\ &= \frac{0.0036 (\text{rating in kilowatt-hours})}{w_L} \end{aligned} \quad (6.5)$$

6.3 Energy Costs of RBHA

It is now possible to make estimates of the cost of energy stored in the RBHA. The method used is described in Appendix A.9 which follows that of references 1, 3, and 4.

The assumptions made are as follows:

Economic Values:

cost of capital	10%
company debt ratio	50%
interest on debt	8%
general escalation	5%
escalation on operations and maintenance	6%
Federal corporate income tax rate	48%
Federal investment credit	10%
other taxes rate	11%
Sum of years digits method for tax depreciation	
straight-line depreciation for company books.	

Plant factors

power rating	300 MW.
storage time	10-180 days
efficiency (Energy out/Energy in)	75%
plant load factor	100%
operations and maintenance	1% of capital invested/year
overhead	0.5% of capital invested/year
average plant life	20 years
cost of land	\$1000/Acre

The above economic factors were chosen based on what are considered conservative values. ^(1,4) The operations, maintenance, and overhead charges are given as lifetime levelized estimates of appropriate values. ^(1,7) Examination of actual utility data shows that a typical lifetime levelized charge for operations and maintenance is 15% of the initial capital per year. However, this 15% is approximately 80% fuel costs so that about 3% of this is the cost of operations and maintenance not directly

spent on fuel.⁽⁸⁾ It is common for solar projects to assume that 1% is closer to the true value.^(7,9) The losses in the storage system will be small, i.e., less than 5%; however to allow for losses due to pump work and in heat exchangers, the efficiency, defined as energy out divided by total energy into the system, was placed at 75%. In a similar manner the plant life was chosen as 20 years as a conservative estimate since this is the minimum life expected. The only parameters still needed are the cost of storage capacity C_s , the cost of energy conversion C_p , and the energy per unit land area.

The plant construction is assumed to be that of Figure 6.1A and the backfill to be standard gravel. The density of the bed is then 1600 kg/m^2 and the energy density for 500°C temperature rise is 0.67 GJ/m^2 . The cost of removal and backfill was estimated to be $\$3/\text{ton}$ or $\$5.30/\text{m}^3$. The manifold is assumed to be 10 cm. in diameter with a cost of $\$6.60$ per meter of length. Referring to Figure 6.1A for a height, H , of 22 meters, spacing between manifolds of 3 meters, and a volume efficiency of 36%; then by Eqn. (6.3), the effective or average energy density is 0.24 GJ/m^2 ; by Equ. (6.2) the cost of storage is $\$23.35/\text{GJ}$; and the energy per unit land area is 5.27 GJ/meter^2 of land surface required. It is then estimated that the cost for heat exchangers and a pump should be between $\$20/\text{KW}$ and $\$35/\text{KW}$. Since there are a number of parameters which are not defined precisely, a sensitivity study will be done similar to references 8 and 10.

These parameters were used in the computer simulation

described in Appendix A.9 and the results shown in Figures 6.2 through 6.6. Figure 6.2 shows the lifetime levelized energy cost as a function of storage time, where the unit is assumed to be continuously fully charged and then discharged at full capacity. The storage time is the time to discharge the unit of full capacity. For a 180-day storage unit, this is a reasonable assumption for a once-a-year complete charge and discharge. However, this may not be appropriate in the operation of smaller units such as a 10 day unit. For this reason a load factor defined in Eqn. (6.6) is used, which is

$$\text{load factor} = \frac{\# \text{ cycles per year}}{\# \text{ cycles possible in a year}} \quad (6.6)$$

basically the energy removed from the unit divided by the maximum amount of energy that could be removed assuming no down time for maintenance. The effect of changing load factor on the lifetime levelized energy cost is shown in Fig. 6.6. Figures 6.3 and 6.4 show the capital investment required in the storage unit with changes in discharge times based on both the thermal power rating and the storage capacity. It is interesting to note that as the discharge time increases, the cost of the storage unit per unit energy stored approaches C_s , which is \$23.35/GJ since C_p is divided over a larger number of days of storage, as shown in Fig. 6.4. The capital cost based on thermal energy transfer capacity, Figure 6.5, shows that the costs go up as storage capacity increases. However, the cost of storage is still considerably below that of any attempts to expand short

term energy storage proposals to long term storage, such as those considered in references 5 and 6, i.e., pumped-hydro storage, compressed air, thermal steam, thermal oil, lead-acid batteries, advanced batteries, flywheel, and hydrogen storage.

Figure 6.5 shows the effect of changing the efficiency on the lifetime levelized energy cost. Since the costs are relatively sensitive to the efficiency, this value must be chosen with care.

6.4 Conclusions

The RBHA has been shown to be economically feasible for long-term energy storage. The maximum cost of thermal energy from the storage unit for \$3/ton removal and backfill cost is 7 mills/KWh for 180 days of storage and continuous operation. The cost of energy from the RBHA is very sensitive to the number of charge/discharge cycles per year if the operation is less than 70% of the maximum possible number of cycles based on continuous operation. The costs are also sensitive to the cost of removal and backfill with gravel and to the average, or effective, energy density in the packed bed.

FIGURE 6.2. Lifetime levelized energy cost of the RBHA unit as a function of storage time assuming continuous charge and discharge of the unit.

FIGURE 6.3. Capital cost of the RBHA unit as a function of storage time.

FIGURE 6.4. Capital cost of the RBHA unit based on the energy storage capacity of the unit as a function of the storage time.

FIGURE 6.5. Sensitivity of the levelized energy cost to the REHA unit overall energy efficiency.

FIGURE 6.6. The sensitivity of the RBHA unit lifetime levelized energy cost to the average annual load factor of the unit.

6.5 References

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Appendix A-1

CATALOG OF COMPUTER MODELS

Case 1: Schematic of calculation domain showing boundary conditions and regions of constant thermal diffusivity for case of constant temperature charging and discharging without convection.

Application: Regions 1, 2, and 3 represent layers of limestone, dry St. Peter sandstone, and wet St. Peter sandstone respectively. Charging and discharging occurs by a compressed water pipe represented by the constant temperature source in region 2. Because there is an array of these pipes, symmetry ($q=0$) boundary conditions occur on the left and right. Because a water table exists above the limestone cap and under the dry sandstone, a constant temperature boundary condition equal to the water temperature exists on the top and bottom.

Results: Thermal diffusivities for regions 1, 2, and 3 were taken as $1.568\text{E-}06$, $6.013\text{E-}07$, and $2.458\text{E-}06 \text{ m}^2/\text{s}$ respectively. Depths were 6, 6, and 2 meters. Charging occurred for six months at a temperature of 350°F ; discharging occurred for

six months at a temperature of 160°F. These would be typical temperatures for recovering waste heat from a steam power plant. Efficiencies for various pipe spacings were found as follows:

Pipe Spacing (width of domain)	Volume Efficiency	1st-Law Efficiency	2nd-Law Efficiency
1 meter	10.4%	15.7%	54.4%
.5	32.5	32.1	54.4
.25	67.8	47.1	54.4

Comments: As the width of the calculation domain increases, the first law efficiency becomes negative. As the width decreases, the first law efficiency approaches one. This is born out by Ingersoll in his analysis of earth heat exchangers.* The single pipe is the best dissipator of heat while the plane surface (which we approach as we decrease the pipe spacing) is the best absorber of heat.

*Ingersoll, L. R., Heat Conduction with Engineering, Geological and other Applications, p. 257, U. of Wisconsin Press, Madison, WI, 1954.

Figure 1: Typical temperature profiles for .5 meter wide calculation domain after charging (top) and discharging (bottom).

Case 2: Schematic of calculation domain showing boundary conditions and regions of constant thermal diffusivity for case of constant temperature charging and discharging. This is a side view of figure shown as case 1. The arrows indicate the one dimensional flow of a fluid which charges and discharges the bed by conduction.

Application: Regions 1, 2, and 3 represent limestone, dry St.

Peter sandstone, and saturated St. Peter sandstone respectively. A pipe carrying compressed water is buried in the dry St. Peter sandstone. The pipe extends between two access tunnels which it is assumed are kept at constant temperatures year round. The top and bottom boundary conditions are due to the assumed constant temperatures of the top and bottom water tables.

Flow analysis: Because of the asymmetry of the calculation domain, the modeling must be done in two parts as indicated by the two origins. Furthermore, a separate calculation

has to be done on the flow of water at each time iteration. This causes a variable temperature boundary condition. Assuming one dimensional plug flow, and negligible dissipation losses, the fluid equation can be written

$$h(T_{\text{top boundary}} - T_f) + h(T_{\text{bottom boundary}} - T_f) \\ = \rho C_f A \frac{\partial T_f}{\partial t} + \rho C_f A u \frac{\partial T_f}{\partial x}$$

with an initial condition of $T_f = T_{\text{COLD}}$ and a boundary condition of $T_f(0, t) = T_{\text{HOT}}$.

For the upper and lower bed calculations we get the usual 2-dimensional conduction equation where one of the boundary conditions is:

$$K_b \frac{\partial T_b}{\partial x} = h (T_{\text{boundary}} - T_f(x))$$

Note that this situation is simplified if we assume the boundary temperature equals the fluid temperature. This occurs when the Biot modulus is large. Thus if we use the empirical correlation of:

$$Nu_d = .023 Re_d^{.8} Pr^{.4}$$

for turbulent, fully developed tube flow and assume a sandstone conductivity of 1 W/mK and typical values for the properties of water, then for a flow rate of 1 m/sec Bi is on the order of 5000 and for .01 m/sec Bi is on the order of 100. We can therefore ignore differences between

the fluid and boundary temperatures for flow rates as low as .01 m/sec without inducing errors of over 1%.

Results: Thermal diffusivities for regions 1, 2 and 3 were taken as $1.568\text{E-}06$, $6.013\text{E-}07$, and $2.458\text{E-}06$ respectively.

Depths were 6, 6, and 2 meters. The width was 250 meters.

Charging occurred for 2 days at a temperature of 350°F ;

discharging occurred for 2 days at 160°F . For longer periods of time, the water would appear to have a constant temperature such that the results given by case 1 would appear to be applicable. Results are as follows:

	<u>Volume</u> <u>Efficiency</u>	<u>1st-Law</u> <u>Efficiency</u>	<u>2nd-Law</u> <u>Efficiency</u>	<u>Water</u> <u>Velocity</u>
Upper case	1.9%	45.2%	54.4%	.01 m/sec
Lower case	1.3	51.8	54.4	.01

Figure 2: Typical temperature profiles for upper case after charging (top) and discharging (bottom).

Case 3: Schematic of calculation domain showing boundary conditions and regions of constant thermal diffusivity for case of source-sink flow.

Application: Regions 1, 2 and 3 represent limestone, dry St.

Peter sandstone, and saturated St. Peter sandstone respectively. A source-sink flow occurs in region as indicated. During discharge the source and sink are switched. Because these sources and sinks are repeated as an array, symmetry ($q=0$) boundary conditions occur on the left and right. The constant temperature boundary conditions on the top and bottom exist because of water tables which act as constant temperature sinks.

Flow analysis: The flow is a potential flow problem. Starting with the x and y Navier-Stokes equations and eliminating body forces, z derivatives, dual-directional derivatives and noting that for porous media $Du/dt=0$, we get the following two equations:

$$0 = -\frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x^2} \right) \quad 0 = -\frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right)$$

By differentiating with respect to the opposite derivative and subtracting, the pressure term cancels out. After substituting for the velocities

$$u = \frac{\partial \phi}{\partial x} \quad v = \frac{\partial \phi}{\partial y}$$

there follows

$$\frac{\partial^4 \phi}{\partial y \partial x^3} + \frac{\partial^4 \phi}{\partial y^3 \partial x} - \frac{\partial^4 \phi}{\partial y \partial x^3} - \frac{\partial^4 \phi}{\partial x \partial y^3} = 0$$

such that the left hand side cancels out. Thus only the continuity equation needs to be solved

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + S = 0$$

Results: Thermal diffusivities for regions 1, 2, and 3 were taken as 1.568E-06, 6.013E-07, and 2.458E-06 respectively. Depths were 6, 6, and 2 meters. The power input to the domain from which the mass flow rate was calculated was 361 watts. Charging occurred for 6 months with an air temperature of 350°F; discharging occurred for 6 months with an air temperature of 160°F. Results and other useful information are listed in the following table:

With of domain	Output/ Input Vel.	1st Law Effic.	2nd Law Effic.	Vol. Effic.	Other
1	2	3.8%	47.1%	2.3%	D=2 meters
1	2	3.6	46.2	2.4	D=3 meters
1	3	6.0	47.8	3.7	D=2 meters
1	2	17.0	53.3	5.6	D=2 meters
					$\alpha_2=6.013E-08m^2/sec$
.5	2	11.2	49.7	7.5	D=2 meters
.25	2	62.0	54.0	17.0	D=2 meters
					$\alpha_2=6.013E-08m^2/sec$

Comments: In the particular application for consolidated St.

Peter sandstone the pumping power required is exceptionally high. This would indicate that a higher input temperature would have to be used (to reduce the mass flow rate) or the properties of the sandstone would have to be changed in some way to make it easier to pump. We can see from the Blake-Kozeny equation that this would probably mean increasing the size of the sand particles by reconsolidating the grains into larger pebbles with some sort of adhesive binder.

Figure 3. Typical temperature profiles after charging (top) and discharging (bottom) for case with $D=2$, width of domain = .25, output/input velocity = 2 and $\alpha_2 = 6.013E-08 \text{ m}^2/\text{sec}$.

Case 4: Schematic of calculation domain showing boundary conditions and regions of constant thermal diffusivity for case of three-dimensional line source in axisymmetric calculation domain.

Application: A long cylindrical shaft with a radius equal to the distance from the axis of symmetry to the left edge of region 2 is dug in a highly porous medium. At the bottom of the shaft hot air travels outward from a thin pipe or line source in all three directions. This is approximated here by a finite number of three dimensional point sources. Typically the air would pass from the pipe through a region of gravel (region 2) into sand (region 1). The gravel or another material of known porosity and permeability is placed at the bottom of the shaft to insure an initial easy flow of air into the undisturbed ground.

For the purpose of this model, the top boundary condition is assumed constant at the average ambient air temperature

(10°). The outer boundary condition is assumed to be infinity (20 meters) and equal to the average annual ground temperature (10°C). The $q=0$ boundary condition on the bottom is due to the symmetry of the calculation domain (i.e. there is a T-const boundary condition due to the water table, a distance equal to the depth of the calculation domain below the bottom boundary). Finally, the symmetry boundary condition on the right side is due to the axisymmetric nature of the problem.

Flow analysis: The line source is approximated by superimposing the velocities due to the three-dimensional point sources in the calculation domain. The velocity from such a source to a point a distance r from it is:

$$V = Q/2\pi r^2$$

where Q is equal to the power into the bed divided by $(\rho c)_b \Delta T$ and the number of point sources.

Results: Thermal diffusivities for regions 1 and 2 were taken as 1.97E-07 and 7.14E-07 respectively. The widths were 19.75 and .25 meters; the heights were 8.5 and 1.5 meters. Charging occurred for six months at 510°C; discharging occurred for six months by suction of air from the ground. The (ρc) of the air was 847 J/m³°C. Results are as follows:

Output/ Input Vel.	Power Input	1st-Law Efficiency	2nd-Law Efficiency	Other
1	4000W	35.3%	73.6%	
2	4000	50.9	69.3	
3	4000	59.0	67.0	
2	4000	17.8	59.1	$\alpha_1 = 1.14E-06$
2	8000	58.6	69.9

Figure 4: Typical temperature profiles after charging (top) and discharging (bottom) for case with output/input velocity ratio equal one.

Case 5: Schematic of domain showing boundary conditions and regions of constant thermal diffusivity for case of parallel trenches heated by convection.

Application: A large array of parallel trenches are dug and filled with gravel. These are heated from the bottom of region 2 by air which blow upward. During discharge this flow is reversed. Symmetric boundary conditions ($q=0$) exist on the right and left sides because it is assumed this basic unit is repeated many times to form a large array. A constant temperature condition exists at the top and takes on the average ambient temperature. A symmetry condition exists on the bottom because it is assumed the calculation domain is reflected about this line. In the reflected portion the constant temperature boundary would be due to the water table and air would flow downward during discharge.

Results: Thermal diffusivities for regions 1 and 2 were taken

as $7.143\text{E-}07$. Depths were 1 and 11 meters respectively. Charging occurred until the exit temperature minus the ambient temperature was less than 50°C ; discharging occurred until the exit temperature minus the ambient temperature was less than 100°C . The (ρc) of air was taken as $1000 \text{ J/m}^3\text{C}$. Air velocity and widths were varied as indicated.

Table I: Air velocity = $5\text{E-}3 \text{ m/sec}$ for both charging and discharging
Volume Efficiency (%)

B1/B2 \ B2	.5	1.	2.	4.	6.	8.
.125	38.	37.8	35.6	28.1	16.6	12.1
.25	37.0	35.4	33.4	24.1	15.1	12.8
.5	33.7	33.5	30.3	24.3	19.3	19.0
.75	31.5	31.5	28.0	27.6	25.0	24.8
1.	50.3	50.3	50.3	50.3	50.3	50.3

Table II: Air Vel. = $5.\text{E-}3 \text{ m/s}$ for both charging and discharging
1st Law Efficiencies (%)

B1/B2 \ B2	.5	1.	2.	4.	6.	8.
.125	89.7	89.5	89.3	88.5	86.5	87.4
.25	96.2	96.3	96.0	94.8	93.2	95.0
.5	98.6	98.5	98.4	98.2	98.2	97.7
.75	99.0	98.9	99.3	99.0	98.9	98.8
1.	99.5	99.5	99.5	99.5	99.5	99.5

Table III: Air Vel. = $5.\text{E-}3$ for both charging and discharging
2cd Law Efficiencies (%)

B1/B2 \ B2	.5	1.	2.	4.	6.	8.
.125	75.0	75.1	75.3	75.3	75.5	75.6
.25	76.5	76.6	76.1	76.0	76.2	76.2
.5	76.4	76.5	76.9	75.8	76.0	76.1
.75	75.5	75.5	75.3	76.2	76.6	76.1
1.	75.9	75.9	75.9	75.9	75.9	75.9

Table IV: Air Vel. = $5.E-3$ for both charging and discharging
Switching times: Charging/Discharging (years)

B1/B2 \ B2	.5	1.	2.	4.	6.	8.
.125	.376 .615	.376 .615	.356 .64	.286 .57	.196 .44	.126 .30
.25	.212 .36	.203 .35	.193 .35	.143 .30	.096 .22	.076 .17
.5	.113 .19	.113 .19	.103 .18	.083 .17	.066 .14	.066 .13
.75	.083 .14	.083 .14	.073 .13	.073 .13	.066 .12	.066 .12
1.	.056 .100	.056 .100	.056 .100	.056 .100	.056 .100	.056 .100

Table V: Six month switching times

B1/B2	B2	Charge vel. Discharge vel.	Vol. Eff.	1st-Law Eff.	2cd-Law Eff.
.25	2.	1.83E-3 3.29E-3	36.65	89.35	74.16
.25	4.	1.64E-3 3.04E-3	31.72	86.93	73.91
.25	6.	1.36E-3 2.97E-3	26.05	82.95	73.41
.5	2.	1.02E-3 1.8E-3	39.32	91.10	74.67
.75	6.	.59E-3 1.16E-3	37.58	89.56	73.48

Figure 5: Typical temperature profiles after charging (top) and discharging (bottom) for module 6 meters wide, $B1/B2 = .25$ and 6 month switching times.

Appendix A-2

LARGE-SCALE THERMAL STORAGE IN ROCK:
CONSTRUCTION, UTILIZATION, AND ECONOMICS

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LARGE-SCALE THERMAL STORAGE IN ROCK:
CONSTRUCTION, UTILIZATION, AND ECONOMICS*

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ABSTRACT

The performance, design constraints, construction methods, and economics of a large-scale rock bed heat accumulator capable of providing 300 MWe at high temperatures (250° to 500°C) for up to 6 months are discussed. Six construction schemes are identified and compared in relation to siting requirements, land use, groundwater interaction, rock quality, and construction cost. Examples are given of the effect of construction cost, construction period and plant life on the present worth of stored energy. For the assumed operating conditions this present worth of stored energy is less than 15 mills/kWht, thus making the concept of a rock bed accumulator economically attractive for long-term storage.

NOMENCLATURE

c_a	specific heat of air, kJ/kg°C
c_b	specific heat of bed, kJ/kg°C
D_p	mean particle diameter, m
g	acceleration due to gravity
h_v	volumetric heat transfer coefficient, W/m ³ °C
L	length of bed, m
l	reduced length of bed, m
k_b	thermal conductivity of bed, W/m°C
k_e	effective thermal conductivity of bed, W/m°C
M	number of hours/day of charging, hours
N	number of months/year charging, months
P	power input, kW
ΔP	pressure drop, Pa
r	ratio pumping to thermal stored energy
S	constant
t	time, s
T	Temperature, °C
ΔT	temperature potential, °C
v_a	superficial air velocity, m/s
v_b	velocity of heat front in bed, m/s
V_b	volume of accumulator, m ³

x	distance in bed, m
x_2	length, m
α	diffusivity, $k_b/c_b \rho_b$, m ² /s
α^*	diffusivity, $k_b/c_a \rho_a$, m ² /s
γ	coefficient of cubic expansion of air, °C ⁻¹
η	volume efficiency
ϕ	porosity of bed
κ	permeability of rock bed, m ²
θ	dimensionless temperature, $(T-T_0)/\Delta T$
μ	air viscosity, Pa-s
ν	kinematic viscosity, m ² /s
ρ_a	air density, kg/m ³
ρ_b	bed density, kg/m ³

INTRODUCTION

Thermal energy storage on a large scale in terms of both time and volume of material can play an important role in energy conservation and management; it is being considered for load leveling by electric utilities, it is an essential component in the utilization of solar energy for heating or power applications. While most of these applications have been concerned with short-term storage (hourly, daily, or weekly), there is a need also to consider long-term or seasonal storage (months). This paper deals with the technical and economic feasibility of large-scale long-term seasonal thermal energy storage at high temperatures (above 250°C) in rock and native earth materials.

The rock bed heat accumulator concept as presented originally by Riaz et al. [1], involves heating crushed or naturally formed porous rock by flow of hot air from a central manifold to collector manifolds located at the top and bottom of the bed. On discharge the air flow is reversed: cold air is introduced through the top and bottom manifolds and hot air harvested through the central manifold. Input and output temperatures of up to 500°C and charging and discharging times of four to six months are possible with storage capacity limited only by the size of the bed. Various configurations of rock bed heat accumulators can be envisioned (see for instance Figures 3 through 8) with local geological conditions, environmental constraints and cost dictating the appropriate selection.

The paper identifies the design parameters and material properties that influence system performance and evaluates the constraints with respect to construction methods and material selection. Preliminary cost estimates are included for various construction schemes from which an assessment of energy storage costs are presented.

THERMAL ROCK-BED ACCUMULATOR PERFORMANCE

The design of the rock bed accumulator is governed by four general considerations:

- o conservation of energy, i.e. first-law efficiency should be high,
- o maintenance of energy quality (output temperature as near as possible to the input temperature),
- o cost, and
- o environmental effects.

These general considerations lead to several more specific requirements:

- o heat loss due to conduction to surrounding soil/rock or groundwater should be minimized,
- o the volume efficiency should be optimized with respect to first-law efficiency,
- o pumping losses in charging and discharging the system should be minimized, and
- o free convection within the bed should be inhibited.

The first of these specific requirements dictates that the input manifolds be placed midway between the collector manifolds. A spherical arrangement of collector manifolds surrounding a central input would be ideal, but this arrangement is not cost effective to construct. Instead an extensive horizontal planar arrangement is favored in which hot air flows up and down from a central hot manifold to the top and bottom cold manifolds. If the horizontal dimensions of this plate are large compared to its thickness the heat losses will be small, since they will be confined to the perimeter.

Basically a rock bed accumulator consists of a gaseous phase and the rock phase. The system is characterized by the gas velocity and temperature, and the rock temperature. Schumann [2] first analyzed such a two-phase model in one dimension and used a gas velocity sufficiently high so that the heat conduction through the solid particles can be ignored.

In the rock bed accumulator, the air velocities are low, typically less than 0.01m/s, $k_a \ll k_b$ and $\rho_a c_b$, and hence the air and rock temperatures are essentially the same and the governing equations can be reduced to a single equation, with

$$T_a = T_b = T$$

$$\frac{\delta T}{\delta t} + v_b \frac{\delta T}{\delta x} = \alpha \frac{\delta^2 T}{\delta x^2} \quad (1)$$

$$\text{where } v_b = v_a \rho_a c_a / \rho_b c_b \quad (2)$$

$$\alpha = k_b / \rho_b c_b \quad (3)$$

For intermediate conditions, Riaz [3] suggests that equation still can be used but with an "effective" thermal conductivity k_e substituted for the thermal conductivity of the bed

$$k_e = k_b + (v_a \rho_a c_a)^2 / h_v \quad (4)$$

For all conditions considered for the rock bed accumulators discussed here, the second term in (4) is small and has been neglected.

Volume Efficiency

The volume efficiency η is defined as the ratio of actual energy stored in the rock bed to the energy stored if all the rock were heated to the input air temperature. Figure 1 shows schematically the temperature profile when the bed is 'full' and 'empty' under operating conditions.

Figure 1. Schematic representation of temperature distribution in bed.

