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High Precision Numerical Solutions to Electric Fields in a
Radial Drift Chamber

by

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High precision numerical solutions to
electric fields in a radial drift chamber

Jonathan D. Faust

Under the supervision of Dr. J. M. Hauptman of the Physics
Department of Iowa State University and Ames Laboratory

A modification to the proportional chamber design makes it possible to achieve $5\mu m$ spatial resolution in particle tracking. Achieving this requires knowing exactly what the electric field is in the chamber, and simple approximations to the field are not accurate enough. The operation of the chamber and techniques which accurately evaluate the electric field are discussed.

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Jonathan D. Faust

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Graduate Faculty in Partial Fulfillment of the
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INTRODUCTION

High Energy Physics is the pursuit of knowledge of the fundamental constituents of the universe. Experimental research work in this area has developed to the point where it is necessary to use large and sophisticated accelerators, detectors, and computers to gather and process information. The study and design of detectors, in particular, has developed into a fine science which incorporates much of atomic, nuclear, and solid state physics.

Detectors used in High Energy experiments vary from scintillators to calorimeters to drift chambers. The function of all types of detectors is to measure properties of the particles which pass through them, e.g., velocity, energy, time of an event, or position. The drift chamber performs the function of reconstructing the path of a particle, which is useful in the identification of the particle as well as locating the interaction point of an event.

Drift chambers have been used successfully for quite some time and their operation is well understood, but one problem with them has been the limits on spatial resolution, that is, how accurately a path can be reconstructed. By making some changes in the conventional design it was postulated [1] that the uncertainty in position could be reduced to $5\mu m$, a factor of 10 better than was previously achievable.

Chapter One is a brief discussion of how a wire chamber detects particles and

an explanation of why it is important that the electric field in the chamber be well known. Chapter Two is an in-depth description of a numerical technique which calculates the monopole charges in the chamber and from that calculates the electric field with a small but unacceptable error. Chapters Three and Four describe two different ways to expand on the technique in Chapter Two to account for multipole moments on particularly problematic wires. Chapter Five describes a different technique which calculates the electric field by first calculating the potential at all points in the chamber. Chapter Six is a brief design study on this chamber and a discussion of the operation of the prototype.

CHAPTER 1. OPERATION OF WIRE CHAMBERS AND DRIFT CHAMBERS

Wire chamber detectors use signals from wires at a positive potential to detect and track the passage of particles through them in two dimensions. The conventional proportional chamber design has multiple wires running parallel to each other in a plane [2]. The wires would be held at a high potential while cathode plates at a low potential are placed parallel to the plane of the wires some distance above and below them. This configuration and its resulting electric field are shown in Figure 1.1. Note the symmetry of the electric field; the field lines emanating from a wire never cross the perpendicular bisector to the line segment connecting a wire and its nearest neighbors. This is an important characteristic of this detector because it makes possible the distinction between events which occur in different regions of the chamber. The region of space in the chamber between the bisectors on either side of a wire is called a cell. Since all cells are essentially identical, the analysis of the detector as a whole simplifies to an analysis of the individual cells.

The z coordinate, which is defined to be parallel to the wires, is lost because a signal on a wire could come from any point on its length. All cross sections of the z -axis are identical so, only the x and y coordinates will be considered. From here on, all diagrams of electric fields and configurations are assumed to be cross sections

Figure 1.1: Proportional chamber electric field; the equipotential lines and the effect of displacing a wire are also shown

Figure 1.2: The development of the electron cascade process

of the z-axis.

The space in the chamber is filled with a gas which is ionized when a charged particle passes through it. The ionization process frees electrons from the gas molecules along the trajectory of the particle; the free electrons are accelerated by the electric field along the field lines to the plane of the wires. When the kinetic energy of the electron reaches the ionization potential of the molecules in the gas, it will ionize another molecule. The two electrons repeat the process as they are both accelerated by the field (see Figure 1.2). At some point along their path the number of free electrons will be:

$$N_e = 2^{\frac{V}{V_i}} \quad (1.1)$$

where V is the change in potential of the initial electron, assumed to be positive, and

V_i is the ionization potential of the molecules in the gas. It is apparent from this equation that most of the electrons are freed very close to the wire; in fact, one-half of them are freed within the last mean free path, and the importance of this will become apparent later.

The cascade continues until the cloud of electrons strikes a wire, and their charge is neutralized. The ionized molecules are also accelerated by the electric field, but their charge is positive so they are accelerated away from the wires to the cathode.

A signal is induced on a wire when it is required to do work on a charged object, and the signal strength is proportional to the amount of work done. Most of the electrons are produced very close to the wires and are accelerated over a small potential difference, so the work done on them by the electric field is small and they induce a negligible signal on the wire. The ionized molecules, however, are accelerated across the full potential difference between the wires and the cathode, and thereby induce a large signal on the wires. Thus, the signal observed on the wires is almost entirely due to the motion of the ions.

The x -coordinate of the particle at some instant is simply the position of the wire, but the y -coordinate is more difficult to evaluate. In most High Energy experiments, the time a molecule is ionized will be known. At some later time the signal is induced on the wires so the time of flight (T) of the electrons in the gas can be evaluated. The y -coordinate can then be found by multiplying the average drift velocity of the electrons in the gas and the drift time (T). Thus, from the wire number (n) and time of flight (T) the x and y coordinates of the ionization event can be found:

$$(n, T) \rightarrow (x, y) \quad (1.2)$$

The information from the cells in the chamber can be compiled to generate sets of

(x, y) coordinates which can be used to reconstruct the path of the particle through the chamber.

The problem with this scheme is that the spatial resolution of the device is limited. The root-mean-square uncertainty in x (σ_x) is equal to $\frac{s}{\sqrt{12}}$, because a signal on a wire has an equal probability of coming from any x coordinate in the cell. Unfortunately, the uncertainty in y is more difficult to evaluate.

Suppose an ionization event occurs and (n, T) are known for a particular cell; the event could have taken place anywhere along a short curve of space shown in Figure 1.3 because an ionization at any point along that path would take the same time to reach the wire. Thus the uncertainty in y (σ_y) is approximately the same as the uncertainty in x .

$$\begin{aligned}\sigma_x &= \frac{s}{\sqrt{12}} \\ \sigma_y &\approx \frac{s}{\sqrt{12}}\end{aligned}\tag{1.3}$$

The spatial resolution can be made small by using small wires and placing them very close to each other, but it will be limited by how small the wires can be made and how closely they can be placed next to each other. A new design for a wire chamber which is not limited by wire size and spacing is needed. Just such a device was invented by Dave Nygren [1]. The new design takes advantage of the signals on more than one type of wire to reconstruct the path of the particle through the chamber.

