178
119/78

1h. 508

ORNL/TM-5804

MASTER

Frequency Analysis of a Thick-Rim Flywheel

D. N. Fanning

OAK RIDGE NATIONAL LABORATORY
OPERATED BY UNION CARBIDE CORPORATION • FOR THE DEPARTMENT OF ENERGY

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

Printed in the United States of America. Available from
National Technical Information Service
U.S. Department of Commerce
5285 Port Royal Road, Springfield, Virginia 22161
Price: Printed Copy \$5.25; Microfiche \$3.00

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, contractors, subcontractors, or their employees, makes any warranty, express or implied, nor assumes any legal liability or responsibility for any third party's use or the results of such use of any information, apparatus, product or process disclosed in this report, nor represents that its use by such third party would not infringe privately owned rights.

ORNL/TM-5804

Dist. Category UC- 948

Contract No. W-7405-eng-26

Engineering Technology Division

FREQUENCY ANALYSIS OF A THICK-RIM FLYWHEEL

D. N. Fanning

Date Published - September 1978

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Department of Energy, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

Prepared by the
OAK RIDGE NATIONAL LABORATORY
Oak Ridge, Tennessee 37830
operated by
UNION CARBIDE CORPORATION
for the
DEPARTMENT OF ENERGY

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

[Signature]

**THIS PAGE
WAS INTENTIONALLY
LEFT BLANK**

CONTENTS

	<u>Page</u>
ABSTRACT	1
INTRODUCTION	1
FINITE-ELEMENT METHOD	1
TRANSFER MATRIX METHOD	7
Background	8
Element transfer matrices	13
Determining frequencies and mode shapes	24
Analysis	27
CONCLUSION	30
ACKNOWLEDGMENT	32
REFERENCES	33
APPENDIX: PROGRAM LISTING	35

FREQUENCY ANALYSIS OF A THICK-RIM FLYWHEEL

D. N. Fanning

ABSTRACT

This report describes the calculation of natural frequencies of a thick-rim flywheel using two analysis methods: the finite-element method and the transfer matrix method. A listing of the computer program developed using the transfer matrix method is included. The results of the finite-element analyses indicate that the computer programs NONSAP and ADINA are incapable of determining out-of-plane natural frequencies of the flywheel. The results of the analysis using the transfer matrix method are satisfactory for out-of-plane as well as in-plane frequencies.

INTRODUCTION

Ensuring the structural integrity and satisfactory dynamic stability of a structure rotating at high speed usually requires some type of dynamic analysis, and often the first step in such an analysis is calculation of the critical frequencies of the system. Appropriate design iterations can then be made to eliminate and/or minimize the effects of these vibration modes in the operating range. [This report investigates the calculation of the critical frequencies of a potential candidate for an automobile flywheel to be used as an energy storage device.] In making this calculation, two independent analysis methods were used: the finite-element method, which used the two three-dimensional static/dynamic codes NONSAP and ADINA, and the transfer matrix method, which employed a code written especially for this application. (See Appendix.) A Kelvar/Epoxy composite was used in construction of the flywheel, which had a thick composite rim with composite straps connecting it to a metallic hub (see Fig. 1). The operational range of the flywheel was <40 krpm (0 to 667 cps), and therefore only natural frequencies in this range were required.

FINITE-ELEMENT METHOD

At the beginning of this project, NONSAP¹ was the only three-dimensional finite-element code internally available which was capable of

Fig. 1. Design of the flywheel.

determining fundamental frequencies. After NONSAP was selected as the analytical tool for this investigation, an analytical model of the flywheel was generated. Only half of the flywheel was modeled because NONSAP's in-core solution routine prevented modeling of the entire structure (a ring structure has an inherently large bandwidth). It seemed possible to obtain all the fundamental frequencies by using appropriate boundary conditions for a series of separate analyses. The three-dimensional model generated for the initial analysis is shown in Fig. 2. This analytical model, although very coarse, adequately models the geometry of the flywheel. The bending action of the straps can be accommodated by the 20-node elements used to model them because this type of element is able to reproduce the linear strain distribution characteristic of flexure.²

ORNL-DWG 77-7690

Fig. 2. Three-dimensional finite-element model of half of the fly-wheel.

Twenty-node elements are also used for the ring portion of the flywheel to allow for accurate modeling of flexure for this part of the structure, although the ring is expected to behave as if it were essentially rigid.

Prior to any analysis, it was necessary to modify the material model library of NONSAP to include an orthotropic elastic material for representing the composite material of the flywheel. The material model implemented is fully orthotropic although the flywheel material is only transversely isotropic (stratified). The following material constants were derived experimentally: $E_1 = 100 \text{ GPa}$ ($14.5 \times 10^6 \text{ psi}$), $E_2 = E_3 = 5.5 \text{ GPa}$ ($8 \times 10^5 \text{ psi}$), $\nu_{21} = \nu_{31} = 0.01875$, $\nu_{23} = 0.34$, $G_{12} = G_{13} = G_{23} = 2.1 \text{ GPa}$ ($3 \times 10^5 \text{ psi}$); the subscript 1 refers to the direction parallel to the fibers, the subscripts 2 and 3 refer to the transverse directions, E is the modulus of elasticity, ν is Poisson's ratio, and G is the shear modulus.

For the first analysis, the ends of the ring were restrained from moving normal to the plane of the cross section, as shown in Fig. 3, which eliminated the odd torsional modes (modes with an odd number of

ORNL-DWG 77-7691

TOP VIEW

SIDE VIEW

Fig. 3. Boundary conditions for the three-dimensional half-ring model. (Straps omitted for clarity.)

wavelengths in a circumference involving tangential movement of the ring). The lowest frequency was expected to occur near 13.13 cps, which corresponds to axial motion of the entire ring as a rigid body. The frequency approximation for this mode was obtained using the one-degree-of-freedom vibration equation, $f = (K/M)^{1/2}$. In this case, M is equal to the mass of the ring plus about half the mass of the straps, and K is equal to the stiffness of the straps. The mass density of the flywheel is 1380 kg/m^3 ($1.295 \times 10^{-4} \text{ lb-sec}^2/\text{in.}^4$), from which M is calculated to be 10.9 kg/m^3 ($0.062 \text{ lb-sec}^2/\text{in.}$). For this mode the straps deflect with no rotation of their ends; thus the stiffness of each strap is $12EI/L^3$, where I is the moment of inertia of the cross section and L is the length of the strap. From this formula, K is determined to be 73.9 kN/m (422 lb/in.). These values of K and M result in an estimated circular frequency of 82.5, which is equivalent to 13.13 cps. Unfortunately, the results of the analysis by NONSAP are completely erroneous. The lowest mode according to NONSAP occurs at a frequency of 46.35 cps, and the corresponding mode shape is the deformation of the cross section at one of the restrained ends of the ring as shown in Fig. 4. The higher modes determined by NONSAP were equally devoid of any physical significance.

A two-dimensional model was generated (see Fig. 5) after the lack of success with the three-dimensional model. The lowest fundamental frequency calculated by NONSAP is 172.3 cps and corresponds to the first torsional mode. The next mode shape is an in-plane longitudinal motion of the ring as a rigid body. However, the frequency corresponding to this mode shape is above the operational range of the flywheel at 879.8 cps. These results appear reasonable.

While working with NONSAP, the three-dimensional finite-element program ADINA³ was acquired, which, like NONSAP, has an eigensystem solution routine for determining fundamental frequencies and mode shapes. ADINA has the advantage of an out-of-core solution routine, which enables it to accommodate a three-dimensional model of the entire flywheel (see Fig. 6). However, analysis of this model using ADINA was not possible due to an error in the code which caused it to use an enormous amount of scratch space during the solution routine. The three-dimensional mesh used with NONSAP was then analyzed using ADINA. As with NONSAP the results using

ORNL-DWG 77-7692

Fig. 4. Lowest mode calculated by NONSAP.

Fig. 5. Two-dimensional finite-element model of the flywheel.

Fig. 6. Three-dimensional finite-element model of the complete flywheel.

ADINA were completely incorrect. Although the frequencies obtained were different from those calculated by NONSAP, the mode shapes were essentially the same.

TRANSFER MATRIX METHOD

Since it did not seem reasonable that the errors in the three-dimensional finite-element codes could be found and corrected in time to make the codes useful, a special-purpose code was written using the transfer matrix technique. This method is ideally suited for structural

problems which consist of a number of structural elements linked end-to-end to form a chain. Using this method the size of the matrices involved in finding a solution is only dependent on the order of the differential equation governing the behavior of the individual elements, not on the number of elements. The code was written using BASIC computer language and the Tektronix 4051 computer.

Background

The state vector at a point "i" of an elastic system is a column vector whose components are the displacements of the point "i" and the corresponding internal forces. The right-hand cartesian coordinate system is used with the X-axis coinciding with the centroidal axis of the elastic body, as in Fig. 7. A cut across the body exposes two faces; the one whose outward normal points in the positive direction of the X-axis is

ORNL-DWG 77-7695

Fig. 7. Coordinate system for the transfer matrix method.

the positive face and other is the negative face. Displacements are positive if they coincide with the positive directions of the coordinate system. Forces are positive if their vectors are in the positive directions when acting on the positive face and if their vectors are in the negative directions when acting on the negative face.

Consider a spring-mass system, as shown in Fig. 8, which is vibrating with circular frequency ω . The state vector just to the right of mass m_i is denoted by Z_i^R and the state vector to the left is denoted by Z_i^L . Isolating spring k_i , the positive forces and deflections are as shown in Fig. 9. From equilibrium,

$$N_{i-1}^R = N_i^L.$$

From the stiffness property of the spring,

$$N_{i-1}^R = N_i^L = k_i (X_i - X_{i-1}).$$

ORNL-DWG 77-7696

Fig. 8. Spring-mass system.

ORNL-DWG 77-7697

Fig. 9. Free-body diagram of an isolated spring.

These can be rewritten as

$$\begin{Bmatrix} X \\ N \end{Bmatrix}_i^L = \begin{bmatrix} 1 & \frac{1}{k_i} \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} X \\ N \end{Bmatrix}_{i-1}^R$$

or

$$Z_i^L = F_i Z_{i-1}^R.$$

Thus the state vector Z_i^L is expressible in terms of the state vector Z_{i-1}^R using the matrix F_i . F_i is called the field transfer matrix because it transfers the state vector from one end of a structural component to the other.

Now consider a free-body diagram of mass m_i , as shown in Fig. 10. In addition to the two spring forces, there is the inertia force $m_i \omega^2 X_i$. Since the mass is rigid,

$$X_i^R = X_i^L.$$

From equilibrium,

$$N_i^R = N_i^L - m_i \omega^2 X_i.$$

Rewriting these equations yields

$$\begin{Bmatrix} X \\ N \end{Bmatrix}_i^R = \begin{bmatrix} 1 & 0 \\ -m_i \omega^2 & 1 \end{bmatrix} \begin{Bmatrix} X \\ N \end{Bmatrix}_i^L$$

ORNL-DWG 77-7698

Fig. 10. Free-body diagram of an isolated mass.

or

$$Z_1^R = P_1 Z_1^L.$$

Thus the state vector Z_1^R can be expressed in terms of the state vector Z_1^L using the transfer matrix P_1 , which is called the point transfer matrix since it transfers the state vector over a point.

Transfer matrices can be used to determine natural frequencies of structural systems such as the simple spring-mass system of Fig. 11. Using the transfer matrices derived above gives

$$Z_1^L = F_1 Z_0^R$$

and

$$Z_1^R = P_1 Z_1^L.$$

Substituting the first equation into the second gives

$$Z_1^R = P_1 F_1 Z_0^R.$$

Since $X_0^R = N_1^R = 0$, the above equation yields

$$X_1^R = N_0^R / k_1$$

and

$$N_1^R = 0 = (1 - m_1 \omega^2 / k_1) N_0^R.$$

ORNL-DWG 77-7699

Fig. 11. One degree-of-freedom spring-mass system.

A nontrivial solution exists only if

$$1 - m_1 \omega^2 / k_1 = 0 .$$

The above equation yields

$$\omega = (k_1 / m_1)^{1/2} ,$$

which is the well-known solution for a 1-DOF spring-mass system.