At the cold manifold the temperature is to be kept to a minimum, so that energy in the form of hot gas is not lost, hence charging must be stopped when this minimum temperature has been reached; at this point not all the rock has reached the input temperature. On the other hand to maintain high energy quality of the outputs discharging of the bed is stopped when the exit temperature drops substantially below the input temperature. The volume efficiency is

$$\eta = \left[\int_0^H (T^f - T^e) dx \right] / H \Delta T \quad (5)$$

The slope of the thermal wavefront can be approximated (Riaz, 1977) by

$$d\theta/dx = -[(v_a \rho_a c_a / k_e) / 4\pi x]^{1/2} \text{ at } \theta = 1/2 \quad (6)$$

As an example of the parameters that influence volume efficiency, consider the idealized temperature distribution shown in Fig. 2. Here,

$$\eta = \frac{L - x_2/2}{L}$$

$$\text{where } x_2 = 1 / \left. \frac{d\theta}{dx} \right|_{\theta=1/2} = 2 \left(\frac{\pi k_e L}{v_a \rho_a c_a} \right)^{1/2} \quad (7)$$

or by substituting from (2) with

$$v_b = 1/(NM \ 108000)$$

$$v_a = \frac{1}{NM \ 108000} \frac{c_b \rho_b}{c_a \rho_a} \quad (8)$$

then

$$\eta = (1 - \frac{582}{l} \sqrt{\frac{k_e NM}{\rho_b c_b}}) \quad (9)$$

where the bed height H is

$$H = l + 582 \sqrt{\frac{k_e NM}{\rho_b c_b}} \quad (10)$$

This expression, though approximate, illustrates the nature of the influence of various parameters on the volume efficiency. For example, it indicates that the volume efficiency increases for long beds, and low effective thermal conductivities. A more complete evaluation of the volume efficiency of the rock bed is given by Blackshear et al. [4].

Pumping Energy

Work is done in pumping the air through the rock bed. The pressure difference across the bed, ΔP , which is proportional to the pumping energy, is related to the air velocity, bed height and permeability. However, as the air velocity is also proportional to the bed height, then

$$\Delta P = \mu S H^2 / \kappa \text{ where } S = c_b \rho_b / (c_a \rho_a \ NM \ 108000) \quad (11)$$

The permeability of the rock bed usually is measured experimentally, but for well-rounded particles of uniform diameter packed with a porosity ϕ , the Kozeny-Carmen equation (Carmen, 5) predicts permeability with acceptable accuracy. It states that

$$\kappa = \frac{D_p^2 \phi^3}{150(1 - \phi)^2} \quad (12)$$

However, in a rock bed made of naturally formed porous material or crushed rock, this relationship can result in significant error. Such factors as particle shape, particle size distribution, and tortuosity influence the permeability. Hazen [6] found that if D_p is taken as the particle diameter for which only 10% by weight of the particles is smaller, then the Kozeny-Carmen equation gives acceptable permeability predictions for well graded materials.

Stability

To minimize thermal losses in the system the bed should be heated from the middle with both up-cast and down-cast air flowing to the cold manifolds. However, for the upcast air there is a possibility that free convection will develop due to the buoyancy of the hot air. Free convection would result in loss of thermal stratification of the bed and hence increased thermal losses at the cold manifold.

The stability of a porous system heated from below can be characterized by the Raleigh number. Elders [7] and Katto and Masuoka [8] have shown

that no free convection is observed when the Raleigh number is less than 40, i.e.

$$R_a = \frac{\kappa \gamma g H \Delta T}{\alpha_m \nu} < 40 \quad (13)$$

ROCK BED ACCUMULATOR DESIGN

Layout Constraints

The rock bed accumulator total volume, V_b , depends on the rock properties, $\rho_b c_b$, storage temperature, volume efficiency η , storage charge time (108000 NM secs) and power input P where

$$\eta \rho_b c_b \Delta T V_b = P \ NM \ 108000 \quad (14)$$

For example, for a 100-MWe system (300-MWt) with a storage charge time of 4 months, 8 hours per day, $\Delta T = 500^\circ\text{C}$, $\eta = 40\%$, the required volume of heated porous rock is approximately 4 million cubic meters.

Heat loss considerations indicate that porous rock should be constructed as a relatively flat plate. For maximum volume efficiency, the thickness of the plate should be maximized within the constraints imposed by the thermal stability or pumping energy criteria.

A significant constraint on the rock bed accumulator site selection, design and cost will be to minimize its interaction with groundwater. As the rock bed accumulator will be heated above 100°C , the rock bed will be dry, but groundwater interaction with the surrounding heated material and infiltration into the bed could cause undesirable effects such as:

- o accelerated weathering and disintegration of the rock bed accumulator
- o breakdown in the structure of the surrounding rock or soil
- o environmental damage due to increased temperature and solubility of the groundwater
- o heat loss to the groundwater, which would reduce the first-law efficiency.

Minimizing the interaction with the groundwater can be achieved by:

- a) site selection: place the bed above the natural groundwater with the bed covered by an impermeable layer for protection against precipitation; construct the bed in a naturally impermeable geologic strata (tight unfractured rock mass).
- b) construction provision: the rock bed accumulator could be protected from groundwater either by groundwater draw-down by pumps and/or by placing the rock bed on an artificially constructed impermeable layer.

Material Constraints

The rock material selected as the storage medium must have low effective thermal conductivity, high permeability, high heat capacity and be both homogeneous and thermally inert.

The thermal conductivity of rock varies considerably from as low as $0.2 \text{ W/m}^\circ\text{C}$ [9].

The effect of porosity on the thermal conductivity is dramatic for dry rocks. For example, the thermal conductivity of sandstone with porosity of approximately 30% is between 0.3 and 0.7 W/m°C whereas the conductivity of quartz, the major constituent of sandstone, ranges from 1 to 5 W/m°C [10].

The thermal stability and pumping energy place a constraint on the bed height for a given bed permeability. From the stability criterion with $\Delta T = 500^\circ\text{C}$, equation (13) gives the constraint $\kappa H < 1.50 \times 10^{11}$.

If the pumping energy associated with charging and discharging is to be kept below a prescribed fraction of the input energy, say, 2%, then the ratio

$$r = \frac{2\Delta P}{\rho_a c_a \Delta T} < 0.02 \quad (15)$$

The factor 2 in (14) appears because pumping energy is expended during both charge and discharge of the bed.

Substituting for ΔP from (11), with $\Delta T = 500^\circ\text{C}$ $M = 8$, $N = 4$, equation (15) becomes

$$H^2/\kappa < 4.65 \times 10^{11}$$

The pumping energy and the thermal stability constraints are shown graphically in Fig. 2, for both $\Delta T = 500^\circ\text{C}$ and $\Delta T = 250^\circ\text{C}$. The curves show that for the assumed material properties and for a 2% pumping energy, the bed height cannot exceed 35 to 40 m.

Figure 2 Interrelationship of bed height and bed permeability.

As the volume efficiency demands maximizing the bed height, this indicates that an optimum permeability exists which is approximately $5 \times 10^{-9} \text{ m}^2$ or 5000 darcys. With the use of the Kozeny-Carmen equation (12) with $\phi = .35$, this permeability corresponds to a particle size $d_p = 0.003 \text{ m}$.

The life of the rock bed is dependent on the long-term mechanical/chemical stability of the bed material under thermal cycling. Any chemical or mechanical changes in the rock almost certainly will result in an increased fraction of fine particles. These small particles will limit the performance of the bed in two ways:

- o Increased pumping energy: A reduction in particle size will decrease the bed permeability and hence increase the pumping energy required.
- o Reduced volume efficiency: Increased particle to particle contacts will increase the effective thermal conductivity of the bed and reduce the volume efficiency.

In addition, any mechanical/chemical changes in the bed could result in slumping, i.e., reduced porosity of the bed. This too will contribute to reduced bed permeability, and to higher effective thermal conductivity.

Such considerations indicate that the rock selected for the bed material must be thermally stable. Unfortunately, little data is available on the effects of thermal cycling on the mechanical/chemical properties of rock in the temperature range of interest. Sommerton et al. [11] have studied the thermal alteration of sandstones in the temperature and pressure range of interest while Murrell and Chabravarty [12] have evaluated the effect of temperature on the mechanical properties of several crystalline rocks. Both authors show very little mechanical damage to the rock in the 0 to 500°C range, but for one temperature cycle only. Thirumalai and Demou [13] observed thermally induced micro-cracking in granite during thermal cycling to 400°C . Several sedimentary rock types such as mudstones, shales, limestone, and evaporites can be eliminated from consideration because of their generally weak nature, and dehydration characteristics. There is a great variety of crystalline and metamorphosed rocks such as granites, quartzites, gabbro and marbles including some sandstones that could be acceptable. An experimental program to test the influence of thermal cycling on the mechanical properties of a number of rock types is underway.

The rock bed must be homogeneous with respect to thermal conductivity, porosity, permeability, weatherability, and, to a lesser extent, particle size. These constraints influence not only the construction method but also the site selection process as the raw materials will have to be relatively homogeneous before construction begins.

CONSTRUCTION METHODS

Many geometric layouts and construction methods can be used to build a rock bed accumulator in a variety of soil/rock types. The size of the bed will influence the construction technique and the unit cost of storage. This discussion will be limited to large-scale storage systems involving rock volumes of over $4 \times 10^6 \text{ m}^3$; this corresponds to about 8×10^6 metric tons of rock or storage of 4 months thermal energy (8 hours/day) for a 100 MWe plant.

Six construction techniques are presented in Figures 3 through 8 in which schematic cross sections of partially completed rock bed accumulators are shown. Table 1 provides an overall comparison. Each scheme could be used in certain circumstances but none could be universally applied. Three techniques use surface methods while the other three are constructed underground.

Most naturally formed geological materials will have to be sorted, fractured or crushed to meet the tight constraints on bed permeability and bed height. Hence one of the major costs associated with creating these large crushed rock beds is materials handling. A system that moves the least tonnage of material will be most cost effective.

Surface Construction Methods

The first method, trenching, as proposed by Riaz et al. [1] requires that a supply of crushed rock is delivered from external sources by a commercial crushed-rock supplier, to back-fill trenches dug in a deep soil deposit. In reviewing the cost of crushed rock it was found that over 80% of the retail price was incurred after the crushed rock left the quarry, i.e., in transportation costs and distributors profits. Hence the trenching method could be made more economical if the quarrying was incorporated into the accumulator construction - this led to schemes 2 & 3 illustrated in Figs. 4 and 5.

Figure 3. Scheme 1. Schematic cross section, bed construction by trenching and backfilling.

CONSTRUCTION SCHEME	CASED ROCK SUPPLY	ROCK QUALITY	TRANSPORTATION	SITING	ENVIRONMENTAL IMPACT	MAJOR COST	COST INDEX	BEST SUITED	CONSTRUCTION DURATION (YRS.)
TRENCHING	EXTERNAL	AS SPECIFIED	OF CRUSHED ROCK	DEEP SOILS MUST BE AVAILABLE	PUMPING	TRANSPORTING CRUSHED ROCK	\$4/TON	SMALL STORAGE	2
STRIP MINING	SORTED ON SITE	MIXED	ON SITE OF CRUSHED ROCK AND FINES	SANDY SOIL MUST BE APPROPRIATE	LAY IMPERVIOUS BED	?	\$2 to \$3/TON	LARGE STORAGE	2
PILE QUARRY	AGGREGATE	GOOD BY SELECTION	ON SITE OF CRUSHED ROCK ONLY	OUTCROPS OF SUITABLE MATERIAL	LAY IMPERVIOUS BED	DRILL AND BLAST IN QUARRY	\$2 to \$3/TON	ALL SIZES	2
CAPITAL PILES ROCK	NATURAL	NATURAL	ROCKING OF ACCESS	LOCALLY FAVORABLE CONDITIONS	PUMPING IF NECESSARY	ACCESS HORIZONTAL DRILL HOLES	\$1 to \$2/TON	MORE LOCAL CONDITIONS APPLY	3
UNDERGROUND WITH PASS BLASTINGS	WASH BLASTING	GOOD	200 to 400 EXTRACTED DISPOSED ON SURFACE	ROCK AND GROUNDWATER CONDITIONS SUITABLE	IN TIGHT ROCK IN PILE LAY	ACCESS AND SIFT IF AT DEPTH	\$3/TON	LARGE STORAGE	2-3
UNDERGROUND, CRUSHED ROCK BLASTING	CRUSHED EXCAVATION MATERIAL	GOOD	ON SITE ALL MATERIAL MOVED	ROCK AND GROUNDWATER CONDITIONS SUITABLE	IN TIGHT ROCK IN PILE LAY	EXCAVATION	\$4 to \$4/TON	LARGE STORAGE	2-3

In both schemes 2 & 3 the rock bed is located near a supply of suitable rock, a gravel or sandy soil deposit, or a suitable rock outcrop. The gravels or sandy soil would have to be washed and screened to remove the fine and coarse material. The manifolds would be emplaced during the bed construction.

Figure 4. Scheme 2. Schematic cross section, bed construction by strip mining.

Figure 5. Scheme 3. Schematic cross section bed construction with rock quarrying.

Groundwater interaction could be a problem in arrangements 1 and 2 as they are both placed in soils that normally would contain groundwater. This could be minimized for the trenching method by pumping to draw down the water table, whereas with strip mining (scheme 2) an impermeable layer

of compacted clay could be used to isolate the accumulator from the groundwater. Scheme 3 could be constructed above the natural ground surface and hence above the groundwater level. All three surface beds should have little interaction with water as long as infiltration from precipitation is prevented by an impermeable top cover.

Underground Construction Methods

The underground construction methods shown in Figs. 6, 7, and 8 have advantages over the surface methods in that land usage is reduced and groundwater interaction in many instances can be less.

Figure 6. Scheme 4. Schematic cross section, bed located in porous sandstone.

Scheme 4 utilizes naturally occurring porous rock as the storage medium. In the Minneapolis/St. Paul area the underlying St. Peter sandstone possibly could satisfy the various conditions imposed by pumping energy, bed height, volume efficiency, etc. Figure 6 shows schematically the geological cross section in the Minneapolis/St. Paul area where a rock bed accumulator could be placed. The St. Peter sandstone is characterized as a fine grained (0.2 mm), highly porous (28%), high silica content (99%) sandstone. Its permeability ranges from 5×10^{-12} to $30 \times 10^{-12} \text{ m}^2$ (5 to 30 darcys) and has a thermal conductivity of approximately $0.7 \text{ W/m}^\circ\text{C}$. Figure 2 shows that the pumping energy criterion places a severe constraint on the bed height; either the pumping energy would have to be higher or the charge time longer (lower air velocity) than assumed in Fig. 2 for an acceptable volume efficiency. Table 2 lists the maximum bed height for various input temperature changes, pumping energy criteria and bed permeabilities. The pumping energies approach 18% for a bed of height greater than 10 m for $\kappa = 10 \times 10^{-12} \text{ m}^2$.

Schemes 5 and 6 shown in Figs. 7 and 8 would be placed in similar rock types, competent crystalline rocks that are best suited to withstand thermal cycling. For scheme 5 the permeability would be produced by drilling and blasting the rock in situ, whereas in alternative 6 the blasted rock would be removed, crushed, screened and the appropriately sized material used to backfill the underground caverns. Scheme 6 allows precise control of the bed

permeability but at the substantial cost of handling the crushed rock.

Figure 7. Scheme 5. Schematic cross section, bed construction by underground mass blasting.

Figure 8. Scheme 6. Schematic cross section of underground bed construction.

Table 2 Maximum bed height in St. Peter Sandstone

Charge Time M (hours)	ΔT ($^\circ\text{C}$)	Pumping Energy Ratio	Bed Height (m)
8	250	0.02	2.7
8	250	0.06	4.6
8	250	0.18	8.0
8	500	0.02	2.1
8	500	0.06	3.6
8	500	0.18	6.3
24	250	0.02	4.5
24	250	0.06	8.0
24	250	0.18	13.5
24	500	0.02	3.6
24	500	0.06	6.3
24	500	0.18	10.8

The optimum design for scheme 5 depends on the predictability of the bed permeability because the maximum allowable bed height decreases rapidly for permeabilities above $5 \times 10^{-12} \text{ m}^2$, see Fig. 2,

but construction costs are reduced with increased height. As a result of in situ blasting, the rock bed would have a large range of particle sizes; the maximum block size can be controlled by the blast hole pattern. Particle size distribution, however, and hence the bed permeability are more difficult to control. Full-scale testing of blast characteristics and bed permeability would have to precede a final design for this scheme.

Arrangement 5 could be constructed at depth in competent bedrock, thereby minimizing the ground-water interaction. Scheme 6 normally would be located closer to the surface so that the rock can be moved economically to the surface for crushing and screening.

Groundwater inflow for both schemes 5 and 6 would be low; local areas of higher inflow could be either plugged by grouting or intercepted by drain holes.

Construction Costs

The construction cost index shown in Table 1 should be used for comparison purposes only as they are neither the result of experience in construction of such beds nor the product of detailed costs estimates. The cost index is based on just the costs associated with the construction of the bed with its air distribution manifolds, and does not include such items as heat exchangers, air pumping equipment, nor the cost of energy production at the input of the system, and cost of surface lands. They have been compiled from comparisons with similar construction projects, such as crushed rock quarrying and pipe laying, but are influenced by local labor costs, availability of equipment and local geologic conditions. Hence, costs can be estimated accurately only for a particular location, design and time.

The trenching operation normally would require shoring, temporary braces between the trench walls, while the trench remained open and as the manifolds were laid; shoring is very expensive and would raise the cost above \$6/ton of bed material. We have assumed that special equipment would be built to mechanize the trenching, shoring, pipe laying and backfilling operations. A major portion of the cost in the trenching alternative is the cost of crushed rock (\$3/ton if close to a supplier).

The overall cost per ton of bed material for scheme 2 is influenced by the particle size distribution of the natural sandy soil or gravel used. If there is a high fines content then more material will have to be transported and screened. In scheme 3, only the rock that will be used in the bed is moved (plus a small proportion of fines) but drilling and blasting costs could offset the cost advantage over scheme 2.

The construction costs for scheme 4 would be low because the storage medium would not be moved. Construction would consist of access tunnels from which long horizontal holes would be drilled for the air distribution manifolds. Costs would be low because the porous sandstones are easily and cheaply excavated using water jets and horizontal drilling costs are low.

For scheme 5 construction costs would be relatively low but the cost of excavating both the top headings for the long hole drilling, and air distribution tunnels would be relatively high (\$5 to \$20/ton). However, because only a small percentage of the rock bed is removed the overall costs/ton of storage material still would be low. The material removed to form the top headings and the tunnels could be sold for concrete aggregate or crushed rock.

The construction costs for scheme 6 are based in part on the costs for construction of large underground storage caverns in Sweden (Jansson 14). The geometry, volume and rock types of these caverns are very similar to the proposed thermal storage excavations. We have added an estimate for rock crushing and placement to arrive at the figure shown in Table 1.

Construction Period

The construction time of a rock bed accumulator influences the overall economics of energy storage. An estimate of the construction time for each scheme is shown in Table 1 for the typical $4 \times 10^6 \text{ m}^3$ bed. The estimate is included only for comparative purposes as the actual construction time will depend on contractual agreements, local conditions, etc.

The surface construction method in general could be completed in shorter periods than the underground methods because of greater mobility and the use of massive equipment. However, as schemes 4 and 5 disturb only part of the bed, time is relatively short.

ECONOMIC CONSIDERATIONS

Construction Costs

The construction cost indices for these six proposed schemes of rock bed heat accumulators are in the range of \$1.77/m³ (\$1.00/ton) to \$10.60/m³ (\$6.00/ton) of rock material. A ΔT of 500°C and a volume efficiency of 40% yield an energy storage density in the rock of 0.24 GJ/m³. The cost of site preparation, backfilling with selected material and manifold costs were estimated from standard cost figures in the Dodge Construction Estimate Guide.

Assumptions, Economic Parameters, and Costs

The assumptions made for the revenue requirements economic analysis are:

Year of initial operation	1977
Construction cost schedule	linear over 2 years
Heat accumulator data:	
Charge/Discharge Power Rating	300 MWt
Cycle Time	180 days
Storage Capacity	150 MW-yrs. (or $4.6656 \times 10^6 \text{ GJ}$)
Depth of bed	22 meters
Plant life	20 years
ΔT	500°C
Land area	89.034 hectares (220 acres)
Land cost (not applicable to schemes 4,5,6)	\$1000/acre
Energy efficiency	100%
Density of bed	1602 kg/m ³

Economic Factors:

Cost of capital	10%
Interest on debt	8%
Debt ratio	50%
Operation and maintenance	10%
Overhead	5%
Inflation rate on operation & maintenance	5%
Federal tax rate on profits	48%
Federal investment credit	10%
Other taxes	11%
Depreciation method	Sum-of-year digits

Based upon the above assumptions and utilizing the revenue requirements economic analysis program, derived from Jeyner [15], the capital costs and energy costs as a function of the cost of material removal and backfill is given in Table 3. The cost figures include interest during construction.

Table 3 Capital Cost and Energy Cost as a Function of Construction Costs.

COST OF REMOVAL AND BACKFILL \$/TON	CAPITAL COST		PRESENT WORTH ENERGY COST	
	\$/GJ	\$/KW	\$/GJ	MILLS/KWhe
1.00	9.54	148	1.37	4.9
2.00	17.66	275	2.53	9.1
3.00	25.78	401	3.69	13.3

The initial capital cost depends on the total volume of material removed and backfilled, and on the cost per ton for removal and backfill. The bed volume is inversely proportional to volume efficiency, and hence the present worth of the energy cost is very sensitive to volume efficiency.

Economic Effect of Construction Period

The construction period for a given project influences the cost factors because of interest during construction. Using the assumptions given previously, and assuming a material removal and backfill cost of \$3.35/ton, the effect of interest during construction is illustrated in Table 4. In each case, the cash flow schedule during construction is assumed to be linear over the construction period.

Table 4 Effect of Interest During Construction

CONSTRUCTION PERIOD-YEARS	CAPITAL COST		PRESENT WORTH ENERGY COST		% INCREASE IN ENERGY COST
	\$/GJ	\$/KW	\$/GJ	MILLS/KWhe	
0	25.98	404	3.90	14.0	0
1	27.26	424	4.09	14.7	4.92
2	28.62	445	4.29	15.5	10.19
3	30.08	468	4.51	16.2	15.8
4	31.63	492	4.75	17.1	21.8
5	33.29	518	4.99	18.0	28.2

Economic Effect of Plant Life

Plant lifetime has an appreciable effect on energy cost. For example, assuming a 2-year construction period and construction cost of \$3.35/ton, a 20-year plant life results in a present worth of energy storage of 15.5 mills/kWhe which drops to 9.6 for a 40-year plant life.

CONCLUSIONS

Because of the inherent low cost of rock bed heat accumulators, they appear eminently qualified to provide high-temperature annual cycle thermal energy storage in a great variety of applications. Consideration of volume efficiency, thermal stability, pumping work with acceptable environmental impact leads to an optimum bed height of about 40 m and to a particle size of 3×10^{-3} m. Construction costs depend on local geological resources and excavation techniques; cost figures are in the range of 5 to 15 mills/kWhe for annual storage.

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CONSTRUCTION TECHNIQUES

- TRENCHING
- STRIP MINING
- ROCK QUARRY
- POROUS UNDISTURBED ROCK
- UNDERGROUND MASS BLASTING
- UNDERGROUND CRUSHED ROCK BACKFILL

COMPARISON OF CONSTRUCTION TECHNIQUES

CONSTRUCTION SCHEME	CRUSHED ROCK SUPPLY	ROCK QUALITY	TRANSPORTATION	SITING	ENVIRONMENTAL IMPACT	MAJOR COST	COST INDEX	BEST SUITED	CONSTRUCTION PERIOD (YRS.)
TRENCHING	EXTERNAL	GOOD AS SPECIFIED	OF CRUSHED ROCK	DEEP SOILS MUST BE AVAILABLE	PUMPING	TRANSPORTING CRUSHED ROCK TRENCHING	\$6/TON	SMALL STORAGE	?
STRIP MINING	SORTED ON SITE	MIXED	ON SITE OF CRUSHED ROCK AND FINES	SANDY SOIL MOST APPROPRIATE	LAY IMPERVIOUS BED	?	\$2 to \$3/TON	LARGE STORAGE	2
ROCK QUARRY	ADJACENT	GOOD BY SELECTION	ON SITE OF CRUSHED ROCK ONLY	OUTCROPS OF SUITABLE MATERIAL	LAY IMPERVIOUS BED	DRILL AND BLAST IN QUARRY	\$2 to \$3/TON	ALL SIZES	2
NATURAL POROUS ROCK	NATURAL	NATURAL	REMOVAL OF ACCESS	LOCALLY FAVORABLE CONDITIONS	PUMPING IF NECESSARY	ACCESS HORIZONTAL DRILL HOLES	\$1 to \$2/TON	WHERE NATURAL CONDITIONS APPLY	3
UNDERGROUND WITH MASS BLASTINGS	MASS BLASTING	GOOD	20% to 40% EXTRACTION DISPOSED ON SURFACE	ROCK AND GROUNDWATER CONDITIONS DICTATE	IN TIGHT ROCK IN FLOW LOW	ACCESS AND SHAFT IF AT DEPTH	\$3/TON	LARGE STORAGE	2-3
UNDERGROUND, CRUSHED ROCK BACKFILL	CRUSHED EXCAVATION MATERIAL	GOOD	ON SITE ALL MATERIAL MOVED	ROCK AND GROUNDWATER CONDITIONS DICTATE	IN TIGHT ROCK IN FLOW LOW	EXCAVATION	\$4 to \$6/TON	LARGE STORAGE	3-6

Scheme 1: Trenching

Scheme 2: Strip mining.

