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The Potential for Reducing the Cost
of a Heavy Ion Accelerator for ICF

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Michael J. Monsler *
Satori Technology Inc.

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Laser Fusion Program
Univ. of California
Lawrence Livermore National Laboratory
Livermore, CA 94550

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* Now at: W. J. Schafer Associates, Inc.
6140 Stoneridge Mall Rd. Suite 590
Pleasanton, CA 94566
(415) 463-1108

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I. Problems and Promise of HIB drivers for ICF

For several years a small but tenacious community of accelerator physicists have urged consideration of heavy ion accelerators as inertial fusion drivers [1-9]. An induction linac would be used to accelerate approximately $1.0E5$ Amperes of heavy ions ($130 < A < 208$) to 10 GeV energies to obtain the required energy and power on target. Although this technology offers high efficiency ($>25\%$), high rep-rate and good reliability and lifetime, the projected cost has been prohibitive (\$1-2 B). Several new pieces of information have recently converged, however, to motivate us to take a fresh and serious look at this technology.

- o In the Single-Beam Experiment LBL has propagated larger currents down a single alternating-gradient focusing-quadrupole channel than the stability theory indicated would be possible [10].
- o The MBE-4 experiment, also at LBL, has demonstrated stable simultaneous propagation of four current-limited beams, side-by-side, down the center of a single row of induction cores [11].
- o Also in MBE-4, rapid bunching (current amplification) exceeding $\Delta\beta/\beta = .2$ has been demonstrated without losing tune.
- o An intense metal ion source has been developed that can produce high currents of heavy ions with high efficiency in single charge states [31].

These experimental results have greatly increased confidence that multiple beamlets of heavy ions can be accelerated in a single large induction linac to obtain very large total currents.

These charged particles must be focused to spot sizes of several millimeters from a distance of 8-10 meters. Ballistic focusing of such space-charge-dominated beams in a vacuum is probably only feasible for low charge states ($Z = +1$ or $+2$). Recent studies of beam neutralization using either a background gas at $1.0E-3$ torr, or co-moving injected electrons, show that neutralization fractions of greater than 90% are both necessary and possible for the higher charge states which are thought to reduce cost [12,13].

On the target physics front, it has been noted that heavy ion deposition can yield high x-ray conversion efficiencies that increase with increasing intensity on target, opposite to the case with lasers. This fact, coupled with the projected classical nature of heavy-ion stopping in dense plasmas, is very favorable to the design of hohlraum targets [14,18].

Armed with the new information that substantially higher currents may be accelerated, focussed on target, and converted to soft x-rays, LBL researchers sought to redesign conceptual linear induction accelerators. They found that costs could be halved by operating at higher charge states ($Z = +3$), principally because operating at a higher current allowed a concomitant reduction in accelerating potential, and hence accelerator length and cost, without incurring a prohibitive rise in current transport cost [15,16].

The present LBL view of a heavy ion driver for an ICF powerplant is reflected in the performance and cost parameters given in Table 1.1 [15-17]. Also, a comparison is provided of costs normalized to a linear induction accelerator with which we have direct experience, ATA [29].

We note that the projected costs are still prohibitively high, though in the range of some proposed laser systems. Then why do we think that the costs can be reduced, particularly since the cost per meter and cost per megavolt are already below those of ATA? The following points are pertinent.

- o ATA is a research accelerator, first of its kind. It has ferrite cores, which are more expensive than Metglas. It is designed for short pulse operation over its entire length, and for high average power. A HIB does not need kilohertz rep rates; it can use much cheaper pulsed power over 90% of its length.
- o The LBL conceptual designs may still be far from optimum. The heavy ion beam community is quite conservative; their strategy seems to be to slowly creep up in current and down in cost, in order to stay very close to today's technology and maximize credibility. A bolder complementary approach is only recently being pursued: to set ambitious cost goals, ask what innovations are required to reach these goals, and define development programs to contain the risk. There are good indications that much higher currents can be accelerated in shorter devices, that lighter ions at lower kinetic energies are favorable, and that some technology innovation can make a big impact compared to present conceptual designs.
- o A HIB driver for a high gain test facility would have different requirements than the power plant driver of Table 1.1. Many of these requirements would lead to reduced cost. For example, 0.1 hertz operation for a 5.0×10^6 shot lifetime is adequate. Since efficiency is not important except as it affects capital cost, perhaps pulsed focusing quadrupoles, driven by cheap Nova flashlamp circuits, could replace superconductors. Perhaps cheaper cores

Table 1.1 Performance and cost parameters for a HIB driver for an inertial fusion power plant. The overnight construction costs are for a $1.0E9$ shot lifetime, tenth of kind facility, in 1986 dollars. No contingency, engineering and design, project management or time-related costs are included [15-18,29].

Performance

Ion charge and mass	$Z = +3, A = 200$ amu
Ion kinetic energy	13.24 GeV
Driver energy	6.57 MJ
Target gain	183
Yield	1200 MJ
Rep-rate	2.5 Hz
Fusion power	3000 MW
Accelerating potential	4.41 GV
Driver length	2.57 km
Driver efficiency	29%
Efficiency-Gain product	53

<u>Driver Costs</u>	<u>\$M</u>
Front end (<50 MV)	160
Main accelerator	750
Final transport and focus	90

Total Driver Cost	\$ 1000M

<u>Normalization with ATA Accelerator</u>	<u>HIB</u>	<u>ATA</u>
Ave. accelerating gradient, MV/m	1.72	0.55
Cost per unit length, \$K/m	389	555
Cost per potential, \$K/MV	227	1000
Cost per energy, \$/joule	152	---

than Metglas can be devised. Can solid-state switching be used for these pulse durations and pulse rates? A factor tending to raise costs, on the other hand, is that this accelerator would be a first-of-a-kind device, rather than tenth-of-a-kind, so that there is little benefit from mass production.

This study was initiated to identify the high leverage areas for reducing the cost of a 10 MJ heavy ion beam driver for a high gain target development facility. Our efforts to innovate to reach affordable cost have been mostly successful, in that it looks like the \$500 M range may indeed be possible. We conclude that heavy ion beams do have substantial promise for an inertial fusion driver. However, the pace of R&D would have to be substantially increased to realize this promise on a timescale necessary for a High Gain Test Facility.

11. Target and Final Focusing Requirements

Our goal is to develop a driver for a target development facility; a driver which will ignite inertial fusion targets with energy gains greater than 100. The driver energy and power needed are set by the implosion physics to minimize the risk of failure of achieving high gain. A cost goal of approximately \$500M is assumed; this is simply an affordability assessment. Together these requirements make driver development extremely challenging. In Table 2.1 we list a minimum set of requirements on the driver, target and focusing geometry for a high gain target development facility. From this set we can infer other parameters of interest. For example, the driver power is in the 500TW range. If we imagine two sided illumination of a radiation driven target, so that a multitude of driver beamlets are focused on two spots, we can achieve intensities of in the range of $1.0E14$ to $2.0E15$ watts/cm² on each spot. For ion beams even higher intensities are favored because x-ray conversion efficiency increases with intensity [14].

Table 2.1. Target and final focusing requirements for achieving high gain with high confidence.

Beam energy	E	= 5-10 MJ
Pulse duration	τ_p	= 10-30 ns
Target radius	r_t	= 3-5 mm
Ion range in target	R_t	= 0.05-0.2 gm/cm ²
Chamber radius	r_{ch}	= 8 m

Given the constraints of Table 2.1, a wide latitude of choice is available in ion charge and mass, and in number of ions versus kinetic energy per ion, to find a design point that minimizes cost.