For a structure composed of n elements,

$$Z_n = U_n U_{n-1} \dots U_2 U_1 Z_0 = T Z_0 ,$$

where U_i is the transfer matrix for element "i" (U_i is used to denote either a field or a point matrix). Since the flywheel is a closed structure (i.e., $Z_n = Z_0$),

$$Z_0 = T Z_0$$

or

$$(T - I)Z_0 = 0 . \tag{1}$$

A nontrivial solution exists only if

$$\det(T - I) = 0 . \tag{2}$$

Since the determinant is a polynomial in ω , it will be represented as $p(\omega)$. The values of ω which satisfy this equation are the natural frequencies of the system. Obtaining the mode shapes corresponding to a natural frequency requires solving Eq. (1) for Z_0 . Once Z_0 is known, the state vectors at the ends of all elements can be determined by successive applications of the recursive relation

$$Z_i = U_i Z_{i-1} .$$

The mode shape is thus completely determined.

Element transfer matrices

The special-purpose code combines the following three basic elements to form one superelement: elastic-massless circular arch, elastic support, and point mass. The transfer matrix for the superelement is simply the product of the transfer matrices of the three basic elements which form it. The elastic-massless circular arch is used to model the elastic properties of the rim of the flywheel. The elastic-massless strap element represents the strap, and the point-mass element is used to approximate the distributed mass of the flywheel. Vibrations in and perpendicular to the plane of the central axis are uncoupled. Figure 12 shows the elements with appropriate direction, displacement, and force (and moment) vectors for the in-plane case. The state vector for in-plane vibrations is

$$Z = \{u \quad -w \quad \psi \quad M_Y \quad V_Z \quad N\} .$$

A similar figure for the out-of-plane case is presented in Fig. 13. For out-of-plane vibrations the state vector is

$$Z = \{\phi \quad v \quad \theta \quad M_Z \quad -V_Y \quad T\} .$$

The transfer matrices in the code were verified by comparing the natural frequencies calculated by the code with those obtained from exact solutions. The transfer matrices for the circular arch elements were also verified by comparing them with an approximation made using straight beam segments and rigid corners. Exact solutions for four kinds of vibrations of a circular ring are given in Ref. 4. It is assumed that the ring has a constant cross section, the cross-sectional dimensions are small compared to the radius of the ring centerline, and one of the principal axes of the cross section is in the plane of the ring. The transfer matrices for all but the in-plane strap were taken from Ref. 5. The out-of-plane strap is represented as an elastic support. The transfer matrix for the in-plane strap was derived from the stiffness matrix of a beam.

Elastic-massless circular arch. The theoretical solution for in-plane flexural vibrations (neglecting rotary inertia and extension of the center

ORNL-DWG 77-7700

Fig. 12. Elements with in-plane force and displacement vectors shown on the positive face.

Fig. 13. Elements with out-of-plane force and displacement vectors shown on the positive face.

line) is given by⁴

$$\omega_n^2 = \frac{n^2(n^2 - 1)^2 EI}{(n^2 + 1)\mu R^4},$$

where n is the number of wavelengths in a circumference and R is the radius to centerline of the ring cross section. The portion of the transfer matrix related to this kind of vibration was verified by calculating the critical frequencies of a circular ring with the properties $E = I = R = \mu = 1$, where μ is the mass per unit length of the ring. The theoretical solution gives $\omega = 2.68328$. Discretizing the ring into 16 identical segments of 22.5° resulted in a calculated value of $\omega = 2.6733$ using the special-purpose code. Examination of the state vectors (see Table 1) reveals that the mode shape has two complete wavelengths in a circumference as expected.

The theoretical solution for in-plane extensional modes is⁴

$$\omega_n^2 = \frac{(n^2 + 1)E}{\rho R^2} = \frac{(n^2 + 1)EA}{\mu R^2},$$

where ρ is density and A is the area of the cross section. The portion of the transfer matrix for in-plane vibration of the arch related to extension was verified by modeling a circular ring with the properties $E = 2.4 \times 10^6$, $A = 112.0998$, $R = 10$, $\mu = 5$. The closed-form solution yields $\omega = 733.5388$, which is very close to the value of 731.113 calculated by the special-purpose code. This mode shape is a purely radial motion (see Table 2).

The theoretical solution for out-of-plane flexural vibration for a circular cross section is⁴

$$\omega_n^2 = \frac{n^2(n^2 - 1)^2 EI}{(n^2 + 1 + \nu)\mu R^4}.$$

To verify the coding of the out-of-plane transfer matrix of the circular arch, a ring with the following properties was modeled using eight segments of 45° : $\nu = 0.2$, $\mu = 5$, $R = 10$, $E = 2.4 \times 10^6$, $I = 1000$. The code calculated the frequencies 574.490 and 1562.84 cps for the lowest flexural modes; corresponding theoretical values from the closed-form solution are

Table 1. Mode shapes for the in-plane flexural natural frequency of the check case.

Point	u	-w	ψ	My	Vz	N
0	-8.389E-02	2.454E-02	-2.469E-01	-7.559E-02	1.000E+00	0.000E-01
1	-6.812E-02	-9.971E-02	-2.005E-01	3.071E-01	6.441E-01	5.738E-01
2	-1.244E-02	-1.656E-01	-3.660E-02	5.099E-01	-8.916E-02	8.115E-01
3	5.053E-02	-1.344E-01	1.487E-01	4.140E-01	-7.702E-01	5.738E-01
4	8.389E-02	-2.454E-02	2.469E-01	7.559E-02	-1.000E+00	6.454E-08
5	6.812E-02	9.971E-02	2.005E-01	-3.071E-01	-6.441E-01	-5.738E-01
6	1.244E-02	1.656E-01	3.660E-02	-5.099E-01	8.916E-02	-8.115E-01
7	-5.053E-02	1.344E-01	-1.487E-01	-4.140E-01	7.702E-01	-5.738E-01
8	-8.389E-02	2.454E-02	-2.469E-01	-7.559E-02	1.000E+00	-8.411E-08
9	-6.812E-02	-9.971E-02	-2.005E-01	3.071E-01	6.441E-01	5.738E-01
10	-1.244E-02	-1.656E-01	-3.660E-02	5.099E-01	-8.916E-02	8.115E-01
11	5.053E-02	-1.344E-01	1.487E-01	4.140E-01	-7.702E-01	5.738E-01
12	8.389E-02	-2.454E-02	2.469E-01	7.559E-02	-1.000E+00	1.874E-07
13	6.812E-02	9.971E-02	2.005E-01	-3.071E-01	-6.441E-01	-5.738E-01
14	1.244E-02	1.656E-01	3.660E-02	-5.099E-01	8.916E-02	-8.115E-01
15	-5.053E-02	1.344E-01	-1.487E-01	-4.140E-01	7.702E-01	-5.738E-01
16	-8.389E-02	2.454E-02	-2.469E-01	-7.559E-02	1.000E+00	2.503E-11
0	-2.454E-02	-2.013E-01	-7.223E-02	6.200E-01	0.000E-01	1.000E+00
1	5.478E-02	-1.766E-01	1.612E-01	5.439E-01	-8.782E-01	7.702E-01
2	1.020E-01	-4.843E-02	3.002E-01	1.492E-01	-1.242E+00	8.916E-02
3	8.949E-02	1.081E-01	2.634E-01	-3.329E-01	-8.782E-01	-6.441E-01
4	2.454E-02	2.013E-01	7.223E-02	-6.200E-01	2.200E-07	-1.000E+00
5	-5.478E-02	1.766E-01	-1.612E-01	-5.439E-01	8.782E-01	-7.702E-01
6	-1.020E-01	4.843E-02	-3.002E-01	-1.492E-01	1.242E+00	-8.916E-02
7	-8.949E-02	-1.081E-01	-2.634E-01	3.329E-01	8.782E-01	6.441E-01
8	-2.454E-02	-2.013E-01	-7.223E-02	6.200E-01	-8.411E-08	1.000E+00
9	5.478E-02	-1.766E-01	1.612E-01	5.439E-01	-8.782E-01	7.702E-01
10	1.020E-01	-4.843E-02	3.002E-01	1.492E-01	-1.242E+00	8.916E-02
11	8.949E-02	1.081E-01	2.634E-01	-3.329E-01	-8.782E-01	-6.441E-01
12	2.454E-02	2.013E-01	7.223E-02	-6.200E-01	3.195E-08	-1.000E+00
13	-5.478E-02	1.766E-01	-1.612E-01	-5.439E-01	8.782E-01	-7.702E-01
14	-1.020E-01	4.843E-02	-3.002E-01	-1.492E-01	1.242E+00	-8.916E-02
15	-8.949E-02	-1.081E-01	-2.634E-01	3.329E-01	8.782E-01	6.441E-01
16	-2.454E-02	-2.013E-01	-7.223E-02	6.200E-01	-1.408E-11	1.000E+00

Table 2. Mode shape for the in-plane extensional natural frequency of the check case.

Point	u	-w	ψ	My	Vz	N
0	-3.558E-19	3.947E-08	9.096E-20	-5.479E-01	4.142E-01	1.000E+00
1	3.067E-19	3.947E-08	7.504E-20	-5.479E-01	4.142E-01	1.000E+00
2	1.717E-19	3.947E-08	1.122E-20	-5.479E-01	4.142E-01	1.000E+00
3	-2.261E-19	3.947E-08	-7.078E-20	-5.479E-01	4.142E-01	1.000E+00
4	9.519E-20	3.947E-08	-3.031E-21	-5.479E-01	4.142E-01	1.000E+00
5	3.202E-19	3.947E-08	3.809E-20	-5.479E-01	4.142E-01	1.000E+00
6	-8.724E-20	3.947E-08	-8.018E-20	-5.479E-01	4.142E-01	1.000E+00
7	7.517E-20	3.947E-08	-1.967E-19	-5.479E-01	4.142E-01	1.000E+00
8	1.258E-18	3.947E-08	-4.632E-19	-5.479E-01	4.142E-01	1.000E+00

576.461 and 1646.39, respectively. The mode shape for the lower frequency has two complete wavelengths in a circumference (see Table 3); the second mode shape has three complete wavelengths in a circumference (see Table 4).

Table 3. Mode shapes for the lowest out-of-plane flexural natural frequency of the check case.

Point	ϕ	v	θ	M_z	$-V_y$	T
0	-2.616E-09	-1.543E-08	7.153E-16	1.000E+00	-1.000E-01	0.000E-01
1	5.165E-16	1.128E-15	3.020E-09	-6.946E-08	-1.000E-01	4.142E-01
2	2.616E-09	1.543E-08	-1.698E-16	-1.000E+00	1.000E-01	-5.542E-08
3	1.095E-19	-3.343E-16	-3.020E-09	4.816E-08	1.000E-01	-4.142E-01
4	-2.616E-09	-1.543E-08	-5.029E-21	1.000E+00	-1.000E-01	-5.258E-13
5	1.023E-19	-3.343E-16	3.020E-09	4.817E-08	-1.000E-01	4.142E-01
6	2.616E-09	1.543E-08	1.698E-16	-1.000E+00	1.000E-01	5.542E-08
7	5.165E-16	1.128E-15	-3.020E-09	-6.946E-08	1.000E-01	-4.142E-01
8	-2.616E-09	-1.543E-08	-7.153E-16	1.000E+00	-1.000E-01	-3.464E-13
0	4.169E-15	-4.278E-20	7.290E-09	0.000E-01	-2.414E-01	1.000E+00
1	6.316E-09	3.725E-08	-4.338E-16	-2.414E+00	2.414E-01	-4.742E-07
2	5.201E-16	-1.034E-15	-7.290E-09	3.230E-07	2.414E-01	-1.000E+00
3	-6.316E-09	-3.725E-08	1.400E-16	2.414E+00	-2.414E-01	-5.661E-08
4	-1.345E-21	-2.059E-20	7.290E-09	2.763E-12	-2.414E-01	1.000E+00
5	6.316E-09	3.725E-08	1.400E-16	-2.414E+00	2.414E-01	-5.661E-08
6	-5.201E-16	1.034E-15	-7.290E-09	-3.230E-07	2.414E-01	-1.000E+00
7	-6.316E-09	-3.725E-08	-4.339E-16	2.414E+00	-2.414E-01	-4.742E-07
8	-4.169E-15	-2.597E-19	7.290E-09	1.121E-12	-2.414E-01	1.000E+00

Table 4. Mode shapes for the second out-of-plane flexural natural frequency of the check case.