The new chamber, called a "radial drift chamber", is based upon three principles. The first is the localization of the electron cluster as it avalanches into a wire. The second is the slow drift of the electrons in a cool gas, and the third is the continuous,

Figure 1.3: Possible locations of events given a cell number and drift time

Figure 1.4: The positions of the Sense, Field, and Pickup wires in the new chamber

high speed waveform sampling of an entire track as its ionization clusters drift along the track direction.

The geometry of the new chamber is very much the same as its ancestor, but the constituent cells are very different. The cathodes are the same but instead of one wire type there are three: the sense, pickup, and the field wires (see Figure 1.4). Each wire has a particular function, but they work together so the cell operates as a single unit to generate (x, y) coordinate pairs which are used to reconstruct the path of the particle. The electric field in the chamber may look like Figure 1.5.

The sense wire performs the same function as the central wire in the conventional chamber. It is at a high potential, typically 1050V, and has a small radius so there will be a strong electric field on its surface.

The wires on the edge of the cell are called the field wires. Their function is to shape the electric field so that it attains a desirable form and to electrically isolate cells from each other. The potential on the field wires changes the form of the electric

Figure 1.5: The electric field in the new wire chamber

field in the chamber, thus it can be tuned to give the best possible electric field; this will be discussed later.

The signals from the field wires are not used for a number of reasons. Relative to the signals on the sense and pickup wires the signals on the field wires would be small because the ion and electron paths are not very close to them. Even more important is the fact that a cell shares its field wires with the neighboring cells, so a signal on a field wire could come from one of two cells. Finally, there is no information from a field wire that isn't obtainable from the sense or pickup wires, so processing the information from it is not practical.

The two wires adjacent to the sense wires are called pickup wires. A gauge is chosen so that their potential is zero. The signals on the pickup wires are used to calculate the angle at which the electrons cascade into the sense wire. As the ions travel back away from the sense wire they will induce a signal on the pickup wires as well as the sense wire. The relative magnitude of the signals on the two pickup wires is directly related to the angle (α) between the y-axis and electron cloud path as it avalanches into the sense wire (see Figure 1.6). Once the angle is known the location of the initial ionization can be calculated by tracing a path back along the electric field line a distance $v_d T$, where v_d is the drift velocity of the electrons in the gas. This is a much more complicated process than can be explained here; it will have to be sufficient to say that the position of the initial ionization can be found from the time (T), the angle (α), and the cell number (n):

$$(n, \alpha, T) \rightarrow (x, y) \quad (1.4)$$

In this case the spatial precision in recreating the path of the particle is much better than the cell size. The spatial precision depends on a number of things, all

Figure 1.6: The angle α an electron cloud would make with the y -axis as it avalanches into the sense wire

relating to how accurately the paths of the electrons and ions can be recreated in the cell. They both travel along the electric field lines, so knowing the electric field precisely, at all points, is essential to accurate tracking of the particle through the chamber.

CHAPTER 2. CALCULATION OF MONOPOLE CHARGES ON WIRES

There are a number of ways to calculate the electric field in a chamber such as this. An analytical solution to the problem would be very difficult to find, because of the complexity of the system. Even simple boundary conditions can result in extremely complicated solutions [3]. This problem has relatively complicated boundary conditions, so a numerical solution was pursued.

One way to solve this numerically is to integrate the contribution to the electric field over all charges in the system. This presents a bit of a problem because the parameters of the system are the potentials of the constituent wires and cathodes, not the charges. This means the charges on the wires and cathodes would have to be calculated.

The calculation of the charges by numerical methods is made easier if the cathodes are represented as a series of lines of charge rather than a plane of charge. These lines of charge are called pseudo-wires because they are not physical wires, but, for calculational purposes, are identical to physical wires.

The experienced electrostatics problem solver will recognize this as a step in the wrong direction; after all, the simplest element in the chamber is the infinite plane of charge. This is true, for the most part, but consider the complications. If the plane

were left as it is, then special consideration would have to be made for it at every stage in programming because there would be two types of cell elements, planes and wires. If the cathodes are represented by a series of pseudo-wires, then only one type of element need be considered, that being the infinitely long wire.

As in all approximations there will be some error introduced in doing this. In the region near the cathodes the electric field due to the pseudo-wires in no way resembles that of an infinite plane of charge, as it should if the approximation is to be accurate. However, as the distance from the plane increases, the series of wires looks more and more like an infinite plane. The region where this is a good approximation varies with the spacing between the pseudo-wires and the distance to the plane they lie in. The region of interest is the space around the sense, pickup, and field wires, because the rest of the chamber has an essentially uniform field. The wires are far from the cathodes so the error can easily be made as small as necessary by making the distance between the pseudo-wires small relative to the distance to the cathodes.

There is no longer a distinction between the physical wires and the lines of charge which represent the cathodes, so the process of calculating the charge on each can begin. This is, however, not an easy task because of the large number of wires in the system, now at 345. If the capacitance per unit length ($C_{i,j}$) of each pair of wires were known then the charge per unit length (λ_i) on each wire could be calculated by:

$$\lambda_i = \sum_{j=1}^n C_{i,j} V_j \quad (i = 1, 2, \dots, n) \quad (2.1)$$

where V_j is the voltage on wire j . In matrix form this equation simplifies to:

$$\lambda = (C) V \quad (2.2)$$

where λ and V are the charge per unit length and voltage vectors and (C) is the

capacitance per unit length matrix. The voltages V_j are parameters of the chamber, determined by settings of power supplies, so they are known. However, the capacitance per unit length between two wires is, unfortunately, a function of all elements in the system. This makes them very difficult to evaluate directly; but the elements of the inverse of the capacitance matrix are relatively easy to find. Let the matrix (P) be defined as the inverse of the capacitance matrix:

$$(P) = (C)^{-1} \quad (2.3)$$

It so happens [4] that $P_{i,j}$ can be evaluated from information on the i and j wires alone:

$$P_{i,j} = \begin{cases} 2 \ln \frac{1}{r_i} & i = j \\ 2 \ln \frac{1}{d_{i,j}} & i \neq j \end{cases} \quad (2.4)$$

where $d_{i,j}$ is the distance between the wires i and j and r_i is the radius of wire i . Once the elements of (P) are known the charges can be found by taking the inverse of the (P) matrix to find the capacitance matrix and multiplying by the voltage vector,

$$\lambda = (P)^{-1} V \quad (2.5)$$

In summation form this is:

$$\lambda_i = \sum_{j=1}^n (P)_{i,j}^{-1} V_j \quad (i = 1, 2, \dots, n) \quad (2.6)$$