In conceptually designing a heavy ion beam accelerator it is easy to lose track of the fact that we are not trying to deliver charge; charge is only a necessary evil in the acceleration and focusing process. We want to deliver a high flux of particles to a small spot. To fix ideas, we will give some characteristic magnitudes for the accelerator parameters of interest.

The total driver energy may be expressed as

$$E = NT = IV\tau = 10 \text{ MJ} \quad (2.1)$$

where N = total number of particles = $6.25E15$
 T = kinetic energy per ion = 10 Gev
 I = total current = $1.0E5$ Amps
 V = total accelerating potential = $3.33E9$ volts
 τ = final pulse duration = 30 nsec

This beam would have a velocity of $9.78E7$ m/sec, so that the relativistic parameters are $\beta = v/c = 0.326$ and $\gamma = 1.0$ (i.e. the beam is nonrelativistic). Noting from the ratio of kinetic energy to voltage potential that the assumed charge state is $Z = +3$, we can also calculate that the total accelerated charge is $Q = ZeN = 3.0E-3$ coulombs. In layman's terms, a heavy ion beam accelerator is a device which will hurl a cloud of particles weighing less than a speck of dust (2 nanograms), at one-third the speed of light, into a spot the diameter of a pea located two car lengths away.

Final Focusing Constraints

In order to hit the target, it is essential that the beam be substantially neutralized or the space charge forces will cause the beam to blow up to many times the target radius. In the space charge dominated regime, and in the absence of externally applied focusing, one can show that the key controlling parameter is the perveance per beam, which is essentially the ratio of beam current to the cube of the ion velocity [19,20]. The normalized, or nondimensional perveance is

$$Q = \frac{2I}{(\beta\gamma)^3 [B\rho]} \left(\frac{1}{4\pi\epsilon_0 c^2} \right) \quad (2.2)$$

where $\beta\gamma$ is a measure of the beam speed and $[B\rho]$ the beam stiffness. In order to separate out the dependence on charge and mass, it is useful to rewrite the perveance using the following substitutions

$$\beta\gamma = \left[\frac{2ZeV}{Am_p c^2} \right]^{1/2} \quad (\text{non-relativistic}) \quad (2.3)$$

and

$$[B\rho] = \frac{\beta\gamma Am_p c}{Ze} \quad (2.4)$$

where Z = charge number
 e = electronic charge = $1.6E-19$ coulombs
 A = mass number
 m_p = mass of a proton = $1.67E-27$ kg
 c = speed of light = $3.0E8$ m/s
 ϵ_0 = permittivity constant = $8.85E-12$ F/m
 V = accelerating potential, volts

Substitution of these into equation (2.1) gives for the perveance per beam

$$Q_b = \frac{I_b}{V^{1/2}} \left(\frac{A m_i}{22e} \right)^{1/2} \frac{1}{4\pi\epsilon_0} \quad (2.5)$$

We envision propagating a beam in a background gas of $1.0E-3$ to $1.0E-2$ torr in order to get substantial neutralization of the ion beam from the background plasma [12]. Alternatively, co-injected electrons moving at the same speed as the ions are being considered. Either way the original current per beam I_{b0} will be reduced. If f is the fraction of neutralization, the net current $I_b = I_{b0}(1-f)$ should be used in the perveance formula.

Note that a limitation on perveance per beam translates to a limit on current per beam that increases with the three-halves power of the beam voltage, or equivalently, with the cube of the particle velocity. We shall now determine the perveance limit of a space-charge limited beam focused on a distant target and propagating through a partially neutralizing medium.

Following Lee [20,21], we note that the envelope equation for a beam of radius a responding only to its own space charge is

$$\frac{d^2 a}{ds^2} = \frac{Q_b}{a} \quad (2.6)$$

The minimum radius achievable (the target radius) resulting from a solution of this equation is

$$r_t = a_{\text{lens}} \exp(-\theta^2/2Q_b) \quad (2.7)$$

where $a_{\text{lens}} = R_{\text{ch}} \theta$. Generally, higher order effects in magnetic focusing lenses limit the maximum convergence half-angle θ to 15-20 milliradians, or lens diameters to 15-20 centimeters for a chamber of the order of 10 meters radius. The perveance limit per beam is then

$$Q_b \leq \frac{\theta^2}{2 \ln \left(\frac{R_{\text{ch}} \theta}{r_t} \right)} \quad (2.8)$$

which is only weakly dependent on target and chamber size, and strongly dependent on the maximum focusing half-angle of the final ion optics.

Because the perveance of each beam is limited, we must divide the total current of the accelerator I into a number of beams, $n_b = I/I_b$, to focus all the particles on target. Combining the above equations allows us to solve for this number of beams.

Since

$$Q_b = 6.5 \times 10^5 \left(\frac{A}{Z} \right)^{1/2} \frac{I(1-f)}{n_b V^{1/2}} \leq \frac{G^2}{2 \ln(R_{ch} G / r_c)} \quad \text{in mks units} \quad (2.9)$$

and since the total beam power is $P=IV$, the number of required beams is

$$n_b \geq 1.3 \times 10^6 \left(\frac{A}{Z} \right)^{1/2} (1-f) \frac{P}{V^{3/2}} \frac{\ln(R_{ch} G / r_c)}{\theta^2} \quad (2.10)$$

To reduce cost and complexity, we would like to minimize the number of necessary beams. From equation 2.10 we see that, from the standpoint of focusing, it is desirable to use a high voltage accelerator focusing at large convergence angles with a high degree of neutralization. In Figure 2.1 we graph the number of focusable beams as a function of final accelerator voltage for three degrees of neutralization. For the purposes of illustration, we have chosen a beam power of 500 TW, an ion of $Z=+3$ and $A=200$, an 8 meter radius chamber with 15 mrad final optics, and a target of 4 mm radius. Note how important it is to get near 90% neutralization to keep the number of beams down to manageable numbers.

In order to put this information in a form useful for accelerator design it is best to display it on a cost-performance map. We can determine the final-focusing constraints on both the ion (Z/A) and the beam voltage V by rearranging equation 2.10 to relate Z/A and V . First, note that the logarithm term doesn't vary much, so we simply evaluate it with the above parameters.

We can then solve for the accelerator voltage required to focus

$$V \geq 455 \left[\frac{(1-f)P}{\theta^2 n_b} \right]^{2/5} \left(\frac{Z}{A} \right)^{-1/5} \quad (2.11)$$

This formula is graphed in the accelerator performance map of Figure 2.2 where it can be seen that the final focusing constraints essentially put a hard lower bound on the acceleration potential - too low a voltage and one cannot focus on target.

Figure 2.1 The number of beams required to focus on target is a strong function of voltage. A neutralization greater than or equal to 90% is likely to be needed.

Figure 2.2 An accelerator performance map showing that, given a choice of ion, the number of beams required to focus decreases with increasing accelerator voltage.

Range Constraints

In Table 2.1 we gave the range of an ion in the target plasma ($0.05 < R < 0.2 \text{ gm/cm}^2$) that is desired to achieve a near-ideal radiator at the power levels of interest.

