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A STATISTICAL ANALYSIS OF AVERAGE LASER DAMAGE IN DIELECTRIC THIN FILMS*

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Abstract

The overall statistical analysis of the theory of extreme events. It is generally accepted that the data arising in the failure process of electronic components are a breakdown of materials.

The distribution of time to failure (pre-to-damage and inter-failure times) for a material was determined thin film coated parallel plate capacitors and diodes with tightly focused laser beams. The data used is furnished by Miles. The fit to the data is excellent; we often obtain least squared correlation coefficients greater than 0.9.

It is found almost universally that statistical models of breakdown, such as the bulk electron theory, oversimplify the damage process by neglecting nonlinear interactions and anisotropies induced by impurities. Thus, the fundamental intensity dependence on pulse length t_p often deviates from the classical $t_p^{-1/2}$ dependence resulting from 2-photon absorption without diffusion, or from linear absorption with diffusion, to dependencies as high as $t_p^{-1.22}$ for the former to $t_p^{-0.44}$ for the latter. This fact, coupled with the experimental nonobservability of higher than 2-photon absorption seems to imply that the avalanche mechanism is the most likely initiator of the plasma requisite for lattice meltdown.

Statistical confidence bands for material survivability as a function of laser intensity and pulse length can be constructed; this lends high practical utility to the Weibull distribution as an engineering diagnostic tool.

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1. Introduction

Laser induced breakdown of solid dielectric films and bulk materials has been a subject of ongoing interest and concern at these symposia. Although the complete theory leading to breakdown will most likely never be understood, i.e., aspects pertaining to imperfections intrinsic and extrinsic to the material, much effort is spent to evaluate two possible physical mechanisms initiating the breakdown process -- avalanche and multi-photon ionization. Both of these theories have been suggested as candidates to provide the requisite number of electrons precursory to lattice melting.

The avalanche theory proposes the existence of loosely bound electrons derived from shallow traps arising from such structural imperfections as F-centers. These electrons interact with the laser field, become excited, and multiply upon impact ionization with the lattice.

When the laser photon energy is $\frac{1}{n}$ times the bandgap energy E_g , n-photon ionization becomes an alternate mechanism for the creation of conduction electrons, but with rapidly diminishing probability as n increases. Thus, the longer the wavelength the laser light is the less effective multi-photon ionization is for exciting valence electrons to the conduction band. For example, a 10.6 μ m laser light source requires a 21-photon process to elevate electrons through the NaCl F-center bandgap of 2.4 eV.

In reality there is probably some augmentation between multiphoton and avalanche ionization which is dependent on photon energy $\hbar\omega$ and E_g . As the photon energy attains a value $\geq E_g/2$ two photon absorption will quite likely assist the avalanche process, for $E_g/3 \leq \hbar\omega < E_g/2$ three photon absorption may occur, but with reduced probability, etc. Liu et al have set up experiments to detect 3-photon absorption at 355 nm in NaCl by direct absorption. [2]

Not only were they unsuccessful, but all studies including those by photo-conductivity have never been able to discover absorption higher than 5-th order in solids. Apparently the major pitfall is that at the intensity levels of observation breakdown usually intervenes. How to deconvolute this presumed joint dependence of breakdown on both mechanisms is a subject of this paper.

As the total number of such conductive electrons generated depends on the 1) laser pulse duration, 2) photon energy of the laser, 3) energy gap and impurity levels, and 4) ambient conditions there is a great deal of latitude for interpreting the damage event statistically. Such a point of view has been corroborated by the experiments of Milam et al.^[3] for dielectric breakdown times of bulk and thin film materials when irradiated at small focal volumes by tightly focused laser beams.

In the following two sections we shall discuss the role and application of the Weibull distribution to the analysis of damage data. This distribution is then fit to data compiled by Picard et al and Milam for various thin film coated samples of SiO_2 , Al_2O_3 , and ZrO_2 .^[4,5] Estimates concerning characteristic lifetimes and breakdown intensities are given as well as a scheme for determining the reliability of those estimates. It is also shown that the dependence of breakdown intensity on laser pulse duration, for the most part, does not deviate markedly from linear or 2-photon absorption. A strong case is then made for the avalanche process.

II. Statistics of Failure: The Weibull Distribution

The use of statistical failure models to make predictions and deductions about material breakdown in the presence of external forces can be traced to the pioneering work of Griffith in his study of flaws and their correlation to fracture.^[16] Griffith theorized that the reason for the difference between calculated and observed values of material strengths resides in the fact that there exist flaws in the body which weaken it. If the flaws are distributed inhomogeneously they will provide a distribution of strengths as a function of material position, each position requiring a different degree of stress to induce fracture. When the actual performance of the material is then predicated on its weakest "link" the use of extreme value statistics is in order, that is, the statistics of maxima and minima.^[17] Some of the experiments of this theory are presented in the Appendix.

