The effects of strain on the superconducting properties of niobium-tin conductors

Ronald Warren Hoard
(Ph.D. Thesis)

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The Effects of Strain on the Superconducting
Properties of Niobium-Tin Conductors

by

Ronald Warren Hoard

A dissertation submitted in partial fulfillment
of the requirements for the degree of

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Department of Nuclear Engineering

Date

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Doctoral Dissertation

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University of Washington

Abstract

THE EFFECTS OF STRAIN ON THE SUPERCONDUCTING
PROPERTIES OF NIOBIUM-TIN CONDUCTORS

By Ronald Warren Hoard

Chairperson of the Supervisory Committee:
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Department of Nuclear Engineering

Typical Nb_3Sn multifilamentary superconducting composites produced by the bronze diffusion process have repetitive fiber elements of niobium embedded in a bronze matrix, with a Nb_3Sn reaction zone serving as the interfacial layer between the two. Clusters of these units are surrounded by tin diffusion inhibitors, such as tantalum, and placed inside a second external copper matrix. The strain degradation of the critical current density has been both analytically and experimentally investigated in an inversion of this configuration. The new geometry consists of bronze filament cores inside of niobium tubes, with the usual Nb_3Sn interface. Since the niobium acts as a diffusion barrier, copper can be directly utilized as the embedding matrix, eliminating the motivation for tantalum addition. Analytic results were obtained from a computer program (MAXIMSUPER) which evaluates the three-dimensional strain fields on the Nb_3Sn zones in a cylindrical repeating element of the composite due to both

residual (thermal contraction) and applied axial loading. The code then incorporates the calculated Nb_3Sn strain values into appropriate scaling laws to predict the critical current density and its degradation with increasing axial strain.

Investigations were performed to ascertain additional information on the connection between the cubic to tetragonal martensitic phase transformation and the phenomenon of superconductivity in Nb_3Sn . Of particular interest is the degradation of the critical parameters, such as T_c , H_{c2} , and J_c , with mechanical straining of the superconductor. These studies yielded information that assisted in the derivation of the critical current-strain scaling laws mentioned above. Since martensitic transitions are also known to be strain sensitive, perhaps the structural change is directly responsible for the anomalous reduction in the superconducting properties of the material. Low temperature X-ray diffraction experiments were performed on Nb_3Sn ribbons (in which the bronze layers had been etched off) mounted to copper and indium sample stages. The cryostat has the unique feature that allows the superconductor to be placed under both compressive and tensile strains, while at low temperatures, by means of a vacuum mechanical insert. Results indicate that the martensitic phase transition temperature, T_m ,

increases with increasing compressive strains. Critical parameters, such as H_{c2} and T_c are thought to vary according to a two-level state model which expresses the Nb_3Sn cubic-to-tetragonal, martensitic-phase transformation as being strain sensitive.

Experimental investigations conducted with a 12 Tesla, low-temperature, tensile-test facility on both the niobium core and bronze core geometries confirm the importance of the tangential strains (in addition to the axial strain) on the critical current properties of the conductors. In particular, resulting higher radial and azimuthal strains yield lower critical current densities in the latter geometry. Under applied axial loading, the increasing tangential strains exhibit contradistinctive behavior from that of the axial z strain, which changes sign from compressive to tensile. The magnitude of these tangential strains depends primarily on the composite geometry and configuration. Good agreement between the MAXIMUPER predictions and experimental results indicates that the program may be suitable for superconducting composite design applications.

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DEDICATION

Perhaps it would be interesting to assign a "label" or descriptive title to every generation of mankind, as if one were reading man's history from the chapters of some omniscient book. As an example, the first ancestors of prehistoric man may have been dubbed "The Tool Makers"; or the most recently passed modern generation could have been called "The Industrialists." Today man is growing more cognizant of his technology's interaction upon the environment. He is now developing a conscious concern in maintaining the earth's atmosphere, which is so vital to his existence; he may even learn to limit careless exploitation of her resources. For these reasons, the present generation may later be cited as "The Ecologists." Only the Lord God Almighty knows what will become of the next procreation. I would like to dedicate this humble work to Him and to all of them--past, present, and awaiting.