Scheme 3: Rock quarry

SECTION AA

MANIFOLD DETAIL

Scheme 4: Porous undisturbed rock.

Scheme 5: Underground mass blasting.

Scheme 6: Underground crushed rock backfill.

Appendix A-3HIGH-TEMPERATURE ENERGY STORAGE
IN NATIVE ROCKS

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HIGH-TEMPERATURE ENERGY STORAGE IN NATIVE ROCKS

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Abstract - The successful utilization of thermal energy derived from solar collectors, from off-peak excess thermal energy of power plants, and from industrial process or waste heat systems is predicated upon the development of suitable storage systems which provide the necessary buffer and matching function between the time-varying thermal energy inputs and the output thermal and electrical loads. This paper explores the technical feasibility of large-volume packed beds using native earth or rock materials for long-duration (months) storage of thermal energy at high temperatures (up to 500°C) and power levels (in the tens of MWt range) compatible with electric power generation and community heating in total energy systems context. The advantages of this storage concept derive from the inherent simplicity of the overall system, from its utilization of low-cost, on-site, native materials resulting in potentially minimum environmental impact, and from the increased energy density obtainable with large temperature swings. A generic configuration of heat accumulators involving arrays of pebble-filled trenches (or holes) surrounded by undisturbed earth and fed by hot air from above or below is examined.

INTRODUCTION

Thermal energy storage will play an important role in energy conservation for such large-scale applications as electric utility systems - where load leveling provides for improved utilization of capital equipment resulting in substantial cost savings - , community or district heating systems - where the use of waste or process heat reduces premium fuel requirements - , and solar heating and solar power systems - where the inherent cyclical and intermittent nature of solar energy requires the use of efficient storage systems for optimum energy utilization. The basic function of the thermal energy storage system is to provide the necessary buffer and match between the time-varying thermal energy inputs and the corresponding output thermal and electrical loads. The development of practical, low-cost methods for storing large amounts of high-temperature energy is becoming of increasing importance for solar-thermal conversion systems adapted to electric power generation and total energy systems.

The paper discusses one concept based on the use of "rock bed heat accumulators" which is presently being studied by an interdisciplinary team of the Institute of Technology at the University of Minnesota under the recently initiated sponsorship of the Division of Conservation at ERDA. Among the objectives of the study are these:

- To investigate the technical and economic feasibility of storing large amounts of thermal energy over extended periods of time using native earth materials that can be prepared at low cost.
- To conduct studies to establish the performance characteristics of large rock bed heat accumulators at various power and temperature levels compatible with thermal conversion systems.
- To assess the materials, geological, and environmental problems associated with the installation and operation of large earth heat accumulators.

The storage of thermal energy as sensible heat in packed beds (pebble bed, rock pile) has been the object of extensive studies in recent years for various applications encountered in chemical systems and in the heating and cooling of solar houses. In the metallurgical industries, gas fired air preheaters for blast furnaces frequently use storage type heat exchangers with brick walls as heat accumulators.

A possible system concept for storing solar energy for electric power generation is illustrated schematically in Fig. 1. The packed-bed heat accumulator is conceived to be formed in situ by appropriate drilling, filling, and blasting. Because of the high temperature conditions, solar energy collection requires the use of concentrators

Fig. 1 A solar-excited earth heat accumulator for electric power generation

which could be in the form of distributed collectors or in a central-receiver arrangement. As shown in Fig. 1, solar radiation is concentrated on a porous type receiver (atop the central tower) through which air is drawn, heated, and directed into the large packed-bed thermal storage unit; thermal energy can then be extracted from the bed over long time periods when solar energy is not available. The arrangement shown can provide storage of 100 MWe for about a week.

Because the heat accumulator must be capable of storing large amounts of energy (in the tens of megawatt-years) at high temperatures (250-500°C) over long duration (up to six months), and, furthermore, must be inexpensive (say less than 50\$/GJ or 0.18\$/kWh) and made of abundant materials that tolerate high temperature cycling, all these considerations have governed the preliminary design decisions concerning the storage system to be studied and have led to the configuration of a pebble-bed module shown in Fig. 2. In this generic arrangement, the accumulator consists of arrays of pebble-filled trenches (or holes) surrounded by undisturbed earth and fed by hot air from above or below. The paper presents some preliminary results applicable to this type of proposed thermal accumulator.

Fig. 2 Generic configuration of pebble-bed heat accumulator

MODEL OF HEAT ACCUMULATOR

Storage Sizing

To establish orders of magnitude for the overall size of an earth-type heat accumulator, the following material properties are assumed for the bed: $\rho_b = 1400 \text{ kg/m}^3$ and $c_b = 0.84 \text{ kJ/kg}^\circ\text{C}$ so that the bed heat capacity is $\rho_b c_b = 1176 \text{ kJ/m}^3^\circ\text{C}$. For a temperature differential $\Delta T = 500^\circ\text{C}$, the energy storage density is then $0.6 \text{ GJ/m}^3 = 0.16 \text{ MWh/m}^3$ (and half that for $\Delta T = 250^\circ\text{C}$). Ignoring losses, minimum accumulator volumes needed to store various thermal energy levels are shown in Fig. 3 for two temperature conditions.

By performing a heat balance on the accumulator of the type shown in Fig. 2, the energy stored can be expressed as the mass rate of flow of air that must be pumped through the pebble area A_a over a period of time t times the specific heat of air and the temperature potential ΔT ; this energy in turn is equal to the volumetric heat capacity of the pebble bed and its earth surroundings times the temperature rise and the total volume. Thus,

$$W = \dot{m} c_a \Delta T t =$$

$$\rho_a c_a v_a A_a \Delta T t = \quad (1)$$

$$\rho_b c_b V_b \Delta T = \xi \rho_b c_b A_b L \Delta T$$

If, furthermore, the energy is derived from solar collection covering an area A_c over the period of time t , it can also be expressed by

$$W = \eta_c A_c I t \quad (2)$$

These relations indicate that the air velocity v_a depends on the ratio A_c/A_a of collector to storage projected areas as well as the ratio A_a/A_b of pebble area to total storage area.

Transient Model

The partial differential equations which govern the performance of a packed bed of the type shown in Fig. 2 can be written as

Fig. 3 Thermal storage sizing

$$\begin{aligned} \rho_a c_a \left(\frac{\partial T}{\partial t} + v_a \frac{\partial T}{\partial x} \right) &= h_v (T_b - T_a) \\ \rho_b c_b \frac{\partial T_b}{\partial t} &= h_v (T_a - T_b) + k_b \nabla^2 T_b \end{aligned} \quad (3)$$

Underlying the formulation of these equations are these assumptions:

- zero heat loss
- constant properties for the system
- zero mass transfer
- zero heat conduction in air
- air flow is one dimensional while heat conduction in the bed is either one - or two dimensional.

The model can be greatly simplified if h_v is assumed to be infinitely large so that the air and bed temperatures are always equal and the coupled equations (3) reduce to the single equation

$$\frac{\partial T}{\partial t} + \frac{r}{1+r} v_a \frac{\partial T}{\partial x} = \frac{\alpha_b}{1+r} \nabla^2 T \quad (4)$$

in which normally the heat capacity ratio $r = \rho_a c_a / \rho_b c_b \ll 1$. The simplified model based on $h_v = \infty$ is reasonably accurate to estimate long-term performance of thermal storage units in which flow velocities are very low.^[1] Interestingly enough, Eq. (4) can be solved for the single blow case with one-dimensional axial bed conduction in the closed form

$$\theta = \frac{T - T_{amb}}{T_{hot} - T_{amb}} = \frac{1}{2} \operatorname{erfc} \left(\frac{x - vt}{2\sqrt{\alpha t}} \right) + \frac{1}{2} e^{\frac{vx}{\alpha}} \operatorname{erfc} \left(\frac{x + vt}{2\sqrt{\alpha t}} \right) \quad (5)$$

where the second term can be ignored. The first term represents a thermal wave in the shape of a complementary error function moving at the velocity $v \approx rv_a$ ($r \ll 1$). The general two-dimensional transient case for repeated charging and discharging of the accumulator requires the use of computer simulation. Some preliminary results are given in the following section.

STORAGE CHARACTERISTICS

Pumping Requirements

The pumping work that must be done to move the air through the pebble bed divided by the energy carried by the air can be written as

$$\frac{\Delta P}{\rho_a c_a \Delta T} = 150 \mu \frac{(1 - \epsilon)^2}{\epsilon^3} \frac{v_a}{\rho_a c_a \Delta T} \frac{L}{D_p^2} \quad (6)$$

for particle Reynold's number less than 100.

Since the velocity will be quite low and, as will be seen, particle size is small, the low Reynolds' number regime is the appropriate one to consider for long-term thermal energy storage. Even when A_c/A_b and A_b/A_a are greater than 1, the Reynolds' number of the flow will for the most part be less than 100. In a perfectly stratified system where $A_b = A_a$, $\xi = 1$, the volume of the bed is

$$LA_b = \frac{\eta_c I A_c t}{\rho_b c_b \Delta T} \quad (7)$$

Heat loss considerations dictate that the pebbles should be heated from below:

- a) Eckert^[2] has shown that heat loss from uninsulated underground structures is inversely proportional to the linear size and that for a large array on the order of thousands of meters in radius, heat loss to the underground would be a negligible fraction of the solar income over the same area.
- b) If the pebbles were heated from above, the heat loss to the surface would be excessive without prohibitively costly insulation.

Elder^[3] has shown that the stability of porous systems heated from below depends upon the Rayleigh number. For Rayleigh numbers less than 40, no free convection columns are observed, and pebble-bed stratification can be maintained. This criterion translates to the following relationship between bed length and particle size:

$$\text{for stability } L \leq \frac{8.6 \times 10^{-4}}{D_p^2} \quad (8)$$

Noting from Eqs. (7) and (8) that

$$L = \frac{A_c}{A_b} \frac{\eta_c I t}{\rho_b c_b \Delta T} \leq \frac{8.6 \times 10^{-4}}{D_p^2} \quad (9)$$

and taking the equality in (8), Eqs. (6) and (7) become in the general case for which $A_a \neq A_b \neq A_c$

$$\frac{\Delta P}{\rho_a c_a \Delta T} = \frac{150\mu}{8.6 \times 10^{-4}} \frac{(1 - \epsilon)^2}{\epsilon^3} \frac{(\eta_c I)^3}{(\rho_a c_a)^2} \left(\frac{t}{\rho_b c_b}\right)^2 \left(\frac{A_c}{A_b}\right)^3 \frac{A_b}{A_a} \frac{1}{(\Delta T)^4} \quad (10)$$

In the above, the insolation I has been averaged over the period t . If I is intermittent and occurs only over a period $t_i < t$, the velocity $v_i = v_a t/t_i$ and Eq. (10) must be modified accordingly as shown below.

To estimate the range of conditions we take:

$$\begin{aligned} I &= 0.3 \text{ kW/m}^2 \\ c_a &= 1 \text{ kJ/kg}^\circ\text{C} \\ c_b &= 1 \text{ kJ/kg}^\circ\text{C} \\ \epsilon &= 0.2 \end{aligned}$$

$$\begin{aligned} t &= 6 \text{ months} \\ \rho_a &= 1 \text{ kg/m}^3 \\ \rho_b &= 2 \times 10^3 \text{ kg/m}^3 \\ \mu &= 0.9 \times 10^{-4} \text{ Pa s} \\ \eta_c &= \xi = 1 \end{aligned}$$

so that

$$\frac{\Delta P}{\rho_a c_a \Delta T} = \frac{0.46 \times 10^6}{(\Delta T)^4} \left(\frac{A_c}{A_b}\right)^3 \frac{A_b}{A_a} \frac{t}{t_1} \quad (11)$$

With the stipulation that pumping work must be less than one percent of the energy flux rate and that the stability criterion must be satisfied, we obtain a relationship for allowable ranges of A_c/A_b and A_a/A_b . This relationship, illustrated in Fig. 4 for $t/t_1 = 5$ and $\Delta T = 250^\circ, 500^\circ\text{C}$, is

$$\frac{A_a}{A_b} = \frac{2.3 \times 10^8}{\Delta T^4} \left(\frac{A_c}{A_b}\right)^3 \quad (12)$$

A wide range of choices is thus allowed to meet the pumping work and stability constraints.

Accumulator Performance

To investigate the temperature stratification and heat loss associated with representative storage systems that meet the previous stability and pumping requirements, a general computer program, originally developed by Professor Patankar for the study of two-dimensional systems, is being adapted to the present problem as governed by partial differential equations of the type expressed in (4). Various test conditions have been defined and the computation is carried to simulate a 20-year period of operation. Typical results are presented in Figs. 5, 6, and 7 which are based on the following data:

$$\begin{aligned} v_a &= 1 \text{ mm/s} \\ \rho_a c_a &= 0.9 \text{ kJ/m}^3^\circ\text{C} \\ \rho_b c_b &= 1400 \text{ kJ/m}^3^\circ\text{C} \\ t &= 6 \text{ months} \end{aligned}$$

Fig. 4 Area ratio constraints meeting flow stability and pumping criteria

$$\begin{aligned} k_b &= 1.38 \text{ W/m}^\circ\text{C} \\ k_{\text{earth}} &= 0.17 \text{ W/m}^\circ\text{C} \\ T_{\text{hot}} &= 509^\circ\text{C} \\ T_{\text{amb}} &= 9^\circ\text{C} \end{aligned}$$

Fig. 5 Temperatures reached at ends of charging and discharging periods for top and bottom of pebble trench.

Figure 5 shows that periodic steady-state conditions are reached in three years after startup. Figure 6 portrays the two-dimensional distribution of temperatures inside an accumulator module in which the pebble-filled trench has a width of 1 m, separation 2 m, and length 11 m below an earth surface cover of 1 m; the simulated condition is a 6-month charging mode corresponding to a steady-state operation attained after three years. Figure 6 gives the temperature time profiles during a 6-month charging period subsequent to twelve years of operation; in this case, however, the accumulator is a single packed bed, with no trenches but a large cavity ($b_1 = b_2$), that can be described as a one-dimensional system. Surface heat losses are of the order of 10% in both cases. Even though such results are preliminary, they do provide an indication of the kind of stratification and first-law efficiency that can be expected from such accumulator designs. If the storage system is to store and supply heat continually, imperfect stratification has to be accepted. By allowing the temperature of the air returning to the solar energy collector to rise to approximately the midpoint temperature and permitting the hot gas during the withdrawal phase to drop to a midpoint temperature, a fraction of the withdrawal period will give temperatures

Fig. 6 Spatial temperature distributions at completion of charging phase after 3 years

Fig. 7 Temperature time profiles during charging phase in the 13th year of operation

very near the maximum year after year once steady-state conditions have been reached. In particular, this fraction increases the longer L and the larger A_a/A_b .

MATERIALS AND ENVIRONMENTAL CONSIDERATIONS

The efficient operation of a low-cost heat storage unit depends to a considerable extent on the geologic-hydrogeologic setting for it determines the availability of low-cost accumulator fill material, the feasibility of site preparation and project operation, and establishes the compatibility with environmental standards and regulations. Three sets of factors are considered:

- a) Scale of reference volumes and material properties.
- b) Nature of transport and rate processes in earth materials.
- c) Environmental considerations.

The concept of scale is best illustrated schematically in Fig. 8. For convenience, three main regions are distinguished. Region I corresponds to the micro-scale of the accumulator; region II defines the geological vicinity to the accumulator and establishes the coupling to region III representing the macroscopic hydro-geological scale.

Region I: The principal problem here is to determine the material properties for heat conduction and mechanical stability of the rock materials over the range of temperatures and the frequency of cycling prescribed by the operation of the accumulator. Figure 2 shows that two different kinds of materials have to be considered: the fragmented rock material in the pebble trenches, and the native rock material (which may also be the source of the trench pebbles) in the surrounding undisturbed earth. Thermal conductivity, heat capacity, density and diffusivity are necessary parameters for heat transport calculations. Table I gives a sample of rock properties for a range of unconsolidated, sedimentary, metamorphic and igneous materials. The average parameters refer to rocks of

Rock Type	Temperature						Ref
	0°C		100°C	200°C	300°C	400°C	
	ρ kg/m ³	k W/m°C	k W/m°C	k W/m°C	k W/m°C	k W/m°C	
Granite	2610	3.51	3.01	2.72	2.47	2.18	6
Gabbro	2870	1.99	1.99	1.99	2.00	2.01	6
Gneiss	2640	2.16	2.02				6
Quartzitic Sandstone	2640	5.48	4.23	3.68	3.22	2.92	6
Limestone	2690	2.55	2.26	2.11	2.09	2.07	6
Fine Sand	1640	0.55					7

Table I: Thermal properties of typical rocks

hand specimen size or laboratory dimensions, for it needs to be pointed out that absolute values can vary with size or reference volume. For example, conductivities of simple crystals generally are higher than in their aggregate form in a rock because of imperfect thermal contact at grain boundaries. Lower values of conductivity are encountered in extended geological units in which joints, cracks, fissures, faults, bedding planes further reduce conductivity. The thermal properties depend also on porosity, saturation of pore space, heat capacity and conductivity of the saturating fluid, and permeability of the rock matrix. It is important to know the mechanical strength and resistance to disintegration of pebble materials under cyclic thermal stresses in order to assess the life cycle of the earth accumulator. Results of thermal cycling experiments performed on different types of rock with temperature variations of up to 600°C (accelerated weathering to simulate lunar conditions) have shown that mechanical resistance of igneous rocks decreases drastically as minerals inside the rock specimen separate; however, total disintegration does not necessarily follow, especially when the rock specimen is under some degree of confining pressure.^[4]

Fig. 8 Heat and mass transport model for regions I, II, and III. (A: atmosphere; WT: water table; a: heat flow; b: evaporation and vapor transport; c: groundwater flow)

Region II: This is the most crucial region for determining the technical feasibility of storage operations. For here geological and engineering processes interface, but in contrast to region I, relatively little process control can be exerted. The mechanisms of heat transport are more complex than in the accumulator itself, because of the co-existence of different saturation regimes. Furthermore, geologic and structural features such as fault zones, high permeability zones, bedding planes, etc. may substantially control the amount of heat loss, whence the overall thermal efficiency of the storage process.

To analyze the complex mechanisms occurring in region II, the following factors must be considered:

1. **Heat transfer in a dry zone:** Surrounding the storage unit, is a dry zone through which heat is conducted via a mechanism very similar to molecular heat transfer, the only difference being that effective

heat transfer coefficients must account for the porous rock aggregate characteristics.

2. Heat transfer in a partially saturated zone: Adjoining the

dry zone are earth materials in which natural soil moisture is present. Such moisture content directly affects the thermal properties as illustrated in Fig. 9.^[5] Temperatures in this zone will range from approximately 1/2 of the maximum storage unit operating temperature to ground-water temperatures. The dependency of thermal conductivity and diffusivity with temperature is sketched in Fig. 10.^[6,7]

3. Exchange processes at boundaries:

The most important soil-air boundary is the land surface near the storage unit where a temperature gradient exists with respect to the near ground atmosphere. Heat loss at this interface is through turbulent transfer mechanisms. An internal boundary between dry and partially saturated zone exists at some distance from the accumulator in region II. In a first approximation this boundary can be treated as a problem of conduction through composite materials with differing thermal properties. In addition, phase change takes place with associated vapor transport along a concentration or partial pressure gradient which, in general, would be opposite to the thermal gradient. The last important boundary for region II is the one with the groundwater reservoir which represents a considerable heat sink with temperatures ranging between 7°C to 15°C. The heat capacity in the groundwater zone is about twice that of dry materials. Furthermore some accelerated evaporation from the water table may take place directly underneath the storage unit, thereby creating a depression in the water surface and possibly local flow reversals.

Fig. 9 Saturation dependence of thermal properties for unconsolidated sandy loam. $\rho c [10^2 \text{ kJ/m}^3\text{°C}]$; $k [10^{-2} \text{ W/m°C}]$; $\alpha [10^{-7} \text{ m}^2/\text{s}]$

Fig. 10 Temperature dependence of thermal properties of rocks (Quartzitic Sandstone: $k_1 [\text{W/m°C}]$, $\alpha [10^{-6} \text{ m}^2/\text{s}]$) - (limestone $k_2 [\text{W/m°C}]$)

4. Associated secondary effects: Hydration of minerals and attendant effects of lowering mechanical strength and changing heat transfer characteristics of earth materials are possible in regions of increased temperature and partial water saturations.

5. Design Parameters: A very simple one-dimensional heat transfer model was used to calculate the "thermal penetration thickness" (δ_t). This is the depth below the accumulator where the isotherm represents a temperature change of less than 1% of the temperature difference between maximum accumulator temperature ($T_1 = 500^\circ$) and the temperature of the groundwater reservoir. ($T_0 = 10^\circ\text{C}$). This represents a local temperature rise above the water table to 15°C , which is still within the normal range of groundwater temperatures and therefore would not be considered an environmentally significant stress. The time of application of the maximum temperature difference is one half of the heating cycle namely 100 days. The effects of cycling are not taken into account therefore the results are on the safe side. A further safety factor is built in by choosing high values of heat diffusivity, $\alpha = 5 \times 10^{-7} \text{ m}^2/\text{s}$. The penetration thickness is then in the order of 10 to 15 m. The distance between the accumulator and water table should be equal or exceed these values in order to minimize heat loss to the groundwater reservoir.

Siting of the accumulator in a location where depth to water table and minimal seasonal fluctuations of the water table guarantee this penetration distance becomes the most important design criterion in region II. The other one is to find geologic units with low conductivity and diffusivity characteristics for the bedrock materials.

Region III: This region comprises the hydrogeologic aquifer system from the point of temperature input into the groundwater reservoir near the accumulator to points of natural or artificial groundwater discharge. The mechanisms of heat transport through this system is by convective dispersion in the flowing groundwater. Its effective heat transfer coefficient depend on the volume of the aquifer, its longitudinal and transversal dispersion characteristics, and the prevailing groundwater flow velocity.

Preliminary calculations indicate relatively rapid heat dissipation and small temperature rises, hence reducing potential environmental degradation. This question has to be explored more fully for the case of non-ideal siting and long term cycling and effects of small but relatively permanent temperature rises of groundwater reservoirs.

CONCLUSIONS

The paper has presented some preliminary results of a first-phase study of a high temperature, large-scale, and long duration thermal storage method employing earth heat accumulators. The advantages of the method stem from the inherent simplicity of the storage system,

from its utilization of low-cost, on-site, native materials resulting in potentially minimum environmental impact, and from the increased energy density obtainable with large temperature savings. The basic configuration as arrays of pebble-filled trenches surrounded by undisturbed earth, fed by hot air from below during charging and ambient air from above during discharge appears to be an attractive design for an earth heat accumulator. The parameters and material properties that govern the behavior of these arrays have been identified. Preliminary results indicate that there is a range of heater size, spacing, pebble size, and mode of operation that offers the potential of stable temperature stratification with acceptably low flow work.

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NOMENCLATURE

A_a = pebble area, m^2
 A_b = accumulator bed area, m^2
 A_c = solar collector area, m^2
 b_1, b_2 = pebble trench width and separation, m
 c_a = specific heat of air, $kJ/kg^\circ C$
 c_b = specific heat of bed, $kJ/kg^\circ C$
 D_p = mean particle diameter, m
 h_v = volumetric heat transfer coefficient, $W/m^3^\circ C$
 I = average insolation, kW/m^2
 k_b = thermal conductivity of bed, $W/m^\circ C$
 L = length of bed, m
 $\dot{m}$ = mass flow rate, kg/s
 ΔP = pressure drop, Pa
 $r = \rho_a c_a / \rho_b c_b$ = heat capacity ratio
 t = time, s
 T = temperature, $^\circ C$
 ΔT = temperature potential, $^\circ C$
 v_a = superficial air velocity, m/s
 V_b = volume of accumulator, m^3
 W = stored thermal energy, J
 $\alpha_b = k_b / \rho_b c_b$ = diffusivity of bed, m^2/s
 ϵ = void fraction
 η_c = solar collector efficiency
 ξ = fraction of bed length usable for storage
 μ = air viscosity, Pa s
 δ_t = thermal penetration thickness, m

Subscripts

a = air
 b = bed
 c = solar collector

Appendix A-4

ON THE TEMPERATURE DEPENDENCE
OF THERMOPHYSICAL PROPERTIES
OF ROCKS IN THE 0° to 500°C RANGE

An abridged version of this appendix appears as a paper entitled "Rock Properties In Thermal Energy Storage Systems in the 0° to 500°C Range" in the Proceedings of 1977 ISES Meeting in Orlando, FL, Vol. 1, pp. 18.5 - 18.9, June 1977.

ON THE TEMPERATURE DEPENDENCE OF THERMOPHYSICAL
PROPERTIES OF ROCKS IN THE 0° to 500°C RANGE

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Thermophysical properties of rocks depend strongly on their mineral composition, the storage temperature and on the history of periodic temperature application (cycling). Long-term and large-scale rock bed thermal storage systems have operating ranges between 200°C and 500°C. Little and only scattered information of rock properties at these temperatures is available. This paper presents a collection and critical review of experimental data from the literature on specific heat, thermal conductivity and diffusivity of representative rock forming minerals and dense (non-porous) rocks between ambient and 773°K. The two most important resistive mechanisms controlling heat conduction are mutual phonon scattering and boundary scattering. Their relative influence in minerals, monomineral aggregates and polymineral rocks varies with temperature and structural properties of the materials such as free mean path, sound velocity, grain size and distribution, form and number of contacts between particles. The form of temperature dependency for thermal conductivity is $K = BT^m$. The empirical coefficients and exponents are presented in tables for mineral aggregates and common igneous and sedimentary rock types, most likely to be used for thermal storage projects. Graphs show the range of measured values, their extension into regions for which extrapolation still gives reasonably accurate values for engineering calculations. Results of cycling experiments show that conductivities and other related properties (sound velocity, expansion coefficient) decrease as a function of cycle frequency especially at the lower temperatures. These effects seem to diminish at higher temperatures.

