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**SUBSURFACE FRACTURE MAPPING FROM GEOTHERMAL
WELLBORES**

Final Report

By
B. A. Hartenbaum
G. Rawson

August 1983

Work Performed Under Contract No. AC04-81AL16863

**H-Tech Laboratories, Inc.
Santa Monica, California**

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PREPARED FOR THE
U.S. DEPARTMENT OF ENERGY
DIVISION OF GEOTHERMAL ENERGY
WASHINGTON, D.C. 20545

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ABSTRACT

To advance the state-of-the-art in Hot Dry Rock technology, an evaluation is made of (i) the use of both electromagnetic and acoustic radar to map far-field fractures, (ii) the use of more than twenty different conventional well logging tools to map borehole-fracture intercepts, (iii) the use of magnetic dipole ranging to determine the relative positions of the injection well and the production well within the fractured zone, (iv) the use of passive microseismic methods to determine the orientation and extent of hydraulic fractures, and (v) the application of signal processing techniques to fracture mapping including tomography, holography, synthetic aperture, image reconstruction, and the relative importance of phase and amplitude information. It is found that according to calculations, VHF backscatter radar has the potential for mapping fractures within a distance of 50 ± 20 meters from the wellbore. A new technique for improving fracture identification is presented. The range of acoustic radar is five to seven times greater than that of VHF radar when compared on the basis of equal resolution, i.e., equal wavelengths. Analyses of extant data indicate that when used synergistically the (1) caliper, (2) resistivity dipmeter, (3) televiewer, (4) television, (5) impression packer, and (6) acoustic transmission are useful for mapping borehole-fracture intercepts. Improvements in both data interpretation techniques and high temperature operation are required. The surveying of one borehole from another appears feasible at ranges of up to 200 to 500 meters by using a low frequency magnetic field generated by a moderately strong dipole source (a solenoid) located in one borehole, a sensitive B field detector that traverses part of the second borehole, narrow band filtering, and special data inversion techniques. A new model of hydraulic fracturing is presented which indicates that a hydraulic fracture is dynamically unstable; consequently, improvements in locating the crack tip may be possible. The importance of phase in signal processing is stressed and those techniques which employ phase data are emphasized for field use.

SECTION 1

INTRODUCTION

This report is submitted under Contract No. DE-AC04-81AL16863 to assist in the planning of future experiments in the Hot Dry Rock program by providing a timely review of (i) the use of both electromagnetic and acoustic radar to map far-field fractures, (ii) the use of conventional well logging tools to map borehole-fracture intercepts, (iii) the use of magnetic dipole ranging to determine the relative positions of the injection well and production well within the fractured zone, (iv) the use of acoustic emissions (microseisms) to map the extent and orientation of hydraulic fractures, and (v) the application of signal processing techniques to fracture mapping. Each topic is examined in some detail. The emphasis in both radar studies, the magnetic ranging study, the acoustic emission study, and the signal processing study is on basic physics, while that of the logging study is on experimental data analysis and interpretation. Important recommendations are made for the better utilization of each technique.

1.1 THE MAPPING OF FRACTURES IN A GEOTHERMAL RESERVOIR BY PULSED ELECTROMAGNETIC RADAR

The widespread use of radar in the everyday world to perform detection, tracking and mapping functions, and some recent experiments underground to explore salt domes /1,2/ and coal mines /3/, and to locate buried objects /4,5/ makes it quite natural to inquire as to whether or not radar can be used in a geothermal reservoir to detect and map the fractures which are so crucial to the operation of a hot dry rock system. The basic operating principles of the above ground and subterranean radars are essentially similar. However, the constitutive properties of the medium, the nature of the radar target, and the borehole geometry introduce physical constraints and technical problems into the design of a subterranean system that are quite different from those of an above ground system. The two most important issues regarding the usefulness of a fracture mapping radar are those of operating range, and the identifying and locating of a fracture. It is shown in this report that according to calculations, the

maximum useful range of a backscatter radar in a granitic medium is of the order of 50 meters. However, more than just a long range is required; prior to hydraulic fracturing, it would be useful to know the location of naturally occurring fractures, but the identification of radar signatures from far-field fractures that do not intersect the borehole is an unresolved problem as is that of separating the signature of a fracture from that of a dielectric or conductive anomaly. On the other hand, fractures that are useful for transferring heat must intersect the borehole, and for this very important class of fractures a positive identification and ranging technique has been devised under this contract: By inflating the fracture with an electrically conducting fluid, say 0.5 molal NaCl, a large dielectric and conductivity contrast is created which greatly increases the strength of the return signal for frequencies in the range of 10^7 to 3×10^8 Hz. These frequencies are quite low compared to those sometimes proposed for fracture detection, but yield a stronger return signal than the higher frequencies for far-field fractures that are 1 to 2 mm in thickness. It is the relatively low frequency that allows the radar to see as far as 50 meters and the high conductivity that reflects waves with wavelength much greater than the fracture thickness. To be truly useful, a backscatter radar must resolve the angular position of a fracture, a task requiring the development of a directive antenna that will fit within the confines of a borehole. Recent experimental efforts have addressed this problem and some success has been claimed /6,7/, although the resolution does not approach that of an above ground antenna. Some theoretical designs for directive borehole antennas have been discussed in the literature /8/. The work described in this report implies that in principle a directional borehole antenna can be obtained by utilizing one or two small magnetic loops in conjunction with electric dipoles. Questions of resonance, matching, and noise level need to be addressed before test antennas are built. Directionality appears to be more important in the receiving antenna than in the transmitting antenna. If the radar is operated with the transmitter and receiver in separate boreholes that straddle the fracture, then the directivity requirements can be relaxed. (In this mode the range is at least twice as great as the backscatter range, which is the range normally quoted in this report.)

The operating range of subterranean radar is analyzed in Section 2. Three different models are used both to bound the problem and to indicate the increase in range that could be obtained by improving antenna performance. Fracture identification is discussed in Section 3, and the use of saline water injection for mapping fractures that intersect the borehole is quantified. Certain aspects of the physics used in Sections 2 and 3 are discussed as needed. These details include electromagnetic propagation in a conductive medium, the conductivity of granitic materials, Rayleigh's criterion, Fresnel's equations, receiver sensitivity, antenna characteristics, temperature effects, the noise fields, the minimum range of a radar, and pulse length. A summary of the radar study is presented in Section 4. The state-of-the-art of radar experimentation and radar hardware is reviewed in Section 8.

1.2 THE MAPPING OF BOREHOLE-FRACTURE INTERCEPTS IN A GEOTHERMAL RESERVOIR USING CONVENTIONAL LOGGING TOOLS

The availability of a large number of well logging tools, developed primarily for characterizing oil and gas bearing sedimentary strata, provides the opportunity to test and evaluate various concepts for detecting fractures in a geothermal well. Because the tools, which we shall call conventional tools, are ready for field use, invoke a wide range of physical phenomena, and have been refined and made more reliable over a span of years, it is wise both from an engineering and economic standpoint to test them in the hot dry rock program. In Section 5, more than twenty tools are evaluated for their ability to accurately map natural fractures, hydraulic fractures, and interwell flow connections. The basis for the evaluation is mainly experimental in nature. It is found that no one tool is able to accurately and unambiguously locate natural fractures. Eight techniques are identified which when used synergistically do a credible job of mapping natural fractures. Three additional methods are quite useful for determining the size and location of hydraulic fractures and interwell connections. The other techniques are of marginal value, although with additional development, one of them might prove useful. In Section 6, the data on natural fractures and weakness planes, obtained from the conventional

logs, are used to develop a fracture model of the Fenton Hill reservoir which has proven useful in a global sense for setting ideas about the structure of the hot dry rock reservoir and in a detailed sense for interpreting micro-seismic data and investigating the performance of backscatter radar. A summary of the work on conventional tools is presented in Section 7. The use of conventional tools in fracture mapping activities in the United States, Canada, and Sweden is reviewed in Section 8.

1.3 DETERMINING THE RELATIVE POSITIONS OF TWO BOREHOLES USING MAGNETIC RANGING TECHNIQUES

The accurate alignment of the injection well, the production well, and the fracture zones, all at great depths in a hot dry rock reservoir, places stringent demands upon conventional wellbore surveying techniques. Experience has shown that as deviations from a straight line build up in a wellbore so do the cumulative errors in measuring the position of the wellbore with a gyroscopic tool. Efforts to map the relative positions of wellbores at depth have resulted in large uncertainties. To look for a remedy to this situation, we examine, in Section 9, the possibility of surveying one wellbore from a second wellbore using magnetic techniques. The main idea of magnetic borehole ranging is that by placing a magnetic dipole of known source strength (a solenoid) in one borehole and by measuring the vector components and the phase of the a.c. magnetic field in a distant borehole, both the range and the orientation of one borehole with respect to another can be determined. It is found that by using a low frequency magnetic field with $2 < f < 100$ Hz, a sensitive B field detector, a moderately strong dipole source with a magnetic moment of 10^4 to 10^5 A-m², and a narrow band filtering, ranges of between 200 to 500 meters appear viable. In our opinion, quite an effort would have to be made to get a reliable system into the field.

1.4 THE MAPPING OF FRACTURES IN A GEOTHERMAL RESERVOIR USING ACTIVE AND PASSIVE ACOUSTIC TECHNIQUES

Unlike electromagnetic radar, probing of the earth using stress waves has been performed for many years using several different techniques. Earth structure has been investigated using extremely low-frequency waves (passive

probing) that propagate around the world. At the other extreme the geology in the immediate vicinity of a borehole has been studied using sonic frequency waves (active probing) that have a relatively short range. In Section 10 we investigate (i) the limitations on both acoustic range and the detectability of underground structures, (ii) what technical means are available for optimizing the usefulness of acoustic mapping, and (iii) the similarities and differences between electromagnetic and acoustic radar. It is shown that for $f < 30$ kHz the minimum detectable signal strength is set by seismic noise; at higher frequencies the limit is imposed by thermal noise. Acoustic dissipation in granite is less than dielectric dissipation in the same medium at the same wavelength. The end result is that for equal wavelengths the probing range of an acoustic system is five to seven times greater than that of an electromagnetic system.

Acoustic emission or microseismicity associated with hydraulic fracturing has been used to map the extent and orientation of fractures. As currently practiced the observed seismic emission almost assuredly has originated from shear slippage on naturally occurring weakness planes that cut across the hydraulic fracture. In Section 11 we show that it is quite likely that hydraulic fracturing is a dynamically unstable process capable of emitting both P- and S-waves of about equal amplitude from tensile failure at the crack tip. The notion of a stable crack tip, which is one of the most commonly employed models of hydraulic fracturing, appears to be incorrect.

1.5 THE PROCESSING OF SIGNALS FOR DETECTING FRACTURES

Interpreting geophysical data is a crucial aspect of mapping fractures. A large number of data processing techniques have been employed by geophysicists, especially those involved with petroleum exploration. In the past twenty years advances in optical imaging and medical diagnostics have sparked the application of new methods to geophysical exploration. In Section 12 we evaluate the usefulness of (i) tomography, (ii) holography, (iii) synthetic aperture, and (iv) wavefront imaging to fracture mapping. We draw heavily on recent work on the importance of phase and amplitude in signal reconstruction. We conclude that phase information is crucial;

consequently, tomography that employs only amplitude (as currently practiced by research geophysicists) is of limited value. The use of holography, which is a compact way of storing both phase and amplitude data, is in our view premature in fracture mapping because of the difficulty in interpreting data, no matter how phase and amplitude data are manipulated. Because of the constraints imposed by a borehole and the geometry of a geothermal reservoir the formation of a large antenna through synthetic aperture techniques appears to be promising.

SECTION 2

ELECTROMAGNETIC RADAR PROBING DISTANCE

The range of an aboveground radar is measured in kilometers, that of a subterranean radar in meters. The thousand-fold reduction in range is caused principally by dissipation in the lossy geologic medium. Therefore, it is worthwhile to examine the effects of losses alone, unencumbered by problems of antenna design and fracture reflectivity, in order to place an upper bound on the maximum useful range that can be attained underground. Once this is accomplished, the extent to which the antenna radiation pattern and fracture reflectivity reduce the range can be clearly seen. That this is a profitable approach is borne out by the results themselves. For example, at 100 MHz, a hypothetical backscatter radar set that produces a perfectly collimated beam has a range of 56 meters. With a quarter-wave dipole antenna, which has rather poor directivity, the range is reduced by a factor of 1.7 to 33 meters. If it were possible to build a high gain antenna which would fit into a practical sonde (and such an antenna does not now exist), then the range would be reduced from the perfect case by a factor of only 1.3 to 42 meters. Thus, the upper bound is sufficiently close to what might be achieved in practice that it does indeed provide a useful estimate. Furthermore, the highly directive antenna provides a 27-percent increase in range over that of a simple dipole so that the effort to produce such an antenna is worthwhile, but not crucial in terms of range. (In terms of direction-finding capabilities in the backscatter mode, a narrow beam antenna is essential.)

2.1 UPPER BOUND RANGE MODEL

It is clear that the greatest range attainable by a radar would occur with a hypothetical beam with zero divergence and a target with perfect reflectivity. The range limitation for a given transmitter and receiver is then fixed mainly by dissipation in the medium, and the situation is entirely analogous to the propagation of plane waves in a conductive medium. To proceed, we write Maxwell's equation in the form of the wave equation for the electromagnetic field as

$$\nabla^2 \vec{E} - \epsilon\mu \frac{\partial^2 \vec{E}}{\partial t^2} - \mu\sigma \frac{\partial \vec{E}}{\partial t} = 0 \quad (2.1)$$

and seek plane wave solutions for the electric field in the form

$$\vec{E} = E_1 e^{-i\omega t + \vec{\gamma} \cdot \vec{r}} + E_2 e^{-i\omega t - \vec{\gamma} \cdot \vec{r}} \quad (2.2)$$

where the constitutive properties of the medium are represented by σ , the conductivity, ϵ , the permittivity, and μ , the permeability, all of which may be functions of frequency, temperature and pressure. Implicit in this formulation of the constitutive properties is that the losses arise from currents controlled by the conductivity and driven by the electric field. An alternate description is often employed for dielectrics in which the permittivity is assumed to be a complex quantity and the conductivity is effectively replaced by the imaginary part of the permittivity. A second source of loss (and range reduction) is associated with the oscillation of magnetic dipoles in ferromagnetic materials, such as magnetite. To include magnetic losses, the permeability should be treated as a complex quantity. We have chosen not to do so because extant data for geologic media do not consistently separate the effects of electric and magnetic losses. To continue,

$$\gamma^2 = \mu\epsilon\omega^2 + i\sigma\mu\omega.$$

For convenience, we set

$$\gamma = \alpha + i\beta,$$

and solving for the attenuation factor, α (which is always a positive quantity), and the phase constant, β , we obtain

$$\alpha = \omega \left[\frac{\mu\epsilon}{2} \left(\sqrt{1 + \left(\frac{\sigma}{\epsilon\omega} \right)^2} - 1 \right) \right]^{1/2}, \quad (2.3a)$$

$$\beta = \omega \left[\frac{\mu\epsilon}{2} \left(\sqrt{1 + \left(\frac{\sigma}{\epsilon\omega} \right)^2} + 1 \right) \right]^{1/2}. \quad (2.3b)$$

The phase velocity is given by

$$c = \frac{\omega}{\beta} = \frac{c_0}{\sqrt{\kappa_e \kappa_m}} \left[\frac{2}{\sqrt{1 + \left(\frac{\sigma}{\epsilon \omega}\right)^2} + 1} \right]^{1/2}, \quad (2.3c)$$

where the specific inductive capacity, κ_e , is the ratio of the permittivity of the medium to that of vacuum (ϵ/ϵ_0), the specific magnetic permeability, κ_m , is the ratio of the permeability of the medium to that of vacuum (μ/μ_0), and c_0 is the speed of light in vacuum ($\approx 3 \times 10^8$ m/s). All three quantities, α , β , and c , are controlled by $\sigma/\epsilon\omega$, which is called the loss tangent. From Maxwell's equation, it is seen that the loss tangent is the ratio of dissipative currents to the vacuum displacement current. Two limiting cases are usually discussed / 9/.

Case I: $(\sigma/\epsilon\omega)^2 \ll 1$. The medium is either only slightly dissipative or the frequency is very high. By expanding α , β , and c in powers of $(\sigma/\epsilon\omega)^2$, the expressions are simplified to yield

$$\begin{aligned} \alpha &= \frac{\sigma}{2} \sqrt{\frac{\mu}{\epsilon}} = 188.3\sigma \sqrt{\frac{\kappa_m}{\kappa_e}} \\ \beta &= \omega \sqrt{\mu\epsilon} \left[1 + \frac{1}{8} \left(\frac{\sigma}{\epsilon\omega}\right)^2 + \dots \right] \\ c &= c_0 / \sqrt{\kappa_m \kappa_e} \end{aligned}$$

The wave is dispersive, i.e., c depends upon ω , only insofar as the constitutive properties are frequency dependent. The spatial attenuation rate is frequency dependent only through the constitutive properties.

Case II: $(\sigma/\epsilon\omega)^2 \gg 1$. The medium is either highly conductive (a metal) or the frequency is low (the situation more commonly studied in geophysical surveying of large areas). In this case

$$\alpha = \beta = \sqrt{\frac{\mu\sigma\omega}{2}} = \frac{1}{\delta} ,$$

$$c = \sqrt{\frac{2\omega}{\mu\sigma}} ,$$

where δ is defined as the skin depth. The wave is dispersive as shown by the expression for the phase velocity. The spatial attenuation rate is explicitly dependent upon frequency, increasing with the square root of the frequency, and is implicitly dependent upon the frequency through the conductivity and permeability.

Because of the great difference in electromagnetic behavior between Case I and Case II, it is worthwhile to determine if the granitic medium at radar frequencies acts either as a low-loss dielectric or as a good conductor. The characterization is done in terms of the loss tangent. For $\sigma/\epsilon\omega < 0.1$, the medium acts as a low-loss dielectric, while for $\sigma/\epsilon\omega > 10$, the medium acts as a good conductor. Using laboratory data for the conductivity of granite /3,10-15/ and for the permittivity /11-15/, the loss tangent is plotted as a function of frequency in Figure 2.1. It is important to bear in mind that the data were obtained from small samples at room temperature and atmospheric pressure and that in situ values in a geothermal reservoir may be different. Using the extant data, we find that at frequencies greater than 2.5×10^7 Hz, wet granite behaves as a low-loss dielectric. Note that for $f \gtrsim 10^8$ Hz, the loss tangent of wet granite is a very slowly decreasing function of frequency and has an average value of 0.045. Thus, the attenuation factor increases directly with frequency, i.e., the data for wet granite can be fit by the expression $\alpha \doteq 1.15 \times 10^{-9} f$, where α is in nepers/meter and f is in Hz.

2.1.1 The Electrical Conductivity of Granite

The conductivity of granite has such a strong effect on radar performance (as shown in Subsections 2.2-2.4) that it is worthwhile to dwell on the subject at greater length. The conductivity data used to plot the loss tangent of wet granite in Figure 2.1 are themselves plotted in Figure 2.2, along with data for dry granite. Three concerns that arise when examining the data are addressed in this section; namely, the scatter in the data, the

Figure 2.1. Loss tangent vs. frequency with conductivity as a parameter. The specific inductive capacity, $\kappa_e = 6$ and the specific magnetic permeability, $\kappa_m = 1$.

Figure 2.2. The conductivity of granite as a function of frequency, measured in the laboratory. Wet granite: $\blacktriangle$ Ref. 3, $\bullet$ Ref. 10, $\blacksquare$ Ref. 14 (Fenton Hill). Dry granite: $\triangle$ Ref. 3, ∇ Ref. 11 (1 Westerly, RI, 2 Llano, TX, 3 Rockport, MA), $\times$ Ref. 12 (black & white), $+$ Ref. 12 (rusty), $*$ Ref. 12 (Raymond), $\diamond$ Ref. 13 (Cold Spring granite), $\square$ Ref. 14 (Fenton Hill), $\circ$ Ref. 14 (1 Westerly, RI, 2 Berkeley).

accuracy with which the laboratory data represents in situ properties, and trends in the data.

Most of the data are plotted in such a way that the scatter arises from genuine differences in constitutive properties between granites from different locations. This is particularly true of the dry granite data from References 11 and 12. Thus, the performance of radar in dry granite would be expected to vary from site to site. A second type of data scatter has been averaged out in most cases and only one datum is plotted for each type of granite at each discrete frequency. This scatter arises from the use of laboratory-size samples which are small compared to the scale of inhomogeneities in naturally occurring granites. Hence, different samples from what is nominally the same piece of granite may have different properties. The effect is quite striking, for example, in waveguide tests where the measured value of the conductivity often depends upon whether the arbitrarily designated front or back surface of the test specimen faces the incident wave. The magnitude of the scatter from sample to sample for specimens extracted from the same location is illustrated by the data plotted from Reference 14. These data are of particular interest to the geothermal program because they were obtained with samples removed from the hot dry rock wellbores at Fenton Hill. The scatter in the data is fairly large, amounting to a factor of nearly three from the lowest to the highest values at X-band (8 to 9 GHz). From what little data are available on repetitive runs on the same sample, it appears that the measurements were made to a precision of about 10 percent. The factor of three scatter thus appears to be the result of geologic variations.

Complete details of experimental procedures were not reported by the experimentalists, so it is not possible to assess the relative accuracy of the measurements. A brief review of the techniques is as follows: The data from Reference 3 were obtained on a Hewlett-Packard type 4815A impedance meter using small samples of gray granite and gneiss removed during construction of a vehicular tunnel in Switzerland. Measurements were made on dry samples which were in an "as received" condition and on wet samples which were prepared by soaking in tap water for an unspecified period of

time. As discussed below, we are concerned about the accuracy of the measurements for $f \geq 2.5 \times 10^7$ Hz. The data from Reference 10 were obtained at $10^3 \leq f \leq 10^5$ Hz using a standard Schering bridge technique (although important details of the sample holder were not revealed) and at $f = 9.15 \times 10^8$ and 2.45×10^9 Hz using a novel microwave heating technique. The low frequency data are not plotted in Figure 2.2, but rather are used as a basis for extrapolating the data to $f \geq 10^6$ Hz; thus, the straight line plot in Figure 2.2 is not a fit to the plotted points but rather is a means for comparing the high frequency and low frequency data. Samples of granite were obtained from a bedrock outcropping in north-central Georgia and for the wet measurements were soaked in a 1.0 N NaCl solution to provide a pore water with a known ionic concentration, even if that concentration were not representative of in situ values. The authors of Reference 10 believe that because of the high salinity their conductivity measurements at microwave frequencies are an order-of-magnitude greater than would occur in situ, but we note that the two data points plotted in Figure 2.2 are not far different from laboratory measurements made by other experimenters. Because of this relatively good agreement and because the high frequency lossiness of water is controlled more by the dipole moment of the water molecule than by ionic conduction, we believe that the data of Reference 11 are accurate to within a factor of two, rather than ten. The data from Reference 11 were obtained using well-established techniques /16/. At 4.5×10^8 Hz, a reentrant cavity forming part of a resonant circuit was employed. At 3.5×10^{10} Hz, voltage standing wave ratio measurements were made on a waveguide with a section loaded with granite and followed by a moveable short. Dry samples were employed from Westerly, Rhode Island, Llano, Texas, and Rockport, Massachusetts. The data from Reference 12 were obtained with samples of black and white, rusty, and Raymond granite. The data from Reference 13 were obtained with Cold Spring granite. Neither of these last two references contained details of the measurement techniques. The data from Reference 14 were obtained with samples extracted from wellbores EE-1

and GT-2 at Fenton Hill, with control samples of Westerly and Berkeley granite. Measurements were made of the voltage standing wave ratio using a granite-loaded short circuited waveguide with a slotted line.

The wet data have been fit by a curve as indicated in Figure 2.2, and this fit has been used in subsequent calculations to describe the conductivity of granite as a function of frequency. A curve has not been fit to the dry data, but the extrapolation of the low frequency data does provide a fair approximation to the high frequency dry data. The conductivity of wet rock is not much different from that of dry rock at frequencies greater than 1 GHz, but at low frequencies the wet rock is considerably lossier than dry rock. Qualitatively, most of the data appears to have the proper trend. One exception is the wet data from Reference 3, which indicates that the conductivity of wet granite approaches a constant value for $f \geq 2.5 \times 10^7$ Hz. This behavior is hard to reconcile with all of the other data and with the behavior of the components of wet granite, i.e., the conductivity of dry granite is an increasing function of frequency, Figure 2.2, as is that of water /16/. The roll-off in conductivity would be significant if it actually occurred because it would allow a greater maximum radar range than if the conductivity continued to increase with frequency. For the reasons previously advanced, we have discounted the data from Reference 3 at $f \geq 2.5 \times 10^7$ Hz and have used instead the fit to the wet data as shown in Figure 2.2.

Laboratory measurements are needed on granite subjected simultaneously to elevated temperature and pressure. Measurements on distilled water at 25°C and 85°C show that the dielectric constant and conductivity is a function of both frequency and temperature. Conductivity measurements on granite at normal atmospheric pressure, but elevated temperature /17/ show that within the range $10 \leq f \leq 10^6$ Hz the conductivity decreases very slowly with increasing temperature up to a limiting temperature; thereafter the ac conductivity has the same behavior as the dc conductivity, that is to say, the ac conductivity increases with increasing temperature and has essentially the same value as the dc conductivity. At $f = 10^6$ Hz the

limiting temperature is approximately 900°C, far in excess of currently utilized geothermal temperatures. Based on this small amount of data the use of room temperature conductivity measurements is a conservative procedure and will not lead to an overestimate of the range of radars. Although it is often risky to predict in situ values from laboratory measurements, we believe that laboratory measurements of the electrical properties of granite at elevated temperature and pressure would increase the accuracy of estimates of radar performance.

2.2 THE RANGE OF A PERFECTLY COLLIMATED ELECTROMAGNETIC RADAR BEAM IN A GEOTHERMAL RESERVOIR

For a perfectly collimated beam and a perfect reflector, the relation between the radar range, R , the transmitter power, P_t , the minimum usable signal power at the receiver, P_r , and α follows directly from Equation 2.2,

$$\frac{P_r}{P_t} = e^{-4\alpha R}, \quad (2.4)$$

where α is given in nepers/meter by Equation 2.3a. In the engineering literature, the attenuation is usually expressed in decibels/meter so that

$$A_{\text{db/m}} = 8.686 \alpha .$$

The upper bound on the maximum range is expressed as

$$R = \frac{5.0}{A} \log_{10} \frac{P_t}{P_r} . \quad (2.5)$$

The transmitter peak power is set by considerations of size, power dissipation, cost, and the fact that the range increases only logarithmically with power. The minimum useful received power depends upon the noise generated external to the receiver and various aspects of receiver design. In a deep subterranean setting, the principal sources of noise at microwave frequencies will arise from the thermal oscillations of the molecules

of the rock and from within the circuits of the receiver. The bandwidth, coupling losses and signal averaging techniques, all affect the minimum power necessary to produce a useful output signal. The received thermal noise is given by

$$N = kT\Delta f$$

where k is Boltzmann's constant ($k = 1.38 \times 10^{-23}$ J/K), T is the effective temperature of the noise source, and Δf is the bandwidth of the receiver. Because the medium is lossy, all of the energy radiated by the transmitter is absorbed within the rock; thus, the medium is a black body at microwave frequencies and is characterized by its actual temperature. (Above ground the effective temperature depends upon the frequency and the inclination of the antenna and can vary between a few degrees Kelvin to several thousand degrees Kelvin.) Because the temperature of a geothermal reservoir is two to five times greater than the noise temperature of a microwave receiver (suitably dewatered in a sonde) the noise power is dominated by the received external noise, and within certain limits is independent of antenna design. This is one of the reasons why it is worthwhile to examine the use of a small magnetic loop for a directional borehole antenna. The bandwidth must be wide enough to respond fully to the energy contained within one pulse, but once this is accomplished, making the bandwidth any wider causes the detected noise to increase in direct proportion to the bandwidth without any further increase in the signal. It turns out that the optimum bandwidth depends upon the transmitter pulse shape and the pass band characteristics of the intermediate frequency amplifier so that $0.5 < \Delta f\tau < 1.5$, where τ is the pulse length [18]. For our purposes, it is sufficient to assume that the bandwidth is equal to the inverse of the pulse length, i.e., $\Delta f\tau = 1$. The receiver noise figure is a measure of how much noise is added by the receiver itself and is defined as the ratio of the input signal-to-noise power ratio, S_i/N_i to the output signal-to-noise power ratio, S_o/N_o .

$$F \equiv \frac{S_i/N_i}{S_o/N_o} .$$

It is readily shown that only the first few stages of a superheterodyne receiver contribute significantly to the noise, for in a cascaded amplifier the noise addition of successive stages is inversely proportional to the gain of the preceding stages /18/. Thus, based on our data for microwave frontends /19/, we chose $F = 1$ db for our calculations. As a rough estimation to actual practice, we have taken coupling losses and signal averaging techniques, lumped them together, and state that a usable signal must be 3 db above the output noise level. Hence, the minimum useful signal in our model is

$$S = 2.5 kT/\tau ,$$

which gives a maximum range of

$$R = \frac{5.0}{A} \log_{10} \frac{P_t \tau}{2.5 kT} \quad (2.6)$$

This form of the range equation emphasizes two important points. First, It is the energy contained within a radar pulse rather than the peak transmitter power that is fundamental in determining the maximum range. Second, the higher the reservoir temperature, the shorter will be the range. The pulse length is set by the minimum range that is to be explored, i.e., the transmitter must be quiescent before a return echo can be detected. For $f \geq 10^8$ Hz, the minimum range has been selected as 2 meters, which gives a pulse length of 33 nsec for $\kappa_e = 6$. At lower frequencies, the minimum range must increase because even a solitary half-sine pulse has a relatively long duration, e.g., at $f = 10^7$ Hz, a half-sine monopulse has a width of 50 nsec, which gives a minimum range of 3.1 meters.

If in actual practice the range were actually limited by noise and not by clutter then the fundamental importance of the signal energy implies that the range could be extended by integrating over many pulses. For example, with the aforementioned pulse lengths, a duty cycle of five, and a one second integration time the range would be extended by a factor of about 1.5. The maximum range as a function of frequency and peak radiated power, subject to the previously discussed conditions, at a reservoir temperature of 280°C (550°K) is plotted in Figure 2.3a. The decrease in range with increasing frequency, Figure 2.3a, reflects the increase in conductivity with increasing

Figure 2.3a. Maximum upper bound radar range vs. frequency and peak transmitter power with $\Delta f \tau = 1$ and a reservoir temperature of 280°C (550°K). The receiver noise figure is 1 db. Losses and the detectability criterion add another 3 db to the minimum usable signal strength. Three cases are considered: (1) a perfectly collimated beam and a perfect specularly reflecting target, (2) a pencil beam and a perfect diffusely reflecting target ($1/R^2$ -law), (3) a dipole radiator and a perfect diffusely reflecting target ($1/R^3$ -law). The degree of uncertainty in the calculated results, arising from scatter of constitutive properties data, is of the same order as the spread between a perfect beam and a dipole radiator, i.e., ± 30 to $\pm 40\%$. For each beam group the upper curve is for a 10 kW transmitter and the lower curve is for 10 watts. Uniform homogeneous rock.

Figure 2.3b. Maximum upper bound electromagnetic radar range vs. frequency and total radiated energy. Other conditions identical with those described in Figure 2.3a.

frequency, Figure 2.2. The scatter in the conductivity data between individual samples taken from the same location /14/ leads to an uncertainty in the calculated range of approximately $\pm 40\%$. It is clear from the results of this simple model that radar can have a potentially useful range in a geothermal reservoir of the order of 10 to 100 meters, and hence it is worthwhile to proceed with the formulation of more accurate models.

2.3 THE RANGE OF A NARROW BEAM RADAR ILLUMINATING A LARGE PLANAR FRACTURE

The analysis of Subsection 2.1 predicts a greater range than can be achieved in practice because it ignores the reduction of signal strength caused by the geometrical spreading of both the transmitted beam and the return wave. This defect is now remedied by estimating the maximum range of a radar set employing a relatively high gain antenna, e.g., one with a beam spread of less than one radian, illuminating a large planar fracture. On physical grounds, it is clear that for a relatively narrow beam incident on a target which is much greater in lateral extent than the beam itself, all of the transmitted power falls on the target, and it is the spreading of the return signal which is responsible for a $1/R^2$ diminution of power at the receiving antenna. This argument can be quantified either exactly by solving a boundary value problem or approximately by adapting a phenomenological theory of sea echoes originally proposed by Goldstein /20/. We adopt the latter approach which is sufficient for indicating the influence of antenna design and large planar fractures on radar range.

