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Material Density Measurements from Dynamic Flash X-Ray Radiographs Using Axisymmetric Tomography

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MATERIAL DENSITY MEASUREMENTS FROM
DYNAMIC FLASH X-RAY RADIOGRAPHS USING
AXISYMMETRIC TOMOGRAPHY

By

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ABSTRACT

The axisymmetric version of the tomographic x-ray reconstruction procedures has been utilized to determine the material density for the impact of a cylinder on a steel plate. Derivations of the reconstruction algorithms relating x-ray radiographic intensities to the material densities are presented. Effects of noise, point spread functions, and motion blur are minimized.

I. INTRODUCTION

One experimental technique to measure the dynamic deformation of a cylindrical projectile impacting a plate is to take several x-ray radiographs during the course of the impact process. These radiographs are projections of the total x-ray attenuation from the x-ray source through the experiment to the film. If the experiment has axial symmetry, it is possible to reconstruct the density distribution in the projectile at the time of exposure from one radiograph.

First, the relationship between film exposure and x-ray attenuation are derived; then the algorithm to reconstruct the material density is developed. Application of this procedure to the impact of an inert mockup of a polymer-bonded explosive (PBX) cylinder against a steel plate yields a material density map as a function of position and allows determination of the various wave fronts and density structure.

II. THEORY

The experimental configuration considered in this analysis for radiographing an axisymmetric experiment has the axis of the experimental setup

perpendicular to the beam axis (Fig. 1). The coordinate z denotes length from the x-ray target along the line through the axis of the device; x and y denote coordinates in the planes perpendicular to z , with x being perpendicular to the apparatus axis and y coinciding with the axis. The x-ray source-to-shot center distance is denoted by z_S , and the shot center-to-film plane distance is denoted by z_F . The radiographic magnification, RM , is defined,

$$RM = (z_S + z_F)/z_S \quad . \quad (1)$$

An intense, highly directional beam of high-energy x rays is generated at the source, passes through the experimental apparatus and arrives at the film. The film is exposed by the electrons emitted by a lead intensifying screen placed in front of the film. The photographic density, D , is a monotonically increasing function of the x-ray exposure, E , incident on the film plane. For the x-ray exposure range of interest, this dependence is approximately,

$$D = k E^\gamma \quad , \quad (2)$$

where k and γ are constants depending on the type of film and the development process, $\gamma \sim 1$. The photographic density of the film is determined after development by a densitometer, which measures the transmission of ordinary light through the film.

$$D = \log \frac{i_0}{i} \quad , \quad (3)$$

where i_0 is the incident light intensity, and i is the transmitted light intensity through the densitometer.

The exposure, E , is proportional to the integral over time of the intensity incident upon the film plane.

$$E = \int I \, dt \quad . \quad (4)$$

If I is a constant for specified duration, Δt , and zero otherwise,

$$E = I \, \Delta t \quad . \quad (5)$$

The value of the intensity, I , which reaches the film plane, is inversely proportional to the square of the target-to-film distance. It also falls off rapidly with angular departure from the beam axis. This beam axis does not necessarily coincide with the mechanical axis of the experiment. The location of the beam axis can be described by two angles ϕ , ψ ,

$$\phi = \tan^{-1} \left(\frac{x_0}{z_S + z_F} \right); \quad \psi = \tan^{-1} \left(\frac{y_0}{z_S + z_F} \right) \quad , \quad (6)$$

where x_0 , y_0 are the coordinates of the beam axis at the film plane. The lateral dependence of the beam intensity is of the form¹,

$$q(\theta) = (\ln b)^{-1} \left[E_1(a\theta^2) - E_1(ba\theta^2) \right] \quad , \quad (7)$$

where a and b are constants dependent on the initial x-ray target configuration and energy level of the resultant x rays and,

$$\theta = \tan^{-1} \sqrt{\frac{(x-x_0)^2 + (y-y_0)^2}{z_S + z_F}} \quad , \quad (8)$$

and $E_1(x)$ is related to the exponential integral,

$$E(x) = -C - \ln x - \sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n n!}, \quad (9)$$

where C is Euler's constant.