INTRODUCTION

Long term and large scale rock bed thermal storage systems are expected to operate efficiently at temperatures up to 773°K. (500°C). Most frequently encountered ranges in and around the accumulator will lie between 200°C and 500°C. From the accumulator site outward temperature fluctuations due to cyclic storage and withdrawal of heat will dampen out with distance and temperatures will eventually reach those of the groundwater controlled temperature field (average annual air temperature). Heat transfer mechanisms from the

accumulator outward include conduction and gaseous convection in the dry zone, complex heat transfer with phase change and mass transport in the partially saturated zone, and a combination of these with forced convection in the saturated zone of the groundwater flow field.

This paper deals with heat transfer under dry conditions that would apply in the accumulator bed itself and in the immediately adjacent regions of native rock or soil from which any possible initial moisture has been expelled by the heat storage process.

Preliminary observations indicate that thermophysical properties of the rock material depend on the mineral composition, the solid-porous phase distribution and the temperature range under consideration. It is important to take this temperature dependence into consideration, or to be able to estimate average values for conductivities or diffusivities when calculating the propagation of the temperature front from the storage site under operating conditions.

The specific purpose of this paper is to extend the data base concerning thermophysical properties of rocks to some rock types considered for heat storage and into temperature ranges for which now little direct experimental information exists. The reasons for this paucity of data is that many basic questions of heat conduction are more easily treated near the absolute zero, geological interest of rock material has mainly been at temperatures near those of melts, and studies of refractory materials also concentrate on higher temperatures (800 - 1200°C). The range between ambient and 500°C has been covered to a lesser degree than most other temperature ranges for natural materials.

The areas investigated are general mechanisms of heat transfer, and determination of the most predominant ones in natural rocks; examination of the basis of equations to estimate bulk conductivities of heterogeneous material on the basis of the individual properties of the phases and their volumetric distribution, and influence of thermal cycling on rock properties.

Experimental results are collected from various sources and compared to predicted values based on various models.

GENERAL MECHANISMS OF HEAT TRANSFER IN NONMETALLIC SOLIDS

The main mode of heat transfer to be discussed in this paper is that of conduction, although at the higher temperatures heat transfer through transparent spaces by radiation has to be considered as a contributing factor.

The three thermophysical properties of interest are specific heat C , thermal conductivity K and thermal diffusivity α . Extensive general discussion of this topic can be found in Eucken (1940), Kingery and McQuarrie (1954), and Eckert and Drake (1972).

Specific Heat [$\text{J mol}^{-1}\text{K}^{-1}$; $\text{J Kg}^{-1}\text{K}^{-1}$]

Specific heat is defined as the amount of heat (q) per unit mass necessary to raise the temperature of a given system by a unit temperature difference (dT):

$$C = \frac{dq}{dT} \quad (1)$$

It is a direct expression of the vibrational energy state of a solid. The definition of C is generally specified for either constant pressure (C_p) or volume (C_v) conditions. Because of the small value of the temperature expansion coefficient of solids one can take as a first approximation $C_v = C_p = C$.

C_v is a constant regardless of temperature provided the measurements are taken above a certain critical temperature, which is different for each substance. The value of C_v over its temperature independent domain can be expressed approximately by:

$$C_v = 3R = 24.9 \text{ J mol}^{-1}\text{K}^{-1}$$

where R is the universal gas constant.

The critical temperature below which C_v is a function of temperature is the Debye temperature θ_D , defined as

$$\theta_D = \frac{h \nu_D}{k} \quad (2)$$

with ν_D : maximum frequency of thermal vibrations to have $3N$ vibrational modes

k : Boltzmann's constant

h : Planck's constant

N : total number of molecules

Below the Debye temperature $C_v = f(T^n)$, where the exponent n may take different values in different temperature domains. $n = 3$ when $T < (\theta_D/50)$ for example.

Some characteristic curves of specific heat (C_p) variation of minerals with temperature are given in figure 2.

Thermal Conductivity [$\text{W m}^{-1} \text{K}^{-1}$]

Heat in solids is conducted by various carriers electrons, lattice waves (phonons), magnetic excitations and if the right conditions prevail by electromagnetic radiation. The total thermal conductivity ($K(T)$) is the sum of the contributions by each carrier:

$$K(T) = \frac{1}{3} \sum_i C_i u_i l_i \quad (3)$$

C_i : contribution of each carrier to the specific heat per unit volume

u_i : velocity of carrier

l_i : suitably defined mean free path

In the context of natural rock materials (dielectrics) it is the lattice wave mode of propagation which is the predominant one and which will be considered here.

The operational definition of conductivity is its identification with the proportionality factor in Fourier's heat law

$$q = K \nabla T$$

$$K = \frac{q}{\nabla T} \quad (4)$$

with q : heat flux [$W L^{-2}$]

∇T : temperature gradient [$^{\circ}K L^{-1}$]

which give K the dimensions [$W L^{-1} K^{-1}$].

Even for engineering calculations K can be considered constant only over a very limited range of temperatures.

The basic mechanism of thermal conduction in nonmetallic solids is due to the atomic motions and their transfer along a temperature gradient. These motions take the form of thermal vibrations that become by superposition thermoelastic or lattice waves. The energy transported by these lattice waves (at fixed frequencies) can also be represented figuratively by energy quanta or phonons. A modified Debye theory leads to a similar expression as equation 3.

$$K = \frac{1}{3} C u l \quad (5)$$

where C : is defined as above

u : is the velocity of the lattice waves (phonons)

l : is the distance which phonons travel before their intensity is attenuated to $1/e$ of their original value.

Scattering of the lattice waves gives rise to thermal resistance and different scattering processes effectively reduce the length of l thereby reducing K .

Thermal resistance R is the inverse of thermal conductance. It is strongly dependent on scattering processes which tend to reduce l and u .

In actual crystals the total thermal resistance R_t can be expressed as the sum of five scattering processes (Kowalczyk, 1955)

$$R_t = R_u + R_B + R_D + R_I + R_M \quad (6)$$

where: R_u : resistance due to mutual scattering of phonons by "three-phonon" collision and "Umklapp" processes. These processes are the most important contributors to thermal resistance in pure and regular crystals.

R_B : resistance due to scattering at the boundaries of a specimen and at the grain boundaries of polycrystalline materials.

R_D : resistance due to lattice defects or "structure" scattering. These include small irregularities in the lattice of atom dimension as well as large scale defects such as mechanical breaks and crystallographic defects.

R_I : resistance due to chemical impurities

R_M : resistance due to higher order scattering (mosaic structure)

The importance and contribution to total resistance of particular scattering processes differs in different temperature ranges, and their temperature dependency varies to different degrees. This is in part due to the fact that the predominant mode of wave transport and therefore conduction is at low frequencies in the low temperature range, and at high frequencies at higher temperatures. This means that at high temperatures mutual scattering becomes more predominant and boundary scattering becomes negligible. For an approximate working classification of temperatures we can use the Debye temperature to define:

low : $T < \theta_D/3$

intermediate: $\frac{\theta_D}{3} < T < \theta_D$

and high : $T > \theta_D$

Most of the data discussed in this paper falls therefore into the intermediate range, since θ_D for minerals is comparatively much higher than for metals. The general aspect of a $K=f(T)$ relationship is given for a real mineral, quartz on Figure 1 where the different predominant contributions for each temperature range are indicated.

At low temperatures we have:

$$R_u = \frac{1}{AT} e^{-\frac{1}{2} \frac{\phi_D}{T}}$$

and at higher temperatures this expression shifts from the exponential form to a direct dependence on T such that

$$R_u = A^1 T$$

with A, A¹ numerical constants

$$R_B = \frac{1}{BLT^3}$$

with B an empirical constant.

L smallest linear dimension of sample

$$R_I = CT \text{ with } C \text{ numerical constant}$$

$$R_M = 1/DT \text{ again } D \text{ is a numerical constant}$$

In amorphous materials such as glasses the less ordered structure leads to higher scattering and thermal resistance. This is evidenced by the rather large difference between thermal conductivities of quartz and fused quartz or glass, which is about one order of magnitude at ambient temperatures. The general form of the resistance function above room temperatures: $R_{\text{Glass}} \propto 1/T$.

Thermal Diffusivity [$m^2 \text{ sec}^{-1}$]

Thermal diffusivity α derived quantity that interrelates the two thermo-physical parameters C_p and K discussed above in the following way:

$$\alpha = \frac{K}{\rho C_p} \quad [L^2 T^{-1}]$$

The property itself determines the rate of heat propagation in transient state processes. It appears in the transient heat diffusion equation as

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \quad (7)$$

when thermal properties can be considered independent of temperature, but has to be written in the following form when C_p , and K vary with temperature and considering one dimensional flow:

$$\rho \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left(\frac{K}{C_p} \frac{\partial T}{\partial x} \right) \quad (8)$$

Solutions of this equation in general are carried out under the assumption of constant and temperature independent diffusivity. The least that should be done, short of solving equation (8) with $\alpha = \alpha(T)$ is to assess how good

the approximation of constant α is for a given operating temperature range.

From its definition and considering the temperature dependency of K and C_v , also setting $C_p = C_v$ for solids (minerals) with a small temperature expansion coefficient we have

$$\alpha = \frac{K}{C_v} = \frac{\frac{1}{3}C_v u l}{C_v} = \frac{u l}{3} \quad (9)$$

with the quantities as defined earlier.

Variation of α with temperature would therefore have the same general form as that of thermal conductivity especially in temperature ranges above the Debye temperature. α would vary with the same factors that influence carrier velocity u and mean free path l . Experimental data plots for three representative minerals show this general agreement. (Figure 3)

In summary it can be said that for the temperature range considered one would expect a $1/T$ relationship of K and α , $K(\frac{1}{T})$, $\alpha(\frac{1}{T})$ and direct relationship ($C \propto$), i.e., $C(T)$. The degree to which the $\frac{1}{T}$ inverse temperature relation is realized depends on the relative contribution and the preponderance of various scattering mechanisms in the different temperature ranges. These will now be examined for real mineral and rock systems, for which the discussion of general phenomena will serve as guidelines and models to determine or estimate thermophysical behavior as a function of composition and temperature.

HEAT TRANSFER IN REAL GEOLOGIC SYSTEMS

The purpose of the next section is to see how natural geological substance conduct heat as a function of composition, texture and structure and temperature, to determine the predominant heat conduction mechanisms in the range of temperatures of interest, and to extend fragmentary information on thermophysical properties to rocks of various mineral compositions and into higher temperature ranges on the basis of bulk composition models.

Rock-Forming Minerals

In the context of this study those basic rock-forming minerals for which values of thermophysical parameters as a function of temperature were available have been reviewed. They constitute about 90 percent of the common rocks. Mineralogic and other physical information is listed on table 1.

Rock forming minerals are generally not in the ideal state of composition or structure as pure crystals for which much of the general theory of heat conduction was developed and for which a fair amount of experiential verification exists.

A number of mineralogic parameters will be correlated to thermophysical parameters in order to check how far the general principles developed for ideal crystals apply to less perfect materials. This is first carried out for experimental data obtained at any temperature often only one point and then extended into temperature ranges by comparing and compiling from different sources, by analogous extrapolation between groups of structurally similar minerals, and by checking against theoretically expected and plausible values.

Most of the following discussion is based on the work of Horai (1971) and Eucken and Kuhn (1928).

Density. Extensive experimental data presented by Horai taken at about 23° C suggest that thermal conductivity is a function of mineral density (ρ) and mean atomic weight (M). When values of K for a given M are plotted against ρ a quasi straight line relationship of the following form occurs:

$$K = a + b\rho \quad (10)$$

where a depends on M, b is independent of M, and has a value of $5.43 \text{ W m}^{-1}\text{K}^{-1}$. The relationship is entirely empirical, some important minerals do not follow this trend, but it seems useful to predict K on the basis of ρ and M which are relatively easy to obtain.

By an analogy with Birch's (1961) discussion about correlation of rock density and compressional-wave velocities it seems that the density effects on K would relate primarily to the increase in carrier velocity u.

Impurities. A number of isomorphous minerals that varied only in their metallic ion content show a decrease in K with increasing mean atomic weight. The minerals are of the garnet, pyroxene and amphibole series. Since the relative weight contribution of the silicon-oxygen group remains the same (isomorphism) the mean atomic weight as a first approximation is proportional to the mass of metallic ions in the mineral.

The effect may be explained by the fact that phonon scattering hence thermal resistance is proportional to the mass of metallic ions. The metallic ions could be viewed as impurities introduced into the lattice and have the effect of point defects due to mass difference and differences in the bonding between the metallic ion and its silicon-oxygen network neighbor. (Klemens 1960). On the other hand one could follow the explanation of Eucken and Kuhn (1928) that wave interference increases due to the creation of two different wave trains (acoustic and optic stems). This is so because mass imbalance between lattice components exist as studies on KCl-K Br crystal systems have shown. For both explanations the effects on K are the same and have a theoretical basis. A very similar effect is observed for carbonates, dolomite and calcite, which is shown in the difference of K between dolomite (sediment) and limestone on figure 7.

Solid Solutions. Solid solutions are mixtures of crystalline phases that have identical or very similar structures, consisting of atoms or ions that have similar dimensions and identical electrical charge. Eucken and Kuhn (1928) treat the first case in a $KCl - KBr$ system, Horai (1971) cites plagioclase, olivine and pyroxene systems, representing the latter more complex case.

For the example of plagioclases, as composition changes from one end number, albite, say, to the other, anorthite, thermal conductivity decreases rapidly for only slight admixtures of anorthite, in the order of 10 to 20%, then passes through a minimum at about a 50-50 composition, and increases again to the value of the other end member when only it is represented. This minimum exists only for thermal conductivity, other properties such as density, refractive index, sound velocity and electrical conductivity (especially in metallic alloys that bear a certain resemblance to mineral solid solutions) vary linearly or at least monotonically over the entire range of composition.

In this case it seems reasonable to attribute the phenomenon to a decrease in free mean path rather than to a change in phonon velocities. Eucken (1928) and Joffe (1956) see this as a manifestation of the insertion of foreign components into the lattice when solid solution occurs, hence as an impurity effect.

Lattice Structure. Eucken (1928) has postulated that the greater the variety of lattice components and the more complex their spatial arrangement the more pronounced the branching of the thermo-elastic wave spectrum (into acoustic and optic wave trains). This means that more interference occurs with subsequent phonon scattering and decrease in K . Horai (1971) explains the phenomenon quantitatively by the increased number of possible pathways afforded phonons in an intricately interconnected structure. This will increase the probability of phonons to meet and collide at junction points. The average time between collisions τ is shortened, and since carrier velocity u remains constant mean free path length have to decrease.

Experimental data for various silicates confirm the postulates. They exhibit great structural variety and in general there is a close correlation throughout the whole class between structure and physical properties.

The complexity of the structure can be expressed by the degree of polymerization, which involves the linking of the basic building blocks, the SiO_4 tetrahedra by means of the common oxygen to form large interconnected crystal structures by ionic bonds. In table 2 the complexity of structure increases from the top to the bottom with the expected decrease in thermal conductivity. (Average for the number for which data was available, Horai, 1971).

Table 2

Silicate Structure	$K[W\ m^{-1}\ ^\circ K^{-1}]$
Ortho and ring Silicates (Olivines)	4.50 ± 2.52
Chain Silicates (Pyroxenes)	3.90 ± 1.07
Sheet Silicates (Kaolinite, mica)	3.21 ± 1.87
Framework Silicates (Feldspars)	1.93 ± 0.57

Very similar findings showing the same trend are reported by Eucken (1928) for minerals of different degree of complexity in structure and atomic composition at a temperature of 273°K. The minerals follow a less ordered progression since their structural relationship is not as obvious as the one above.

Velocity of Sound. Since Debye's equation (5) relates conductivity to some carrier velocity u it seems reasonable to expect a correlation between velocity of sound and K . Horai (1971) obtains a simple expression by using equation 10 and substitution for the density, by combining Birch's (1961) relationship for compressional wave velocity and an extension to shear wave velocity to read

$$V_p = a_1(M) + b_1K \quad (11a)$$

$$V_s = a_2(M) + b_2K$$

where V_p : compressional wave velocity

V_s : shear wave velocity

$a_1(M); a_2(M)$: empirical parameters that depend on the mean atomic weight (M)

$b_1; b_2$: is a rate of change factor independent of M

K : thermal conductivity

For a group of silicate minerals good correlation was obtained, the coefficient obtained by a least square analysis are

$$a_1 = 6.07 \pm 0.17 \quad \text{km/sec}$$

$$a_2 = 3.37 \pm 0.13 \quad \text{km/sec}$$

$$b_1 = 0.063 \pm 0.008 \quad (\text{km/sec})/(\text{W/m } ^\circ\text{K})$$

$$b_2 = 0.034 \pm 0.004 \quad (\text{km/sec})/(\text{W/m } ^\circ\text{K})$$

The temperature and pressure conditions were for ambient, 1 atm (extrapolated) and 23°C.

Hardness. Eucken (1928) has shown qualitatively that a relationship between mineral hardness and K can be established. It remains somewhat difficult to quantify since among other things the Mohs hardness scale itself is only relative.

Both correlations deal with the compactness and structural bonding properties, which are directly related to V_p , V_s , ρ and hardness. The parameter affected here is the phonon velocity u , which indeed depends on the interatomic force constant. It is large when the forces acting between the atoms in the crystal or the bonding strength are large.

The discussion above is given to relate certain physical properties of minerals to thermal conductivity. Most of the experimental data is available only for a given temperature and does not permit to see temperature dependencies. Some knowledge of the mechanisms of heat transport involved and their relative contribution to total conductivity will be useful when this discussion is extended into higher temperature ranges, for which relatively few data points can be found.

The next section will present a summary of available thermophysical mineral properties and their dependency on temperature.

MINERALS AND MONOMINERAL AGGREGATES

The following discussion concentrates and focusses on thermal conductivity of real minerals and aggregates. This is because most basic principles and trends of temperature dependence of thermo physical parameters can be adequately shown with this parameter. It is also the one which is very sensitive to small disturbances such as solid solution effects and experimental conditions. Therefore it has a determining effect on derived parameters such as diffusivity. Comparability of experimental results obtained by different investigators is problematic for these reasons.

Specific heat measurements, on the other hand, are less sensitive to these factors, therefore literature data is more compatible and can be used on materials for which the exact compositions including trace elements are not known.

Thermal Conductivity

Experimental Data Compilation. Experimental data for thermo-conductivity of minerals and rock as a function of temperature is relatively scarce. It is scattered throughout the literature and in most cases the entire range of temperatures of interest in the present context is not covered. In most cases no or only partial information about the exact mineralogic composition is given. As pointed out earlier, behavior of similar rock types may vary widely because of the great influence of small perturbations introduced by impurities, compositional changes of solid solutions. Different methods of sample preparation and measuring techniques of K introduce different experimental biases and uncertainties. Hence comparability and transfer of data from different sources, and the precision of averaged values becomes questionable. Even in the most careful experiment precision and accuracy is in the 5% range and 10% for more problematic work. This means that accuracy of global engineering composite values should not be expected to be more than 10% - 15%, or even 20%.

The method of interpretation of measurements introduces further biases. They essentially are due to the fact that data reduction makes use of different equations to separate bulk from individual mineral conductivity in multicomponent systems. Measurements on the same specimen that gave similar bulk results resulted in differences of about 10% after data reduction between a USGS and MIT method (Sass et al. 1971).

The purpose of the present work is to collect thermal conductivity data from different literature sources for rock forming minerals, monomineralic aggregates, and polymineralic rocks. The intention is to select representative values to cover the temperature range of ambient to 500°C, where necessary by extrapolation. These are presented in a simple expression suitable for engineering applications, and for the calculation of whole rock data for which much fewer experimental results are available.

The basic sources of information are the classical work of Birch and Clark (1940), Clark (1966), Morovelli and Veith (1965), Kawada (1964) and the compilations of Touloukian et al. (1970 and 1973).

Information about the most common rock forming minerals over a range from 273°K to 800°K is available for quartz, plagioclase feldspars, hornblende (pyroxene), olivine, calcite and dolomite. Information about orthoclase, augite, muscovite and biotite - all important constituents of igneous rocks of the granitic suite - are either missing, of limited representativity or unreliable.

Data Evaluation. Information was combined together according to mineralogic grouping as far as possible and plotted as logarithmic graphs. For the intermediate temperature range considered an inverse relationship for K with T has been postulated by various authors. The plots were checked for compliance with this relation, and those that could not fit a straight line relationship, or that

fluctuated wildly, or were totally out of the range that most results for the particular group fell within were not considered for the averaging process. Examples are the fayalite (17*) and fosterite curves (16*) which were eliminated for the olivine group. Simple arithmetic averages were formed for those groups of minerals lying reasonably close together and behaving as expected by theory. This was the procedure to obtain averaged aggregate values for quartz (30*), olivine (31*) and calcite (33*).

In other cases only one set of results was available. The information was included if it conformed to theoretical expectations as outlined in the first section of this paper. Dolomite may serve as an example. Only one set of measurements was available (Birch and Clark 1940). However, according to Horai (1971) its K is expected to be higher than that of calcite because of lower atomic weight and density. This is borne out by the graph, and the plot show reasonable relative agreement with Horai's data at 299°K. Although the experimental temperature range for dolomite only extends to 473°K it is extended to 773°K by analogy with data of calcite, which mineralogically is in the same group. Calcite can be represented by a reasonable straight line plot up to 773°K. This procedure is reflected on tables 3 and 4, where the column "Range" indicates the temperature interval for which actual experimental points are on a straight line plot, "extended" indicates that the line was extended further by extrapolation. The expected error from using this extrapolation should remain within a range of 10%. On Figure 4 this portion of the curve is indicated by broken lines.

Presentation of Results. Results are plotted on Figure 4 and tabulated in tables 3 and 4. Eucken (1932) suggests a relationship of the form where

$$K = \frac{1}{B'T^m} + C' \quad (12)$$

where K is the thermal conductivity, T temperature, B' , C' and m empirical constants. His expected value for m is in the order of 1 for crystals. If the temperature is expressed in degrees Kelvin the relation simplifies to

$$K = \frac{B}{T^m} = BT^{-m} \quad (13)$$

For the present data only the relatively pure and simple crystals quartz and halite have m values of about -1. The other crystals plotted and especially the aggregates have values of m much lower ranging between -0.6 to -0.1. In analogy with Horai's finding that conductivities are inversely related to structural complexity of the minerals (see also table 2) a similar statement can be suggested about the behavior of temperature coefficients m . In table 2a the order of increasing structural complexity of silicate minerals is from the top to the bottom. Crystal data where available were used from table 3.

Footnote: asterisked numbers in parentheses refer to the numbering system of minerals in tables 3 and 4.

Table 2a

		^m x-tal. aggr.
Ortho and ring silicates	Olivine (18* + 31*)	.670 (.578)
Chain Silicates	Pyroxenes (Bronzite (12* + 32*))	.522 (.467)
Sheet Silicates	No data	
Framework Silicate	Syenite (9*)	.138
	Albite (8*)	.004

Further refinement and additional data on crystals is needed to verify this relation. The thermodynamic interpretation would be that as temperature increases only the anharmonic scattering processes contribute to the conduction process, and that conductivities of those materials that were initially high at lower temperatures converge faster.

Two types of minerals deviate from the temperature relationship. This is first the orthoclase, calculated from data by Kanamori et al. (1968) and GSA special paper #36 (Birch et al. 1942). Its temperature coefficient is positive, behavior otherwise only expected with glasses. For subsequent calculations the values for an Augite-Syenite were used for orthoclase. This follows the reasoning of Birch and Clark (1940) for the same choice. Even so its temperature coefficient m is much smaller than expected. The same situation prevails with the albite, and one of the relative pure end members of the olivine series forsterite shows the same trend between 273 and 473°K. The limited information about exact chemical composition, mineralogic structure and experimental conditions does not permit to formulate any hypothesis about this phenomenon grounded on experimental evidence in this context.

Data about micas is insufficient to detect any temperature relationship. It is however intuitively obvious that strong anisotropy according to direction of measurement along or across the principal mineralogic axis is a very important factor. The values in table 3 are for a direction perpendicular to the principal axis and are very low when compared with Harai's muscovite (2.49 W/m°K) and biotite 2.34 (W/m°K) at 299°K. These are the values that should be used for aggregates, but no information about their temperature dependence is available.

In summary table 3 gives the values of m and B to be inserted into equation 13 to calculate conductivity of a number of rock forming minerals in the temperature range between 273 to 773°K. The accuracy of extrapolated temperature range is in the order of 10% for the composition and structure of minerals plotted.

Specific Heat (Cp)

Tabulated data for specific heat of minerals over the temperature range of interest is much more available in published compilations. (Touloukian

and Buyco, 1970). It is less sensitive to small compositional variations and experimental method. Therefore the scatter of values is much less than for K , and results from different sources much more comparable and compatible. Figure 2 shows a plot of experimental data of minerals for which K - data also have been plotted in figures 4. The minerals under consideration are quartz, olivine, albite, calcite, muscovite. All values fall within a relatively narrow range. The dashed line represents an average value.