For the setup sketched in Figure 2.4, assume that the fracture plane is composed of a large number of independent scattering sites. The power incident on any one scatterer (the j^{th} scatterer) is

$$P_{ij} = \frac{G_t F^2(\theta, \phi, R_j) P_t \sigma_j}{4\pi R_j^2} .$$

The pattern propagation factor, F , is defined as the ratio of the amplitude of the electric field that exists under actual operating conditions at some point in the medium (specified by the coordinates θ, ϕ, R with respect to the electric axis of the antenna) to the amplitude of a hypothetical electrical field that would exist in free space if the antenna were directed at

Figure 2.4. Narrow beam antenna illuminating a large planar fracture. Special case with beam lying in the plane of the borehole and the normal to the fracture surface from the radar set.

the point under observation. In symbols,

$$F \equiv \left| \frac{E}{E_0} \right| ,$$

where E is the actual electric field at the point under observation, and E_0 is the hypothetical field along the beam axis in free-space. Because of the importance of absorption, F is separated into two parts,

$$F = f(\theta, \phi) e^{-\alpha R} .$$

The antenna pattern function, $f(\theta, \phi)$, is defined as the ratio of the free-space electric or magnetic field strength in the direction (θ, ϕ) to the free-space field in the direction of maximum radiation

$$f(\theta, \phi) \equiv \frac{E(\theta, \phi)}{E_0} \leq 1 .$$

The antenna gain for transmission, G_t , is defined as the ratio of the power radiated by an isotropic antenna, P_1 , necessary to produce a given field strength or Poynting vector, S_1 , at some given distance to the power radiated by a directional antenna, P_2 , producing the same field strength or Poynting vector at the same distance in the direction of maximum transmission. Thus,

$$G \equiv \frac{P_1}{P_2} = \frac{S_1 \int_0^{4\pi} d\Omega}{S_1 \int_0^{4\pi} |f(\theta, \phi)|^2 d\Omega} = \frac{4\pi}{\int_0^{4\pi} |f(\theta, \phi)|^2 d\Omega}$$

The antenna gain depends only on the pattern function. P_1 is the value of the transmitter power, which is assumed here to have a constant value for the duration of one pulse. The cross section of a scatterer σ_j is defined for a linearly polarized wave as the area intercepting that amount of power which, when scattered isotropically, produces a return signal equal to that observed from the target. Symbolically,

$$\bar{S}_r = \frac{\bar{S}_1 \sigma}{4\pi R^2} ,$$

where $\bar{S}_r$ and $\bar{S}_i$ are the average values of the Poynting vector at the receiver and target, respectively.

The power reflected from the j^{th} scatterer and incident upon a receiving antenna with an effective absorption area or aperture A_e , is

$$P_{rj} = \frac{G_t^2 f_t^2(\theta, \phi) f_r^2(\theta, \phi) e^{-4\alpha R_j} j P_t A_e \sigma_j}{(4\pi)^2 R_j^4} .$$

By assuming that the transmitter and receiver antenna are identical and using the definition of gain for a resonant, matched antenna [21/

$$G_r = \frac{4\pi A_e}{\lambda^2} ,$$

the expression for received power is

$$P_{rj} = \frac{G^2 \lambda^2 f^4(\theta, \phi) e^{-4\alpha R_j} j P_t \sigma_j}{(4\pi)^3 R_j^4}$$

which follows directly from the preceding definitions. The assumption of independent scatterers (and this is the crucial assumption of the analysis), allows the total received power from the fracture surface to be considered as the sum of all of the individual returns,

$$P_r = \frac{G^2 \lambda^2}{(4\pi)^3} \sum_{j=1}^n \frac{f^4(\theta, \phi) e^{-4\alpha R_j} j P_t \sigma_j}{R_j^4} .$$

Assuming a large number of scatterers uniformly distributed with a density, $N(\sigma)$, and using the (z, θ, ϕ) coordinates of Figure 2.4 with the fracture plane making an angle, ζ , with respect to the borehole, the summation is replaced by the integral

$$P_r = \frac{P_t G^2 \lambda^2}{(4\pi)^2 z^2} \int_{-\pi}^{+\pi} \int_0^{\pi/2} \int_0^{\infty} f^4(\theta, \phi) e^{-4\alpha z / \sin(\theta + \zeta)} \sigma N \sin(\theta + \zeta) d\theta d\phi d\sigma ,$$

where we have considered only the special case for which the radar beam lies in the plane containing the borehole and the normal to the fracture surface from the position of the radar set. The integral over σ is a dimensionless cross section whose value cannot be determined by this simple theory. Although not exact, we shall identify the dimensionless cross section with the plane wave reflection coefficient, r , which is defined as the ratio of the normal components of the reflected Poynting vector to the incident Poynting vector. On physical grounds, it is clear that because of the reduction in signal strength arising from dissipation the upper limit on the integral over ϕ need only extend to some finite (but as yet unspecified) value. The integral over ϕ defines a beam width, ϕ . For a very narrow beam in the θ direction, the function, $f^4(\theta)$ varies much more rapidly than the other quantities in the integrand and they may therefore be taken outside the integral. The remaining integral over $f^4(\theta)$ defines a beam width, θ . This result holds only for a narrow beam. If $f^4(\theta, \phi)$ varies over a large region of space, then the particular antenna pattern function must be integrated along with the other terms in the integrand. Thus, the received power from a narrow beam reflected from a large plane surface is

$$P_r = \frac{P_t G^2 \lambda^2}{(4\pi)^3 z^2} \theta \phi r e^{-4\alpha R} \sin(\theta + \zeta) . \quad (2.7)$$

At near normal incidence, the result is

$$P_r = \frac{P_t G^2 \lambda^2 \theta \phi r e^{-4\alpha R}}{(4\pi)^3 R^2} . \quad (2.8)$$

In addition to the attenuation caused by the complex permittivity, the received signal is further reduced by the square of the distance between the radar set and the reflecting surface. The reduction in range caused by $1/R^2$ -spreading can be seen in Figure 2.3a by comparing this result with that of the perfectly collimated beam. In evaluating Equation 2.8, we have assumed that $\theta = \phi = 1$ radian, and that $G = 4\pi/(\theta\phi)$.

In addition the minimum useable signal power was set equal to 4 db above the noise level as discussed in Section 2.2. For a resonant, matched antenna the received noise power is

$$N = \int_0^{4\pi} \frac{4\pi}{\lambda^2} kTBA_e d\Omega = \int_0^{4\pi} GkTBd\Omega = kTB ,$$

where B is the bandwidth, i.e., $B = \Delta f$. For nonresonant operation both the received signal and received noise are reduced in the same proportion. Over the frequency band considered $10^7 \leq f \leq 10^{10}$ Hz, $1/R^2$ spreading results in a reduction of range that is less than 25 percent of that of a perfectly collimated beam. The $1/R^2$ dependence for near-normal incidence on the surface of the sea has been confirmed by experiments with airborne radar; we expect the same relationship to hold in a subterranean setting if a narrow beam antenna can be devised to fit within a sonde.

2.4 THE RANGE OF A WIDE BEAM RADAR ILLUMINATING A LARGE PLANAR FRACTURE

We now turn to estimating the maximum range of a radar with a relatively wide beam such that (i) only a fraction of the radiated power falls on the fracture plane, and (ii) the antenna pattern function is essentially constant, i.e., the antenna is very nearly an isotropic radiator. We do so for several reasons. At the present time, a narrow beam borehole radar for $f \leq 10^9$ Hz does not exist. For $f > 10^9$ Hz, where the wavelength is comparable to or less than the borehole diameter, it should be entirely possible to build a directional antenna, but because of the relatively short range of radar operating at $f \geq 10^9$ Hz and because of a new fracture identification technique reported in Section 3 which must operate with $f \leq 3 \times 10^8$ Hz, we concentrate on VHF radar. By considering a fracture plane to have effective radar backscatter cross section, σ , and setting $f(\theta, \phi) = 1$, the expression for the received power is

$$P_r = \frac{P_t G^2 \lambda^2 e^{-4\alpha R} \sigma}{(4\pi)^3 R^4} . \quad (2.9)$$

The problem is now one of determining σ . We employ three different models in an attempt to bound σ and to understand the effects of various models on the calculated range. The first model is derived from geometrical optics and places an upper bound on the cross section. If it is assumed that the fracture plane is a perfect reflector of great lateral extent then the strength of the received backscattered signal is the same as that produced by an image transmitter located a distance $2R$ from the receiver, i.e.,

$$P_r = \frac{P_t G^2 \lambda^2 e^{-4\alpha R}}{64\pi^2 R^2} .$$

Comparison with Equation 2.9 yields $\sigma = \pi R^2$. The expression for the received power is formally identical with Equation 2.8 if $\theta = \sqrt{\pi}$ radians.

A model that apparently was first developed to explain reflections from the earth's surface /20/ and later employed to set the backscatter cross section in subterranean radars /3/ involves the use of Fresnel zones. In calculating reflections from the earth's surface, the analysis of wave interference effects leads to the result that the main contribution to the reflection arises from the first Fresnel zone. Recall that Fresnel zones in a lossless medium are constructed in such a way, either on a wave front or on a reflecting surface, that the amplitude of excitation associated with each zone varies slowly from zone to zone, but that a phase reversal occurs between adjacent zones so that at an observation point the contributions of successive zones essentially cancel each other except for the radiation from half the area of the first zone and from the entire last zone. The cancellation comes about, because between the transmitter and the receiver the path lengths to and from adjacent zones differ by $\lambda/2$. In the backscatter mode the path length difference to adjacent zones is $\lambda/4$; the remaining $\lambda/4$ is obtained on the return path. It is usually supposed that a lack of longitudinal coherence causes the contribution from the last zone to vanish, leaving only the contribution from one-half of the first zone /22/. Thus, in the backscatter mode $\sigma \doteq \frac{R\lambda}{4}$. The Fresnel zone argument holds for both electromagnetic and acoustic radiation (Section 10.1.3) and strictly applies

only to normal incidence in a lossless medium. At oblique incidence, the zones are elliptical, and cancellation is not assured between adjacent zones. In a lossy medium, the amplitude varies between adjacent zones and exact cancellation will not occur, but only a relatively small number of zones need be considered in the determination of the cross section because of the reduction in amplitude as the zone number increases. To account for imperfect reflection we reduce the Fresnel zone backscatter cross section by a reflection coefficient, r . Equation 2.9 then becomes

$$P_r = \frac{P_t G_{\lambda}^2 R^3 e^{-4\alpha R}}{256\pi^2 R^3} \quad (2.10)$$

The received signal is reduced by the cube of the range between the radar set and the reflecting surface, over and above that caused by dielectric losses. The Fresnel zone backscatter cross section is not only less than that of a large plane reflector in the ratio $\lambda/4R$, but at the maximum estimated range it is also less than the normal incidence backscatter cross section of an isolated reflector such as might occur in a hot dry rock reservoir. The backscatter cross section of a perfect, planar, circular reflector is $\lambda^2/20$

$$\sigma(\theta) = \frac{4\pi A^2}{\lambda^2} \left\{ 4 \left(\frac{J_1(2ka \sin \theta)}{2ka \sin \theta} \right)^2 \right\} \cos^2 \theta \quad ,$$

where A is the area of the reflector, θ is the angle of incidence, and J_1 is the Bessel function of the first kind of order 1. At normal incidence the cross section is a maximum and is given by

$$\sigma_{\max} = \frac{4\pi A^2}{\lambda^2} \quad .$$

The formulas are strictly valid only for plane waves; we thus restrict the lateral extent of the reflector to ensure that phase variations across the reflector are less than $\lambda/36$. Hence, the radius of the planar reflector cannot exceed $\sqrt{R\lambda/18}$, which is one-third the radius of the first Fresnel

zone. The radius $\sqrt{R\lambda/18}$, its associated $\sigma_{\max}$, the radius of the first Fresnel zone, one-half the area of the first Fresnel zone, and the maximum radius of a planar reflector inferred from geophysical logs (Section 6) are listed in Table 2.1. At $f = 10^7$ Hz the $\lambda/36$ criterion leads to a radius that exceeds that permitted by the geophysical logs. If the radius is set equal to the maximum permitted by the logs then the backscatter cross section still exceeds half the Fresnel zone area by a factor of almost two. For the case at hand it appears that the backscatter cross section lies between that of a large planar reflector and one-half the area of the first Fresnel zone. The reduction in range caused by the $1/R^3$ behavior can be seen in Figure 2.3a by comparing this result with that of a perfectly collimated beam and with a narrow beam. The effect of radiating a fixed amount of energy e.g., 1 joule or 100 joules, is illustrated in Figure 2.3b. One way to increase the radiated energy in a pulsed system is to integrate over many pulses, rather than to increase the peak power or the pulse length, both of which are disadvantageous. The plot in Figure 2.3b affords a useful way to compare electromagnetic radar with acoustic radar (see Figure 10.8). In evaluating Equation 2.10, it was assumed that the antenna was a resonant, matched half-wave dipole with $G = 1.64$. Over the frequency band considered, $10^7 \leq f \leq 10^{10}$ Hz, the use of a half-wave dipole reduces the range by less than 25 percent of that of a highly directive antenna. The main problem with the dipole is not the reduction in range, but rather the inability to resolve the angular position of a fracture.

Table 2.1. Cross Section and Fracture Dimensions

f Hz	$\sqrt{R\lambda/18}$ (m)	$\sigma(R\lambda/18)$ (m ²)	Fresnel Zone Radius (m)	$\frac{1}{2}$ Fresnel Zone Area (m ²)	Radius Planar Reflector (m)
10^7	11.5	1.44×10^4	34.4	1.86×10^3	10
10^8	1.7	7.0×10^2	5.1	40.9	10
10^9	0.21	14.7	0.6	0.59	4
10^{10}	0.021	0.18	0.06	6×10^{-3}	1

2.5 THE EFFECTS OF THE REFLECTION COEFFICIENT, RESERVOIR TEMPERATURE AND FRACTURE ROUGHNESS ON RADAR PERFORMANCE

The calculations of Sections 2.2-2.4 are useful for estimating the maximum range of a subterranean radar and for comparing the relative effects of dielectric losses, beam spreading and transmitter power. The calculations were made with $r = 1$ and $T = 280^{\circ}\text{C}$ (550°K). Since the reflection coefficient will, in general, be less than one, and the temperature can be either greater or smaller than 280°C , it is important to estimate the variation in range with r and T . Moreover, in one of the calculations, it was assumed that the reflection was specular, while in the other two, it was assumed that the reflection was diffuse. We now quantify the effects of the reflection coefficient, the reservoir temperature and the nature of the reflecting surface.

2.5.1 The Variation of Range with Reflection Coefficient

In the formulas for the range, the transmitter power and the reflection coefficient always appear as a product so that the reduction in range caused by the reflection coefficient having a value less than 1 can be estimated immediately from Figure 2.3 if the plots for $P_t = 10$ watts, $r = 1$ are used to represent $P_t = 10^4$ watts, $r = 10^{-3}$. For the $1/R^2$ law, a 0.1% reflection coefficient reduces the range to between 70% to 80% of its value with $R = 1$, while for the $1/R^3$ -law, the reduction is between 65% to 75%. The actual value of r that occurs at a fracture depends upon the geometry of the fracture, the loss tangent, differences in ϵ and σ between the fracture material and the surrounding rock, the polarization and the angle of incidence. In general, a boundary value problem in which all of these variables are accounted for must be solved to obtain r . On physical grounds, if the fracture has a large lateral extent compared to the beam width or if the loss tangent causes a large decay in the signal strength across the lateral face of a fracture, then Fresnel's formula /9/ for plane wave reflection coefficients should provide a good approximation to r . In this section we assume that the fractures of interest fulfill one or both of these conditions, and accordingly,

we employ Fresnel's results. To further simplify the problem, only the case of normal incidence is considered. For the time being, we restrict our attention to water inflated cracks, deferring a discussion of naturally occurring fractures to Section 3. The fracture model is particularly simple; namely, a single planar crack with thickness, d , exists within a mass of rock whose boundaries are far away from the crack. Because water has a much greater conductivity than rock, Fresnel's formulas are implemented by assuming that the host rock is lossless, while the fracture is characterized by a finite loss tangent. Under these conditions, the reflection coefficient is expressed by the relation

$$r = \frac{r_{12} \left[\left(1 - e^{-2\alpha_2 d} \right)^2 + 4e^{-2\alpha_2 d} \sin^2 \beta_2 d \right]}{\left(1 - r_{12} e^{-2\alpha_2 d} \right)^2 + 4r_{12} e^{-2\alpha_2 d} \sin^2 (\delta_{12} + \beta_2 d)}, \quad (2.11)$$

where for a nonmagnetic material

$$r_{12} = \frac{(\beta_1 - \beta_2)^2 + \alpha_2^2}{(\beta_1 + \beta_2)^2 + \alpha_2^2}$$

$$\delta_{12} = \tan^{-1} \left[\frac{2\beta_1 \alpha_2}{(\beta_2^2 + \alpha_2^2) - \beta_1^2} \right],$$

d is the crack thickness, α and β are defined in Subsection 2.1, and the subscripts 1 and 2 refer to rock and water, respectively.

The reflection coefficient, evaluated from Equation 2.11, is plotted in Figures 2.5 through 2.7 as a function of crack thickness with the frequency, water purity and to a limited extent the temperature as parameters. Within the framework of a single crack model (which is but a crude idealization of a hot dry rock reservoir), an inflated fracture might have a thickness of 1 to 2 mm. The plots extend to 10 mm, not because it is believed that a crack could have such a great thickness,

Figure 2.5. Plane wave reflection coefficient for normal incidence on a planar fracture filled with distilled water at 25°C as a function of crack thickness and frequency.

Figure 2.6. Plane wave reflection coefficient for normal incidence on a planar fracture filled either with fresh water, or Fenton Hill water recycled for ten days, or 0.5 molal NaCl as a function of crack thickness and frequency.

Figure 2.7. Plane wave reflection coefficient at high frequencies for normal incidence on a planar fracture filled with 0.5 molal NaCl at 25°C as a function of crack thickness.

but rather to illustrate trends. The frequency range, $10^7 \leq f \leq 10^{10}$ Hz, illustrates the need for a high frequency to obtain a large reflection coefficient from very narrow cracks with $d < 1$ mm. The water purity considered ranges from that of distilled water to highly saline water and includes effluent from the Fenton Hill Hot Dry Rock System. Calculations based upon the properties of distilled water are unrealistic for a geothermal fracture system because the ions present in the circulating fluid will markedly increase the electrical conductivity over that of pure water. Nevertheless, the data for distilled water are quite well known and provide an accurate lower bound on the reflective power of water filled cracks. At the other end of the purity scale, a 0.5 molal NaCl solution (2.24×10^4 ppm), which also has accurately measured properties, provides a good upper bound on the reflective power. In between these two limits lie the results for Fenton Hill effluent. Lack of data has restricted calculations of temperature effects to 25°C and 85°C for distilled water only. Data for liquid water at $200 \leq T \leq 400^\circ\text{C}$ are needed, but are apparently unavailable.

Calculations based upon the properties of distilled water at 25°C , Figure 2.5, indicate that the frequency should be 10^8 Hz or greater to have $r \geq 10^{-3}$. At $f = 10^{10}$ Hz, the reflection coefficient is large even for $d = 0.2$ mm, but the range in granite at this frequency is less than 1 meter. The reflection coefficient is a damped oscillatory function of the crack thickness as evidenced by the curve for $f = 10^{10}$ Hz. The three lower frequency curves are still in the resonance region for $d < 10$ mm and have not yet decayed to their asymptotic value of $r = 0.3$.

The increased conductivity of water that had been recycled for ten days /23/ causes the reflection coefficient at $f = 10^7$ Hz to be three times that of fresh water newly added to the Fenton Hill heat transfer loop, Figure 2.6. At 10^8 Hz, recycled water and fresh water yield identical values of r . A significant result which appears to have very important practical implications is that for 0.5 molal NaCl, the reflection coefficient is quite high and is essentially invariant with frequency for

$10^7 \leq f \leq 3 \times 10^8$ Hz. The rather remarkable behavior of the reflection coefficient for saline solutions prompted us to extend the calculations to higher frequencies, Figure 2.7. At $f \geq 10^9$ Hz, the reflection coefficients for saline water and distilled water are not much different, implying that the role of ionic conduction is no longer crucial. This result is discussed at greater length in Section 3. Upon comparing Figures 2.5 and 2.6, it is noted that the reflection coefficient at 10^8 Hz for distilled water is calculated to be greater than that for either fresh or recycled water. This inversion of physical reality almost assuredly stems from assuming in the calculation of the loss tangent of Fenton Hill effluent that the conductivity is a constant rather than an increasing function of frequency. (Neither the frequency nor the temperature at which the measurements were made were reported in Reference 23.)

An increase in temperature from 25°C to 85°C reduces the reflection coefficient by a greater amount at low frequencies than at high frequencies, e.g., a factor of two at 10^8 Hz, but only fifteen percent at 10^{10} Hz, Figure 2.8. It is important to recognize that test measurements made at room temperature may yield overly optimistic range predictions. For completeness, the electrical properties of water under various conditions of purity and temperature are listed in Table 2.2.

The reduction in maximum range caused by reflection coefficients that might be experienced in practice can now be calculated using the preceding results. Choosing a fracture thickness of 2mm, the farthest range for the detection of a fracture filled with (i) a 0.5 molal NaCl solution, and (ii) Fenton Hill effluent recycled for ten days is plotted and compared with that of a perfect reflector in Figures 2.9 and 2.10 for narrow beam and wide beam antennas, respectively. The reduction in range is not significant.

2.5.2 The Variation of Range With Reservoir Temperature

The reservoir temperature affects the maximum range by setting the level of thermal noise generation and by altering the dielectric properties of the rock itself and of the water within the fractures. By noting that

Figure 2.8. Plane wave reflection coefficient at slightly elevated temperature for normal incidence on a planar fracture filled with distilled water at 85°C as a function of crack thickness and frequency.

Table 2.2. Electrical Properties of Water

	f (Hz)	κ_e	$\tan \delta$
Distilled @ 25°C	10^7	78.2	4.6-03
	10^8	78.0	5.0-03
	10^9	76.9	5.3-02
	10^{10}	55.0	5.4-01
Distilled @ 85°C	10^7	58.0	1.3-02
	10^8	58.0	3.0-03
	10^9	56.8	2.1-02
	10^{10}	54.0	2.6-01
0.5 Molal NaCl @ 25°C	10^7	72.9	1.1+02
	10^8	70.3	1.1+01
	10^9	68.0	1.5
	10^{10}	51.0	6.3-01
Fresh Effluent Fenton Hill	10^7	72.9	3.5-01
	10^8	70.3	3.6-02
	3×10^8	69.0	1.2-02
Recycled 10 Days Fenton Hill	10^7	72.9	1.5
	10^8	70.3	1.5-01
	3×10^8	69.0	5.2-02

Figure 2.9. Reduction in maximum range based on $1/R^2$ -law caused by imperfect reflection at a 2 mm thick planar fracture.

Figure 2.10. Reduction in maximum range based on $1/R^3$ -law caused by imperfect reflection at a 2 mm thick planar fracture.

the reduction in received signal strength caused by imperfect reflection can be accounted for in Equation 2.6 by multiplying P_t by r , it is seen that the effect of temperature is not a strong one insofar as the reflection coefficient and the thermal noise are concerned: The range varies only logarithmically with these quantities. However, any variation of the attenuation coefficient with temperature has a direct effect on the range. Data apparently are not available to allow $r(T)$ to be estimated. At least for $f \leq 10^6$ Hz and $T < 900^\circ\text{C}$ the data discussed in Section 2.1 indicate that the attenuation coefficient does not increase with increasing temperature. The reduction in range caused by an increase in thermal noise at elevated temperatures is entirely negligible over the interval $25 < T < 400^\circ\text{C}$, $10^7 \leq f \leq 10^9$ Hz, amounting to less than 5 percent for both the $1/R^2$ and $1/R^3$ laws.

2.5.3 The Effect of Fracture Roughness on Radar Scattering

The angular dependence of an electromagnetic wave reflected from a fracture surface depends upon the roughness of the surface compared to a wavelength. The term roughness is used to describe the height of irregularities, their lateral extent and spatial distribution. Irregularities can be thought of as arising from two sources, either grain breakage or block fault motion. The former produce asperities that are less than about five millimeters in size for a coarse grained granite and are not important within the frequency range that we have considered. The latter are important and their characteristics have been estimated using (i) logs from Fenton Hill to obtain the block spacing, and (ii) general field experience to set the relative slip along a fracture as a function of fracture tightness, as discussed at some length in Section 7. Pertinent results from that section are summarized in Table 2.3, in which five types of weakness planes, identified from Fenton Hill borehole logs in terms of the strength of their signatures, are characterized by a differential slip and an average fracture spacing in a random direction. It is assumed that a producing fracture will be one of the more prominent fractures, i.e., type 1. Thus, the roughness on the surface of the type 1 fracture will be caused by offsets arising from slip along the other weakness planes which intersect the type 1 fracture. The distance between these intersections on the type 1 fracture varies between 2 and 55 meters, Table 2.3.

The effective roughness height is reduced from the slip offset by the sine of the intersection angle (see Figure 7.1) which is inferred to be approximately 0.75 for vertical wellbores in the Fenton Hill geology. In summary, the height of the irregularities is caused by fault slippage, while the lateral extent and the spatial distribution of the irregularities are the inverse of one another and are set by the block size.

Table 2.3. Fracture Plane Offsets and Spacings

Weakness Type	Differential Slip (m)	Average Spacing Between Fractures (m)	Spacing on Main Fracture (m)
1	10^{-1} - 1.0	40	55
2	10^{-2} - 10^{-1}	10 - 13	20
3	10^{-2} - 10^{-1}	10 - 40	40
4	10^{-3} - 10^{-2}	4 - 7	8
5	0 - 10^{-3}	1.3	2

If an electromagnetic wave can maintain phase coherence as it traverses a fracture system, then a reflection will tend to be specular; if not, the reflection will tend to be diffuse. If we assume for the moment that irregularities are uniformly distributed over the reflecting surface, then the Rayleigh criterion /22/ can be used to determine if the reflection will be specular or diffuse. Rayleigh's criterion is formulated in the following way. Let h be the height of an irregularity and ψ the grazing angle (the complement of the angle of incidence). Then the path difference between the ray that strikes the top of an irregularity and the ray that strikes its base is $\Delta R = 2h \sin \psi$, and the phase difference is

$$k\Delta R = \frac{4\pi h}{\lambda} \sin \psi .$$

If the phase difference is small, the surface is effectively smooth. As the phase difference approaches π , the two rays interfere destructively. Although there is no well established practice in the matter, the crossover between specular and diffuse reflection has been placed at a phase difference of $\pi/2$, which gives the criterion

$$h \sin \psi < \lambda/8$$

for the reflection to be considered specular. In view of the uncertainty attendant to establishing h in a geothermal reservoir, the crude Rayleigh theory is sufficient for our purposes. We further simplify the problem by considering only normal incidence ($\psi = \pi/2$) and plot in Figure 2.11 the boundary between specular and diffuse reflection in terms of the size of surface irregularities and the radar frequency. With a maximum roughness height of 7.5 cm (obtained under the previously described conditions), specular reflection will occur for $f < 2 \times 10^8$ Hz. The average roughness may well be less than 7.5 cm in which case the crossover between specular and diffuse reflection will move to a higher frequency. For frequencies greater than the crossover frequency, reflections will be truly diffuse only if there are a large number of irregularities within the illuminated region. If the primary reflecting region is taken to be the first Fresnel zone, then the number of irregularities within the zone must be large for the reflection to be diffuse. The diameter of the first Fresnel zone as a function of frequency is plotted in Figure 2.12. Only the region $f > 2 \times 10^8$ Hz needs to be investigated for at lower frequencies the reflection is always specular. Based on the model used to calculate Table 2.2, it is clear that for $2 \times 10^8 < f \leq 10^9$ Hz there is at most one offset within the main reflecting zone. The reflection will assuredly not be diffuse, but rather will exhibit a scattering pattern caused by a step offset. Implicit in this deduction is the assumption that the cancellation of the reflections from the higher order Fresnel zones will not be markedly altered by a step which cuts across the zones. A detailed calculation of the reflection from a stepped surface appears to be necessary for interpreting scattering from a fracture plane.

Figure 2.11. Boundary Between Specular and Diffuse Reflection for Normal Incidence as a Function of Surface Roughness and Frequency, Based on Rayleigh's Criterion.

Figure 2.12. Diameter of First Fresnel Zone at Maximum Range vs. Frequency.

SECTION 3

ELECTROMAGNETIC DETECTION OF FRACTURES

Rather than being a monolithic block of granite, a hot dry rock reservoir is likely to be crisscrossed by a large number of fractures and dikes. This is the picture that has been pieced together from conventional logging measurements made at the Fenton Hill site (Section 7). Thus, the task of a radar mapping tool is not only to detect fractures that can provide potentially useful heat transfer surfaces, but also to distinguish between a fracture and an electrical anomaly associated with a dike or other intrusive material. The latter task is complicated because a narrow water-filled fracture and a mineral intrusion can have the same value of the reflection coefficient (at least up to some limiting value set by the properties of the intrusion and the host rock). Because the dielectric properties of an intrusive material like quartzite are more nearly like those of granite than are those of water, it is clear that for equal values of the reflection coefficient a dike must be much thicker than a water-filled fracture. For example, upon comparing Figures 2.5, 2.7, 2.8 and 3.1, it can be seen that the minimum thickness of a quartzite intrusion ($\kappa_e = 4$) in granite ($\kappa_e = 6$) that produces a one percent reflection coefficient for normal incidence at $f = 10^9$ Hz must be thirty times greater than that of a water-filled fracture, i.e., the thickness of a water-filled fracture is approximately 0.4 mm, but the thickness of a quartzite intrusion must be at least 12.5 mm. The reflection coefficient at the granite-quartzite interface is an oscillatory function of the intrusion thickness which has a maximum value of 0.04 at a thickness of 40 mm and then gradually approaches $r = 0.01$ as the intrusion becomes very thick. Thus, if backscatter signals are compared solely on the basis of amplitudes, it is not possible to differentiate between a thick quartzite intrusion and a thin water-filled fracture with a thickness less than approximately 0.7 mm. However, cracks and intrusions which produce identical reflection coefficients will, in general, have

Figure 3.1. Minimum Intrusion Thickness for a One Percent Radar Reflection Coefficient. Ratio of Intrusion κ_e to Host $\kappa_e = 2/3$.

different wave transit times across the feature. In the previous example, the crack and intrusion thickness differ by a factor of 31.3, while wave speeds differ by a factor of 4.1 to yield a transit time difference of 7.6. In principle, it should be possible to differentiate between a fracture and an intrusion on the basis of their backscattered waveforms. At the present time, the analyses necessary to implement such a procedure have apparently not been performed. Moreover, it is far from certain that water naturally exists between the faces of a fracture in a hot dry rock reservoir. It is quite possible that the largest backscatter signals would arise from relatively wide intrusions and that tight dry fractures would be relatively transparent to radar.

The uncertainty that currently exists in distinguishing between fractures and electrical anomalies can be eliminated, and positive identification can be assured by the use of a rather simple technique for that class of fracture which intersect the borehole. By inflating a fracture with an electrically conducting material, say 0.5 molal or greater saline water and by using a frequency of approximately 3×10^8 Hz, the reflection coefficient at the fracture is greatly increased over what it would be under normal conditions. Three radar scans are possible: (i) uninflated to yield a background measurement, (ii) inflated with either fresh or recycled water to provide a return signal from wide fractures, and (iii) inflated with saline water to provide an 8- to 10-db increase in signal strength over (ii). The main idea is to produce an enhanced signal from a fracture that will stand out from the backscatter produced either by electrical anomalies or by numerous, but minor, fractures. For $f \leq 3 \times 10^8$ Hz, the reflection coefficient for normal incidence is greatly increased when saline water is injected into a planar fracture as is readily seen by comparing Figures 2.5 through 2.8. Thus, permeable planar fractures can be made to stand out from background noise by injecting an electrically conducting fluid into the fracture and by using a suitable frequency. At very high frequencies, the contrast disappears as the lossiness switches from ionic conduction to molecular

excitation, which restricts the frequency band to less than approximately 3×10^8 Hz. Not only do the lower frequencies at $f \leq 3 \times 10^8$ Hz provide the necessary contrast, but they also provide a longer range than can be attained at higher frequencies. To take full advantage of this technique requires the development of a directional borehole antenna for $f \leq 3 \times 10^8$ Hz.

SECTION 4

RADAR SUMMARY

According to calculations, VHF radar has the potential for mapping fractures within a region of a hot dry rock reservoir that extends away from the wellbore for a distance of approximately 50 ± 20 meters. This range is sufficiently great to make radar a useful tool if two other requirements can be satisfied. To realize the full potential of backscatter radar as a mapping tool, a directional VHF receiving antenna must be developed to provide both azimuthal resolution and the ability to detect fractures that are nearly normal to the borehole. Partly because of its own inherent characteristics and partly because of the nature of the noise field one or two small magnetic loops and a small electric dipole appear to constitute a directional antenna suitable for borehole use. Certain design questions must be answered before such an antenna should be built. As the angle between a borehole and a fracture becomes larger and approaches $\pi/2$, the backscatter signal becomes weaker. Thus, the geometry of the second phase Fenton Hill system which employs boreholes intersected by nearly perpendicular fracture planes is less well-suited to radar backscatter experiments than is the geometry of the first phase system where the borehole and fractures were more nearly parallel. Radar transmission through the fracture from one borehole to another may be more appropriate for fractures that are nearly perpendicular to the borehole. However, wellbore-to-wellbore separation distances in excess of 100 meters may preclude the use of radar transmission as a diagnostic tool.