A graph of ion range in hot dense matter is given in Figure 2.3, for a selection of ions, as a function of the ion kinetic energy. For the heavy ions of interest ($A > 84$) and ranges of

interest the curves are well fit by the formula

$$R = (4.51 \times 10^{-13} \frac{\text{gm}}{\text{cm}^2}) \left(\frac{ZV}{A} \right)^{3/2} \quad (2.12)$$

where V is expressed in volts, as always. In this form, we can see that it is possible to put the target range constraint on the same accelerator performance map in (Z/A vs V) space. This is done in Figure 2.4, where we can see that the design of the accelerator is already quite constrained.

It is useful to recognize that the target range constraint, by a interesting coincidence, also fixes the ion velocity. Because

$$\beta = \frac{v}{c} = \left(\frac{2ZeV}{A m_p c^2} \right)^{1/2} = 4.61 \times 10^{-5} \left(\frac{ZV}{A} \right)^{1/2} \quad (2.13)$$

fixing the range is equivalent to fixing the velocity. The dependence on Z, V, A is identical. A range of $R=0.1 \text{ gm/cm}^2$ corresponds to $\beta = 0.279$. Therefore all of the heavy ion accelerators we conceptualize, having different ions accelerated to different potentials, will end up with an output velocity somewhere between one-fourth and one-third of the speed of light.

Returning to the performance map of Figure 2.4, we are now free to design accelerators that fit within the operating space shown. In the next sections we will vary the ion (Z, A) and the accelerator design (I vs V) to find the lowest cost combination within the allowed operating space.

Figure 2.3 The range of ions in hot dense aluminum increases with increasing ion energy. [from LLNL]

Figure 2.4 A performance map showing the allowed operating region defined by final focus and ion deposition constraints.

III. Current transport in linear induction ion accelerators

An induction linac ion accelerator consists of a large number of closely packed beamlets travelling in parallel through a long row of induction cores [20-21]. The bunches of charge are accelerated simultaneously as they pass through a series of potential differences. This is illustrated in Figure 3.1. The voltage across each acceleration gap is supplied by an inductive circuit that is dominated by a massive ferromagnetic core. The voltage, supplied by a pulse forming network, appears as a steady potential difference across the gap until the core is magnetically saturated, at which time the current starts to rise exponentially and the pulse is terminated [22-25]. The pulse duration is timed to coincide with the transit time of the beamlets.

Each beamlet is transported in a channel consisting of an alternating focusing-defocusing system of electric or magnetic quadrupoles. The separate channels are stacked radially in a close-packed geometry, in a vacuum of approximately 1.0×10^{-8} torr. This arrangement of channels and quadrupoles is called a transport lattice because of its periodicity in all directions.

The transport system is required to apply centering and restoring forces to each beam as it is accelerated. In a properly designed and tuned transport lattice, the normalized emittance is a constant of the motion [30]. There is an excellent and instructive analogy to laser amplifier chains here. In a properly designed master oscillator-power amplifier chain, the beam is alternately focused, defocused and amplified so that an image is propagated down the chain. If this "image relaying" is performed correctly no additional diffraction is introduced. The analogy between emittance growth in ion optics and diffraction in photon optics is useful.

Transport Equations

The particle paths in an ion transport channel are shown in Figure 3.2, both for a single particle beam, and a beam with a high current density. In general, a single particle will make a long wavelength betatron oscillation around the centerline, taking perhaps a half dozen periods of the focusing-defocusing system for one period of oscillation. Accelerator physicists measure this by the tune, σ_0 , which is the phase advance per lattice period. A tune of $\sigma_0 = 60$ degrees means there are six (360 degrees/ 60 degrees) lattice periods per betatron wavelength. The tune is

$$\sigma_0 = \frac{\eta [B' \text{ or } E'/v] L^2}{[B\rho]} \quad (3.1)$$

$$\oint \mathbf{E} \cdot d\mathbf{l} = - \frac{\partial}{\partial t} \mathbf{B} \cdot d\mathbf{A}$$

$$\Delta V = \frac{\Delta B_{\text{core}} A_{\text{core}}}{\tau_{\text{pulse}}}$$

$$\Delta \text{K.E.} = I_{\text{beam}} \Delta B_{\text{core}} A_{\text{core}}$$

Figure 3.1 The principle of an induction accelerator is illustrated by this schematic of ATA. In a heavy ion beam accelerator, several beams of ions are accelerated in parallel instead of a single electron beam. [from LLNL]

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The tune is the phase advance per lattice period

$\sigma_0 = 60^\circ$ means there are 6 lattice period lengths per betatron wavelength

$$\sigma_0 = \frac{\eta B' L^2}{\left(\frac{mv}{q}\right)} = \frac{\text{focusing strength}}{\text{beam stiffness}}$$

Figure 3.2 The effect of space charge on particle paths in a transport channel. Space charge increases the betatron wavelength, depressing the phase advance per lattice cell. [from LBL]

where

- L = half period length of the lattice
- η = occupancy factor = (length of quadrupoles)/ L
- B', E' = quadrupole field gradient
- v = particle velocity

The particle rigidity is given by

$$[B\rho] = \frac{\beta\gamma A m_p c}{Ze} \quad (3.2)$$

and the relativistic factor approximated by

$$\beta\gamma = \left(\frac{2ZeV}{A m_p c^2} \right)^{1/2} \quad (3.3)$$

From equation 3.1 we see that the tune is increased (betatron wavelength decreased) for stronger focusing systems or slower, less rigid beams.

When space charge density is large, and defocusing forces are significant, the betatron wavelength is increased, since the outward space charge force continuously acts to counter the quadrupole focusing force. This is referred to as depressing the tune. As the current is increased, and the tune depressed, a limit is reached where the transport channel can no longer carry the current in a stable manner. This limit was first derived by A. Maschke.

The depressed tune is

$$\sigma^2 = \sigma_0^2 - \left(\frac{2L}{a} \right)^2 Q \quad (3.4)$$

where a = the radius of the ion beam and Q is the dimensionless perveance per beam previously encountered in Section II:

$$Q = \frac{2I}{(\beta\gamma)^2 [B\rho] (4\pi\epsilon_0 c^2)} \quad (3.5)$$

The normalized emittance of the beam is related to the depressed tune by

$$\epsilon_n = \beta\gamma\epsilon = \frac{\beta\gamma\sigma a^2}{2L} \quad (3.6)$$

The three equations 3.1, 3.4 and 3.6 can be solved for the maximum transportable current per beamlet, I_b , the half-period length L , and the beam radius a . The results are:

$$I_b = (2\pi\epsilon_0 c^2) \left(\sigma_0^2 - \sigma^2 \right) \left(\frac{\epsilon_n}{\sigma} \right) \left[\frac{A m_p c \eta B'}{Ze \sigma_0} \right]^{1/2} (\beta\gamma)^{3/2} \quad (3.7)$$

$$L = \left[\frac{\beta \delta A m_p c \sigma_0}{\eta B' z e} \right]^{1/2} \quad (3.8)$$

$$a = \left(\frac{2 \epsilon_0}{\sigma} \right)^{1/2} \left[\frac{A m_p c \sigma_0}{\eta B' z e \beta \gamma} \right]^{1/4} \quad (3.9)$$

These equations are somewhat different from those of Lee, because he assumes that $B'a = B$, which results in a somewhat different functional form for the transportable current. We take

$$B' = B_q/b = \text{constant},$$

where B_q is the maximum field at the pole tip and b is the inner radius of the quadrupole bore.