The electric field in the chamber can be calculated by assuming that the wires and pseudo-wires are infinite lines of charge. By Gauss' law, the electric field at a point $\vec{r}$ due to an infinite line of charge (density λ , position $\vec{r}'$) is:

$$\vec{E}(\vec{r}) = \frac{2\lambda(\vec{r} - \vec{r}')}{4\pi\epsilon_0 |\vec{r} - \vec{r}'|^2} \quad (2.7)$$

This can be generalized to include all wires in the system:

$$\vec{E}(\vec{r}) = \sum_{i=1}^n \frac{2\lambda_i(\vec{r} - \vec{r}_i)}{4\pi\epsilon_0 |\vec{r} - \vec{r}_i|^2} \quad (2.8)$$

Equation 2.8 is a good approximation to the electric field in a wire chamber, but it is only an approximation because there is still something that hasn't been taken into account. What has been neglected is the fact that the charge on a wire is not at its center but is distributed over its surface. This would still be exact if the charge distribution on the surface were uniform, but that is not true either. A wire is influenced by the electric fields from other charges in the chamber resulting in the separation of charge on its surface. Charges will move where necessary on the surface of a conductor in such a way that all points on the surface of a conductor are held at the same potential. After this has happened, the wires will have multipole moments which will affect the electric field, and, if corrections are not made for them, introduce errors in the recreation of the particle paths.

The boundary condition on the electric field at the surface of a conductor is that the electric field must be perpendicular to it, in this case a wire in the chamber. This is a powerful tool which can be used to find and evaluate the seriousness of trouble spots in an electric field diagram. By examining the electric field generated by a particular method on the surface of a wire it can immediately be seen if there are any obvious problems. Figure 2.1 is a plot of the electric field in a cell using Equation 2.8, not having made corrections for the multipole moments of the wires. There seems to be very little trouble with the sense and field wires, which is to be expected because the electric field around them, due to external sources, is small. The pickup wires, however, appear to have problems which is better seen in Figure 2.2, a blowup of the

Figure 2.1: The electric field in a cell without having made corrections for the multipole moments

pickup wire region. This was also to be expected because they reside in the strong electric field emanating from the charge on the sense wire. Thus, it can be concluded that a multipole moment capable of significantly altering the electric field will be induced on the pickup wires, but not the sense or field wires.

There are a number of ways to solve for the multipole moment and two of them are discussed in the following two chapters.

Figure 2.2: A blowup of the electric field in the region of the right pickup wire; no corrections made for the multipole moments

CHAPTER 3. THE CORRECTIONS TO THE ELECTRIC FIELD USING THE METHOD OF IMAGES

An easy and fairly accurate way to correct for the multipole moments on the pickup wires is to use the method of images. If a conducting cylinder is placed near a line of charge (λ), then the multipole induced on the cylinder can be accounted for by placing a line of charge inside the cylinder (see Figure 3.1). The line of charge will have position and charge density per unit length chosen so that the wire surface is an equipotential.

The electric field in the region will be the sum of the contributions to the electric field of the source and the image. Once this is done there is no distinction between the image and other wires in the cell, so Equation 2.8 can be used to calculate the electric field. The image charges, like the charges that represent the cathodes, are not physical wires so they, too, are called pseudo-wires.

It can be shown that the charge on the pseudo-wire must have the same magnitude, but the opposite sign as the source,

$$\lambda_{image} = -\lambda_{source} \quad (3.1)$$

and its position must be along a line between the source and the wire a distance

$$\rho = \frac{a^2}{R} \quad (3.2)$$

Figure 3.1: The position of an image which makes the surface of the conductor an equipotential

from the center of the wire, where a is the radius of the wire and R is the distance from the center of the wire to the source.

Other problems are introduced when this is applied to the problem of the pickup wire. The potential at the pickup wire surface will be an equipotential, but will not be zero. This happens because the monopole charge of the wire was changed. To assure charge conservation, and to set the potential on the cylinder surface back to zero, a charge equal in magnitude but opposite in sign to the image must be added somewhere inside the pickup wire. The charge is added at the center of the wire because placing it anywhere else would change the fact that the surface is an equipotential.

The sense wires are very close to the pickup wires and have a large charge density, thus they will induce the strongest multipole moments on the pickup wires.

Figure 3.2: The components of the electric field on a pickup wire

This means that the best place to apply the method of images is on the sense wires. When this is done, the error in the approximation of the electric field at the pickup wire surface drops to 4% of its original value. The error is evaluated by examining plots of the component of the electric field parallel to the surface of the wire ($E_{\parallel}$, see Figure 3.2) at a point described by the angle θ , in radians. Figure 3.3a is a plot of $E_{\parallel}$ without having made any corrections, and Figure 3.3b is a plot made after applying the method of images.

The graph in Figure 3.3b is not zero everywhere because there are other charges in the system which influence the pickup wires. The field wires, in particular, will significantly contribute to the pickup wire multipole moments. This is because they have charges comparable in magnitude to the charge on the sense wires and are only

Figure 3.3: $E_{\parallel}$ after various applications of the Method of Images

twice as far away from the pickup wires as the sense wires. This implies the method of images should also be applied to the field wires, but there is no reason to stop at that. The technique applies to all wires and pseudo-wires in the chamber so it was done. A pseudo-wire was added to each pickup wire for every other wire in the system. The effect of every charge on each pickup wire should then be considered and the $E_{||}$ graph should go to zero.

After this was done the maximum value of $E_{||}$ dropped to less than .5% of its original value (see Figure 3.3c). Though small, this was a disappointing result; it being nonzero implies there are other, significant, charges to be considered.

To find the images in the pickup wires, it was again assumed that all wires, except the pickup wire of interest, are lines of charge; but that is not true. The sense wires, field wires, and especially the opposite pickup wires have multipole moments on them which will also effect the moments on the pickup wire. These multipole moments can influence other wires just as much as monopoles can; so a truly accurate evaluation of the electric field would require the addition of images in all wires.

Unfortunately, it doesn't stop at that. These images will affect the multipole moments on other wires; in other words, the images have images. Thus, for each of these images it is necessary to place another image in each of the other wires, but each of these second order images will have images in the other wires as well. There is no end to how many images could be added.

As can be seen, getting an exact solution to the problem using this method is quite impossible because it would require an infinite number of images. Just the first set of images, described above, would require the addition of $344 * 345$ additional pseudo-wires. If we assume the cathodes won't have multipoles then the number

drops to $20 * 345$ but this is still beyond what is practical, and it would still not give a good result because higher order corrections need to be made.

The number of pseudo-wires to be added is brought down to a reasonable number if the multipole effects of all wires except those of the pickup wires are ignored. This should be a good assumption because no wire in the system has nearly as large a multipole moment as the pickup wires. In this case, each image will only have one image of the next higher order, that image being in the pickup wire opposite the original. The total number of images after working out n corrections will be $2 * 344 * n$, which is a number small enough to allow calculations, at least for reasonably small values of n .