The second unfulfilled requirement levied on a radar system is target identification, which is in a rudimentary state. Experiments have shown that the clear-cut identification of subterranean targets is a formidable task. Based on an analysis of fracture reflectivity, it is believed that fracture identification can be greatly enhanced by inflating a fracture with a conductive fluid, e.g., salt water. A comparison of radar echoes in the pressurized and non-pressurized states should make a fracture appear quite prominent.

SECTION 5

CHARACTERIZATION OF BOREHOLE FRACTURE INTERCEPTS USING CONVENTIONAL BOREHOLE LOGGING SURVEYS

Conventional borehole logging techniques are an important and necessary part of a fracture mapping program. As shown in this and the subsequent section, conventional techniques can be employed to (1) determine the number and size of fractures and weakness planes intersecting a borehole, (2) define the fracture orientation at the borehole, (3) develop an approximate model of the natural fracture pattern around the borehole, (4) suggest regions for inducing hydraulic fractures, (5) determine the location and width of hydraulic fractures, and (6) locate connection points between wellbores. Aside from temperature limitations which restrict the use of all borehole tools, the most serious shortcomings of conventional techniques are (1) an inability to determine the far-field extent and orientation of hydraulically induced fractures, and (2) the failure of any one tool to provide an accurate and unambiguous determination of fracture location and orientation.

Before proceeding, it is appropriate to note that the term conventional borehole techniques is used to mean those methods normally employed by logging companies and drilling contractors to service the petroleum industry. In almost all cases, wireline techniques would be more useful for geothermal logging if the equipment could handle higher temperatures. In some cases, data reduction would be improved if analyses were developed specifically for unfolding data obtained in fractured hard rock rather than relying on methods developed for use in sedimentary strata. Certain techniques which are commonly used for logging in sedimentary rock have been found to be of marginal value in a hot dry rock reservoir. However, even without improved analyses, fracture locations can often be inferred by crosscorrelating records from a suite of appropriate tools as indicated in Subsection 5.2.

For the purpose of fielding an effective fracture mapping program, it is useful to divide the construction of a hot dry rock reservoir into four operations: (1) drilling, (2) pre-hydraulic fracture, (3) post-hydraulic fracture, and (4) two-well completion. The type of data that can be obtained during

drilling is usually related to the lithology and is rather general in nature, except for those gathered from an occasional traverse by a sonde during a break in drilling operations. (Logging-while-drilling via a telemetry link would improve the situation, but it is not yet widely used.) The types of logs that can be obtained during a drilling operation and their usefulness are discussed in Subsection 5.1. Knowledge of the natural structure of a reservoir, i.e., the pattern of fractures and dikes, is valuable for planning hydraulic fractures, interpreting test data, and inferring the existence of leakage paths. A number of techniques employing mechanical, electrical, optical, and radiochemical methods can map borehole fracture intercepts when used synergistically during the pre-hydraulic fracture phase as discussed in Subsection 5.2. Mapping after hydraulic fractures have been created is crucial both for setting the direction of subsequent drilling and for determining the position of likely flow connections. Techniques used during this phase are similar to those of the pre-hydraulic fracture phase except that fluid flow measurements become more viable, as discussed in Subsections 5.3 and 5.4. The final operation is one of measuring the position and size of flow connections between wells. Tracer methods, reviewed in Subsection 5.4, along with temperature and flow measurements complete the discussion of conventional techniques.

For convenience of reference, all of the techniques discussed in this section are listed in Table 5.1 along with the operation during which they would be utilized.

5.1 LOGGING DURING DRILLING OPERATIONS

Logs recorded during drilling operations provide the initial characterization of subsurface fracturing. The logs are obtained from the assay of material removed from or circulated through the wellbore, the monitoring of drill performance, and an occasional sonde traverse when the drill bit has been removed for servicing. The logs are normally run to assist drilling; from the standpoint of fracture mapping, they are rather coarse. Nonetheless, if the logs are carefully taken, they provide the first indication of either fracture or injection zones, and may help to resolve anomalies that appear during fracture mapping operations.

Table 5.1. Techniques for mapping borehole fracture intercepts and their application during the four main operations associated with constructing a hot dry rock geothermal reservoir.

Log	Operation			
	Drilling	Pre-Hydraulic Fracture	Post-Hydraulic Fracture	Two-Well Completion
Drill Cutting	X			
Drill Rate	X			
Fluid & Mud	X			
Core	X			
Caliper	X	X	X	
Resistivity		X	X	
Televiewer		X	X	
Television		X	X	X
Impression Packer		X	X	
Acoustic Transmission		X	X	
Density		X	X	
Spontaneous Potential			X	
Gamma Ray		X		
Spectralog		X		
Silicon/Calcium		X		
Temperature	X			
Temperature (Differential)		X	X	X
Spinner			X	X
Tracer			X	X

5.1.1 Drill Cutting Log

Circulating drill fluid carries rock chips or drill cuttings from the bit to a surface settling pond where samples are either taken at intervals or extracted continuously. Petrological analyses of the chips are used to differentiate between rock units. Since the chips are small, color changes tend to be the dominant first indicator of rock unit boundaries. The identification of minerals that are known to deposit in fractures is a potential indicator of fracture zones. Because of mixing and the inaccuracies in determining the transit time of a chip from the bit to the surface, the order in which chips were excavated and the depth at which a chip originated cannot be known exactly. Therefore, the depth of a rock unit or an inferred fracture zone is usually uncertain to within 2 to 10 feet, depending on the drilling rate, sampling frequency, circulation pump rate, and the care exerted by the logging crew. An example of a chip log delineating rock type as a function of depth at the Fenton Hill reservoir is shown in Figure 5.1, along with three other types of logs. Rock types and rock boundaries based upon the chip log as reported by West /24/ are indicated between a depth of 2500 feet to 4300 feet. Without going into the details of the other logs (see Subsection 5.2), it is noted that the chip log correctly identified two major zones containing dikes (weakness planes), but was not useful for predicting the locations of the fractures.

To refine and clarify interpretations based on the drill cutting log, and to provide almost incontrovertible evidence of subsurface properties, core samples are taken occasionally. Direct fracture characterization would be possible if the borehole were completely core drilled with an orientation scribe and the core 100 percent recovered. Expense prohibits such drilling, and 100 percent core recovery is virtually impossible.

5.1.2 Drill Rate Log

The rate of descent of a drill into the ground is commonly recorded automatically. By monitoring pull-down pressure and torque, the drill rate log can be a sensitive indicator of changes in rock resistance to drilling when combined with data on bit wear rates under similar drilling conditions. Changes in apparent hardness can be compared with logs taken later on during

Figure 5.1. Fracture distribution inferred from crosscorrelation of four different conventional borehole logs. Data from upper portion of hole GT-2 at Fenton Hill obtained from chip analyses, silicon log, background gamma ray, and two-independent-arm caliper measurements. Nomenclature: Gr (granite), Grd (granodiorite), Mgr (Monzogranite), Sgr (Syenogranite), FZ (zone of multiple fractures), CA (zone of chemical alteration usually characterized by borehole erosion), ? (interpretation uncertain). To aid crosscorrelation of contact alterations with eroded zones, the silicon log is modified with the following visual aid: Si < 6, Si ~ 6, Si ~ 7, and Si > 7.

the pre-hydraulic fracture phase to assist in locating chemically altered or fracture zones.

5.1.3 Fluid and Mud Logs

A detailed record of the analyses of drilling-fluid samples, and notations of even small fluid losses are important to fracture mapping and weakness characterization. Analyses of fluid samples can determine changes in the chemistry of the fluid arising from interactions with the rock. Zones of lost fluid indicate penetration into the surrounding rock, an early signal of open, permeable fracture zones. These logs appear to lose their effectiveness at depths in excess of about 8000 feet because the lithostatic pressure decreases the permeability of natural fractures.

5.1.4 Initial Sonde Traverse

The first sonde that is run into an unstudied hole is usually rather simple, consisting in most cases of not more than two tools: an inexpensive two-independent-arm caliper and a maximum temperature recording thermometer. The caliper is used primarily to measure borehole geometry and to detect obstructions that could cause a sonde to become lodged in the drill hole. (At worst, if the first sonde gets stuck in the drill hole, it is milled out, and the economic loss is kept to a minimum.) A maximum temperature recording thermometer is used to give a rough indication of formation temperature and is attractive from an operational standpoint because of its simplicity, although rough handling may result in shock-induced errors. A continuous temperature measurement is of considerably more value for fracture mapping than a peak record because of its ability to detect temperature anomalies which may occur at permeable, fractured zones. This usefulness for fracture detection is undiminished by the fact that during drilling the borehole is not in temperature equilibrium with the surrounding reservoir, i.e., for the purpose of thermal engineering, a continuous temperature record during drilling is not especially useful in a hot dry rock reservoir. (The continuous record is quite important in a hydrothermal reservoir.) The caliper and temperature logs that are obtained during the drilling operation are useful for providing a first rough picture of fracture intercepts at the borehole. They

are, however, capable of considerably better precision than is usually attained during the first logging runs, as discussed in Subsections 5.2 and 5.4.1, respectively.

5.2 LOGGING DURING PRE-HYDRAULIC FRACTURE OPERATIONS

Fourteen different techniques were evaluated for their effectiveness in mapping the intersections of natural fractures with a borehole. The techniques are listed in Table 5.2. Variation of the basic methods increase the number of techniques available to more than thirty. Based mainly on experimental performance, six techniques appear quite useful: (1) caliper, (2) resistivity dipmeter, (3) televIEWer (acoustic imaging), (4) television, (5) impression packer, and (6) acoustic transmission. The dipmeter and acoustic transmission methods require improved analyses, while all six could benefit from improved high temperature capabilities. A seventh technique, the silicon/calcium log, is useful for differentiating between fractures and zones of chemical alteration, but is in itself not capable of detecting fractures. Six other techniques appear to be of marginal value for locating fractures at the present state-of-the-art. They are the (1) density log, (2) spontaneous potential, (3) resistivity methods that sample large volumes of rock, (4) gamma log, (5) spectralog, and (6) temperature log. A differential temperature measurement might conceivably record temperature changes arising from fluid seepage at natural fractures, but the technique has not been tested in a hot dry rock reservoir. The temperature logs appear useful for mapping hydraulic fractures under forced flow conditions; discussion of these logs is deferred to Subsection 5.4. Because of its impact on cost and the compatibility of different tools on a single sonde, the logging rate for each tool is listed in Table 5.2. A persistent problem in crosscorrelating logs to locate fractures in a deep, hot geothermal well is an apparent depth offset of several feet between logs; a remedy to the problem is suggested in Subsection 5.2.9.

5.2.1 Caliper Logs

After the drill bit, the rotating drill pipe, and the drilling fluid have produced wear on the borehole wall, caliper logs provide a sensitive

Table 5.2. The use and limitations of logging techniques for the detection of natural fracture intercepts at a wellbore reviewed in Subsection 5.2. The logging rate gives a rough indication of cost and compatibility of different tools on the same sonde.

Technique		Usefulness and Limitations	Logging Rate, fps
Caliper	Three independent arms required		70
Spontaneous Potential	No direct relation between SP and fractures.		100
Resistivity Diplog	Detects fluid-filled fractures. Standard data reduction codes suspect. Additional work warranted.		50
Normal Electric	Diffuse record. Cannot locate isolated fractures.		100
Induction	Fair correlation with caliper. Additional work required, but dipmeter appears superior.		100
Televiewer	Useful acoustic image of fracture intercepts and borehole surface.		6
Television	Useful optical image of fracture intercepts, fluid flow, and borehole surface. Not used in hot dry rock program. Requires thermal hardening. Photography may be more expedient.		
Impression Packer	Three-dimensional record of fracture size and orientation. Unreliable operation even at modest temperatures (150°C).		
Acoustic Logs	Cycle skipping: Irregular and often unreproducible effect. Uncertain results. Amplitude reduction: Useful for correlation. No clear-cut advantage over waveform recording. Interpretation uncertain. Waveform analysis: Contains all acoustic information but interpretation uncertain. Phase Splitting: Interpretation uncertain. Requires additional work.		60

Table 5.2. Continued

Technique	Usefulness and Limitations	Logging Rate, fps
Radiochemical	Silicon/calcium: Distinguishes chemically altered zones from fracture.	25
	Gamma Ray: Poor correlation with caliper log.	30
	Spectralog: As with Si/Ca and γ -ray logs, no close correlation with fractures.	11
Formation Density	Spurious responses. Requires calibration tests in hard rock of known fracture porosity.	30
Temperature	Used for locating fractures under forced fluid flow conditions. Differential temperature not tested.	65

measure of the resistance of the rock to erosion. Commonly, the caliper logs correlate well with the drill logs, since they both reflect hardness, degree of fracturing, and easily eroded zones that in igneous rock may be zones of chemical alteration. By itself, a caliper cannot distinguish between erosion resulting from fracturing or from chemical alteration. Calipers commonly used in commercial practice to measure the size of a borehole have either two or three independent arms; it is clear that to measure the orientation of a fracture plane, three independent measurements are necessary. LASL has built and fielded a high temperature (200°C) oriented caliper with three independent arms that are capable of resolving changes of 0.2 mm in radius. The LASL tool is about five times more sensitive than commercial tools.

An example of the type of fracture mapping data obtained from a three-independent-arm-caliper is shown in Figure 5.2, which is an isometric replot of standard logging data. The existence of a fractured zone about 0.5 to 0.7 feet wide cutting across the borehole is easy to visualize from the isometric plot, as is the dip and the strike. Because a good visual display often enhances the usefulness of many types of data, we believe that caliper data intended for fracture mapping should be plotted in isometric form rather than in the standard form used by commercial logging companies. To do so requires only a straightforward computer plotting routine. The improvement in the visualization of fracture planes afforded by the isometric plot is clearly illustrated by comparing Figure 5.2 of this report with Figure 6.1 of Reference 25. The value of the isometrics is made even clearer when it is realized that Figure 6.1 of Reference 25 is plotted with an inverted depth scale, i.e., distance increases from the bottom of the wellbore to the surface; the orientation of the fracture plane indicated by the isometric plot makes all depth and angular relationships quite clear.

When data from a two-independent-arm-caliper are all that are available, no more than a rough indication of whether fractures have a shallow or steep dip can be obtained. An indication of a steep dip will be qualitatively correct, but an indication of a shallow dip may be misleading for it may arise

Figure 5.2. Isometric replot of LASL three-independent-arm caliper data. The fracture plane, dip, and strike are readily visualized when data are plotted in this manner. LASL data at 8920 feet in hole GT-2B /23/. The erosion patterns are drawn with an exaggerated scale in the radial direction.

from a pathological case in which the caliper arms see two eroded points at the same elevation in the same quadrant, but miss the low (or high) eroded point. In principle, the trace of the fracture plane should be detectable but in practice, it may be overlooked. For the purpose of interpreting two-arm caliper data in this review, we assume that the pathological case is rare and that most indications of shallow dip do, in fact, arise from nearly horizontal weakness planes. An example of raw, two-independent-arm unoriented caliper data with fracture location and estimated shallow or steep dip angles is shown in Figure 5.3. The results of interpreting commercial two-independent arm caliper data over an 1800-foot interval from 2500 to 4300 feet in hole GT-2B at Fenton Hill are shown in Figure 5.1.

Figure 5.3. Dresser Atlas two-independent-arm caliper log of a portion of GT-2B modified to illustrate inferred fractured locations and estimates of shallow or steep dips /26/.

5.2.2 Electrical Properties Logs

Water in contact with rock will contain dissolved salts and act as an electrolyte, eventually attaining a conductivity of up to several S/m. Because fractures in hard rock provide most of the permeability, the fluid-filled fractures are relatively good electrical conductors in comparison with the adjacent low porosity rock. Several different types of tools make measurements based on the conductivity of the medium (the rock and fluid) in the immediate vicinity of a borehole. Historically, tools that measure the d.c. or low frequency conductivity of the medium have been called electrical tools. The measurement requires a continuous conducting path extending outward from the tool electrode into the formation and back to the return electrode. Another class of tools, which is referred to as induction tools by the logging industry, use high frequency inductive circuits to make measurements within the formation without requiring a continuous conducting path. A third type of measurement senses the voltage that arises from electrochemical and electrokinetic forces. This is the spontaneous potential method (SP). The SP log will be reviewed first because it is commonly used by the logging industry, although it does not appear particularly useful for locating natural fractures. Electrical and induction tools will then be discussed. Although induction logs have been highly developed in recent years, it appears that a specialized electrical tool--the resistivity dipmeter--is especially useful for locating fracture intercepts at the borehole.

5.2.2.1 Spontaneous Potential Log

The spontaneous potential or SP log is a recording versus depth of the potential difference between an electrode moving in the borehole and a fixed electrode placed at a distant location, usually the ground surface. The differences in potential are thought to be related to differences in ionic activity throughout the formation and to "streaming" potentials produced by the flow of electrolytes through the rock pores and fractures. Typically, there is not a direct observable relation between SP deflection and fractures, porosity, and permeability /27/. Based on many years of empirical development in the petroleum industry, the logs are useful for distinguishing sandstones and shales and are often run as part of other geophysical logs to

assist correlations and interpretations of other logs. This experience, however, is not directly transferrable to geothermal logging in hard rock.

The SP records at Fenton Hill indicate no significant sensitivity to fracture location prior to hydraulic fracturing, as illustrated in Figure 5.4.

5.2.2.2 Electric Logs

Electric tools vary mostly in the number and location of electrodes. The design configurations depend primarily on whether the purpose is a vertical or a lateral or an omnidirectional investigation, or shallow or deep sensing beyond the wellbore. The different designs are reviewed in some detail in Reference 27, where the emphasis is on tool operation in strata of interest to the petroleum industry. We shall not duplicate that work, but rather shall extend it to examine the ability of tools to locate fractures in hard rock. Before proceeding, however, certain tool names and trade names will be defined. Two of the earliest logs used by the logging industry as part of a conventional electric survey rely on the normal and the lateral resistivity tools. The normal tool injects current between an electrode on the sonde and the uninsulated portion of the cable armor, which is some distance from the sonde. One measuring electrode is located on the sonde, and the other is on the insulated portion of the cable. The spacing between injection and measuring electrodes is normally 16 inches or 64 inches. For specialized logs, an ultra-long spacing ranging from 75 to 1000 feet is used. The lateral log has both current injection electrodes on the sonde. The spacing between the midpoint of the injection electrodes and the measuring electrode is 18' 8". To provide better definition of the measurement region than is afforded by the normal and lateral tools, focusing or guard electrodes are employed to control the potential distribution and the current paths in the ground. Several different types of Laterologs (a Schlumberger trade name) produce a relatively narrow transverse current beam. For example, the approximate thickness of the current sheet produced by a Laterolog 7 is 32 inches, that of the Laterolog 3 is 12 inches, and that of the Laterolog 8 is 14 inches. Not only does current from the Laterolog 8 not extend as far from the borehole as does that from the Laterolog 7, but in an orthotropic

Figure 5.4. Spontaneous potential log in hole EE-1 at Fenton Hill prior to hydraulic fracturing. No natural fractures are apparent /28/.

medium, the response of the Laterolog 8 is more dependent upon the component of conductivity parallel to the borehole. A Dual-Laterolog employs yet two other variations of the focused electric log which simultaneously provide both deep and shallow penetrations. As the name implies, the spherically-focused log produces spherical equipotential surfaces. Without guard electrodes, the borehole would distort the current paths making data interpretation difficult. The Microlog (also a Schlumberger trade name) uses small electrodes on a pad to sample a small region, while the Microlaterolog is a laterally-focused version of the Microlog. With the jargon of the trade defined, we proceed to review electric tool operation in hard rock.

Fracture Identification from Electric Logs

The diplog--an oriented, four-independent-arm microlaterolog device--is of particular interest for its apparent usefulness in fracture mapping. Attached to each arm are current injection and potential measurement electrodes in pads that contact the wall. This close-spaced resistance measuring tool details both minor and major zones that are permeable to fluids in the borehole wall. The pad electrodes produce four traces at each quadrant of the borehole with resistivity changes at permeable zones appearing as spikes on each trace. If the resistivity spikes can be reliably correlated between the four records, then a permeable plane can be identified, and the orientation can be calculated for the strike and dip of an identified plane. Generally, the correlations and determinations of orientation are made with service company computer programs originally developed for the petroleum industry. However, programming dip calculations for bedded strata is, at present, considerably more reliable than programming for the much less systematically oriented fractures. Almost assuredly, no existing program is adequate for producing dependable answers. Thus, computer-reduced displays of fractured basement rock may contain considerable error, even though the raw data are very accurate. Scott Keys of the U.S.G.S. has confirmed this difficulty after observing poor correlations between computerized diplog output and the data obtained from the sonic televiewer /29/.

Figure 5.5 is a sample of four diplog traces indicating a varying resistivity with depth in GT-2. A portion of the log is shown where a correlation appears to exist and where the record is relatively simple. Because an accurate diplog correlation algorithm has apparently not been devised for hard rock, we have employed a very simple method for locating fractures and compared the results with a caliper log. The raw diplog resistivities were combined by summing the values of the less resistive areas recorded by the four pads and plotting these values on the corresponding caliper log.

Figure 5.6 shows a sampling of the GT-2 caliper log cross-plotted with the summed relative resistivity values. Also shown are the approximate locations and indicated relative dips for the fractures. Note that about a four foot depth correction is needed to bring the correlation between the diplog and caliper log into better alignment. It is believed that the depth inconsistency has arisen more from the depth logging than from the reduction of the caliper and diplog traces. To wit, the caliper measures features directly at the borehole wall. The spacing between electrodes on a Microlog is of the order of one inch; thus, the diplog samples a volume of rock with a characteristic height of no more than a few inches. The offset between a caliper and a diplog should be less than a few inches at most--not several feet. The need for a depth correction when trying to interpret logs is a disturbing aspect of the analysis which occurs all too often as will be seen in subsequent subsections. The apparent depth inconsistency is emphasized because it is a problem that can be solved in a straightforward manner both by improved data taking and improved hardware, as indicated in Subsection 5.2.9.

The diplog seems to have consistent and reliable traces from a depth of 5786 feet (near the top of the log) to about 6160 feet. After that, the records do not correlate as well with the indicated weaknesses on the caliper log. The log was terminated at 6460 feet, even though the hole was open to 8960 feet. Whether or not the diplog would perform well at depths to 9000 feet is not known with certainty, but considering the fact that a Laterolog was used to this depth, there is no fundamental reason for not expecting the diplog to be useful. The Laterolog record is shown in

Figure 5.5. Portion of the four-independent-arm diplog resistivity survey in GT-2 on 12/17/74 showing tentative fracture plane correlations of the 4 traces. Pads 1, 2, 3, and 4 are on successive quadrants.

Figure 5.6. Sample diplog and caliper-log correlation showing approximate fracture locations. Based on Dresser-Atlas data obtained at Fenton Hill in hole GT-2 on 12/17/74 (diplog) and 2/11/72 (caliper). The correlation is improved by offsetting logs four feet.

Figure 5.7. This record is consistent with the corresponding caliper log (Figure 5.3) if a depth correction of about 4 feet is made.

Figure 5.7. Focused resistivity log (Laterolog) obtained at great depths in hole GT-2B at Fenton Hill /26/.

The diplog appears to be a very useful tool for mapping borehole fracture intercepts. Two reservations in our recommendations for its use are (1) the diplog apparently was not used below 6460 feet, and (2) the correlations between diplog and caliper decreased below 6160 feet. Additional diplog experiments are warranted.

The resistivity records taken at Fenton Hill include not only the microspacings of the diplog, but the standard source-receiver spacings of 16 inches, 64 inches and 18' 8" as well. With increased spacing, the resistivity electrodes investigate farther from the borehole but cannot locate individual fractures or permeable zones. The record from a short normal log, Figure 5.8, is rather diffuse and does not define isolated fracture planes.

Fracture Identification From Induction Logs

The laterally focused induction log is a special tool that was originally developed for sites where nonconducting oil-based muds had to be used, for example, to stabilize weak clay zones, but induction devices also work with borehole fluids having a low to moderate conductivity. The operating principle is based upon the sensing of eddy current fields induced in a conductive medium. The measured signal is proportional to the conductivity of the medium surrounding the sonde. Focusing is used to minimize the effects

Figure 5.8. The Schlumberger Induction Electric Log showing short normal, induction resistivity and conductivity traces in Hole EE-1 at Fenton Hill. Over the interval 6420 to 6870 feet, there are twenty-one regions of increased conductivity and twenty-eight zones of caliper offset. Seventeen zones of increased conductivity and caliper offset correlate with one another. A one-to-one correspondence between amplitudes in the conductivity and caliper logs does not exist. The normal electric log is too diffuse to be useful. The conductivity and induction resistivity plots are the inverse of one another. (The induction resistivity plot is faint in the original field log and does not reproduce well.) The corresponding SP log is plotted in Figure 5.4.

of borehole fluids. When induction-logging devices are used with source and receiver coils oriented both parallel and perpendicular to the axis of the sonde, it is also theoretically possible to determine formation dip. Values of horizontal and vertical conductivities can, in principle, be derived from the measured values, but in practice, anisotropy greatly complicates the responses of induction as well as direct current and alternating current electrode borehole logging devices /30/.

One run of an induction log for depths of 6420 to 6880 feet in EE-1 (which was repeated three times for validation) is shown in Figure 5.8. Spontaneous potential (SP), short normal, induction resistivity and conductivity traces were all recorded from the single sonde (the SP trace is shown in Figure 5.4). Because the conductivity trace is shown at a small scale, relative values and peaks are poorly resolved. The induction resistivity record is the inverse of the conductivity record, but the resistivity record is faint and consequently difficult to interpret. However, twenty-one locations of increased conductivity are identified in Figure 5.8. Of these twenty-one, seventeen are identified as fractures based on a correlation with hole erosion patterns on the corresponding caliper log.

5.2.3 Acoustic Televier Log

The televier is an acoustic device that produces a picture that images the elastic properties of the borehole surface. It is thus able to see cracks, offsets, and zones of varying acoustic impedance. The televier is the U.S. Geological Survey high temperature version of the commercial seivier. The operating principle of the televier is as follows: A narrow acoustic beam is emitted from a rotating transmitter to scan the borehole surface. Irregularities on the borehole wall cause the reflected signal to vary in intensity. The amplitude modulated acoustic signal is then used to produce an intensity-modulated display on a cathode ray tube. A coherent picture is produced by synchronizing the display with the rotating transmitter. The signal frequency is in the one to two megahertz range to allow a resolution of approximately one millimeter. The pulse repetition rate is approximately one kilohertz, and the transmitter rotation is 3 rps /31,32/.

As is the case with most sophisticated tools, the televiewer is limited by temperature. A dewar protects electronics to a down-hole temperature of approximately 260°C, but it appears that the operation of the magnetic orientation device may have been unreliable on some tests. The technique of acoustic imaging makes the tool complementary to several other logs, especially the caliper and television, which operate on different principles. Televiewer data have proven especially useful in several parts of Fenton Hill boreholes /26/. However, the televiewer system is generally not a feasible method for logging an entire borehole because a slow logging rate (about 6 feet/min) makes the traverse expensive. Nonetheless, the tool appears to be extremely useful for confirming the presence of fractures and measuring their size and orientation.

5.2.4 Television

Borehole television has been used as an inspection device on several underground tests, and more importantly, for the present purpose, it has been used not only to detect permeable gas bearing fractures intersecting boreholes in shale /33/, but also to locate borehole-fracture intercepts in granite /34/. Based on the experience gained in shale during the coal degassification project, it is believed that borehole television could be used to resolve uncertainties in interpreting caliper and televiewer data. Permeable fractures were located in the coal degassification program by scanning the borehole for signs of fluid turbulence caused by gas injection. Although we are getting a little ahead of our story, we note that efflux points between boreholes could be mapped by a similar technique, provided the borehole fluid contained a small number density of particulates to make flow visualization possible. Prior to its use in a geothermal reservoir, a television camera would require thermal hardening. The expense of doing so, however, makes it prudent to first examine the feasibility of using downhole photography.

5.2.5 Impression Packer

In principle, special inflatable packers made of rubber-like material can be inserted in a borehole to provide a three-dimensional molded record of the borehole surface. The molded impression can be used to confirm the existence of fractures and to determine their size and orientation. In

practice, the use of this conceptually simple device has not always yielded good results. The impression packer was found to be unsatisfactory for fracture mapping at normal temperatures in the Wattenberg Gas Field north of Denver, Colorado /34/, perhaps because of relatively soft sedimentary rock formations. At elevated temperatures in the Fenton Hill reservoir, special impression packers rated at 200°C failed at 150°C. Surprisingly, ordinary packers of the type used to seal-off sections of wellbore proved capable of retaining impressions and were so used at Fenton Hill /36/. The main uncertainty in the use of ordinary packers at the 150°C temperature range is associated with the unreliable operation of magnetic orientation sensors. The effective use of impression packers in a hard, hot rock requires improved high temperature capabilities.

5.2.6 Acoustic Logs

In single hole acoustic logging, the basic procedure is to transmit a sonic wavetrain ($5 < f < 15$ kHz) from a sonde out into the formation and to listen for the direct transmitted wave and reflected waves at a receiver several feet away on the same sonde. Although the hardware of different manufacturers may differ in a number of ways, e.g., receiver Q, center frequency, pulse shape, isolation of receiver from transmitter, and the number and spacing of transmitters and receivers on a single sonde, we shall not examine hardware in detail, but rather will concentrate on reviewing four principal data analysis techniques that have been used with varying degrees of success in locating fractures. The methods are: (1) cycle skipping, (2) amplitude reduction, (3) waveform analysis, and (4) phase splitting. The basic procedures were introduced about twenty years ago, but the complexity of acoustic wave interactions with geologic formations and what appears to be a lack of active research have conspired to prevent any one of the techniques from unambiguously locating fractures. At the present time, the logs are perhaps most useful simply because they provide what a log analyst calls "correlatable features," i.e., a distinct irregularity in the record that at least in some cases can be related to fractures.

5.2.6.1 Cycle Skipping

Cycle skipping is an effect observed with the interval transit time (Δt) sonic log. The sonic tool usually employs one transmitter above and one transmitter below two pairs of receivers to effect a partial compensation for borehole irregularities and sonde tilt. The spacing between a transmitter and its nearest receiver is of the order of several feet, and between pairs of receivers about half the transmitter-receiver spacing, e.g., a typical Dresser-Atlas BHC Acoustilog has a transmitter-receiver spacing of four feet and a receiver-receiver spacing of two feet. The log is customarily used to record the reciprocal of the formation P-wave velocity parallel to the sonde by averaging the transit times measured by each set of receivers. Time interval measurements are made by setting voltage trigger levels that correspond to preselected P-wave amplitudes.

If the acoustic path between the transmitter and receiver encounters fractures, then the first arriving signal may be so attenuated as to be incapable of triggering the timing circuits in either of the two receivers. Rather, a higher amplitude signal later in the wave train will cause triggering, resulting in an abrupt and clearly anomalous increase in the recorded time interval measurement. The discontinuity in Δt arises from what is referred to as a cycle skip. The relationship between P-wave amplitude reduction and fracturing is complicated /37/, so that a simple adjustment of triggering levels to provide a unique indication of fracturing is apparently not possible at the present state-of-the-art. Based upon experimental evidence gathered in hole GT-2B at Fenton Hill, cycle skips were frequently recorded, but their locations were not reproducible over the interval 8500 to 8900 feet. The raw data from 8700 to 8800 feet are shown in Figure 5.9 for three different runs.

When all of the cycle skips were combined in a single record, their locations correlated with irregularities indicated by a two-independent-arm caliper. Note that the record not only exhibits increases, but also decreases in Δt . A decrease following an increase might be explainable as arising from circuit overload recovery were it not for an occasional solitary spike indicating a decrease in Δt . This anomalous behavior requires a detailed analysis of circuit operation.

Figure 5.9. Cycle skipping observed on a Δt log in hole GT-2B at Fenton Hill. Three different logging runs over the same depth interval indicate that cycle skipping is not reproducible. Log run on 8/12/78.

5.2.6.2 Amplitude Reduction

Historically, the phenomenon of cycle skipping was recognized through observation of field logs taken for the purpose of determining formation P-wave velocity. For detecting fractures, researchers decided that the more fundamental approach of directly recording the amplitude of both the first positive compressional wave and first positive shear-wave (or borehole-surface wave) would lead to more useful results than a technique that depended upon the uncertain settings of gain controls. Thus, the amplitude reduction log consists of a plot of wave amplitudes versus depth, which can be obtained, for example, by using a sample-and-hold boxcar circuit. Theoretical, laboratory, and field studies have shown that an amplitude reduction is experienced by a P-wave as it crosses a fracture having a steep dip with respect to the borehole. Theory indicates that a maximum reduction occurs for dip angles between 33 and 78 degrees. Of particular interest for logging from vertical boreholes in a deep geothermal reservoir, theory predicts that the attenuation decreases rapidly for dips greater than 78 degrees, with no attenuation of P-waves for exactly vertical fractures /37/. S-wave amplitudes suffer their greatest reduction in amplitude when crossing fractures that are nearly perpendicular to the borehole /37/.

Based on work performed by Schlumberger in limestone /37/, the reduction in P-wave amplitude within a region known through core recovery to contain "near-vertical" fractures is illustrated in Figure 5.10. The core samples were not analyzed with precision so the dip angle associated with the term "near-vertical" is not known. (It would be of great usefulness to know if the dip exceeded 78 degrees.) The core did not extend into the region characterized by a sharp decrease in S-wave amplitude, Figure 5.10, therefore, the existence of near-horizontal fractures cannot be confirmed.