Note that as the speed of the particles increases along the accelerator, the maximum transportable current and the required half-period length both increase, and the beam radius decreases, as

$$I_b \sim (\beta \delta)^{3/2} \quad L \sim (\beta \delta)^{1/2} \quad a \sim (\beta \delta)^{-1/4} \quad (3.10)$$

if $\eta = \text{constant}$. However we are also free to vary the occupancy fraction of quadrupoles η along the accelerator. Remember that (ηL) is the physical length of the quadrupoles. A particularly simple and elegant special case arises if instead of keeping η constant we decrease the occupancy fraction as the speed of the particles increases, such that $\eta \beta \delta = \text{constant}$.

From an inspection of the above equations we see that this new condition makes the length of the quadrupoles and the beam radius constant over the entire length of the accelerator. This would simplify magnet manufacturing requirements enormously; we could produce thousands of identical parts, significantly driving down the cost per part.

The major transport parameters will then vary as follows:

$$\eta \sim (\beta \delta)^{-1} \quad I \sim \beta \delta \quad L \sim \beta \delta \quad \eta L = \text{const.} \quad a = \text{const.} \quad (3.11)$$

In subsequent analysis we shall use the cumulative accelerating potential V as the independent variable, as well as the ion charge and mass numbers. We therefore replace $\beta \delta$ in equations (3.7-3.9) by equation (3.3) and evaluate the physical constants using SI units. We then obtain

$$I_b = (2.78 A) (\sigma_0^2 - \sigma^2) \left(\frac{\epsilon_0}{\sigma} \right) \left[\frac{\eta B' \sqrt{V}}{\sigma} \right]^{1/2} \left(\frac{Z}{A} \right)^{1/4} V^{1/4} \quad (3.12)$$

$$L = (1.20 \times 10^{-2} \text{ m}) \left[\frac{\sigma_0}{\eta_i B' \sqrt{V_i}} \right]^{1/2} \left(\frac{Z}{A} \right)^{-1/4} V^{1/2} \quad (3.13)$$

$$a = (22.8 \text{ m}) \left(\frac{\epsilon_n}{\sigma} \right)^{1/2} \left[\frac{\sigma_0}{\eta_i B' \sqrt{V_i}} \right]^{1/4} \left(\frac{Z}{A} \right)^{-3/8} \quad (3.14)$$

$$\eta = \eta_i \left(\frac{V}{V_i} \right)^{-1/2} \quad (3.15)$$

where the subscripts on the occupancy factor indicate the initial values at the entrance to the accelerator (e.g. $\eta_i = 0.9$ at $V_i = 2 \text{ MV}$).

Let us evaluate the current, half-period length, beam radius and quadrupole length at the end of the accelerator to get a feel for the physical magnitudes, using the parameters given in Table 3.1, typical of a heavy ion beam accelerator.

Table 3.1 Typical heavy ion beam parameters

$\epsilon_n = 1.0\text{E-}5 \text{ m-rad}$	$Z = +3$
$\sigma_0 = 80^\circ = 1.40 \text{ rad}$	$A = 200$
$\sigma = 8^\circ = 0.140 \text{ rad}$	$V_i = 2.0\text{E}6 \text{ volts}$
$B' = 2 \text{ Tesla}/0.15\text{m} = 13.3 \text{ T/m}$	$V_f = 3.3\text{E}9 \text{ volts}$
$\eta_i = 0.9$	

We then calculate that at the end of the accelerator we would have

Current per beam	$I_b = 1700 \text{ A}$
Half period length	$L_f = 17.9 \text{ m}$
Beam radius	$a = 8.84 \text{ cm}$
Occupancy factor	$\eta_f = 0.022$

It should be clear from equation (3.7) that one wants to operate at the largest possible tune and the smallest possible depressed tune to maximize the current per channel. Tiefenback and Keefe [1.] have experimentally determined that $\beta_L = 80$

and $\sigma = 8$ are acceptable values allowed by beam stability. A stability map is shown in Figure 3.3.

Returning to the above example, notice that the beam fills the fraction $a/b = 60\%$ of the quadrupole bore and that the length of each quadrupole ($\eta L = 0.396$ m) is constant over the entire length of the device. Let us rewrite the expression for the length of a quadrupole to examine the parameter dependence:

$$\eta L = (1.20 \times 10^{-2} \text{ m}) \left(\frac{\eta \sigma}{B'} \right)^{1/2} \left(\frac{AV_i}{Z} \right)^{1/4} \quad (3.16)$$

To minimize the length and therefore the mass and cost of the quadrupoles, we want to increase the magnetic field gradient B_q/b and use lighter, more highly charged ions.

Number of Beams Required

The accelerator must provide a higher current than can be carried by a single transport channel. Since the total desired current is the heavy ion beam power divided by the final voltage, the number of transportable beams that need to be stacked radially in the transport lattice is

$$\eta_{tb} = \frac{I}{I_b} = \frac{E}{I_b V_f \tau_f} = 23.6 \frac{E}{\tau_f} \left(\frac{Z}{A} \right)^{-1/4} V_f^{-1/2} \quad (3.17)$$

for the parameters of our example accelerator.

The number of transportable beams decreases rapidly as the final voltage is increased; at a fixed voltage, the required number of beams is slightly less for lighter ions with higher charge states. Although the scaling with charge to mass ratio may seem counter-intuitive, note that the number of required beams also scales like

$$\eta_{tb} \sim \left(\frac{Z}{A} \right)^{5/4} (\beta\gamma)^3 \quad (3.18)$$

so that, if a particular final speed is desired, a larger number of beams is needed for higher (Z/A) ions, as expected.

The number of transportable beams scales differently than the number of focusable beams. Although both are space-charge dominated, a ballistically focused beam is free of confining quadrupoles and will therefore have a stronger dependence on voltage. To illustrate this, we have graphed in Figure 3.4 the number of transportable beams as an overlay on Figure 2.4, which illustrated the number of focusable beams. We see that for a given ion there is only one voltage for which the number of transportable beams equals the number of focusable beams. All

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Figure 3.3 We want to operate at the highest tune σ_0 and lowest depressed tune σ allowed by experimental stability.

Figure 3.4 A comparison between the number of transportable beams required and the number of beams required to focus. In general the number of transportable beams is greater, so that further beam splitting will not be necessary in order to focus.

other operating points will require beam splitting or beam combining in the final focusing region. There is an advantage to avoiding such beam manipulations, since they increase the emittance and add additional magnets, length, and cost to the accelerator. Because the number of transport beams usually exceeds the number required to focus it appears that no further beam splitting is necessary to focus on target.

Beamlet Stacking Geometry

The number of beamlets needed to transport a given amount of charge directly affects the size of the cores needed to accelerate the charge. Imagine that we stack n_b beamlets in a close-packed geometry as shown in Figure 3.5. The inner radius of the surrounding core is then determined by the smallest circle that will hold the stack of quadrupoles. Define the following geometrical factors:

n_b = number of beamlets (or quads) per core

a = radius of each beamlet

b = bore radius of a quadrupole

b_0 = outer radius of a quadrupole

R_i = inner radius of surrounding core

$\xi = b_0/a$ = ratio of outer quadrupole radius to beam radius

The area packing fraction of a large number of hexagonally close-packed small circles inside a large circle is 75%. Therefore

$$\text{Packing fraction} = 0.75 = n_b \left(\frac{\pi b_0^2}{\pi R_i^2} \right) \quad (3.19)$$

Solving for the inner core radius we have

$$R_i = 1.15 n_b^{1/2} b_0 = 1.15 n_b^{1/2} \xi a \quad (3.20)$$

showing that the core size is proportional to the beamlet size and the square root of the number of beamlets. These quantities are given in equations 3.14 and 3.17 respectively. Combining these we get for the inner core radius:

$$R_i = 15.7 \xi \left[\frac{E \sigma_0}{(\sigma_0^2 - \sigma^2) \eta_i B' V_i \tau_f} \right]^{1/2} \left(\frac{A}{Z} \right)^{1/2} V_f^{-3/4} \quad (3.21)$$

Reducing the final voltage of the accelerator from 10GV to 3.3GV increases the core radius from 0.53 to 1.22 meters.