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CHAPTER 1. INTRODUCTION AND PURPOSE OF THE STUDY

A. Introduction to the Chapter

The need for high-field magnets ($B > 10$ Tesla) to be utilized in the magnetic fusion energy program may well be satisfied with the development of superconducting composite current conductors. These composites are formed from the extrusion of a normal metal matrix with superconducting multifilaments (Nb-Ti), or two metals are extruded and reacted at some elevated temperature to form a superconducting interface (Nb_3Sn), between the filament core and matrix by means of the solid-state diffusion process. Throughout the 1970's, multifilamentary Nb-Ti has become the most commercially available superconductor, with planned additional applications that extend well into the 1980's, including the newly started construction of the Magnetic Fusion Test Facility (MFTF) at Lawrence Livermore National Laboratory (Fig. 1.). Currently Nb-Ti is deployed in fusion machines that produce magnetic fields up to 9 Tesla with the superconductor maintained at 4.2 K liquid helium temperature. We can anticipate that future reactor scale facilities may require larger fields of 12 Tesla and above; therefore, commercial

efforts have to be expanded to include the fabrication of a newer high-field superconducting material. Niobium-tin (Nb_3Sn) seems to be a suitable next generation material having an upper critical field H_{c2} of 23 Tesla at 4.2 K; however, the production of Nb_3Sn conductors introduces three additional problems that are not inherent with Nb-Ti superconductors. Nb_3Sn is a brittle material, fracturing at elongation strains of approximately 0.2 percent, while Nb-Ti is ductile up to about 10 percent elongation. Because of this ductile property of Nb-Ti, it can be extruded along with the pure OFHC copper matrix without producing filament breakage. Figure 2 shows the cross section of such a composite conductor. However, Nb_3Sn must be formed after the composite is made by producing a reaction of the niobium filaments with the tin in a bronze region surrounding the filaments. Circumscribing the large clusters of superconducting filaments imbedded in bronze are tin diffusion inhibitors, such as tantalum, which prevent the migration of tin from the bronze to contaminate the high purity of the copper outer zones. The copper acts to stabilize the conductor, allowing for a place in which the current can transfer during the accidental case of a quench (transition of the Nb-Ti or Nb_3Sn from the superconducting phase to the normal state). The third difficulty associated with

Nb_3Sn is that its intrinsic superconducting properties are strain sensitive. Therefore, it requires special handling and processing to prevent the degradation of its critical parameters, such as the upper critical field H_{c2} , the transition temperature T_c , and the critical current density J_c , by the application of unwanted straining of the conductors.

The mechanical interaction caused by the differing thermal expansion coefficients, Young's moduli, and Poisson's ratios between the composite matrix and the filament cores often induces residual stresses and strains upon the superconductor or superconducting filament-matrix interface. This can occur when the composite is either cooled from its initial fabrication temperature and/or placed under axial stress. In the case of Nb-Ti filament cores, the superconducting properties are only slightly affected by strains of less than 1 percent elongation. Higher field, less ductile conductors, such as the Nb_3Sn filament-matrix interfaces which have larger critical current densities J_c and greater critical temperatures, are much more sensitive to the effects of strain on their superconducting properties. Unfortunately, the degradation of superconducting properties with increasing strain seems to plague all hard Type II superconducting compounds with crystal

structures belonging to the class A15 group (see Fig. 3). These include all of the A_3B high-field compounds of both niobium and vanadium. Since these materials are the most promising for future high-field magnet development, additional research is needed to investigate the strain dependence of their superconducting properties.

The current method of experimentation involves cooling the composites down to liquid helium temperature and then applying axial strains while measuring the corresponding critical currents. The strains are prototypic of those expected to arise from the coil winding tension of the magnet and those induced by the Lorentz forces during magnet operation. Figure 4 represents a conglomeration of such data taken for Nb_3Sn composite conductors at various applied magnetic field strengths, compiled by Lawrence Livermore National Laboratory and other sources. There appears to be wide scatter in the measurements, but several predominate trends seem to be consistent. The first observation is that nearly all of the data suggest that the critical current is at first enhanced with axial strain, reaches a maximum, and then decreases with further increasing strain. A second indication of the data trend is that there is a dependence on the applied magnetic field strength H . Previous investigators have concluded that the Nb_3Sn layers were under

compression after the composite was cooled to liquid helium temperature, due to the differing thermal expansion coefficients of the Nb_3Sn , niobium, and bronze regions in the composite.¹⁻⁹ Subsequent applied axial strains would at first relieve the compressive strain causing the observed maximum in the critical current. Continued straining would place the Nb_3Sn zones under tensile stress and hence decrease the critical current. Consequently, authors have assumed that the peak in critical current occurs at near zero strain values of the Nb_3Sn layers.