Several factors other than fractures also reduce sonic amplitudes. In a hot dry rock reservoir, these factors include poor tool centralization, hole size variation, and borehole rugosity. The wavelength of a 15-kHz sonic wave in a borehole fluid (water) is only four inches. Roughly speaking, a two inch eccentricity would give complete destructive interference between waves on opposite sides of the borehole resulting in a sharp decrease in

received amplitudes. A one-quarter inch offset is reported to result in a 50 percent decrease in amplitude /37/. Hole size variations lead to interference effects, while borehole roughness scatters waves leading to a reduction in intensity. Most of the research on amplitude reduction for fracture detection appears to have been carried out circa 1963. At that time, it was felt that no conclusive results had been reached concerning the advantage of signal attenuation measurements over straight amplitude logging (waveform recording). Apparently, little has changed during the intervening years.

Figure 5.10. Schlumberger record of a confirmed vertical fracture reducing the compressional wave amplitude and a suspected flat angle fracture affecting the shear wave amplitude /37/.

5.2.6.3 Waveform Analysis

The waveform record of a received acoustic pulse is the most fundamental of the acoustic logs. It contains all of the information regarding elastic wave propagation in the formation, i.e., it is a complete set of amplitude and phase data. Decreases in amplitude that gave rise to cycle skipping and dips in the amplitude log can be detected by analysis of the waveform. Interference effects arising from reflections can be detected as changes in amplitude or phase.

A portion of a waveform log (Dresser-Atlas Signature Log) between 8700 and 8800 feet in hole GT-2B at Fenton Hill is shown in Figure 5.11. The logging tool employed a six-foot and a four-foot spacing between the transmitter and receiver. The two spacings correspond to the spacing used in the Δt Acoustilog. The waveforms are, in fact, the raw signals that are used to generate a Δt log (although the data of Figure 5.11 were not taken during the same logging run as those of Figure 5.9). With appropriate data manipulation, the waveform log can be used to produce the other three logs discussed in this subsection.

Qualitatively, one can observe that the region of heavy cycle skipping between 8750 and 8770 feet, Figure 5.9, is marked both by phase shifts and amplitude variations, Figure 5.11. Depending upon the details of the Δt logging circuitry, both the phase shift and amplitude reduction can produce cycle skipping. The state-of-the-art has not yet reached the level of sophistication that allows fractures to be located unambiguously and with high confidence from waveform analyses.

5.2.6.4 Phase Splitting

Instead of viewing amplitude versus time waveforms, it is possible to produce an entirely different type of record that enhances the visual perception of phase shifts. The waveform display of Figure 5.11 was produced by displaying signal amplitude versus time on an oscilloscope and then recording successive cathode ray tube (CRT) images on a continuous strip of 35 mm film. By disconnecting the vertical amplifier from the vertical deflection plates and then using the vertical output to modulate a control grid in the CRT, a straight-line, intensity modulated display is generated to represent the acoustic signal. The commercial techniques produce a faint spot with a zero voltage waveform, a spot intensity which increases with increasing positive voltage, and beam cut-off (no spot) for all negative voltages. A recent innovation at the Lawrence Livermore Laboratory is to color modulate the display. The utility of this new scheme is still being evaluated by its developers /38/. With the proper exposure, the images of successive intensity modulated sweeps blend together to form a continuous display of alternating

DEPTH	4' SPACING SIGNATURE	500 μ SEC / 2.25 IN.	6' SPACING SIGNATURE
	SIGNATURE SAMPLING : 4 ft.		

Figure 5.11. Waveform log (Dresser-Atlas Signature Log) in hole GT-2B at Fenton Hill. The amplitude reduction and phase shifts between 8750 and 8770 feet correspond to the cycle skips in Figure 5.9.

dark and light bands. In a homogeneous medium, the record consists of uniform dark and light bands running parallel to the borehole. In an inhomogeneous medium, the banding is irregular, and it is the irregularities which are interpreted to indicate fracturing and lithologic variations. The commercial logs are trade named Variable Density Log or VDL (Schlumberger), Micro-Seismogram (Welex), and 3-D Log (Birdwell).

The interpretation of the logs is not straightforward; we feel that at the present state-of-the-art many of the irregularities in the records must fall into the category of "correlatable features." For example, data taken in limestone, Figure 5.12a, illustrate the phenomena of phase splitting which has been associated with vertical fracturing. Note that at 1080 ms and at 30 feet above the location marked B, the dark band splits and then rejoins at location B. Because this phase splitting occurs without any appreciable change in time of the first arrival, Welex log analysts believe that the phase split results from the signal on one side of the borehole being out of phase with that from the other side. Because of the long interval over which the split occurs, they have deduced that the cause is a fracture or other large discontinuity more nearly vertical than horizontal /39/, but they did not deduce the distance of the discontinuity from the well. To complement the phasing information of the Micro-Seismogram, an amplitude reduction curve (called a Fracture Finder by Welex) is also displayed. When taken during the first positive pulse, the amplitude is that of the P-wave; when taken near the first direct S-wave arrival, the curve can be contaminated with P-wave returns from reflections at some distance from the well. The plot in Figure 5.12a is termed the S-wave amplitude. Within the phase-split interval, the signal amplitude is not reduced as much as it is at the ends of the interval. A significant S-wave reduction from vertical fractures is not expected as discussed in Subsection 5.2.6.2. and thus, it is believed that an adequate data display must include a P-wave Fracture Finder. Whether the phase split is caused by a P-wave returning from a fracture at some distance from the wellbore and interacting with the

Figure 5.12. Phase-splitting technique using Micro-Seismograms and acoustic amplitude log (Fracture Finder). Tests performed in limestone /38/.

(a) Vertical fracture inferred from phase-split at $t = 1080$ ms ranging from B to 30 feet above B.

(b) Vertical fracture verified by core indicated by phase splits, phase irregularities, and dip in acoustic amplitude log at indicated location.

direct S-wave is not known. The amplitude curve of Figure 5.12a does not appear useful for locating the fracture inferred from the Micro-Seismogram. That a fracture did, in fact, exist at some distance from the wellbore was not determined by independent means.

A case where vertical fracturing was observed from a core is illustrated in Figure 5.12b. In this case, the fracture cut across the wellbore. Phase-splittings are clearly evident on the Micro-Seismogram at the location indicated, as are a number of other well defined interference effects. The amplitude reduction curve is not identified as either a P-wave or S-wave, but it too clearly indicates a fractured zone.

An example of phase-splitting observed with a Variable Density Log (VDL) at hole GT-2B at Fenton Hill is illustrated in Figure 5.13. Distinct phase-splits occur near 8740, 8768, 8790, and 8805 feet, and at several times from the onset of the first direct P-wave arrival. The orientation of the fractures and their distances from the wellbore are uncertain. Without an underlying theoretical analysis and an extensive catalog of experimentally verified VDL patterns, it is not possible to infer the details of fracture geometry with a high confidence level. The simplest interpretation of Figure 5.13, based upon the logs of Figure 5.12, is that the fractures are near-vertical and intersect the wellbore at the indicated depths. Additional support for this simple interpretation comes from a core sample which verifies the fracture at 8740 feet /40/. We believe that considerably more work must be done before the phase-splitting techniques can be justified for fracture mapping.

5.2.7 Radiochemical Logs

The contact surfaces between different rock types are often susceptible to erosion when cut by a borehole. To differentiate between erosion arising at a fracture and at a contact alteration, both of which appear the same to a caliper, it is necessary to sense the chemical properties of the rock. Geochemistry and petrology are extensive subjects which we shall not dwell upon; for logging purposes, we merely note that chemical differences between rocks arise from metamorphism, crystallization, diffusion, hydrothermal alteration, and fractionation. Thus, logs which sense major and minor element concentrations help to identify rock units and interpret various fracture-sensitive logs. The most common tool is the gamma ray log, which integrates the

Figure 5.13. Dresser Atlas Variable Density Acoustic Log (VDL) of a portion of hole GT-2B interpreted to locate fractures by phase shifts. Phase shifts occur near 8740, 8768, 8790, and 8805 feet. Log run 6/14/77.

total background gamma radiation emitted from the rock. More chemically selective is the spectralog, a tool using radiation spectral analysis to sense radioactive potassium, thorium, and uranium. A third is the silicon/calcium log, which employs pulsed neutron sensing. A correlation of chemical variations with caliper log indications of hole erosion can distinguish weak zones indicating dikes and changes in rock type from fractures within given rock units.

All three chemical sensing logs--gamma ray, spectralog, and silicon/calcium--were recorded at Fenton Hill. Figure 5.1 illustrates a portion of hole GT-2 where logs sensitive to rock chemistry were used to supplement the chip log and to locate dikes (igneous intrusive tabular bodies). The qualitative variations in silicon content derived from the silicon/calcium log are shown as relative concentrations. The dikes of low-silicon content seem to have been injected in such a way that chemical alteration occurred. Dikes are commonly confined by old fractures, often associated with a weakening of both the injected and bounding rocks by hydrothermal fluids. The weaknesses are recorded as washouts by the caliper log. Once a depth correction of about 6 to 10 feet is made between the silicon/calcium and the caliper logs, these zones of lower silicon content often correspond with symmetric hole erosion indicated by the caliper logs, Figure 5.14. This method of locating rock units is more accurate than the chip log and can precisely pinpoint unit boundaries, particularly those of dikes and other thin rock units.

Abnormally high total gamma radiation measurements were also plotted to aid in determining the lithology and to seek a correlation between gamma sources and fractures. Gamma spikes appear to occur in thicker rock zones of constant silicon content and rarely show signs of contact alteration between rock units. The spikes in Figure 5.1 are interpreted as representing either a localized crystallization band of trace elements within a major rock unit or as a fracture where trace elements have condensed out of fluids moving through the fracture. Based upon spectralog data, the dominant radioactive trace element is uranium. Only occasionally do gamma

Figure 5.14. Comparison of silicon and caliper logs in zone of heavy contact alteration. Note 6 to 10 foot offset in depth scale needed to improve correlation between logs. Zones of low silicon and enlarged diameter interpreted to be dikes and contact alterations. Zones of borehole erosion without prominent silicon content interpreted to be fractures. Fracture location and size indicated by F2 or F4 (see Subsection 6.1 for definitions). Dikes and contact alterations indicated by CA,3 (see, also, Subsection 6.1).

spikes correlate with fracture locations inferred from the caliper log as indicated in Figure 5.1. The silicon/calcium and gamma ray logs were found quite helpful in identifying rock contacts and zones of chemical alteration of the rock. However, no chemical indicator seemed to correlate closely with fractures.

5.2.8 Formation Density Compensated Log

The formation density compensated log is used mainly as a porosity logging tool in the petroleum industry. Whether or not a commercial tool is sensitive enough to detect borehole fracture intercepts or accurate enough to measure fracture porosity is addressed in this subsection by examining field logs.

Using a cobalt 60 source, the density logging tool emits gamma rays in the 1 Mev range that collide with the orbital electrons of the formation elements and are scattered principally through the Compton effect. A crucial aspect of the tool design is the two detectors and compensating circuits that are used to make a correction for the effects of mudcake and borehole rugosity. The backscattered rays that reach the main detector, which is spaced about two feet from the source, are counted and related to the rock density through a knowledge of the scattering process and the atomic properties of the elemental constituents of the rock. The value of the density is inaccurate because of failure to account for the thickness and density of borehole fluids interposed between the tool and the formation. A density correction, computed using the signals from the main detector and the secondary detector, which is spaced about one foot from the source, is applied to the uncompensated value derived from the main detector. Intuitively, it can be appreciated that by employing two detectors, one that sees a long path through the formation because it is spaced away from the source, and one that sees only a short path through the formation because of its close spacing to the source, effects near the borehole can be eliminated from the final result. The total depth of investigation is limited to about seven inches into the formation. It is clear that the accuracy of the correction must depend upon each detector being spaced the same distance from the borehole wall, i.e., sonde tilt or surface

irregularities will reduce the accuracy of the compensation. Under average operating conditions, presumably meaning a smooth borehole, the standard deviation of a compensated tool is of the order of 0.01 to 0.015 gm/cc /41/.

The ability of the formation density compensated log to detect fractures can be evaluated by comparing the density log with a caliper log. A Dresser-Atlas Compensated Densilog between 8600 and 8800 feet in hole EE-1 at Fenton Hill is shown along with a two-independent-arm caliper log in Figure 5.15. Borehole erosion patterns from the caliper log at 8694, 8699, 8718, 8727, and 8732 feet match irregularities in the formation density log to within about two feet, although the density log feature at 8712 feet is quite weak and almost surely would not be associated with a fracture if it were not for the caliper log. The inverse process of picking fractures from the density log and matching them with caliper erosion zones does not work well. The prominent pulse at 8687 feet on the density log occurs within a region of the caliper log which appears relatively smooth. Because the density and caliper logs fail to correlate at 8687 feet, and because the density correction is negative, we are led to believe that the apparent large decrease in density is an artifact which may have arisen from tool tilt. An example of density log artifacts--large non-reproducible excursions of the measured density--is shown in Figure 5.16 for a logging run made in an enlarged and rugose shale borehole.

Taken at face value, the density log yields the following fracture porosities at the six aforementioned depths, based upon grain density of 2.60 gm/cc: 0.130, 0.064, 0.061, 0.026, 0.034, 0.076, and 0.091. The grain density of 2.60 gm/cc is a value consistent with the density log between 8600 and 8670 feet, and the in situ matrix porosity of 0.001 derived from laboratory tests /25/. The density log implies an even smaller value of the grain density (2.55 gm/cc) between 8670 and 8800 feet; both values are less than that usually associated with granite (2.66 gm/cc) and raises a question about the accuracy of the density log. The inferred fracture porosities are considerably greater than the 0.001 in situ matrix porosity. The value of 0.130 appears to be associated

Figure 5.15. Comparison of compensated formation density log (Dresser-Atlas Densilog) with two-independent-arm caliper log in hole EE-1 at Fenton Hill. Fractures inferred from caliper log correlate with pulses in density log to within 2 feet, but the inverse does not hold. The large Densilog spike at 8687 feet is believed to be a spurious signal arising from tool tilt. Densilog run 9/28/75, caliper log run 9/27/75.

with a spurious response; establishing the validity of the other measurements requires, in our opinion, calibration tests in a medium of known fracture porosity.

Figure 5.16. Non-reproducible artifacts on formation density compensated log in shale caused by borehole enlargement and rugosity /41/.

5.2.9 Depth Measurement Offsets

The elimination of depth uncertainties when crosscorrelating logs requires a precision of the order of 0.02 percent, i.e., it is believed that a depth accuracy of ± 1 foot near the bottom of a geothermal well is needed for relating sharply defined features, such as fractures, from one log to another. This high degree of precision requires that the cable reel

be accurately calibrated and that corrections be made for thermal expansion and elastic stretch. For example, a change in the mean temperature of the borehole of 25°F, which can easily occur if logging runs are made at different times and under different operating conditions, will result in a change in cable length of between one to two feet (depending on cable alloy) at a depth of 8000 feet. The required precision appears to be greater than most logging service companies are accustomed to, as evidenced by their standard logging records. Although more detailed record keeping concerning reel calibration, sonde weight, borehole temperature, and cable specifications would reduce uncertainties, it is believed that depth uncertainties could almost assuredly be greatly reduced by installing a fiducial ring as a means of referring all measurements to a fixed depth in the borehole. The device could be an attachment at the bottom of the drill hole casing. Identifiers attached to the ring--such as a caliper offset, radiation source, magnetic source, and an electrically conductive sleeve--would enable each log to self-record a depth measurement based on a common, fixed reference point. The use of hardware to provide a common depth reference is appealing because it is essentially fool proof.

5.3 LOGGING DURING POST-HYDRAULIC FRACTURE OPERATIONS

In a reservoir of economic size, it is almost certain that a series of steeply dipping hydraulically induced fractures will form the heat transfer surfaces and that the fractures will be cut by two near-horizontal wellbores. To ensure that the wellbores are essentially perpendicular to the heat transfer surfaces, the strike of hydraulically induced fractures must be known in advance of directional drilling operations. One method for satisfying this requirement is to create a hydraulic fracture in a still vertical wellbore just above the intended operating depth and then to measure the orientation of the fracture with an appropriate tool, as discussed in Subsection 5.2. The measurements are limited in working depth only by the maximum operating temperature of logging tools and are limited in accuracy by possible variations in strike with distance from the borehole. The latter limitation needs to be investigated with long

range mapping methods, such as radar (Sections 2 and 3), and with theoretical analyses. A second method, which does not depend upon borehole tools and appears potentially useful, is to determine the average strike of a hydraulic fracture using surface tilt measurements. At the present time, the tilt technique has been demonstrated to depths of 10,000 feet. The method is reviewed briefly in Appendix A.

The second application of post-hydraulic fracture mapping is to locate and determine the orientation of possible well interconnections. The techniques discussed in Subsection 5.2 can also be used to locate hydraulically induced fractures. In most cases, the locations of induced fractures are known from the position of straddle-packers used in the fracturing operation, so a greater emphasis is placed on determining fracture orientation and size. The strike and dip can be determined in principle with a televiewer, television, three-independent arm caliper, four-independent-arm resistivity dipmeter, and impression packer. In practice, high temperatures restrict their use. Fracture size can be estimated using several other techniques which depend mainly on the fluid flow or heat transfer properties of a fracture. With a single borehole and a blind fracture, all fluid flow processes are inherently unsteady; namely, the fracture must be inflated at high pressure and then be allowed to collapse at reduced pressure. The logs which work under these conditions are (1) temperature, (2) flow meter, and (3) tracer. These three techniques are evaluated in the following subsection. Television, discussed in Subsection 5.2.4, might be practical for detecting fluid mixing at fracture sites if the borehole fluid contained enough particulates to make flow visualization possible, while still retaining sufficient optical clarity.

5.4 LOGGING DURING TWO-WELL COMPLETION

The most fundamental way to map fracture connections between wells is by flow testing. The main emphasis is on determining the location and flow capacity of fractures. The techniques involving the use of temperature probes, flow meters, and radioactive tracers are reviewed in this subsection.

5.4.1 Temperature Logs

Although the temperature level is needed for estimating the capacity of a geothermal reservoir, a logging tool that records the temperature difference between two sensors is more capable of detecting flow locations at low fluid flow rates than one that measures the temperature level with the same type of transducer. A conventional temperature log and a caliper log run over the same interval in borehole CH-3 at the Raft River Geothermal Reservoir /42/ are compared in Figure 5.17. The correlation between apparent flow entries and the two major hole enlargement (fracture) portions of the caliper log is quite good. The differential temperature measurements were accomplished using two sensors spaced two feet apart on a sonde. The differencing was accomplished numerically. The accuracy of the temperature measurement was not reported, i.e., the separate temperature measurements corrected for the two-foot offset was not compared for the two transducers.

At Fenton Hill, single-transducer temperature logs have been used to infer flow conditions at well-to-well fracture connections. Temperature logs have been taken during pressurization, production, and thermal recovery periods and have been able to locate fractures and porous zones /40/. Temperature surveys made in hole GT-2B during shut-in and at different times during a subsequent flow test are summarized in Figure 5.18. The temperature profile during shut-in shows the effects of residual heating from a previous flow test at the locations marked 1, 2 and 3. Heat transfer calculations based upon the flow test measurements indicate that the main connection is at location 2, whilst the secondary connection is at 1, and the tertiary (and quite minor) connection is at 3. The LASL temperature probe is quite accurate. It has been calibrated to have an error or no more than 0.26°C and a resolution of 0.01°C , which is considerably better than that ascribed to thermistor probes in Reference 43.

5.4.2 Flow Meter Logs

A number of flow meters, each employing a different operating principle, are available for measuring the velocity of internal flows such as is found in process piping /44/. Only one type of flow meter currently

Figure 5.17. Temperature log, computer-derived differential temperature log and caliper log of Borehole CH-3 at Raft River Geothermal Reservoir /42/.

Figure 5.18. GT-2B temperature surveys during shut-in (background survey) showing residual heating from previous flow tests and at increasing times during subsequent flow test. The main flow connection is at position 2 with subsidiary connection at 1 and 3 /26/.

exists for borehole use, and that is the spinner type /43/. A spinner flow meter contains an impeller or turbine rotor that spins at a rate proportional to the velocity of fluid flowing through the meter. At low velocities near the minimum detection threshold, some nonlinearity will be present in the response because of bearing friction. The minimum flow velocity that will turn the impeller is about 0.4 ft/s. This threshold sensitivity is about the same for commercial tools /45/ and a Well Worth tool modified by LASL /25/. Low volume flow rates in large boreholes can be measured by decreasing the flow area, either by packers or with a nozzle, e.g., LASL has reported measuring a flow of 0.017 ft³/s near the bottom of hole GT-2B at Fenton Hill. By way of comparison, this flow is about three-quarters of what can be supplied by a household faucet. Schlumberger rates its spinner at 260°C; LASL rates its tool at 200°C. An example of the usefulness of a spinner for logging flow entries is illustrated in Figure 5.19.

Figure 5.19. Reported results of a LASL-built spinner survey tool run in hole GT-2B showing the major flow entry regions as steps of decreasing total flow as the tool is lowered down past each major flow entry /25/.

5.4.3 Tracer Logs

Tracers are particularly useful in steady flow situations where they effectively tag individual fluid particles whose subsequent motion can then be studied as a function of time and position. Intricate flow paths can be discerned and transit times measured. Tracers are indeed a powerful diagnostic tool. The most common tracer logs make use of radioactive materials such as I^{131} or Br^{82} , which have a half-life of 8 days and 35.5 hours, respectively, and can be detected by conventional gamma-ray logging tools. An isotope such as tritium is not as useful for borehole logging as either I^{131} or Br^{82} , because its 12.1 year half-life would result in the contamination of subsequent surveys. In addition to radioactive tracers, sodium fluorescein dye has also been used at Fenton Hill. A dye requires down-hole sampling and laboratory analyses. It is believed that a radioactive tracer is a superior diagnostic tool. Radioactive tracers can be used to identify flow entries into a well as indicated in Figure 5.20. The entries are sharply defined when the tracer first appears, but diffusion and transport smear the record at late times. Transport also causes the gamma pulse to move up the well so that only the onset of a gamma pulse gives an accurate location for the inter-well connection. Flow exits out of a well can also be identified from the residual radioactivity of tracers adsorbed onto the fracture surface.

Figure 5.20. Tracer log showing the sequential appearance of BR⁸² at several well interconnections. LASL experiment in hole GT-2B at Fenton Hill /46/.

SECTION 6

RESERVOIR FRACTURE STRUCTURE BASED ON CONVENTIONAL BOREHOLE LOGS

Test borings at Fenton Hill and several surface exposures of the basement rock in New Mexico lead to the view that the rock in the Fenton Hill reservoir is not a recent monolithic intrusion of granite, but rather is an igneous metamorphic rock that evolved over millions of years and underwent several periods of deformation and igneous rock injection /26,47/. It follows that the hot dry rock reservoir is likely to have a complex structure that may influence not only the interpretation of data used to map the fractures which form the heat transfer surfaces between wellbores, but also the formation of the hydraulically induced fractures themselves. Thus, for the purpose of evaluating techniques for mapping fractures and for testing the consistency of measurements, it is useful to have a detailed structural model of the geothermal reservoir. The development and applications of such a model along with certain unresolved matters are described in this section. Conventional borehole logs were sufficient to provide data for inferring the dip, strike, number, spacing, and size of fractures and other weakness planes which intersect the borehole. It is assumed that the fractures and dikes which intersect the borehole are not simply local phenomena, but are representative of the rock structure at some distance from the wellbore. As it now stands, most conventional logs are not adequate for determining the lateral extent of fracturing. As a first approximation, the intercepts at the borehole are mapped into the far-field using a planar geometry for fractures and dikes. Details of the model are presented in Subsection 6.1. Results concerning fracture roughness and fracture frequency--the initial reason for developing the model--are given in Subsection 6.2. Certain questions have not been completely answered, especially those concerning the direction of permeable fractures, and the extent of hydraulically induced fractures. These topics are discussed in Subsection 6.3.

6.1 RESERVOIR FRACTURE MODEL

Fractures and other weakness planes, inferred from logging data, were categorized as types 1 through 5 on the basis of their size estimated primarily by crosscorrelating caliper, electric, and acoustic logs, and by making use of some supplementary data from gamma ray, silicon/calcium, density, and drilling-related logs.

6.1.1 Fracture Size

It was assumed that when the larger erosion zones (caliper deflections) coincide with the regions of greatest fluid penetration (resistivity deflections) and greatest acoustic attenuation (zones of low velocity), the major fractures of type 1 have been identified. Similarly, when small breaks occurring in smooth portions of the borehole correspond with small indications of fluid penetration and with minor perturbations or disruptions of the acoustic records, type 5 or very minor fractures are identified. Types 2 and 4 are intermediate categories of fracture deformation. Type 3 is less specific. It refers to significant borehole erosion identified in the caliper log associated with rock contacts, usually dikes.

The size associated with each type of fracture is indicated in Table 6.1. The magnitudes of the differential slip are, in fact, not known from measurements, but rather are estimates based upon experience with underground mapping in hard rock in the western United States. For example, type 1 displacements have been set equal to those of large fractures which might possibly extend a few kilometers. Types 2 and 4 are associated with progressively smaller and shorter fractures. Type 3 are assumed equivalent to type 2 fractures in terms of offsets and width because they are permeable weaknesses of considerable extent. Because of the cementitious nature of injected dikes filling in weakness voids, they are not considered equivalent to type 1. The type 5 weaknesses are thought to represent cracks with a limited length of perhaps a few tens of meters.

Table 6.1. Characterization of fracture type based upon subjective analyses of Fenton Hill logs below 5800 feet. Differential slip and opening dimensions are assumed; frequency is determined from Figure 6.1 and Table 6.3.

Relative Weakness Type	Symbol	Estimated Magnitude		Average Spacing Between Fractures (m)
		Differential Slip (m)	Fracture Opening, Max. (mm)	
1		$10^{-1} - 1.0$	3	40
2		$10^{-2} - 10^{-1}$	0.5	10 - 13
3	Altered Zones Dike Injection		0.5	10 - 40
4		$10^{-3} - 10^{-2}$	0.05	4 - 7
5	Not Shown	$0 - 10^{-3}$	0.005	1.3

6.1.2 Fracture Orientation

In principle, the directions of a planar weakness--the dip and strike--can be determined from borehole logs. In practice, this can be a difficult task, and the results will be quite uncertain if the logging runs were not designed to yield the necessary data. Such is the case with some of the Fenton Hill data obtained with commercial tools. Fortunately, several logging runs were made with tools that did allow the dip and strike to be measured and the error associated with the measurements to be estimated. A considerable amount of the data in our possession were obtained with commercial two-independent-arm calipers which cannot establish the dip of a weakness plane with any certainty. What can be obtained from the commercial caliper log is a rough indication of whether a fracture intersects a borehole in a low or steep angle. An indication of a low dip can be quite unreliable because it is entirely possible

for a steeply dipping fracture to give the appearance of a horizontal fracture to a device that makes only two independent measurements. The best that can be done with a two-independent-arm caliper is to assume that if enough measurements are made, then a high percentage of indicated shallow dips will, in fact, arise from near-horizontal fractures. An indication of a steep angle, on the other hand, can only arise from a steeply dipping fracture. As it turned out, the two-independent-arm caliper registered a large number of steep dips with about an equal number of dips having apparently opposite sign. Because the tool did not contain an orientation measuring device, it cannot be stated with certainty that fractures did actually crisscross. We merely assume this to be the case rather than being an artifact of tool rotation, which has been measured to be small effect with an oriented diplog. To actually set the value of the angle associated with the steep dips, recourse had to be made to other measurements.

Based on measurements employing six different techniques, the dip of fractures has been inferred to vary from 66 degrees to near-vertical, Table 6.2. A seventh technique employing gamma ray detection determined the dip of the granite/gneiss contact surface to be 63 degrees, which suggests that the reservoir has a structural weakness with a dip of approximately 63 degrees. For the intended purpose of the fracture structure model, a dip angle of ± 70 (± 10) degrees is considered to be sufficiently accurate. An additional feature of the weakness plane structure is a set of shallow angle dip planes which are inferred from the two-independent-arm caliper measurements. The three sets of weaknesses crisscross to produce a reservoir that is both fractured and penetrated by dike-like intrusions, as shown schematically in Figure 6.1.

The applications for which the structural weakness model was developed required no knowledge of fracture strike. In actual practice, the determination of strike is crucial for the placement of boreholes. Measurements indicate two general trends for fracture strike: one to the northwest and the other to the northeast, as indicated in Table 6.2. The experimental results indicate a fair uncertainty in the value of the strike angle at essentially the same locations. It appears that there are no basic physical

Figure 6.1. Schematic vertical section illustrating fracture and weakness structure inferred from single borehole geophysical log data below 5800 feet at Fenton Hill.

reasons why a televiewer, an impression packer, and a three-arm-caliper should not yield accurate determinations of the strike. Experimental uncertainties are discussed in Subsection 5.2, while matters concerning the two different strikes are discussed in Subsection 6.3.1.

Table 6.2. Fracture orientations at the borehole. Referenced data reports do not consistently distinguish between true North and magnetic North (a difference of approximately 17 degrees).

Hole	Depth	Dip (deg)	Strike	Tool	Comment	Reference
EE-1	6438	~ 90	N70°W	Impression Packer	Suspect by LASL	35
	6449	~ 90	N70°W	Impression Packer	Suspect by LASL	35
	6530		N25°E	Impression Packer		39
	6533		N3°E	Televiewer	Suspect by USGS	28
	6560		N25°E	Impression Packer		39
	6562		N12°E	Televiewer	Suspect by USGS	28
	9650		N53°W	Temperature and Tracer	Dip Assumed Vertical	35
	9670	80	N70°W	Televiewer	Cross Fracture	25
	9670	90	N20°E	Televiewer	Cross Fracture	25
GT-2	8640	71	N27°W	Televiewer		25
GT-2B	5840	75		4-Arm Diplog		
	8852	66	N18°E	3-Arm Caliper	True North	24

6.1.3 Measured Fracture Distribution

The data supporting the fracture distribution model are listed in Table 6.3. These were compiled while doing the crosscorrelation of the principal geophysical logs of holes GT-2, GT-2B, and EE-1. Data between the intervals of 5800 feet to 9400 feet in the three holes were tabulated by apparent fracture and

Table 6.3. Frequency of fracture and weakness plane intercepts with vertical boreholes at the Fenton Hill hot dry rock reservoir as a function of fracture size. Fracture size (type) defined in Table 6.1.

- (a) Number of intercepts per 100 ft interval in three different holes inferred from several different types of logs. Hole GT-2: two-independent-arm caliper and four-arm diplog used from 5800 to 6600 ft; caliper only below 6600 ft. Hole GT-2B: VDI acoustic and laterolog used from 6000 to 8800 ft. Hole EE-1: Two-independent-arm caliper from 6500 to 9500 ft.

Depth Intervals (ft.)	Hole GT-2				Hole GT-21				Hole KI-1			
	Fracture Type											
	1	2	3	4	1	2	3	4	1	2	3	4
5800 - 5900	0	2	0	6								
5900 - 6000	0	1	1	3								
6000 - 6100	1	1	3	4	2	1	0	3				
6100 - 6200	0	0	0	5	1	2	0	3				
6200 - 6300	0	2	1	10	0	0	0	4				
6300 - 6400	0	2	0	7	0	0	0	4				
6400 - 6500	0	0	0	8	0	1	0	4				
6500 - 6600	0	3	3	6	2	2	2	4	0	1	0	4
6600 - 6700	0	1	0	4	2	2	2	4	0	0	0	2
6700 - 6800	1	1	1	4	2	2	3	4	0	0	0	2
6800 - 6900	1	0	1	7	2	2	2	4	2	0	1	4
6900 - 7000	0	0	0	2	2	2	2	4	0	0	0	4
7000 - 7100	1	1	1	3	2	2	2	4	0	1	0	3
7100 - 7200	0	1	0	2	2	2	2	4	1	0	1	3
7200 - 7300	0	1	1	3	2	2	2	4	0	0	0	4
7300 - 7400	0	0	1	0	2	2	2	4	0	0	0	3
7400 - 7500	0	0	0	3	2	2	2	4	0	0	0	1
7500 - 7600	0	0	0	3	1	1	1	4	0	0	0	2
7600 - 7700	0	1	0	1	0	1	0	4	0	0	0	4
7700 - 7800	0	0	0	3	0	0	0	2	1	0	0	3
7800 - 7900	0	0	0	2	0	0	0	0	0	0	0	2
7900 - 8000	0	0	0	2	0	1	0	4	0	1	0	1
8000 - 8100	1	0	0	3	0	0	2	1	0	0	2	4
8100 - 8200	0	0	0	2	0	0	0	5	0	2	2	2
8200 - 8300	0	0	0	4	1	0	2	5	0	0	0	0
8300 - 8400	1	1	1	2	0	0	1	7	0	1	0	2
8400 - 8500	0	0	0	1	0	4	0	1	0	0	0	2
8500 - 8600	0	0	0	2	0	2	0	4	0	0	0	5
8600 - 8700	0	0	0	2	0	0	1	5	0	1	0	5
8700 - 8800	0	0	0	0	0	3	2	5	0	1	1	3
8800 - 8900	0	1	0	6					0	1	0	7
8900 - 9000	0	0	0	4					0	1	2	3
9000 - 9100	0	1	2	3					1	1	1	3
9100 - 9200	0	1	1	2					0	1	0	8
9200 - 9300	0	0	0	1					0	1	2	2
9300 - 9400	0	0	1	2					0	0	2	2
9400 - 9500	1	1	0	2					0	2	1	3

- (b) Average intercept frequency per 100 ft interval with logging tools as a parameter.