Figure 3.5 Many beamlets are accelerated simultaneously; the core radius required varies as the square root of the number of beamlets.

Remember that our goal is to calculate the total volume of induction cores and total volume of magnetic quadrupoles in order to estimate their cost. To calculate the core volume we need to find the outer dimensions of the core, and the number of cores in the entire accelerator. To calculate the number of quadrupoles we need to know the length of the accelerator and the distribution of quadrupoles along the accelerator. Both of these calculations depend critically on the acceleration process and its physical limits, which we investigate in the next section.

IV. Acceleration and Final Transport

Our objective in this section is to examine the physical effects which constrain the maximum allowable voltage gradient in different regions of the device, since the voltage gradient has the greatest effect on the length and cost of the accelerator. We will calculate the accelerator length, and then the length of the final transport region.

Surprisingly, the voltage gradient is not limited by the amount of volt-seconds supplied by the induction cores. If this were the only limit we could make an induction linac as short as 100 meters, consisting entirely of closely packed cores one meter thick. The two major constraints on accelerator length and cost that effectively limit the voltage gradient of the device are the rate of current amplification (beam bunching) and the breakdown threshold of insulators. Both constraints are a function of the pulse duration of the ion bunch, or equivalently, of the duration of the applied high voltage.

An inductive core of cross-sectional area A_c and total magnetic field swing ΔB_c will supply a voltage increment ΔV for a time τ as follows [25]:

$$\tau \Delta V = \Delta B_c \cdot A_c \quad (4.1)$$

The area of the core is

$$A_c = (R_o - R_i) \alpha dz \quad (4.2)$$

where α = the fraction of the length taken up by solid core material. Combining equations 4.1 and 4.2 we can solve for the thickness of core required to obtain a given voltage gradient

$$(R_o - R_i) = \frac{\tau}{\alpha \Delta B_c} \left(\frac{dV}{dz} \right) \quad (4.3)$$

Since we already know how to calculate the inner core radius required to encompass the quadrupoles, we can use equation 4.3 to solve for the outer radius of the cores. Because the core thickness is proportional to pulse width, and the voltage gradient is a function of pulse width, the core thickness needed will vary along the accelerator as the pulse is compressed.

Let us now direct our attention to the limitations on voltage gradient.

Insulator Flashover Limit

High voltages can cause insulators in a vacuum to breakdown along their surfaces, due primarily to field emission and field enhancement on the highly stressed surfaces [26]. Even with properly shaped surfaces, grading rings, vacuum bakeout of impurities, surface treatment, etc., insulators are a terrible weakpoint in accelerator design. Humphries [24, p 229] says that steady-state (>1 microsec.) operating levels for insulators are in the 2 MV/m (20kV/cm) range, whereas short pulse (<100 ns) breakdown levels are in the 10-15 MV/m (100 kV/cm) range. Conservative design practice suggests selecting operating levels a factor of two to or more below this.

We are going to parameterize the insulator breakdown field as follows

$$\text{Steady state } (\tau \geq 1 \mu\text{sec.}) \quad \phi_{ss} = 0.5 \text{ MV/m} \quad (4.4a)$$

$$\text{Short pulse } (\tau < 1 \mu\text{sec.}) \quad \phi(\tau) = \phi_{ss} \left(\frac{\tau}{10^{-6} \text{ sec.}} \right)^{-1} \text{ MV/m.} \quad (4.4b)$$

This leads to a 100 ns breakdown threshold that is an order of magnitude above the steady-state threshold. Later, we will examine the effect on cost of varying ϕ_{ss} . These vacuum insulator breakdown thresholds are graphed in Figure 4.1, along with other thresholds about to be discussed.

Bunching Rate Limit

C.H. Kim [27,28] and others have pointed out that there is a limit on how fast one can increase the current by accelerating the tail of the ion bunch more than the head. Too great a differential acceleration, or "velocity tilt" as it is called, will disrupt the longitudinal stability and will cause the tail of the pulse to have a disproportionate size and therefore a different tune than the head, interfering with good transport.

The limit on velocity tilt is usually taken to be $\Delta\beta/\beta = 20\%$, although higher tilts have already been achieved on MBE-4. Until we have better experimental data, we shall limit the rate of change of current with distance to conform to such a limit. The relationship between current amplification and velocity tilt is given by Kim to be

$$\frac{l}{I} \frac{dI}{dz} \leq \left(\frac{\Delta\beta}{\beta} \right)_{\text{max}} \quad (4.5)$$

where $l = \beta c \tau$ is the length of an ion bunch.

Figure 4.1 The maximum voltage gradient is limited by bunching rate early in the accelerator, then by insulator breakdown over the majority of the device.

In Section III. we discussed a mode of transport in which the current rises as the voltage to the one-half power; other modes are possible depending on the way the quadrupole occupancy varies along the accelerator. A power law form fits all cases of interest though, so let us express the relationship between current, pulse duration, and voltage in the form

$$\frac{I}{I_f} = \left(\frac{V}{V_f}\right)^m, \quad \frac{\tau}{\tau_f} = \left(\frac{V}{V_f}\right)^{-m} \quad (4.6)$$

where the subscript f indicates final conditions, as usual. Then

$$\frac{dI}{I} = -\frac{d\tau}{\tau} = m \frac{dV}{V} \quad (4.7)$$

and the limit on voltage gradient is

$$\frac{dV}{dz} \leq \frac{V}{ml} \left(\frac{\Delta\beta}{\beta}\right)_{\max} \quad (4.8)$$

The bunch length can be expressed in other terms as follows

$$l = v\tau = \left(\frac{2ZeV}{Amp}\right)^{1/2} \tau \quad (4.9)$$

Now by combining equations 4.6 through 4.9 we can determine the voltage gradient limit as a function of pulse duration:

$$\left(\frac{dV}{dz}\right)_{\text{bunching}} = \left(\frac{\Delta\beta}{\beta}\right)_{\max} \frac{1}{m\tau} \left[\frac{AmpV_f}{2Ze}\right]^{1/2} \left(\frac{\tau}{\tau_f}\right)^{-(1+\frac{1}{2m})} \quad (4.10)$$

For our current transport mode $m = 1/2$, so the voltage gradient varies as τ^{-2} . This is graphed on the Figure 4.1, for the accelerator parameters given in Table 3.1, and a final pulse duration of $\tau_f = 100$ ns. Clearly the bunching limit, not insulator breakdown threshold, is the determining constraint early in the acceleration, for $\tau \leq 300$ ns.