The curves of Fig. 4 are further normalized with respect to the strain which produces the maximum critical current. The intrinsic strain is assumed to be zero at the peak critical current and is defined as $\epsilon_{\text{intr}} = \epsilon - \epsilon_{\text{icmax}}$. The results of Fig. 4 are replotted with the intrinsic strain in Fig. 5. Not only does Fig. 5 display more detail of the dependence of the applied magnetic field on the critical current strain degradation rate, but it also indicates (along with Fig. 4) that geometrical factors, such as the Nb_3Sn layer thickness and the ratio of the bronze cross-sectional area to that of the Nb_3Sn , may also contribute to the observed effects.

In addition to the impact of strain-sensitive superconductors on future magnetic fusion magnet design, recent investigations have further probed the underlying physical cause of the phenomenon. Perhaps the first vital clue was illuminated by Testardi, who observed, using ultrasonic attenuation techniques, a temperature sensitive softening of the lattice elastic constants in several of the A15 compounds, such as vanadium silicon (V_3Si), niobium-tin (Nb_3Sn), and niobium-germanium (Nb_3Ge).¹⁰⁻¹⁴ It was suggested that the unusual temperature dependence was indicative of a low-temperature phase transition occurring for the superconductors of this group. The transformation was later observed by Batterman, who (using low-temperature x-ray diffraction techniques) noted that the normally cubic A15 lattices undergo a cubic-to-tetragonal phase transition. The temperature of the start of transition seems to occur several (approximately 50 K) degrees higher than the superconducting-to-normal transition temperature T_c (17.8 K for Nb_3Sn).¹⁵⁻¹⁶ Previous authors have repeated the experiments on Nb_3Sn and have found that in general the transformation temperature T_m varied with the purity and stoichiometry of fabricated Nb_3Sn samples. Indeed, some Nb_3Sn samples have been reported not to undergo the phase transition at all, while others

have either partially or completely transformed the entire crystal volume between the starting transformation temperature T_m and a transition arrest temperature T_a .¹⁷ These results bear a close resemblance to the martensitic phase transformations known to occur in various stainless steels; particularly since at such low temperatures, diffusion controlled transformations are ruled out as unlikely.¹⁸⁻¹⁹ Currently, the exact linkage between the material superconducting properties and the martensitic phase transformation is uncertain, although several efforts have been made to connect the two phenomena by theoretical models based on calculations of the material density of states and a form of Peierls instability.²⁰⁻³⁵ Hopefully, continued research and investigations of these materials will yield sufficient information possibly to explain the even more fundamental question of why the Al5's crystal structure contains several of the highest known transition temperature superconducting compounds.

B. Purpose of the Study

The purpose of this study is to investigate further the phenomenon of strain sensitivity in superconducting Nb_3Sn , both from the practical viewpoint of its future application in magnetic fusion magnets and as an approach to extract additional information on the underlying physics of the connection between superconductivity and

strain. In general, the techniques used for the two explorations will be different because applied engineering magnet design encompasses the macroscopic properties of the compound, such as information on the critical parameters: the upper critical magnetic field H_{c2} , the critical temperature T_c , the maximum critical current density J_c , and the degradation of these values with applied and/or residual thermal strains on the conductor. On the other hand, investigation of the physics of the strain-superconducting interaction involves factors such as the superconducting Cooper electron pairs, the effect of the crystal lattice on the electrons, the cubic-to-tetragonal martensitic phase transition, the electron-phonon interaction between the lattice and the electrons, and other such considerations which are predominately microscopic in origin. Nevertheless, the success of such explanations is indicated by the amount of agreement and overlap between the results.

Using observations obtained from the experimental results, the author will attempt an analytical determination of the strain dependence of the critical current. This mathematical function could later assist in the design of multifilamentary superconductors, in which a predetermined strain could be induced to yield the

desired critical current. The method of attack for this problem can be divided into two stages:

1. A program must be developed which predicts the stresses and strains in composites as a result of thermal and axial loading.
2. The functional dependence of the critical current on strain must be derived.

Chapter 2 documents the development of the computer program MAXIMSUPER, which has been applied with considerable success to predict the state of strain on the Nb_3Sn layers in the composites. Chapter 3 discusses experimental methods which were used to verify the analytical predictions of the program.