Point	ϕ	v	θ	M_z	$-V_y$	T
0	-8.163E-10	-4.170E-09	4.262E-16	1.000E+00	-2.000E-01	0.000E-01
1	5.772E-10	2.949E-09	5.661E-10	-7.071E-01	8.284E-02	1.213E-01
2	-3.304E-17	-5.701E-16	-9.420E-10	1.588E-07	8.284E-02	-1.716E-01
3	-5.772E-10	-2.949E-09	5.661E-10	7.071E-01	-2.000E-01	1.213E-01
4	8.163E-10	4.170E-09	1.112E-17	-1.000E+00	2.000E-01	2.119E-09
5	-5.772E-10	-2.949E-09	-5.661E-10	7.071E-01	-8.284E-02	-1.213E-01
6	-4.965E-17	-6.673E-16	9.420E-10	1.829E-07	-8.284E-02	1.716E-01
7	5.772E-10	2.949E-09	-5.661E-10	-7.071E-01	2.000E-01	-1.213E-01
8	-8.163E-10	-4.170E-09	-4.492E-16	1.000E+00	-2.000E-01	-9.242E-10
0	1.489E-14	-2.660E-17	5.490E-09	0.000E-01	-4.828E-01	1.000E+00
1	3.364E-09	1.719E-08	-3.882E-09	-4.121E+00	1.166E+00	-7.071E-01
2	-4.758E-09	-2.431E-08	7.818E-16	5.828E+00	-1.166E+00	-4.701E-07
3	3.364E-09	1.719E-08	3.882E-09	-4.121E+00	4.828E-01	7.071E-01
4	1.897E-18	9.518E-18	-5.490E-09	-2.259E-09	4.828E-01	-1.000E+00
5	-3.364E-09	-1.719E-08	3.882E-09	4.121E+00	-1.166E+00	7.071E-01
6	4.758E-09	2.431E-08	7.863E-16	-5.828E+00	1.166E+00	-4.691E-07
7	-3.364E-09	-1.719E-08	-3.882E-09	4.121E+00	-4.828E-01	-7.071E-01
8	-1.488E-14	4.757E-17	5.490E-09	1.033E-08	-4.828E-01	1.000E+00

The theoretical solution for out-of-plane torsional vibrations for a circular cross section is⁴

$$\omega_n^2 = \frac{(n^2 + 1 + \nu)G}{\rho R^2}.$$

The coding pertaining to this kind of vibration was verified by modeling a circular ring with the following properties: $E = 2.4 \times 10^6$, $I = 1000$, $R = 10$, $G = 10$, $J = 2000$, $\mu = 5$, where J is the polar moment of inertia of the cross section. Discretizing the structure into 80 segments of 4.5° , the lowest torsional mode was found at $\omega = 518.53$ using the code. The corresponding theoretical value is 518.690.

To provide additional verification of the transfer matrices of the circular arch element, a program was written which calculates the transfer matrix of the arch and then approximates this element by using a finite number of straight beam segments rigidly coupled at appropriate angles (see Fig. 14). Results for the in-plane and out-of-plane cases are presented in Tables 5 and 6, respectively. In both cases, the transfer matrix of the approximation approaches that of the actual arch as the number of segments is increased.

Strap. Since it was necessary to derive the transfer matrix for the in-plane strap, that derivation will be included herein. The force-displacement relation⁶ for the beam element used to represent the strap (see Fig. 15a) is

$$\begin{Bmatrix} V_{Z1} \\ N_1 \\ -M_{Y1} \\ V_{Z2} \\ N_2 \\ -M_{Y2} \end{Bmatrix} = \frac{E}{L} \begin{bmatrix} \left(A\lambda^2 + \frac{12I}{L^2}\mu^2\right) & \left(A - \frac{12I}{L^2}\right)\lambda\mu & \left(A\mu^2 + \frac{12I}{L^2}\lambda^2\right) & 0 & 0 & 0 \\ \frac{6I}{L}\mu & -\frac{6I}{L}\lambda & 4I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -\left(A\lambda^2 - \frac{12I}{L^2}\mu^2\right) & -\left(A - \frac{12I}{L^2}\right)\lambda\mu & -\frac{6I}{L}\mu & \left(A\lambda^2 + \frac{12I}{L^2}\mu^2\right) & 0 & 0 \\ -\left(A - \frac{12I}{L^2}\right)\lambda\mu & -\left(A\mu^2 + \frac{12I}{L^2}\lambda^2\right) & \frac{6I}{L}\lambda & \left(A - \frac{12I}{L^2}\right)\lambda\mu & \left(A\mu^2 + \frac{12I}{L^2}\lambda^2\right) & 0 \\ \frac{6I}{L}\mu & -\frac{6I}{L}\lambda & 2I & -\frac{6I}{L}\mu & \frac{6I}{L}\lambda & 4I \end{bmatrix} \begin{Bmatrix} w_1 \\ u_1 \\ -\psi_1 \\ w_2 \\ u_2 \\ -\psi_2 \end{Bmatrix}, \quad \text{Sym.}$$

where $\lambda = \cos \beta$ and $\mu = \sin \beta$. Since end 2 is fixed ($w_2 = u_2 = \psi_2 = 0$),

Table 5. Comparison of the in-plane transfer matrix of a circular arch (a) with approximations using 2 straight beam segments (b), 5 straight beam segments (c), and 10 straight beam segments (d).

μ	$-w$	ψ	M_y	V_z	N
8.660E-01	-5.000E-01	-1.340E+00	-1.180E-09	6.391E-08	2.385E-07
5.000E-01	8.660E-01	5.000E+00	6.699E-09	2.328E-08	6.391E-08
0.000E-01	0.000E-01	1.000E+00	2.618E-09	6.699E-09	-1.180E-09 (a)
0.000E-01	0.000E-01	0.000E-01	1.000E+00	5.000E+00	-1.340E+00
0.000E-01	0.000E-01	0.000E-01	0.000E-01	9.660E-01	-5.000E-01
0.000E-01	0.000E-01	0.000E-01	0.000E-01	5.000E-01	8.660E-01
8.660E-01	-5.000E-01	-1.340E+00	-1.319E-09	6.337E-08	2.394E-07
5.000E-01	8.660E-01	5.000E+00	6.641E-09	2.450E-08	6.337E-08
0.000E-01	0.000E-01	1.000E+00	2.611E-09	6.641E-09	-1.319E-09 (b)
0.000E-01	0.000E-01	0.000E-01	1.000E+00	5.000E+00	-1.340E+00
0.000E-01	0.000E-01	0.000E-01	0.000E-01	9.660E-01	-5.000E-01
0.000E-01	0.000E-01	0.000E-01	0.000E-01	5.000E-01	8.660E-01
8.660E-01	-5.000E-01	-1.340E+00	-1.202E-09	6.382E-08	2.387E-07
5.000E-01	8.660E-01	5.000E+00	6.690E-09	2.347E-08	6.382E-08
0.000E-01	0.000E-01	1.000E+00	2.617E-09	6.690E-09	-1.202E-09 (c)
0.000E-01	0.000E-01	0.000E-01	1.000E+00	5.000E+00	-1.340E+00
0.000E-01	0.000E-01	0.000E-01	0.000E-01	9.660E-01	-5.000E-01
0.000E-01	0.000E-01	0.000E-01	0.000E-01	5.000E-01	8.660E-01
8.660E-01	-5.000E-01	-1.340E+00	-1.186E-09	6.389E-08	2.386E-07
5.000E-01	8.660E-01	5.000E+00	6.696E-09	2.332E-08	6.389E-08
0.000E-01	0.000E-01	1.000E+00	2.618E-09	6.696E-09	-1.186E-09 (d)
0.000E-01	0.000E-01	0.000E-01	1.000E+00	5.000E+00	-1.340E+00
0.000E-01	0.000E-01	0.000E-01	0.000E-01	9.660E-01	-5.000E-01
0.000E-01	0.000E-01	0.000E-01	0.000E-01	5.000E-01	8.660E-01

Table 6. Comparison of the out-of-plane transfer matrix of a circular arch (a) with approximations using 2 straight beam segments (b), 5 straight beam segments (c), and 15 straight beam segments (d).

ϕ	v	θ	M_z	V_y	T
8.660E-01	0.000E-01	5.000E-01	1.636E-09	2.909E-09	4.709E-09
-1.340E+00	1.000E+00	5.000E+00	2.965E-09	5.495E-09	-2.909E-09
-5.000E-01	0.000E-01	8.660E-01	9.591E-10	2.965E-09	-1.636E-09 (a)
0.000E-01	0.000E-01	0.000E-01	8.660E-01	5.000E+00	-5.000E-01
0.000E-01	0.000E-01	0.000E-01	0.000E-01	1.000E+00	0.000E-01
0.000E-01	0.000E-01	0.000E-01	5.000E-01	1.340E+00	8.660E-01
8.660E-01	0.000E-01	5.000E-01	1.632E-09	2.401E-09	4.717E-09
-1.340E+00	1.000E+00	5.000E+00	3.033E-09	5.779E-09	-2.401E-09
-5.000E-01	0.000E-01	8.660E-01	9.348E-10	3.033E-09	-1.632E-09 (b)
0.000E-01	0.000E-01	0.000E-01	8.660E-01	5.000E+00	-5.000E-01
0.000E-01	0.000E-01	0.000E-01	0.000E-01	1.000E+00	0.000E-01
0.000E-01	0.000E-01	0.000E-01	5.000E-01	1.340E+00	8.660E-01
8.660E-01	0.000E-01	5.000E-01	1.635E-09	2.828E-09	4.710E-09
-1.340E+00	1.000E+00	5.000E+00	2.976E-09	5.550E-09	-2.828E-09
-5.000E-01	0.000E-01	8.660E-01	9.552E-10	2.976E-09	-1.635E-09 (c)
0.000E-01	0.000E-01	0.000E-01	8.660E-01	5.000E+00	-5.000E-01
0.000E-01	0.000E-01	0.000E-01	0.000E-01	1.000E+00	0.000E-01
0.000E-01	0.000E-01	0.000E-01	5.000E-01	1.340E+00	8.660E-01
8.660E-01	0.000E-01	5.000E-01	1.636E-09	2.900E-09	4.709E-09
-1.340E+00	1.000E+00	5.000E+00	2.966E-09	5.501E-09	-2.900E-09
-5.000E-01	0.000E-01	8.660E-01	9.586E-10	2.966E-09	-1.636E-09 (d)
0.000E-01	0.000E-01	0.000E-01	8.660E-01	5.000E+00	-5.000E-01
0.000E-01	0.000E-01	0.000E-01	0.000E-01	1.000E+00	0.000E-01
0.000E-01	0.000E-01	0.000E-01	5.000E-01	1.340E+00	8.660E-01

Fig. 14. Circular arch (a) and approximation using three straight beams (b).

the above system of equations is reduced to

$$\begin{Bmatrix} V_{Z1} \\ N_1 \\ -M_{Y1} \end{Bmatrix} = \frac{E}{L} \begin{bmatrix} A\lambda^2 + \frac{12I}{L^2}\mu^2 & & \\ \left(A - \frac{12I}{L^2}\right)\lambda\mu & A\mu^2 + \frac{12I}{L^2}\lambda^2 & \\ \frac{6I}{L}\mu & -\frac{6I}{L}\lambda & \end{bmatrix} \begin{Bmatrix} w_1 \\ u_1 \\ -\psi_1 \end{Bmatrix} \quad \begin{matrix} \text{Sym.} \\ \\ 4I \end{matrix}$$

Fig. 15. Force and displacement vectors on beam element representation of a strap.

Inspection of Fig. 15b yields

$$u_i = u_{i-1} \quad M_{Y_i} - M_{Y_{i-1}} = M_{Y_1} ,$$

$$w_i = w_{i-1} \quad V_{Z_i} - V_{Z_{i-1}} = V_{Z_1} ,$$

$$\psi_i = \psi_{i-1} \quad N_i - N_{i-1} = N_1 .$$

Using these equations with the force-displacement equations gives

$$\begin{Bmatrix} u_i \\ -w_i \\ \psi_i \\ M_{Yi} \\ V_{Zi} \\ N_i \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \frac{6EI}{L^2}\lambda & \frac{6EI}{L^2}\mu & \frac{4EI}{L} & 1 & 0 & 0 \\ \left(\frac{AE}{L} - \frac{12EI}{L^3}\right)\lambda\mu & -\left(\frac{AE}{L}\lambda^2 - \frac{12EI}{L^3}\mu^2\right) & -\frac{6EI}{L^2}\mu & 0 & 1 & 0 \\ \left(\frac{AE}{L}\mu^2 + \frac{12EI}{L^3}\lambda^2\right) & -\left(\frac{AE}{L} - \frac{12EI}{L^3}\right)\lambda\mu & \frac{6EI}{L^2}\lambda & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} u_{i-1} \\ -w_{i-1} \\ \psi_{i-1} \\ M_{Yi-1} \\ V_{Zi-1} \\ N_{i-1} \end{Bmatrix},$$

or $Z_i = U_i Z_{i-1}$, where U_i is the desired transfer matrix.