Volt-seconds Limit

We need to check to see if there isn't indeed a limit on velocity gradient due to the amount of core material we can provide, both radially and per unit length. Recasting equation 4.3 as follows

$$\frac{dV}{dz} \leq \alpha \frac{(R_o - R_i)_{\max}}{\tau} \Delta\beta_c \quad (4.11)$$

and using values of $\alpha = 0.5$, $(R_1 - R_2) = 1$ meter, and $\Delta E = 2.4$ Tesla characteristic of large Metglas cores, we see that the core limit on velocity gradient is very generous, and that the bunching limit is more constraining. We can now feel confident that our accelerator will operate at the maximum voltage gradient over its entire length.

Our Objective Revisited

At this point it is good to remember where we are going. Our objective is to find the lowest cost way to deposit the required energy on target with a heavy ion beam. We are looking for the ion (Z vs A) and induction accelerator (V, I, n_p) that will minimize the cost within the constraints. We can't think of a better way to approach this goal than to have every inch of the device operate simultaneously at the maximum current carrying limit and at the maximum possible voltage gradient. This leads to the best utilization of expensive hardware.

Accelerator Length

The accelerator can be thought of as having a number of distinct regions corresponding to the dominant physics or engineering constraints, as follows:

- a. Multiple beamlet ion source
- b. Bunching-rate-limited region
- c. Insulator-breakdown-limited region
- d. Final transport and beam compression region
- e. Final focus in chamber

Regions a and e do not have sufficient lengths to significantly affect costs. Region b, in which the acceleration gradient is limited by the bunching rate, is very short and corresponds to voltages between $2 < V < 5.5$ MV. Region c, limited by insulator breakdown, is the majority of the accelerator.

Referring to Figure 4.1, we see that region c actually consists of two subregions corresponding to the steady-state and short-pulse constraints on accelerator breakdown. We should properly calculate the length of each of these regions and subregions independently, since the power-law dependence in each is different. In practice, we found that the total length of the accelerator is quite accurately represented by using the short-pulse insulator-breakdown-limited voltage gradient for the entire accelerator, from 2 MV to the final voltage. In Figure 4.1, this corresponds to using the dotted line all the way to pulse widths of 4 microseconds, instead of following the steady-state breakdown limit and the bunching limit. This enormously simplifies the scaling algebra, while introducing an inaccuracy less than 10%, which is within our cost estimating tolerance.

Proceeding to calculate the length of the accelerator, we substitute the voltage gradient limit of equation 4.4b into an integral for length to obtain

$$L = \int dz = \int \left(\frac{dV}{dz} \right)^{-1} dV = \frac{10^6}{\Phi_{ss}} \int \tau(V) dV \quad (4.12)$$

Using the relation between pulse length and voltage of equation 4.6, and integrating between the initial and final voltage, we get for the length

$$L = \frac{10^6 V_f \tau_f}{\Phi_{ss} (1-m)} \left[1 - \left(\frac{V_i}{V_f} \right)^{1-m} \right] \quad (4.13)$$

For the case of constant quadrupole length and constant beam radius, $m = 1/2$, and $V_i/V_f \ll 1$. The approximate length may then be written

$$L_a = \frac{(2 \cdot 10^6) V_f \tau_f}{\Phi_{ss}} \quad (4.14)$$

For our example conditions, the length of this accelerating region is 1320 meters. The length varies directly with the final voltage and inversely with the maximum allowed voltage gradient.

Final Transport Length

The length of the final transport region is determined by the distance needed to collapse the beam. In exact analogy to the initial bunching region, we give the tail of the beam a slight increase in velocity so that it starts to catch up with the head. The ion beam is bunched in a way that the tail essentially overtakes the head at the center of the chamber. Of course, in reality a great deal of careful manipulation and pulse shaping is done in this region of a few hundred meters. At the same time the beam is split into two clusters of beamlets and steered around to provide two-sided illumination for the target.

To calculate the length of the final transport and drift region, we recognize that the tail, traveling at higher velocity for a longer distance (only by the bunch length), must have the same travel time to the target as the head.

$$\text{Travel time} = \frac{L + \ell}{v_f + \Delta v} = \frac{L}{v_f} \quad (4.15)$$

where v_f = the final velocity and $\ell = v_f \tau_f$ is the bunch length at the end of the accelerator, when $\tau_f = 100$ ns.

Solving equation 4.15 for the length of the final transport region, we obtain

$$L_t = \frac{v_t \tau_t}{(\Delta\beta/\beta)_t} \quad (4.16)$$

where we have substituted $(\Delta\beta/\beta)_t$ for $\Delta v/v$, the maximum allowed velocity tilt in the transport region. This limit will be set by the longitudinal dynamics of the bunch, and the effect of fast beam bunching on the focusability of the final optics. It is currently felt that $(\Delta\beta/\beta)_t < 2-3\%$ is tolerable. Selecting $(\Delta\beta/\beta)_t = 0.025$ leads to a final transport length (of each arm) of about 400 meters. During this entire length the beam is transported at the current transport limit, so the density of quadrupoles must increase as the current increases. There is no net acceleration during the final transport however; the only induction cores needed are the few required to provide the small amount of velocity tilt.

V. Volume of Induction Cores and Quadrupoles

An induction linac ion accelerator is dominated by the induction cores and quadrupoles needed to transport and add energy to the beam. This hardware also dominates the cost. If we have good estimates of the mass or volume of these items, and how they scale with the choice of ion and accelerator voltage, we will be well on our way to understanding what drives ion beam cost.

Volume of Induction Cores

The calculation of total core volume is complicated somewhat by the fact that the outer radius of the cores, and the distribution of cores with distance, varies along the accelerator as the pulse length is decreased. Fortunately the inner radius is constant. The best way to proceed is to perform an integral over voltage, rather than over distance, to sum the core volume.

The differential volume of cores required is

$$d(\text{Volume}) = \pi(R_o^2 - R_i^2) \alpha dz = \frac{\pi(R_o + R_i)(R_o - R_i) \alpha dV}{(dV/dz)_{\max}} \quad (5.1)$$

The core thickness, from the acceleration equation 4.3, is

$$(R_o - R_i) = \frac{\tau(V)}{\Delta B_c} \frac{1}{\alpha} \left(\frac{dV}{dz} \right)_{\max} \quad (5.2)$$

We can then rewrite the differential volume as

$$d(\text{Volume}) = \frac{\pi}{\Delta B_c} \left[2R_i + \frac{\tau}{\alpha \Delta B_c} \left(\frac{dV}{dz} \right) \right] \tau dV \quad (5.3)$$

Substitution of the pulse length from equation (4.6) and the velocity gradient from equation (4.4) and integrating over voltage gives for the volume of cores

$$\text{Volume} = \frac{2\pi V_f \tau_f}{\Delta B_c} \left[2R_i + \frac{10^{-6} d_{11}}{\alpha \Delta B_c} \right] \left[1 - \left(\frac{V_i}{V_f} \right)^{1/2} \right] \quad (5.4)$$

Numerically it turns out that the second term in both parentheses is of second order and can be ignored. In addition, we have not properly accounted for the extra core material needed to provide the final bunching, so we will multiply equation (5.4) by a factor of 1.2. Then the volume of cores is well approximated by the expression

$$\text{Volume} = \frac{15.1 V_f \tau_f R_i}{\Delta B_c} \quad (5.5)$$

which yields a total volume of around 3.0E3 cubic meters for a Metglas core of 1.5 meters inner radius.