The second step required a further investigation into the nature of the sensitivity of the critical parameters on strain. This was accomplished by studying the effects of strain on the martensitic phase transformation in superconducting Nb_3Sn . Chapter 4 describes this experiment using low temperature x-ray diffraction techniques. The diffractometer cryostat has the unique feature that allows the Nb_3Sn to be placed under both compressive and tensile strains, while at low temperature, by means of a vacuum mechanical insert. A theoretical model to interpret the results of the investigation is developed in Chapter 5. From the theory emerges the critical-

current scaling law, required for step 2, used in the MAXIMUPER computer program.

The author anticipates that the program will not only be useful as an aid in accounting for the observed trends in the strain- I_c data, but should also find its greatest applicability in the theoretical design of future superconducting composites, since it allows the convenience of being able to model mathematically each proposed conductor design and to optimize (or adjust) parameters, such as the bronze-to-niobium ratio, Nb_3Sn layer thickness, and then produce conductors with predetermined critical current characteristics.

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CHAPTER 2. CODE MAXIMSUPER

A. Introduction to the Chapter

In essence, the general method of fabricating multifilamentary superconductors is to use trial and error approaches. One designs a conductor which has the required proportions of niobium and bronze to form an adequate amount of Nb_3Sn , as a result of the diffusion reaction. Included in the considerations are the amounts of copper which must be added to the composite matrix to ensure stabilization against accidental superconducting to normal quenches, and the question of whether tantalum should be included to prevent contamination of the high-purity, high-conductivity copper by diffusing tin during the reaction. A superconductor manufacturer takes these things into account, along with the mechanical constraints and limitations of his fabrication techniques, to produce a prototypic sample, which is then tested for its superconducting properties.¹⁻⁴ Typically, the average time between conductor design to fabrication and testing is approximately from six months to one year for each composite.

The conductors are tested for various superconducting and mechanical properties, such as the critical

temperature T_c , the critical current density J_c , the upper critical field H_{c2} , resistance to breakage under applied stress, resistivity of the copper stabilizer as a function of temperature, and the dependence of the critical parameters on mechanical strains. These strains are representative of those which would be induced in the conductor as the result of coil winding, mechanical stress from the magnet and support structure weight, and the Lorentz magnetic pressure induced once the coil is energized. Earlier investigators have shown that the multifilamentary conductor critical currents I_c , critical temperature T_c , and upper critical field H_{c2} appear to have a parabolic dependence on the axial tensile strain applied to short test lengths of superconductor (see Figs. 5, 6, and 7).⁵⁻⁸ In addition, certain properties, such as the initial amount of compressive pre-strain, could be varied by changing some of the conductor's internal parameters like the volume ratio of bronze to niobium, diffusion reaction time and temperature, and the layer thickness of the Nb_3Sn zones in the composite.^{5,9-11}

Since the observed critical properties in Nb_3Sn are indeed strain sensitive, one must develop a method of obtaining knowledge of the strain state of the Nb_3Sn zones in the fiber composite, first as a result of the

differing thermal expansion coefficients of the conductor components (copper, niobium, Nb_3Sn , tantalum, and bronze) and second as the result of the different mechanical properties of the materials under the influence of the axial strains placed on the conductor. Presently, the standard procedure is to affix to short sample lengths of the conductors resistive strain gages which yield the axial ϵ_z strain on the conductor while it is being loaded at liquid helium temperature by a low-temperature tensile tester. This method is inadequate for the following three reasons: first, the resistive strain gages are temperature sensitive and, therefore, cannot give any information on the initial strain state produced by thermal contraction in the conductor as the result of cooldown from the Nb_3Sn diffusion reaction temperature (~ 1023 K); second, there are no existing strain gages that work from such elevated temperatures down to and including 4.2 K; third, the strain gages are attached to the composite surface and, hence, only give information on the applied ϵ_z strain as the result of axial loading (the cooldown thermal expansion strain history is lost) while the Nb_3Sn filaments are actually under a three-dimension state of strain (axial, radial, and azimuthal).

Commercially available composite conductors contain several thousand filaments of Nb_3Sn (see Fig. 8). There

is no available way to measure the strain configuration of each filament experimentally; therefore, analytical methods must be considered.