To validate the coding of the transfer matrix for the elastic support, the test problem consisted of a single support member and a mass. This structure should behave as a massless cantilever beam with a concentrated mass at the end. The in-plane modes of the elastic support include extensional vibrations and flexural vibrations. For the test problem used, $AE = 72,000$, $EI = 5 \times 10^8$, $L = 10$, and $M = 6$. The exact solution for extensional vibration of a 1-DOF problem such as this is $\omega = (K/M)^{1/2}$. For the uniaxial bar, $K = AE/L$; therefore, $\omega = 10\sqrt{12}$. The code determined this frequency correctly to the limit of its accuracy (about 12 digits). The exact solution for the flexural natural frequency is

$$\omega^2 = \frac{3EI}{L^3 M}.$$

For the test problem, this formula yields $\omega = 500$, which is the value calculated by the code.

Point mass. The transfer matrix for this element was implicitly verified during the verification of the other elements since it was used for those verifications.

Determining frequencies and mode shapes

Solving for a natural frequency is done by a determinant search method which requires interaction with the user. The first step in searching for a root involves inputting an initial value of ω and a $\Delta\omega$. The program then computes the value of the determinant at ω [$p(\omega)$] and increments ω by $\Delta\omega$ solving for the determinant at the new ω , etc. Two criteria exist for determining if a root has been bracketed by two points ω_1 and ω_2 , where $\omega_1 < \omega_2$:

$$p(\omega_1) \cdot p(\omega_2) < 0 ,$$

$$p'(\omega_1) \cdot p'(\omega_2) < 0 \text{ and } p(\omega_1) \cdot p'(\omega_1) < 0 ,$$

where $p'(\omega)$ is the first derivative of $p(\omega)$ with respect to ω . If the first criterion is satisfied, a root must be in the interval (ω_1, ω_2) according to the intermediate value theorem for continuous functions; however, the second criterion does not guarantee that a root exists in the interval. Experience with typical problems has indicated that a root (probably a double root) does exist in the interval if criterion 2 is satisfied. After a root is bracketed, the code converges on the root by approximating a portion of the function $p(\omega)$ with a parabola. If criterion 1 is satisfied, the following algorithm is used:

1. Let $\omega_3 = [p(\omega_1) \cdot \omega_2 + p(\omega_2) \cdot \omega_1] / [p(\omega_1) - p(\omega_2)]$.
2. Determine the parabola $p^*(\omega) = a\omega^2 + b\omega + c$ which passes through the points $[\omega_1, p(\omega_1)]$, $[\omega_2, p(\omega_2)]$, $[\omega_3, p(\omega_3)]$.
3. Calculate α such that $p^*(\alpha) = 0$ and $\omega_1 \leq \alpha \leq \omega_2$.
4. If $p(\alpha) \cdot p(\omega_2) > 0$, then set $\omega_2 = \omega_3$ and $\omega_3 = \alpha$; otherwise if $p(\alpha) \cdot p(\omega_1) < 0$, set $\omega_1 = \omega_3$ and $\omega_3 = \alpha$.
5. Repeat steps 2 through 4 until the convergence criteria are satisfied.

If criterion on 2 is satisfied, the following method is used:

1. Let $\omega_3 = (\omega_1 + \omega_2) / 2$.
2. Determine α such that the parabola $p^*(\omega) = \beta(\omega - \alpha)^2$ passes through the points $[\omega_1, p(\omega_1)]$, $[\omega_2, p(\omega_2)]$, $[\omega_3, p(\omega_3)]$.

3. If $\alpha > \omega_3$, then $\omega_1 = \omega_3$ and $\omega_3 = \alpha$; otherwise $\omega_2 = \omega_3$ and $\omega_3 = \alpha$.
4. Repeat steps 2 and 3 until the convergence criteria are satisfied.

These algorithms seem to work well. The iterative solution terminates when $p(\omega)$ is less than a small value ϵ , which is several orders of magnitude less than the nearest relative extrema of $p(\omega)$, or when the number of iterations exceeds 10. If the solution has not converged after 10 iterations, the user has the option to continue the iterations.

After finding a natural frequency, the next step is usually finding the corresponding mode shape. This is accomplished by solving Eq. (1) using Gauss-Jordan reduction with the maximum pivot strategy. Consider a system of l simultaneous equations in l unknowns,

$$At = 0,$$

where A is the matrix of coefficients and t is the solution vector. Let a_{ij} be the term in the i th row and j th column of A . After the i th pass of the elimination, the matrix A is modified as shown [assuming the pivot elements $(d_1, d_2, \dots, d_i)$ are the successive diagonal elements $(a_{11}, a_{22}, \dots, a_{ii})$ for simplicity]:

$$\begin{bmatrix} 1 & 0 & \dots & 0 & c_{1,i+1} & \dots & c_{1,l} \\ 0 & 1 & \dots & 0 & c_{2,i+1} & \dots & c_{2,l} \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ 0 & 0 & \dots & 1 & c_{i,i+1} & \dots & c_{i,l} \\ 0 & 0 & \dots & 0 & c_{i+1,i+1} & \dots & c_{i+1,l} \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ 0 & 0 & \dots & 0 & c_{l,i+1} & \dots & c_{l,l} \end{bmatrix}$$

If $c_{jk} = 0$ for all $j > i, k > i$, then A appears as follows:

$$\begin{bmatrix} 1 & 0 & \dots & 0 & c_{1,i+1} & \dots & c_{1,\ell} \\ 0 & 1 & & 0 & c_{2,i+1} & \dots & c_{2,\ell} \\ \cdot & \cdot & & \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot & \cdot & & \cdot \\ 0 & 0 & \dots & 1 & c_{i,i+1} & \dots & c_{i,\ell} \\ 0 & 0 & \dots & 0 & 0 & \dots & 0 \\ \cdot & \cdot & & \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot & \cdot & & \cdot \\ 0 & 0 & \dots & 0 & 0 & \dots & 0 \end{bmatrix}.$$

The rank of A is i and there are $n - i$ linearly independent solutions. The solutions may be obtained by assuming the values of the solution vector as follows:

$$x_j^k = 0 \quad j = i + 1, i + 2, \dots, \ell \quad j \neq i + k,$$

$$x_{i+k}^k = 1,$$

where x_j^k is the value of the j th component of the k th solution vector. After assuming the values of the above components, it follows that

$$x_j^k = -c_{j,i+k} \quad 1 \leq j \leq i \quad 1 \leq k \leq \ell - i.$$

The complete set of solution vectors is

$$\{X\}^1 = \begin{Bmatrix} -C_{1,i+1} \\ -C_{2,i+1} \\ \vdots \\ \vdots \\ -C_{i,i+1} \\ 1 \\ 0 \\ \vdots \\ \vdots \\ \vdots \\ 0 \end{Bmatrix}, \{X\}^2 = \begin{Bmatrix} -C_{1,i+2} \\ -C_{2,i+2} \\ \vdots \\ \vdots \\ -C_{i,i+2} \\ 0 \\ 1 \\ 0 \\ \vdots \\ \vdots \\ \vdots \\ 0 \end{Bmatrix}, \dots, \{X\}^{h-i} = \begin{Bmatrix} -C_{1,\ell} \\ -C_{2,\ell} \\ \vdots \\ \vdots \\ -C_{i,\ell} \\ 0 \\ \vdots \\ \vdots \\ \vdots \\ 0 \\ 1 \end{Bmatrix}.$$

The major problem with this technique is determining the rank of the matrix A (the multiplicity of the root is the order minus the rank of A). Ideally, the criterion for terminating the reduction scheme would be to stop at the end of the i th pass when the $i+1$ th pivot element (d_{i+1}) is zero. In practice, however, the pivot element is rarely identically zero since we cannot solve Eq. (2) exactly. This difficulty is overcome by arbitrarily using the following termination criteria: stop at the end of the i th pass if the value of the determinant of the $i+1$ by $i+1$ submatrix is less than 0.005 times the first pivot. If a root of the equation is sufficiently accurate, this procedure gives the correct number of solution vectors. Thus it is important to specify a small ϵ when solving Eq. (2).

Analysis

The flywheel is composed of eight identical 45° segments, each of which is modeled by three superelements as shown in Fig. 16. The input data necessary for this problem are shown in Tables 7 and 8 for the out-of-plane and in-plane cases, respectively. The values of the point masses include half the mass of the attached straps where appropriate (i.e., superelements 1 and 3 only). The mass moments of inertia do not

Fig. 16. Model of the flywheel for the transfer matrix technique.

Table 7. Input data for the flywheel (out-of-plane case).

INPUT THE NUMBER OF SEGMENTS	: 3
INPUT THE RADIUS	: 8.75
INPUT E AND I	: 14.5E6, 8.18833
INPUT G AND J	: 0.3E6, 12.6154

INPUT THE FOLLOWING

ANGLE	: 14.774
ROTARY SPRING STIFFNESS (X)	: 7.105E5
LONGITUDINAL SPRING STIFFNESS (Y)	: 26.3964
ROTARY SPRING STIFFNESS (Z)	: 7105
MOMENT OF INERTIA (X)	: 3.68772E-3
MASS	: 2.54238E-3
MOMENT OF INERTIA (Z)	: 3.44765E-3

ANGLE	: 15.113
ROTARY SPRING STIFFNESS (X)	: 0
LONGITUDINAL SPRING STIFFNESS (Y)	: 0
ROTARY SPRING STIFFNESS (Z)	: 0
MOMENT OF INERTIA (X)	: 3.77234E-3
MASS	: 2.54172E-3
MOMENT OF INERTIA (Z)	: 3.57681E-3

ANGLE	: 15.113
ROTARY SPRING STIFFNESS (X)	: 7.105E5
LONGITUDINAL SPRING STIFFNESS (Y)	: 26.3964
ROTARY SPRING STIFFNESS (Z)	: 7105
MOMENT OF INERTIA (X)	: 3.77234E-3
MASS	: 2.59940E-3
MOMENT OF INERTIA (Z)	: 3.57681E-3

Table 8. Input data for the flywheel (in-plane case).

INPUT THE NUMBER OF SEGMENTS	: 3
INPUT THE RADIUS	: 8.75
INPUT E AND I	: 14.5E6, 4.42708
INPUT THE AREA	: 8.5
INPUT THE FOLLOWING	
ANGLE	: 14.774
A*E FOR THE STRAP	: 1.74E6
E*I FOR THE STRAP	: 81562.5
LENGTH OF THE STRAP	: 7.5
ANGLE - STRAP AND NORMAL TO RING	: -7.387
MASS	: 2.54238E-3
MOMENT OF INERTIA (Y)	: 2.34817E-3
ANGLE	: 15.113
A*E FOR THE STRAP	: 0
E*I FOR THE STRAP	: 0
LENGTH OF THE STRAP	: 7.5
ANGLE - STRAP AND NORMAL TO RING	: 0
MASS	: 2.54172E-3
MOMENT OF INERTIA (Y)	: 2.45210E-3
ANGLE	: 15.113
A*E FOR THE STRAP	: 1.74E6
E*I FOR THE STRAP	: 81562.5
LENGTH OF THE STRAP	: 7.5
ANGLE - STRAP AND NORMAL TO RING	: 7.387
MASS	: 2.59940E-3
MOMENT OF INERTIA (Y)	: 2.45210E-3

include the effects of the curvature of the elements. One end of each strap is assumed to be rigidly attached to the rim of the flywheel, while the other is assumed to be fixed to the hub. The effective length of the strap is approximated as 29.5 mm (7.5 in.). Interaction caused by overlapping of the straps is neglected.

Two natural frequencies for out-of-plane modes were found in the range of interest, 13.19 and 223.5 cps. The mode shape for the lower frequency is purely axial motion (see Table 9). The second frequency corresponds to a "wobble" mode [rotation of the ring about an imaginary line in the plane of the ring which passes through the axis of the ring

Table 9. Mode shape for the lowest out-of-plane natural frequency of the flywheel.