It was concluded, after some experimentation, that considering the effect of the other charges on the pickup wires to third order was sufficient. That is, for every wire in the chamber an image was placed in each of the pickup wires and, for each of those images another image was placed in the opposite pickup wire, and again, for each of those images another image was placed in the original pickup wire. This brought the value of the maximum of $E_{||}$ down to 10^{-4} its original value (see Figure 3.3d). The remaining residue of $E_{||}$ is due to weak multipoles in the sense and field wires and higher order effects from the pickup wire multipoles. The process started here could be continued to make the error smaller, if desired, but this error is small enough so that it isn't necessary.

With this correction made, the electric field in the cell and the right pickup wire region looks like Figures 3.4 and 3.5. This can be compared to the electric field diagrams for other techniques described later.

Figure 3.4: The electric field in a cell using the method of images

Figure 3.5: The electric field in the region of the right pickup wire using the method of images

CHAPTER 4. CORRECTIONS TO THE ELECTRIC FIELD FOR THE MULTIPOLE MOMENTS BY CALCULATING THE CHARGE DISTRIBUTION ON THE PICKUP WIRES

The method of images gives a solution to the problem of the multipoles on the pickup wires with a tolerable error, but there is another way to get the solution which has some advantages over it. The technique described here solves for the charge distribution on the circumference of the pickup wires. If the charge distribution on the pickup wires is known, then an accurate evaluation of electric field is possible. But it also allows us to properly calculate the electric field line density, so the electric field plotting programs can calculate where the field lines should begin.

The main reason for calculating the charge distribution is, of course, to account for the multipoles. If the charge distribution on the wire is known, the multipole corrections to the electric field will be taken into account when the contribution to the electric field due to the charge on the wire is integrated over the wire surface.

As before, numerical evaluation is made easier if the charge distribution is expressed as a series of pseudo-wires. These pseudo-wires are equally spaced along the perimeter of the real pickup wire (see Figure 4.1). For this particular problem, 1000 pseudo-wires were used to represent each pickup wire (though Figure 4.1 shows much fewer than that). As usual, there is no distinction between the pseudo-wires and the

Figure 4.1: The positions of the pseudo-wires which will represent the pickup wire physical wires in the chamber, at least as far as Equation 2.8 is concerned.

It will be necessary to decide what is to happen to the charge on the old pickup wire. It would be possible to eliminate the old pickup wire element from the array and place its charge on the pseudo-wires which now represent it. This means the pseudo-wires would entirely define the presence of the pickup wire in the chamber, that is, they would express all of the moments of the wire, including the monopole. Instead of doing this, the charge on the original pickup wire was left as it was so the pseudo-wires would only represent the higher order multipoles.

This option was chosen because it makes programming easier. The sum of the charges on the 1000 pseudo-wires should be zero. This makes errors easier to find; if the charges on the pseudo-wires don't add up to zero, then something has gone wrong.

The task of finding the charge on each wire employs the boundary condition discussed earlier: the electric field must be perpendicular to the surface of a conductor. If, for some charge distribution, the electric field is perpendicular to the surface of the wire at all points on its perimeter and the monopole charge has not been changed, then uniqueness guarantees that the correct solution has been found [3].

The charge on each pseudo-wire is calculated using an iterative technique, that is, successive corrections to the charge distribution are made until the correct one is found to within a reasonable error. An iteration, or correction, is the process by which an amount of charge, q_{move} , is moved from wires to their neighbors in such a way that $E_{||}$ is made smaller. Iterations are made until $E_{||}$ is essentially zero at all points on the surface. An iteration consists of a couple of steps.

The first step is the evaluation of $E_{||}$. $E_{||}$ is calculated 1000 times, once at each pseudo-wire position (see Figure 4.2) generating an array of $E_{|| i}$ values. $E_{||}$ is evaluated by adding up the components to the electric field using Equation 2.8, and multiplying by the cosine of the angle (α) the electric field vector makes with the tangent to the wire surface.

The second step is to move charge from pseudo-wire to pseudo-wire. The amount of charge moved from pseudo-wire _{$i-1$} to pseudo-wire _{$i+1$} is

$$q_{move i} = k * E_{|| i} \quad (4.1)$$

where k is a constant to be determined later. Note that the value of $E_{||}$ is fundamental to the operation of the algorithm, unlike its use for the method of images, which used $E_{||}$ calculations only to assess the accuracy of its approximations.

Figure 4.3 is a number of graphs of $E_{||}$ as a function of the angle θ , as defined in Figure 3.2, after several iterations. Note that the graphs are not scaled the same

Figure 4.2: The components of the electric field near a pseudo-wire.

but to the maximum value the function attains. The shape of $E_{\parallel}$ changes very little, but the amplitude drops rapidly as iterations proceed, reaching a level of 2.5×10^{-6} of its initial value, at which point the error due to numerical roundoff becomes a limiting factor. Figure 4.4 shows the charge distribution around the wire after the same number of iterations. The charge distribution changes very little after a few iterations because the corrections get smaller as more iterations are performed. It is interesting to note that it is not perfectly sinusoidal as it would be if the field were uniform, but has more charge near $\theta = \pi$, the point closest to the sense wire.

The constant k determines the convergence rate of the algorithm, larger values corresponding to faster rates of convergence. However, using values of k that are too large results in the failure of the algorithm to converge to a solution. Figure 4.5 is a number of plots of $E_{\parallel}$ as a function of the angle θ where the value of k was

Figure 4.3: $E_{\parallel}$ after various iterations, solution converges

Figure 4.4: Charge distribution after various iterations, solution converges

too large. Note that the graph crosses the axis more than twice, and its amplitude increases. The corresponding charge distribution (Figure 4.6) has the same problem: it too crosses the axis four times and increases in amplitude. If an overcompensation for $E_{||}$ is made, the $E_{||}$ graph will cross the axis four times in the following iterations. As the program tries to correct this error it repeatedly overapproximates, resulting in the $E_{||}$ and charge distribution graphs getting larger in amplitude and changing sign with every iteration.

The convergence could be made arbitrarily slow by choosing small values of k , but a solution which uses the least resources, i.e., CPU time, is usually desired, so an optimum value of k needs to be found. Let k_0 be defined as the value of k for which the algorithm will give the exact (dipole) solution in one iteration if the external field were uniform. As was noted, overapproximations to the amount of charge to move introduce serious problems, and for this reason values of k somewhat less than k_0 are used.

This results in much slower convergence, but problems due to overapproximations are not apt to occur. Through trial and error, it was determined that a good, conservative value for k is $.4k_0$. With this value of k the algorithm typically converges in 35 to 45 iterations, at which point the error due to numerical roundoff becomes significant, disallowing a better approximation to be made.