To pursue the analogy of the foregoing to laser breakdown of optical materials we postulate that a damage event is due to a number of underlying causes. Not only is it a function of laser intensity, impurity concentration, and structural defects, but interposed with these causes is the unifying factor that damage may be the result of cumulative laser pulses. This implies that damage is not necessarily an all or none event but most probably depends on the material's previous history; this includes weakest "links". Inhomogeneities in the material and external fluctuations subsumes the use of extreme value statistics.

An extreme event distribution which has been previously found to fit the failure data of electrical components and of D.C. breakdown of materials is the Weibull distribution.^[8,9] Asymptotically it is given by the distribution function

$$F(y) = 1 - e^{-(y/\alpha)^B} \quad \begin{matrix} y \geq 0 \\ \alpha, B > 0 \end{matrix} \quad (1)$$

where y denotes the random variable

$$y = \lim_{n \rightarrow \infty} y_n = \lim_{n \rightarrow \infty} \min_{i=1, \dots, n} \{X_i\}, \quad (2)$$

consisted of the minimum of a large number of independently distributed damage random variables X_i whose distributions are given by (A1b). The parameters α and β are respectively, defined as scale and shape parameters, with α being a characteristic failure coefficient since it is the 10% y ($1 - e^{-1}$) = 63.2 percentile of the distribution for any value of β .

In an attempt to understand the meaning of (1) we examine plots of the density function $f(y)$

$$f(y) = \frac{1}{\alpha} \Gamma(\beta/\alpha) y^{-\beta} e^{-(y/\alpha)^\beta} \quad (3)$$

in figure 1. For values of β in the range $0 < \beta \leq 1$ $f(y)$ is an amodal function of y with mean α and variance α^2 (eqs. A2) ranges, respectively, $\alpha = 1$ and $(\sigma^2/\alpha^2) = -1/2$ $\beta = 0 \rightarrow 1$. At $\beta = 1$ (1) is exponentially distributed. For values of $\beta > 1$, $f(y)$ is unimodal, resembling a Gaussian at $\beta \approx 3.6$. The limiting values of α and σ^2 are

$$\lim_{\beta \rightarrow \infty} \begin{cases} \alpha = 1 \\ \sigma^2 = 0 \end{cases} \quad (4)$$

In plotting failure data y_i , $i = 1, \dots, N$, to (1) we rank the y_i 's from smallest to largest and assign values to $F(y_i)$ as

$$F(y_i) = \frac{i}{N+1} \quad (5)$$

By placing these points on Weibull graph paper (fig. 2) and drawing a

straight line through them rough estimates of α and β can be obtained. The α value is found by entering the probability scale at the 63.2% point and then reading its projection on the abscissa. β is found by drawing a line parallel to the fitted straight line through the point marked origin; the value of its intersection with the vertical shape parameter scale at the left equals β . An alternate but more accurate procedure for determining α and β is to least squares fit a straight line to

$$\ln n [1 - F(y)]^{-1} = \ln y + \ln \alpha \quad (6)$$

One of the drawbacks to utilizing (1) occurs when failure events pile up at the small values of x . This tends to shift the weight of the distribution to the left, away from the true mode which varies as $N^{-1/2}$, resulting in correspondingly poorer fits. This drawback can be remedied by choosing as a variant of (1) the distribution

$$f(z) = e^{-(\alpha'/z)^{\beta'}} \quad (7)$$

z is the random variable defined as

$$z = \lim_{n \rightarrow \infty} z_n = \lim_{n \rightarrow \infty} \max_n \{X_1, \dots, X_n\} \quad (8)$$

the X_i 's being independent and identically distributed according to (18b).

α' and β' are the counterparts to the α and β of (1). This distribution is also known as the Pareto distribution after the economist Pareto.^[10] Thus, instead of asking what the minimum value of y for which failure occurs is, we now invert the question to the maximum value of z . Pileups of z_i for small values of z will better the fit to (7). The parameters α' and β' are found by least squares fitting a straight line to

$$\ln n F(z)^{-1} = \beta' \ln \alpha' - \beta' \ln z \quad (9)$$

III. Damage Statistics of Thin Film Dielectrics

In this section we apply the formulas of Section II to damage data compiled by Milan for various sputtered thin film dielectric samples of SiO_2 , ZrO_2 , and Al_2O_3 . These films were prepared on CG7059 electronic substrates with various percentage of sputter bias and sputtering gas (O_2 or Ar) concentrations. The materials were irradiated at a number of separate sites with 20-30 msec. pulses focused to approximately 6 mm focal spots. The pulses were obtained by switching out the central portion of the output from a single longitudinal mode, single transverse mode Q-switched laser by means of a Pockels cell shutter. For a given laser pulse intensity, I , the time delay to damage, or survival time t_d was measured from the width of the transmitted pulse, and beam power from the height of a reference pulse.* The t_d values were obtained to a $\pm 2.5\%$ accuracy. As this I was varied these measurements were equivalent to those of breakdown threshold energy density I_B versus t_p .