The temperature dependency can be expressed as theoretically expected by the relationship:

$$C_p = F T^n$$

where C_p is the specific heat at constant pressure, F an empirical constant, T temperature in $^{\circ}\text{K}$, and n an empirical exponent. For the average graph of figure 2 F is 41.3, and the slope is $n = 0.515$.

Furthermore the combination rule to find the specific heat of a poly-mineral aggregate, $C_p(A)$ is by simple summation of the values for individual specific heats, $C_p(I)$, weighted by their proper volume fraction $P(I)$:

$$\begin{aligned} C_p(A) &= \sum P(I) C_p(I) \\ \sum P(I) &= 1 \end{aligned} \tag{14}$$

Availability of specific heat data is relatively abundant, scatter of data is low, sensitivity to perturbations is also low, a simple additive rule for aggregate specific heat exists. Therefore averaged values are expected to have better reliability and transfer value than any of the other thermophysical parameters.

Diffusivity

The thermal diffusivity is the thermal parameter necessary for the calculation of transient or cyclic heat flow situations. It is a derived parameter and hence will be as sensitive to scatter as the input parameters. The definition equation is:

$$\alpha = \frac{K}{\rho C_p} \tag{15}$$

The variability of density (ρ) is related to the bulk modulus of thermal expansion (β) which is very small for minerals. (Quartz, $\beta \approx 30 \cdot 10^{-6} \text{ } ^{\circ}\text{K}^{-1}$). Thus the error of assuming ρ to remain constant for the temperature range between 273 to 773 $^{\circ}\text{K}$ is small compared to the errors introduced in the averaging process for the other parameters.

Derived values of α for three representative minerals, quartz, olivine and calcite are plotted on figure 3. The graph shows the great variability of values and the expected trends where $\alpha \approx f(1/T^{1.5} \text{ to } 1/T)$ depending on the temperature exponent m of the conductivity of the mineral in question.

At this point it is not useful to give averaged diffusivity values for various rock forming minerals. Rather it should be calculated for each case for which composition of minerals and rocks are known from the values of K , as given here and in the literature, C_p and ρ as found in the tables of Birch and Touloukian.

Thermal Conductivity of Natural Rocks

Information about thermal properties of natural rocks is even more limited than that for minerals. The only parameter for which direct experimental measurements are available is thermal conductivity. The present section will therefore discuss this parameter only.

The factors involved in determining conductivities for mineral aggregates can be directly applied in the estimation of rock parameter. In addition bulk mineral composition, petrographic fabric especially mineral grain size, intergranular contact, and spatial dispersion, and microstructural features of rocks such as fissures, cleavage and other phenomena that tend to weaken rock cohesion have an influence on aggregate behavior.

The aim of the present research is to present data for common rock formations that would be likely sites for the accumulator emplacement or that would be generally available for accumulator charge material.

Conductivity Data. The three basic groups that are investigated are granites, as representatives of silicic igneous material, the gabbro-basalt or subsilic group and a number of sedimentary rocks of varied composition. No adequate information on the intermediate igneous suites such as the dioritic group, or about higher grade metamorphic rocks was available in suitable form.

The curves have been plotted on figure 5 for granites, figure 6 for gabbro-basalts, and figure 7 for the sedimentary rocks.

In general a relationship of the form $K = B T^m$ can be defined as for minerals and monomineral aggregates. The values for B and m are given in table 4. Slopes for the granites vary from $m = -.228$ to $-.658$ approximately ranked in order of increasing quartz-pyroxene content. The sedimentary rocks show close relation with the principal mineral component. The basalts and gabbros however show a somewhat unexpected temperature behavior. m is either very small (Dresser basalt (39*) shows no temperature dependency ($m=0$) (Diabase (41*) and gabbro (40*)), or have positive temperature coefficients

(Anorthosite (44*) and gabbro (43*)). A satisfactory explanation for this behavior cannot be given on the basis of the behavior of the mineral constituents alone. Some of the rocks, especially the basalts and diabase may contain some minerals in amorphous form which has a tendency to increase K with T.

In this context the results are simply presented in graphic and tabular form and should be used for order of magnitude indications of thermal conductivity of the three general rock types.

Composite Conductivity Models. It was further attempted to calculate rock conductivities from mineral composition and monomineralic values. The three simplest models treat heat flow as in parallel, or in series through the mineral constituents (P(I)) which act as elements with different resistances (1/K(I)). The geometric mean model allows for a contribution by both processes.

$$\text{Parallel Model} \quad K(A)_p = \sum P(I) K(I) \quad (16)$$

$$\text{Series Model} \quad \frac{1}{K(A)_s} = \sum \frac{1}{P(I) K(I)} \quad (17)$$

$$\begin{aligned} \text{Geometric Mean} \quad K(A)_g &= K(A)_p K(A)_s \\ \sum P(I) &= 1 \end{aligned} \quad (18)$$

The parallel model will give the highest values for aggregate conductivity K(A), the series model the lowest. Other more sophisticated models have been proposed based either on a model by Maxwell or on variational principles (Hutt and Berg 1968). In view of the overall accuracy of the input data their use is not warranted in this context.

$K(A) = f(T)$ has been calculated for the granites and for a basalt, by the series model. Agreement of calculated data with the basalt is good (figure 6, points are calculated values for Dresser basalt (39*)). For the granites agreement is not better than about 20%. The calculated values for Barre granite composition are shown as points in figure 5 as an example. Its composition is 26% quartz, 9% pyroxene equivalents, 35% albite and 25% orthoclase.

For the latter Birch and Clark's (1940) values for Augite syenite (9*) and their justification for this substitution is accepted and used.

The discrepancy indicates that other structural parameters such as boundary configuration, contact between mineral grains and micro structure such as cleavage and fissures need to be incorporated into the aggregate conduction model. The limited and averaged data presented here do not permit further re-

finement. One general influence of grain size can be deducted from the calcite group. Data for the pure crystal shows the highest conductivity, it decreases for the coarse grained marble and is lowest for the fine grained Solnhofen limestone as expected. This relationship is also retained throughout the temperature range. Too little information is available from mineralogically homogeneous igneous suites to verify this relationship.

The rock data as presented in this paper can be used as guidelines and order of magnitude estimates for thermal conductivity in engineering calculations. The gaps in information and the low reliability of the aggregate models indicate where more research is needed.

Radiation Component of Conductivity in Rocks

Radiation occurs through transparent specimens or through air filled spaces. It follows Stefan Boltzmann's law and is proportional to T^4 . If it becomes a significant contributing factor in a conduction experiment it is measured as an effective addition to conductivity. When radiation is across one pore only then the effective radiative contribution to conductivity of a pore can be written (Loeb 1954):

$$K_R = 4\mu de\sigma T^3 \quad (19)$$

where: d : largest dimension of gap in direction of heat flow
 μ : geometrical pore factor
 e : emissivity of radiating surface (about 0.5 for mineral materials)
 σ : Stefan's radiation constant
 T : average absolute temperature of region across which radiation occurs

In porous materials the total effective conductivity K_p is given in terms of the matrix conductivity K_m , which may be substituted by the aggregate conductivity $K(A)$ from above if one deals with a polycrystalline matrix, and in terms of the relative distribution of the pores as expressed by porosity factors ϕ . One obtains with (Loeb 1954):

$$K_p = K_m (1 - \phi_C) + \frac{\phi_C}{\frac{1}{K_m} (1 - \phi_L) + \frac{\phi_L}{4\mu e\sigma T^3}} \quad (20)$$

with ϕ_C : fractional areal porosity of cross section perpendicular to heat flow

ϕ_L : fractional lineal porosity in direction of heat flow

$\bar{T}$: mean temperature of sample

and the other quantities as defined above.

Radiation can also take place across transparent minerals such as quartz and calcite for example. In that case the ϕ values have to be corrected or replaced by the volumetric fraction of transparent minerals, and a transparency factor has to be added to the radiative term.

Predicted effects in porous and transparent materials will be in the order of 3% - 10% at temperatures of about 773°K. The same order of magnitude figures are also quoted by Birch and Clark (1940).

In the context of this review it seems that radiative contribution may become a factor at the higher end of the temperature range, when exact evaluation is necessary. The quartz and calcite curves seem to flatten out at the higher temperatures, possibly indicating a radiative component to K. For the first order of magnitude approximations needed for engineering calculating this effect, however, should be neglected.

Effects of Thermal Cycling

Only a few experimental investigations deal with the evolution of thermal properties with thermal cycling. Therefore two other parameters from which inference about thermal properties can be drawn are included in this discussion.

Thermal Conductivity. Although the effects of thermal cycling on the conductivity of rocks has been mentioned by some authors (Birch and Clark, 1940), the only articles that actually show the magnitude of the effect are two early articles by Poole (1912 and 1914). Table 5 contains a list of data for thermal cycline of Newry granite between 352°K and 789°K which is illustrated in figure 8.

Analysis of the data shows that at the lower temperatures (about 273°K) the first cycle with a maximum temperature of 789°K caused a 23.5% drop in conductivity, the second cycle to 686°K caused a 0.5% drop, and the third cycle to 668°K caused almost no change. At the higher temperatures (about 673°K) the first cycle to 789°K caused a 4.5% drop in conductivity, the second cycle to 686°K caused a 0.5% drop, and the third cycle to 668°K left almost no change. The cumulative decrease in conductivity was 24% for the lower temperatures and 5% for the higher temperatures.

Thermal Crack Development: Poole proposes that the irreversible decreases in conductivity during thermal cycling are caused by the formation of microcracks due to the differential expansion of minerals. Crack counts were not made in the studies by Poole, but another study has been made recently relating irreversible volume changes to the formation of microcracks (Thirumalai and

Demou, 1973). Figure 9 shows the effect of thermal cycling on the thermal expansion of charcoal granite. The rock increases in volume by 0.1% after the first cycle to 400°C, and then retains a constant volume and a stable expansion curve after the third cycle. Crack density measurements of acetate peels by the random line method established that the amount of irrecoverable volume increase is indicative of the intensity of thermally induced damage.

Sound Velocity. One property which is theoretically related to conductivity of rocks is the sound velocity. In a study by Ide (1937) sound velocity as a function of temperature was measured in Quincy granite, and figure 10 shows Ide's data for two cycles, the first to 573°K and the second to 773°K. The first cycle reduced the sound velocity by 19% and the second cycle by 61% for a cumulative decrease in sound velocity of 80% at 100°C. After the rock was cycled to 773°K it showed almost no recovery of sound velocity upon cooling.

Conclusions. Even though the data above is not for identical rocks, the following general assumptions may be made:

1. Permanent lowering of conductivity in rocks during thermal cycling is caused by the formation of microcracks due to the differential expansion of mineral constituents. Even a monomineral aggregate will form microcracks since mineral crystals expand at different rates in different crystal directions.
2. Since thermal cycling can lower conductivity by 25% at low temperatures in the range 273 - 773°K, it can be expected that estimates of rock conductivity using the series model will be too high at low temperatures. At higher temperatures (673°K) the series model may be accurate, since the lowering of conductivity is less.
3. A comparison of Figures 8 and 9 indicates that the conductivity of packed beds will not have a stable variation with temperature until the rocks have been cycled through the required range of temperatures at least three times.
4. The lack of recovery of some of the sound velocity in the Quincy granite as compared to the partial recovery of the conductivity in the Newry granite may indicate that there are mechanisms other than lattice wave vibrations which allow heat conduction across boundaries or micro fissure planes.

SUMMARY

The main purpose of this work has been to assess the availability of literature data on thermophysical properties of natural geologic materials and to evaluate their reliability and precision for engineering type heat flow calculations. All show a certain temperature dependence. Heat capa-

city shows the least scatter, thermal conductivity is very sensitive to small perturbances in the conduction process and therefore shows some discrepancies. It changes by a factor of 2 to 3 in the temperature range under consideration.

The discrepancies were reduced by choosing representative values based on analogous behavior of minerals or rocks within a generic group and by checking and selecting those values by comparison with expected values by application of theoretical considerations.

Conductivity data was extended into the temperature range for which it did not exist by extrapolating experimental findings if this could be justified by comparison with values of similar materials for which this information was at hand, by following established linear trends and by conforming with theoretical expectations. The overall accuracy is judged sufficient for first order approximations.

Values for polycrystalline rocks can be obtained by properly combining values of the constituent minerals. At the present level of precision a simple series model seems to be sufficient. More sophisticated models are not justified because of the low precision of input data.

The effect of thermal cycling has been shown on a few samples for which this information was available. It shows that properties deteriorate rapidly in the first few passes but stabilize later to a more or less constant value. These effects are less pronounced at the higher level of the temperature scale.

The rock types suitable for the kind of thermal storage project discussed should have a high mechanical stability under thermal expansion and cycling. They should have a high conductivity and diffusivity at lower temperatures to permit rapid and efficient thermal charge of the accumulator, and low conductivity at the high temperature end to minimize heat loss during the storage period. The prime candidates would be granites and quartzitic rocks.

The exact choice for a given project depends on the local situation and availability. It also depends on the total heat transfer through the entire rock unit and will therefore differ from case to case.

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Table 1. Physical Properties of Minerals Used in This Study
(Data from Horai 1971, and Horai & Simmons, 1970)

Mineral and it's Chemical Formula	Density	Mean Atomic Weight	P-Wave Velocity	S-Wave Velocity	Debye Temp.
Quartz SiO_2	2.648	20.02	9.06	6.05	935
Orthoclase KAlSi_3O_8	2.583	21.41			
Microcline KAlSi_3O_8	2.596	21.41	7.00	14.00	551
Albite $[(\text{NaSi})_{0.88}(\text{CaAl})_{0.12}] \text{AlSi}_2\text{O}_8$	2.646	20.32	6.63	3.60	521
Anorthite $[(\text{NaSi})_{0.18}(\text{CaAl})_{0.82}] \text{AlSi}_2\text{O}_8$	2.754	21.18	7.02	3.72	530
Pyroxene					
Bronzite $(\text{Mg}_{0.90}\text{Fe}_{0.10})\text{SiO}_3$	3.284	20.71	7.64	4.59	691
Muscovile $\text{K}_3\text{Al}_4[\text{Si}_6\text{Al}_2\text{O}_{20}](\text{OH})_4$	2.872	20.96			
Phlogopite $\text{K}_2\text{Mg}_5\text{Fe}[\text{Si}_6\text{Al}_2\text{O}_{20}](\text{OH})_4$	2.864	21.65	6.43	3.71	
Olivine					
Fosterite $(\text{Mg}_{0.93}\text{Fe}_{0.07})_2\text{SiO}_4$	3.276	20.73	7.91	4.39	665
Calcite CaCO_3	2.721	20.02	6.63	3172	
Dolomite $\text{CaMg}(\text{CO}_3)_2$	2.857	18.44	5.5		

Table 3 Thermal Conductivity of Crystals and Monomineral Aggregates (K=B T^m)
Coefficients and Range for Thermal Conductivity Calculations

No	Mineral or Rock	m	B	Range °K	Extended °K	Ref. (No.)
1	Quartz (Crystal, parallel)	-1.052	4148	250-600	273-773	Touloukian (1970)
2	Quartz (Crystal, perpendicular)	-.946	1381	200-550	200-773	T (1970)
3	Quartz (ave. from above)	-.988	2053	200-600	200-700	calc
4	Quartzitic Sandstone (para.)	-.733	341	273-473	273-773	Birch & Clark (1940)
5	Jasper Quartzite, portion 1	-.907	1056	273-473	273-600	} Marovelli and Veith (1965)
6	Jasper Quartzite, portion 2	-1.230	7681	473-773	473-773	
7	Orthoclase (Aggregate)	+.486	0.08	273-773	273-773	Kanamori (1969)
8	Albite (Aggregate)	-.044	2.6	273-473	273-773	B & C (1940)
9	Syenite	-.138	4.8	323-473	273-773	B & C (1940)
10	Pyroxene, portion 1	-.3	312	273-373	273-473	B & C (1940)
11	Pyroxene, average	-.58	124	273-473	273-700	B & C (1940)
12	Bronzite (Aggregate)	-.522	84	323-623	290-773	B & C (1940)
13	Bronzite (Rock)	-.411	41	323-473	290-737	B & C (1940)
14	Muscovite	0	0.67	273-773	273-773	} Powell and Griffiths (1937)
15	Phlogopite	0	0.23	500-773	500-773	
Olivine-						
16	Fosterite (crystal)	-.53	170	473-773	473-773	T (1970)
17	Fayalite (cryst.)	-.47	47	473-773	473-773	T (1970)
18	Olivine (cryst.)	-.67	211	323-473	273-737	B & C (1940)
19	Olivine (cryst.)	-.50	95	400-600	300-700	Kanamori (1969)
20	Dumite 1	-.547	92	373-523	300-773	B & C (1940)
21	Dumite 2	-.557	106	323-473	300-773	B & C (1940)
22	Dumite 3	-.590	127	273-473	290-773	B & C (1940)
23	Dumite	-.602	120	366-873	273-773	Kawada (1964)
Calcite-						
24	Calcite (cryst. parallel)	-.600	104	373-623	283-773	B & C (1940)
25	Calcite (cryst. perpendic)	-.523	59	373-623	273-723	B & C (1940)
26	Calcite (average from above)	-.563	78	373-623	273-723	B & C (1940)
27	Marble	-.555	66	315-473	273-773	calc
28	Solnh. Limestone	-.659	116	323-473	273-773	B & C (1940)
29	Dolomite (rock)	-.670	205	273-473	273-773	B & C (1940)

Table 4 Thermal Conductivity of Rocks (K=B T^m)
Coefficients and Range for Thermal Conductivity Calculations

No.	Mineral or Rock	m	B	Range °K	Extended °K	Ref. (No.)
Calculated Average Aggregate Values						
30	Quartz Aggregate	-.894	1010	273-600	273-773	calc
31	Olivine Aggregate	-.578	122	300-600	290-773	calc
32	Bronzite Aggregate	-.467	59	323-623	290-737	calc
33	Calcium Carbonate Aggregate	-.560	75	373-623	273-723	calc
34	St. Cloud Granite	-.477	38	373-673	273-773	M & V (1965)
35	Rockville Granite	-.658	116	373-773	273-773	M & V (1965)
36	Leucocratic Granite	-.507	33	473-1073	333-1073	Moyseyenko (1972)
37	Barre Granite	-.366	21	273-473	473-773	B & C (1940)
38	Westerly Granite	-.228	8.2	273-473	273-773	B & C (1940)
39	Dresser Basalt	-.177	6.2	273-673	273-773	M & V (1965)
40	Gabbro Wisc. (2)	0	2.0	273-623	273-773	B & C (1940)
41	Diabase	0	2.1	273-773	273-773	B & C (1940)
42	Basalt	-.490	37	374-900	273-900	B & C (1940)
43	Gabbro	+.261	.254	473-873	473-873	Moys. (1972)
44	Anorthosite	+0.156	.68	323-573	273-773	B & C (1940)
Sedimentary-						
45	Limestone (Penn.)	-.416	35	323-473	273-773	B & C (1940)
46	Slate	-.281	13	385-700	273-773	Kawada (1964)
47	Slate	-.222	6.6	273-473	273-773	B & C (1940)
5	Jasper Qtz. (average)	-1.16	4706	273-773	273-773	M & V (1965)
48	Quartz (average)	-.963	1060	273-773	273-773	calc

Appendix A-5

TRANSIENT ANALYSIS OF PACKED BED
THERMAL STORAGE SYSTEMS

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TRANSIENT ANALYSIS OF PACKED-BED THERMAL STORAGE SYSTEMS *

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ABSTRACT

An equivalence between single- and two-phase models of packed beds is established in which an effective heat transfer coefficient including axial conductivity effects is introduced in the Schumann model and, correspondingly, an effective conductivity is used in the single-phase model that includes the air-rock heat transfer. Closed-form solutions for the responses produced by unit impulse, step, ramp, and arbitrary time-varying inlet air temperatures are obtained for the single-phase semi-infinite one-dimensional model. Plots of the step response portraying the motion and dispersion of the thermal wavefront separating the hot and cold zones of the bed - the thermocline - are given for design purposes over a large range of space-time dimensionless variables.

INTRODUCTION

The heat transfer to and from a flowing fluid to a packed bed (a porous solid) has been the subject of many theoretical and experimental investigations since Schumann's original work [1]. This interest stems from the many industrial processes which utilize fluid flow in porous materials for such widely different applications as thermal regenerators and absorbers, catalytic reactors, blast furnaces, nuclear reactors, and thermal storage units, to cite only a few. In particular, interest has focussed recently on the utilization of air-rock or gravel packed beds (rock pile) for the thermal storage of energy derived from process or waste heat systems or from solar collector systems. This paper which concerns more specifically this latter type of applications presents a simple model for the packed bed which accounts for the fluid convective motion, the air-rock heat transfer, axial bed conduction, and internal particle conduction. Based on a single-phase conductivity model of packed beds, analytical solutions are given in simple closed forms for a variety of inlet temperature conditions. The significance of the developed analytical results is that they provide a simple picture of the underlying physical processes occurring in a packed bed as well as a means of validating more complex computer solutions. The analytical solutions are well suited to establish the long-term dynamic performance of packed beds and to devise design procedures for the utilization of air-rock packed beds in thermal energy storage systems.

MODELS OF PACKED BEDSTwo-Phase Models

In the one-dimensional two-phase model of a packed bed, an energy balance is performed on each of the fluid and solid phases, yielding a coupled set of partial differential equations:

$$\rho_a c_a \left(\frac{\partial T_a}{\partial \tau} + v \frac{\partial T_a}{\partial \xi} \right) = h_v (T_b - T_a) + k_a \frac{\partial^2 T_a}{\partial \xi^2} \quad (\text{fluid}) \quad (1a)$$

$$\rho_b c_b \frac{\partial T_b}{\partial \tau} = h_v (T_a - T_b) + k_b \frac{\partial^2 T_b}{\partial \xi^2} \quad (\text{solid}) \quad (1b)$$

subject to appropriate boundary conditions. To simplify notation, the void fraction or porosity is considered to be already included in the ρ and k parameters.

Underlying the describing equations are the following assumptions:

- the fluid is in plug flow
- the physical parameters are considered to be constant
- thermal gradient within solid particles is neglected
- radiation effects are negligible
- radial heat transfer does not occur
- losses to the environment are ignored
- internal heat generation is absent

Even though the equations are linear and time invariant, they cannot be solved exactly in simple analytical forms so that digital computer solutions are usually sought [2]. In the case of an air-rock packed bed, it is justified to ignore the thermal capacity and the axial conduction terms in the gas phase equation (1a). If, in addition, the axial conduction in the bed material is also neglected (an assumption reasonably well warranted at large Reynold numbers), the Schumann model of a packed bed results as described by the simplified equations

$$\rho_a c_a v \frac{\partial T_a}{\partial \xi} = h_v (T_b - T_a) \quad (2a)$$

$$\rho_b c_b \frac{\partial T_b}{\partial \tau} = h_v (T_a - T_b) \quad (2b)$$

With the introduction of

$$x_{N2} = \frac{\xi}{x} = \frac{v \rho_a c_a}{h_v} \quad \text{and} \quad t_{N2} = \frac{\tau}{t} = \frac{\rho_b c_b}{h_v} \quad (3)$$

as characteristic distance and time respectively,

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the equations can be rewritten in terms of the dimensionless space time variables x and t as

$$\frac{\partial T_a}{\partial x} = T_b - T_a; \quad \frac{\partial T_b}{\partial t} = T_a - T_b \quad (4)$$

Using infinite series and Bessel functions, Schumann [1] originally solved these equations for the single-blow case (step change of inlet gas temperature) of a semi-infinite packed bed initially at a uniform temperature. Other analytical solutions [3,4,5,6] have been obtained for single blow and other inlet conditions. Plots of the step response of a packed bed are given in the original Schumann paper and, for an extended range of variables x and t , in a paper by Furnas [7].

Single-Phase Conductivity Model

A single-phase conductivity model can be derived formally from the two-phase equations by letting the volumetric heat transfer coefficient h_v tend to infinity. In this limit, the fluid and solid temperatures coalesce to the same value T and Eqs. (1) reduce to the single equation:

$$(\rho_a c_a + \rho_b c_b) \frac{\partial T}{\partial t} + v \rho_a c_a \frac{\partial T}{\partial x} = (k_a + k_b) \frac{\partial^2 T}{\partial x^2} \quad (5)$$

If additionally $\rho_a c_a \ll \rho_b c_b$ and $k_a \ll k_b$ (as is the case of an air-gravel system), then Eq. (5) can further be simplified to

$$\frac{\partial T}{\partial t} + v \frac{\partial T}{\partial x} = \alpha \frac{\partial^2 T}{\partial x^2} \quad (6)$$

in which $v = v_a \rho_a c_a / \rho_b c_b$ and $\alpha = k_b / \rho_b c_b$, the axial diffusivity of the bed. When the conduction term is zero ($\alpha=0$), the equation reduces to the purely convective motion of a thermal wave traveling at the reduced velocity v . In addition to Eq. (6), boundary conditions expressing the energy balance at the entrance and exit of the bed must be imposed. Known as Danckwerts' boundary conditions, they are in the case of the semi-infinite bed

$$-k_b \frac{\partial T}{\partial x} = v_a \rho_a c_a (T_{in} - T) \text{ at } x = 0 \quad (7)$$

and

$$\frac{\partial T}{\partial x} = 0 \text{ at } x = \infty.$$

Introducing

$$x_{N1} = \frac{x}{v} = \frac{\alpha}{v} = \frac{k_b}{v_a \rho_a c_a} \quad (8)$$

$$t_{N1} = \frac{t}{\alpha} = \frac{v}{2} = \frac{k_b \rho_b c_b}{(v_a \rho_a c_a)^2}$$

as the characteristic distance and time, Eqs. (6) and (7) in dimensionless x and t variables become

$$\frac{\partial T}{\partial t} + \frac{\partial T}{\partial x} = \frac{\partial^2 T}{\partial x^2} \quad (9)$$

and

$$-\frac{\partial T}{\partial x} = T_{in} - T \text{ at } x = 0; \quad \frac{\partial T}{\partial x} = 0 \text{ at } x = \infty. \quad (10)$$

Simple closed-form solutions of these equations will be presented in a later section for a variety

of specified input temperatures. Plots of the step response will also be given in Figures 2 and 3. If these plots are compared with those of Schumann [1] and Furnas [7], it can be verified that they coincide for x and t greater than 10 while, for lower values, the agreement is substantially close especially for the distribution of the bed temperature. This coincidence of the response for basically two different models of the dispersion - convection of a packed bed suggests that the single- and two-phase models are equivalent and can therefore be effectively combined.