Further, there is an attenuation in intensity as the x-ray passes through the material of the experimental apparatus. The attenuation of the intensity as the x-ray beam passes through the medium is,

$$I = I_0 \{ S(e) \exp[-\int_0^L \mu(e, \xi) \rho(\xi) d\xi] \} / \{ S(e) de \}, \quad (10)$$

where $\rho(\xi)$ is the material density, $\mu(e, \xi)$ is the x-ray absorption coefficient, which is a function of x-ray energy, e , and $S(e)$ is the distribution function of the energy of the x-ray source.

If either μ is independent of the x-ray energy or the x-ray source is monochromatic, i.e., $S(e) = S_0 \delta(e - e_0)$, where $\delta(x)$ is the Dirac delta function.

If the absorption coefficient is not independent of the x-ray energy and the source is not monochromatic radiation, it is possible to calculate an effective constant absorption coefficient through a nonlinear stretching transformation, utilizing the property that the transmitted intensity decreases monotonically with increasing thickness.

The intensity can be written in a simpler form,

$$I = I_0 \exp(-\int \rho \mu d\xi). \quad (11)$$

Thus the intensity at the film plane is,

$$I(x, y, z_S + z_F) = I_0 q(\theta, x_0, y_0, z_S + z_F) \exp\{\int \rho \mu d\xi\} / \left[(z_S + z_F)^2 + x^2 + y^2 \right]. \quad (12)$$

Substituting this expression into the expression relating photographic density to intensity and taking its logarithm, [with $x, y \ll (z_S + z_F)$]

$$-\ln D = -\ln k - \gamma \ln q \int \mu \rho d\xi . \quad (13)$$

Ignoring for the moment other variations in q and with $q(0) = 1$,

$$-\ln D = k' + \gamma \int \mu \rho d\xi , \quad (14)$$

where k' is a new constant and ξ is integrated along the line connecting the target and $(x, y, z_S + z_F)$.

Consider now the values at the film plane of $\ln D$ along a thin slice, Δy , about $y = 0$. This function of x is fan beam projection through the device. The density and absorption coefficient are functions of r from the center of the device.

$$r^2 = x^2 + y^2 \text{ at } z = z_S . \quad (15)$$

The ray from the x-ray source to the film is equivalent to a ray parallel to the mechanical axis, with a scaled value of x' at the film plane.

$$x' = x/RM + 0 \left(\left(\frac{x}{z_S + z_F} \right)^3 \right) . \quad (16)$$

Denote by $g(x') = \gamma \int \mu(r) \rho(r) dy'$ and $f(r) = \gamma \mu(r) \rho(r)$. In the resulting parallel projection, we have,

$$g(x') = 2 \int_0^{\sqrt{a^2 - x'^2}} f(x,y) dz \quad , \quad (17)$$

or,

$$g(x') = 2 \int_{x'}^a \frac{rf(r)dr}{\sqrt{a^2 - x'^2}} \quad , \quad (18)$$

where a is some radius beyond which $f(r)$ vanishes, i.e.,

$$g(a) = 0 \quad g(x') = 0 \quad \text{for } x' > a \quad . \quad (19)$$

Given $g(x')$, $f(r)$ can be calculated. Analytically,

$$f(r) = \frac{\pi}{2} \int_r^a \frac{dg}{dx'} \frac{1}{\sqrt{a^2 - x'^2}} dx' \quad . \quad (20)$$

The function $f(r)$ that is generated is related to the actual product of density and absorption coefficient, $F(r)$, through the integral,

$$f(r) = \int F(\xi) p(r - \xi) d\xi = F(r) * p(r) \quad , \quad (21)$$

where $p(r)$ is the point spread function. If the x-ray propagation were ideal, the point spread function would be a delta function. However, as

the x-ray propagates through the medium, it is scattered by a number of physical processes. Hunt, Janney, and Ziegler² have computed this function for Compton scattering and pair production at the lead intensifying screen only. Other processes, such as x-ray scattering in the experimental apparatus itself, also contribute to the total point spread function. The point spread function describes the blurring of the geometrical optical image. It may arise from x-ray scattering and other attenuation mechanisms as the x ray passes through the object or as the x ray passes through the intensifying screens at the film plane. The precise form is dependent on the geometry of the test and must be determined for each experimental configuration; for the present the general form is sufficient.