A statistical ensemble to which eqs. (1) and (7) are fit is obtained by ranking all survival times t_d for specific values of I_B , or alternatively by

*The relation between laser pulse energy E , intensity I , and power P , is formally given as

$$E = \iint dx dy \int dt I(x, y, t)$$

where x and y are coordinates in the plane normal to the light propagation axis. For a constant intensity pulse this reduces to

$$E = I t_p A$$

where t_p is pulse duration and A is focal spot area.

ranking all I_B for specific values of t_d . The results of least squares fitting this data to these distributions are given in Tables I-III. Parts A of these tables correspond to distributions in t_d , $F(t_d)$. The first and second columns contain individual breakdown intensities I_B with the t_d values of I_B (given in parentheses) to be fit to $F(t_d)$ and their respective t_d values. The $\bar{t}_d$ denote characteristic breakdown intensities, $\bar{I}_B$. The entries in the last two tables correspond to distributions in I_B , $F(I_B)$. Now the first and second columns contain individual lifetimes t_d with the I_B values of t_d to be fit to $F(I_B)$. The $\bar{I}_B$ denote characteristic breakdown intensities, $\bar{t}_d$. The entries in the 3, 4, 5, and 6 are shape parameter, correlation coefficient, standard error, and variance. The entries in the 7 and 8 columns are not to be literally interpreted as the 7 and 8's from (1). They may be of any β values, depending on whether β was larger for the fit to (9) than it was for (1). This statement is more conditionally true for the calculated mean and variance values.

The dependencies to be drawn from these tables are best summarized as follows:

- i) The least squares correlation coefficients r indicate that the fits of this film damage data to the Weibull distribution are excellent.
- ii) Parts A imply that increasingly larger values of I_B lead to decreasingly smaller values of $\bar{t}_d$.
- iii) Parts B imply that increasingly larger values of t_d lead to decreasingly smaller values of $\bar{I}_B$; this is true since the sample experiences the damaging pulse for longer times.
- iv) The β coefficients from parts B will in general lead to much narrower

distributions for $F(I_B)$ than for $F(t_d)$.

- v) In some cases the variances σ^2 are not sufficiently broad to distinguish statistics from experimental error.
- vi) There may be a stabilization in the range of σ values as a function of sputter bias and sputtering gas concentration. More data is needed to investigate this point.

If we now plot for each sample category the median I_B values as a function of $(\bar{t}_d)^q$, and least squares fit the result to

$$\ln I_B = q \ln \bar{t}_d + C_0 \quad (10)$$

the ensuing results for q and r^2 are presented in table IV. By arguing that breakdown occurs when a fixed amount of heat Q_B is deposited per unit volume of the sample, the relevance of these q values can be discerned from the heuristic expression

$$Q_B = K I_B (t_d) \quad (11)$$

where K is an effective multiphoton absorption coefficient and the function $q(t_d)$ may be expressed as^[4]

$$q(t_d) = \begin{cases} t_d^{-q} & \text{absence of thermal diffusion} \\ t_d^{-2.1} & \text{for thin films or long laser pulse durations in the presence of thermal diffusion} \end{cases}$$

Thus, for the samples studied, in the absence of thermal diffusion breakdown incurred by pure multi-photon ionization leads to a maximal 5-photon process.

This upper limit is reduced in the presence of diffusion to a 2-photon process. Since the true picture is probably somewhat less than these upper limits due to the availability of conduction electrons from avalanche processes or from other excited states (excitons, for example) we feel that multi-photon events greater than 2 probably play an insignificant role in the damage process.

IV. Discussion

As demonstrated in this paper the Weibull distribution is perfectly suited as a statistical tool for the analysis of dielectric breakdown of thin film optical coatings. The authors have performed a similar analysis on gas breakdown for Ar at 500 psi.^[3,5] The same excellence of fit was attained, but with the major difference that β was considerably less than those values reported here, consequently leading to much larger values of σ^2 . It is the contention of these authors that the Weibull distribution can be utilized to characteristically fit laser induced breakdown of any material. We just wait for experimental verification of this conjecture.

We have shown in Section III that characteristic lifetimes $\bar{t}_d$ and breakdown intensity $\bar{I}_B$ are determinable from α (or β). This has high practical significance since with their respective confidence bands delimits on the reliability of the material can be established.^{[11]*} This, of course, enables computation of confidence bands around $\bar{t}_d$ and $\bar{I}_B$ for any material under investigation.