Tools	Fracture Type				Comment
	1	2	3	4	
Caliper & Diplog	0.13	1.4	1.0	6.4	GT-2 5800 - 6000 ft
Caliper	0.21	0.38	0.34	2.7	GT-2
	0.17	0.50	0.50	3.0	EE-1
VDI & Laterolog	0.25	1.2	1.0	3.75	GT-2B

weakness type over 100-foot depth intervals. The methods used to estimate fracture existence and location are described in Section 5. The tabulated results not only reveal some interesting and important features of the fracture pattern, but also provide an indication of the sensitivity of different tools to fractures. On the average, very large fractures (type 1) intersect a near-vertical borehole only infrequently--about once every 400 to 500 feet. The wide spacing arises in part because the fractures themselves are nearly vertical. The normal distance between fracture planes is much less than the borehole intersection distance, being reduced by the secant of the dip angle to about 170 feet. The fracture-borehole intersections become more frequent as the size of the fracture is reduced, e.g., for a type 4 fracture the intersection distance is approximately 32 feet, while the spacing between fractures is about 11 feet. The variability arising from the subjective method of picking fractures is less between different boreholes using the same tool than it is in essentially the same borehole using different tools. For example, the average number of fractures and weakness planes inferred from a two-independent-arm caliper varies by less than 20 percent between the GT-2 and EE-1 boreholes at Fenton Hill, while the averages vary by 40 to 140 percent in GT-2 and GT-2B when both VDL acoustic and Laterologs are used instead of a caliper. This result implies that the weakness structure is rather uniform at Fenton Hill, but that the criteria used to pick fractures with the deeper sensing tools are not in concert with that used with the caliper log

6.2 FRACTURE ROUGHNESS AND FRACTURE FREQUENCY

If a hot dry rock reservoir were a monolithic block of granite penetrated by only a few nonintersecting fractures, then certain problems regarding fracture mapping could be resolved quite simply. For example, the origin of backscattered electromagnetic or acoustic energy could only be a fracture heat transfer surface. Multiple scattering and multiple mode conversion between P- and S-waves would not exist. For almost all wavelengths of practical interest, reflections would be specular. The evidence presented in Subsection 6.1 indicates that the Fenton Hill reservoir is highly fractured, and consequently rather complicated wave interaction

phenomena may be expected. A fundamental question that arises when probing for fracture planes with either electromagnetic or acoustic waves is whether reflections are specular or diffuse. The model developed in Subsection 6.1 can be used to answer this question by establishing the roughness of a major fracture plane, i.e., a type 1 fracture. The surface roughness of a fracture is caused by offsets arising from slip along the other weakness planes which intersect the fracture in question. The distance between fracture intersections, indicated schematically in Figure 6.1, varies between 2 and 55 meters along a type 1 fracture. The slip offsets and the spacing between offsets are listed in Table 6.4. The effective roughness height is reduced from the slip offset by the sine of the intersection angle (Figure 6.1) which is approximately 0.75 for the Fenton Hill geology. When searching for a major fracture plane with electromagnetic or acoustic energy, waves will be reflected back to the receiver by the numerous minor fractures which crisscross the reservoir. To estimate the amount of noise or clutter produced as a ray traverses minor fractures, it is useful to know the number of reflecting planes or fractures that intersect the ray. To simplify matters, we have eliminated the angular dependence by averaging over a large number of angles. The average spacing between fractures along a hypothetical average ray probing outward from a vertical borehole varies between 1.3 and 40 meters, depending upon the type of weakness as indicated in Table 6.4.

Table 6.4. Fracture plane offsets and spacings

Weakness Type	Differential Slip (m)	Average Spacing Between Fractures (m)	Spacing on Main Fracture (m)
1	10^{-1} - 1.0	40	55
2	10^{-2} - 10^{-1}	10 - 13	20
3	10^{-2} - 10^{-1}	10 - 40	40
4	10^{-3} - 10^{-2}	4 - 7	8
5	0 - 10^{-3}	1.3	2

6.3 UNRESOLVED ISSUES OF FRACTURE STRUCTURE

The fracture structure model derived from conventional borehole logs (Subsection 6.1), does not address the question of how far fractures extend from the borehole. Indeed, most conventional borehole logs measure reservoir properties within less than a meter of the borehole and, consequently, are incapable of yielding information on the lateral extent of either natural or hydraulically-induced fractures. In this section, we discuss certain problems concerning fracture structure that cannot be resolved by use of conventional borehole logs, and also introduce some information based on techniques still under development in an attempt to clarify the status of fracture mapping research.

6.3.1 Direction of Permeable Zones.

The LASL model of seismicity implies that slippage can occur along planes trending to the northeast. From other investigations of seismic activity following fluid injection into fractured prestressed regions, it appears likely that fluid will migrate along the most permeable paths, and when the paths intersect regions of critical stress balance, the fluid pressure will trigger microseisms/48/. Continued injection should then produce microseisms at increasingly distant locations. Borehole measurements have confirmed the existence of fractures striking to the northeast. Attempts to relate the direction of permeable fractures within the Fenton Hill reservoir to either current or ancient stress fields outside of the Fenton Hill caldera almost certainly rest on tenuous ground. Nonetheless, it is interesting to note that basement outcrops near Albuquerque indicate an old fracture pattern trending northeast /49/, while a recent determination of in situ stresses near Socorro suggest a fracture pattern trending northwest. These observations are in qualitative agreement with the current view that major hydraulic fractures trend northwest, while subsidiary leakage paths trend northeast.

6.3.2 Extent of Hydraulic Fracturing

It is thought that major weaknesses, when oriented to in situ stresses so that tensile openings can occur, will inflate first to become the major initial hydraulic fractures. Even if packers were used to force the

hydraulic fracture to originate in a naturally unfractured section and even if the local state of stress at the borehole were such as to cause the borehole wall to fracture in a direction different from that of natural weaknesses, it seems likely that a fracture would develop only a short distance before intercepting a favorably oriented natural weakness, which would then take over as the major hydraulic fracture path. Other weaknesses would also intersect the major initial hydraulic fracture path creating crossconnections to other favorably oriented weaknesses roughly perpendicular to the minimum horizontal component of in situ stress.

At the present time, three types of logs have been tested for their ability to define the extent of fracturing away from the borehole. One is a conventional technique: the induced potential. The other two are not normally used in commercial practice, but they appear to be quite useful; namely, passive microseismic detection and borehole-to-borehole acoustic transmission. One test of the induced potential method demonstrated that a fluid-filled fractured region gives rise to a measurable potential variation. Thus, the extent of fracturing parallel to the borehole can conceivably be related to the potential distribution, but the necessary analysis and confirming experiments have apparently not been performed. The microseismic technique has been used to provide fracture maps, although the current method provides a rather diffuse map (see Section 11). The active acoustic method is made complicated by the strong interactions and mode conversions that occur when acoustic waves traverse a fractured region. Cutting off shear wave propagation by fracture inflation has been shown by experiment to be a powerful method for mapping fractured regions. The limits of resolution of acoustics for mapping fractures needs to be determined, i.e., can acoustics map large discrete fractures within a region containing many fractures or can only the boundaries of the region be mapped (see Section 10).

SECTION 7

CONVENTIONAL LOG SUMMARY

With extant borehole logging techniques, it is possible to crosscorrelate a set of conventional logs to infer the location and orientation of fractures where they intersect a borehole. The usefulness of conventional tools is diminished, however, by two principal factors: temperature limitations and uncertain interpretations of the data. The situation can be improved by (1) upgrading the temperature capabilities of tools, (2) calibrating tools in a hard rock of known fracture structure, (3) developing models of tool operation in a fractured medium appropriate for hot dry rock operations, and (4) requiring greater attention to logging details in the field.

Logs obtained during drilling should, in principle, assist in inferring subsurface fracture conditions, but during operations at Fenton Hill, only core recovery provided substantial data. Based upon experimental evidence, six logs, when used synergistically, appear useful for mapping the intercepts of natural fractures with a borehole once drilling has ceased. They are the (1) caliper, (2) resistivity dipmeter, (3) televIEWer (acoustic imaging), (4) television, (5) impression packer, and (6) acoustic transmission. The dipmeter and acoustic transmission methods require improved analyses, while all six would benefit from improved high temperature capabilities. A seventh technique, the silicon/calcium log, is useful for differentiating between fractures and zones of chemical alteration, but is in itself not capable of detecting fractures. Six other techniques appear to be of marginal value for locating natural fractures at the present state-of-the-art. They are the (1) density log, (2) spontaneous potential log, (3) resistivity methods that sample large volumes of rock, (4) gamma-ray log, (5) spectralog, and (6) temperature log. The density log has shown spurious responses thought to arise from tool tilt. Problems with cross-correlating a density log with a caliper log have raised questions about inferring fracture porosity from a formation density compensated log. The density log, in our opinion, requires

testing and calibration in a granitic medium with a known fracture structure. A differential temperature measurement might conceivably record temperature changes arising from fluid seepage at natural fractures, but the technique has not been tested in a hot dry rock reservoir. Temperature logs appear to have their main use in mapping hydraulic fractures under forced flow conditions.

After hydraulic fractures have been produced, the techniques that work for mapping natural fractures can again be employed. In addition, temperature, flow meter, and tracer techniques can be used to determine the size of fractures. Well interconnections can also be located with the latter three methods. An uncertainty in tool depth that is apparent when correlating different logs can be prevented in future runs by installing a special fiducial collar and by keeping more detailed logging records.

The conventional logs have been used to develop a model of the fracture structure of the Fenton Hill reservoir, and this model shows the reservoir to be crisscrossed by a large number of natural fractures and dikes. The accuracy of the model rests on the validity of extrapolating near-wellbore data to the far-field and can be tested only by obtaining far-field data with new techniques. By serving as a first approximation, the model has been helpful for designing advanced mapping methods.

SECTION 8

FRACTURE MAPPING ACTIVITIES IN THE UNITED STATES, CANADA AND SWEDEN

The use of radar and conventional logging techniques to map fractures from a geothermal wellbore has been dealt with in this report either from first principles or from the analysis of experimental data. To ensure that we have not taken a parochial view of fracture mapping operations, we have reviewed other fracture mapping activities in the United States, Canada, and Sweden via field trips, telephone conversations, technical reports, and journal articles. The results of the review are presented below.

8.1 MAJOR PROGRAMS FOR MAPPING FRACTURES

Four major programs, in addition to the one sponsored by the Division of Geothermal Energy, are concerned with subterranean fracture mapping. To evaluate the suitability of a hard rock setting for a nuclear waste repository, both the Department of Energy, Office of Nuclear Waste Isolation, and Atomic Energy of Canada Limited are actively engaged in developing techniques for mapping fractures in granite. Methods for detecting fractures, voids and water-filled cavities are being investigated both by the Department of Transportation (Federal Highway Administration) and the Bureau of Mines. The former agency is interested in improving tunnel boring operations both from an economic and safety standpoint, while the latter agency is directing its efforts to improving the economic recovery of coal and the safety of mining operations.

Office of Nuclear Waste Isolation (ONWI) /50/. ONWI through its contractor, the Lawrence Berkeley Laboratory (LBL), and in cooperation with both the Swedish Nuclear Fuel Safety Board (KBS) and the Swedish Geologic Survey (SGV) has conducted a series of fracture mapping tests in a granitic setting at a depth of approximately 340 meters in an abandoned iron mine at Stripa, Sweden /34,51-55/. Thirteen different types of measurements were made to investigate fracturing. LBL fielded nine tools: (1) gamma-ray, (2) formation density (uncompensated), (3) porosity (neutron),

(4) caliper, (5) resistivity, (6) temperature, (7) intra-hole acoustic, (8) cross-hole acoustic, and (9) mechanical modulus. SGV fielded both normal and lateral electric logs, and a differential resistance log. In addition, use was made of core recovery and borehole television. LBL believes that the sonic waveform, caliper, and differential resistance tool were the most useful for locating fractures, but that all three tools need to be improved. The acoustic logs were displayed in a form similar to a VDL, and we believe that the comments of Section 5.2.6.4 also apply to the LBL tool. LBL mapped borehole-fracture intercepts by crosscorrelating signals from different tools in a manner similar to that done in the DGE program (Section 5.2). LBL observed that only a small fraction of the fractures logged from core or television produced a sonic waveform that indicated a fracture. The SGV differential resistance tool is quite different in construction from a resistivity diplog, but the action of the two tools is similar. Additional development work for the SGV tool is recommended by LBL--a recommendation similar to ours for the resistivity diplog. SGV believes that the application of electrical methods to crystalline rock is unsatisfactory for neither is the analysis well-developed nor are the tools designed for operation in a rock matrix of high resistivity ($\rho \approx 10^5 \Omega\text{-m}$), instead having been designed for use in sedimentary rock ($\rho \leq 10^3 \Omega\text{-m}$). To wit, the low frequency electrical properties of crystalline rock are governed by pore space and fracture characteristics that depend heavily on the fracture pattern and conducting material within the fracture, but in a manner that is incompletely understood.

The operating principles of twelve of the tools used at Stripa are similar to those discussed in Section 5, although the implementation of the techniques was in some cases different. One tool used at Stripa but not at Fenton Hill was a Colorado School of Mines Young's modulus cell. Although a decrease in modulus correlated with an increase in fracturing, the tool was not used extensively because the region of rock sensed by the cell was quite small. Other techniques can map borehole fracture intercepts in a more expeditious manner. LBL used a single thermistor to make

temperature measurements. The comments of Section 5.4.1 regarding differential temperature measurements also apply to the LBL experiments as do the comments of Section 5.2.1 concerning the use of a single trace caliper by LBL.

Atomic Energy of Canada Limited (AECL) /56/. AECL, working principally with Energy, Mines, and Resources Canada and the Geological Survey of Canada, is engaged in a program of airborne, ground, and borehole measurements to (1) determine the size and shape of rock masses that might be suitable for a nuclear waste repository, and (2) map and characterize fractures within the rock masses /57/. Before reviewing the borehole measurements, we mention one surface technique examined by AECL. Using powerful navy transmitters operating in the 15 to 25 kHz (VLF) band as a source of electromagnetic radiation, measurements were made of the magnetic field induced at the ground surface by subterranean return currents. It is hypothesized that the underground currents are channeled by conductive fractures; hence, the location and size of fractures are inferred from the magnitude and phase of the components of the magnetic field vector. The depth of investigation is limited by the skin depth with both the incident and secondary waves each being attenuated by ground loss. For example, at $f = 10^4$ Hz with $\sigma = 10^{-4}$ S/m, the skin depth is 503 meters. Whether satisfactory operation could be achieved at $f < 10^2$ Hz where $\delta = 5030$ meters has apparently not been investigated.

The borehole measurements include seismics, radar, and temperature. Additional techniques under study include gamma-ray, radioactive tracers, and borehole acoustics. The seismic studies uncovered large variations in P-wave velocity, but these variations often did not correlate with the borehole geology determined from core samples /58/. It was suggested in Reference 58 that the seismic survey was influenced by a volume of rock with a radius of 8 to 15 meters surrounding the borehole, i.e., substantial variations in rock properties at some distance from the borehole influence the received signals. The seismic data were also interpreted to yield borehole-fracture intercepts from the arrival times of tube waves /59/. It was

suggested in Reference 59 that inasmuch as tube waves arise from the squeezing effect of compression waves on fractures that intersect a borehole, it might be possible to relate the size of the fracture to the tube wave signature. The seismic data were further interpreted on the basis of the theory of seismic velocities in cracked solids to yield the density of (small) cracks surrounding a borehole /60/. Because the attenuation of seismic signals is believed to be caused principally by fluid flow between cracks, the possibility was raised of estimating rock permeability by using seismic techniques. For the geothermal reservoir program, we note that the relative magnitude of the effects on seismic signals arising from a few very large fractures and from numerous small-scale cracks must be quantified.

By emplacing a 100-MHz radar system in boreholes spaced 10 and 46 meters apart in a low magnetic susceptibility granodiorite, it was determined that a one-way range of at least 46 meters could be achieved in the transmission mode /61/. If accurate performance figures for the radar were available, the backscatter range could be calculated.* The best that can be said based on available data is that the backscatter range is less than 23 meters, which is about one-half that determined from Figure 2.3. Reflections recorded with the transmitter and receiver in the same borehole were interpreted to arise from fractures. By extrapolating the received signal to zero time, the borehole-fracture intercepts were determined and found to be in good agreement with those inferred from television, density, resistivity, and gamma-ray logs.

Federal Highway Administration (FHWA) /62/. The FHWA has emphasized hardware, field demonstrations, and data manipulation. Data interpretation

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*It can be shown that the backscatter range, R_2 , for a large diffusely scattering fracture plane is related to the one-way transmission range, R_1 , by $R_2/R_1 = (\lambda/8 R_1)^{1/3} \exp(\alpha R_1(1 - 2 R_2/R_1)/3)$. If the range were controlled solely by dissipation in a lossy medium, then the backscatter range would be one-half the one-way transmission range. In general, the backscatter range will be less than one-half the one-way transmission range because of scattering effects.

is recognized to be a crucial aspect of the detection problem, although it has not yet been possible to give it adequate attention. The basic idea behind the agency's work is that rock quality ought to be determinable by deploying a small diameter geotechnical sensing tool within a long, horizontal, air-filled borehole instead of employing the established and expensive practice of driving an exploratory inspection tunnel. Operation with single and multiple boreholes has been considered; the latter is more expensive than the former, but is more versatile for it allows transmission measurements to be made. Attention has focused on placing radar, acoustic and resistivity instruments within the tool. Radar and acoustics are complementary techniques in that both should detect voids and fractures, but should respond differently to anomalies. Resistivity was selected for a less fundamental reason; namely, the large extant data base.

The basic operating principles of the tools are the same in the tunneling and geothermal business, but certain aspects of the design are different, mainly, because in tunneling the borehole is air-filled, while in geothermal mapping the wellbore is water-filled under normal circumstances. The acoustic instruments must employ pads pressed against the borehole wall to couple energy into the rock. If the acoustic design is somewhat more complicated in the tunneling project, then the radar design is a bit simpler inasmuch as a lossy water medium does not surround the antenna. The resistivity measurements are made with a magnetic induction technique to obviate the need for contact between the tool and borehole wall. Four field experiments using various techniques were carried out, but in only one was a mine-back possible. Excavation verified a tomographic VHF electromagnetic map of grout injection into a sand pit, but at a construction site where mine-back was not possible, an anomaly mapped by VHF tomography was unseen by acoustics.

Bureau of Mines /63/. The Bureau of Mines, Denver, has employed pulsed surface radars with a center frequency of 30 MHz. The radar data have been interpreted to indicate the occurrence of more fractures than are indicated by, say, resistivity measurements. Furthermore, it is

believed that echoes from fractures arise from a change in material properties across the interface, quite unlike the model used in Subsection 2.5.1 to describe a geothermal reservoir. The Bureau of Mines model has not been verified by mine-back. Attempts to delineate anomalies (cavities and weak zones) have been hampered by the omni-directional properties of antennas. The main effort until recently has been the production of clean, well-damped pulses. The Bureau of Standards is currently developing a directional surface antenna for the Bureau of Mines. Bureau of Mines personnel believe that their efforts have not yet yielded practical tools. As is the case with most users of radar data, they feel that data interpretation is the biggest unresolved problem.

United States Geologic Survey (USGS) /64/. VHF pulsed radar experiments have indicated that volume scattering is the major contribution to signal attenuation under the conditions of the USGS experiments. Clays and water bearing strata also reduce the range. A range of 70 to 80 meters was attained in a New England granite quarry and a range of 4 to 20 meters in a basalt lava flow on the island of Hawaii. Fractures in the basalt were observable only after a rainstorm.

8.2 OTHER FRACTURE MAPPING ACTIVITIES

The analysis of subterranean radar data has until recently usually focused on picking reflections from individual traces of received signal amplitude vs. time. The received echoes are seldom replicas of the transmitted pulse, but rather are modified by a number of effects which include dispersion by the medium and interference from separate multiple reflectors or from different parts of a large reflector. The origin of the reflections is often unknown, and it is difficult to distinguish between clutter and an echo that has arisen from the object of the radar mapping, e.g., a material interface or a fracture. Various assumptions and hypotheses often find their way into the data reduction process. Verification of fracture geometry, either via mining or drilling, is seldom attempted, except in adequately funded programs. Signature analysis is a subject which is receiving increased attention. At the present time, we are unaware of a workable inverse scheme

for target identification. For the near future, at least, we believe that for applications within a geothermal reservoir, the use of a conducting fluid to map fractures that intersect the borehole is the most promising radar detection and identification technique. Examples of data and the difficulties experienced in their interpretation can be found in References 1, 2, 4 and 65-69. Precise of these references pertinent to the geothermal program are set forth below as are some recent developments obtained via telephone conversations and field trips.

Holser, et al., /1/ experienced considerable difficulty in determining the geometry of the boundaries of a salt dome because of the lack of azimuthal directivity from a borehole antenna. Data from other geophysical logs were necessary to form a consistent picture of the location of the flank of the salt dome. The reflecting surfaces themselves did not produce a signal which correlated for any appreciable distance along the borehole traverse; in fact, correlations extended only over a region comparable to the beam width. Stewart and Unterberger /2/, using a narrow beam produced by a large antenna stationed within a salt mine, observed numerous reflections originating from within the salt, but the nature of the reflectors was unknown. Moffatt and Puskar /4/, employing surface radars to look below the surface of a quarry, observed large increases in echo amplitude under rain soaked conditions compared to dry conditions, presumably arising from water-filled fractures. Confirmation of the existence of a radar-mapped fault was foiled by extensive fracturing during subsequent quarrying operations.

Ensco, Inc., /65-68/ has displayed radar returns, not only as individual traces, but has also investigated the use of record sections, common depth point reflection techniques, and signal stacking. Additional signal processing, such as the migration schemes used in reducing seismic reflection data, might improve the quality of the radar displays. The use of record sections to aid the detection of fractures in a quarry using borehole radar indicated that at the present time the delineation of the exact configuration of a reflector is a very difficult task. Ensco has found that an impedance match cannot be obtained between a radar antenna

with fixed characteristics and the ground over an appreciable length of borehole, a problem they attribute to seemingly random changes in the impedance of in situ geologic materials. To overcome this problem, they are working on a tunable antenna. To alleviate problems associated with the propagation of pulses in dispersive, lossy media, they are investigating the use of synthetic pulse forming, i.e., forming a pulse from a large number of essentially continuous sine waves. A 600 MHz, 3-inch diameter borehole antenna with directional properties is also currently under development.

Southwest Research Institute /69-71/ has been involved with the design and fielding of three types of VHF radars: surface backscatter, subsurface borehole backscatter, and subsurface borehole-to-borehole transmission, and one type of CW FM UHF radar: high resolution, limited range (~ 1 foot) subsurface backscatter. The surface radar was used principally to detect tunnels and buried objects. The subsurface VHF radar is potentially useful to the geothermal fracture mapping program and was employed in the following ways: Anomalous regions, perhaps faults and sand channels, were detected within a radius of approximately 17 meters from a borehole passing through sedimentary rock and coal. Signals interpreted to arise from near-surface jointing and broken gneiss and schist were detected near the Forest Glen Metro station in Washington, D.C., but a planned mine-back to verify the interpretations was never performed. Near-surface tunnel detection was carried out by borehole-to-borehole transmission. The UHF radar was designed to measure the thickness of coal surrounding a tunnel during mining operations. In those cases where the interface between the coal and shale was sharp, the resolution was of the order of $\pm 1/4$ inch, but when the interface was diffuse, it could not be detected. A similar unit might be able to detect fracture intersections with a geothermal wellbore, but several conventional logging tools can already perform this task (Section 5). Like Ensco, Southwest Research is also developing a 3-inch diameter directional VHF antenna. Although not a pencil beam by any means, the antenna should be good enough to resolve the quadrant from which a

signal originated. In most cases, an 8 to 10 nsec, unipolar, VFH video pulse (30 to 300 MHz) is used to excite their antennas. A monocyclic pulse, which has the advantage of a narrower spectrum than a half-cycle pulse (there is no DC component), has been used in some cases to improve resolution, but perhaps with some decrease in range. Southwest Research's main efforts have been directed towards instrumentation development with only a subsidiary amount of labor expended on analyses. Data interpretation has usually been based on a pulse-echo time analysis. SWRI personnel believe that radar interpretations must be corroborated by other measurements, such as acoustics.

Sandia Laboratories is involved in the mapping of hydraulic fractures for the enhanced gas recovery program, and has also participated in programs initiated by the petroleum industry /72/. Three main techniques have been investigated: the measurement of the surface potential arising from passing a current between two wellbores and the subsequent distortion of the potential field by current conducted along the hydraulically induced fracture, the measurement of seismic disturbances (Section 11), and the measurement of surface tilt (Appendix A). Although the potential method proved technically feasible in the Sandia application, the cost to implement it led to it being abandoned in favor of the other two techniques /73/.

SECTION 9

MAGNETIC BOREHOLE RANGING

The problems experienced while attempting to locate the GT-2 wellbore with respect to the EE-1 wellbore at Fenton Hill have led us to look for an alternate means of surveying wellbores. If a technique were available that could determine the position of one borehole, while it was being drilled, relative to an already completed borehole, then it might well be possible to increase the accuracy of drilling and to reduce the uncertainty in the determination of the separation distance between the completed boreholes. Based upon our experience with certain subterranean magnetic ranging techniques used to determine displacements resulting from large explosions /74/, and from the results of tests recently completed for the Bureau of Mines /75/, we examine the possibility of surveying wellbores in the Hot Dry Rock Program using magnetic techniques.

9.1 MAGNETIC RANGING AND ORIENTATION TECHNIQUES

The main idea of magnetic borehole ranging is that by placing a magnetic dipole of known source strength (a solenoid) in one borehole and by measuring the vector components and the phase of the a.c. magnetic field in a distant borehole, both the range and the orientation of one borehole with respect to another can be determined. Under what conditions this can be accomplished will be set forth in this section. Our main emphasis will be on determining the maximum distance for which magnetic ranging will work and what type of hardware is required.

The field of a time invariant magnetic dipole expressed in terms of polar coordinates is

$$\vec{H} = \frac{m}{4\pi R^3} (\hat{i}_r 2 \cos \theta + \hat{i}_\theta \sin \theta) ,$$

where m is the dipole moment and θ is the angle between the z axis and the axis of the dipole. The field is measured in actual practice, however, in a Cartesian coordinate system fixed to an instrument package which, in general, is rotated with respect to the axis of the dipole. To locate

the solenoid in, say, borehole 1 from borehole 2 requires not only the determination of three rectangular coordinates, but also the two Euler angles specifying the rotation of the axis of borehole 1 with respect to borehole 2. Since the dipole moment of the solenoid is known, there are five unknowns to be determined from three vector measurements. Hence, several measurements at different locations must be made in borehole 2 to locate the solenoid in borehole 1. The problem is then overdetermined, and a data fitting process must be employed to reduce the data. Other than stating that the positions of buried magnets have been determined successfully using a magnetometer in a borehole /74/, we shall not go into the details of coordinate transformations and the accuracy of data fitting procedures. We shall instead treat the problem of data inversion accuracy in a parametric manner by performing borehole ranging calculations with different signal-to-noise ratios and by stating (without proof) that for signal-to-noise ratios of between 20 to 40 db the data inversion will be accurate to within at least a few percent.

9.2 MAXIMUM BOREHOLE SEPARATION

We now turn to estimating the maximum borehole separation that can be detected by magnetic ranging. To facilitate the analysis and to make the physical phenomena stand out, we simplify the mathematics by neglecting the effects of orientation and consider only the measurement of weak magnetic fields. Thus, we relate range, magnetic flux density, and magnetic moment by

$$R \doteq \left(\frac{\mu_0 m}{4\pi B} \right)^{1/3} . \quad (9.1)$$

The minimum useful B field is set by transducer noise and by magneto-telluric noise. The equivalent B field noise for three different magnetometers is listed in Table 9.1. By using a.c. excitation of the solenoid not only can the effects of both sources of noise be greatly reduced, but so can the contamination of the solenoid signal by the d.c. geomagnetic background. Since the magnetometer noise is broadband, the detected noise

Table 9.1. Equivalent B field noise for magnetometers.
 NOL is the Naval Ordnance Laboratory, and
 JPL is the Jet Propulsion Laboratory.

Magnetometer	Type	Equivalent B Field Noise	Reference
Develco	Fluxgate	$3.3 \times 10^{-2} \gamma/\sqrt{\text{Hz}}$	74
NOL	Fluxgate	$3.10 \times 10^{-3} \gamma/\sqrt{\text{Hz}}$	75
JPL	Vector Optically Pumped Helium	$10^{-2} \gamma/\sqrt{\text{Hz}}$	76

level is decreased in direct proportion to the bandwidth. Thus, it is advantageous to use narrow band circuits; in the calculations we assumed $Q = 100$. Secondly, by using a.c. coupling the spatial variations of the geomagnetic field caused, say, by an irregular distribution of magnetite will not affect the data reduction process. Moreover, temporal variations in the geomagnetic field can, to a large degree, be filtered out. (The amount of field distortion at the signal frequency caused by patches of magnetite remains to be quantified.) At frequencies greater than about 1 Hz, magneto-telluric noise is dominated by atmospheric effects, i.e., lightning discharges. To estimate atmospheric noise, we use Maxwell's data for the mean value of the horizontal magnetic field noise density at the surface of the earth within a 1-Hz bandwidth during the summer in Boulder, Colorado /78/. Between 20 to 100 Hz, the spectrum is flat with a value of 117 db below 1 A/m. The noise density of the horizontal magnetic flux density is then $B_a = 1.77 \times 10^{-3} \gamma/\sqrt{\text{Hz}}$, where $\gamma = 10^{-9}$ tesla. This value, which is several db greater than measurements made at other locations by Maxwell and by other investigators, is a conservative estimate even when extrapolated (neglecting Schumann resonances) to 2.5 Hz (the lowest frequency that we have considered). We neglect the slight attenuation of the low frequency atmospheric noise with depth and calculate the total noise field from the rms value of the transducer noise and the

low frequency atmospheric noise. A third advantage of using a.c. excitation is that it opens the possibility of using a magnetic loop to detect the B field instead of a magnetometer.

The use of a.c. signalling is not without its complications. Because the field polarity reverses every half-cycle, a 180 degree ambiguity in the direction of the solenoid can be resolved only by measuring the instantaneous phase of the detected signal and comparing it with the phase of the solenoid excitation. When the magnetic field detector is moved to a new location in the borehole, the use of resonant circuits leads to a so-called "build-up time" during which the measurement approaches its steady state value. The build-up time constant is equal to $2/\Delta f$, where Δf denotes the bandwidth, so that the narrower the bandwidth the larger the build-up time. To make an accurate measurement, the settling or waiting time at each new location would be at least five time constants or $10/\Delta f$. For example, with $\Delta f = 0.025$ Hz, the waiting time is about seven minutes. The excitation frequency cannot be made too high without introducing propagation effects into the analysis. Propagation modifies the static analysis with terms proportional to $2R/\delta$, when δ is the skin depth. For $\sigma = 10^{-4}$ S/m, which is a low frequency conductivity typical of wet granite, the skin depth is plotted as a function of frequency in Figure 9.1.

Figure 9.1. Skin depth vs. frequency. $\sigma = 10^{-4}$ S/m, $\sigma/\epsilon\omega \gg 10$. The magnetostatic analysis is accurate for $R \ll \delta$.

For a signalling range of approximately 300 meters, a frequency of 100 Hz will begin to introduce small modifications to the static results. At all frequencies of interest to magnetic borehole ranging, the ground acts as a good conductor, i.e., $\sigma/\epsilon\omega \gg 10$.

To continue, the maximum borehole separation for which the magnetic ranging technique will work is estimated from Equation 9.1. The minimum useful B field signal is set equal to either ten or one-hundred times the noise field to ensure that the signal strength is adequate to accurately invert the data. By parametrizing the signal-to-noise ratio, we also obtain a feel for how this quantity affects the range. The noise field for magnetometers is taken from Table 9.1. For a magnetic loop, the noise voltage arises from thermal fluctuations and is given by $\bar{v}_n = \sqrt{4kTR(\Delta f)}$, where R is the loop resistance. To make a calculation, we must choose a specific loop size: We select no. 20 wire and a diameter, D, equal to six inches. The signal voltage is given by $v = (\pi D)^2 f B/2$. Calculations were performed for three different dipole source strengths, $m = 10^3$, 10^4 , and 10^5 A-m^2 . The value of 10^3 A-m^2 represents a rather modest source, while $m = 10^4 \text{ A-m}^2$ is quite practical. Magnetic antennas with $m = 10^7 \text{ A-m}^2$ have been constructed /79/. The maximum range between boreholes is plotted as a function of detector noise in Figure 9.2. The symbols indicate the maximum ranges that can be achieved by specific types of detectors. The magnetometers are characterized by their inherent noise level (Table 9.1) and by a detector bandwidth of either 0.1 or 0.025 Hz. The magnetic loop is characterized by $Q = 100$ and a frequency of either 10 or 100 Hz. A range of several hundred meters can be attained with several different types of hardware. The departure of some of the calculated points from the straight lines indicates that atmospheric noise is beginning to limit the range. Thus, the range cannot be extended to arbitrarily large values by building a noise-free transducer. It is thought that the most expeditious way to boost range is to increase the source strength.