To understand the scaling of the core volume, however, we should include the dependence of the core inner radius on the ion charge to mass ratio and accelerator voltage, remembering that as we decrease voltage we must also increase the total current, the number of beamlets, and the core size required to encircle them. From equation (3.21) the inner radius is

$$R_i = 15.7 \xi \left[\frac{\sigma_o E}{(\sigma_o^2 - \sigma^2) \eta_i B' \sqrt{V_i} \tau_t} \frac{A}{Z} \right]^{\frac{1}{2}} V_f^{-\frac{3}{4}} \quad (5.6)$$

Substituting into equation (5.5) we find that the core volume is

$$V_c = \frac{238 \xi}{\Delta B_c} \left[\frac{\sigma_o E \tau_t}{(\sigma_o^2 - \sigma^2) \eta_i B' \sqrt{V_i}} \right]^{\frac{1}{2}} \left(\frac{Z}{A} \right)^{-\frac{1}{2}} V_f^{\frac{1}{4}} \quad (5.7)$$

To attain a given driver energy E , one minimizes the volume of induction cores by decreasing the voltage and increasing the charge to mass ratio. The dependence on voltage is quite weak, since the majority of the length reduction gained by decreasing the voltage is given back by the need to increase core radius as the current is increased.

Volume of Quadrupoles

To calculate the total volume of quadrupoles necessary to carry the ions, we will do an integral similar to that performed in the core volume calculation. This is necessary because the quadrupole occupancy fraction varies with distance (or voltage) in both the accelerator and the final transport regions. In the accelerator section the occupancy fraction decreases from 90% to 2.2% as the voltage increases down the accelerator. In the final transport section the occupancy fraction increases again a factor of 25 as the current is ramped up a factor of 5 (because the pulse length is reduced in the final transport section from 100 ns to 20 ns). In both regions the cross-sectional area of the transport lattice, and therefore the quadrupoles, is constant.

The cross-sectional area of the quadrupole array is just the inner core area less the total area of the bores:

$$\text{Net area} = \pi R_i^2 - n_b (\pi b_o^2) \quad (5.8)$$

But from equation 3.19 we know that the

$$\text{Packing fraction} = 0.75 = \frac{n_b \pi b_o^2}{\pi R_o^2}$$

A reasonable estimate of the ratio of inner bore radius to outer radius of a well designed pulsed magnet is $b_i/b_o = 0.75$. The net cross-sectional area is then

$$A_q = 0.58 \pi R_o^2 \quad (5.9)$$

We will use this cross-sectional area for both the accelerator and final transport sections. In the transition between the two, the beamline will be split, for example, from one 72 beamlet line to two 36 beamlet lines in order to have two opposed beamlines enter the chamber. We will treat it the two final transport lines with half the area as one line at full area.

To get the total volume of quads we need to add up the total length of quadrupoles over both the accelerator (a) and one final transport (t) section

$$\text{Quad Volume} = A_q (L_{qa} + L_{qt}) \quad (5.10)$$

Let's find the total length of the final transport quadrupoles first. This length is

$$L_{qt} = \int \eta(z) dz = \frac{V_t}{(\Delta\beta/\beta)_t} \int_{z_t}^{z_f} \eta_t \left(\frac{z_t}{z} \right)^{-2} dz \quad (5.11)$$

where we have used the fact that the occupancy must vary as

$$\eta \sim I^2 \sim z^{-2}$$

in the final transport section in order to operate at the current limit (see equation 3.7). Performing the integral, we get

$$L_{qt} = \frac{\eta_t V_t z_t}{(\Delta\beta/\beta)_t} \left[\left(\frac{z_t}{z} \right) - 1 \right] = \eta_t \left[\frac{z_t}{z_t} - 1 \right] L_t \quad (5.12)$$

Physically, we find that the final transport line needed to compress the pulse from $\tau_f = 100$ ns at the end of the accelerator to $\tau_t = 20$ ns at the target is 400 m long, of which 35.2 m (or 8.8%) is in quadrupoles. In actuality, there would be two such lines, each having half the cross-sectional area.

In the accelerator region the situation is slightly more complex since the occupancy fraction, pulse length and breakdown strength all vary with distance. The simplest way to calculate the length is to integrate over voltage. The length is

$$L_{qa} = \int_i^f \eta(z) dz = \int_i^f \eta(V) \left(\frac{dV}{dz} \right)^{-1} dV \quad (5.13)$$

Into this integral we substitute the occupancy fraction, breakdown strength, and pulselength as a function of voltage, given here:

$$\eta = \eta_i \left(\frac{V}{V_i} \right)^{-1/2} \quad \tau = \tau_f \left(\frac{V}{V_f} \right)^{-1/2} \quad \frac{dV}{dz} = \phi_{ss} \left(\frac{\tau}{10^{-9}} \right)^{-1}$$

to obtain for the total length of quadrupoles

$$L_q = \eta_f \tau_f V_f \frac{10^6}{\phi_{ss}} \ln \left(\frac{V_f}{V_i} \right) \quad (5.14)$$

This works out to 108 m, or 8.6% of the accelerator. Remarkably, and coincidentally, this is about the same average occupancy fraction as that in the final transport region.

The total volume of quadrupoles can now be calculated by multiplying the cross-sectional area (5.9) by the sum of the accelerator plus final transport lengths to get

$$V_q = \frac{4\pi S^2 \sigma_c E}{(\sigma_s^2 - \sigma_c^2) B'} \left[\frac{A}{Z} V_f^{-1} \left(\frac{10^6}{\phi_{ss}} \right) \ln \left(\frac{V_f}{V_i} \right) + \left(\frac{2e}{m_p} \right)^{1/2} \left(\frac{A}{Z} \right)^{1/2} \left(\frac{\tau_f}{\tau_c} - 1 \right)^{-1/2} \frac{V_f}{\left(\Delta p / p \right)_c} \right] \quad (5.15)$$

This equation looks quite complicated because we have explicitly displayed the dependence of the core radius and final ion velocity on the Z/A of the ion and the final voltage, V , of the accelerator. These are our two independent variables. It is evident that the quadrupole volume is minimized by operating at high voltage, and therefore low current, as expected. At a given voltage and current however, note that it is more favorable to use a multiply charged, lower mass ion.

We now have all the quantities need to develop cost estimates of the ion accelerator. This is done in the next section.

VI. Cost Minimization

Our objective here is to estimate the procurement costs of the family of induction linac ion accelerators that we have designed, and then to find the parameters which minimize cost by varying the choice of ion (Z vs A) and the accelerator (I vs V) within the acceptable operating regime given in Fig. 2.4. It is not necessary or desirable to calculate the total construction cost to optimize the design. We have included only those costs which are both significant and vary significantly with the choice of ion or final voltage in order to find the "sweet spot" in design space. Engineers and designers can then develop point designs and detailed engineering cost estimates of a system that they are confident is near optimum. A list of items we have included and excluded is provided in Table 6.1.

Table 6.1 List of items included and excluded from the hardware cost index used for optimizing an ion accelerator.

<u>Included</u>	<u>Excluded</u>
Quadrupoles	Ion source
Induction cores	Turning magnets in final
Insulators	transport section
Support structure for cores	Final focus magnets
and quadrupoles	Target chamber and its
Outer steel vacuum vessel	building
Cryopanel vacuum pumps	Office building
Power supply and conditioning	Control room
system	Target diagnostics
Concrete pad and roof structure	Overhead and indirect
Distributed controls and	costs
diagnostics	R & D costs
All other conventional	Contingency.
facilities for the accelerator	
and final transport.	