Point	ϕ	v	θ	M_z	$-V_y$	T
0	-1.857E-07	1.411E-01	-6.995E-08	1.000E+00	2.060E-02	0.000E-01
1	-1.177E-07	1.411E-01	-7.915E-09	1.013E+00	-1.239E+00	1.773E-01
2	-5.548E-09	1.411E-01	-1.823E-09	-1.895E+00	1.225E+00	6.031E-02
3	-4.771E-08	1.411E-01	-5.755E-09	9.492E-01	2.060E-02	-9.906E-02
4	-2.966E-08	1.411E-01	2.614E-08	9.892E-01	-1.239E+00	1.312E-01
5	5.888E-08	1.411E-01	1.161E-08	-1.906E+00	1.225E+00	9.559E-03
6	-1.311E-08	1.411E-01	-5.549E-09	9.521E-01	2.060E-02	-1.263E-01
7	-1.179E-08	1.411E-01	1.969E-08	9.989E-01	-1.239E+00	1.183E-01
8	6.756E-08	1.411E-01	1.893E-09	-1.893E+00	1.225E+00	-3.638E-04
9	-1.213E-08	1.411E-01	-1.623E-08	9.668E-01	2.060E-02	-1.318E-01
10	-1.565E-08	1.411E-01	9.726E-09	1.015E+00	-1.239E+00	1.139E-01
11	5.992E-08	1.411E-01	-6.173E-09	-1.877E+00	1.225E+00	-5.072E-04
12	-2.037E-08	1.411E-01	-2.182E-08	9.825E-01	2.060E-02	-1.336E-01
13	-2.484E-08	1.411E-01	6.750E-09	1.030E+00	-1.239E+00	1.097E-01
14	4.902E-08	1.411E-01	-6.115E-09	-1.861E+00	1.225E+00	-5.416E-04
15	-2.957E-08	1.411E-01	-1.872E-08	9.981E-01	2.060E-02	-1.360E-01
16	-3.311E-08	1.411E-01	1.246E-08	1.046E+00	-1.239E+00	1.055E-01
17	4.134E-08	1.411E-01	2.082E-09	-1.845E+00	1.225E+00	-4.687E-04
18	-3.347E-08	1.411E-01	-8.618E-09	1.014E+00	2.060E-02	-1.344E-01
19	-3.216E-08	1.411E-01	2.330E-08	1.061E+00	-1.239E+00	1.117E-01
20	4.998E-08	1.411E-01	1.197E-08	-1.832E+00	1.225E+00	9.394E-03
21	-1.563E-08	1.411E-01	-1.983E-09	1.024E+00	2.060E-02	-1.089E-01
22	2.438E-09	1.411E-01	2.328E-08	1.064E+00	-1.239E+00	1.635E-01
23	1.145E-07	1.411E-01	-1.284E-09	-1.842E+00	1.225E+00	6.017E-02
24	7.254E-08	1.411E-01	-3.588E-08	1.000E+00	2.060E-02	-1.022E-10

(see Table 10)]. The next out-of-plane natural frequency is near 955 cps. The only natural frequency for an in-plane mode found in the range of interest was 175.0 cps. The corresponding mode shape is rotation of the entire rim relative to the shaft [the first torsional mode (see Table 11)]. The next higher in-plane natural frequency is about 874 cps.

CONCLUSION

The two-dimensional NONSAP solution verified that the results of the in-plane transfer matrix method are very good. Whereas NONSAP calculated the values of 172.3 and 879.8 cps for the two lowest in-plane modal frequencies, the transfer matrix method obtained the values of 175.0 and 874 cps, respectively. The out-of-plane transfer matrix solution seems to

Table 10. Mode shapes for the second out-of-plane natural frequency of the flywheel.

Point	ϕ	v	θ	M_z	$-v_y$	T
0	5.081E-08	-8.905E-08	-5.994E-06	0.000E-01	1.000E+00	1.404E-01
1	-1.338E-06	-1.347E-05	-5.803E-06	2.194E+00	9.328E-01	-5.161E-01
2	-2.873E-06	-2.622E-05	-5.167E-06	4.417E+00	8.015E-01	3.774E-01
3	-3.483E-06	-3.707E-05	-4.203E-06	5.994E+00	6.125E-01	-6.904E-01
4	-4.336E-06	-4.529E-05	-3.039E-06	7.338E+00	3.867E-01	-2.011E+00
5	-5.549E-06	-5.064E-05	-1.544E-06	8.501E+00	1.330E-01	1.299E-01
6	-4.977E-06	-5.233E-05	5.005E-08	8.477E+00	-1.338E-01	-1.117E+00
7	-4.793E-06	-5.057E-05	1.505E-06	8.183E+00	-3.859E-01	-2.328E+00
8	-4.975E-06	-4.540E-05	2.984E-06	7.606E+00	-6.134E-01	-1.936E-01
9	-3.555E-06	-3.694E-05	4.274E-06	5.994E+00	-8.017E-01	-8.890E-01
10	-2.443E-06	-2.623E-05	5.167E-06	4.235E+00	-9.325E-01	-1.281E+00
11	-1.486E-06	-1.356E-05	5.764E-06	2.255E+00	-1.000E+00	-4.038E-01
12	-5.081E-08	8.905E-08	5.994E-06	-4.173E-08	-1.000E+00	-1.404E-01
13	1.338E-06	1.347E-05	5.803E-06	-2.194E+00	-9.328E-01	5.161E-01
14	2.873E-06	2.622E-05	5.167E-06	-4.417E+00	-8.015E-01	-3.774E-01
15	3.483E-06	3.707E-05	4.203E-06	-5.994E+00	-6.125E-01	6.904E-01
16	4.336E-06	4.529E-05	3.039E-06	-7.338E+00	-3.867E-01	2.011E+00
17	5.549E-06	5.064E-05	1.544E-06	-8.501E+00	-1.330E-01	-1.299E-01
18	4.977E-06	5.233E-05	-5.005E-08	-8.477E+00	1.338E-01	1.117E+00
19	4.793E-06	5.057E-05	-1.505E-06	-8.183E+00	3.859E-01	2.328E+00
20	4.975E-06	4.540E-05	-2.984E-06	-7.606E+00	6.134E-01	1.936E-01
21	3.555E-06	3.694E-05	-4.274E-06	-5.994E+00	8.017E-01	8.890E-01
22	2.443E-06	2.623E-05	-5.167E-06	-4.235E+00	9.325E-01	1.281E+00
23	1.486E-06	1.356E-05	-5.764E-06	-2.255E+00	1.000E+00	4.038E-01
24	5.081E-08	-8.904E-08	-5.994E-06	-6.644E-11	1.000E+00	1.404E-01
0	-5.863E-07	-6.175E-06	-8.872E-08	1.000E+00	0.000E-01	-1.295E-01
1	-5.866E-07	-6.179E-06	8.591E-08	1.000E+00	-3.080E-02	-2.828E-01
2	-6.322E-07	-5.769E-06	2.705E-07	9.670E-01	-5.971E-02	-1.689E-02
3	-4.744E-07	-4.943E-06	4.379E-07	8.017E-01	-8.491E-02	-1.158E-01
4	-3.567E-07	-3.809E-06	5.616E-07	6.155E-01	-1.039E-01	-1.829E-01
5	-2.629E-07	-2.399E-06	6.556E-07	4.002E-01	-1.159E-01	-4.558E-02
6	-8.455E-08	-8.156E-07	7.079E-07	1.338E-01	-1.201E-01	-3.420E-02
7	8.222E-08	7.911E-07	7.083E-07	-1.296E-01	-1.161E-01	2.414E-02
8	2.604E-07	2.376E-06	6.567E-07	-4.010E-01	-1.042E-01	-4.757E-02
9	3.548E-07	3.790E-06	5.633E-07	-6.125E-01	-8.491E-02	6.742E-02
10	4.729E-07	4.928E-06	4.401E-07	-7.988E-01	-6.034E-02	2.170E-01
11	6.312E-07	5.760E-06	2.731E-07	-9.673E-01	-3.148E-02	-2.170E-02
12	5.863E-07	6.175E-06	8.872E-08	-1.000E+00	-1.213E-10	1.295E-01
13	5.866E-07	6.179E-06	-8.591E-08	-1.000E+00	3.080E-02	2.828E-01
14	6.322E-07	5.769E-06	-2.705E-07	-9.670E-01	5.971E-02	1.689E-02
15	4.744E-07	4.943E-06	-4.379E-07	-8.017E-01	8.491E-02	1.158E-01
16	3.567E-07	3.809E-06	-5.616E-07	-6.155E-01	1.039E-01	1.829E-01
17	2.629E-07	2.399E-06	-6.556E-07	-4.002E-01	1.159E-01	4.558E-02
18	8.455E-08	8.156E-07	-7.079E-07	-1.338E-01	1.201E-01	3.420E-02
19	-8.222E-08	-7.911E-07	-7.083E-07	1.296E-01	1.161E-01	-2.414E-02
20	-2.604E-07	-2.376E-06	-6.567E-07	4.010E-01	1.042E-01	4.757E-02
21	-3.548E-07	-3.790E-06	-5.633E-07	6.125E-01	8.491E-02	-6.742E-02
22	-4.729E-07	-4.928E-06	-4.401E-07	7.988E-01	6.034E-02	-2.170E-01
23	-6.312E-07	-5.760E-06	-2.731E-07	9.673E-01	3.148E-02	2.170E-02
24	-5.863E-07	-6.175E-06	-8.872E-08	1.000E+00	1.179E-12	-1.295E-01

Table 11. Mode shape for the lowest in-plane natural frequency of the flywheel.

Point	u	-w	ψ	My	Vz	N
0	-4.696E-05	-1.569E-08	5.375E-06	1.000E+00	-8.815E-01	1.154E-01
1	-4.696E-05	1.569E-08	5.375E-06	-1.187E+00	4.960E-01	-2.093E-01
2	-4.697E-05	-7.398E-14	5.354E-06	-7.934E-03	5.334E-01	7.163E-02
3	-4.696E-05	-1.569E-08	5.375E-06	1.000E+00	-8.815E-01	1.154E-01
4	-4.696E-05	1.569E-08	5.375E-06	-1.187E+00	4.960E-01	-2.093E-01
5	-4.697E-05	-7.369E-14	5.354E-06	-7.934E-03	5.334E-01	7.163E-02
6	-4.696E-05	-1.569E-08	5.375E-06	1.000E+00	-8.815E-01	1.154E-01
7	-4.696E-05	1.569E-08	5.375E-06	-1.187E+00	4.960E-01	-2.093E-01
8	-4.697E-05	-7.400E-14	5.354E-06	-7.934E-03	5.334E-01	7.163E-02
9	-4.696E-05	-1.569E-08	5.375E-06	1.000E+00	-8.815E-01	1.154E-01
10	-4.696E-05	1.569E-08	5.375E-06	-1.187E+00	4.960E-01	-2.093E-01
11	-4.697E-05	-7.423E-14	5.354E-06	-7.934E-03	5.334E-01	7.163E-02
12	-4.696E-05	-1.569E-08	5.375E-06	1.000E+00	-8.815E-01	1.154E-01
13	-4.696E-05	1.569E-08	5.375E-06	-1.187E+00	4.960E-01	-2.093E-01
14	-4.697E-05	-7.418E-14	5.354E-06	-7.934E-03	5.334E-01	7.163E-02
15	-4.696E-05	-1.569E-08	5.375E-06	1.000E+00	-8.815E-01	1.154E-01
16	-4.696E-05	1.569E-08	5.375E-06	-1.187E+00	4.960E-01	-2.093E-01
17	-4.697E-05	-7.387E-14	5.354E-06	-7.934E-03	5.334E-01	7.163E-02
18	-4.696E-05	-1.569E-08	5.375E-06	1.000E+00	-8.815E-01	1.154E-01
19	-4.696E-05	1.569E-08	5.375E-06	-1.187E+00	4.960E-01	-2.093E-01
20	-4.697E-05	-7.366E-14	5.354E-06	-7.934E-03	5.334E-01	7.163E-02
21	-4.696E-05	-1.569E-08	5.375E-06	1.000E+00	-8.815E-01	1.154E-01
22	-4.696E-05	1.569E-08	5.375E-06	-1.187E+00	4.960E-01	-2.093E-01
23	-4.697E-05	-7.441E-14	5.354E-06	-7.934E-03	5.334E-01	7.163E-02
24	-4.696E-05	-1.569E-08	5.375E-06	1.000E+00	-8.815E-01	1.154E-01

be good because the lowest out-of-plane modal frequency calculated by the transfer matrix method was 13.19 cps, very close to the one-degree-of-freedom vibration solution of 13.13 cps. According to these calculations, only three modal frequencies are within the operating range of the flywheel.

ACKNOWLEDGMENT

This work was conducted for and funded by the Composite Flywheel Development Program within the Y-12 Development Division. C. E. Knight served as the coordinator for this portion of the program.