The value of k_0 can be calculated by comparing the charge distribution generated by the algorithm with the known solution to the problem of a wire in a previously uniform electric field $\vec{E}_0$ (see Figure 4.7). It can be shown that the analytical solution to this problem is:

$$\sigma(\theta) = 2\epsilon_0 E_0 \cos \theta \quad (4.2)$$

Figure 4.5: $E_{\parallel}$ after various iterations, solution diverges

Figure 4.6: Charge distribution after various iterations, solution diverges

Figure 4.7: Conducting cylinder in a uniform electric field; the equipotentials are also shown

where σ is the charge density per unit area on the wire.

The charge distribution per unit length on a pseudo-wire at position θ will be the total charge that would reside on the arc of length $r d\theta$, where r is the wire radius. For $d\theta$ sufficiently small this is approximately:

$$\begin{aligned}\lambda_i &\simeq \sigma(\theta_i) r d\theta \\ &\simeq 2\epsilon_0 E_0 \cos \theta_i r \Delta\theta\end{aligned}\tag{4.3}$$

The solution to the problem that the algorithm will generate can be found as

well by making a few assumptions. The function $E_{||}(\theta)$ will be perfectly sinusoidal,

$$E_{||}(\theta_i) = E_0 \sin \theta_i \quad (4.4)$$

The amount of charge moved is proportional to $E_{||}$ (see Equation 4.1), so the charge on pseudo-wire_{*i*}, after one iteration, can be calculated. It will be the difference between the charge moved onto it and the charge moved off of it to another wire. Application of Equation 4.1 and several substitutions yields:

$$\begin{aligned} \lambda'_i &= k_0(E_{||\ i-1} - E_{||\ i+1}) \\ &= k_0 E_0 (\sin \theta_{i-1} - \sin \theta_{i+1}) \\ &= k_0 E_0 (\cos \theta_i \ 2\Delta\theta) \end{aligned} \quad (4.5)$$

Equating Equations 4.3 and 4.5 and solving for k_0 yields:

$$k_0 = \epsilon_0 r \quad (4.6)$$

One problem with using pseudo-wires in this way to simulate a charge distribution is that the electric field contribution from charge on that wire (Equation 2.5) goes to infinity at the pseudo-wire position. This is not what happens near a uniform charge distribution; thus, the approximation is not a good one near the pseudo-wires. This is a particularly serious problem because the algorithm solving for the charge distribution calculates the electric field at the pseudo-wire positions to evaluate $E_{||}$.

The error can be reduced by using a more accurate expression for the contribution to the electric field near the pseudo-wires. The wire has a non-uniform, continuous distribution of charge on a cylindrical surface, but as the scale gets small the charge distribution looks more uniform and the surface of the cylinder looks flat. Thus, if the

number of pseudo-wires is sufficiently large then each pseudo-wire can be assumed to be a narrow, infinitely long, plane of uniform charge density. Thus the electric field near a pseudo-wire can be approximated by integrating over the plane of charge, so that the electric field near a pseudo-wire is:

$$\vec{E}(x', y') \simeq \frac{\lambda}{L} \left[\frac{1}{2} \ln \left| \frac{(x' + L)^2 + y'^2}{(x' - L)^2 + y'^2} \right| \hat{i} + \left(\arctan \frac{x' + L}{y'} - \arctan \frac{x' - L}{y'} \right) \hat{j} \right] \quad (4.7)$$

where variables are defined in Figure 4.8. This expression should be used instead of Equation 2.7 in the summation Equation 2.8; in general, a coordinate transformation will be necessary to get this into the appropriate form.

Even with this correction it is necessary to place the pseudo-wires slightly below the surface of the pickup wires. This is necessary because the $E_{||}$ values are very sensitive to the amount charge on the individual wires near them. When $E_{||}$ is calculated, there are small errors introduced from numerical roundoff and other sources which create small discontinuities in the charge distribution. Unless the individual effect of a pseudo-wire is extremely small, the discontinuities will grow larger in successive iterations. This occurs because of the positive feedback from $E_{||}$, similar to the feedback which disallowed convergence when k was too large, discussed above. In this case, it happens on the scale of a few pseudo-wires, so the resulting $E_{||}$ and charge distribution graphs get very erratic, in as few as three iterations. Figure 4.9 is a plot of $E_{||}$ and the charge distribution made after eight iterations.

One might think that increasing the number of pseudo-wires would eliminate the problem, but it actually makes it worse. This behavior depends entirely on what happens at the level of the individual pseudo-wires. At that level there is no difference made by adding more wires, except that more calculations are made which increases the error due to numerical roundoff. Thus, the pseudo-wires were placed

Figure 4.8: The contribution to the electric field of one pseudo-wire

Figure 4.9: The charge distribution and $E_{\parallel}$ graphs when the pseudo-wires are placed too close to the surface

1-2 μm below the surface of the pickup wire. Since there are no pseudo-wires very close to the point where the field is being evaluated, the problems discussed above are essentially eliminated.

As in all numerical approximations this will introduce some error. In this case the error is insignificant because the distance they are placed below the wire is small relative to the radius of the pickup wire (75 μm). To verify this, the electric fields were calculated with the pseudo-wires buried different distances below the surface of the pickup wire. The distances were as large as 10 μm below the surface of the wire and as small as 1 μm . There were small, almost negligible differences in the electric fields at the 10 μm depth, and they dropped to zero quadratically as the distance was made small. Thus, it is safe to say that this will not introduce significant errors.

When these corrections to the electric field are made, the electric field looks much more physical near the pickup wires. Figure 4.10 is a diagram of the electric field in the same cell observed earlier (Figure 2.2) with the corrections made for the multipole moments; and Figure 4.11 is a blow up of the right pickup wire region.

Figure 4.10: The electric field in a cell using charge distribution method

Figure 4.11: The electric field in the region of the right pickup wire using the charge distribution method

CHAPTER 5. EVALUATION OF THE ELECTRIC FIELD USING ELECTROSTATIC POTENTIALS

The two techniques discussed thus far give very accurate solutions to the electric field, but they are both built on the matrix inversion technique which was described in Chapter Two. Another solution to the problem not based on this matrix inversion would be nice to have because it would be an independent check on the first two methods. There is another way to solve the problem which avoids the calculation of any charges.

The way to do this is to use electrostatic potentials. If the potential (ϕ) is known at all points, then the electric field can be found precisely using:

$$\vec{E} = -\vec{\nabla}(\phi) \quad (5.1)$$

An analytic solution to the potential distribution would have the advantage of yielding an analytic electric field, but the complexity of the system made it too difficult and it was abandoned. Even in simple geometries, the analytic solution is rather complex as was noted earlier. A recent thesis [3] on this topic describes how to find an analytical solution to the potential distribution in a coaxial counter with square cross sections. As difficult as this problem is, it is far simpler than the one dealt with here.