Equivalent and Combined Models

This equivalence of single- and two-phase models has been established by various authors [7,8,9,10] after comparing the mathematical solutions of the models or by transforming the differential equations from the two- to the single-phase model. The validity of the results is predicated upon a physical interpretation of the heat transfer occurring in a packed bed in which the various individual dispersive mechanisms involved in stagnant bed conduction, air-rock heat transfer, and internal particle conduction act separately (i.e., they are statistically independent) and can be combined by superposition. In accordance with this view, it is appropriate to define the characteristic parameters of a combined heat-transfer, conductivity model of a packed bed as the sum of the individual characteristic quantities so that using Eqs. (3) and (8)

$$x_N = \frac{x}{v} = x_{N2} + x_{N1} = \frac{v_a \rho_a c_a}{h_v} + \frac{k_b}{v_a \rho_a c_a} = \frac{v_a \rho_a c_a}{h_{eq}} = \frac{k_{eq}}{v_a \rho_a c_a} \quad (11)$$

and

$$t_N = \frac{t}{\alpha} = t_{N2} + t_{N1} = \frac{\rho_b c_b}{h_v} + \frac{k_b \rho_b c_b}{(v_a \rho_a c_a)^2} = \frac{\rho_b c_b}{h_{eq}} = \frac{k_{eq} \rho_b c_b}{(v_a \rho_a c_a)^2} \quad (12)$$

These relations indicate that the effect of conductivity of the bed can be included in the Schumann model (this is especially relevant at low Reynolds numbers) by introducing a modified heat transfer coefficient given by

$$\frac{1}{h_{eq}} = \frac{1}{h_v} + \frac{k_b}{(v_a \rho_a c_a)^2} \quad (13)$$

Similarly, in the single-phase model, the effect of air-rock heat transfer can be accounted for by introducing an effective thermal conductivity defined by

$$k_{eq} = k_b + \frac{(v_a \rho_a c_a)^2}{h_v} \quad (14)$$

To account for temperature gradients within the rock particles, the value of h_v may be reduced by a factor which incorporates the Biot number such as $1 + Bi/5$ proposed by Jeffreson [9].

It is of interest to point out that the ratio

$$\frac{x_N}{t_N} = \frac{\xi/x}{\tau/t} = v_a \frac{\rho_a c_a}{\rho_b c_b} = v \quad (15)$$

is independent of both heat transfer coefficient and bed conductivity.

SYSTEM RESPONSE OF THE SINGLE-PHASE EQUIVALENT MODEL

The dynamic behavior of the single-phase model of the packed-bed system governed mathematically by Eqs. (9) and (10) is analyzed by developing analytical solutions for these equations. The one restriction imposed on the linear packed-bed system is that it be initially relaxed, i.e., the bed is assumed to be at a uniform temperature T_r at $t=0$. No loss of generality follows from taking this reference temperature to be, for convenience, zero. At time $t=0$, a given time varying function for the air inlet temperature $T_{in}(t)$ (measured with reference to T_r) is prescribed as the input of the packed-bed system at $x=0$. The response $T(x,t)$ of the system will now be given explicitly for five input conditions.

Impulse Response

As developed in the Appendix, the impulse response is derived by obtaining the inverse Laplace transform of the packed-bed system function, namely:

$$H(s,x) = \frac{\bar{T}_{in}(s,x)}{\bar{T}_{in}(s)} = \frac{1}{\frac{1}{2} + \sqrt{s+1/4}} e^{x(\frac{1}{2} - \sqrt{s+1/4})} \quad (16)$$

with the result

$$\theta_\delta(t,x) = L^{-1}H(s,x) = \frac{1}{\sqrt{\pi t}} e^{-\frac{(x-t)^2}{4t}} - \frac{1}{2} e^x \operatorname{erfc}\left(\frac{x+t}{2\sqrt{t}}\right) \quad (17)$$

in which erfc denotes the complementary error function. These two expressions which provide a simple characterization of the packed-bed system in the frequency and time domains respectively can then be used to determine the response of the system to various types of inputs.

Step Response

Thus the response to a unit step input $u(t)$ can be obtained by performing either of two operations as indicated by

$$\begin{aligned} \theta_u(t,x) &= \int_0^t \theta_\delta(t,x) dt = L^{-1} \frac{H(s,x)}{s} \\ \text{so that} \\ \theta_u(t,x) &= \frac{1}{2} \operatorname{erfc}\left(\frac{x-t}{2\sqrt{t}}\right) + \sqrt{\frac{t}{\pi}} e^{-\frac{(x-t)^2}{4t}} \\ &\quad - \frac{1}{2}(1+x+t) e^x \operatorname{erfc}\left(\frac{x+t}{2\sqrt{t}}\right) \quad (18) \end{aligned}$$

The significance of this solution will be examined in the next section.

Ramp Response

In a similar manner, the response of the system to a unit ramp $r(t) = tu(t)$ can be derived from

$$\theta_r(t,x) = \int_0^t \theta_u(t,x) dt = L^{-1} \frac{H(s,x)}{s^2}$$

with the result

$$\begin{aligned} \theta_r(t,x) &= -\frac{1}{2}(1+x-t) \operatorname{erfc}\left(\frac{x-t}{2\sqrt{t}}\right) \\ &\quad - \frac{1}{2}(-1+t+xt + \frac{x^2}{2} + \frac{t^2}{2}) e^x \operatorname{erfc}\left(\frac{x+t}{2\sqrt{t}}\right) \\ &\quad + \frac{1}{2}(2+x+t) \sqrt{\frac{t}{\pi}} e^{-\frac{(x-t)^2}{4t}} \quad (19) \end{aligned}$$

Arbitrary Inlet Temperature

Because the packed bed constitutes a linear system, its response to a prescribed time-varying inlet air temperature $T_{in}=f(t)$ (at $x=0$) may be obtained by applying the convolution process to the previous elementary responses so that

$$T(t,x) = \int_0^t f(\tau) \theta_\delta(t-\tau,x) d\tau \quad (20a)$$

$$= T_0 \theta_u(t,x) + \int_0^t \frac{df}{d\tau} \theta_u(t-\tau,x) d\tau \quad (20b)$$

These relations may readily be implemented on a digital computer (Eq. 20b is often referred to as Duhamel's integral).

In many cases, the excitation function may be approximated over a finite time interval by a piecewise linear function that is composed of a finite number of steps (having different amplitudes), and a finite number of ramps (of different slopes). For such a piecewise linear approximation of the input, the system response can be explicitly written for $t_{n-1} \leq t \leq t_n$ as the combination of

$$T_u(t,x) = T_0 \theta_u(t,x) + \sum_{j=0}^{n-1} (T_{j+1} - T_j) \theta_u(t-t_j,x) \quad (21)$$

establishing the response to step functions of amplitude T_j at the breakpoint t_j , and of

$$T_r(t,x) = \sum_{j=0}^{n-1} (M_{j+1} - M_j) \theta_r(t-t_j,x) \quad (22)$$

giving the response of linear segments of slope $M_j = (T_j - T_{j-1}) / (t_j - t_{j-1})$ at the breakpoint t_j .

Sinusoidal Steady-State Response

Consider a sinusoidal input excitation of the form $T_{in} = T_0 \cos \omega t = \operatorname{Re}[T_0 e^{j\omega t}]$ where ω is a dimensionless frequency; with a characteristic frequency $1/t_N = \sqrt{2}/a$, the actual frequency is then $\Omega = \omega \sqrt{2}/a$. The sinusoidal steady-state response of the packed bed is given by

$$T(t,x) = \operatorname{Re}[T_0 H(j\omega,x) e^{j\omega t}] \quad (23)$$

in which the transfer function, by virtue of Eq. (16)

$$H(j\omega, x) = \frac{2}{1 + \sqrt{1 + j4\omega}} e^{\frac{x}{2}(1 - \sqrt{1 + j4\omega})} \quad (24)$$

a polar plot of which is shown in Fig. 1.

CHARACTERISTICS OF THE STEP RESPONSE

The response of a packed bed to a sudden change in inlet temperature affords a convenient experimental procedure for determining the parameters and for assessing the dynamic performance of the system. The step (single-blow) response is thus of particular significance because it constitutes a theoretical basis for the analysis and design of packed-bed systems. For such design purposes, it is desirable to present the single-blow response of a packed bed in the form of plots of Eq. (18) as illustrated in Figs. 2 and 3. The dimensionless temperature indicated is the generalized temperature $\theta = (T - T_r)/(T_0 - T_r)$ where T_r is the initial bed temperature and T_0 is the step inlet temperature. These plots which give the space-time distribution of temperature in the bed $\theta(t, x)$ clearly portray the motion of a dispersing thermal wave. Viewed as a function of distance, the curves of Fig. 2 represent so-called thermoclines that define the demarcation between hot and cold zones in the bed at various times. Clearly, the sharpness of the thermocline wavefront decreases with time. The same behavior is observed in Fig. 3 where temperature is plotted as a function of time with distance the parameter. Furthermore, in Fig. 3 the time rate of change or slope of a curve at a specific distance x establishes the impulse response at that x , while the area under the curve gives the corresponding ramp response.

For x and t larger than 10, the last 2 terms in the step response of Eq. (18) can be ignored compared to the first; in this case

$$\theta(t, x) \approx \frac{1}{2} \operatorname{erfc}\left(\frac{x-t}{2\sqrt{t}}\right) \text{ for } t, x > 10$$

As a function of distance, this expression can be simply visualized as constituting one half a complementary error function which moves at unity dimensionless velocity (or v actual) and which spreads out with increasing time.

The sharpness of the thermocline wavefront may be expressed mathematically as

$$\frac{\partial \theta}{\partial x} = \left(\frac{t}{\pi}\right)^{1/2} e^{-\frac{(x-t)^2}{4t}} - \frac{1}{2}(2+x+t)e^x \operatorname{erfc}\left(\frac{x+t}{2\sqrt{t}}\right) \quad (25)$$

Of special interest are the values of θ and $\frac{\partial \theta}{\partial x}$ at $x=t$ for large x since, in this case, the following approximations hold

$$\theta \approx \frac{1}{2} - \frac{1}{2\sqrt{\pi x}} \quad \text{and} \quad \frac{\partial \theta}{\partial x} \approx -\frac{1}{2\sqrt{\pi x}}$$

providing simple measures of the thermocline location and sharpness.

CONCLUSIONS

The following conclusions may be drawn from the results of this transient analysis of packed bed storage thermal systems: (1) The established equivalence between the two-phase Schumann model and the single-phase conductivity model allows the representation of linear packed beds to be performed in terms of a simple but modified single-phase model that uses an effective conductivity (diffusivity) accounting for various axial dispersion mechanisms and for the convective motion of the thermal wave. (2) The differential equations governing the single-phase model of the semi-infinite initially relaxed one-dimensional packed bed system are solved in simple closed forms, yielding the temperature response of the bed to a variety of inlet air temperature conditions: impulse, step, ramp, arbitrary time-dependent function, piecewise linear input, sinusoidal steady-state excitation. (3) The characteristics of the step response are presented in the form of generalized plots portraying the time-space profile of the thermal waves. These curves representing so-called thermoclines may be used for preliminary design purposes and for estimating the dynamic performance of packed-bed storage systems. (4) From a mathematical modeling standpoint, the present transient analysis is limited to the case of a linear semi-infinite initially relaxed packed-bed system. The more complicated situation in which the packed bed is not only of finite length but has an arbitrary initial temperature distribution should be further investigated analytically.

APPENDIX: System Function and Impulse Response of a Packed Bed

By taking the Laplace transform of Eq. (9) subject to zero initial condition, the following second-order differential equation in x is obtained

$$\frac{d^2 \bar{T}}{dx^2} - \frac{d\bar{T}}{dx} - s\bar{T} = 0$$

A solution satisfying the boundary condition at infinity is

$$\bar{T} = A e^{\frac{x}{2} - x\sqrt{s+1/4}}$$

To determine the constant A , the inlet boundary condition (10) is used after taking its Laplace transform

$$\frac{d\bar{T}}{dx} = \bar{T} - \bar{T}_{in} \quad \text{so that } A = \frac{\bar{T}_{in}}{\frac{1}{2} + \sqrt{s+1/4}}$$

Hence, the system function describing the packed bed can be written as

$$H(x, s) = \frac{\bar{T}}{\bar{T}_{in}} = \frac{1}{\frac{1}{2} + \sqrt{s+1/4}} e^{\frac{x}{2} - x\sqrt{s+1/4}}$$

The impulse response is then derived as follows

$$\begin{aligned} \theta_0(t, x) &= L^{-1}(H, s) = e^{\frac{x}{2} - \frac{t}{4}} L^{-1} \frac{e^{-x\sqrt{s}}}{\sqrt{s+1/4}} \\ &= e^{\frac{x}{2} - \frac{t}{4}} \left[\frac{1}{\sqrt{\pi t}} e^{-\frac{x^2}{4t}} - \frac{1}{2} e^{\frac{x}{2} + \frac{t}{2}} \operatorname{erfc}\left(\frac{x}{2\sqrt{t}} + \frac{\sqrt{t}}{2}\right) \right] \end{aligned}$$

which reduces to Eq. (17) and where, in the first step, use is made of the s-shifting property of Laplace transforms, and, in the second, reference is made to standard tables of transform pairs.

For completeness, the dimensionless equations (18), (19), and (24) are here rewritten in terms of the actual physical variables as

$$\theta_{\delta}(\tau, \xi) = \frac{1}{v} \left(\frac{a}{\pi \tau} \right)^{1/2} e^{-\frac{(\xi - v\tau)^2}{4a\tau}} - \frac{1}{2} e^{\frac{v\xi}{a}} \operatorname{erfc} \left(\frac{\xi + v\tau}{2\sqrt{a\tau}} \right) \quad (26)$$

for the impulse response,

$$\theta_u(\tau, \xi) = \frac{1}{2} \operatorname{erfc} \left(\frac{\xi - v\tau}{2\sqrt{a\tau}} \right) + v \left(\frac{\tau}{\pi a} \right)^{1/2} e^{-\frac{(\xi - v\tau)^2}{4a\tau}} - \frac{1}{2} \left[1 + \frac{v}{a} (\xi + v\tau) \right] e^{\frac{v\xi}{a}} \operatorname{erfc} \left(\frac{\xi + v\tau}{2\sqrt{a\tau}} \right) \quad (27)$$

for the step response, and

$$H(j\Omega, \xi) = \frac{2}{1 + \sqrt{1 + \frac{4a}{v^2} j\Omega}} e^{\frac{\xi v}{2a} (1 - \sqrt{1 + \frac{4a}{v^2} j\Omega})} \quad (28)$$

for the frequency response.

NOMENCLATURE

c	- specific heat	(J/kg°C)
h_v	- volumetric heat transfer coefficient	(W/m ³ °C)
k	- thermal conductivity	(W/m°C)
s	- dimensionless Laplace operator	
t	- dimensionless time	
x	- dimensionless distance	
H(s, x)	- system function for packed bed	
T	- temperature response	(°C)
T_{in}	- air temperature at entrance of bed	(°C)
T_r	- reference bed temperature	(°C)
u(t)	- unit step function	
v_a	- velocity of air flow	(m/s)
v	- velocity of thermal wavefront	(m/s)
α	- diffusivity	(m ² /s)
θ	- generalized temperature	
θ_{δ}	- unit impulse response	
θ_u	- unit step response	
θ_r	- unit ramp response	
ρ	- density	(kg/m ³)
ξ	- distance	(m)
τ	- time	(s)
Ω	- frequency	(rad/s)
ω	- dimensionless frequency	

Subscripts

a	- air (fluid)
b	- bed (porous or packed solid)

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Figure 1 Polar plot of frequency response of packed bed.

Figure 2 Step response of packed bed (as function of distance)

Figure 3 Step response of packed bed (as function of time)

Appendix A-6

ANALYTICAL SOLUTIONS FOR
SINGLE- AND TWO-PHASE MODELS
OF PACKED-BED THERMAL STORAGE SYSTEM

This appendix appears as a Technical Note in the Journal of Heat Transfer, ASME Transactions, Vol. 99, pp. 489-492, August 1977.

Analytical Solutions for Single- and Two-Phase Models of Packed-Bed Thermal Storage Systems

M. Riaz¹

Nomenclature

c = specific heat
 h_v = volumetric heat transfer coefficient
 k = thermal conductivity
 t = dimensionless time, τ/θ
 x = dimensionless streamwise coordinate, ξ/λ
 T = temperature
 T_{in} = temperature at inlet of bed
 v_a = velocity of air flow
 v = velocity of thermal wavefront, $v_a \rho_a c_a / \rho_b c_b$

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Fig. 1 Step response of packed bed (as a function of time)

Fig. 2 Step response of packed bed (as a function of distance)

$$T_r(x, t) = -\frac{1}{2}(1+x-t) \operatorname{erfc}\left(\frac{x-t}{2\sqrt{t}}\right) - \frac{1}{2}\left(-1+t+xt + \frac{x^2}{2} + \frac{t^2}{2}\right) e^x \operatorname{erfc}\left(\frac{x+t}{2\sqrt{t}}\right) + \frac{1}{2}(2+x+t) \sqrt{\frac{t}{\pi}} e^{-(x-t)^2/4t} \quad (10)$$

individual characteristic quantities associated with each model so that

$$\lambda_{eq} = \lambda_1 + \lambda_2 = \frac{u_a \rho_a c_a}{h_{eq}} = \frac{k_{eq}}{u_a \rho_a c_a} \quad \theta_{eq} = \theta_1 + \theta_2 = \frac{\rho_b c_b}{h_{eq}} = \frac{k_{eq} \rho_b c_b}{(u_a \rho_a c_a)^2} \quad (11)$$

where

$$\frac{1}{h_{eq}} = \frac{1}{h_o} + \frac{k_b}{(u_a \rho_a c_a)^2} \quad k_{eq} = k_b + \frac{(u_a \rho_a c_a)^2}{h_o} \quad (12)$$

Equivalent and Combined Models

The close agreement between the transient responses of the single- and two-phase models of a packed bed suggests that they are equivalent and can, therefore, be effectively combined. This equivalence, established differently by several authors [7-10], is predicated upon a physical interpretation of the heat transfer occurring in a packed bed in which the various individual dispersive mechanisms involved in stagnant bed conduction, air-rock heat transfer, and internal particle conduction act separately (i.e., they are statistically independent) and can be combined by superposition. In accordance with this view, the characteristic distance and time for a combined heat-transfer, conductivity model of a packed bed may be defined as the sum of the

These relations indicate that the effect of conductivity of the bed can be included in the Schumann model (this is especially relevant at low Reynolds numbers) by introducing the modified heat transfer coefficient h_{eq} . Similarly, in the single-phase model, the effect of air-rock heat transfer can be accounted for by introducing an effective thermal conductivity k_{eq} . To account for temperature gradients within the rock particles, the value of h_o may be reduced by a factor which in-

α = diffusivity, $k_b/\rho_b c_b$
 θ = characteristic time
 λ = characteristic distance
 ρ = density
 ξ = distance
 τ = time

Subscripts

a = air (fluid)
 b = bed (porous or packed solid)
 1 = single-phase model
 2 = two-phase model
 eq = equivalent combined model

Introduction

Single- and two-phase models are commonly used to describe the dynamic behavior of rock-bed thermal storage systems. The classical Schumann two-phase model normally ignores axial conduction while the single-phase model is essentially a simple convection-conduction representation in which rock and air are at the same temperature. This note presents explicit analytical solutions for the response of these two models to a step change in inlet air temperature assuming that the semi-infinite bed is initially at a uniform temperature. The close agreement between the two responses, especially for large time, justifies the establishment of a combined equivalent model which accounts for both air-rock heat transfer and axial bed conductivity. The developed closed-form solutions for the transient response of the equivalent single-phase model of a packed bed provide a simple and convenient means for estimating the long-term dynamic performance of thermal storage systems without resorting to complex computer simulations.

Two-Phase Schumann Model

In the usual description of an air-rock packed bed as a one-dimensional two-phase linear model, an energy balance is performed on each of the fluid and solid phases yielding two coupled partial differential equations

$$\begin{aligned} \rho_a c_a v_a \frac{\partial T_a}{\partial \xi} &= h_v (T_b - T_a) & (\text{air}) \\ \rho_b c_b \frac{\partial T_b}{\partial \tau} &= h_v (T_a - T_b) + k_b \frac{\partial^2 T_b}{\partial \xi^2} & (\text{bed}) \end{aligned} \quad (1)$$

For an air system, it is justified to ignore the thermal capacity and conduction terms in the fluid phase equation. In addition, if axial conduction in the bed is also neglected (setting $k_b = 0$), the Schumann model of a packed bed is obtained [1]² and is governed by two coupled equations normally written in dimensionless space-time variables x and t as

$$\frac{\partial T_a}{\partial x} = T_b - T_a = -\frac{\partial T_b}{\partial t} \quad (2)$$

with the introduction of $\lambda_2 = v_a \rho_a c_a / h_v$ and $\theta_2 = \rho_b c_b / h_v$ as characteristic distance and time, respectively. For the single-blow operation in which a semi-infinite bed initially at a uniform reference temperature (chosen here, for convenience, to be zero) is suddenly subjected to a unit step of air inlet temperature, the solutions of equations (2) can be obtained [3] using Laplace transform methods as

$$\begin{aligned} T_b(x, t) &= e^{-x} \int_0^t e^{-\lambda} I_0(2\sqrt{x\lambda}) d\lambda \\ T_a(x, t) &= T_b + e^{-x} - I_0(2\sqrt{x t}) \end{aligned} \quad (3)$$

where I_0 denotes a modified Bessel function of the first kind of zero order. Plots of the step responses were given originally by Anzelius

[2] and Schumann [1], and, for an extended range of variables x and t , by Furnas [3]. Other analytical solutions have since been derived corresponding to various input conditions [4-6].

Single-Phase Conductivity Model

A single-phase conductivity model can be derived formally from the two-phase equations by letting the volumetric heat transfer coefficient h_v tend to infinity. In this limit, the air and bed temperatures coalesce to the same value T and equations (1) reduce to the single equation

$$\frac{\partial T}{\partial \tau} + v \frac{\partial T}{\partial \xi} = \alpha \frac{\partial^2 T}{\partial \xi^2} \quad (4)$$

subject to a Danckwert type of inlet boundary condition, namely $\partial T / \partial \xi = (v/\alpha)(T - T_{in})$. The parameter $v = v_a \rho_a c_a / \rho_b c_b$ represents the reduced velocity of a purely convective thermal wave and $\alpha = k_b / \rho_b c_b$ is the axial diffusivity of the bed. By introducing $\lambda_1 = \alpha/v$ as characteristic distance and $\theta_1 = \alpha/v^2$ as characteristic time, the equations of the single-phase model can be written in terms of dimensionless variables x and t as

$$\frac{\partial T}{\partial x} + \frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} \quad \text{with} \quad \frac{\partial T}{\partial x} = T - T_{in} \quad \text{at } x = 0 \quad (5)$$

The solution of this equation for the unit step response can be shown, with the use of Laplace transform methods, to be expressible in the closed form

$$\begin{aligned} T(x, t) &= \frac{1}{2} \operatorname{erfc} \left(\frac{x-t}{2\sqrt{t}} \right) + \sqrt{\frac{t}{\pi}} e^{-(x-t)^2/4t} \\ &\quad - \frac{1}{2} (1+x+t) e^x \operatorname{erfc} \left(\frac{x+t}{2\sqrt{t}} \right) \end{aligned} \quad (6)$$

in which erfc denotes the complementary error function. Plots of equation (6) as shown in Figs. 1 and 2 illustrate the space-time distribution of temperature inside the packed bed for the single blow case.

Comparison of the Two Models

By comparing these plots of equation (6) with those of Furnas [3], it can be verified that they are nearly identical. For values of time greater than ten, the curves practically coincide; for small values of t , the agreement between T of equation (6) and T_b of equation (3) is within ten percent; clearly, for small t , T_a differs appreciably from T or T_b as expressed by the second equation (3).