REFERENCES

1. Klaus-Jürgen Bathe, E. L. Wilson, and R. H. Iding, *NONSAP: A Structural Analysis Program for Static and Dynamic Response of Nonlinear Systems*, Report UC SESM 74-3, University of California, Berkeley (February 1974).
2. R. H. Gallagher, *Finite Element Analysis Fundamentals*, p. 302, Prentice-Hall, Englewood Cliffs, N.J., 1975.
3. Klaus-Jürgen Bathe, *ADINA: A Finite Element Program for Automatic Dynamic Incremental Nonlinear Analysis*, Report 82448-1, Massachusetts Institute of Technology (1975).
4. D. Young, *Handbook of Engineering Mechanics*, chap. 61, pp. 19-20, ed. by W. Flugge, McGraw-Hill, New York, 1962.
5. E. C. Pestel and F. A. Leckie, *Matrix Methods in Elastomechanics*, pp. 338-95, McGraw-Hill, New York, 1963.
6. H. C. Martin, *Introduction to Matrix Methods of Structural Analysis*, p. 161, McGraw-Hill, New York, 1966.

THIS PAGE
WAS INTENTIONALLY
LEFT BLANK

Appendix

PROGRAM LISTING

File 1: Driver

```

1 SET NOKEY
2 PRINT @32,26:0
3 GO TO 3000
4 REM *** ROOT SEARCH ***
5 GO TO 4790
8 REM *** DISPLAY ***
9 GO TO 6790
12 REM *** PRINT MODE SHAPES ***
13 GO TO 7880
20 END
100 REM *****
110 REM
120 REM          *** CALCULATE TRANSFER MATRIX FOR RING ***
130 REM
140 REM *****
150 CALL "IDN",P
160 REM *****
170 REM          *** CALCULATE TRANSFER MATRIX FOR ONE SEGMENT ***
180 REM *****
190 FOR NO=1 TO N
200 GOSUB 2000
210 Q1=U MPY P
220 P=Q1
230 NEXT NO
240 U=P
250 REM *****
260 REM          *** MULTIPLY TRANSFER MATRICES FOR ALL SEGMENTS TOGETHER ***
270 REM *****
280 CALL "IDN",P
290 L1=M
300 L2=0
310 L2=L2+1
320 IF L1/2\L2=>2 THEN 310
330 Q1=U
340 FOR L3=1 TO L2
350 Q2=Q1 MPY Q1
360 Q1=Q2
370 NEXT L3
380 Q2=P MPY Q1
390 P=Q2
400 L1=L1-2\L2
410 IF L1=>2 THEN 300
420 IF L1<1 THEN 440
430 P=U MPY Q2
440 FOR L1=1 TO 6
450 P(L1,L1)=P(L1,L1)-1
460 NEXT L1
470 RETURN
1000 REM *****
1010 REM
1020 REM          *** EQUATION SOLVER ***
1030 REM
1040 REM          *** GAUSS-JORDAN REDUCTION WITH MAXIMUM PIVOT STRATEGY ***
1050 REM
1060 REM *****
1070 R1=1

```

```

1080 R2=6
1090 C1=1
1100 C2=6
1110 S=1
1120 FOR L3=1 TO 6
1130 T(L3)=L3
1140 NEXT L3
1150 FOR L3=C1 TO C2
1160 Q=0
1170 FOR L1=L3+R1-C1 TO R2
1180 FOR L2=L3 TO C2
1190 IF ABS(P(L1,L2))<ABS(Q) THEN 1230
1200 Q=P(L1,L2)
1210 L4=L1
1220 L5=L2
1230 NEXT L2
1240 NEXT L1
1250 IF L3<>C1 THEN 1270
1260 Q0=0.005*ABS(Q)
1270 IF L4=L3+R1-C1 THEN 1290
1280 S=-S
1290 IF L5=L3 THEN 1310
1300 S=-S
1310 S=S*Q
1320 IF ABS(S)<=Q0 THEN 1610
1330 L6=T(L5)
1340 T(L5)=T(L3)
1350 T(L3)=L6
1360 FOR L2=C1 TO C2
1370 L6=P(L4,L2)
1380 P(L4,L2)=P(L3+R1-C1,L2)
1390 P(L3+R1-C1,L2)=L6
1400 NEXT L2
1410 FOR L1=R1 TO R2
1420 L6=P(L1,L5)
1430 P(L1,L5)=P(L1,L3)
1440 P(L1,L3)=L6
1450 NEXT L1
1460 IF L3+1>C2 THEN 1500
1470 FOR L2=L3+1 TO C2
1480 P(L3+R1-C1,L2)=P(L3+R1-C1,L2)/Q
1490 NEXT L2
1500 P(L3+R1-C1,L3)=1
1510 FOR L1=R1 TO R2
1520 IF L1-L3+R1-C1 THEN 1590
1530 IF P(L1,L3)=0 THEN 1590
1540 IF L3+1>C2 THEN 1580
1550 FOR L2=L3+1 TO C2
1560 P(L1,L2)=P(L1,L2)-P(L1,L3)*P(L3+R1-C1,L2)
1570 NEXT L2
1580 P(L1,L3)=0
1590 NEXT L1
1600 NEXT L3
1610 FOR L1=L3+R1-C1 TO R2
1620 FOR L2=L3 TO C2
1630 P(L1,L2)=0
1640 NEXT L2
1650 P(L1,L1+C1-R1)=-1
1660 NEXT L1
1670 IF R1=C1 THEN 1740
1680 FOR L1=R1 TO R2
1690 FOR L2=L3 TO C2
1700 P(L1-R1+C1,L2)=P(L1,L2)
1710 P(L1,L2)=0
1720 NEXT L2

```

```

1730 NEXT L1
1740 L1=0
1750 L1=L1+1
1760 IF L1>6 THEN 1870
1770 IF T(L1)=L1 THEN 1750
1780 L4=T(L1)
1790 T(L1)=T(L4)
1800 T(L4)=L4
1810 FOR L2=L3 TO C2
1820 L5=P(L1,L2)
1830 P(L1,L2)=P(L4,L2)
1840 P(L4,L2)=L5
1850 NEXT L2
1860 GO TO 1770
1870 RETURN
2000 REM      *** TRANSFER MATRIX WILL BE APPENDED HERE ***
3000 PAGE
3010 DIM P(6,6),P0(6),Q1(6,6),Q2(6,6),S0(2),T(6),U(6,6),X(2),R0(3),D0(3)
3020 DIM K9(3,3),A$(1),Y$(1)
3030 F1=0
3040 F2=0
3050 PRINT "THIS PROGRAM CALCULATES THE NATURAL FREQUENCIES AND MODE"
3060 PRINT "SHAPES FOR A RING STRUCTURE USING THE TRANSFER MATRIX"
3070 PRINT "TECHNIQUE."
3080 PRINT
3090 PRINT "DO YOU WANT IN-PLANE MODES? ";
3100 INPUT A$
3110 I=3
3120 IF A$="N" THEN 3150
3130 IF A$<>"Y" THEN 3050
3140 I=2
3150 FIND I
3160 DELETE 2001,2999
3170 APPEND 2000
3180 REM *****
3190 REM
3200 REM      *** READ INPUT DATA ***
3210 REM
3220 REM *****
3230 REM      N      - NUMBER OF ELEMENTS IN A SEGMENT
3240 REM      R      - RADIUS OF RING CENTER-LINE
3250 REM      X(1)    - MODULUS OF ELASTICITY * MOMENT OF INERTIA OF
3260 REM                CROSS-SECTION
3270 REM      A(I)    - CENTRAL ANGLE SUBTENDED BY ELEMENT 'I'
3280 REM
3290 REM      *** OUT-OF-PLANE CASE ***
3300 REM
3310 REM      X(2)    - SHEAR MODULUS * POLAR MOMENT OF INERTIA
3320 REM      B(1,I)  - ROTARY SPRING STIFFNESS ABOUT X-AXIS FOR
3330 REM                ELEMENT 'I'
3340 REM      B(2,I)  - LONGITUDINAL SPRING STIFFNESS ALONG Y-AXIS FOR
3350 REM                ELEMENT 'I'
3360 REM      B(3,I)  - ROTARY SPRING STIFFNESS ABOUT Z-AXIS FOR
3370 REM                ELEMENT 'I'
3380 REM      W(1,I)  - MASS MOMENT OF INERTIA ABOUT X-AXIS FOR
3390 REM                ELEMENT 'I'
3400 REM      W(2,I)  - MASS OF ELEMENT 'I'
3410 REM      W(3,I)  - MASS MOMENT OF INERTIA ABOUT Z-AXIS FOR
3420 REM                ELEMENT 'I'
3430 REM
3440 REM      *** IN-PLANE CASE ***
3450 REM
3460 REM      X(2)    - MODULUS OF ELASTICITY * AREA OF CROSS-SECTION
3470 REM      B(1,I)  - MODULUS OF ELASTICITY * AREA OF STRAP FOR
3480 REM                ELEMENT 'I'

```

```

3490 REM      B(2,I) - MODULUS OF ELASTICITY * MOMENT OF INERTIA OF
3500 REM      CROSS-SECTION FOR STRAP OF ELEMENT 'I'
3510 REM      B(3,I) - LENGTH OF STRAP FOR ELEMENT 'I'
3520 REM      W(1,I) - MASS OF ELEMENT 'I'
3530 REM      W(2,I) - ANGLE BETWEEN STRAP AND NORMAL TO THE RING FOR
3540 REM      ELEMENT 'I'
3550 REM      W(3,I) - MASS MOMENT OF INERTIA ABOUT Y-AXIS FOR
3560 REM      ELEMENT 'I'
3570 REM
3580 REM *****
3590 SET RADIANS
3600 S0=0
3610 PRINT
3620 PRINT "INPUT THE NUMBER OF SEGMENTS      : ";
3630 INPUT N
3640 PRINT "INPUT THE RADIUS                    : ";
3650 INPUT R
3660 PRINT "INPUT E AND I                      : ";
3670 INPUT T(1),T(2)
3680 IF A$="Y" THEN 3730
3690 PRINT "INPUT G AND J                      : ";
3700 INPUT T(3),T(4)
3710 X(2)=T(3)*T(4)
3720 GO TO 3760
3730 PRINT "INPUT THE AREA                      : ";
3740 INPUT T(3)
3750 X(2)=T(1)*T(3)
3760 X(1)=T(1)*T(2)
3770 DELETE A,W,B
3780 DIM A(N),W(3,N),B(3,N)
3790 Q=0
3800 IF A$="Y" THEN 4060
3810 REM *****
3820 REM      *** OUT-OF-PLANE CASE ***
3830 REM *****
3840 PRINT
3850 PRINT "INPUT THE FOLLOWING"
3860 FOR I=1 TO N
3870 PRINT
3880 PRINT "ANGLE                                : ";
3890 INPUT A(I)
3900 PRINT "ROTARY SPRING STIFFNESS (X)         : ";
3910 INPUT B(1,I)
3920 PRINT "LONGITUDINAL SPRING STIFFNESS (Y) : ";
3930 INPUT B(2,I)
3940 PRINT "ROTARY SPRING STIFFNESS (Z)         : ";
3950 INPUT B(3,I)
3960 PRINT "MOMENT OF INERTIA (X)                : ";
3970 INPUT W(1,I)
3980 PRINT "MASS                                : ";
3990 INPUT W(2,I)
4000 PRINT "MOMENT OF INERTIA (Z)                : ";
4010 INPUT W(3,I)
4020 Q=Q+A(I)
4030 A(I)=A(I)*PI/180
4040 NEXT I
4050 GO TO 4340
4060 REM *****
4070 REM      *** IN-PLANE CASE ***
4080 REM *****
4090 PRINT
4100 PRINT "INPUT THE FOLLOWING"
4110 FOR I=1 TO N
4120 PRINT
4130 PRINT "ANGLE                                : ";