For the dynamic radiograph, the image $f(r)$ of the moving part is the result of a convolution of rectangular pulse of width, $V\Delta t$, where V is the average particle velocity at r , and Δt is the duration of the pulse. Thus, $f(r) = F(r)*p(r)*R(r)$, where,

$$\begin{aligned} R(r) &= 1/V\Delta t & 0 \geq r \geq V t \\ &= 0 & \text{otherwise} \end{aligned} \quad (22)$$

In addition to the deterministic corrections to the image listed above, procedure must be devised to account for noise in the image. The main source of noise is slight variations in the recorded intensity due to film grain size. Several approaches to suppression of this noise can be utilized. These methods would be applied to a digitized version of the measured intensity data. The first is to average several closely spaced lines of the digitized data. A second method is to apply a low pass spatial filter to the data. A third technique is to apply a median filter, either to a line of scanned data or to a rectangular region of the data.

III. EXAMPLE

Two flash radiographs from a set of radiographs of the impact of an inert PBX mockup on a steel plate were selected for study. The experimental data was provided by Russ Jensen of the Hercules, Inc. The projectile was a right circular cylinder 8.85 mm radius, $L/D \sim 1.2$, the impact

velocity was 677 m/sec. The composition of the cylinder is listed in the table. The x-ray source was 150 kev; radiographic times after initial contact with the steel plate were 4.8 and 11.2 μ sec.

These two radiographs were digitized on a PDS scanning microdensitometer at 50 micron spacing with a 50 micron by 50 micron aperture; 512 lines with 1024 pixels per line were obtained. Figures 2 and 3 show the digitized radiographs.

These radiographs were then displayed on the Comtal 8000 digital image display and an interactive program was utilized to determine the centerline of the projectile. The maximum tilt of the projectile from ideally normal impact was measured and was less than 2° . The needed projections, at fixed values of z along the axis were then extracted from the digitized radiograph.

Two methods were tried to reduce film grain noise. First, three, five and ten lines were averaged; second, a median or Tukey filter was used on each line. The latter method gave qualitatively superior results if the window for the filter was slightly larger than the grain size of the film. Figure 4a shows one line of the original input data of the 4.8 microsecond radiograph with film noise. This line is near the top of the projectile (about ten lines below the top). Figure 4b shows the same data, after the median filter has been applied. Figure 4c shows the negative logarithm of the same data with the background intensity subtracted. The intensity outside the projectile is nearly constant throughout this line and throughout the entire radiograph. This mean background value was subtracted from the entire digitized projection. The resulting projection was then inverted numerically to give an estimate of the material density along that line (Figure 4d). The resulting curve shows an increased level of noise.

Figures 5a through 5d show the same sequence for another line on the 4.8 microsecond radiograph, taken about twenty lines above the plate and thus including the spray region. The calculated density profiles show a number of small features. Interpretation on these features as noise or systemic density variations must be done by assembling the whole picture. Filtering of the reconstructed density profiles was not done at this time.

Figures 6 and 7 show the images (512 x 512) of the reconstructed density profiles for the 4.8 and 11.2 microsecond radiographs; higher densities are darker. The reconstructed images show one half of the projectile; the left side is the axis of symmetry.