* A confidence band around a distribution function $F(x)$ provides an interval by interval estimate of $F(x)$. The width of the interval depends upon the confidence level $(1 - \epsilon) \times 100\%$, $0 < \epsilon \leq 1$. In terms of the confidence level one can be $(1 - \epsilon) \times 100\%$ sure that $F(x)$ lies between some upper and lower boundary curves, $L(x)$ and $U(x)$, respectively. For the Weibull distribution these curves are usually found in terms of the relations

$$L(x) = H_x^{-1}(\epsilon/2); \quad U(x) = H_x^{-1}\left(\frac{1-\epsilon}{2}\right) \quad (12)$$

where $H_x(y)$ is the probability distribution function of the beta distribution.^[12]

In regard to ascertaining the correctness of previous physical or statistical models of breakdown it seems fairly clear that coupled with experimental nonobservability of higher than 2-photon absorption^[1] the $I_B (\bar{t}_d)^{-q}$ fits seem to imply that the avalanche mechanism is the most likely initiator of the plasma requisite for lattice meltdown. These fits also imply that the only other statistical avalanche mechanism for plasma creation, the lucky electron theory, is wrong since this model predicts a failure distribution λ which varies on t and the electric field $\mathcal{E} \propto I^{1/2}$.^[2,13]

$$\lambda = 1 - e^{-t/\tau_d} \quad (13)$$

where

$$\tau_d = k I^{-1/2} \quad (14)$$

k being a constant greater than zero. At damage threshold this implies for a constant value of I_B a linear dependence on t_d , while for a constant value of t_d a Gumbel-like dependence on I_B (i.e., $e^{-e^{-I_B}}$).^[7] It seems fairly clear that the linear dependence on t_d is incorrect. The exponential dependence of (11) on I_B , however, often seems to fit our Weibull description for the large values of λ obtained. This can be seen upon recasting (11) into the form

$$\lambda = (I/\gamma)^{-\gamma} \quad (15)$$

with $\gamma = I$. Then λ must be

$$\lambda = \frac{k I^{-1/2}}{\ln(\gamma/I)}$$

By defining γ as a characteristic damage intensity, $\gamma = I = \bar{I}_B$, then as $I \rightarrow \bar{I}_B$, $\lambda \rightarrow \infty$. Suitable values of I can be found to match the λ values we found.

The strong power-law dependence exhibited here may be due to small, nonlinear microscopic mechanisms, which in the macroscopic limit are averaged out, once thermal diffusion is included, to produce near-linear absorption. Unfortunately it is impossible for the analysis here performed to make any definitive prediction as to the exact microscopic mechanism, other than the improbability of high-order multiphoton ionization.

Appendix

The Weibull distribution has frequently been employed as a lifetime failure model of electrical components and is of breakdown at potential V . It is also a well-known distribution derived from the theory of extreme values and is given by the probability function $f(x)$:

$$f(x) = \frac{1}{\sigma} \left(\frac{x}{\sigma} \right)^{\frac{1}{\sigma}-1} \exp \left(- \left(\frac{x}{\sigma} \right)^{\frac{1}{\sigma}} \right) \quad (A1)$$

where σ is a parameter of the distribution and σ^{-1} is a variance $^{-1}$.

$$F(x) = \begin{cases} 1 - \exp \left(- \left(\frac{x}{\sigma} \right)^{\frac{1}{\sigma}} \right) & x \geq 0 \\ 0 & \text{elsewhere} \end{cases} \quad (A2)$$

$$F(x) = \left(\frac{x}{\sigma} \right)^{\frac{1}{\sigma}} - \left(\frac{x}{\sigma} \right)^{\frac{1}{\sigma}+1}$$

The theory of extreme events poses the following problem:

Given n samples $\{X_1, \dots, X_n\}$ drawn randomly and independently from a population with density function $f(x)$ [distribution function $F(x)$] let us define y_n such that

$$y_n = \min_n \{X_1, \dots, X_n\} \quad (A3)$$

The distribution function for y_n is formally

$$\begin{aligned} G_n(y) &= \text{Prob} (y_n \leq y) \\ &= 1 - \text{Prob} (y_n > y) \end{aligned} \quad (A4)$$

From (A3) and the independence of all the x_i , the second equation in (A4) may be rewritten as

$$\begin{aligned} G_n(y) &= 1 - \text{Prob} \left(\bigcap_{i=1}^n x_i > y \right) \\ &= 1 - [1-F(y)]^n \end{aligned} \quad (A5)$$

For relatively simple distribution functions $F(y)$ such as uniform and exponential distributions, $G_n(y)$ has a rather simple structure. In general, however, it is a fairly complicated function. In the limit as n becomes large it is therefore much more convenient to utilize an asymptotic relation for $G_n(y)$. This is derivable by defining the new random variable

$$z_n = nF(y_n) \quad (A6)$$

such that for any fixed u , $0 \leq u \leq n$

$$\begin{aligned} \text{Prob} (z_n \leq u) &= \text{Prob} (nF(y_n) \leq u) \\ &= \text{Prob} (y_n \leq F^{-1}(u/n)) \\ &= G_n(u) \end{aligned} \quad (A7)$$