Having settled on finding a numerical solution, the problem of calculating the

potential was approached using a mesh technique. If the electric field is to be known at all points, then the potential must also be known at all points. Representing a potential field in a numerical form may be done with by using an array of values, each being the presumed potential at a specified lattice point.

The lattice is a two-dimensional array of real variables which contains the numerical values of the potential at selected (x, y) positions in the chamber. In general, the relationship between the lattice array point (i, j) and the real coordinate point (x, y) is linear:

$$(s * i + x_0, s * j + y_0) = (x, y), \quad (5.2)$$

where s is the distance between the lattice points in real space. This transformation is simplified by making the array origin $(i = 0, j = 0)$ the same point as the chamber coordinate system origin. In that case $(x_0, y_0) = (0, 0)$.

The potential at an arbitrary point in space can be found from the potentials at the lattice points by doing a four-point interpolation on the nearest lattice points (see Figure 5.1):

$$\begin{aligned} \phi(x, y) = & [\phi(i, j)(s - dx)(s - dy) + \phi(i + 1, j)(dx)(s - dy) \\ & + \phi(i, j + 1)(s - dx)(dy) + \phi(i + 1, j + 1)(dx)(dy)]/s^2. \end{aligned} \quad (5.3)$$

The electric field can be found, once the potentials are known, by evaluating the gradient of the potential distribution (Equation 5.1). The potential field is not an analytic function, so a form of the gradient must be used which takes advantage of the geometries we have defined. The approximations,

$$\left(\frac{d\phi}{dx}\right)_{i,j} \simeq \left(\frac{\phi(i + 1, j) - \phi(i - 1, j)}{2s}\right) \quad (5.4)$$

Figure 5.1: Four-point interpolation

$$\left(\frac{d\phi}{dy}\right)_{i,j} \simeq \left(\frac{\phi(i, j+1) - \phi(i, j-1)}{2s}\right) \quad (5.5)$$

can be used to find the gradient. The electric field at a lattice point will be:

$$\begin{aligned} \vec{E}(i, j) \simeq & - \left[\frac{\phi(i+1, j) - \phi(i-1, j)}{2s} \right] \hat{i} \\ & - \left[\frac{\phi(i, j+1) - \phi(i, j-1)}{2s} \right] \hat{j} \end{aligned} \quad (5.6)$$

To find the electric field at an arbitrary point in space, four-point interpolation can be used, but in this case it must be used to calculate both the x and y components of the electric field,

$$\begin{aligned} \vec{E}(x, y) = \frac{1}{s^2} [& \vec{E}(i, j)(s-dx)(s-dy) + \vec{E}(i+1, j)(dx)(s-dy) \\ & + \vec{E}(i, j+1)(s-dx)(dy) + \vec{E}(i+1, j+1)(dx)(dy)] \end{aligned}$$

$$\begin{aligned}
= & \frac{1}{2s^3} [\{(\phi(i+1, j) - \phi(i-1, j))\hat{i} + (\phi(i, j+1) - \phi(i, j-1))\hat{j}\}(s-dx)(s-dy) \\
& + \{(\phi(i+2, j) - \phi(i, j))\hat{i} + (\phi(i, j+1) - \phi(i, j-1))\hat{j}\}dx(s-dy) \\
& + \{(\phi(i+1, j+1) - \phi(i-1, j+1))\hat{i} + (\phi(i, j+2) - \phi(i, j))\hat{j}\}(s-dx)dy \\
& + \{(\phi(i+2, j+1) - \phi(i, j+1))\hat{i} + (\phi(i+1, j+2) - \phi(i+1, j))\hat{j}\}dx dy] \quad (5.7)
\end{aligned}$$

Laplace's equation must be satisfied for the region in the chamber where there is no charge, that is, everywhere except on the surfaces of the wires and the cathode plates:

$$\nabla^2 \phi = 0 \quad (5.8)$$

If Laplace's equation can be satisfied for some set of lattice potentials and boundary conditions, then uniqueness guarantees that the solution has been found [3].

Electric field lines emanate from locations of high potential and end at locations of low potential, just like a fluid would travel from a location of high pressure to one of low pressure, or heat from a location of high temperature to one of low temperature. The pressure in a fluid flow problem or the temperature in a heat flow problem is analogous to the potential in an electrostatic problem.

This should be no surprise, because the diffusion equation which governs fluid and heat flow,

$$k \nabla^2 u = \frac{du}{dt} \quad (5.9)$$

where u is the pressure or temperature and k is the diffusivity constant, is identical to Equation 5.8 for the equilibrium condition, $\frac{du}{dt} = 0$. This permits the use of the diffusion equation to solve for electrostatic potentials, the correct solution having been found when the equilibrium condition is met.

This may not appear to be particularly helpful because there is no time axis in this problem. But, the analogy made earlier can be extended to introduce a time parameter. A fluid would diffuse through a chamber, as time passes, if pressure is applied, that is, if new boundary conditions are imposed. In the same fashion, the potential can be allowed to diffuse through the system until the equilibrium condition is met.

The diffusion process can be simulated by successively looping over all values of i and j while averaging the potential at the lattice points. That is, the new potential at a lattice point $(i, j; n + 1)$ will be the replaced by the average of the potentials of its four nearest neighbors:

$$\phi(i, j; n + 1) = \frac{\phi(i + 1, j; n) + \phi(i - 1, j; n) + \phi(i, j + 1; n) + \phi(i, j - 1; n)}{4} \quad (5.10)$$

The numerical Laplacian of a potential field of this sort is given by [5]:

$$\nabla^2 \phi(i, j) = \frac{\phi(i + 1, j) + \phi(i - 1, j) + \phi(i, j + 1) + \phi(i, j - 1) - 4\phi(i, j)}{s^2} \quad (5.11)$$

which reduces to Equation 5.10 in the equilibrium condition: $\nabla^2 \phi = 0$. Special consideration must be given to points which lie on the boundaries of the system. Boundary points are lattice points which lie on or inside a wire, as well as some other points which will be discussed later. The potentials on the wires are known; thus, the potentials at these lattice points should be fixed.

Consider an example of how the looping changes the values of the potentials: assume all potentials are initially zero, that is, $\phi(i, j; 0) = 0$ for all (i, j) , except the boundary points. After one iteration the potentials next to the boundaries will be between one-quarter and three-quarters of the boundary potential, depending on how many of its neighbors are boundary points. On the next iteration the points next to

these will be influenced, and on later iterations those farther away will be affected. The process continues until the equilibrium condition is met.