We estimate hardware costs in four major categories that account for the majority of the costs of a heavy ion beam. These are:

1. Quadrupoles
2. Induction cores
3. Length-related costs
4. Power conditioning

The costs of induction cores and pulsed quadrupoles are estimated by simply using the "cost per pound" of fabricated material. Metglas(tm) amorphous iron cores can be purchased, wound and insulated for \$4/lb in large quantities [32]. These cores are principally iron, so the specific gravity is 7.8.

The pulsed electromagnetic quadrupoles consist of copper windings and insulation required to generate the magnetic field, but the weight is dominated by the iron pole pieces and flux return paths which shape and channel the magnetic flux. The specific gravity of iron is also a good estimate for the average density of quadrupole materials. We have estimated that the cost of fabricating hundreds of quadrupoles is \$10/lb.

A reasonable estimate for the length related costs (support structure, concrete pad and enclosure, the cylindrical steel vacuum enclosure, the vacuum cryo-pumping system, distributed controls,etc.) is \$60K/meter [33].

The cost of pulse power is difficult to estimate, because the voltage pulse to be supplied varies from 4 microseconds at the beginning of the accelerator to 100 nanoseconds at the end. There is no satisfactory substitute for actually designing and costing a power conditioning system to do this, but the following method gives an adequate estimate for our purposes. For very long pulses (1 millisecond) characteristic of laser flash lamp circuits, we are confident that \$0.10/joule systems can be built [34]. In the range of a few microseconds pulse power systems can be purchased for \$1-\$2/joule. The cost of very short pulse systems (20-70 nanoseconds) can be estimated from the Sandia Particle Beam Fusion Accelerator (PBFA-II) technology, which is in the \$10-\$20/joule range. These costs are illustrated in Figure 6.1, where we can see that there is a reasonable fit to a cost scaling algorithm of

$$\text{Cost} = 10 \left(\frac{\tau}{10^{-7} \text{ sec}} \right)^{-3/4} \text{ \$ / Joule} \quad (6.1)$$

We believe that this equation estimates the maximum costs; lower costs are certainly conceivable.

The average cost of an accelerator would therefore be between \$1.5 and \$10/electrical joule. To improve this estimate we performed an integral that sums up the energy added to the beam as the voltage, current and pulse length change along the accelerator. This gives us an expression which depends on the initial (τ_i) and final (τ_f) pulselengths, the final energy of the beam (E) and the electrical efficiency of the inductive circuit (η_e). We obtain

$$\begin{aligned} \text{Pulser cost} &= \frac{10}{\eta_e} \int_{\tau_i}^{\tau_f} \left(\frac{\tau}{10^{-7}} \right)^{-3/4} V(\tau) I(\tau) d\tau \quad \text{dollars} \\ &= \left(\frac{40}{11} \right) \frac{V_f I_f \tau_f}{\eta_e} \left[1 - \left(\frac{\tau_i}{\tau_f} \right)^{-1/4} \right] \end{aligned} \quad (6.2)$$

Figure 6.1 The cost of pulsed power increase rapidly as the output pulsewidth is decreased.

We can substitute the total ion beam energy E for $V_f I_f \tau$, and ignore the last term in the brackets because it is much less than unity. A good estimator of the pulser cost is then simply

$$\text{Pulser cost} = \frac{3.6 E}{\eta_e} \quad \$ \quad (6.3)$$

For example, a 10 MJ ion beam that is 25% efficient would have a maximum pulsed power cost of \$144M. Volume production of many components should reduce this cost.

Now we are ready to sum up the major costs of our ion beam driver in our four categories. The cost of the induction cores is determined by the bulk cost times the volume of cores (Eqn. 5.7). The cost of quadrupoles is the bulk cost times the volume of quadrupoles (Eqn. 5.15). The length related cost is the unit cost times the sum of the accelerator and final transport lengths (Eqns 4.14 and 4.16). The pulse power cost is given by equation 6.3 above. The sum of these costs is the hardware cost index which we will use as the figure of merit to be minimized.

In Figure 6.2 we have graphed the hardware cost index for an ion having $Z/A = 0.015$ (characteristic of $^{208}\text{Pb}^{+3}$ or $^{131}\text{Xe}^{+2}$) as a function of the final accelerator voltage. There is a cost minimum between 2 and 2.5 GV, determined by a balance between large quadrupole costs at high current and low voltage, and large length related costs at high voltage and low current. The costs of pulsed power or induction cores are not strong contributors to determining the minimum. We conclude that, to minimize costs, we should: Operate at a final voltage between 2 and 3 GV to optimally balance current-carrying and accelerating costs.

What is the best choice of ion? Is there also an ion choice which minimizes cost? To answer these questions we graph the hardware cost index as a function of the ratio of charge number to mass number in Figure 6.3. The driver energy and accelerator voltage are fixed at 10 MJ and 2 GV respectively. We see that the costs fall rapidly and monotonically as the charge to mass ratio increases. There seem to be diminishing returns after $Z/A > 0.02$ or 0.025, at which the cost index is reduced to below \$40/joule. Note that both the core cost and quadrupole cost are decreasing with increasing charge to mass ratio. That the quadrupole cost is also dropping may be somewhat counter-intuitive, but the reasons are apparent from equation 3.21. A higher charge to mass ratio leads to a small outer radius of each bundle of quadrupoles, or equivalently, to a small inner radius of the induction core.

Figure 6.2 The cost minimum near 2.5GV is determined by a balance between quadrupole cost at high current and long accelerator lengths at high voltage.

Figure 6.3 Costs are reduced with highly charged, lower mass ions (holding voltage fixed). Diminishing returns set in above a $Z/A = 0.025$ or so.

There is a limit to how large the charge to mass ratio may be however. The upper limit is set by ion deposition in an inertial fusion target, as we discussed in Section II. This is best seen in the performance map of Figure 6.4. We can only minimize cost within the allowed region. In voltage we found that there was a cost valley around ($2.0 < V < 2.5$ GV). In the ion charge/mass direction one desires as large a Z/A as possible, which puts us up against the maximum ion range constraint of 0.2 gm/cm^2 , beyond which it is difficult to design targets. This occurs at $Z/A = 0.025$, which is also approximately the onset of diminishing returns in cost reduction. We can state our second important conclusion for minimizing cost: Operate at the largest Z/A allowed by ion deposition in the target. Ions ranging from

$^{208}\text{Pb}^{+5}$ to $^{40}\text{Ar}^{+1}$ might be suitable. The actual choice is probably most dependent on the design of the ion source.

While we now understand how to find the minimum cost operating point for an induction ion accelerator, we have not determined the actual total costs of an ion beam for a High Gain Test Facility. This must be done by expert hardware engineers for a specific point design. But the total costs of the driver can be roughly estimated by adding such things as the ion source and the final focusing optics at the target chamber. These will increase the total cost by several tens of million dollars. This would be added to the \$420 M cost of the 2GV, 10MJ, $Z/A = 0.025$ accelerator taken from Figure 6.3. We can then estimate that a 10 MJ ion beam might have a total hardware cost of roughly \$500M, for a normalized cost of \$50/joule on target. This would make a very interesting 10MJ inertial fusion driver to use for developing a high gain target.

Figure 6.4 A performance map showing the allowed operating region defined by final focus and ion deposition constraints, as well as the preferred operating point defined by minimum cost.

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