There are two ways to model mathematically multifilamentary conductors such as the ones shown in Figs. 8 and 9. It is possible to use finite element and grid techniques along with basic relations of elastic and plastic analysis to treat clusters of Nb_3Sn filaments and, hence, calculate the strains present on the Nb_3Sn zones. Alternatively, one can fashion each filament and its surrounding nearest neighbor environment of bronze and copper as a cylinder (infinitely long in the z direction to exclude end effects) with concentric annular rings composed of the various composite materials. The author has chosen to use the latter approach, because of the availability of previously existing work done on stress strain relations in fiber composites and the ease of understanding the physical and mathematical foundations of the resulting theory.

One numerical solution to the problem of finding strains in filament composites was treated by S. Hecker, C. Hamilton, and L. Ebert, in which the elastic solutions were obtained and then iterated against plasticity data according to an elasto-plastic analysis developed by Hecker.¹²⁻¹⁴ Their results yielded a computer program

capable of calculating the stress and strain states of a single composite filament as the result of thermal contraction and applied loading. Experimental verification of the code predictions was performed first by using two component monofilaments of OFHC copper core with a 4340 steel and another sample with an 18 percent Ni maraging steel case (see Hecker Ref. No. 13). Hamilton later extended the experimental test of the code performance by investigating three material component composite filaments. He selected two such systems: a beryllco 25 alloy core and case, with an OFHC copper central ring, and a maraging steel core and case with an OFHC copper ring.¹⁵ All results yielded predicted versus experimental unaxial stress-strain curves that were closer in agreement than curves predicted from the "rule of mixtures" calculations.¹⁶

MAXIMSUPER is a Fortran modification of the original ALGO general fiber composite program written by Hecker.¹³ First, the code calculates the residual strains (due to the different thermal expansion coefficients of the composite components) produced when the Nb_3Sn conductor is cooled from its initial fabrication temperature (the Nb_3Sn diffusion reaction temperature is approximately 1023 K) to 4.2 K. It is assumed that the initial stress-strain state on the conductor is zero at the elevated

reaction temperature because of the plastic mechanical properties of the bronze matrix (10% Sn - 90% Cu bronze has a solidus temperature of approximately 1118 K).¹⁷ Then axial strain loading is applied and the code calculates the three-dimensional states of stress and strain for each region of the repeating composite element. Finally, an effective strain is calculated from the triaxial strain components on the Nb₃Sn zones and this is incorporated into appropriate scaling laws to predict the resulting critical current density J_c of the conductor. The details of the functional dependence of the critical parameters J_c , T_c , and H_{c2} on strain are given in Chapter 5, which describes the critical current density subroutine used in the program.

MAXIMSUPER treats a single cylindrical repeating element of the entire multifilament composite as shown in Figs. 8 and 9. This single superconducting filament represents the average geometry and configuration (niobium-core radius, Nb₃Sn layer thickness, bronze-to-niobium ratio, and the prototypic amount of copper matrix separating each filament from the nearest neighbors) surrounding a typical filament. Since this average filament is in an approximately equivalent environment of perhaps thousands of its companions, its resulting strain fields from thermal expansion and axial loading should be fairly

representative of those on the average superconducting filament.

However, there are several problems which arise from this modeling which can provide erroneous strain results. MAXIMSUPER treats repeating cylindrical elements of the composite, mainly because the cylindrical symmetry yields strain solutions that are constant with respect to axial z and azimuthal θ directions in the conductor. Fig. 10 shows such a repeating element; while Fig. 11 and 12 indicate that, although the cylindrical symmetry is approximately maintained for composites with large matrix to fiber volume fractions, considerable distortions (due to fabrication techniques) can occur in the symmetry for conductors with closely packed filaments. Also, a monofilament cylinder has the boundary condition that its radial stresses vanish at its outer radius. Actual fabrication tends to deform the cylindrical elements into hexagonally close-packed structures. This destroys the axial symmetry, imposing an azimuthal dependence on the radial stresses. Ebert et al. have compared the results of their cylindrical code to those of a hexagonal repeating element program and have found that the reported states of strain show little difference for fiber to matrix volume ratios up to 0.5.¹⁸ Therefore, the slightly hexagonally distorted filaments may be approximated as

cylindrical repeating elements with an outer radius equal to one-half the average distance to its surrounding nearest neighbors, as shown in Fig. 10.