By virtue of (A4) and (A5) $G_n(u)$ is

$$G_n(u) = 1 - (1 - u/n)^n \quad (A8)$$

which in the limit as $n \rightarrow \infty$ converges in distribution to Z ,

$$\lim_{n \rightarrow \infty} F_n(u) = F(u) = 1 - e^{-u} \quad (A9)$$

By now choosing the underlying distribution of the X_i 's as a power law

$$F\left(\frac{x}{r}\right) = \begin{cases} 0 & x/r < 0 \\ \left(\frac{x}{r}\right)^{-1} & 0 \leq x/r < \infty \end{cases} \quad (A10)$$

then

$$\begin{aligned} z_n &= n \left(x_n^{-1} \right) \\ x_n &= \left(r^{-1} / n \right)^{-1} \end{aligned} \quad (A11)$$

and therefore

$$F_n(z) = \begin{cases} 1 - n^{-1} z^{-1} & z \geq 0 \\ 0 & \text{elsewhere} \end{cases} \quad (A12)$$

One should note that when $r = 1$, $F(X_i)$ is uniformly distributed and (A1) reduces to the Poisson distribution characteristic of the lucky electron theory (as a function of time).

As (A1) is derivable from an asymptotic analysis of minima, a similar procedure given the random variable

$$z_n = \max_n \{X_1, \dots, X_n\} \quad (A13)$$

with distribution function

$$\begin{aligned} H_n(z) &= \text{Prob} (z_n \leq z) \\ &= [F(z)]^n \end{aligned} \quad (A14)$$

will upon defining $\hat{r}_n$ as

$$\hat{r}_n = n \left(1 - \hat{F}(x_n) \right) \quad (A15)$$

result in an asymptotic distribution function $\hat{F}_n(v)$, $0 \leq v \leq n$, of maxima

$$\begin{aligned} \hat{F}_n(v) &= \text{Prob}(\hat{r}_n \leq v) \\ &= 1 - (1-v/n)^n \end{aligned} \quad (A16)$$

As $n \rightarrow \infty$, $\hat{F}_n(v)$ also converges in distribution to

$$F(v) = \lim_{n \rightarrow \infty} \hat{F}_n(v) = 1 - e^{-v} \quad (A17)$$

Corresponding to the Weibull distribution, if as our underlying density for the X_j we choose

$$\begin{aligned} f(X_j) &= 0 \quad X_j \leq 0 \\ &= (c'/X_j)^{c'+1} \end{aligned} \quad (A18a)$$

then

$$F(X_j) = 1 - (c'/X_j)^{c'} \quad (A18b)$$

and

$$f(z) = e^{-(c'/z)^{c'}} \quad (A19)$$

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NOTES

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Bibliography

1. Reviews of laser-induced breakdown in optical materials are given in:
H. Bloembergen, IEEE J. Quant. Elect. QE-10, 375 (1974); W. L. Smith,
Opt. Engr. 17, 489, 1978.
2. P. Liu, W. L. Smith, H. Lotem, J. H. Bechtel, H. Bloembergen, and
R. S. Adhav, Phys. Rev. B 1978 (in press).
3. D. Milam, R. A. Bradbury, and P. H. Picard, in Laser Induced Damage in
Optical Materials : 1975 (NBS Special Publication 435), p. 347 (1975).
4. R. H. Picard, D. Milam, R. A. Bradbury, and J. C. C. Fan, in Laser
Induced Damage in Optical Materials : 1975 (NBS Special Publication 435),
p. 272 (1975).
5. Data supplied by D. Milam, private communication.
6. A. A. Griffith, Phil. Trans. Roy. Soc. 221A, 163 (1920).
7. F. Gumbel, "Statistical Theory of Extreme Values and Some Practical
Applications, Appl. Math. Series 33, NBS (1954).
8. J. H. K. Kao, Technometrics 1, 389 (1959).
9. P. H. H. Fischer and K. W. Nissen, IEEE Trans. Elect. Insul. EI-11, 57
(1976).
10. H. T. Davis, "The Theory of Econometrics (Bloomington, Ind. 1941) p. 23.
11. G. J. Hahn and S. S. Shapiro, "Statistical Models in Engineering", (John
Wiley and Sons, New York, 1968).
12. J. H. K. Kao, Proc. 6th Nat. Symp. Reliability Quality Control in
Electron., pp. 190-201, 1960.
13. M. Bass and H. H. Barrett, IEEE J. Quant. Electron. QE-8, 338 (1972).

FIGURE CAPTIONS

Figure 1. Graphs of the Weibull density function $f(x)$ for $\lambda = 1$ and $\frac{1}{\beta}$, 1, 3, and 10. λ and $\frac{1}{\beta}$ are, respectively, the mean and variance for this distribution.

Figure 2. A plot on Weibull graph paper of the cumulative distribution of t_d (in nanoseconds), from smallest to largest, at a fixed breakdown intensity I_B of 3.1 GW/cm². The sample irradiated is a $t_{1/2}$ thin film deposited without sputter bias or O_2 sputtering gas on CG 7519 substrate. N is the total number of spots irradiated, of which 16 damaged, λ is the characteristic sample lifetime, and β the shape fitting parameter.