The set of potentials $\phi(i, j; n)$ will then be the potential distribution after some time has elapsed; more specifically, n will be proportional to time. In terms of these parameters the derivative can be taken:

$$\frac{d\phi}{dt} = C \frac{\phi(n+1) - \phi(n)}{dn} \quad (5.12)$$

where C is a proportionality constant relating n and t . As time progresses, the system will eventually reach equilibrium, that is, $\phi(n+1) = \phi(n)$. This implies $\frac{d\phi}{dt} = 0$, so Equation 5.8 will be satisfied and the correct potential distribution has been found.

The value of C depends on a number of parameters, including the speed at which the potential diffuses in the chamber and the spacing between the lattice points. This makes it extremely difficult to evaluate. However, it doesn't matter what C is because it affects the rate of convergence, not the convergence itself.

There may be a strong temptation to solve explicitly for C and use this technique for time varying potentials, but that should be avoided. This is a crude way of simulating a fluid flow which will not be accurate in a time dependent way. In one iteration the algorithm will make a correction to its previous guess by, more or less, wandering in the right direction. This may sound unscientific, but the purpose of this algorithm is to find the equilibrium condition only, which it will eventually converge to and is verified when $\frac{d\phi}{dt} = 0$. Thus, the set of potentials $\phi(n)$ will not necessarily represent the potential distribution at some particular time and should not be looked at with hopes of finding something useful until the equilibrium condition has been found.

As was noted, the values of the lattice points which lie in, or on, the wire surface

are the boundaries and are to remain unchanged, but there are other boundaries as well. Other boundaries which require attention are the edges of the chamber. To accommodate these boundaries the arrays are made one element larger on each side of the chamber.

The top and bottom sets of boundary points will be determined by the cathode potentials and should be fixed, but the sides are different. They are not technically boundaries because they have values that will change when the program is run; but still, special consideration must be made for them, so they are called boundaries. They are conditioned by the potential on the opposite side of the chamber, which makes it possible to accurately account for the influence of other cells in the system when only one cell is covered by lattice points. This will not adversely affect anything else in the system because of the symmetry about the y-axis in the chamber, and so the potential on opposite sides of the cell should be the same.

One problem with this technique is that the lattice spacing needs to be small relative to the physical size of the objects in the chamber, or the results will not be accurate. The smallest object in the chamber is the sense wire with a radius of $10\mu m$. On the basis of this it would follow that the lattice spacing (s) should be of the order $1\mu m$. The cell width is $2000\mu m$ so there would be 2000^2 or 4×10^6 lattice points. This is an impractically large array, assuming the use of single precision real variables, storage of this array would require 16 megabytes of memory and would require years of CPU time to establish equilibrium.

The way to get around this problem is windowing. Instead of using one two-dimensional array to store the lattice points in the cell, a window with small spacing can be used where there are objects with fine features, such as around the wires, and

one with larger spacing where there are no fine features, such as in the rest of the cell. To keep track of the lattice points in multiple windows a three-dimensional array is used, two indices for the (x, y) or (i, j) locations and one for the window number.

For this particular application windows with 81^2 lattice points were used. This makes the lattice spacing in the cell, that is, in the primary window, equal to $25\mu m$. The spacings in the secondary windows were chosen to be $1.25\mu m$ around the sense wire and $2.5\mu m$ around the pickup and field wires.

Unfortunately, this creates some more problems, the most serious of which is how to get the windows to communicate with each other. If the windows aren't communicating properly then there will be discontinuities in the potential distribution at window boundaries. This is a particularly difficult problem to solve because the windows have different spacings.

This problem is simplified if the parameters of the system are carefully chosen so that the ratio between the spacings of the secondary and primary windows is an integer. If the origins of the windows are chosen wisely, then the primary and secondary windows have some of lattice points in common.

The boundaries of the secondary windows and the corresponding holes in the primary window are difficult to keep track of. The easiest way to do this is to update the boundaries between iterations, that is, loop over the edges of the windows while updating the values of the potentials as need be. Unfortunately, this resulted in there being discontinuities in the potential distribution at the boundary. So a different approach was taken; the values on the edges of the windows were updated when the algorithm would normally have updated that lattice point. That is, when the algorithm loops to a value in a secondary window which lies on its edge (see Figure

5.2), the value at that point is set at:

$$\phi_{sec}(i, j) = \frac{\phi_{sec}(i, j+1) + \phi_{sec}(i, j-1) + \phi_{sec}(i-1, j) + \phi_{pri}(x', y')}{4} \quad (5.13)$$

where $\phi_{pri}(x', y')$ is evaluated using the four-point interpolation Equation (5.3) on the primary window.

The value of the edge of the corresponding hole in the primary window is also updated when the algorithm would normally have evaluated the potential at that lattice point. It is set at the value:

$$\phi_{pri}(p', q') = \phi_{sec}(p, q) \quad (5.14)$$

The electric field generated by this technique agrees with the previous techniques to within 5% in the magnitude of the electric field and 2 degrees of its direction. Figure 5.3 is a plot of the electric field in the wire region which can be compared with Figure 4.10, a plot of the same cell having used the charge distribution method to calculate the electric field. There are noticeable differences which are disappointing, since an exact match was desired.

The source of the error is possibly due to too large a lattice spacing. A run with the lattice spacing twice as large was performed; if the error on this set of lattice potentials was sufficiently larger than what it was for the previous case, then it could be concluded that the error varies with some power of the lattice spacing and it could be made smaller by making the spacing small.

Unfortunately, the error for the second case was nearly identical to that of the first. This implies there is either something basically wrong with the technique or some fine point on its implementation. If there were something wrong with the implementation, it would probably show up in the form of a nonsense solution. The

Figure 5.2: Lattice points on and near a boundary between two windows

Figure 5.3: The electric field in a cell using the potential method

solution being as close as it is to the previous two results implies something correct is happening, so there must be a small flaw in the technique.

CHAPTER 6. A BRIEF DESIGN STUDY: THE PROTOTYPE FIELD

Optimization of the chamber is the process of finding the configuration which results in the smallest uncertainty in particle path reconstruction. There are several parameters which affect the resolution. Some of them are wire positions, wire diameters, cathode potential, sense and field wire potentials, and the type and pressure of the gas used.

As was noted earlier, the potential on the field wire has a significant effect on the electric field. This is shown in Figures 6.1 and 6.2, a number of electric field diagrams of cells and the right pickup wire region for field wire potentials ranging from -250V to -750V. As can be seen, the electric field can be manipulated into a wide variety of forms by changing the potential on the field wires. Even more possibilities exist when different potentials on the cathodes and sense wires are also allowed to vary, so there is a lot of room to work with.