```

```

4140 INPUT A(I)
4150 PRINT "A*E FOR THE STRAP           : ";
4160 INPUT B(1,I)
4170 PRINT "E*I FOR THE STRAP           : ";
4180 INPUT B(2,I)
4190 PRINT "LENGTH OF THE STRAP         : ";
4200 INPUT B(3,I)
4210 B(3,I)=1/B(3,I)
4220 B(1,I)=B(1,I)*B(3,I)
4230 B(2,I)=B(2,I)*B(3,I)
4240 PRINT "ANGLE - STRAP AND NORMAL TO RING : ";
4250 INPUT W(2,I)
4260 W(2,I)=W(2,I)*PI/180
4270 PRINT "MASS                         : ";
4280 INPUT W(1,I)
4290 PRINT "MOMENT OF INERTIA (Y)         : ";
4300 INPUT W(3,I)
4310 Q=Q+A(I)
4320 A(I)=A(I)*PI/180
4330 NEXT I
4340 M=INT(360/Q+1.0E-4)
4350 IF M*Q=360 THEN 4390
4360 PRINT
4370 PRINT "THE INPUT ANGLES ARE INCORRECT - TRY AGAIN"
4380 GO TO 3610
4390 PRINT
4400 PRINT "INPUT FILE NUMBER FOR STORING DETERMINANT VS. FREQUENCY: ";
4410 INPUT F1
4420 IF F1<4 THEN 4390
4430 IF F1=F2 THEN 4390
4440 PRINT "IS THIS FILE EMPTY? ";
4450 INPUT Y$
4460 IF Y$="Y" THEN 4530
4470 FIND F1
4480 ON EOF (0) THEN 4520
4490 READ @33:O,S
4500 S0(1)=S0(1)+1
4510 GO TO 4490
4520 OFF EOF (0)
4530 PRINT
4540 PRINT "INPUT FILE NUMBER FOR STORING MODE SHAPES: ";
4550 INPUT F2
4560 IF F2<4 THEN 4530
4570 IF F2=F1 THEN 4530
4580 PRINT "IS THIS FILE EMPTY? ";
4590 INPUT Y$
4600 IF Y$="Y" THEN 4700
4610 FIND F2
4620 ON EOF (0) THEN 4690
4630 READ @33:O,J,K,L
4640 FOR I=1 TO L*J*K+L
4650 READ @33:P0
4660 NEXT I
4670 S0(2)=S0(2)+1
4680 GO TO 4630
4690 OFF EOF (0)
4700 PRINT
4710 PRINT "THIS PROGRAM IS CONTROLLED BY FUNCTION KEYS AS FOLLOWS:"
4720 PRINT
4730 PRINT "KEY 1 - ROOT SEARCH"
4740 PRINT "KEY 2 - DISPLAY DETERMINANT VS. FREQUENCY"
4750 PRINT "KEY 3 - PRINT MODE SHAPES"
4760 PRINT "KEY 5 - STOP EXECUTION"
4770 PRINT
4780 PRINT "BEGINNING ROOT SEARCH ..."

```

```

4790 REM *****
4800 REM
4810 REM *** ROOT SEARCH ***
4820 REM
4830 REM *****
4840 SET KEY
4850 PRINT @32,26:2
4860 FIND F1
4870 IF S0(1)=0 THEN 4910
4880 FOR I=1 TO S0(1)
4890 READ @33:0,S
4900 NEXT I
4910 R0=0
4920 D0=0
4930 PRINT
4940 PRINT "INPUT STARTING FREQUENCY AND INCREMENT: ";
4950 INPUT O,00
4960 GO TO 4980
4970 O=O+00
4980 GOSUB 100
4990 P=INV(P)
5000 S=DET
5010 WRITE @33:0,S
5020 S0(1)=S0(1)+1
5030 PRINT USING 5040:"FREQUENCY =",O," DET =",S
5040 IMAGE 12A,9E,9A,3E
5050 R0(1)=R0(2)
5060 D0(1)=D0(2)
5070 R0(2)=R0(3)
5080 D0(2)=D0(3)
5090 R0(3)=O
5100 D0(3)=S
5110 IF SGN(D0(2))=-SGN(D0(3)) THEN 5260
5120 IF D0(1)=0 THEN 4970
5130 IF SGN(D0(2)-D0(1))<>SGN(D0(2)-D0(3)) THEN 4970
5140 IF SGN(D0(1))=SGN(D0(2)-D0(1)) THEN 4970
5150 PRINT "CRITERIA 2 IS MET -- ITERATING FOR ROOT"
5160 L=0
5170 L0=10
5180 E=1.0E-12*ABS(D0(1)+D0(2)+D0(3))
5190 O=R0(3)
5200 S=D0(3)
5210 R0(3)=R0(2)
5220 D0(3)=D0(2)
5230 R0(2)=O
5240 D0(2)=S
5250 GO TO 5720
5260 PRINT "CRITERIA 1 IS MET -- ITERATING FOR ROOT"
5270 L=0
5280 L0=10
5290 E=1.0E-10*ABS(D0(2)-D0(3))
5300 R0(1)=R0(2)
5310 D0(1)=D0(2)
5320 R0(2)=R0(3)
5330 D0(2)=D0(3)
5340 REM *****
5350 REM *** POINTS SATISFY CRITERIA 1 ***
5360 REM *****
5370 O=R0(2)+D0(2)*(R0(2)-R0(1))/(D0(1)-D0(2))
5380 GOSUB 100
5390 P=INV(P)
5400 S=DET
5410 WRITE @33:0,S
5420 S0(1)=S0(1)+1
5430 PRINT USING 5040:"FREQUENCY =",O," DET =",S

```

```

5440 IF ABS(S)<E THEN 6180
5450 L=L+1
5460 IF L<L0 THEN 5490
5470 I=1
5480 GO TO 6120
5490 IF L=1 THEN 5530
5500 I=1+(S*D0(2)=>0)
5510 R0(I)=R0(3)
5520 D0(I)=D0(3)
5530 R0(3)=0
5540 D0(3)=S
5550 T(4)=D0(1)*(R0(3)-R0(2))
5560 T(5)=D0(2)*(R0(1)-R0(3))
5570 T(6)=D0(3)*(R0(2)-R0(1))
5580 T(1)=T(4)+T(5)+T(6)
5590 T(2)=-T(4)*(R0(3)+R0(2))-T(5)*(R0(1)+R0(3))-T(6)*(R0(2)+R0(1))
5600 O2=2*T(1)/(4*T(1)*O*(T(1)*O+T(2))+T(2)*T(2)+1)|1.5
5610 IF O2<1.0E-20 THEN 5680
5620 T(3)=R0(2)*R0(3)*T(4)+R0(3)*R0(1)*T(5)+R0(1)*R0(2)*T(6)
5630 T(6)=SQR(T(2)*T(2)-4*T(1)*T(3))
5640 T(4)=(-T(2)+T(6))/(2*T(1))
5650 T(5)=(-T(2)-T(6))/(2*T(1))
5660 O=T(5-(T(4)>R0(1) AND T(4)<R0(2)))
5670 GO TO 5380
5680 I=2-(SGN(S)=SGN(D0(1)))
5690 D0(I)=S
5700 R0(I)=0
5710 GO TO 5370
5720 REM *****
5730 REM *** POINTS SATISFY CRITERIA 2 ***
5740 REM *****
5750 O=D0(1)*(R0(3)|2-R0(2)|2)
5760 O=D0(2)*(R0(1)|2-R0(3)|2)+O
5770 O=D0(3)*(R0(2)|2-R0(1)|2)+O
5780 O2=R0(2)*D0(1)+R0(3)*D0(2)+R0(1)*D0(3)
5790 O2=R0(1)*D0(2)+R0(2)*D0(3)+R0(3)*D0(1)-O2
5800 O=0.5*O/O2
5810 GOSUB 100
5820 P=INV(P)
5830 S=DET
5840 WRITE @33:O,S
5850 S0(1)=S0(1)+1
5860 PRINT USING 5040:"FREQUENCY =",O," DET =",S
5870 IF ABS(S)<E THEN 6180
5880 L=L+1
5890 IF L<L0 THEN 5920
5900 I=2
5910 GO TO 6120
5920 IF SGN(S)=SGN(D0(1)) THEN 5990
5930 PRINT
5940 PRINT "MORE THAN ONE DISTINCT ROOT IN THIS INTERVAL"
5950 PRINT "WILL DETERMINE THE SMALLER ROOT"
5960 R0(2)=R0(3)
5970 D0(2)=D0(3)
5980 GO TO 5340
5990 IF S<D0(3) THEN 6040
6000 I=2-(O<R0(3))
6010 R0(I)=0
6020 D0(I)=S
6030 GO TO 5750
6040 I=2-(O>R0(3))
6050 O2=R0(3)
6060 R0(3)=0
6070 R0(I)=O2
6080 O2=D0(3)

```



```

6090 DO(3)=S
6100 DO(I)=O2
6110 GO TO 5750
6120 PRINT
6130 PRINT "ITERATIONS EXCEEDED -- MORE ITERATIONS? ";
6140 INPUT Y$
6150 IF Y$<>"Y" THEN 6200
6160 L0=L0+5
6170 GO TO I OF 5460,5890
6180 PRINT
6190 PRINT "SOLUTION CONVERGED -- WILL DETERMINE MODE SHAPE(S)"
6200 REM *****
6210 REM
6220 REM          *** DETERMINE MODE SHAPE(S) ***
6230 REM
6240 REM *****
6250 FIND F2
6260 IF S0(2)=0 THEN 6330
6270 FOR I=1 TO S0(2)
6280 READ @33:S,J,K,L
6290 FOR N1=1 TO L*J*K+L
6300 READ @33:P0
6310 NEXT N1
6320 NEXT I
6330 REM *****
6340 REM          *** DETERMINE NUMBER OF ROOTS ***
6350 REM *****
6360 GOSUB 100
6370 GOSUB 1000
6380 R1=C2+1-L3
6390 WRITE @33:O,M,N,R1
6400 PRINT USING 6410:R1," ROOT(S)"
6410 IMAGE 4D,8A
6420 FOR L=1 TO R1
6430 IF A$="Y" THEN 6490
6440 PRINT
6450 PRINT "POINT      THETA      Y      PHI";
6460 PRINT "          M          -V      TORQUE"
6470 PRINT
6480 GO TO 6530
6490 PRINT
6500 PRINT "POINT      X      -Z      SI";
6510 PRINT "          M          V      N"
6520 PRINT
6530 REM *****
6540 REM          *** DETERMINE STATE VECTOR AT POINT 0 ***
6550 REM *****
6560 FOR I=1 TO 6
6570 P0(I)=-F(I,L3+I-1)
6580 NEXT I
6590 WRITE @33:P0
6600 J=0
6610 PRINT USING 6620:J,P0
6620 IMAGE 3D,3(1X,3E),3(1X,2E)
6630 REM *****
6640 REM          *** DETERMINE STATE VECTOR AT ALL OTHER POINTS ***
6650 REM *****
6660 FOR I=1 TO M
6670 FOR N0=1 TO N
6680 GOSUB 2000
6690 T=U MPY P0
6700 P0=T
6710 J=J+1
6720 WRITE @33:P0
6730 PRINT USING 6620:J,P0

```

```

6740 NEXT NO
6750 NEXT I
6760 NEXT L
6770 S0(2)=S0(2)+1
6780 END
6790 REM *****
6800 REM
6810 REM          *** PLOT DETERMINANT VS. FREQUENCY ***
6820 REM
6830 REM *****
6840 PRINT @32,26:2
6850 FIND F1
6860 PRINT
6870 PRINT "INPUT MINIMUM AND MAXIMUM FREQUENCY FOR PLOT: ";
6880 INPUT T(5),T(6)
6890 T(1)=1.0E+300
6900 T(2)=-T(1)
6910 T(3)=T(1)
6920 T(4)=T(2)
6930 IF S0(1)<>0 THEN 6960
6940 PRINT "NO POINTS ON TAPE"
6950 END
6960 FOR I=1 TO S0(1)
6970 READ @33:O,S
6980 IF O>T(6) THEN 7040
6990 IF O<T(5) THEN 7040
7000 T(1)=T(1) MIN O
7010 T(2)=T(2) MAX O
7020 T(3)=T(3) MIN S
7030 T(4)=T(4) MAX S
7040 NEXT I
7050 REM *****
7060 REM          *** DETERMINE PLOT LIMITS ***
7070 REM *****
7080 O=T(1)
7090 S=T(2)
7100 GOSUB 7670
7110 T(1)=O
7120 T(2)=S
7130 J=I
7140 O=T(3)
7150 S=T(4)
7160 GOSUB 7670
7170 T(3)=O
7180 T(4)=S
7190 VIEWPORT 18,125,10,100
7200 WINDOW T(1),T(2),T(3),T(4)
7210 PAGE
7220 AXIS (T(2)-T(1))/J,(T(4)-T(3))/I,T(1),T(3)
7230 AXIS (T(2)-T(1))/J,(T(4)-T(3))/I,T(2),T(4)
7240 AXIS
7250 FIND F1
7260 REM *****
7270 REM          *** PLOT POINTS ***
7280 REM *****
7290 FOR K=1 TO S0(1)
7300 READ @33:O,S
7310 IF O>T(6) THEN 7410
7320 IF O<T(5) THEN 7410
7330 MOVE O,S
7340 SCALE 1.5,1.5
7350 RMOVE 1,0
7360 RDRAW -1,-1
7370 RDRAW -1,1
7380 RDRAW 1,1