Table I. Thin Films of alumina prepared under various sputter bias and sputtering gas conditions.

- A. Distribution in t_d : The 1st and 2nd columns rowise pair individual breakdown intensities I_B with the N values of t_d (given in parentheses) to be fit to $F(t_d)$ and their respective t_d ranges. The λ 's denote characteristic sample lifetimes, t_d .
- B. Distributions in I_B : The 1st and 2nd columns rowise pair individual lifetimes t_d with the N values of I_B to be fit to $F(I_B)$. The λ 's denote characteristic breakdown intensities, $\bar{I}_B$, s , r^2 , $\bar{t}_d$, and s^2 are, respectively, shape parameter, correlation coefficient squared, mean, and variance.

Table II. Thin films of zirconia -----, Legend description same as table I.

Table III. Thin films of silica ----, Legend description same as table I.

Table IV. Scaling law for breakdown intensity as a function of laser pulse length.

WEIBULL DENSITY FUNCTIONS -- $\alpha = 1$

$$\delta(y) = (\beta/\alpha)(y/\alpha)^{\beta-1} e^{-(y/\alpha)^\beta}$$

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Λe_2^{03} TABLE 1A

GH/c^2	N (no.)	B	$\mu = \bar{t}_d$	r^2	μ	σ^2
NO BIAS, NO O_2						
$7 \leq I_B \leq 7.4$	$6.8 \leq t_d \leq 10.2$ (12)	7.46	7.94	0.819	7.46	1.77
$7.7 \leq I_B \leq 8.1$	$6.2 \leq t_d \leq 8$ (17)	15.64	6.74	0.884	6.57	0.17
$8.1 \leq I_B \leq 8.5$	$6 \leq t_d \leq 7$ (11)	15.96	6.68	0.885	6.66	0.17
$9.3 \leq I_B \leq 9.8$	$4.6 \leq t_d \leq 7.2$ (10)	7.15	6.77	0.826	5.71	0.29
$10 \leq I_B \leq 10.5$	$4.7 \leq t_d \leq 7.4$ (9)	5.73	5.72	0.917	4.77	1.1
$10.7 \leq I_B \leq 11.3$	$4.8 \leq t_d \leq 6$ (10)	14.05	5.61	0.834	4.83	1.1
10% BIAS, NO O_2						
$6.5 \leq I_B \leq 6.9$	$12 \leq t_d \leq 19$ (10)	5.61	17.54	0.946	16.27	11.7
$7 \leq I_B \leq 7.4$	$11 \leq t_d \leq 19$ (10)	6.63	12.68	0.927	11.83	4.3
$7.6 \leq I_B \leq 8$	$9 \leq t_d \leq 16$ (15)	6.50	13.60	0.917	12.67	5.7
$8.6 \leq I_B \leq 9$	$8 \leq t_d \leq 11.2$ (12)	9.47	9.83	0.978	9.16	1.1
$9.2 \leq I_B \leq 9.7$	$6.2 \leq t_d \leq 12$ (10)	5.14	7.41	0.952	6.81	2.7
$10.8 \leq I_B \leq 11.3$	$5 \leq t_d \leq 7.3$ (9)	7.48	5.72	0.938	5.77	0.17
$11.7 \leq I_B \leq 12.4$	$4.3 \leq t_d \leq 7.3$ (8)	5.21	4.62	0.913	4.25	0.17
10% BIAS, 10% O_2						
$8 \leq I_B \leq 8.4$	$10.8 \leq t_d \leq 15.5$ (12)	5.73	12.69	0.971	16.17	10.9
$9.3 \leq I_B \leq 9.8$	$7.2 \leq t_d \leq 15.7$ (12)	5.03	9.61	0.976	8.28	3.58
$11.6 \leq I_B \leq 12.2$	$4.6 \leq t_d \leq 7.6$ (10)	4.93	5.28	0.914	4.84	1.26
$12.3 \leq I_B \leq 12.9$	$3.6 \leq t_d \leq 7$ (12)	5.03	5.72	0.984	5.44	1.57
$13 \leq I_B \leq 13.7$	$3.4 \leq t_d \leq 7.7$ (11)	3.68	4.23	0.913	3.79	1.3