Of course, this modeling is more accurate for conductors with uniform filament spacings and densities across the composite cross-sectional area. Composite superconductors with clusters of filaments, grouped into small islands, separated by large areas of bronze or copper (see Fig. 13), present calculational problems in that the strain states for the filaments near the "coasts" of the islands are drastically different from those in the island centers. Fortunately, the critical-current density of the coastal filaments is lower than those in the body of the cluster (because of the higher strains imposed by the larger surrounding copper or bronze regions). Therefore, most of the current is transferred into the inner filaments, making the modeling of an island central filament a reasonable approximation, provided the number of inner filaments is large.

B. Mathematical Formulation: Strains Produced From Thermal and Axial Loading

The problem of calculating the strains present on the representative cylindrical repeating element is solved using the theory of elasticity with an iterative procedure to account for plastic effects developed by Hecker.¹³

The element is divided into a multi-N-ring composite cylinder, with a specified material composition present in each zone (i.e., niobium, Nb_3Sn , bronze, and copper), as shown in Fig. 14. The zone radii are in the exact dimensions specified by the conductor design or the configuration revealed by microscopic examination of a fabricated conductor. The mathematical model makes the following assumptions about the composite cylinder.

1. The cylinder is infinitely long, ignoring end effects (typical filaments have a diameter of about 10 microns, with continuous lengths of several meters).
2. The axial strain ϵ_z is constant across the filament cross-sectional area.
3. There is perfect bonding at the interfaces of all material zones (no slippage or separation of the N-rings).
4. No external body forces act on the composite.

The general approach to solving this problem is to express the equilibrium-of-forces equation in terms of the displacements and then to solve the resulting differential equation subject to imposed boundary conditions. The equilibrium of forces equation briefly states that for a polar element of unit thickness to be in a state of equilibrium, the sum of all the forces in the radial and azimuthal directions must be equal to zero. From

elasticity theory, this summation in polar coordinates is known to yield:¹⁹

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r} \frac{\partial \tau_{r\theta}}{\partial \theta} + \frac{1}{r} (\sigma_{rr} - \sigma_{\theta\theta}) + F_r = 0 \quad (2-1)$$

for the equation of equilibrium in the radial direction. In the absence of body forces, using the principle stresses, and accounting for the axial symmetry, equation 1 reduces to:

$$\frac{d\sigma_r}{dr} + \frac{1}{r} (\sigma_r - \sigma_\theta) = 0 \quad (2-2)$$

This differential equation for the stresses can be expressed in terms of the strains by using Hooke's law of elasticity including the effects of thermal strain,

$$\begin{aligned} \sigma_r &= K \left[\epsilon_r (1 - \nu) + \nu \epsilon_\theta + \nu \epsilon_z - (1 + \nu) \alpha \Delta T \right] \\ \sigma_\theta &= K \left[\epsilon_\theta (1 - \nu) + \nu \epsilon_r + \nu \epsilon_z - (1 + \nu) \alpha \Delta T \right] \\ \sigma_z &= K \left[\epsilon_z (1 - \nu) + \nu \epsilon_r + \nu \epsilon_\theta - (1 + \nu) \alpha \Delta T \right] \end{aligned} \quad (2-3)$$

in which α is the thermal expansion coefficient, ΔT is the temperature difference between the actual temperature and a reference temperature T_0 , and ν is Poisson's ratio. For this problem T_0 is chosen as the Nb_3Sn diffusion

reaction temperature (~1023 K) because the "putty-like" yielding of the bronze matrix at these elevated temperatures initially places the composite in an approximately stress-free state. Cooling from this temperature supplies the thermal loading stresses and strains.

The triaxial strains can be expressed in terms of axial and radial displacements by:

$$\begin{aligned}\epsilon_r &= \frac{du}{dR} \\ \epsilon_\theta &= \frac{u}{R} \\ \epsilon_z &= \text{Constant}\end{aligned}\tag{2-4}$$

These relations are inserted into equation 3 to yield Hooke's law, which expresses the principle stresses in terms of displacements.

$$\begin{aligned}\sigma_r &= K \left[(1 - \nu) \frac{du}{dR} + \nu \frac{u}{R} + \nu \epsilon_z - (1 + \nu) \alpha \Delta T \right] \\ \sigma_\theta &= K \left[\nu \frac{du}{dR} + (1 - \nu) \frac{u}{R} + \nu \epsilon_z - (1 + \nu) \alpha \Delta T \right] \\ \sigma_z &= K \left[\nu \frac{du}{dR} + \nu \frac{u}{R} + (1 - \nu) \epsilon_z - (1 + \nu) \alpha \Delta T \right]\end{aligned}\tag{2-5}$$