9.3 SUMMARY

By making measurements of the a.c. magnetic field arising in one borehole from a solenoid located in another borehole, it is possible to determine the relative positions of the two boreholes. By using sensitive mag-

netic field detectors, bandpass filtering, and a magnetic source of moderate strength, the measurement of borehole separations of 200 to 500 meters appears feasible. The implementation of the magnetic ranging technique will require a considerable engineering effort, both in terms of hardware design and mathematical data reduction procedures. Nonetheless, the concept appears quite viable.

Figure 9.2. Maximum range of magnetic surveying technique. The range depends upon the detector design, the magnetic source strength, atmospheric noise, and the data inversion process (the signal-to-noise ratio). The departure of the calculated points from the straight lines indicates that atmospheric noise is beginning to limit the range.

SECTION 10

ACTIVE ACOUSTIC FRACTURE MAPPING

Acoustic probing of the earth has long been used in two quite different regimes. Seismologists work in the millicycle range to delineate earth structures over distances comparable to the size of the earth itself. Petroleum prospectors employ sondes that emit signals in the range $5 < f < 15$ kHz to diagnose rock structure in the immediate vicinity of a borehole (see Section 5.2.6). Recently applications in geothermal energy and tunnelling have produced requirements to detect and map fractures, mining hazards, and other structures over distances of hundreds of meters. It is appropriate, therefore, to ask what limits both the acoustic range and detectability of underground structures, what technical means are available for optimizing the usefulness of acoustic mapping, and finally to compare acoustic with electromagnetic diagnostics. The analysis in this section parallels that of Section 2, "Electromagnetic Radar Probing Distances," and is principally directed toward geothermal applications. For an acoustic transmitter radiating a fixed amount of energy the amplitude of the received signal is affected by geometrical spreading, dissipation in a lossy earth, the scattering cross section of the fracture, and the aperture of the receiver. The detectability of a signal depends upon the noise level and signal processing techniques. All of the aforementioned factors are considered from first principles in this section. The effects of clutter in the form of minor fractures, joints, blocks, and lenses is a topic of current research and is not addressed from a first principles viewpoint. We note that clutter both decreases the amplitude of the received signal and increases the difficulty of interpreting signals through the action of mode conversion, scattering, and refraction. A technique for circumventing some of the problems introduced by clutter as practiced by the geothermal group at Los Alamos is discussed in Section 10.4. Before proceeding we remark that both electromagnetic and acoustic probing techniques are beset with problems of signal interpretation. However, if judged solely on the basis of probing

range then for equal resolution, i.e., equal wavelength, acoustic radar is superior to electromagnetic radar by a factor that varies from approximately five to almost seven. We present calculations only for backscatter acoustic radar. Calculations for transmission between boreholes are done in a similar manner, but the forward scattering cross section must be used. Roughly speaking the cross-borehole range is approximately twice the backscatter range reported in this section (see footnote on page 8-3).

10.1 THE RANGE OF BACKSCATTER ACOUSTIC RADAR

Before presenting the results of acoustic radar calculations we shall discuss three factors that restrict the range of acoustic radar. They are dissipation, noise, and the scattering cross section. For completeness a fourth topic is also discussed; namely, receiver aperture. If the region of acoustic probing is seismically quiet, that is to say, if the seismic noise is weaker than internal receiver noise, or if the seismic noise occurs in bursts, then the receiver aperture plays an important role in determining the range of acoustic radar.

10.1.1 Acoustic Absorption in a Lossy Medium

Dissipation in the earth causes the amplitude of the stress, σ_{ij} , and particle velocity, u , to be attenuated with distance according to the relation

$$u = u_o e^{-\alpha R}$$

$$\sigma_{ij} = (\sigma_{ij})_o e^{-\alpha R},$$

where α is the absorption factor expressed in nepers/meter, and R is the distance travelled by the wave. It follows that the flow of acoustic power per unit area, P , is attenuated according to

$$P = P_o e^{-2\alpha R}.$$

The absorption factor is determined from experiments, usually laboratory experiments. It has been found that for frequencies up to several hundred kiloHertz the absorption factor in dry granite is directly proportional to

the frequency /80/. It is somewhat more convenient to describe the loss characteristics of rock by a frequency independent Q , defined as $Q = \omega/(2\alpha c)$, where c is the phase velocity of either P- or S-waves. It is also more convenient to refer to the absorption factor A in terms of db/meter as was done in Section 2.2. As will be shown it appears that at the present state-of-the-art the scattering properties of S-waves are crucial for interpreting signals backscattered from fractures. Hence we concentrate on the propagation properties of S-waves, which at either comparable frequency or wavelength are subject to greater loss than P-waves. Extant laboratory data for granite indicate that /80,81/

$$2.71 \times 10^{-5} \leq Af^{-1} \leq 9.6 \times 10^{-5}, \text{ db/(m-Hz)} .$$

$Q_s = 80$ is a good representative value for shear waves in granite, with a spread given by $30 < Q_s < 110$. The average value of the absorption coefficient of acoustic S-waves is less than that for electromagnetic waves at all wavelengths of interest (see Figure 10.1).

Because the data on α are for dry granite the question naturally arises as to what effects the water found in a geothermal reservoir has on the absorption coefficient. Neglecting ionic relaxation (no $MgSO_4$ is reported for Fenton Hill effluent /23/), the acoustic absorption coefficient of water is

$$\alpha = \frac{8\pi^2 f^2 \eta_s}{3\rho c} \left(1 + \frac{\eta_v}{\eta_s} \right), \text{ nepers/length ,}$$

where η_s is the shear viscosity, η_v is the dilational viscosity, and $\eta_v/\eta_s \approx 2.81$ /82/. Over the frequency range of interest the attenuation of dry rock is much greater than that of water, Table 10.1. Therefore the intrinsic lossiness of water itself cannot affect the overall loss in wet granite, unlike the situation in the electromagnetic case. We pursue the matter one step further because wet sandstone exhibits a lower Q than dry

Figure 10.1. Absorption coefficient for acoustic S-waves in granite as a function of wavelength and a comparison with electromagnetic absorption.

Table 10.1. The S-wave absorption coefficient A of dry granite, pure water, and (theoretical) in situ Fenton Hill granite as a function of frequency.

Attenuation Coefficient, A db/m			
Frequency Hz	S-wave dry rock	Water	Theoretical S-wave in situ water-saturated
10^2	5.78×10^{-3}	2.1×10^{-6}	1.3×10^{-8}
10^3	5.78×10^{-2}	2.1×10^{-4}	1.3×10^{-6}
10^4	5.78×10^{-1}	2.1×10^{-2}	1.3×10^{-4}
10^5	5.78	2.1	1.3×10^{-2}

sandstone and moreover the wet Q is directly proportional to the frequency. (We refer to the properties of sandstone because we have not located attenuation data for wet granite.) To estimate the lossiness of wet granite we use Biot's theory of propagation of elastic waves in fluid-saturated porous solids /83/ in the form given by White /81/:

$$\alpha_{s,w} = \frac{2\pi^2}{c_s} \frac{\rho_f}{\rho} \frac{K\rho_f}{\eta} f^2,$$

where the properties of the saturated materials carry no subscript and the properties of the fluid are denoted by the subscript f. The meaning of the symbols is as follows: $\alpha_{s,w}$ is the absorption coefficient of S-waves in a water-saturated porous rock caused by viscous dissipation and expressed in nepers/meter, K is the permeability, and η is the viscosity of water. The in situ permeability is estimated to be $K \leq 5 \times 10^{-18} \text{ m}^2$ and the viscosity of water at 200°C is $\eta = 1.4 \times 10^{-4} \text{ Nsec/m}^2$ /26/. The formula for $\alpha_{s,w}$ is valid for $f < 0.15 \eta\phi/(2\pi\rho_f K)$. With $\phi \doteq 10^{-3}$ (see Section 5.28) $f < 6.7 \times 10^7 \text{ Hz}$. Thus, $A_{s,w} = 1.3 \times 10^{-12} f^2 \text{ db/m}$, which is less than that of either constituent, Table 10.1. The small value of $A_{s,w}$ arises from the low value of the in situ permeability. We conclude that viscous dissipation arising from the oscillation of water in the pores of the reservoir rock

is not of practical importance. It is possible, however, that dissipation might occur in the fluid contained within the fractures (see Section 8.1).

10.1.2 The Acoustic Noise Level

The minimum detectable signal at a measurement station is controlled by the noise level generated either in the earth by natural or man-made activity or inside the receiver circuitry. The external noise characteristics of the earth itself is produced by wind, ocean waves, tectonic strain release, and thermal oscillations. The contribution of each source to the total noise level at a sonde depends on the depth, the distance from the seacoast, local tectonic activity, and the temperature. The former three sources are usually lumped together and called seismic noise. The determination of the strength of the seismic noise requires recourse to experiment. A fit to the results of a number of seismic noise measurements made on quiet bedrock, far from a seacoast /84-87/, plotted as the mean square velocity spectral density is shown in Figure 10.2. The plot rests on shaky ground because in the frequency band of interest, $10^2 < f < 10^5$ Hz, we rely on an extrapolation of low frequency data that cuts off at 32 Hz. The fit to the data and the extrapolation are given by

$$\langle u^2 \rangle = \frac{3 \times 10^{-17}}{f^{1.849}} , \frac{m^2}{s^2-Hz} . \quad (10.1)$$

The noise data are a composite of P-, S- and Rayleigh waves /85/. The main justification for using the extrapolation is that the procedure gives a seismic noise that is greater than the thermal noise; hence, the range calculations will be more conservative than if thermal noise were used as the limiting criterion.

The thermal noise, expressed in terms of a mean square velocity, is calculated in the following way. Following Oliver /88/ the number of acoustic modes per unit volume per unit bandwidth, i.e., the mode density is:

$$m = \frac{4\pi f^2}{c^3} ,$$

Figure 10.2. Seismic noise and thermal noise vs. frequency.

where c is the velocity of P-, S- or Rayleigh waves. Since an acoustic wave stores energy quadratically in both kinetic and elastic fields the energy per mode is $E = kT$. The thermal energy density per Hz is

$$\bar{E} = \frac{4\pi f^2 kT}{c^3}, \quad \frac{\text{joules}}{\text{m}^3 \text{-Hz}}.$$

The kinetic energy density per unit bandwidth of the seismic noise is $\rho \langle u^2 \rangle / 2$. Invoking equipartition of energy the mean square thermal velocity field is:

$$\langle u^2 \rangle = \frac{4\pi f^2 kT}{\rho c^3} \quad (10.2)$$

Because the data are a composite of P-, S- and Rayleigh waves we have set (solely for convenience) $c = c_p$ in the calculation. Using Equations 10.1 and 10.2 we find that in a geothermal reservoir at 550°K the seismic noise field and the thermal noise field have the same amplitude at 30.16 kHz. Hence, the external noise is dominated by seismic sources for $f < 30$ kHz and is dominated by thermal oscillations for $f > 30$ kHz. The relations 10.1 and 10.2 can be used to define an equivalent noise temperature for the seismic noise. The temperature of the seismic noise can be quite high, Figure 10.3. Note from Figures 10.2 and 10.3 that some of the advantage of acoustic radar over electromagnetic radar in terms of absorption coefficient is given up in terms of noise temperature. For well designed circuitry the receiver noise temperature will be $100 < T < 300^\circ\text{K}$. It is clear that the detection of acoustic signals is controlled by the external noise field. For critical design work it is necessary to measure the seismic noise field in a geothermal reservoir rather than to rely on the extrapolation of Figure 10.2.

10.1.3 The Backscatter Cross Section

The three geometrical approximations used to bound the electromagnetic backscatter cross section in Section 2 are appropriate for the acoustic case. However, the details of calculating the cross sections are different

Figure 10.3. Noise temperature vs. wavelength for acoustic and electromagnetic radars in a deep geothermal reservoir.

for the two cases. Although both the transverse electromagnetic field and the shear field are governed by the same vector wave equation

$$\frac{\partial^2 \vec{A}}{\partial t^2} - c^2 \nabla^2 \vec{A} = 0 ,$$

the boundary conditions on the vector potential $\vec{A}$ for the electromagnetic and acoustic cases are different for identical shapes, say, a plane reflector. In the electromagnetic case the tangent electric field is zero at the surface of a perfect conductor, which leads to a doubling of the magnetic field at the reflector surface, $H_t = (\nabla \times \vec{A})_t$. In the acoustic (or more precisely, the elastodynamic case) the boundary conditions at a solid-fluid interface involve the derivatives of the displacement $\vec{s}_t = (\nabla \times \vec{A})_t$, i.e., $\tau_{xy} = \mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) = 0$, where u and v are the Cartesian components of $\vec{s}$. In most cases mode conversion occurs at an elastic interface and for either an incident P-wave or S-wave there will be both reflected P- and S-waves. The generation of different modes reduces the backscatter cross section. For example, in the electromagnetic case a large perfectly reflecting plane has a backscatter cross section $\sigma = \pi R^2$. In the acoustic case (with Poisson's ratio $\nu = 1/3$) mode conversion yields a P-wave backscatter cross section $\sigma = 0.61\pi R^2$ for a traction-free surface. Before proceeding we note several results from elastic scattering theory. If an S-wave is polarized in the plane of a fluid-filled interface then the S-wave is reflected merely with a change of sign. A fluid-filled fracture can support compressional waves, so that for an S-wave polarized in the plane of incidence there will be transmitted P- and S-waves, a reflected S-wave, and up to a critical angle of incidence a reflected P-wave. We believe that understanding the nature of both backscattered and transmitted signals is an essential part of developing both the electromagnetic and acoustic fracture mapping techniques, but must leave the solution of this problem to future work. We now move on to estimating the probing range of acoustic radar and to comparing it with the electromagnetic case.

10.1.4 Receiver Aperture

Consider for the moment an isotropic acoustic radiator emitting a power P_t into a lossless homogeneous medium. At a receiver located a distance R from the transmitter the power per unit area is $P = P_t / (4\pi R^2)$. The power absorbed by an acoustic receiver depends upon the mechanical and electrical design of the receiver, and the acoustic properties of the elastic medium and the emplacement borehole. The absorbed power is expressed by $P_r = P A_o / (4\pi R^2)$, where A_o is the absorption cross section, or as it is often called, the receiver aperture. In the remainder of this section we shall show how to calculate A_o .

Before proceeding we shall discuss some preliminary considerations. Rather than work with the transmitted power it is more convenient to express P in terms of field variables. For an elastic solid the time-average power per unit area carried by an S-wave which is the quantity ultimately needed in evaluating A_o is /89/

$$\langle P_s \rangle = \frac{\mu}{2} \frac{U_o^2}{c_s}, \quad (10.3)$$

where U_o is the amplitude of the particle velocity.

The method for analyzing electroacoustic devices involves describing all of the elements of the problem, i.e., the acoustic field the electroacoustic transducer, and the electronic amplifier and detector, in terms of an equivalent lumped element circuit model /90/. To use a lumped circuit model for an electroacoustic receiver we must define a constant drop generator such that the time average power is $\frac{1}{2} pu$, where p is the drop across the generator and u is the flow variable. Clearly $u = U_o$; hence, from Equation 10.3 $p = \rho c_s U_o$. As will be seen p drops out of the determination of A_o (as it must). The crucial quantity in the definition of the field variables is ρc_s , which is a factor used to normalize the complex impedance seen by the receiver and is the impedance of a plane wave.

From circuit theory it is immediately clear that the maximum power transfer to a detector and hence the maximum aperture will occur when the

impedance of the entire electroacoustic circuit is purely real, i.e., the circuit is operated at resonance, and is equal to the impedance of the acoustic field. Nonresonant operation will reduce the aperture. The actual calculation of aperture, restricted to piezoelectric transducers, proceeds as follows.

The equivalent circuit of an electroacoustic receiver is shown in Figure 10.4. The circuit elements have the following meaning: the amplitude of the incoming wave is modelled as a source with a constant pressure drop, p . The volume velocity associated with the wave is denoted by U with dimensions of cubic meters per second (m^3/s). The acoustic resistance, R_A , has units of newton-seconds/meter⁵, or acoustic ohms. S is the physical area of the transducer and acts as a transformer with a turns ratio, S , to convert pressure to force. R_m is the mechanical resistance (lossiness) of the transducer in mechanical ohms, i.e., newton-seconds/meter (N-s/m). M_m is the mechanical mass of the moving parts of the transducer in kilograms. C'_m is the short-circuit mechanical compliance in meter/newton (m/N). τ is the electromechanical coupling coefficient with dimensions of newton/coulomb (N/q) and is not universally defined in the same way in the literature /90/. C'_e is the open circuit capacitance of the crystal in coulombs/volt (q/V). R_l is the electrical load resistance of the receiver circuitry. In actual practice the piezoelectric transducer is often built in the form of a ring, with an outer radius of 2×10^{-2} meters. The acoustic model of the transducer used to calculate the radiation impedance is not particularly crucial as long as the model properly accounts for $1/R^2$ spreading in the far field. It is well known that the exact shape of an acoustic generator or receiver is much less important than its average dimensions in determining the radiation impedance /90a/. As can be seen from Figure 10.5 there is little difference between the specific acoustic impedance of a sphere and of a piston in a large rigid baffle. The resistance and reactance of a piston in a long tube differs by factors of 2.02 and 1.38 respectively from that of a piston in a large baffle. Insofar as the aperture is concerned it will be shown that for a resonant circuit in the long wavelength limit the aperture is independent of the exact shape of the transducer.

Figure 10.4. Lumped element circuit model of acoustic piezoelectric transducer.

Figure 10.5. Specific radiation impedance of a sphere and a circular piston in a rigid baffle $/90a/$.

Figure 10.6. Thevenin equivalent circuit for acoustic transducer.

The values of the acoustic elements in the long wavelength limit ($ka < 1.0$) are

$$R_A = 0.479 \rho c / a^2$$

$$M_A = 0.195 \rho / a$$

For the previously mentioned ring transducer, $ka < 1$ corresponds to $f < 47$ kHz for P-waves and $f < 28$ kHz for S-waves. The mechanical impedance of the acoustic medium seen at the left hand terminals of the acoustical-to-mechanical transformer is

$$Z_a = 0.785 \frac{\rho a^4 \omega^2}{c} + j 1.927 \rho a^3 \omega = R_a + jX_a$$

The circuit of Figure 10.4 can be transformed into the Thevenin equivalent, Figure 10.6, with the impedance

$$Z_{Th} = \frac{Z_1 - j\omega(C_e \tau^2)^{-1} |Z_1|^2}{1 + \omega^2 (C_e \tau^2)^{-2} |Z_1|^2} = R_{Th} + jX_{Th}$$

where

$$Z_1 = (R_a + R_M) + j \left(X_a - \frac{1 - \omega^2 M_m C_m}{\omega C_m} \right)$$

For a resonant circuit $Z_{Th} = R_{Th}$. The maximum power transfer from the acoustic generator to the load resistor occurs when $(C_e \tau)^2 R_L = R_{Th}$. Carrying through the algebra we find that the average power absorbed in the load is

$$\langle P_L, \text{ resonant} \rangle = \frac{p_S^2}{8Z_{Th}} = \frac{p_\lambda^2}{8\pi \rho c}$$

The average power incident on the receiver from a plane wave is

$$\langle P_{\text{incident}} \rangle = \frac{p^2 A_o}{2\rho c} .$$

Therefore, at resonance

$$A_o = \frac{\lambda^2}{4\pi} . \quad (10.4)$$

To demonstrate that the result is independent of the receiver geometry we evaluate A_o using the acoustic impedance of a sphere

$$Z_A = \frac{jka\rho c}{1 + jka} ,$$

which at resonance in the long wavelength limit yields

$$R_M = 4\pi(ka)^2 \rho c a^2 .$$

Substituting R_M for a sphere into the expression for the average power absorbed in a load at resonance we find that $A_o = \lambda^2/4\pi$, which is a fundamental property of matched, resonant antennas /87/. At extremely low frequencies the response of a piezoelectric transducer is controlled by its compliance only. In this case it can be shown that the nonresonant aperture is

$$\lim_{\omega \rightarrow 0} A_o = \frac{b_1 t S}{\lambda}$$

where b_1 is a constant, t is the thickness of the piezoelectric ring, and S is the surface area of the transducer. For nonresonant operation the variation of aperture with frequency is best handled by numerical calculations. We will not pursue the matter further in this report because as will be shown in the next section for a system dominated by external noise the signal-to-noise ratio is independent of receiver aperture down to some limiting value of the aperture.

10.2 THE RANGE OF A PERFECTLY COLLIMATED ACOUSTIC RADAR BEAM IN A GEOTHERMAL RESERVOIR

Both to isolate the effects of dissipation in the earth from geometrical spreading and radiation pattern (but not from the noise field), and to compare the maximum range of acoustic radar with electromagnetic radar we examine the range of a perfectly collimated beam incident on a perfect reflector embedded in a lossy granite. The analysis is restricted to the propagation of S-waves. The signal power at the receiver is

$$\frac{P_r}{P_t} = e^{-4\alpha R} .$$

Rearranging terms, the maximum range is

$$R = \frac{5.0}{A} \log_{10} \frac{P_t}{P_r} . \quad (10.5)$$

The minimum usable signal strength is set by the received seismic noise (Section 10.1.2),

$$N = \frac{\rho c_p \langle u^2 \rangle B A_o}{2} , \quad (10.6)$$

where we assume that the receiver responds only to the kinetic energy noise field. For a resonant, matched antenna the received seismic noise is

$$\begin{aligned} N(f) &= \frac{\rho c_p^3 \langle u^2 \rangle B}{8\pi f^2} \\ &= \frac{6.618 \times 10^{-4} B}{f^{3.849}} , \quad 1 < f < 3 \times 10^4 \text{ Hz} . \end{aligned}$$

Above 30 kHz the received noise is dominated by thermal noise and for a resonant matched receiver is given by

$$N = \frac{4\pi}{\lambda^2} kTBA_o = kTB .$$

Because the seismic noise spectrum is frequency dependent the receiver should, in principle, employ filtering appropriate for non-white noise. To simplify the computations without sensibly affecting the calculated range we shall consider only narrow band operation, i.e., $B = 0.1 f$ and thus avoid the need for a whitening filter. This condition does affect, however, the minimum detectable range because the pulse length $\tau = 1/B = 10/f$. To compare the acoustic range with the electromagnetic range we shall use the same noise figure for both systems, i.e., $F = 4\text{db}$. Over the frequency range $10^2 < f < 10^5$ Hz the maximum range varies from approximately 10^4 meters to approximately 15 meters, Figure 10.7. The corresponding variation in the transverse wavelength is $35 > \lambda > 3.5 \times 10^{-2}$ meters. For comparable wavelength the mechanical dissipation in anelastic granite is sufficiently less than the ohmic dissipation in a lossy, dielectric granite that even with a much greater noise temperature the acoustic range is considerably greater than the electromagnetic range.

10.3 THE RANGE OF AN ISOTROPIC ACOUSTIC RADAR ILLUMINATING A LARGE PLANAR FRACTURE

We restrict the analysis to the propagation of S-waves. The acoustic analysis is analogous to the electromagnetic case discussed in Section 2.4. The power absorbed by a receiver is

$$P_r = \frac{P_t e^{-4\alpha R} r \lambda A_o}{32\pi R^3} ,$$

where we assume that the scattering cross section is one-half the area of the first Fresnel zone and r is the reflection coefficient. The minimum detectable signal power is set equal to 2.5 times the received noise power.

Figure 10.7. Maximum acoustic radar S-wave range vs. frequency and peak transmitter power with $(\Delta f)\tau = 1$ and $\Delta f = 0.1 f$. Range limited by external noise. $Q_{s, \text{granite}} = 80$. Uniform homogeneous rock. Shear wave reflection coefficient $r = 0.6$.

$$P_r = 1.25 \rho c_p \langle u^2 \rangle B A_o, f < 30 \text{ kHz}$$

$$P_r = 2.5 \left(\frac{4\pi}{\lambda^2} kTB \right) A_o = 2.5 kTB, f > 30 \text{ kHz}.$$

The maximum range is obtained from

$$R^3 e^{4\alpha R} = \frac{P_t \tau r \lambda}{40 \pi \rho c_p \langle u^2 \rangle}, f < 30 \text{ kHz}$$

$$R^3 e^{4\alpha R} = \frac{P_t \tau r \lambda^3}{128 \pi^2 kT}, f > 30 \text{ kHz},$$

which is plotted in Figure 10.7 for $P_t = \{10, 10^4\}$ watts and $\tau = 10/f$. In all cases the reflection coefficient $r = 0.6$ (a crude approximation to actual mode conversion phenomena), the reservoir temperature, $T = 280^\circ\text{C}$, and the lossiness of the granite is characterized by $Q = 80$. Note that because the signal-to-noise ratio is dominated by external noise the receiver aperture plays no role in determining the range. In fact, the receiver aperture may be made smaller and smaller without diminishing the maximum range up to the point where the internal circuit noise becomes appreciable compared with the external noise. The gain of the acoustic source and receiver has been set to $G = 1$ because in the frequency range of interest, borehole and sonde dimensions ensure that the radiation patterns are nondirectional, i.e., $ka < 1$. Geometrical spreading and a finite cross section reduce the maximum range from the perfectly collimated case by a factor that varies from approximately 2.5 to 5. A thousand-fold increase in peak radiated power increases the range by a factor that is no greater than approximately two. In the frequency range $10^2 < f < 10^5$ Hz the maximum range varies from $1.77 \times 10^3 > R > 5.8$ meters for $P_t = 10$ watts to $3.57 \times 10^3 > R > 8$ meters for $P_t = 10^4$ watts. If the radiated energy is invariant with frequency and fixed at one joule then $2.93 \times 10^3 > R > 9.59$ meters, Figure 10.8a. At 100 joules $4.24 \times 10^3 > R > 11.1$ so little is

Figure 10.8a. Maximum acoustic radar S-wave range vs. frequency for 1 joule of radiated S-wave energy. The upper and lower curves correspond to $Q = 110$ and 30 which bound the lossiness of granite. The middle curve is for $Q = 80$ which is a typical value for granite.

gained by building a more energetic transmitter, Figure 10.8b. (Extant acoustic transmitters radiate approximately one joule in S-wave energy.) All of the aforementioned values for maximum range are based on a Q_s of granite equal to 80. Because of the variability found in geologic properties and the sensitivity of maximum range to dissipation we have calculated the maximum range corresponding to what are almost assuredly the upper and lower bounds on Q_s , and plotted the results in Figure 10.8. Increasing the loss by decreasing Q from 80 to 30 decreases the range by approximately a factor of two. Decreasing the loss by increasing Q from 80 to 110 increases the range by a factor that is less than 1.3. The range of acoustic radar radiating one joule (Figure 10.8a) is compared to that of electromagnetic radar (Figure 2.3b) in Figure 10.9. For $0.1 < \lambda < 10$ meters the ratio of the acoustic range to the electromagnetic range is $5.2 < R_{ac}/R_{em} < 6.8$. If the lower bound on the maximum acoustic range is compared with the electromagnetic range then $2.1 < R_{ac}/R_{em} < 2.9$.

10.4 ACOUSTIC DETECTION OF FRACTURES

With electromagnetic radar we are concerned mainly with a transverse wave. In acoustics we must be concerned both with longitudinal P-waves and transverse S-waves. The presence of two types of waves complicates the analysis, but at the same time the properties of shear waves provide a valuable diagnostic tool. The complications arise from mode conversions between S-waves and P-waves, and vice-versa at geologic structures that constitute both the target, i.e., the fracture, and the clutter, i.e., secondary fractures, joints, and blocks. The diagnostic tool is provided by the behavior of a shear wave at a solid-fluid interface; namely, as is well known from the solution of Stokes' second problem in fluid mechanics the depth of penetration of shearing motions into a fluid is $\delta = 2\sqrt{\pi\nu/f}$, where ν is the kinematic viscosity [91]. For $10^3 < f < 10^4$, δ is of the order of a few tens of microns. Thus, if a shear wave is polarized in the plane of a fluid-filled fracture and if the fracture is not bridged by asperities, then a shear wave incident on the fracture will be reflected. If the shearing motion has a component normal to the plane of the fracture (as will be the case in most geothermal reservoirs), or if there is

Figure 10.8b. Maximum acoustic radar S-wave range vs. frequency for 100 joules of radiated S-wave energy. See Figure 10.8a.

Figure 10.9. Range of acoustic radar compared to range of electromagnetic radar vs. wavelength in homogeneous, lossy granite.

substantial solid-to-solid contact, then part of the S-wave will be transmitted. The behavior of S-waves at fluid-filled fractures has been exploited by the geothermal group at Los Alamos to map the vertical extent of fractures in the Fenton Hill reservoir /92,93/. The mapping has been performed in the bistatic mode with the transmitter and receiver in separate boreholes. In some cases the mapping occurred in the transmission mode, i.e., the fracture was between the transmitter and the receiver. In other cases the receiver was located such that the mapping occurred in the reflection mode, i.e., the receiver was above the inclined fracture.

In the transmission mode the receiver is fixed at discrete locations in one borehole and the seismic source is moved along the other borehole. A traverse is made with the fracture uninflated (the hydrostatic pressure is less than the lithostatic stress). The fracture is then opened by increasing the pressure to a few hundred psi above the least principal stress. A second acoustic traverse is then made. With the fracture inflated the amplitude of P- and S-waves received below a certain depth is less than that of the first traverse. This depth defines the top of the fracture (except for size and wavelength dependent diffraction effects). The attenuation observed at Fenton Hill is stated to be greater than that which is calculated for a single crack; hence, it is believed that reflections or scattering from multiple cracks is responsible for the loss in received signal strength /92/.

In the reflection mode the received signal was found to be quite frequency dependent, and was attributed to Fresnel zone backscatter. We note however that the same frequency dependence arises from an isolated plane scatterer (Sections 2.4, 2.5, and 10.1.3). With the reservoir unpressurized a strong reflection at 12 kHz was observed and attributed to S-S wave reflection ($\lambda = 0.3$ m) at a geologic discontinuity known to exist from other measurements. Lower frequency signals were not reflected. By moving the source a family of ellipsoids of constant wave transit times can be constructed. An osculating region on the ellipsoid defines a reflecting surface, which in the Fenton Hill reservoir is a gneiss-granodiorite contact surface. The distance to the reflecting surface from the receiver is not

uniquely fixed by the acoustic measurements because the reflection can arise from P-S, P-P, S-P, and S-S interactions. With the reservoir pressurized the received signals exhibited no clear cut P- and S-waves, which was attributed to scattering and reverberations from numerous inflated fractures which apparently surround the receiver. The effect of moving the receiver out of the inferred rubble zone is not known.

The most important result for diagnostic purposes derived from the Fenton Hill experiments is that it is indeed possible to define the vertical extent of fracturing from acoustic measurements. Moreover it appears necessary to alter the fracture zone in as nearly a reversible manner as possible to clearly identify the fracture. Certain features of the observed signals are subject to multiple interpretations. The increase in the clutter signal upon pressurization implies that there are numerous subsidiary fractures, even though their exact configuration is not uniquely defined.

SECTION 11

PASSIVE ACOUSTIC HYDRAULIC FRACTURE MAPPING

Acoustic emission or microseismicity associated with hydraulic fracturing has been used to map the extent and orientation of the hydraulic fracture itself. Conceptually, the technique is quite attractive for fracture mapping at large distances from a borehole, especially from a solitary borehole. The effectiveness with which microseismic techniques can be applied to determine the location and size of a hydraulic fracture is the subject of this section. The basic scheme of microseismic mapping is reviewed in Section 11.1. To see how the fracture process is related to the radiation of acoustic energy, the state-of-the-art of hydraulic fracture modeling is reviewed in Section 11.2. Based upon extant work, the properties of the signal radiated from a tensile failure are presented in Section 11.3. Finally, the interpretation of microseismic signals and their relation to the hydraulic fracturing process is discussed in Section 11.4.

Based upon calculations and experimental observations, we have developed a qualitative model of hydraulic fracturing that indicates that the interplay between the fluid mechanics and fracture mechanics might create a dynamically unstable jerky tensile failure. The notion of an unwetted crack tip arising from viscous effects during a quasi-static fracturing operation is shown to be at variance with some straightforward fluid mechanics calculations. An unwetted crack tip is shown to be possible only under dynamic conditions and that it is not necessary to invoke viscous effects, i.e., an inviscid fluid will allow a transient low pressure region to form at the crack tip. Because of complete wetting of the crack tip under quasi-static conditions for any real material, we believe that it is not admissible to neglect the cohesive strength of competent rock during the fracturing process.