$Z_r O_2$ TABLE IIA

GK/cm^2	N (nsec)	β	$\alpha = \bar{t}_d$	r^2	μ	σ^2
NO BIAS, A_r ONLY						
$7 \leq I_B \leq 7.4$	$7 \leq t_d \leq 9.8$ (14)	8.14	8.89	0.933	8.38	1.49
$7.5 \leq I_B \leq 7.9$	$2 \leq t_d \leq 8.8$ (10)	1.99	8.19	0.732	7.26	14.52
$8.3 \leq I_B \leq 8.7$	$4.4 \leq t_d \leq 7.7$ (10)	5.67	5.12	0.993	4.73	0.93
$8.8 \leq I_B \leq 9.2$	$4.2 \leq t_d \leq 5.8$ (13)	10.06	4.70	0.938	4.47	0.29
$9.4 \leq I_B \leq 9.9$	$1 < t_d \leq 5.1$ (9)	7.63	4.09	0.739	3.84	0.36
$9.8 \leq I_B \leq 10.3$	$1 < t_d \leq 4.3$ (11)	1.43	3.72	0.775	3.38	5.75
30% BIAS, NO O_2						
$8.8 \leq I_B \leq 9.2$	$13.5 \leq t_d < 20$ (10)	6.62	15.34	0.956	14.31	6.41
$9.3 \leq I_B \leq 9.8$	$10.5 \leq t_d \leq 17$ (12)	7.33	14.94	0.974	14.01	5.08
$10.1 \leq I_B \leq 10.6$	$7.2 \leq t_d \leq 15$ (23)	5.29	9.63	0.961	8.87	3.72
$10.7 \leq I_B \leq 11.2$	$1 < t_d \leq 10.3$ (16)	4.04	6.08	0.959	5.51	2.35
$11.3 \leq I_B \leq 11.9$	$1 < t_d \leq 9$ (20)	3.10	6.26	0.966	5.60	3.90
$12.1 \leq I_B \leq 12.7$	$1 < t_d \leq 6$ (11)	1.15	4.14	0.818	3.94	11.80
30% BIAS, + O_2						
$9.3 \leq I_B \leq 9.8$	$12 \leq t_d < 20$ (12)	7.95	17.09	0.919	16.09	5.76
$9.9 \leq I_B \leq 10.4$	$13 \leq t_d < 20$ (14)	10.57	14.17	0.953	13.51	2.38
$10.7 \leq I_B \leq 11.2$	$10 \leq t_d \leq 15$ (13)	4.32	9.94	0.984	9.05	5.61
$11.3 \leq I_B \leq 11.9$	$1 < t_d \leq 13$ (11)	0.79	7.92	0.817	9.06	133.9
$12.2 \leq I_B \leq 12.8$	$1 < t_d \leq 10.3$ (13)	1.02	8.35	0.803	8.28	65.94

Al_2O_3 TABLE IB

nsrc	N (GW/cm ²)	B	$\alpha = T_B$	r^2	μ	σ^2
NO BIAS, NO O ₂						
5 $\leq t_d \leq 5.3$	8.7 $\leq I_B \leq 11.3$ (13)	11.90	10.68	0.969	10.23	1.09
5.4 $\leq t_d \leq 5.7$	8.8 $\leq I_B \leq 11.2$ (9)	9.62	9.12	0.905	8.66	1.17
6 $\leq t_d \leq 6.3$	7.9 $\leq I_B \leq 11$ (10)	8.70	8.49	0.949	8.03	1.21
6.7 $\leq t_d \leq 7$	7.4 $\leq I_B \leq 10.2$ (22)	13.20	7.89	0.900	7.59	0.49
7.2 $\leq t_d \leq 7.6$	6.3 $\leq I_B \leq 10.3$ (9)	6.19	7.29	0.984	6.77	1.63
10% BIAS, NO O ₂						
7.2 $\leq t_d \leq 7.6$	9.2 $\leq I_B \leq 11.7$ (11)	13.21	9.65	0.969	9.28	0.73
14 $\leq t_d \leq 14.7$	6.5 $\leq I_B \leq 8$ (9)	14.35	7.73	0.919	7.45	0.40
10% BIAS, 10% O ₂						
5.5 $\leq t_d \leq 5.8$	11.3 $\leq I_B \leq 14.2$ (8)	12.26	12.22	0.963	11.72	1.35
10.5 $\leq t_d \leq 11$	8.2 $\leq I_B \leq 10.5$ (11)	11.20	9.88	0.967	9.44	1.04

$Z_{r_2} O_2$ TABLE IIB

nsec	N (GH/cm^2)	β	$\alpha = I_B$	r^2	μ	σ^2
NO BIAS, A_r ONLY						
$4.1 \leq t_d \leq 4.4$	$8.6 \leq I_B \leq 9.9$ (9)	20.77	9.63	0.976	9.38	0.31
$4.8 \leq t_d \leq 5.1$	$8.1 \leq I_B \leq 9.6$ (9)	19.96	9.07	0.950	8.83	0.30
30% BIAS, NO O_2						
$t_d < 1$	$10.8 \leq I_B \leq 16.7$ (16)	10.49	12.29	0.901	11.72	1.81
30% BIAS, + O_2						
$t_d < 1$	$11.2 \leq I_B \leq 16.3$ (28)	11.10	14.30	0.967	13.66	2.22