An analysis which does justice to all these variables is far beyond the scope of this discussion, but a lot can be determined without lengthy calculations. Many of the parameters listed above, particularly the potentials and wire positions, affect two important descriptors of the system, α_{max} and x_{max} . x_{max} is defined as the maximum value of x an electric field line emanating from the sense wire can attain.

Figure 6.1: The electric field in cells and the right pickup wire region for various field wire potentials

Figure 6.2: More electric fields in cells and the right pickup wire region for various field wire potentials

It is often limited by the pickup wires, which get in the way of the electric field lines going from the sense wires to the cathode plates. Figure 6.3 is an electric field diagram of a cell where x_{max} is $907\mu m$, which is less than $1000\mu m$, the half-width of the cell.

This is an undesirable characteristic of a radial drift chamber; the electrons freed in events which take place in the region outside of x_{max} will not cascade into the sense wire and be detected. This means a portion of the chamber is not being utilized, which is inefficient. For this reason, configurations resulting in $x_{max} < 1000\mu m$ can be immediately eliminated from prospective candidates for the optimum chamber.

α_{max} is defined as the maximum angle α the electron cloud can make with the y-axis as it avalanches into the sense wire. Note that the path, or electric field line, that determines α_{max} also determines x_{max} , so α_{max} is limited by the presence of the pickup wire in the same way x_{max} is.

The range of possible values of α are $-\alpha_{max}$ to $+\alpha_{max}$. The angle α corresponds to an electric field line or path in the cell, so a larger range of values of α imply the uncertainty in α (σ_α) would correspond to a smaller uncertainty in the x position of an electron cluster. Thus, α_{max} is optimally large.

Figures 6.4 and 6.5 are graphs of x_{max} and α_{max} as a function of the field wire potential for some different cathode, or drift, potentials. For all of these cases the sense wire potential was held at $1050V$ and the pickup wire position at $(300,0)\mu m$. Based on the discussion above, it would appear that the best configuration represented is when the field wire is at $-750V$ and the drift potential is at $-17000V$. This combination results in the electric field shown in Figure 6.6. To keep the reader from thinking that this is the optimum chamber, it should be noted that this particular

Figure 6.3: The trajectory of an electron cloud resulting in x_{max} less the $1000\mu m$

Figure 6.4: x_{max} values for various field wire and cathode potentials

Figure 6.5: α_{max} values for various field wire and cathode potentials

configuration has some undesirable characteristics which are beyond the scope of this discussion.

The purpose of this discussion is to demonstrate the power of these programs as a design tool. They can be used to solve a wide variety of two-dimensional electrostatics problems without requiring significant modification. This became an important feature when the program was used to evaluate the electric field in the prototype chamber. The prototype was built in 1986, before a design study had been done. It was operated with the sense wire at 1050V, the field wires at -450V, the drift potential at -15000V, and the pickup wires at the positions $(\pm 300, 55)\mu m$. The electric field for the prototype is shown in Figure 6.7.

The chamber is not a particularly good one because α_{max} and x_{max} will be substantially less than optimum; in this case x_{max} is only $947\mu m$ which means data will not be read from 5.3% of the chamber and α_{max} is only 1.16 radians.

Nevertheless, the data acquired in this prototype has been analyzed using this electric field. A second prototype, if built, could use these algorithms, in particular the robust and simple algorithms of Chapter 4, to optimize the electric field configuration as a function of the pickup wire position and the potentials on the sense wires, field wires, and cathodes.

Figure 6.6: The electric field in the chamber which has maximized x_{max} and α_{max} values

Figure 6.7: The electric field in a cell of the prototype chamber

APPENDIX A. GENERALIZATION OF THE POTENTIAL ALGORITHM TO INCLUDE NONZERO VOLUME CHARGE DENSITIES

It is interesting to note that the technique described in Chapter 5 can be used to solve a broader range of problems. This comes from the fact that Equation 5.8 is merely a special case of the more general Poisson equation:

$$\nabla^2 \phi = \frac{\rho}{\epsilon_0} \quad (\text{A.1})$$

This technique can be used to solve problems where there is a nonzero volume charge density ρ . The algorithm will function as described above with the exception that the equilibrium condition will be:

$$\nabla^2 \phi = \frac{d\phi}{dt} = \frac{\rho}{\epsilon_0} \quad (\text{A.2})$$

which means Equation 5.10 will change to:

$$\phi(i, j; n+1) = \frac{1}{4} [\phi(i+1, j; n) + \phi(i-1, j; n) + \phi(i, j+1; n) + \phi(i, j-1; n)] + \frac{\rho(i, j)}{\epsilon_0} \quad (\text{A.3})$$

Another generalization to this technique allows solving electrostatic problems in three dimensions. There are no surprises when this is done; in this case, Equation 5.10 will read:

$$\begin{aligned}
\phi(i, j, k; n+1) &= \frac{1}{4} [\phi(i+1, j, k; n) + \phi(i-1, j, k; n) \\
&+ \phi(i, j+1, k; n) + \phi(i, j-1, k; n) + \phi(i, j, k+1; n) \\
&+ \phi(i, j, k-1; n)] + \frac{\rho(i, j, k)}{\epsilon_0}
\end{aligned} \tag{A.4}$$

APPENDIX B. NOTES ON VARIABLE TYPES AND ACCURACY OF CALCULATIONS

For most of the calculations made here, single precision real variables were used. However, there was one case where considerable errors were encountered because this was not sufficient.

When a large number of pseudo-wires are placed in a small area there can be significant errors in the calculation of the electric field. This is exactly what happens when it is calculated near the pickup wires, particularly when the method of images or the charge distribution method is used to account for the multipoles. It is a particularly serious problem when calculating $E_{||}$ in the latter case because the program uses that value to determine the amount of charge to be moved (see Equation 4.1). This is also intolerable because getting an accurate numerical value of the electric field in this region is exactly what these techniques are trying to do.

In the region of the pickup wires the actual electric field may be small while the contributions from the individual pseudo-wires may be large. For example, the method of images algorithm placed several hundred pseudo-wires in each pickup wire. Several of these pseudo-wires have a large charge on them while the net charge of all the pseudo-wires and the pickup wire is small. This means there are many large charges in the pickup wires which sum to a small charge, and there will likewise

Figure B.1: $E_{\parallel}$ after an application of the method of images with single precision variables

be a small electric field in the region which is the sum of a several large, canceling contributions from the individual wires and pseudo-wires.

The computer only keeps a fixed number of significant digits, so, if the individual contribution of a particular wire is large, then several digits which are important to the calculation will be rounded off. Figure B.1 is a plot of $E_{\parallel}$ which was made using single precision variables in an application of the method of images. It should be compared to Figure 3.3d which is a plot made using double precision variables.

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