```

```

7390 RDRAW 1,-1
7400 WINDOW T(1),T(2),T(3),T(4)
7410 NEXT K
7420 VIEWPORT 0,130,0,100
7430 WINDOW 0,130,0,100
7440 S=(T(2)-T(1))/J
7450 O=T(1)-S
7460 K=107/J
7470 L=10-K
7480 FOR IO=1 TO J+1
7490 O=O+S
7500 L=L+K
7510 MOVE L,6
7520 PRINT O
7530 NEXT IO
7540 S=(T(4)-T(3))/I
7550 N1=INT((T(3)+1.0E-3*S)/S)-1
7560 K=90/I
7570 L=9-K
7580 FOR IO=1 TO I+1
7590 O=S*(N1+IO)
7600 L=L+K
7610 MOVE O,L
7620 PRINT O
7630 NEXT IO
7640 MOVE 41,2
7650 PRINT "DETERMINANT VS. CIRCULAR FREQUENCY";
7660 END
7670 REM *****
7680 REM *** CALCULATE PLOT LIMITS ***
7690 REM *****
7700 L=1
7710 C=2*10|INT(LGT(S-O)-1.0E-3)
7720 I=INT((S-C*1.0E-6)/C)-INT((O+C*1.0E-6)/C)+1
7730 IF I>4 THEN 7850
7740 IF L<>1 THEN 7780
7750 L=2
7760 C=C*0.5
7770 GO TO 7720
7780 IF L<>2 THEN 7820
7790 L=3
7800 C=C*0.5
7810 GO TO 7720
7820 L=1
7830 C=C*0.4
7840 GO TO 7720
7850 S=C*INT((S-C*1.0E-6)/C+1)
7860 O=C*INT((O+C*1.0E-6)/C)
7870 RETURN
7880 REM *****
7890 REM
7900 REM *** PRINT MODE SHAPES ***
7910 REM
7920 REM *****
7930 PAGE
7940 PRINT "ROOT NO.      VALUE      MULTIPLICITY"
7950 FIND F2
7960 IF S0(2)<>0 THEN 7990
7970 PRINT "STOP -- NO MODE SHAPES ON TAPE"
7980 END
7990 FOR I=1 TO S0(2)
8000 READ @33:O,J,K,L
8010 PRINT USING 8020:I,O,L
8020 IMAGE 8D,2X,3E,4D
8030 FOR N1=1 TO L*J*K+L

```

```

8040 READ @33:P0
8050 NEXT N1
8060 NEXT I
8070 PRINT
8080 PRINT "FOR WHICH ROOT DO YOU WANT THE MODE SHAPES? ";
8090 INPUT N2
8100 FIND P2
8110 IF N2=1 THEN 8180
8120 FOR I=1 TO N2-1
8130 READ @33:O,J,K,L
8140 FOR N1=1 TO L*J*K+L
8150 READ @33:P0
8160 NEXT N1
8170 NEXT I
8180 PRINT @32,26:0
8190 PAGE
8200 READ @33:O,J,K,L
8210 FOR I=1 TO L
8220 IF A$="Y" THEN 8280
8230 PRINT
8240 PRINT "POINT      THETA      Y      PHI";
8250 PRINT "          M          -V      TORQUE"
8260 PRINT
8270 GO TO 8310
8280 PRINT "POINT      X      -Z      SI";
8290 PRINT "          M          V      N"
8300 PRINT
8310 L1=0
8320 READ @33:P0
8330 PRINT USING 6620:L1,P0
8340 FOR L2=1 TO J
8350 FOR L3=1 TO K
8360 L1=L1+1
8370 READ @33:P0
8380 PRINT USING 6620:L1,P0
8390 NEXT L3
8400 NEXT L2
8410 NEXT I
8420 END

```

File 2: In-Plane Transfer Matrix

```

2000 REM *****
2010 REM
2020 REM          *** IN-PLANE TRANSFER MATRIX ***
2030 REM
2040 REM *****
2050 O2=0*0
2060 Q=A(N0)
2070 S=SIN(Q)
2080 C=COS(Q)
2090 T(1)=0.5*(Q*C+S)
2100 T(3)=0.5*(S-Q*C)
2110 T(5)=0.5*(2-2*C-Q*S)
2120 T(6)=0.5*(Q*(2+C)-3*S)
2130 T(2)=1/X(2)
2140 T(4)=R/X(1)
2150 U(3,4)=Q*T(4)
2160 T(4)=R*T(4)

```

```

2170 U(1,1)=C
2180 U(1,2)=-S
2190 U(1,3)=R*(C-1)
2200 U(1,4)=(S-Q)*T(4)
2210 U(1,5)=R*(Q*S*0.5*T(2)-T(5)*T(4))
2220 U(1,6)=R*(T(1)*T(2)+T(6)*T(4))
2230 U(2,1)=S
2240 U(2,2)=C
2250 U(2,3)=R*S
2260 U(2,4)=(1-C)*T(4)
2270 U(2,5)=R*T(3)*(T(2)+T(4))
2280 U(2,6)=U(1,5)
2290 U(3,1)=0
2300 U(3,2)=0
2310 U(3,3)=1
2320 U(3,5)=U(2,4)
2330 U(3,6)=U(1,4)
2340 T(1)=COS(W(2,N0))
2350 T(2)=SIN(W(2,N0))
2360 T(3)=B(3,N0)*B(3,N0)
2370 T(4)=T(1)*T(1)
2380 T(5)=T(2)*T(2)
2390 K9(1,1)=6*B(2,N0)*B(3,N0)*T(1)
2400 K9(1,2)=6*B(2,N0)*B(3,N0)*T(2)
2410 K9(1,3)=4*B(2,N0)-W(3,N0)*02
2420 K9(2,1)=(B(1,N0)-12*B(2,N0)*T(3))*T(1)*T(2)
2430 K9(2,2)=-B(1,N0)*T(4)-12*B(2,N0)*T(3)*T(5)+W(1,N0)*02
2440 K9(2,3)=-K9(1,2)
2450 K9(3,1)=B(1,N0)*T(5)+12*B(2,N0)*T(3)*T(4)-W(1,N0)*02
2460 K9(3,2)=-K9(2,1)
2470 K9(3,3)=K9(1,1)
2480 FOR L1=4 TO 6
2490 FOR L2=1 TO 6
2500 U(L1,L2)=0
2510 FOR L3=1 TO 3
2520 U(L1,L2)=U(L1,L2)+K9(L1-3,L3)*U(L3,L2)
2530 NEXT L3
2540 NEXT L2
2550 NEXT L1
2560 U(4,4)=U(4,4)+1
2570 U(4,5)=U(4,5)+U(2,3)
2580 U(4,6)=U(4,6)+U(1,3)
2590 U(5,5)=U(5,5)+C
2600 U(5,6)=U(5,6)-S
2610 U(6,5)=U(6,5)+S
2620 U(6,6)=U(6,6)+C
2630 RETURN

```

File 3: Out-of-Plane Transfer Matrix

```

2000 REM *****
2010 REM
2020 REM          *** OUT-OF-PLANE TRANSFER MATRIX ***
2030 REM
2040 REM *****
2050 O2=0*0
2060 Q=A(N0)
2070 S=SIN(Q)
2080 C=COS(Q)

```

```

2090 T(1)=0.5*(Q*C+S)
2100 T(3)=0.5*(S-Q*C)
2110 T(5)=0.5*(2-2*C-Q*S)
2120 T(6)=0.5*(Q*(2+C)-3*S)
2130 T(2)=1/X(2)
2140 T(4)=1/X(1)
2150 U(1,2)=0
2160 U(3,2)=0
2170 U(2,2)=1
2180 U(1,3)=S
2190 U(3,1)=-S
2200 U(2,3)=R*S
2210 U(1,1)=C
2220 U(3,3)=C
2230 U(2,1)=R*(C-1)
2240 U(1,4)=R*Q*S*0.5*(T(2)+T(4))
2250 U(3,6)=-U(1,4)
2260 U(1,5)=R*R*T(3)*(T(2)+T(4))
2270 U(2,6)=-U(1,5)
2280 U(1,6)=R*(T(1)*T(2)-T(3)*T(4))
2290 U(2,4)=R*R*(Q*S*T(4)*0.5-T(5)*T(2))
2300 U(3,5)=U(2,4)
2310 U(2,5)=R*R*R*(T(3)*T(4)-T(6)*T(2))
2320 U(3,4)=R*(T(1)*T(4)-T(3)*T(2))
2330 T(4)=B(3,N0)-W(3,N0)*02
2340 T(5)=W(2,N0)*02-B(2,N0)
2350 T(6)=B(1,N0)-W(1,N0)*02
2360 FOR L1=4 TO 6
2370 FOR L2=1 TO 6
2380 U(L1,L2)=T(L1)*U(7-L1,L2)
2390 NEXT L2
2400 NEXT L1
2410 U(4,4)=U(4,4)+C
2420 U(4,5)=U(4,5)+U(2,3)
2430 U(4,6)=U(4,6)-S
2440 U(5,5)=U(5,5)+1
2450 U(6,4)=U(6,4)+S
2460 U(6,5)=U(6,5)-U(2,1)
2470 U(6,6)=U(6,6)+C
2480 RETURN

```

**THIS PAGE
WAS INTENTIONALLY
LEFT BLANK**

ORNL/TM-5804
Dist. Category UC-77

Internal Distribution

1. H. E. Alvey (Y-12)	52. D. L. Mason (Y-12)
2. E. F. Babelay, Jr. (Y-12)	53. W. J. McAfee
3. N. K. Bernander (Y-12)	54. J. G. Merkle
4. R. B. Burditt (Y-12)	55. F. R. Mynatt
5-8. J. P. Callahan	56. R. K. Nanstad
9. D. A. Canonico	57. D. J. Naus
10. J. A. Clinard	58. C. B. Oland
11. J. A. Conlin	59. G. W. Oliphant (Y-12)
12. W. E. Cooper	60. A. E. Perry (Y-12)
13. J. M. Corum	61. L. R. Phillips (Y-12)
14. W. G. Dodge	62. D. W. Post (Y-12)
15. J. R. Dougan	63. H. Postma
16. H. G. Duggan (Y-12)	64. G. C. Robinson
17-20. D. N. Fanning	65. J. P. Sanders
21. D. L. Foulk (Y-12)	66. J. M. Schreyer (Y-12)
22. R. J. Fraser (Y-12)	67. M. R. Sheldon
23. Uri Gat	68. H. F. Smith, Jr. (Y-12)
24. D. W. Goodpasture	69. J. E. Smith
25. J. R. Gray (Y-12)	70. R. D. Smith (Y-12)
26. W. L. Greenstreet	71. H. H. Stoner (Y-12)
27. V. B. Gritzner (Y-12)	72. W. E. Tewes (Y-12)
28. R. C. Gwaltney	73. W. F. Thompson (Y-12)
29. J. F. Harvey	74. F. V. Tilson (Y-12)
30. C. E. Hensley (Y-12)	75. H. E. Trammell
31. R. L. Huddleston	76. D. B. Trauger
32. F. W. Jones (Y-12)	77. H. H. Tunnell (Y-12)
33. K. G. Kahl (Y-12)	78. J. R. Weir, Jr.
34. P. R. Kasten	79. B. J. White (Y-12)
35. A. Keith (Y-12)	80. G. D. Whitman
36. J. J. Kelley (Y-12)	81. W. K. Whitson (Y-12)
37. H. T. Kite (Y-12)	82. W. J. Yaggi (Y-12)
38-47. C. E. Knight, Jr. (Y-12)	83. ORNL Patent Office
48. M. Levenson	84-85. Central Research Library
49. A. L. Lotts	86. Document Reference Section
50. R. E. MacPherson	87-89. Laboratory Records Department
51. W. R. Martin (Y-12)	90. Laboratory Records, ORNL-RC

External Distribution

- 91-92. Director, Division of Nuclear Research and Applications, DOE, Washington, D.C. 20545
93. Director, Reactor Division, DOE, ORO
94. Research and Technical Support Division, DOE, ORO
- 95-271. For distribution as shown in TID-4500 under category UC-77, Gas-Cooled Reactor Technology