For large values of t , the following approximations hold

$$T(x, t) \approx 0.5 \operatorname{erfc} \left(\frac{x-t}{2\sqrt{t}} \right) \quad (t > 10) \quad (7)$$

and, at $x = t$,

$$\left. \begin{aligned} T &\approx 0.5[1 - (\pi x^3)^{-1/2}] \approx T_b \\ \frac{\partial T}{\partial x} &\approx -0.5(\pi x)^{-1/2} \\ T_a - T_b &\approx 0.5(\pi x)^{-1/2} \end{aligned} \right\} \quad (8)$$

Equation (7) for the temperature profile may be visualized as constituting one half a complementary error function which moves in the x -direction at unity dimensionless velocity (or v actual) and which spreads out with increasing time. Equations (8) provide simple measures of the location and sharpness of the moving wavefront.

Clearly the single-phase model of a packed bed is simpler than the two-phase Schumann model. The closed-form solution for the transient step response as given by equation (6) makes it readily adapted for obtaining the response to arbitrary time-varying inlet temperatures through the use of convolution techniques. The unit impulse and ramp responses can be derived in closed forms by differentiating and integrating in turn equation (6) with respect to time so that:

$$T_b(x, t) = \frac{1}{\sqrt{\pi t}} e^{-(x-t)^2/4t} - \frac{1}{2} e^x \operatorname{erfc} \left(\frac{x+t}{2\sqrt{t}} \right) \quad (9)$$

² Numbers in brackets designate References at end of technical note.

corporate the Biot number such as $1 + Bi/5$ proposed by Jeffreson [10]. It is of interest to point out that the ratio $\lambda_{eq}/\theta_{eq} = v$ and is independent of both h_v and k_b .

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Appendix A-7

GRAVEL-FILLED TRENCHES IN EARTH
FOR ANNUAL THERMAL ENERGY STORAGE

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GRAVEL-FILLED TRENCHES IN EARTH FOR ANNUAL THERMAL ENERGY STORAGE

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ABSTRACT

The use of gravel-filled trenches for annual thermal energy storage is analyzed via digital computer. These results are compared to the performance estimates given by the single blow model. Constraints of pressure drop, stability, and trench geometry are considered with respect to volumetric, first and second law efficiencies.

INTRODUCTION

This paper discusses some characteristics of a system that permits the containment of stored thermal energy in which air passing through rock-filled trenches is directed to minimize heat leaks. This "dynamic containment" offers the possibility of low cost and is thus a candidate for annual cycle storage. One of the constraints for continued use of the process (e.g., 20 years) with low heat loss is that the bed be nearly emptied of heat each cycle. Thus, single blow calculations are potentially of value in estimating performance. Such estimates are made here and compared with the results of a computer model of rock-filled trenches in undisturbed earth.

DYNAMIC CONTAINMENT

Storing energy with minimum heat loss in an uninsulated layer of ground by means of dynamic containment is illustrated in figure 1. Hot air is introduced into the central manifold and flows to the two cold manifolds. The design is such that as the temperature starts to rise at the cold manifold, the flow is reversed and cold air is heated in passing through the porous media and harvested at the hot manifold. The typical performance of such a single cycle bed after simulated operation of manifolds in a uniformly porous medium is shown in figure 2. Here it is seen that the "hot" manifold temperature falls to approximately one-third the inlet temperature. Unless the withdrawal process continues until a substantial lowering of the "hot" manifold temperature is achieved, the temperature gradient in the bed soon fills the entire bed and after two to three years operation it would be no longer operable without substantial heating of the cold manifolds regardless of the length of the system. Thus, if the constraint of a cold exterior to the bed is observed, continuous operation of the bed is possible only if a substantial reduction of temperature at the so-called hot manifold is imposed following each cycle. A plot of the temperature versus time leading to steady state operation

after four years is shown in figure 3.

CONSTRAINTS IMPOSED BY STABILITY AND PRESSURE DROP

In constructing the beds, it appears to be economical to construct them perpendicular to the surface of the earth. In this orientation one-half of the bed represents a special classification of potentially unstable flow regimes, that of a porous medium heated from below with through flow. There are at present no experimental examinations of the stability of the bed with through-flow on which to base design limitations. Homsy and Sherwood (1) have recently reviewed the stability literature and developed the criteria for stability. When their criteria are applied to a system where air (with its dependence of viscosity on temperature) is the working fluid, it appears that the most demanding stability criterion is that for the case of vanishing flow. This is the familiar constraint that the Rayleigh number be less than 40. Hardy and his colleagues (2) have employed this criterion and the constraint that the pumping work be less than two percent of the energy stored and have identified the region where both the stability and the flow work limitations are met. This constitutes an acceptable domain of operation. It is shown in figure 4.

PERFORMANCE ESTIMATES FROM THE SINGLE BLOW MODEL

In the limit that the temperature draw between cycles is complete, the closed form solutions of the one-dimensional single fluid storage systems, published by Riaz, become important tools in anticipating the behavior of the continuously cycled storage system. The generalized plot of the dimensionless temperature profile for the single phase (gas temperature = solid temperature) model is shown in figure 5. If the assumption is made that the cycle is reversed upon achieving ten percent of the temperature rise at the cold end, the energy harvested from the bed is greater than 90 percent of the energy introduced into the bed. The volume utilization efficiency of the bed can be estimated by comparing the energy stored in a prescribed length with the energy that could have been stored had the entire volume been raised to the temperature of the incoming gas. Figure 6 shows a plot of volume efficiency as a function of the dimensionless filling time. An inspection of figure 5 shows that there is a relationship between the dimensionless

bed length, x , and the dimensionless time, t , for each of the isocromes shown in figure 5 such that there is a particular bed length, x , corresponding to the particular dimensionless time, t , for θ to be reduced to any value, say $\theta = 0.1$. The value of x (where $\theta = 0.1$) is also shown on the abscissa of figure 6. It is apparent from figure 6 that an increase in bed length causes a rapid increase in volume use efficiency until the dimensionless time of approximately 4 is reached. Beyond this time (hence bed length) increasing the length produces relatively modest increases in volume efficiency. Thus, if the cost of the bed is proportional to its height raised to a power greater than 1, beyond a certain height the yield, as far as the energy storage capacity per unit cost is concerned, increased rapidly with increasing height for $t > 4$. To explore the design implications of the existence of the "elbow" in figure 6 consider the dimensional filling time to be six months (15.768×10^6 seconds) and the thermal diffusivity α , to be $.1 < \alpha < 1 \times 10^{-6} \text{ m}^2/\text{s}$. Then, for dimensionless time = 4 (and dimensionless length = 7.2) dimensionless time $t' = t(\alpha/U_s^2)$ and actual distance $x' = x(\alpha/U_s)$.

The values of U_s and $x' = L$, for two values of α , and for a period of six months are shown in table 1 for $t = 4$. The exquisite relationship of the bed length at $t = 4$ and thermal diffusivity can be seen in this table. The values of x' that correspond to a dimensionless time = 4 for two different thermal diffusivities are also shown in figure 4. It can be seen that naturally occurring porous media like the St. Peter Sandstone might be attractive as storage media as it exists.

TABLE 1

Thermal Diffusivity, $\alpha, \text{m}^2/\text{s}$	t', s	$U_s = \sqrt{\frac{4\alpha}{t'}} \text{ m/s}$	$x' = \frac{L\alpha}{U_s} \text{ m}$
$0.1 \cdot 10^{-6}$	$15.768 \cdot 10^6$	$1.59 \cdot 10^{-7} \text{ m/s}$	4.5
$1.0 \cdot 10^{-6}$	$15.768 \cdot 10^6$	$5.04 \cdot 10^{-7} \text{ m/s}$	14.3

(Wave velocity, U_s , and length x' for $t = 4$, $x = 7.2$)

A third form of rating the dynamic storage efficiency is a consideration of the availability of the heat stored. A definition which has been employed by Ross and Williams (3) to rate the availability of power is to multiply the power flow rate by the Carnot cycle efficiency. The power extracted from storage multiplied by the Carnot cycle efficiency then divided by the power of the Carnot engine efficiency at the inlet temperature yields the measure of availability employed here. It is called Second Law Efficiency.

Because the heat sink temperature can be a variable, the single blow system doesn't lead to a simple general second law efficiency. The second law efficiency has been found to be weakly dependent upon the temperature-time history of the harvested gas and falls between 50 and 75 percent.

THE INFLUENCE OF TRENCH SPACING

In the considerations thus far the porous media has been treated as a homogeneous, one-dimensional system. Employing the computer computational procedure developed by Professors Spalding and Patankar (4) Emerson and Baliga have calculated the behavior of the system composed of gravel-filled trenches interspersed with undisturbed soil (ref. 2 in fig. 6). Sample temperature profiles for filling and emptying the beds following the stabilization of the process after six years of cycling are shown in figure 7. It can be seen that substantial temperature lag is developed between the beds only after the spacing approaches roughly one-half the length of the bed. A summary of the efficiencies obtained is given in table 2.

TABLE 2

Bed Width Module Width	Module Width	Volume Eff.	1st Law Eff.	2nd Law Eff.	Single Blow Vol. Eff.
.25	2.0	36.65	89.35	74.16	36.70
.25	4.0	31.72	86.93	73.91	29.13
.25	6.0	26.05	82.95	73.41	23.04

It can be seen that the first law efficiency remains high whereas second law efficiency and volume efficiency are reduced as spacing increases. In effect, the increase in spacing can be treated much the way an increase in thermal diffusivity would influence the bed. In his development of the single blow model (5) Riaz showed that the system also describes the two-fluid model in which heat exchange between the fluid and the stationary porous medium would be characterized by heat transfer coefficient h_v . In the present case h_v is the coefficient of heat transfer from trenches to surrounding dirt. The equivalent thermal conductivity that is employed in the single fluid model is:

$$k_e = k + \frac{(v_{air} \rho_{air} C_{air})^2}{h_v}$$

This can be recast in the form of a ratio of thermal diffusivities by approximating the volumetric heat transfer coefficient by $h_v = k/w^2$. Then $v_{air} \rho_{air} C_{air} = U_s \rho_s C_s$ and $U_s/\alpha = x/L$, the above equation becomes:

$$\frac{k_e}{k} = \alpha_e/\alpha = 1 + x^2 \frac{w^2}{L^2}$$

With this value of α_e inserted into the dimensionless time,

$$t = \frac{t'}{\alpha_e/U_s^2}$$

we obtain a new value for the final column of the volumetric efficiency. The values obtained by this method are shown in table 2. To a first approximation, then, it would appear that the design varia-

bles of the gravel-filled trench system should be explored by simply making use of the single blow model providing the operation conditions imposed by the environmental constraints result: in virtually complete thermal draw-down of the bed with each cycle.

SUMMARY

Storage of thermal energy by dynamic containment can be accomplished over many cycles if certain constraints are met. If heat leaks are to be minimized, the temperature of the bed must be substantially reduced each cycle. Thus, the single blow solutions lead to useful predictions of performance. The constraints for stability of the bed heated from below and for pressure drop militate in favor of short beds. Volume efficiency rises sharply with increasing length until a length $x' = (7.2 \cdot \alpha)/U_s$ is reached. Longer beds have slightly higher efficiencies but may involve higher per length construction costs. A trench system designed to meet the $x' = (7.2 \cdot \alpha)/U_s$ constraint shows good first and second law efficiencies but reduced volume efficiency as trench spacing increases. The single fluid plane, one-dimensional, single blow solution can be employed to estimate the effect of trench spacing if an effective thermal diffusivity defined by

$$\alpha_e = \alpha(1 + x^2 \frac{v^2}{L^2})$$

is substituted for the α of the homogeneous case.

CONCLUSIONS

All of the considerations discussed here apply to the annual cycle. It may have occurred to the reader that the trenches could be used for rapid charge-discharge systems and the leaks to the surroundings recovered later. This appears promising and will be treated elsewhere.

APPENDIX

The computational procedure used was a time implicit finite difference approximation of the energy equation using a modified Spaulding hybrid scheme for spatial interpolation. Solution to the nodal equations was done via the tridiagonal matrix method on the University of Minnesota CDC 6400 computer system.

NOMENCLATURE

C - specific heat at constant pressure
 k - thermal conductivity of gravel
 k_e - equivalent thermal conductivity for single fluid model
 L - trench length
 t - dimensionless time per Riaz
 t' - dimensional time
 U_s - thermal wave velocity
 V - velocity of air

w - trench width
 x - dimensionless distance per Riaz
 x' - dimensional distance
 α - thermal diffusivity in gravel
 α_e - equivalent thermal diffusivity for single fluid model
 ρ - density
 θ - dimensionless time per Riaz

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Fig. 1: Dynamic Containment of Thermal Energy

Fig. 2: Typical Bed Performance Showing Increase of Temperature Gradient with Time (Times t_4 and t_5 are during discharge)

Fig. 3: Temp. vs Time Leading to Periodic Steady State (Module width = 2 m. Bed width/module width = .25)

Fig. 4: Bed Permeability Versus Bed Height Showing Bounds of Acceptable Thermal Stability, Pumping Energy, and 50% Volumetric Efficiency (for $\alpha = 10^{-7} \text{ m}^2/\text{s}$; $h = 4.5 \text{ m}$)

Fig. 5: Step Response of a Packed Bed (Single Blow Model)

Fig. 6: Volumetric Efficiency vs Dimensionless Time and Distance (Single Blow Model)

Fig. 7: Sample Charging and Discharging Temperature Profiles (Computer Model)

Appendix A-8

THERMAL ENERGY STORAGE IN
LARGE UNDERGROUND SYSTEMS
AND BUILDINGS

This appendix appears as a paper in the Proceedings of Rockstore 1977 Conference, Stockholm, Sweden, September, 1977, pp. 43-51.

THERMAL ENERGY STORAGE IN LARGE UNDERGROUND SYSTEMS AND BUILDINGS

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Proceedings of 1st International Symposium on Storage in Excavated Rock Caverns, Rockstore '77, Stockholm, Sweden, September 1977

Systems for storing thermal energy at high temperatures (250 to 500°C) and at freezing temperatures are described. Examples of underground residential and commercial buildings are given to illustrate their energy and land conserving aspects. Finally, a concept for energy independent buildings is presented.

Des moyens pour l'accumulation d' energie thermique aux temperatures hautes (250 - 500°C) aussi bien qu'aux temperatures basses sont decrit. Nous donnons des cas concrets des maisons et des batiments commercial pour illustrer leur aspects de le point de vue de la conservation d' energie et de terre. En fin, nous presentons un idee generale, pour la constructun des batiments sans energie de l'exterieur.

Es werden Systeme zur Speicherung von Wärme bei hohen Temperaturen (250 bis 500°C) und beim Eispunkt beschrieben. Biespiele von Untergrund-Wohnungen und Geschäftsgebäuden illustrieren mögliche Energieeinsparungen und Erhaltung des Landschaftsbildes durch diese Bauweise. Zum Schluss wird ein Vorschlag zum Bau eines energieunabhängigen Gebäudes diskutiert.

INTRODUCTION

Soon after the turn of this century, vast reserves of crude oil and natural gas were discovered. This, supposedly inexhaustible, supply of low priced fuel led to a prodigal expansion in the use of energy which has come to dominate every walk of technologically developed societies. Recently, as petroleum consumption has begun dramatically to exceed production, we have come to realise that world fossil fuel resources are finite and that we now confront a new era of increasingly expensive energy.

Of all the energy used in the USA and other industrially developed countries, about 40% is used at low temperatures of less than 100 C; half of this amount is used for space heating residential and commercial buildings and the balance for low temperature industrial process heat [1].

With the impending energy scarcity, we can no longer afford to use finite fossil or nuclear fuels directly for these low temperature requirements but must develop electrical generating plants

U.S. Energy Production and Consumption, 1947-1973

Fig. 1 US Petroleum production and consumption.

which convert these fuels to electrical energy and use the "waste" heat which now is discarded to the atmosphere for low temperature energy needs. All other sources such as solar, tidal, hydro geothermal and wind energy will have to be developed; structures and industrial processes must be designed to be as energy conserving as possible.

Here a major difficulty arises in that the need for heat does not always coincide with its production. Thermal energy then has to be stored on both a short term and on a long or seasonal basis. This paper outlines a number of such techniques being developed at the University of Minnesota.

UNDERGROUND STRUCTURES

Underground structures have a very large thermal mass which can be used as an energy storage system to smooth demand fluctuations. This allows the heating and cooling system to operate at a more constant load with a concomitant increase in efficiency and a reduction in the size of the equipment.

In addition, by reducing heat transferred to and from the surroundings, less energy is consumed to maintain desired conditions within the structure. The heat loss or gain from a structure principally depends on the ventilation load (including intake and infiltration air) and heat transmission through the structure envelope.

The infiltration losses are reduced greatly or eliminated by underground construction since most walls are surrounded by earth. Ventilation air then can be used effectively to capture the heat generated by lighting, people and machines from the exhaust air, and to reintroduce it into the cold make-up air supply.

The transmission losses depend on the wall and soil thermal properties and the difference between the inside and outside temperatures. The thermal transmission coefficient U for a typical underground wall with no insulation is comparable to or better than the best above ground insulation, [1], and soil requires no manufacturing energy, unlike the artificial insulations.

Above ground the temperature difference is determined by local weather conditions; in Minneapolis, for example, the temperature swings from 38°C in summer to a low of -35°C in winter. On the other hand, the earth averages the temperature fluctuations both on a daily and yearly basis. Seasonal temperature fluctuations of the soil reach a depth of several meters whereas the penetration of short-term temperature fluctuations over a period of hours or days is almost negligible Fig.3. The soil temperature in Minneapolis varies slightly either side of 10°C, a mere 10°C colder than that required for comfort. Thus, because of the good insulating properties and mild soil temperatures the heat loss from below grade walls is exceptionally low. For these reasons below ground buildings use typically 50 to 90% less energy for heating and cooling than equivalent above ground facilities [1].

"Is Underground Architecture the only way to build without destroying the land? Of course not. It's

Fig.2 Heat transmission losses for above and below ground structures. Note: the U values given here in SI metric units, were calculated from values listed in ASHRAE [3].

Fig. 3 Average monthly temperature variation of soil with depth near St. Paul, Minnesota [1].

simply one of the most promising (and overlooked) of ways. Sunny, dry and pleasant, it offers huge fuel savings and a silent, green alternative to the asphalt society". Malcolm Wells, 1977 [2]

UNDERGROUND HOUSES

Earth protection offers not only a splendid opportunity to save energy but creates a well tempered space, free of vibration and noise, an environment in keeping with nature, preserving open or historical space. These structures are almost maintenance free, long lasting, far more resistant to earthquake, fire and tornado.

Earth sheltering is as old as human habitat, going back to the prehistoric use of caves to protect early man from ravages of beast and climate.

Earth sheltered buildings may take on a variety of forms. They may be developed for minimal perimeter protection, by berming. Obviously, the more complete the envelope, the more effectively the tempering properties of the earth can contribute to the control of heat transfer. However, the design of human habitat is a complex process, requiring a delicate balance among the factors which satisfy technical goals and those which respond to human needs.

The reaction of many potential occupants, when confronted with the concept of an earth-sheltered habitat, is to respond with apprehension at the prospect of "being stuck in a hole in the ground", living or working "in a basement" or being "buried". Consequently, earth sheltered design has both real and imagined constraints which must be addressed if it is to offer a significant alternative to more conventional contemporary architectural design. Most important of these is the introduction of sunlight into all livable space.

Figure 4 shows the plan and cross-section of a preliminary design for an earth sheltered house. The rooms surround and open onto a bright, sunlit court or atrium. The cross-section shows the dining room with the living room above and the large glass French windows leading from the court into the dining room and bedroom beyond. Horizontal access to daylight eliminates any psychological feeling of being underground. The south facing court provides a large private, sunlit outdoor space which flows naturally into the surrounding parklike area.

The plan can be varied easily from having one bathroom and 2 or 3 bedrooms to 3 bedrooms plus a spare bedroom/bathroom suite.

The building is constructed with the lower floor only two thirds of a floor below the original ground level. The excavated material is not hauled away, but used subsequently to berm up and over the building. This has the advantage of saving hauling costs and of keeping the floor as shallow as possible.

The shell must be insulated on the outside of the building to preserve the thermal mass of the structure. At present we would recommend covering the outside of the roof and walls with Butyl sheeting, a synthetic rubber compound designed for severe-duty waterproofing. It can be obtained in large sheets 6x30 meters to minimize the number of seams. The seams must be meticulously bonded. Styrofoam 25mm thick should be placed beneath the floor slab and up the walls to within 2 meters of the surface. The roof should have a minimum of 100 mm and the

Fig. 4a Earth protected house section, a preliminary design (see acknowledgement)

Fig. 4b Earth protected house plan, a preliminary design (see acknowledgement)

upper 6 meter section of the walls: 50 mm of styrofoam placed directly over the Butyl rubber. Water resistant plywood 2.5mm thick should be placed over the roof styrofoam for protection during backfilling. The plywood is followed by layers of gravel subsoil and mulch to a depth sufficient to grow local wild plants, flowers and vegetables.

The cross-section shows solar collector panels over the living room south wall. These collectors would shade the living room windows during the summer when the sun is high. During winter, the lower sun angle would provide direct sun warmth to help the 'heating'. On mild sunny winter days, excess energy from the solar collectors would provide heat for snow melting in the court, if a transparent inflatable dome, as shown in Figure 5, is not used. The dome can be removed easily during summer.

By far the greatest heat loss from this type of building will occur through the windows and doors into the courtyard. (The building has the same window area as conventional above ground houses). The windows therefore, must be well-sealed against air leaks and, for winter nights, must have folding or rolling thermal shutters as illustrated in Figure 15.

In a very well-sealed building such as this, sufficient ventilating air would have to be introduced and exhausted. A heat recovery system would collect heat from the hot exhaust to warm the cold incoming air. In conventional houses, which have high infiltration losses, heat recovery is not effective.

A number of earth protected housing developments are discussed in reference [1]. There is an enormous potential for reducing the cost of housing while preserving open, green land space and using a small fraction of the energy normally required for conventional housing.

The Department of Civil and Mineral Engineering at the University of Minnesota has recently received a grant from the Legislative Commission on Minnesota Resources to study the design of earth-sheltered buildings. The study, the Earth-

Fig. 5 An inflatable transparent dome such as could be used to cover the courtyard in winter.

Sheltered Energy Independent (ESEI) Building Design Project, is being administered by the Minnesota Energy Agency and the University is working with several local consulting firms to ensure that the research is practical and capable of immediate application.

The primary goal of the project is to demonstrate that the energy requirements for earth-sheltered structures are sufficiently reduced so that these buildings can be made almost totally independent of fossil fuel supplies for their heating and cooling. Heating could be provided by a small bank of solar collectors and/or passive solar design. Cooling could be provided in northern states by the ice air-conditioning system being developed at the University of Minnesota and currently being prototype tested under a separate

Fig. 6 Cross-section of Williamson Hall. Note the sunken court which provides direct sunlight to all the office space (Myers & Bennett Architects BRW).

grant from the Minnesota Energy Agency, as discussed later.

The residential study will be completed by January 1978 and the results of the study will be incorporated into a design manual for earth-sheltered houses. This will provide a wide range of information on the design, construction and energy efficiency of various types of earth sheltered dwellings. The manual will include a set of sample layouts for various site configurations and a separate section with plans, construction details and photographs of existing examples. The study is also addressing building code and financial problems that can occur with this type of construction.

Williamson Hall

There is a shortage of good data for underground building design calculations. In order to generate this data, Williamson Hall, the University of Minnesota's Bookstore/Admissions and Records building, a large (7700 m²) building which is 95%

underground has been instrumented fully under an NSF contract (see Acknowledgement 2). A cross-section of this building is shown in Figure 6. The building has two full floors below grade, plus the mechanical section three floors down. Figure 8 shows the sunlight flooding the minus 2 level of the Admissions and Records section. A full description of Williamson Hall and the energy systems is given in reference [4].

To illustrate the energy efficiency of this building compare it to a similar above grade building with walls having 150mm of glass wool insulation. Assume the same ventilation requirements, the same window area and the same heat loss through the floor and roof (in fact earth protected roofs lose considerably less heat than conventional roofs). Then the only difference will be due to the perimeter walls. For an average January day at -10 C, 55% less total energy will be required for the underground building. The actual savings will be greater still. The above grade section of Williamson Hall amounts to 5% of the floor area but this accounts for 34% of the total conduction heat loss.

Fig. 7 Open space on Williamson Hall roof, open court and diagonal walkway.

Fig. 8 Sunlight flooding Admissions/Records Bldg.

In the past little attention has been paid to life cycle costs because of the low price of energy; this must change. Fig. 9 illustrates the dramatic advantage of earth sheltered buildings. As can be seen, if the underground building were to cost more initially, and this is by no means always the case, after a few years the total cumulative cost

Fig. 9 Cumulative life cycle cost comparison between the earth sheltered Williamson Hall and equivalent above ground buildings [4].

is a great deal less than even the best energy conserving above ground building. The curves were calculated for Williamson Hall using an energy price escalation of 5% per annum, which by all accounts, is lower than expected so that the actual cost advantage will be greater yet. It is worth noting that most of the building is not subjected to the harsh weathering of freeze/thaw so that external maintenance should be much reduced.

The thermal properties of the earth surrounding a building strongly depend on the type of soil and its moisture content. The heat transfer characteristics of the building are being monitored through five different backfill soils, one set of measurement probe groups being installed in each backfill. The backfill materials vary widely.

A probe group consists of four temperature probes, the first three placed one, two and three meters from the building wall and extending to a depth of 7.5 m. The fourth probe is situated 9.3 m from the building in line with the other probes in the group and extends to a depth of 17 m. Each temperature probe consists of a 60mm O.D. plastic pipe, the thermal properties of which are similar to those of the soil, and contains seven thermocouple sensors for determining the soil temperature profiles. Included in each group of probes are three 50mm aluminum tubes paralleling the temperature probes. The aluminum tubes provide access for a nuclear moisture meter.