S_iO_2 TABLE IIIA

GW/cm^2	N (nsec)	B	$\alpha \approx E_d$	r^2	μ	σ^2
NO BIAS, NO O_2						
$7.4 \leq I_B \leq 7.8$	$7 \leq t_d \leq 19$ (15)	9.27	17.93	0.938	17.00	4.83
$9.6 \leq I_B \leq 10$	$6.2 \leq t_d \leq 11.2$ (15)	6.78	9.36	0.983	8.70	2.61
$10 \leq I_B \leq 10.4$	$2.7 \leq t_d \leq 9$ (13)	2.74	7.78	0.865	6.92	7.44
$11.8 \leq I_B \leq 12.2$	$3.3 \leq t_d \leq 4.8$ (13)	8.90	4.40	0.977	4.16	0.31
BIAS SPUTTERED						
$6.4 \leq I_B \leq 6.8$	$7.7 \leq t_d \leq 10$ (7)	9.19	8.10	0.922	7.68	1.00
$6.7 \leq I_B \leq 7.1$	$7.4 \leq t_d \leq 9$ (11)	17.89	7.81	0.915	7.58	0.27
$9.4 \leq I_B \leq 10.2$	$3.4 \leq t_d \leq 4.5$ (8)	12.20	4.18	0.935	4.01	0.15
BIAS SPUTTERED, + O_2						
$6.8 \leq I_B \leq 7.2$	$8.8 \leq t_d \leq 12$ (11)	10.20	9.69	0.925	9.23	1.19
$9.8 \leq I_B \leq 10.2$	$4.3 \leq t_d \leq 5.8$ (11)	9.96	5.36	0.927	5.10	0.38
$10.9 \leq I_B \leq 11.3$	$3.2 \leq t_d \leq 4.3$ (10)	10.45	4.12	0.919	3.93	0.20
$11 \leq I_B \leq 11.4$	$3.2 \leq t_d \leq 4.3$ (9)	10.14	4.09	0.936	3.89	0.21
$11.7 \leq I_B \leq 12.1$	$2.8 \leq t_d \leq 4$ (11)	10.37	3.56	0.930	3.39	0.16

S_iO_2 TABLE IIIB

nsec	N (GM/cm ²)	β	$\alpha = \bar{I}_B$	r^2	μ	σ^2
NO BIAS, NO O ₂						
$3 \leq t_d \leq 3.3$	$11.3 \leq I_B \leq 13.3$ (10)	19.32	12.89	0.912	12.54	0.64
$3.6 \leq t_d \leq 4$	$11.8 \leq I_B \leq 12.4$ (8)	45.68	12.24	0.895	12.09	0.11
$7.8 \leq t_d \leq 8.2$	$7.2 \leq I_B \leq 10.7$ (8)	25.51	9.92	0.951	9.71	0.22
BIAS SPUTTERED						
$3.8 \leq t_d \leq 4.2$	$9.4 \leq I_B \leq 10.7$ (7)	20.24	9.55	0.834	9.30	0.32
$7.7 \leq t_d \leq 8.4$	$6 \leq I_B \leq 7.2$ (10)	18.26	6.97	0.737	6.77	0.21
BIAS SPUTTERED, + O ₂						
$2.8 \leq t_d \leq 3.2$	$11 \leq I_B \leq 13.2$ (11)	19.76	12.56	0.966	12.22	0.59
$3.3 \leq t_d \leq 3.7$	$11 \leq I_B \leq 12$ (8)	30.37	11.81	0.874	11.60	0.23
$4 \leq t_d \leq 4.3$	$10.2 \leq I_B \leq 11.7$ (10)	20.50	11.31	0.853	11.07	0.44
$4.5 \leq t_d \leq 5$	$8.6 \leq I_B \leq 10.8$ (9)	13.00	10.48	0.921	10.07	0.89

TABLE IV

$$g(t_d) = \begin{cases} t_d^{-q} & \text{no diffusion} \\ t_d^{-2q} & \text{diffusion} \end{cases}$$

	<u>$I_B \ g(t_d)$</u>	<u>$q =$</u>	<u>$r^2 =$</u>	Maximal multiphoton, no diffusion	Maximal multiphoton diffusion
SiO ₂ no bias, no O ₂	-0.325	0.992	3	1-2	
SiO ₂ bias sputtered, no O ₂	-0.582	0.997	2	1	
SiO ₂ bias sputtered, +O ₂	-0.532	0.993	2	1	
ZrO ₂ no bias, Ar only	-0.350	0.966	3	1-2	
ZrO ₂ 30% bias, no O ₂	-0.221	0.950	5	2	
ZrO ₂ 30% bias, +O ₂	-0.252	0.740	4	2	
Al ₂ O ₃ no bias, no O ₂	-0.992	0.940	1	0	
Al ₂ O ₃ 10% bias, no O ₂	-0.442	0.973	2	1	
Al ₂ O ₃ 10% bias, 10% O ₂	-0.408	0.956	2-3	1	