11.1 THE MICROSEISMIC FRACTURE MAPPING METHOD

In principle, locating the origin of a microseismic event is a straightforward process employing two or more sets of triaxial geophones, emplaced at different positions in a borehole, as indicated in Figure 11.1. Because a P-wave is a longitudinal wave, the velocity vector can be reconstructed from any one triaxial geophone record, at least to within a 180° ambiguity. A 180° ambiguity may arise because it is not known a priori if the first motion is toward or away from the source (see Section 11.3). If it is definitely known that the geophones are located outside the region of seismic activity, then the source position can be determined uniquely using only one receiving station. It is clear from Figure 11.1 that the ambiguity is removed by using at least two stations. More than two stations provide data on the precision of locating any one event as do deviations from linear polarization at one station. Alternately, the distance between each geophone station and the microseismic source can be obtained from the difference between P-wave and S-wave arrival times, and sonic log measurements of P-wave and S-wave velocities. Since the use of two or more triaxial geophone stations unambiguously locates the source of microseismic activity, the difference between P-wave and S-wave arrival times yields the average values of the elastic wave speeds along the entire path as opposed to the near-borehole data provided by a sonic log. The use of two or more triaxial geophone packages offers the potential for improving mapping accuracy and increasing confidence in the microseismic mapping technique. Until quite recently, the practice at Fenton Hill was to employ only one triaxial geophone package. In this case, the standard deviation of the position of microseismic sources about the inferred fracture plane was 28 meters /94/. The precision of locating any one source is stated to be ± 5 meters /94/. In terms of mapping a fracture over a region that extends hundreds of meters from a borehole, the microseismic technique is quite good; in terms of being able to intersect a fracture with a second borehole, there is room for improvement. As will be reviewed in Section 11.4, part of the data scatter arises because, in fact, the strongest microseismic activity

Figure 11.1. Use of a triaxial geophone array to locate the origin of a microseismic signal.

arises not from tensile failure during the hydraulic fracture process but rather from shear failure along slip planes that cut across the hydraulic fracture. Other problems with applying microseismic techniques arise from signals that are simply hard to interpret; indeed, from all of the microseismic data obtained at Fenton Hill, only between 5 and 10 percent were used for locating the origin of acoustic activity.

11.2 HYDRAULIC FRACTURING

The hydraulic fracturing process is often stated to be aseismic /95/. It is worthwhile to examine the basis for this statement because the microseismic technique depends upon the radiation of acoustic energy for the mapping of a hydraulic fracture. To understand the source mechanism for radiation (or lack of radiation as the case may be) from a hydraulic fracture, we examine the extant knowledge of the hydraulic fracturing process.

Because all of the details of the hydraulic fracturing process have not been clarified at the present state-of-the-art, and because the details influence the manner in which acoustic radiation is emitted, we examine how the state-of-the-art evolved. In so doing, we shall clarify why certain descriptions are used, their limitations, and suggest some new approaches. The intent of this section is to allow the reader to develop a physical feel for the mechanisms that control hydraulic fracturing and the emission of acoustic energy.

11.2.1 Griffith Theory

The starting point of modern brittle fracture theory is Griffith's analysis which is reviewed in References 96 to 98. In essence, Griffith equated the decrease in elastic strain energy resulting from the increase in the size of an elliptical crack,

$$dU = - \frac{\pi \sigma^2 (1-\nu^2)}{E} 2ada \quad (11.1)$$

to the increase in the free energy arising from the creation of new surfaces, $4\gamma da$. The energy per unit area is denoted by γ and the crack half-length by a as indicated in Figure 11.2. The other symbols have their usual meaning,

i.e., ν is Poisson's ratio, E is Young's modulus, and σ is a uniform stress imposed at some distance from the crack. Thus, if the stress is greater than that given by Equation 11.2, then the crack will grow. Griffith's

$$\sigma^2 = \frac{2E\gamma}{\pi a(1-\nu^2)} \quad (11.2)$$

analysis is based on the supposition that cracks have an elliptical shape, i.e., a narrow crack is idealized as an ellipse, as indicated in Figure 11.2. (An ellipse is a convenient shape for representing a crack because the minor axis can be made arbitrarily small compared to the major axis, whilst all of the important results for other crack shapes is given in Reference 99.) We thus come to the first flaw in Griffith's theory; namely, elastic or small deformation theory had been used to calculate the strain energy, but the deformations are anything but small at the crack tip. A second flaw is implied by Figure 11.2, that is, an ellipse cannot change continuously into a longer ellipse without the removal of material from the crack tip /96/.

Elasticity theory can be used to calculate the stress distribution along the prolongation of the crack /96/. Let $T_0(x)$ be the normal stress distribution that would exist along the axis if there were no cracks and which arises from external loads (τ_{xy} is zero by symmetry), i.e., $(\sigma_y)_1 = T_0(x)$, $-a < x < a$. By applying an equal and opposite force distribution to the crack faces (the crack faces are now free of stress by superposition), the stress field outside the crack along the crack axis is

$$(\sigma_y)_2 = \frac{2|x|}{\pi(x^2 - a^2)^{1/2}} \int_0^a \frac{T_0(t)(a^2 - t^2)^{1/2}}{(x^2 - t^2)} dt \quad (11.3)$$

where t is a dummy variable running along the x axis. To clarify the nature of the stress field near the crack tip, new variables are defined by

$$\xi = a - t, \quad s_1 = x - a. \quad (11.4)$$

Figure 11.2. Elliptical crack and postulated crack extension.

(a) Narrow Crack

(b) Large Deformation at Tip

Figure 11.3. Idealized crack and crack tip deformation implied by extrapolation of linear theory /96/.

Equation 11.3 now becomes

$$(\sigma_y)_2 = \frac{2}{\pi} \frac{(a+s_1)}{[s_1(2a+s_1)]^{1/2}} \int_0^a \frac{T_o(\xi) [\xi(2a-\xi)]^{1/2}}{[(s_1+\xi)(2a+s_1-\xi)]} d\xi, \quad s_1 > 0 \quad (11.5)$$

which describes the stress field outside the crack. For $s_1/a \ll 1$, it is useful to expand Equation 11.5 in a power series in s_1 . The result /96/ is

$$(\sigma_y)_2 = \frac{N_o}{s_1^{1/2}} - T_o(a) + O(s_1^{1/2}), \quad (11.6)$$

where

$$N_o = \frac{\sqrt{2a}}{\pi} \int_0^a \frac{T_o(t)}{(a^2-t^2)^{1/2}} dt. \quad (11.7)$$

That the stress increases without bound as the crack tip is approached as predicted by Equation 11.6 is the third failing of the Griffith theory.

11.2.2 Barenblatt's Theory

The three flaws in the Griffith theory are not present in a theory proposed by Barenblatt /97/, which yields quite a different geometry at the crack tip. As will be seen, the new shape at the crack tip produces some interesting implications concerning fluid flow during a hydraulic fracture. To proceed, Barenblatt argues correctly that atomic cohesive forces must be accounted for as material tears apart. The effect of the atomic forces along the crack, denoted by $g(t)$, on the stress distribution outside the crack is given formally by Equation 11.3 with $T_o(t)$ replaced by $-g(t)$. Thus,

$$(\sigma_y)_3 = \frac{N_c}{s_1^{1/2}} + g(a) O(s_1^{1/2}), \quad (11.8)$$

$$N_c = \frac{\sqrt{2a}}{\pi} \int_0^a \frac{g(\xi) d\xi}{[\xi(2a-\xi)]^{1/2}}. \quad (11.9)$$

Since $T_o(t)$ is continuous, it can be expanded in powers of s_1 to yield

$$(\sigma_y)_1 = T_o(a) + O(s_1^{1/2}) . \quad (11.10)$$

The complete stress distribution along the prolongation of the crack is, consequently,

$$\sigma_y = (\sigma_y)_1 + (\sigma_y)_2 + (\sigma_y)_3 = \frac{(N_o + N_c)}{s_1^{1/2}} + g(a) + O(s_1^{1/2}) . \quad (11.11)$$

If $(N_o + N_c) = 0$, then the singularity vanishes. This is precisely what Barenblatt postulates. Note that the vanishing of $N_o + N_c$ cannot be proved. Barenblatt himself /97/ has glossed over this point which has been made quite clear by Goodier /96/. A second postulate and one that is quite reasonable is that the cohesive force distribution is significant only over a small distance d near the crack tip. Thus, the integrand in Equation 11.9 is expanded in power of ξ to yield

$$N_c = \frac{1}{\pi} \int_0^d \frac{g(\xi)}{\xi^{1/2}} \left[1 + \frac{1}{4} \frac{\xi}{a} + \dots \right] d\xi . \quad (11.12)$$

Because $\xi/a \ll 1$, only the leading term need be retained, so that

$$N_c = - \frac{1}{\pi} \int_0^d \frac{g(\xi)}{\xi^{1/2}} d\xi . \quad (11.13)$$

As the loads on a cracked material increase, $g(\xi)$ increases up to a limiting distribution $g_m(\xi)$, which in turn leads to a limiting value of the cohesive stress

$$N_{cm} = \frac{1}{\pi} \int_0^d \frac{g_m(\xi)}{\xi^{1/2}} d\xi = - \frac{1}{\pi} K . \quad (11.14)$$

K is called the modulus of cohesion and is taken to be a property of the material, much like the surface energy is a property of the material in Griffith's theory. In fact, it can be shown /96/ that Barenblatt's modulus of cohesion and Griffith's surface energy are related by

$$K^2 = \frac{\pi \gamma E}{(1-\nu^2)} . \quad (11.15)$$

From the condition $(N_o + N_c) = 0$, it is clear that the critical loads which cause crack extension can be determined from $N_o = K/\pi$. Central to the description of crack propagation, especially during hydraulic fracture, is the shape of the crack tip. For a planar crack, the displacement of the upper (+) and lower (-) crack faces caused by external loads is /96/,

$$v_2(x) = \pm \frac{4(1-\nu^2)}{\pi E} \int_{|x|}^a \frac{t}{(t^2-x^2)^{1/2}} dt \int_0^t \frac{T_o(s)}{(t^2-s^2)^{1/2}} ds , \quad |x| \leq a . \quad (11.16)$$

The displacement (closure) resulting from the forces of cohesion on the upper (-) and lower (+) crack faces is

$$v_3(x) = \mp \frac{4(1-\nu^2)}{\pi E} \int_{|x|}^a \frac{p}{(p^2-x^2)^{1/2}} dp \int_0^w \frac{g(t)}{(p^2-t^2)^{1/2}} dt , \quad |x| < a . \quad (11.17)$$

where p is a dummy variable. It can be shown that by integrating by parts, making a change of variable to $s_2 = a-x$, and for $s_2/a \ll 1$, the crack face displacement near the tip is

$$v(s_2) = v_2(s_2) + v_3(s_2) = \pm \frac{4(1-\nu^2)}{E} (N_o + N_c) s_2^{1/2} + O(s_2^{3/2}) . \quad (11.18)$$

Following the previous reasoning that the stress singularity must vanish, i.e., $(N_o + N_c) = 0$, the derivatives of the crack displacement with respect to distance along the crack is zero at the crack tip. Thus, according to Barenblatt's theory, the crack does not have an elliptical shape as was assumed in earlier work, but rather the crack tip has a constant, non-self-similar wedge-like or cusped shape.

11.2.3 Fracturing at Depth

According to Barenblatt, a cusped shape was also deduced for hydraulic fractures by Khel'tov and Khristianovich /97/ who were interested in determining the size of hydraulic fractures. Their mathematical scheme appears to have been the forerunner of Barenblatt's analysis, but instead of using the cohesive forces of the rock to resist fracturing, they assumed that the overburden or tectonic stresses would act to close a fracture. Based, apparently, upon the notion that a fluid would experience a very high resistance when flowing into an extremely narrow converging channel, they state that the entire pressure drop in the fracturing fluid would occur near the entrance to the cusp. Hence, the tip of the cusp would effectively be free of hydraulic pressure ("unwetted"). As indicated in Figure 11.4, the lithostatic stress, S_1 , would serve the same function as the cohesive stresses discussed in Section 11.2.2, and the same formulas can be used to describe an equilibrium hydraulic fracture. The effective fracturing pressure, Δp , is the difference between the hydrostatic pressure, p , and the lithostatic stress, while a_o is the wetted length of the fracture. By equating the stress singularity factor arising from the hydraulic overpressure (using Equation 11.7),

$$N_o = \frac{2a}{\pi} \int_0^{a_o} \frac{\Delta p dt}{\sqrt{\frac{a}{2} - t}} \doteq \sqrt{\frac{a}{2}} \Delta p \quad (11.19)$$

to that arising from the overburden stress acting over the unwetted crack tip (using Equation 11.13)

$$N_c = -\frac{1}{\pi} \int_0^{\Delta a} \frac{S_1}{s^{1/2}} ds = -\frac{2}{\pi} \sqrt{\Delta a} S_1, \quad (11.20)$$

a relation is obtained between the crack half length, the unwetted length at the crack tip, Δa , the hydraulic overpressure and the overburden stress.

$$\frac{\Delta a}{a} = \frac{\pi}{2\sqrt{2}} \frac{\Delta p}{S_1}. \quad (11.21)$$

Figure 11.4. Unwetted crack tip model. The diagram is misleading for two reasons: High pressure fluid can penetrate cracks as fine as 15 Å. The cusped shape implies crack openings less than the lattice spacing.

Although the theory of Zheltov and Khristianovitch is rather ingenious in that it eliminates the stress singularity at the crack tip, we believe that it would be remarkable, indeed, if the fluid mechanics operated in such a way as to leave part of the crack tip essentially unwetted in order to eliminate a stress singularity in the solid and maintain a condition of static equilibrium. It seems quite likely to us that the situation depicted in Figure 11.4 cannot exist for any real material. Consider an equilibrium crack that has attained a half length, a . Since the tip is stationary, the fluid is in static equilibrium with the surface tension, T_s , just balancing the hydrostatic pressure. Using a two-dimensional macroscopic model, we estimate the diameter, D , of the fluid at the crack tip as a function of pressure by formally applying the equation

$$D = \frac{2T_s}{p}, \quad (11.22)$$

with $T_s \doteq 60$ dyne/sec /100/ and plot the result of Figure 11.5. At a nominal fracturing depth of 10,000 feet for hot dry rock operations, the minimum radius of curvature of the fracturing fluid is approximately 8 Å , which is about five times the lattice half-spacing in a solid. If we assume for the moment that the motion of the crack tip does, indeed, cause a large pressure

drop behind the crack tip, then as the crack stops the full fluid pressure can fill voids as small as $15 \text{ \AA}$, which almost surely will reinitiate fracturing if cohesive forces are ignored.

Figure 11.5. Wetted crack width vs. fracturing pressure under static conditions considering the effects of surface tension. The equilibrium lattice spacing at a solid is $\approx 10 \text{ \AA}$.

In a geologic setting, the overburden stress, S , increases with increasing depth, h , according to the relation

$$S = \rho gh , \quad (11.23)$$

where ρ is the average density of the overburden. Since the intensity of the cohesive forces at the tip of the crack depends only on the material and the length of the crack, it is clear that below a certain depth the magnitude of the overburden stress at the crack tip can be many times that of the cohesive forces. For example, using Equations 11.7, 11.14 and 11.23, the depth at which the effects of the vertical overburden stress are ten times those of the cohesive forces is $h = 10K/(\pi\rho g\sqrt{a/2})$. For granite, h

is estimated to be between 150 ± 78 meters per meter of fracture half-length, the uncertainty arising in the determination of the surface energy. Orowan has estimated $\gamma = 1200$ dyne/cm (for silicon and magnesium oxide) /101/, but Bureau of Mines data are ten times greater /102/. Other data used to calculate K are $E = 4.6 \times 10^{11}$ dynes/cm and $\nu = 0.21$ /103/. K is estimated to be $8.83 \times 10^7 \pm 4.6 \times 10^7$ dynes/cm^{3/2}.

To proceed, we note that if the hydraulic fluid completely fills the crack, as it almost assuredly does at least in the static case, then it is not permissible to neglect the cohesive forces as is done in most of the extant literature /104/. Using a different line of reasoning, Abe', et al. have arrived at the same conclusion /105/. As indicated in Figure 11.6, by superposition the cohesive stress distribution acts only against the hydraulic overpressure and must not be neglected simply because it is much smaller in magnitude than the full overburden stress. In this case, a stress balance at the crack tip leads to

$$N_o = \sqrt{\frac{a}{2}} \Delta p$$

$$N_{cm} = -\frac{K}{\pi},$$

so that

$$\Delta p_f = \sqrt{\frac{2}{a}} \frac{K}{\pi}.$$

The hydraulic pressure in excess of the lithostatic stress needed to induce fracturing, Δp_f , is plotted as a function of crack half-length in Figure 11.7.

Figure 11.6. Wetted crack tip model. The cohesive forces balance the excess hydraulic pressure provided $\Delta p < \sqrt{2/d} K/\pi$.

Figure 11.7. Hydraulic pressure in excess of the in situ stress required to continue fracturing as a function of crack half-length.

So far in this section, we have been rather noncommittal regarding the lithostatic stress that the hydraulic pressure must overcome to induce fracturing, and we have not addressed the issue of fracture direction. Recall that in Section 6 we noted that if natural planes of weakness are oriented close to the perpendicular to the minimum in situ principal stress, then hydraulic fracturing will almost assuredly occur along the weakness planes. It seems unlikely that a neat penny shaped fracture will occur in a geologic setting. Hubbert and Willis /106/ were the first to point out that hydraulic fractures occur very nearly perpendicular to the minimum principal stress. In fact, hydraulic fracturing is now used as a geotechnical tool to determine the value of the principal stresses /107/. We note that hydraulic fracturing along weakness planes is probably modelled more accurately by ignoring cohesion, which implies that a weakness plane will eventually open along its entire length if fluid losses are negligible. If virgin material must be cracked to allow communication along nearly parallel weakness planes of finite extent, then the cohesive model is appropriate. A model that takes into account fluid flow in a system of finite size weakness planes does not appear to have been developed in the extant literature.

11.2.4 Fluid Flow During Fracturing

In this section we examine two important aspects of the hydraulic fracturing process; namely, the pressure exerted by the fluid at the crack tip and the speed at which hydraulic fluid can drive a fracture. In the former case, our analysis differs from that found in the literature because we believe it is unrealistic to allow $w \rightarrow 0$. By requiring that w be greater than the inter-atomic spacing, the singularity in the pressure gradient is eliminated. In the latter case, we find that the maximum flow velocity of hydraulic fluid is much less than the maximum propagation speed of a crack. Therefore, a crack either propagates in a slow quasi-static manner or in a jerky fashion controlled by the way in which the fluid catches up with the crack tip.

11.2.4.1 Pressure Drop at the Crack Tip

The fluid flow at the crack tip is modelled using one dimensional, variable area, unsteady, incompressible flow equations. The fluid mechanics is coupled to the solid mechanics mainly through the crack width function $w(x,t)$. Coupling via longitudinal, x -directed wall motion almost assuredly is negligible for a cusp-like crack tip whose phase velocity, i.e., the propagation velocity, is much greater than the y -directed wall motion which is itself much greater than the x -directed wall motion. The continuity equation

$$\frac{\partial \rho w}{\partial t} + \frac{\partial (\rho w u)}{\partial x} = 0 \quad (11.24)$$

is combined with the momentum equation

$$\frac{\partial (\rho u w)}{\partial t} + \frac{\partial (\rho u^2 w)}{\partial x} = - w \frac{\partial p}{\partial x} - 2\tau_o, \quad (11.25)$$

to yield

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = - \frac{1}{\rho} \frac{\partial p}{\partial x} - \frac{2\tau_o}{\rho w}, \quad (11.26)$$

where for the purposes of setting ideas, we have employed Cartesian rather than cylindrical coordinates. We assume that the shear stress is adequately described by

$$\tau_o = c_1 \frac{6\mu u}{w}, \quad (11.27)$$

which with $c_1 = 1$ is derived from Poiseuille flow between stationary parallel plates. The critical Reynolds number for transition from laminar to turbulent flow based upon crack aperture, $Re = 2300$, is reached at the following hydraulic fracturing conditions: $w = 1$ mm and a fracture propagation rate of 70 cm/sec. For a mobile equilibrium crack propagating at 1 cm/sec /104/, the fluid flow can be considered to be laminar, at least in the sense that flow disturbances created by rock asperities are damped. Laboratory tests have indicated that for flow in a cracked rock $1.04 < c_1 < 1.65$ /108/. The effect of regular (non-geologic) protuberances

on fluid flow is discussed in a number of references /109-113/. Thus it is appropriate to use the modified Poiseuille formula.

The pressure gradient is calculated from

$$\frac{\partial p}{\partial x} = - \frac{12}{w^2} c_1 \mu u - \rho \left(\frac{\partial u}{\partial t} + u \frac{\delta u}{\delta x} \right) . \quad (11.28)$$

The pressure gradient arises from both viscous forces and inertia. The magnitude of the inertial term depends on the form of the unsteady flow. Two effects of inertia are of interest. Upon crack initiation, inertia makes it more difficult for the fluid to remain in contact with the crack tip. On the other hand, if the fluid attains even a modest velocity, inertia forces will increase the effective fracturing pressure when the fluid catches up with the tip. The viscous term depends explicitly on the width of the crack tip, and we focus our attention on the viscous pressure drop near the crack tip. We estimate the pressure drop from

$$\Delta p = 12\mu u \int_{s_{\min}}^s \frac{ds_2}{w^2} ,$$

where u is the fracture propagation rate, and $s_2 = a - x$. Note that the flow is unsteady when viewed in an earth-fixed coordinate system and that we have imposed the condition that the fluid velocity is equal to the phase velocity of the crack tip. Based upon the solution for a cusped crack tip /96/, we estimate the width of the crack as a function of distance from the crack tip as

$$w = \frac{s_1}{E} \frac{s_2^{3/2}}{\tilde{d}^{1/2}}$$

where $\tilde{d}$ is a small distance near the crack tip over which the loads vary rapidly. Substituting this value into the integrand, we find that the viscous pressure drop is

$$\begin{aligned}
 p &\doteq \frac{6\mu u \tilde{d}}{(S_1/E)^2} \left[\frac{1}{s_2^2} - \frac{1}{s_{2\min}^2} \right] \\
 &\doteq \left[\frac{\tilde{d}/w_{\min}}{(S_1/E)^2} \right]^{1/3} \frac{6\mu u}{w_{\min}} .
 \end{aligned} \tag{11.29}$$

Since $\tilde{d}$ is rather ill defined, we will make a rough estimate of the bracketed term. We will then evaluate how crucial the accuracy of the estimate is by comparing the numerical value of the pressure drop to the total pressure used in fracturing operations. At a depth of 10,000 ft, the vertical overburden stress is 11,000 psi. S_1 might be one-half this value or approximately 5,500 psi. We expect the cohesive forces to vary significantly over distances comparable to the lattice spacing, say 5 Å. It is clear that even if continuum theory were to apply, the minimum crack opening must be greater than the lattice spacing. We will assume that $w_{\min}$ is five times the lattice spacing or approximately 15 Å. Thus,

$$\left[(\tilde{d}/w_{\min}) / (S_1/E)^2 \right]^{1/3} = 75 .$$

Formal application of Equation 11.29 results in a viscous pressure drop vs. minimum crack opening that is plotted in Figure 11.8. Note that at an aperture of 15 Å the viscous stresses can resist only 130 psi for a crack propagating at 1 cm/sec, but recall that in Section 11.2.3 we estimated that at the same crack width surface tension can balance a static pressure of more than 11,000 psi. The calculated viscous pressure drop is so low in comparison with fracturing pressures that the exact value of $\tilde{d}/w_{\min}$ is not crucial. Note that the pressure drop is controlled by the minimum value of the crack aperture, but that it is inadmissible to allow $w \rightarrow 0$. Furthermore, it appears necessary to include surface forces as the crack aperture becomes small. One additional caveat concerning the use of the Navier-Stokes equations to describe phenomena that vary over atomic distances is appropriate at this point. However, rather than belaboring the point, we note that the results of mineback

experiments described at the end of this section lead us to believe that the phenomena of interest are macroscopic in nature.

Figure 11.8. Viscous pressure drop vs. minimum crack width and crack propagation speed.

11.2.4.2 Speed of a Hydraulically Driven Fracture

The maximum flow velocity of hydraulic fluid at typical fracturing pressures cannot exceed a few hundred meters per second as indicated in Figure 11.9. We have shown in Section 11.2.4.1 that under quasi-static conditions essentially the full hydraulic pressure is exerted at the crack tip. According to equilibrium theory, hydraulic pressure in excess of the sum of the minimum principal lithostatic stress and the cohesive strength will produce a fracture in rock. As indicated in Section 11.3, the maximum speed at which a fracture can propagate is somewhat less than the Rayleigh

speed or about 3000 m/s in granite, some ten times faster than the speed of the hydraulic fluid. It seems entirely possible that a crack can extend in a jerky fashion, but for reasons quite different than that proposed by Aki /95/. It is the fluid itself that drives the jerky extension. We hypothesize the following sequence of events: The full hydrostatic pressure initiates the fracture, but the fluid cannot keep up with the motion of the crack tip (even if viscous losses are ignored). Now the vacuum in the space between the tip and the lagging fluid acts to arrest the crack motion. Eventually, the fluid catches up with and fills the entire crack to reinitiate fracturing. The presence of a vacuum in contact with a high pressure fluid suggests that the fluid interface will undergo an explosive decompression and gasification. The physical processes at the interface have apparently not been addressed in the literature of hydraulic fracturing. Unsteady pressure waves may augment the fracturing process, but their existence does not appear necessary for jerky extension.

Figure 11.9. Maximum fluid flow velocity vs. pressure.

Whether hydraulic fracturing proceeds in a quasi-static or in a dynamically unstable manner is a question that remains to be answered. In our opinion, extant evidence points to an unstable propagation. Computer calculations of gas driven fractures using the KRAK code have shown

unstable behavior /114/, but at the time the calculations were made, it was not certain whether the instability was reflecting physical behavior or was arising from the numerics.

Minebacks which allow visual observation of hydraulic fractures have been performed by Sandia Laboratories. By dyeing the hydraulic fluid, it is possible to observe the extent to which the fluid penetrated the fracture. The mineback revealed that sections of the fracture were not penetrated by dyed fluid /115/. Because these regions are readily visible to the naked eye, we are not viewing a submicron region near the crack tip. The question then arises as to how a fracture was formed without hydraulic fluid filling what is inferred to be a substantial portion of the crack, a situation that is untenable under quasi-static conditions. A mechanism for such behavior can be found in the stopping process of an unstable crack. For the conditions of the Sandia experiment, the maximum crack speed was 1000 m/s (the experiment was performed in tuff), but the maximum fluid flow velocity was less than 120 m/s (the hydraulic pressure in the formation was 1000 psig). Thus, we infer that when the last increment of tuff was fractured, the crack raced ahead of the hydraulic fluid, but as the hydraulic pump was shut down, the crack closed and as a result of the diminished pressure, the fluid could not penetrate the crack tip.

11.3 RADIATED SIGNAL PROPERTIES

We are interested in the intensity of acoustic radiation as a function of the type of fracture, fracture propagation speed, and angle. For tensile and shear failures, we make use of Haskell's analytical model /116/. For jerky extension, recourse must be made to numerical computation /95/. To employ Haskell's results, refer to the coordinate system in Figure 11.10. The direction of fault propagation is along x_1 . The normal to the fault plane lies along x_3 . The length of the fault is l , and its width in the fault plane is W . The fault displacement, $\bar{D}$, is represented as a wave propagating with velocity, v , along ξ_1

$$\bar{D}(\xi, t) = F(\xi) G(t - \xi/v) ,$$

where $\bar{D}$ is the displacement averaged over the fault width, and s_1 is the coordinant of the source point. For simplicity, $F(s)$ is considered to be a constant. The displacement time function is specified by Haskell to be a ramp of finite duration, T ,

$$G(t) = \begin{cases} 0 & t < 0 \\ t/T & 0 \leq t \leq T \\ 1 & t > T, \end{cases}$$

although Ben-Menahem and Toksöz deduced

$$G(t) = 1 - e^{-t/T} \quad (11.30)$$

from earthquake data. Haskell states that the results differ very little between the two functions. We shall use the exponential form because for our purpose they are slightly more convenient. The duration of slip, T , is set equal to $W/2v$.

Figure 11.10. Coordinate system showing fault plane and direction to field point.

11.3.1 Tensile Failure

A hydraulic fracture arises from tensile failure. Therefore, we are most interested in determining the acoustic signature of a tensile failure. The velocity components in the far-field are given by Haskell as:

$$\dot{u}_r = \frac{1}{4\pi\beta r} \left(\frac{\beta}{\alpha}\right) WI_\alpha \left[1 - 2 \left(\frac{\beta}{\alpha}\right)^2 + 2 \left(\frac{\beta}{\alpha}\right)^2 \sin^2\theta \sin^2\phi \right] \quad (11.31a)$$

$$\dot{u}_\theta = \frac{1}{4\pi\beta r} WI_\beta \sin 2\theta \sin^2\phi \quad (11.31b)$$

$$\dot{u}_\phi = \frac{1}{4\pi\beta r} WI_\beta \sin\theta \sin 2\phi. \quad (11.31c)$$

Since geophones sometimes appear to be located in quiet zones, it is worthwhile to determine if nodes exist in the radiation pattern. Following Haskell, the coordinate system is rotated such that the new polar axis lies along the x_3 axis and θ' coincides with γ_3 . The condition that a node exists is expressed as

$$1 - 2 \left(\frac{\beta}{\alpha}\right)^2 + 2 \left(\frac{\beta}{\alpha}\right)^2 \cos^2\theta' = 0$$

$$\cos \theta' = \frac{1}{2} \left[1 \pm \sqrt{1 - 2 \left(\frac{\alpha}{\beta}\right)^2} \right].$$

For constitutive properties found in nature, no real solution exists, thus acoustic radiation emitted from a tensile failure has no nodes. The term I_α is an integral of the displacement time function defined as

$$I_\alpha = \int_0^\ell \ddot{D}(\xi, t - r/\alpha) d\xi$$

$$= \frac{\alpha D_0}{q_\alpha} \int_{\tau - q_\alpha \ell / \alpha}^\tau \ddot{G}(x) dx = \frac{\alpha D_0}{q_\alpha} [\dot{G}(\tau) - \dot{G}(\tau - q_\alpha \ell / \alpha)]$$

where $q_\alpha = \frac{\alpha}{v} - \cos \theta$, $\tau = t - r_o/\alpha$, and $F(\xi) = D_o$. We evaluate the integral using Equation 11.30 and find for Equation 11.31a, b, c:

$$\dot{u}_r = \frac{D_o}{2\pi r} \left(\frac{v}{\alpha} \right) \frac{v}{\left(1 - \frac{v}{\alpha} \cos \theta\right)} \left[1 - 2 \left(\frac{\beta}{\alpha} \right)^2 + 2 \left(\frac{\beta}{\alpha} \right)^2 \sin^2 \theta \sin^2 \phi \right] \left[\exp \left(1 - \frac{v}{\alpha} \cos \theta \right) \frac{2\ell}{W} - 1 \right] \exp \left[- \frac{(t - r_o/\alpha)}{T} \right] \quad (11.32a)$$

$$\dot{u}_\theta = \frac{D_o}{2\pi r} \left(\frac{v}{\beta} \right) \frac{v \sin 2\theta \sin^2 \phi}{\left(1 - \frac{v}{\beta} \cos \theta\right)} \left[\exp \left(1 - \frac{v}{\beta} \cos \theta \right) \frac{2\ell}{W} - 1 \right] \exp \left[- \frac{(t - r_o/\beta)}{T} \right] \quad (11.32b)$$

$$\dot{u}_\phi = \frac{D_o}{2\pi r} \left(\frac{v}{\beta} \right) \frac{v \sin \theta \sin 2\phi}{\left(1 - \frac{v}{\beta} \cos \theta\right)} \left[\exp \left(1 - \frac{v}{\beta} \cos \theta \right) \frac{2\ell}{W} - 1 \right] \exp \left[- \frac{(t - r_o/\beta)}{T} \right]. \quad (11.32c)$$

The longitudinal velocity, $\dot{u}_r$, has its maximum amplitude in the forward direction. The transverse velocity, $\dot{u}_\theta$, has zeroes at $\theta = 0, \pi/2, \pi, 3\pi/2$ and $\phi = 0, \pi$, while $\dot{u}_\phi$ has zeroes at $\theta = 0, \pi$ and $\phi = 0, \pi/2, \pi, 3\pi/2$. The radiative output is proportional to v/α . Mansinha's theoretical analysis indicates that the maximum propagation speed of a fracture is less than the shear wave speed, with the speed of a tensile failure being less than that of a shear failure /116/. If a tensile failure propagates in a quasi-static manner, then the radiated power should be quite low ($v/\alpha \ll 1$). If, on the other hand, a hydraulic fracture is dynamically unstable, then the radiated signal must be calculated in the manner discussed in Section 11.3.4.