In addition to the temperature probes, heat flux meters are fixed to the building walls, inside and outside, opposite each probe group. It is important to know how much heat entering the wall passes into the soil and how much goes up the wall (particularly through steel reinforcing) into those parts of the building which are exposed above grade.

All data from the temperature and heat flux

measurements, and from the automatic weather station, are collected by a Fluke 2240A data logger interfaced with a Kennedy 1600/360 incremental magnetic tape recorder. The data logger is programmed to scan the transducer field at selected intervals and to transfer this information to magnetic tape which transfers the data to the CDC 6600 computer for analysis.

To complement the field results, two laboratory methods are being used to measure the thermal properties of soil samples. The first, a steady state method which takes many hours, is based on the traditional design [5]. The second is a transient method, and takes a few minutes for each test.

The soil thermal conductivity can vary by a factor of 7 or more over a relatively small range of moisture contents. During the long tests with the classical steady state method, moisture migrates from the central heater to the colder outer jacket where it condenses. The water then travels back by capillary action. This method, thus, cannot be used reliably except for dry samples. The transient method, however, is completed in a matter of minutes and hence moisture migration is negligible and reliable data for moist samples can be obtained. In addition, this test gives the specific heat capacity; with a simple density measurement, the thermal properties thus are known.

In addition, it will be useful to be able to measure rapidly the in-situ diffusivity, namely the product of thermal conductivity and specific heat per unit volume, of the concrete walls, floor or roof at different locations. Moreover, since diffusivity varies strongly with the moisture content of the wall, the measurement can give an indication of the moisture content at different times and parts of the building. To this end, a unique thermal diffusivity sampler has been designed and manufactured.

Commercial Space

Commercial space can be built from the surface such as Williamson Hall or the office of architect Malcolm Wells shown in Figure 10.

However in many areas, geological conditions are such that mined space can be used for cold storage, storage, manufacturing or a host of other uses. Figure 11 shows a cost comparison [1] of a precision manufacturing company in the Kansas City area of their facilities underground in mined space and of a similar above ground facility. The two left hand bars are for capital cost of heating and cooling equipment and the right hand bar indicates the extraordinary reduction in fuel costs for the underground case.

LARGE-SCALE THERMAL STORAGE IN ROCK

Storage of thermal energy for short periods of time has long been recognised as a vital component in matching energy supply with demand for optimising electrical plant capacity, or for solar energy use for example. However, few energy storage systems have been considered for long-term seasonal energy storage. This may be economically feasible in heated rock bed accumulators. The performance, design constraints, construction methods, and economics of a large-scale rock bed heat accum-

Fig. 10 Underground office of architect Malcolm Wells in New Jersey [2].

Fig. 11 1973 Cost and energy comparison of Bruson Co., Kansas City [1].

ulator capable of providing 300MW Thermal at high temperatures (250 to 500 C) for up to six months are discussed in reference [7]. Six construction schemes are identified and compared. Examples are given of the effect of construction cost, construction period and plant life on the present worth of stored energy. For the assumed operating conditions this present worth of stored energy is less than 15 mills/kWh, thus making the concept of a rock bed accumulator economically attractive for long-term storage.

The rock bed heat accumulator concept [8] involves heating crushed or naturally formed porous rock by flow of hot air from a central manifold (from a solar power tower or power plant) to collector manifolds at the top and bottom of the bed. On discharge the air flow is reversed: cold air is

introduced through the top and bottom manifolds and hot air harvested through the central manifold. Input and output temperatures of up to 500 C and charging and discharging times of 4 to 6 months are possible with storage capacity limited only by the size of the bed. Various configurations of rock bed accumulators can be envisioned (see for instance Figures 12, 13 and 14) with local geological conditions, environmental constraints and cost dictating the appropriate selection. The paper is limited to large-scale storage systems involving rock volumes of over $4 \times 10^6 \text{ m}^3$, this corresponds to about 8×10^6 metric tons of rock. This research is supported by ERDA (see acknowledgement 3).

Fig. 12 Schematic cross-section of bed construction by trenching and backfilling.

Figures 12 and 13 are surface techniques. It is worth noting that the configuration of Figure 12 has been used successfully on a very small scale for storing solar energy for a home by Parke Engineering of Jackson, Wyoming.

Fig. 13 Schematic cross-section of bed construction by strip mining.

FREE ENERGY ICE/AIR CONDITIONING SYSTEM

Figure 15 outlines a system in which winter cold can be stored for subsequent cooling and dehumidification during the summer. In this system anti-freeze circulates from the copper pipe heat exchanger in a shady area to the copper coil in the underground water tank. All through the winter the water is cooled slowly and frozen by the winter cold. The flexible insulation and plastic liner easily deform as the ice expands so that no stress is placed on the tank, and an inexpensive tank construction can be used. It is possible that the antifreeze will circulate naturally; if not, a small pump will be used. When the water in the storage tank has frozen solid, the valves are closed, and the winter cold remains in storage until summer.

When air conditioning is required, the chiller coil valves are opened, and cold anti-freeze is circulated through the chiller coil which cools and dehumidifies the air in the ventilation air duct exactly as does a conventional air conditioner. The thermostat control would simply operate the pump. All through summer the ice slowly melts to water,

Fig. 14 Schematic cross-section of underground bed construction.

while air conditioning the building. At the end of summer, the chiller coil valves are closed and the heat exchanger valves are opened so that the water can be frozen during the following winter. In climates not blessed with sufficient winter cold, a small solar collector could run a small cooling plant to store ice throughout the winter.

It is worth noting that no heat pump, which requires electrical energy and hence cannot be energy independent, is required. The technology is all simple and well established. The small circulating pump uses less than 1% of the energy that would be used by a conventional air conditioner. This research is funded by the Minnesota Energy Agency and a prototype is under construction.

ENERGY INDEPENDENT BUILDINGS

By combining the energy conserving aspects of underground construction with a small and hence economic solar collector system, the heating and hot water requirements of a building can be met. However, it is difficult to meet the cooling load

Fig. 15 Free energy ICE air conditioning system (see Acknowledgement 4)

Fig. 16 A Proposal for an Energy Independent underground building for the Minnesota Historical Society at Fort Snelling, Minneapolis. (Myers and Bennett, Architects, BRW)

unless a much larger and more expensive solar collector system is used. By storing winter cold in ice to meet the cooling load not supplied by the solar collectors, we have a real and economic possibility of designing buildings large and small to be energy independent for heating and cooling.

Figure 16 shows the proposed energy independent underground building for the Minnesota Historical Society which will use this concept.

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Appendix A.9

A COMPUTER PROGRAM TO DETERMINE
THE MINIMUM REVENUE REQUIREMENTS
OF AN ENERGY STORAGE SYSTEM

Appendix A.9

REVREQ: A Computer Program to Determine the Minimum Revenue Requirements of an Energy Storage System

A.9.1 Introduction

This program was written to determine the revenue requirements of the Rock Bed Heat Accumulator (RBHA) energy storage unit by the Minimum Revenue Requirements Method of analysis.⁽¹⁾ The model is versatile in that allowance is made to vary any input parameter at will over a specified range with a constant increment between values of the variable. The major limitation is that no cost of heat energy input is included in the model, although the cost of any essential conversion equipment is a part of the final cost found. The cost of input heat energy may be included in the operations and maintenance portion of the expenses, provided it may be approximated as a fixed percentage of the plant initial cost on an annual basis. REVREQ is the main program and controls the calls to all other programs. Figure 1 is the flow chart of REVREQ in which the subprograms are represented as single boxes. Figures 2 through 5 are the flow charts of the subprograms listed in Table I with the calculations that each subprogram accomplishes. The main program, in Figure 1, can be divided into five primary sections; an input section, calculation of year zero costs, calculation of annual costs, calculation of lifetime levelized figures, and an output section. The annual figures are intended to give an estimate of the cash flow on a

TABLE I. The following FUNCTIONS and SUBROUTINES are used in the main program REVREQ to perform the economic analysis. Those names followed by "*" are subject to change for different plant types and assumptions made.

FUNCTIONS:

CHARGE *	This function determines the annual energy that is used to charge the energy storage unit.
ESC	This function calculates the escalation factor for the specified number of years and escalation rate.
PW	This function finds the present worth of the specified array at the rate of the cost of capital.
Y *	This represents a function giving the probability of survival to the given year, where the year is expressed as a per unit value based on the average life of the unit.

SUBROUTINES:

AVG	This program finds the average capital investment in the portion of the plant in service for each year based on the FUNCTION Y.
DPRTAX	This program calculates the tax depreciation in accordance with the straight-line, double-rate declining-balance, or sum-of-years'-digits method as specified in the input data.
PRINPL	This program finds the total investment in the plant at the start of commercial operation, and the interest during construction.
VARY	This program changes the input plant parameters one at a time and causes REVREQ to be run again to allow testing of parameter sensitivity.

FIGURE 1. Flowchart for the program REVREQ.

yearly basis, and the lifetime figures are intended to give a basis for comparison of various construction techniques and operating parameters. The remainder of this appendix is devoted to the description of the program.

A.9.2 Calculation of Year Zero Costs

The calculation of year zero costs is the determination of the total capital necessary at the start of commercial operation to cover all expenses incurred prior to that time. This calculation is done primarily in the subprogram PRINPL which includes the determination of land cost, construction cost, and interest during construction. The flow chart for this operation is given in Figure 2. The output to the main program is the total year zero cost, CAPC; the land area the storage unit will require, ACRES; and the array giving the capital expenditures for each year prior to the start of commercial operation at year zero. The land area is found by equation 1:

$$\text{ACRES} = \frac{\text{SIZE}}{\text{SQRATE} * 4046} \quad (1)$$

The land cost then can be found using equation 2 and equation 3 which includes the escalation of land prices at the rate EPROP as a correction of land prices to the start of construction. The

$$N = \text{NCO} - \text{NOW} + \text{TCON} - 1 \quad (2)$$

$$\text{COSLAN} = \text{ACRES} * \text{CLAND} * (1 + \text{EPROP})^N \quad (3)$$

cost of construction is then found by equation 4 which divides

FIGURE 2. Flow chart for the program PRINPL.

the cost into a storage cost based on the storage capacity of the unit and an energy transfer cost which is based on the power transfer capacity of the unit. The annual expenditures are then

$$\text{COST} = \text{CONCST} * \text{SIZE} * (1 + \text{ECONS})^N + \text{CCONV} * \text{XKW} * (1 + \text{ECONS})^N \quad (4)$$

found along with interest during construction. The capital expenditure curve is approximated as a straight line of constant slope, with the slope equal to one when per unit quantities are used as in Figure 3, and equations 5, 6, 7, and 8. The bases for the per unit system are the construction period, TCON, for time, and the cost of construction for the capital expenditure, PCON. The total cost at year zero is then found by adding the land, construction, and interest costs.

$$\text{TIME}_I = \frac{\text{TCON} - I}{\text{TCON}} \quad (5)$$

$$\text{TLAST}_I = 0 \quad (6)$$

$$\text{PCON}(I) = (\text{TLAST}_I - \text{TIME}_I) * \text{COST} \quad (7)$$

$$\text{TLAST}_{I+1} = \text{TIME}_I \quad (8)$$

$$\text{CIDC} = \frac{\text{COST} * ((1 + \text{AMAR})^{\text{TCON}} - 1)}{\text{TCON} * \text{AMAR}}$$

This method of analysis can give accurate results if the capital expenditure as a function of time during construction is a good approximation of the actual expenditures. (1)

FIGURE 3. Capital expenditure as a function of time during the construction period.

A.9.3 Average Plant in Service for Each Year

The subprogram AVG (Figure 4) is used to find the value of the plant in service for each year, and the last year in which there is any of the original plant in service. This is accomplished by knowing the probability of survival to the given year as a function of time, and integrating this curve over the year. The foregoing operation is performed for each year from the first year until the value of plant in service is insignificant compared to the original value of plant at the start of commercial operation. The survival probability function is contained in the function Y (Table I) which uses a per unit year for time. The per unit base is the average life of the unit so that equation 9 defines the per unit

$$\text{TIME} = \frac{\text{YEAR}}{\text{NAVG}} \quad (9)$$

time. The integral of this survival curve is then approximated over a year at a time using Simpson's rule with a step size of 0.1 year in real time (7). This is expressed in equation 10.

$$\begin{aligned} \text{SUMA} &= \sum_{J=1}^4 \frac{2*j*0.1 + I-1}{\text{NAVG}} \\ \text{SUMB} &= \sum_{J=1}^5 \frac{(2*J-1)*0.1 + I-1}{\text{NAVG}} \\ \text{PAVG}(I) &= \frac{Y\left(\frac{I-1}{\text{NAVG}}\right) + \text{SUMA} + \text{SUMB} + Y\left(\frac{I}{\text{NAVG}}\right)}{3} \end{aligned} \quad (10)$$

The last year of service is set to the first year in which the PAVG value is less than 0.0005 times the capital cost at the year of the start of commercial operation and assigned the name

FIGURE 4. Flow chart for the program AVG.

NLAST. The largest revenue requirement is associated with a plant for which the date of retirement is known with 100% accuracy and is represented in the function Y as a step type function. This step representation was used for the analysis in this paper as a worst case approximation to the actual RBHA unit behavior.

A.9.4 Calculation of Annual Charges

A.9.4.1 Depreciation

Depreciation is used in the calculation of taxes, and to set aside funds for the return of the capital invested in the unit over the life of the unit. There are then two calculations of depreciation to be made, and a multitude of methods available for this purpose. This program allows a choice of three methods of depreciation which are commonly used, the straight-line (SL) method, the double-rate-declining-balance (DRDB) method, and the sum-of-the-years-digits (SYD) method. However, an analysis of the economics of the choices for company book depreciation reveals that it is detrimental to use any method other than the SL method (25). Therefore, this program was written under the assumption that book depreciation is accrued at the SL rate. Using the SL rate for company books is a common practice, and a reasonable assumption. Tax depreciation may be calculated by any of the three methods. The use of the two more liberalized methods of depreciation for computing tax depreciation is advantageous in that it will reduce the present worth of tax revenue requirements by recovering a larger portion of the

investment during the first few years of operation, which have greater present worth factors. The annuity that is equivalent to taxes is then less for the liberalized methods of finding tax depreciation, and the present worth of all future revenue requirements is also less when compared to the SL method.

The SL method of depreciation is the simplest to calculate, and has become the standard against which the other methods are compared. It is calculated according to equations 11, 12 and 13 (1).

$$SLD = \frac{1}{NAV\bar{G}} \quad (11)$$

$$DBOOK(I) = SLD * PAVG(I) * (1 - SALVG) \quad (12)$$

$$DTAX(I) = SLD * PAVG(I) * (1 - SALVG) \quad (13)$$

The depreciation for tax may be the SL method as shown in equation 13 above, or it may be calculated by either of the two other available methods. The choice of tax depreciation method is indicated by the value assigned to the variable IDOPT, which is interpreted as:

IDOPT = 1 = straight-line depreciation
 = 2 = double-rate-declining-balance depreciation
 = 3 = sum-of-the-years-digits depreciation

The DRDB method uses a rate of twice the SL rate with a declining balance until the year in which the SL rate based on the remaining life is greater than the double rate. After this time, the SL rate based on the remaining life is always greater than the double rate, so that it is used until the plant is fully

retired. The declining balance is defined in equations 14 and 17, the remaining life in equation 15, and the depreciation for tax in equation 16.

$$\text{DECBAL}(1) = \text{CAPC} * (1 - \text{SALVG}) \quad (14)$$

$$\text{REMLIF}_I = \frac{\sum_{J=1}^{\text{NLAST}} \text{PAVG}(J)}{\text{CAPC} * Y \left(\frac{I-1}{\text{NAVG}} \right)} \quad (15)$$

$$\text{DTAX}(I) = 2 * \text{SLD} * \text{DECBAL}(I) \text{ or } \frac{\text{DECBAL}(I)}{\text{REMLIF}} \text{ whichever is greater} \quad (16)$$

$$\text{DECBAL}(I+1) = \text{DECBAL}(I) - \text{DTAX}(I) \quad (17)$$

The SYD method is generally the most liberal method of the depreciation methods available in that it will usually result in the least present worth for federal income tax. It is computed by finding the remaining plant life for each year, and computing the rate from the remaining life (25). The remaining life is calculated from equations 18, 19 and 20; the SYD rate is found from equation 21, and the tax depreciation for the year and the tax depreciation accrual fund balance are found using equations 22 and 23. The calculation of the tax depreciation is done in the

$$\text{RSLD}_0 = 0 \quad (18)$$

$$\text{RSLD}_I = \text{RSLD}_{I-1} + \text{PAVG}(I) - \text{PAVG}(I-1) + \text{SLD} * \text{PAVG}(I) \quad (19)$$

$$\text{REMLIF} = \left(\frac{\text{PAVG}(I) - \text{RSLD}_I}{\text{PAVG}(I)} \right) * \text{NAVG} \quad (20)$$

$$\text{DEP} = \frac{2 * \text{REMLIF}}{(\text{INT}(\text{REMLIF}) + 2 * \text{FRAC}(\text{REMLIF})) * (\text{INT}(\text{REMLIF}) + 1)} \quad (21)$$

$$DTAX(I) = DEP*(PAVG(I)*(1-SALVG)-RES) \quad (22)$$

$$RES_I = RES_{I-1} + DTAX(I) - (PAVG(I-1) - PAVG(I)) * (1-SALVG) \quad (23)$$

subroutine DPRTAX to simplify the main program (see Figure 5).

A.9.4.2 Minimal Gross Income

The minimal gross income is the return on the net plant which must be earned to satisfy the demands of the investors, i.e., to supply the minimum acceptable return to the investors. The net plant is defined as the value of the plant in service in the given year less the balance in the book depreciation accrual fund. The minimal gross income is calculated by Eqn. 24.

$$XMGI(I) = AMAR*(PAVG(I) - DEPRESS(I)) \quad (24)$$

A.9.4.3 Taxes

Calculation of taxes is made so that the previously found value of the minimal gross income remains after taxes are paid. In the annual calculation of federal income tax, the investment credit alters only the first year calculation if it is claimed. If the investment credit is not claimed, a value of zero should be used for the statutory investment credit rate in the data, resulting in simplification of the first year equation to that used for the other years. The investment credit for the first year is calculated in Eqn. (25), and Eqn. (26) is used for all other years.

FIGURE 5. Flow chart for the program DPRTAX.

$$\text{TAXIC} = \text{FEDIC} * \text{CAPC}$$

$$\begin{aligned} \text{TAX(I)} = & \frac{\text{FEDTAX}}{1 - \text{FEDTAX}} (\text{XMGI(I)} * (1 - \frac{\text{DEBTR} * \text{DINT}}{\text{AMAR}}) \\ & + \text{DBOOK(I)} - \text{DTAX(I)} - \text{TAXIC}) \end{aligned} \quad (25)$$

$$\begin{aligned} \text{TAX(I)} = & \frac{\text{FEDTAX}}{1 - \text{FEDTAX}} (\text{XMGI(I)} * (1 - \frac{\text{DEBTR} * \text{DINT}}{\text{AMAR}}) \\ & + \text{DBOOK(I)} - \text{DTAX(I)}) \end{aligned} \quad (26)$$

The other taxes that may apply are the state and local taxes on income or property. The method used to approximate the tax revenue requirements for these taxes is to use a lifetime levelized rate based on the other fixed charges discussed above. Equation (27) is the equation that applies to this calculation:

$$\text{OTAX}_I = \frac{\text{OTHTAX}}{1 - \text{OTHTAX}} (\text{XMGI(I)} + \text{DBOOK(I)} + \text{TAX(I)}) \quad (27)$$

The annual expenses discussed to this point make up the fixed charges which are calculated according to Eqn. (28):

$$\text{FIXCH(I)} = \text{DBOOK(I)} + \text{XMGI(I)} + \text{TAX(I)} + \text{OTAX}_I \quad (28)$$

A.9.4.4 Other Charges

The remaining costs for which a revenue requirement must be found are the overhead costs, and the operations and maintenance costs. The overhead cost is treated as always being a constant percentage of the plant in service, while the operations and maintenance cost is treated as a quantity that escalates at a constant rate. This is the only difference between Eqns. (29) and (30) which are used to find these two quantities. These costs are added together.

$$\text{OHCOST}_I = \text{OHRATE} * \text{PAVG}(I) \quad (29)$$

$$\text{OMCOST}_I = \text{OMRATE} * \text{PAVG}(I) * (1 - \text{EMAINT}) \quad (30)$$

$$\text{OTHRCH}(I) = \text{OHCOST}_I + \text{OMCOST}_I \quad (31)$$

and stored in the OTHRCH array as shown in equation 31. The total revenue requirements for a given year can then be found by taking the sum of the FIXCH and OTHRCH arrays for each year as in equation 32.

$$\text{RR}(I) = \text{FIXCH}(I) + \text{OTHRCH}(I) \quad (32)$$

The energy cost for each year can be found by dividing the energy output from the unit into the total revenue requirements for that year. The energy out from the unit is determined in Eqn. (38) by multiplying the efficiency of the unit times the energy sent into the unit specified in the function CHARGE. The cost of energy

$$\text{ENERGY}_I = \text{EFF} * \text{CHARGE}(I) \quad (33)$$

$$\text{YEGCO}(I) = \frac{\text{RR}(I)}{\text{ENERGY}_I} \quad (34)$$

$$\text{XKWH}(I) = \text{YEGCO}(I) * 0.36 \quad \text{in } \text{¢/KWh} \quad (35)$$

can then be found in \$/GJ by equation 34, or in ¢/KWh by equation 35.

A.9.5 Lifetime Levelized Figures

The Lifetime levelized figures are broken down into two main categories, the present worths and the lifetime levelized

annuities. The calculation of the lifetime levelized annuities is simple to perform manually, but are included in the program as a convenience to the user. The method used to find the equivalent annuities is based on the calculation of the present worths, so that there is no need for a table lookup. Equation (36) is the general equation for the present worth of the ARRAY which may be any array used in the program where the consecutive elements represent the costs for each year from 1 to NLAST. The equivalent lifetime levelized

$$PW(ARRAY) = \sum_{I=1}^N \frac{ARRAY(I)}{(1+AMAR)^I} \quad (36)$$

annuities are then found for depreciation, federal investment credit, federal income tax, and the other taxes from equations 37, 38, 39

$$ADEP = \frac{CAPC - AMAR * PW(PAVG)}{PW(PAVG)} \quad (37)$$

$$XIC = \frac{FEDIC * (AMAR + ADEP)}{1 + AMAR} \quad (38)$$

$$ATAX = \frac{FEDTAX}{1 - FEDTAX} ((AMAR + (ADEP - SLD) * (1 - SALVG)) * (1 - \frac{DEBTR * DINT}{AMAR}) - DT - XIC) \quad (39)$$

$$AOT = \frac{OTHTAX}{1 - OTHTAX} (AMAR + ADEP + ATAX) \quad (40)$$

and 40. The annual levelized energy cost represents a weighted average of the annual energy cost in current year dollars, where the weighting factors are the present worth factors (9). This cost, calculated in equation 41, can be used for comparison to other

$$ALEC = \frac{(1+EPLANT)^{NOW-NCO} ((ADEP+ATAX+AOT+AMAR) CAPC + \frac{AMAR*PWOTHR}{1-(1+AMAR)} LIFE))}{ENERGY} \quad (41)$$

methods of generating energy.

Figure 6 is a flow chart of the subroutine VARY, and Table III is a listing of nomenclature used in the program.

FIGURE 6. Flow chart for the program VARY.

TABLE III. Nomenclature. Below is the nomenclature used in this appendix and the program.

ACRES =Land area needed to construct the unit (acres)
 ADEP =Annuity equivalent to book depreciation (fraction)
 AOT =Annuity equivalent to other taxes (fraction)
 ARRAY =any dimensioned array, used as a general array parameter for function and subprogram calls
 ATAX =Annuity equivalent to federal income tax (fraction)
 CAPC =Capital cost at year zero (\$)
 CD =Capital cost at year zero excluding interest during construction
 COST =Cost of construction only at year zero
 DBOOK =Annual book depreciation array (\$)
 DECBAL=Declining-balance for DRDB depreciation (\$)
 DTAX =Tax depreciation array (\$)
 FIXCH =Annual fixed charges array (\$)
 I =Year index
 NLAST =Index value for the last year any portion of the plant is in service
 OHCOST=Overhead cost (\$)
 OMCOST=Operations & maintenance cost (\$)
 OTAX =Other taxes for year (\$)
 OTHRCH=Annual total for other charges than fixed charges (\$)
 PAVG =Average plant in service array for each year
 PCON =Capital expenditure in years prior to operation
 REMLIF=Remaining life (years)
 RR =Array of annual revenue requirements (\$)
 SIZE =Storage rating of the unit (GJ)
 SLD =Straight-line depreciation rate (fraction)
 TAX =Federal income tax for each year (\$)
 TIME =Year during the construction period
 TLAST =Previous year during the construction period
 XKWH =Cost of energy for the year (¢/KWh)
 XMGI =Minimal Gross Income for year (\$)
 YEGCO =Yearly energy cost (\$/GJ)

All other variables used are defined in Table II, except that the variable SIZE is calculated as:

$$SIZE = 0.0864 * XKW * STIME$$

In most arrays utilized, the index corresponds to the time in years of operation.

References

1. Jeynes, P. H.; Profitability and Economic Choice, Iowa State University Press, 1969.