The density for the first example is reasonably constant in the center and the edge structure closely approximates that predicted by the point spread function convolution. The x-ray absorption coefficient for this material in the range of the incident x-ray spectrum are essentially independent of x-ray energy³; therefore, this effect of beam hardening should be negligible. This was confirmed by examination of the reconstructed density profile in a region of the projectile above the shock wave. These two figures show little variation in the region of the projectile. Since the expected variation of density in a shock wave or rarefaction wave is of the order of 0.5% to 10%, the images were enhanced by a linear stretch from 87.5% peak density to 100% peak density in eight grey level increments (Figures 8 and 9). Each grey level represents a 1.56% relative density step in (16/8). This enhancement brings out significant detail. In the 4.8 microsecond picture, a slightly curved shockwave is visible near the top of the projectile and a rarefaction wave is seen to extend from the intersection of the shockwave and the outer edge to the axis and then continues on from the axis (the picture looks like the wave reflects from the axis). There is also a circular feature whose center is at the intersection of the edge of the projectile and the plate. Higher density regions appear behind the top shock and the reflected rarefaction. Densities in the spray are very low. The enhanced picture for the 11.2 microsecond radiograph shows no definable wavefronts, but shows a pronounced density structure.

IV. Conclusions

A methodology for calculation of the material density from a static or a dynamic radiograph is developed. The radiograph is digitized, with appropriate slices (projections) chosen to coincide with an axisymmetric section of the experiment. The data is then treated with an appropriate Tukey filter to minimize film grain noise and then transformed through the relationships among photographic density, x-ray intensity, density and radius.

The projections correspond to a physical quantity, the density. The density versus radius is then obtained from the numerical solution or an integral equation relating the density to its projection. The collection of successive slices is then output on the Comtal image display device and the image enhanced to emphasize small density differences.

Regions of higher and lower densities can be ascertained and shock and rarefaction wave fronts can be delineated. For the experiment considered here, the density measurements are away from a narrow border surrounding the edges of the projectile; this border is due to finite point spread effects in the creation of the radiographic image. This effect can be eliminated by inverting the convolution of the measured image with the point spread function, but the noise inherent in the remainder of the image is increased.

TABLE I

Composition By Weight
of the Inert Polymer-Bonded Explosive (PBX) Mockup Cylinders

<u>Constituent</u>	<u>Wt. %</u>
Cross linked polymers	19.00
Plasticizer	10.00
Powdered Aluminum (5 m)	19.50
Talc	14.12
Salt (100 m)	17.38
Salt (50 m)	20.0

REFERENCES

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2. B. R. Hunt, D. H. Janney, and R. K. Zeigler, "Introduction to Restoration and Enhancement of Radiographic Images," LA-4305 (1969).
3. Seigbahn, K., ed., "Alpha, Beta and Gamma Ray Spectroscopy, Vol I, North Holland (1965).

Fig. 1. Experimental configuration for x-ray radiography of an axisymmetric device.

Fig. 2. Digitized radiographic of the mockup PBX cylinder striking a steel plate at 4.8 μ sec after impact. Striking velocity is 677 m/sec.

Fig. 3. Digitized radiograph of the mockup PBX cylinder striking a steel plate at 11.2 μ sec after impact. Striking velocity is 677 m/sec.

Fig. 4a. Radiographic film density versus radius for one line of the scanned data 1 mm from the top of the projectile. Time of radiograph is 4.8 μ sec.

Fig. 4b. The radiographic film density versus radius for the above figure after being subjected to a Tukey filter.

Fig. 4c. The negative logarithm of the data in Fig. 4b versus radius.

Fig. 4d. Reconstruction of the density profile as a function of radius.

Fig. 5a. Radiographic film density versus radius for one line of the scanned data 1 mm from the bottom of the projectile. Time of radiograph is 4.8 μ sec.

Fig. 5b. The radiographic film density versus radius for the above figure after being subjected to a Tukey filter.

Fig. 5c. The negative logarithm of the data in Fig. 5b versus radius.

Fig. 5d. Reconstruction of the density profile as a function of radius.

Fig. 6. Reconstructed density for the projectile at 4.8 μsec after impact.

Fig. 7. Reconstructed density for the projectile at 11.2 μsec after impact.

Fig. 8. Enhanced image (linear stretch) for the projectile at 4.8 μsec after impact. A shock wave and two rarefaction waves are now visible.

Fig. 9. Enhanced image (linear stretch) for the projectile at 11.2 μsec after impact. No specific wavefront structure is visible, but the density gradients are quite pronounced.