11.3.2 Longitudinal Shear Failure

When shear slippage occurs in the direction of propagation, the velocity components as calculated by Haskell are

$$\dot{u}_r = \frac{1}{4\pi\beta r} \frac{\beta^3}{\alpha} W I_\alpha \sin 2\theta \sin \phi \quad (11.33a)$$

$$\dot{u}_\theta = \frac{1}{4\pi\beta r} W I_\beta \cos 2\theta \sin \phi \quad (11.33b)$$

$$\dot{u}_\phi = \frac{1}{4\pi\beta r} W I_\beta \cos \theta \cos \phi. \quad (11.33c)$$

Using the exponential displacement function, we find that

$$\dot{u}_r = \frac{D_o}{2\pi r} \left(\frac{v}{\alpha}\right) \left(\frac{\beta}{\alpha}\right)^2 \frac{v \sin 2\theta \sin \phi}{\left(1 - \frac{v}{\alpha} \cos \theta\right)} \left[\exp \left(1 - \frac{v}{\alpha} \cos \theta\right) \frac{2\ell}{W} - 1 \right] \exp \left[-\frac{(t - r_o/\alpha)}{T} \right] \quad (11.34a)$$

$$\dot{u}_\theta = \frac{D_o}{2\pi r} \left(\frac{v}{\beta}\right) \frac{v \cos 2\theta \sin \phi}{\left(1 - \frac{v}{\beta} \cos \theta\right)} \left[\exp \left(1 - \frac{v}{\beta} \cos \theta\right) \frac{2\ell}{W} - 1 \right] \exp \left[-\frac{(t - r_o/\beta)}{T} \right] \quad (11.34b)$$

$$\dot{u}_\phi = \frac{D_o}{2\pi r} \left(\frac{v}{\beta}\right) \frac{v \cos \theta \cos \phi}{\left(1 - \frac{v}{\beta} \cos \theta\right)} \left[\exp \left(1 - \frac{v}{\beta} \cos \theta\right) \frac{2\ell}{W} - 1 \right] \exp \left[-\frac{(t - r_o/\beta)}{T} \right]. \quad (11.34c)$$

The longitudinal velocity has zeroes at $\theta = 0, \pi/2, \pi, 3\pi/2$ and at $\phi = 0, \pi$.

The transverse velocity, $\dot{u}_\theta$, has zeroes at $\theta = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4$ and

at $\phi = 0, \pi$, while $\dot{u}_\phi$ has zeroes at $\theta = \pi/2, 3\pi/2$, and at $\phi = \pi/2, 3\pi/2$.

The transverse velocities have a maximum along the direction of propagation, $\theta = 0$.

11.3.3 Transverse Shear Fault

When shear slippage is orthogonal to the direction of propagation, the velocity components as calculated by Haskell are:

$$\dot{u}_r = \frac{1}{4\pi\beta r} \left(\frac{\beta}{\alpha}\right)^3 W I_\alpha \sin^2 \theta \sin 2\phi \quad (11.35a)$$

$$\dot{u}_\theta = \frac{1}{8\pi\beta r} W I_\beta \sin 2\theta \sin 2\phi \quad (11.35b)$$

$$\dot{u}_\phi = \frac{1}{4\pi\beta r} W I_\beta \sin \theta \cos^2 \phi. \quad (11.35c)$$

Employing Equation 11.30, we find

$$\dot{u}_r = \frac{D_o}{2\pi r} \frac{v}{\alpha} \left(\frac{\beta}{\alpha}\right)^2 \frac{v \sin^2 \theta \sin 2\phi}{\left(1 - \frac{v}{\alpha} \cos \theta\right)} \left[\exp \left(1 - \frac{v}{\alpha} \cos \theta\right) \frac{2\ell}{W} - 1 \right] \exp \left[-\frac{(t - r_o/\alpha)}{T} \right] \quad (11.36a)$$

$$\dot{u}_\theta = \frac{D_o}{4\pi r} \left(\frac{v}{\beta}\right) \frac{v \sin 2\theta \sin 2\phi}{\left(1 - \frac{v}{\beta} \cos \theta\right)} \left[\exp \left(1 - \frac{v}{\beta} \cos \theta\right) \frac{2\ell}{W} - 1 \right] \exp \left[-\frac{(t - r_o/\beta)}{T} \right] \quad (11.36b)$$

$$\dot{u}_\phi = \frac{D_o}{2\pi r} \left(\frac{v}{\beta}\right) \frac{v \sin \theta \cos 2\phi}{\left(1 - \frac{v}{\beta} \cos \theta\right)} \left[\exp \left(1 - \frac{v}{\beta} \cos \theta\right) \frac{2\ell}{W} - 1 \right] \exp \left[-\frac{(t - r_o/\beta)}{T} \right] \quad (11.36c)$$

The longitudinal velocity has zeroes at $\theta = 0, \pi$ and at $\phi = 0, \pi/2, \pi, 3\pi/2$. The transverse velocity, u_θ , has zeroes at $\theta = 0, \pi/2, \pi, 3\pi/2$ and at $\phi = 0, \pi/2, \pi, 3\pi/2$, while u_ϕ has zeroes at $\theta = 0, \pi$ and at $\phi = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4$. In contrast with the longitudinal shear fault, the transverse velocities, $\dot{u}_\theta$ and $\dot{u}_\phi$, have a maximum at 45° and 90° to the direction of propagation, respectively.

11.3.4 Jerky Tensile Failure

The analysis of jerky tensile failure is of interest to us principally because it suggests a means for identifying and mapping a hydraulic fracture during the fracturing process. Jumping ahead for the moment it is shown in Section 11.4 that the ratio of SH-wave displacement amplitude to P-wave displacement amplitude for a tensile failure does not exceed three. Thus, it is believed that the identification in the time domain of waves arising from tensile failure will be difficult. The problem is one of distinguishing tensile failure signals from either noise or mode conversion signals in a cracked medium. The main result from tensile failure analysis for our purpose is obtained from Aki's work /95/ and is that the far-field displacement spectrum has one or more peaks. Thus, if Aki's model is correct, the Fourier transform of geophone signals should allow us to distinguish between jerky tensile failure (a peaked spectrum), noise (a flat spectrum), and shear failure (a flat spectrum at low frequency followed by a $1/\omega^n$ roll off above some corner frequency /117/). In our opinion the accuracy of Aki's results is uncertain because of the numerical methods employed. On the other hand if the results are corroborated by more refined calculations then they may have a useful application in actual fracturing operations. It is worthwhile to briefly review Aki's methods.

Following Aki, the magnitude of the Fourier transform of the far-field displacement amplitude for P- and S-waves (as described by Haskell for a solitary fracturing event /118/) is given by

$$\left| u^P(x, \omega) \right| = \frac{\lambda + 2\mu \cos^2 \theta'}{4\pi\rho\alpha^3 r_o} W \left| \Delta\dot{w}\left(\frac{\omega \cos \theta}{\alpha}, \omega\right) \right|$$

$$\left| u^S(x, \omega) \right| = \frac{\sin 2\theta'}{4\pi\beta r_o} W \left| \Delta\dot{w}\left(\frac{\omega \cos \theta}{\beta}, \omega\right) \right| ,$$

where we have changed Aki's angular notation to be consistent with Figure 11.10, i.e., θ' is the angle between the crack plane normal and the direction to the field point, and θ is the angle between the axis of the crack (direction of fracturing) and the direction to the field point.

The quantity

$$\Delta\dot{w}(k, \omega) = \iint \Delta\dot{w}(\xi_1, t) \exp(-i\omega t + ik\xi_1) d\xi_1 dt$$

is the Fourier transform of the discontinuity in particle velocity across the crack. In the far-field approximation its spectrum is imaged directly at each field point except for some factors that depend on position. From the velocity-time history of the crack width, $\Delta\dot{w}(\xi, t)$, the spectrum can be calculated as a function of (ω, k) . The spectrum associated with P- and S-waves propagating in the direction of fracture ($\theta = 0$) can then be obtained by plotting $|\Delta\dot{w}(k, \omega)|$ in the k, ω plane and noting the intersections between $|\Delta\dot{w}(k, \omega)|$ and straight lines with slopes $k/\omega = \pm 1/\alpha$ and $k/\omega = \pm 1/\beta$, i.e., the P- and S-waves. The spectrum of waves propagating at an angle to the fault plane can be obtained from the intersection of $|\Delta\dot{w}(k, \omega)|$ and straight lines with the slopes $k/\omega = \pm \cos \theta / \alpha$ and $k/\omega = \pm \cos \theta / \beta$. Following this procedure for a crack that suddenly opens by one zone width Aki found the magnitude of the Fourier transforms of the P- and S-waves displacements to have one or more spectral peaks, in qualitative agreement with measurements of volcanic tremor. Thus, it is thought /95/ that jerky fracturing partially explains some of the observed phenomena associated with volcanic tremor and by implication is a reasonable model of hydraulic fracturing.

The principal source of our reluctance in applying Aki's results resides in the apparently arbitrary manner in which the fracture was extended by one zone width in the numerical calculations. Nonetheless, Aki's work appears to be a good starting point for additional analysis.

11.4 INTERPRETATION OF MICROSEISMIC SIGNALS

Of the several methods for interpreting microseismic signals, we concentrate our efforts on inferring source mechanisms from the ratio of the amplitude of S- to P-waves. This is the technique used by LASL. Using Ben-Menahem's results for the vertical radiation pattern of SH-waves and P-waves for both strike-slip and tensile failures /118/, we find that the displacement amplitude of shear waves is less than three times that of P-waves for tensile failure, whilst the same ratio can be quite large for shear failure, Figure 11.11. Using a similar procedure, LASL concluded that the ratio of displacement amplitudes does not exceed four for tensile failure /94/. It is useful to also plot the S- to P-wave ratios for velocity signals such as would be observed directly from geophones. Using Equations 11.32 and 11.34, we plot $\dot{u}_\theta / \dot{u}_p$ for both longitudinal shear failure and tensile failure as a function of θ with $\phi = \pi/4$. In this case, the ratio of the amplitude of shear to compressional wave velocities is less than two, Figure 11.12.

The data selected for analysis by LASL showed that on the average the ratio of S- to P-wave amplitudes was eleven. Their conclusion, which is almost certainly correct, is that the analyzed signals arose from shear failure /94/.

At first, the LASL analysis would seem to invalidate the concept of a dynamically unstable tensile crack extension. We believe that this is not the case for the following reasons. It is noted that between 90 and 95 percent of all the microseismic data recorded at Fenton Hill was not selected for analysis either because of low S-wave amplitudes /119/ or because clear P- and S-waves were not obtained from each of the three orthogonal geophones /94,120/. There is a distinct possibility that signals originating from a tensile failure simply were not analyzed. Consider the following. The data selected for processing at LASL were those

Figure 11.11. Ratio of SH-wave displacement amplitude to P-wave displacement amplitude for tensile and shear failures. Vertical radiation pattern based on the results of Ben-Menahem.

Figure 11.12. Ratio of particle velocity in θ direction associated with S-wave to that in longitudinal direction arising from P-wave for both longitudinal shear failure and tensile failure. Based on Haskell's analysis /117/ $\phi = 45^\circ$.

that had an amplitude (usually that of an S-wave) that exceeded some pre-set voltage level. As already mentioned, the selected data yielded a ratio of S-wave to P-wave amplitude of approximately eleven. Note from Figure 11.12 that this large ratio implies polar angles of either less than 15 degrees or more than 75 degrees. Within these angular bands, the ratio of S-wave amplitude from a tensile failure to that from a longitudinal shear failure is less than 0.41 (assuming equal fault displacements and equal fault propagation speeds) as shown in Figure 11.13. Thus, it is quite possible that signals originating from a tensile failure would not pass the automated selection process. Moreover, as is clear from Figure 11.12, these same signals would appear quite non-classical to the eye, exhibiting no prominent onset of S-waves, i.e., the P-waves and S-waves have about the same amplitude.

In summary, we conclude that the analyzed signals arose from shear failure, but that signals arising from a tensile failure may have been overlooked.

Figure 11.13. Ratio of shear wave amplitude arising from a tensile failure to that arising from a longitudinal shear failure. Based on Haskell's results. $\phi = 45^\circ$. In region of observed seismicity $(\dot{u}_\theta)_T / (\dot{u}_\theta)_S \leq 0.41$.

SECTION 12

PROCESSING OF SIGNALS FOR DETECTING FRACTURES

In this section we review several techniques that either have been or could be employed in the mapping of fractures from boreholes. Specifically we review tomography, holography, synthetic apertures, wavefront imaging, and the relative importance of phase and amplitude in signal reconstruction. Results from the last mentioned topic appear to be crucial and cast doubt on the usefulness of tomography as it has been employed in subterranean diagnostics. Because linear detectors are usually used in subterranean signalling and because subterranean wavefront imaging is in a nascent state the use of holographic techniques appears to us to be premature. A synthetic aperture, which by its very nature requires the utilization of phase information, appears useful, but is as yet untried in a deep borehole.

12.1 TOMOGRAPHY

In its original form tomography was associated with the formation of images using only intensity information derived from thin monochromatic x-ray beams. If P_f is the logarithm of the intensity ratio, i.e., $P_f = \ln(-I/I_0)$, then

$$P_f(L) = \int_L f(x,y) ds, \quad (12.1)$$

where $f(x,y)$ is the linear absorption coefficient which is characteristic of both the x-ray energy and the medium. Following the work of Shepp and Kruskal /121/ we find from the Radon inversion formula that the absorption coefficient can, in principle, be reconstructed from the intensity measurements according to

$$f(Q) = -\frac{1}{\pi} \int_0^\infty \frac{dF_Q(q)}{q}, \quad (12.2)$$

where Q denotes the point of interest and $F_Q(q)$ is the average value of $P_f(L)$ over all lines a distance q from Q . In practice the accuracy of the reconstruction depends upon the stability of the inversion process and how

well the discrete data approximate the integral. Moreover for a restricted set of lines there are usually many f that yield the given projection; hence, a unique reconstruction can be obtained only by placing an optimality criterion on the reconstructed f , denoted by f_r . One such criterion is that

$$\iint f_r^2 dx dy = \text{minimum} . \quad (12.3)$$

12.1.1 Practical Algorithms

Practical algorithms for reconstructing f are either convolutional or iterative. The convolutional method inverts a set of uniformly spaced measurements, for example in a mode employing parallel beams, by linear mapping in which the measurements $P_f(L)$ are weighted to estimate the density function $F_q(Q)$. Many different weighting functions have been employed, but in general the weighting is a function only of the distance from the point Q to the line L as suggested by Radon's formula. One convolutional method is based on Fourier transformation and, very briefly, is derived in the following way. The Fourier transform pair of the attenuation function is given by

$$\hat{f}(\omega, \theta) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) e^{-j\omega(x \cos \theta + y \sin \theta)} dx dy \quad (12.4)$$

$$f(x, y) = \frac{1}{(2\pi)^2} \int_0^\pi \int_{-\infty}^{\infty} \hat{f}(\omega, \theta) e^{j\omega(x \cos \theta + y \sin \theta)} |\omega| d\omega d\theta , \quad (12.5)$$

where $|\omega|$ arises from the transformation from Cartesian to polar coordinates. If a function ϕ exists such that $\hat{\phi}(\omega) = |\omega|$ then the Fourier integral becomes a convolution via Parseval's identity

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{P}(\omega) \hat{\phi}(\omega) e^{j\omega t} d\omega = \int_{-\infty}^{\infty} P(t) \phi(\tau - t) dt , \quad (12.6)$$

where $\tau = x\cos\theta + y\sin\theta$. The reconstruction formula is given by

$$f(x,y) = \frac{1}{2\pi} \int_0^\pi \int_{-\infty}^{\infty} P_f(t,\theta) \phi(x\cos\theta + y\sin\theta - t) dt d\theta, \quad (12.7)$$

where $x\cos\theta + y\sin\theta - t$ is the distance from x,y to $L(t,\theta)$. For what applied mathematicians refer to as an "honest" function ϕ tends to zero as $\omega \rightarrow \infty$ and the replacement of $|\omega|$ by ϕ is not possible unless restrictions are placed on ϕ . Usually ϕ is set equal to zero outside some domain where $P_f(t,\theta)$ is small to ensure that the error in reconstruction is small. The convolutional method is routinely applied to fan-beam geometries and has been extended to an arbitrary distribution of rays /122/.

The iterative reconstruction techniques approximate Equation 12.1 by a system of linear equations /123/

$$P_k = \sum_{i=1}^I \sum_{j=1}^J f_{ij} \Delta S_{ijk}, \quad k = 1, 2, \dots, K. \quad (12.8)$$

In the reconstruction of x-ray images it was found necessary to weight ΔS_{ijk} according to the fractional area actually intersected by a narrow, but finite width beam /121/. In subterranean imaging with electromagnetic or acoustic beams the beams are wide and no such weighting is performed. Apparently no weighting has been applied to account for antenna pattern functions /123/. The algebraic reconstruction technique, commonly referred to as ART, treats one equation at a time thereby eliminating the need for storing the coefficient matrix in Equation 12.8. Using either an initial estimate or the values for f_{ij} along the k^{th} path from the preceding n^{th} iteration the discrete line integral is calculated as

$$\hat{P}_k^{(n)} = \sum_i^I \sum_j^J f_{ij}^n \Delta S_{ijk}. \quad (12.9)$$

If the true or measured line integral is expressed as

$$P_k = \sum_i^I \sum_j^J (f_{ij}^n + \Delta f_{ijk}^n) \Delta S_{ijk} , \quad (12.10)$$

then a correction term Δf_{ijk}^n can in principle be calculated from the difference between the measured value and the k^{th} calculated value for the line integral,

$$\sum_i^I \sum_j^J \Delta f_{ijk}^n \Delta S_{ijk} = P_k - \hat{P}_k . \quad (12.11)$$

This system of equations has a non-unique solution thereby requiring the imposition of a minimization criterion similar to Equation 12.3,

$$\min \left\{ C = \sum_i^I \sum_j^J (\Delta f_{ijk}^n)^{2p} \right\} , \quad (12.12)$$

where p is an integer. The only constraint placed on the integer p is that it provides a good reconstruction. In fact, the goodness of all of the tomography methods cannot be predicted from basic theory, but must be demonstrated experimentally by reconstructing a known test object. If $p = 1$ then the reconstruction is the ART formalism and weighting of the correction terms is on the basis of the fractional length of the cell intercepted to the total path length. In practice the detailed calculation of Δf_{ijk}^n is often dispensed with and an average correction term is applied to all computational cells intercepted by the k^{th} ray. The average correction is the difference between the measured P_k/L and the average value of the calculated f_{ijk}^n per unit length along the k^{th} ray. If ray-by-ray processing is employed with the f_{ijk}^n updated at each iteration it is found that the solution often hunts or oscillates because of noisy data. If updating is done after all the rays are processed then convergence is often improved [123]. This technique is known as simultaneous iterative

reconstruction (SIRT). The ART formalism is analogous to the use of geometrical ray optics.

12.1.2 Application to Borehole Fracture Mapping

The application of tomography as commonly practiced to fracture mapping faces two major difficulties. The resolution of a tomogram is proportional to the ray density or the size of the cells; hence, the number of ray paths must be made inversely proportional to the width of a fracture zone. Whether it is practical to resolve a fracture with tomography remains to be determined. The second problem arises from the different physics that controls the propagation of the x-ray beam used in medical diagnostics and the microwaves and acoustic waves that might be used in geophysical tomography. In medical acoustic diagnostics, for example, it is recognized that geometrical optics tomography cannot describe the important diffraction effects that occur in biological media and that it is necessary to employ the wave equation if an adequate tomographic reconstruction is to be obtained /124/. This last observation appears to us to be crucial. Indeed, based upon Section 12.5 it is essentially certain that phase information, if available, is more important than amplitude information for reconstructing a wide variety of images and, perhaps especially, images of fractures.

12.2 HOLOGRAPHY

A considerable literature exists on holography. The principles have been amply discussed /125-128/. We shall here concentrate our attention on the application of holography to underground imaging. The principal advantage and the remarkable aspect of holography is that both the amplitude and phase of a wavefront can be recorded by a device that is sensitive to intensity only, and that the amplitude and phase of the wavefront can be reconstructed from the recording. In short, a square-law (intensity) detector such as photographic film can be made to record the phase information carried by a light wave. In the fields of microwaves and seismics the phase of signals is readily recorded because most of the commonly used detectors are inherently linear. Thus there is no imperative to resort to holography to record phase information. Seismologists, for example, have always recorded amplitude and phase information, and have

sought means for reconstructing or imaging subterranean structures. To be sure, a hologram is a compact way of recording both amplitude and phase, but it is our opinion that until subterranean image reconstruction can be accomplished it is best to address the imaging problem rather than to divert attention to the compact recording of data that are difficult to interpret.

12.3 SYNTHETIC APERTURE

The requirements of long range signal propagation and relatively small diameter boreholes results in signalling wavelengths that are greater than the diameter of a sonde. Thus, as explained in the preceding section, both electromagnetic and acoustic antennas are small compared to a wavelength and consequently have poor directionality. One way of increasing the size of an antenna and thereby improving its resolution, at least in the direction of the borehole, is by employing a synthetic aperture. Synthetic aperture techniques have been employed successfully in such diverse fields as radar mapping /129/ and non-destructive testing /130/. The basic principles of a synthetic aperture are best explained in terms of a simple example given in Reference 131. Consider an antenna of length L composed of an array of N ordinary dipoles, uniformly illuminated, Figure 12.1a. All of the dipoles radiate simultaneously and then receive simultaneously the signal backscattered by the target. The simultaneous radiation by individual dipoles results in an interference pattern that gives rise to a nonuniform radiation pattern that has a first null half-angle given by $\theta = \lambda/L$. For a stationary target the antenna of Figure 12.1a can be replicated by a single dipole that radiates and receives a signal in a fixed location and then is moved successively to each of the locations that would be occupied by a dipole comprising the antenna of Figure 12.1a. Each dipole transmission in the synthetic case, Figure 12.1b, has a very broad radiation pattern. For the synthetic antenna the solitary dipole receives radiation backscattered from the target that was transmitted by itself. At each of the N locations comprising the synthetic antenna the radiated signal is identical and both the amplitude and phase of the received signal is stored. After N stations have been occupied the stored amplitude and phase

Figure 12.1. Schematic of angular resolution of fixed N-element antenna and synthetic dipole antenna /131/.

information is read out. For a target normal to the path of the dipole the phase of all N signals is identical. For a target that makes an angle θ to the normal each of the N received signals will exhibit a phase shift with respect to each other. When reproduced simultaneously a null will occur for the synthetic array at angle $\theta = \lambda/2L$. Thus in this simple case the synthetic array yields better angular resolution than a physical array of the same dimensions. The factor of two improvement arises because in Figure 12.1a the array is focused at infinity and the entire phase shift occurs in the backscattered signal; in the synthetic array half of the phase shift occurs along the transmission path and the other half along the backscattered path. The synthetic array offers greater versatility than a fixed array because it can be focused at all ranges at once, in other words, the amplitude and phase data which are stored individually for each

dipole location can be utilized in conjunction with the other (n-1) data points to focus the antenna at any range /130/.

The resolution of a synthetic array is determined in the following way /132/. Two point scatterers can be resolved if the difference in total path length (along the transmit and receive paths) is $\lambda/2$. Referring to Figure 12.2 the range to a scatterer B located a distance s from scatterer A is

$$R = \sqrt{R_o^2 + \left(\frac{L_{sa}}{2} - s\right)^2}, \quad (12.13)$$

where L_{sa} is the length of the synthetic aperture. The incremental change in range, ΔR , with change in scatterer offset, Δs , at $s = 0$ is

$$\frac{\Delta R}{\Delta s} = -\frac{L_{sa}}{2R_o}. \quad (12.14)$$

To resolve points A and B, $\Delta R = \lambda/4$ on both the transmit and scattering paths. Thus

$$\Delta s = \frac{R_o \lambda}{2L_{sa}}. \quad (12.15)$$

In airborne radar L_{sa} is set equal to the length of the illuminated path at each range so that the resolution is one-half the diameter of the physical antenna independent of range, i.e., $\Delta s = L/2$.

A number of conditions must be placed on the synthetic aperture if it is to perform as intended. We shall discuss only one. To prevent ambiguities in determining the location of a scatterer the solitary antenna that forms the synthetic antenna must occupy stations that are sufficiently close to one another to prevent a 2π phase shift from arising between successive stations that illuminate a fixed scatterer /132/. From Figure 12.3 we obtain the relation between the ranges R and R_o , the angular offset of the scatter θ_R , and the incremental displacement of the physical

Figure 12.2. Scatterers A and B can be resolved if the path length difference of the synthetic aperture is $\lambda/2$ /132/.

Figure 12.3. Diagram for calculating maximum antenna displacement without causing angular ambiguity in the synthetic antenna.

antenna Δx ,

$$R = R_0 + \frac{(\theta R_0 - \Delta x)^2}{2R_0} . \quad (12.16)$$

Setting the difference in range $\Delta R = R - R_0$ equal to $\lambda/2$ (for two-way propagation the difference is a full wavelength) we find that the maximum Δx is

$$\Delta x = \frac{\lambda R_0}{L} - \sqrt{R_0 \lambda} , \quad (12.17)$$

where L is the aperture of the physical antenna.

The application of synthetic aperture techniques to borehole mapping requires the accurate measurement of sonde displacements Δx to within about $\lambda/10$. The required accuracy should be attainable if the displacements are measured with respect to an arbitrary datum at depth rather than to the surface.

12.4 IMAGE FORMATION

The signal g returning to a location (x, y, z) that was backscattered from a target plane situated at $(x_0, y_0, 0)$ is given by /133/

$$g(x, y, z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(x, y, z; x_0, y_0) g(x_0, y_0, 0) dx_0 dy_0 , \quad (12.18)$$

where $h(x, y, z; x_0, y_0)$ is a propagation function and $g(x_0, y_0, 0)$ is a target function. The return signal is composed of a superposition of elemental spherical waves generated in the target plane. For a backscattered acoustic wave propagating in a uniform, homogeneous medium

$$h(x, y, z; x_0, y_0) = \frac{-j\omega \cos(\vec{k} \cdot \vec{r}) e^{j\omega r/c}}{2\pi c r} . \quad (12.19)$$

For near-normal incidence on the scattering plane $\cos(\vec{k} \cdot \vec{r}) \doteq 1$. Setting r in the denominator of Equation 12.18 equal to z does not greatly affect the resultant image, independent of range, but r in the argument of the

exponential controls the phase and must be set accurately. At all ranges

$$r = \left[z^2 + (x_0 - x)^2 + (y_0 - y)^2 \right]^{1/2} . \quad (12.20a)$$

In the Fresnel region, $L \leq r \leq L^2/\lambda$, where L is the size of the antenna,

$$r \doteq z \left[1 + \frac{1}{2} \left(\frac{x_0 - x}{z} \right)^2 + \frac{1}{2} \left(\frac{y_0 - y}{z} \right)^2 \right] . \quad (12.20b)$$

Equation 12.18 can be cast in the form of a spatial convolution

$$\begin{aligned} g(x, y, z) &= - \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{j\omega}{2\pi cz} \left\{ \exp \frac{j\omega}{c} \left[z^2 + (x-x_0)^2 + (y-y_0)^2 \right]^{1/2} \right\} g(x_0, y_0) dx_0 dy_0 \\ &= h(x-x_0, y-y_0, z) * g(x_0, y_0) . \end{aligned} \quad (12.21)$$

Fourier transforming Equation 12.21

$$G(f_x, f_y, z) = H(f_x, f_y, z) G_0(f_x, f_y, z) , \quad (12.22)$$

where $f_x = \frac{x\omega}{2\pi cz}$ and $f_y = \frac{y\omega}{2\pi cz}$.

Inverting H , i.e., $H^{-1} H = 1$ and taking the inverse Fourier transform of Equation 12.22 the image is

$$g(x_0, y_0) = F^{-1} \left[H^{-1}(f_x, f_y, z) G(f_x, f_y, z) \right] , \quad (12.23)$$

where $H^{-1}(f_x, f_y, z) = \exp \left\{ jkz \left[1 - \left(\frac{2\pi f_x}{k} \right)^2 - \left(\frac{2\pi f_y}{k} \right)^2 \right]^{1/2} \right\} .$

In the context of subterranean mapping Equation 12.23 must be regarded as a highly idealized result. The propagation function, Equation 12.19, does not account for inhomogeneities in the rock between a sonde and the target

plane. The use of a line array required by the geometry of a borehole prevents resolving detail in the lateral direction.

12.5 RELATIVE IMPORTANCE OF PHASE AND AMPLITUDE IN SIGNALS

By reconstructing signals using (1) the correct Fourier transform phase and amplitude (2) the correct Fourier amplitude with zero phase and (3) the correct Fourier phase with either a constant, weighted average, or arbitrary (from an entirely different signal) amplitude Oppenheim and Lim /134/ have shown that many important features of a signal are preserved if only the phase is retained, while retention of only amplitude results in almost a complete loss of information. Their results are particularly important to fracture mapping because the relative importance of phase appears to be emphasized for objects with sharp boundaries, such as fractures. Results are presented for several type of signals including optical wavefront reconstruction, x-ray diffraction crystallography, and speech synthesis. One of their examples for optical wavefront reconstruction is particularly striking, Figure 12.4. Their results are experimental in that their conclusion is derived from the analysis of many examples. A complete theoretical treatment of the results is not yet at hand, but the results have been justified in a number of ways. For example, that phase-only reconstructions preserve much of the correlation between signals implies that the location (in time and/or in space) of events tends to be preserved. Much of the intelligibility of reconstruction signals that have been studied is related to the location of lines, points, and boundaries. It is generally plausible that phase reflects the location of events more than does the magnitude (for some examples this is exactly true), e.g., a translation in time or position in space does not alter the Fourier magnitude, but does shift the phase. There are, however, many cases in which phase-only is insufficient for reconstructing a signal. A simple example of a signal that is not uniquely specified by its phase is the convolution of one signal with any other signal having zero phase. Oppenheim and Lim propose that for speech and pictures, if the long-term Fourier transform is "sufficiently smooth," then intelligibility will be retained in phase-only reconstructions.

Figure 12.4. Comparison of signal reconstruction from Oppenheim and Lim /134/. (a) Original image. (b) Image synthesized from the Fourier transform magnitude of (a) and zero phase. (c) Image synthesized from the Fourier transform phase of (a) and unity magnitude. (d) Image synthesized from the Fourier transform phase of (a) and a magnitude averaged over an ensemble of images.

SECTION 13

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APPENDIX

FRACTURE MAPPING FROM THE SURFACE USING TILT MEASUREMENTS

A.1 EXISTING CAPABILITY

M. D. Wood, Inc., offers a commercial service employing surface measurements for determining the azimuth of vertical fractures. A circular array of shallow-buried tiltmeters is usually used to measure the rotation of the earth's surface produced by a fracture at depth. For a vertical fracture, the maximum surface tilt is orthogonal to the fracture plane and occurs at a distance on the surface equal to 0.38 times the depth of the fracture. The azimuthal variation in signal strength around the array allows the strike of the fracture to be inferred. Whether or not uncertainties in the determination of azimuth arise from a non-vertical dip or from multiple fractures has not been tested. Signal processing is used to remove the effects of cyclical disturbances such as earth tides and diurnal heating. Based on the internal consistency of the data, aperiodic or random disturbances such as weather fronts have a negligible effect on the final answer. It is crucial to ensure that the fracture signal is not contaminated with noise having the same spectral energy content as the fracture signal itself. For example, fracture inflation must occur within a time that is short compared to natural cyclical fluctuations, i.e., one-quarter of one day. Background noise data should be acquired over a period of several days (perhaps as much as ten days), which spans the time of fracture inflation. With current techniques, $1/R^2$ fall-off in signal strength limits the depth of hydraulic fracturing to about 10,000 feet. Based on Wood's reports of (i) successfully intersecting a fracture at the predicted location with a second borehole, and (ii) agreement of azimuth determinations using oriented core as an independent check, the technique appears successful.

A.2 IMPROVEMENTS IN TECHNIQUE

Wood has suggested that the maximum mapping depth could be extended by (i) employing a linear array of tiltmeters and stacking the data, or (ii) burying the sensors. A linear array can be used effectively only if the fracture azimuth is known fairly well prior to laying out the array. Thus, to lay out

a linear array for working at great depths, it would first be necessary to produce a hydraulic fracture at a shallow depth, say 9,000 feet, determine the strike and then assume that the strike does not vary rapidly with depth. The success of the method rests on the validity of this last assumption. Linear arrays have been used, but stacking to enhance signal strength has not been employed. Burying the sensors would allow at least a one-to-one increase in fracture mapping depth, but techniques for the remote emplacement of tiltmeters have not been tested by M. D. Wood, Inc. In addition, extant sensor packages have not been designed to work at temperatures and pressures in excess of surface ambient.

A.3 FRACTURE EXTENT DETERMINATION USING THE WOOD METHOD

A.3.1 Existing Capabilities

At the present time, Wood's technique has been employed only in bedded sedimentary strata where determining the vertical extent of fracturing has not been of commercial interest. No proven capability exists to map the vertical size of a fracture in a granitic medium.

A.3.2 New Methods Required

If the vertical extent of fracturing can be inferred from tiltmeters, then a model fitting process is almost assuredly required. Additional work is needed to assess the likely success of such an endeavor.

A.4 SUMMARY AND RECOMMENDATIONS

Wood's commercial tiltmeter service appears useful for determining the strike of a vertical fracture to depths of 10,000 feet. Working at greater depths requires a research and development program. A method for determining the vertical extent of fracturing using tiltmeters does not now exist.

Prior to employing directional drilling in a setting where fracture orientation is unknown, it appears worthwhile to use Wood's service in addition to borehole techniques to determine fracture azimuth. At the very least, the technique should provide an independent check of other geophysical measurements. Because of the potential usefulness of Wood's measurements, it is worthwhile to determine analytically if fracture extent can be inferred from surface tilt data.