Equations 5 are directly substituted into the force equilibrium equation (equation 2), which gives the controlling

differential equation for the radial displacements in the cylindrical composite element.

$$R^2 \frac{d^2 u}{dR^2} + R \frac{du}{dR} - u = 0 \quad (2-6)$$

Equation 6 is an equidimensional linear differential equation having the solution

$$u = C_1 R + C_2 / R \quad (2-7)$$

This equation is solved for each of the N radial zones of the composite element shown in Fig. 14; therefore, it is re-expressed as

$$u_i = C_{1i} R + C_{2i} / R \text{ where } i = 0, 1, 2, 3, \dots, N \quad (2-8)$$

The solution of the problem is obtained when the constants C_1 and C_2 are found for each of the material zones and radial rings of the composite; in the case of thermal loading, ϵ_z , which is uniform across the cross-sectional area, is also an unknown. These constants are determined subject to the following boundary conditions:

- A. For axial loading, ϵ_z is known because it is the strain produced by the tensile test. It is mathematically modeled in the code by inserting predetermined values into the program.

- B. All displacements at the zone interfaces must be continuous, or:

$$u_i = u_{i+1} \text{ at } R = R_i \left(R_i \text{ goes from } R_i \text{ to } R_{i+1} \right) \quad (2-9)$$

$$C_{1i}R_i + C_{2i}/R_i = C_{1i+1}R_i + C_{2i+1}/R_i$$

$$i = 0, 1, 2, 3, \dots, N-1$$

- C. Radial stresses across the interfaces must be continuous (equilibrium of forces), or:

$$\sigma_{ri} = \sigma_{ri+1} \text{ at } R = R_i \quad (2-10)$$

Inserting equation 8 into the σ_r relationship of equation 5 implies that boundary condition 10 gives the following result:

$$\begin{aligned} K_i C_{1i} - K_{i+1} C_{1i+1} - \left(1/R_i^2 \right) \left[K_i (1 - 2\nu_i) C_{2i} \right. \\ \left. - K_{i+1} (1 + 2\nu_{i+1}) C_{2i+1} \right] \\ - \epsilon_z \left[K_{i+1} \nu_{i+1} - K_i \nu_i \right] = T \left[K_i \nu_i (1 + \nu_i) \right. \\ \left. - K_{i+1} \nu_{i+1} (1 + \nu_{i+1}) \right] \end{aligned} \quad (2-11)$$

for $i = 0, 1, 2, 3, \dots, N$; these constitute $N + 1$ linear equations.

- D. To exclude nonphysical results, all displacements must be finite, implying that:

$$C_{2i} = 0 \text{ for } i = 0 \text{ (at the composite central core)} \quad (2-12)$$

- E. The radial stress vanishes at the cylinder outer surface. This is satisfied by assuming the material external to the cylinder is vacuum, setting all the material constants (K_{n+1} , ν_{n+1} , and α_{n+1}) and hence the radial stress equal to zero.
- F. For thermal loading, ϵ_z is unknown, but may be derived from the condition of keeping an equilibrium of forces in the axial direction:

$$\sum_{i=0}^N \int_{R_{i-1}}^{R_i} \sigma_z^i R dR = 0 \quad \text{with } R_{-1} = 0 \quad (2-13)$$

Equations 13, 5, and 8 give:

$$\begin{aligned} \sum_{i=0}^N \left[(R_i^2 - R_{i-1}^2) / R_n^2 \right] K_i \left[2\epsilon_i C_{1i} \right. \\ \left. + (1 - \nu_i) \epsilon_z - (1 + \nu_i) \alpha_i T \right] = 0 \end{aligned} \quad (2-14)$$

Therefore, the boundary conditions have been used to transform equation 8 into $2N + 3$ simultaneous linear equations with $2N + 1$ unknown constants (C_{1i} , C_{2i} , and ϵ_z for

$i = 0, 1, 2, 3, \dots, N$). These are solved by a Gaussian elimination routine enabling the three dimensional elastic stress state for each zone to be calculated by substituting equation 7 into equation 5.

$$\begin{aligned}\sigma_r &= K \left[C_1 - \left(C_2 / R^2 \right) (1 - 2\nu) + \nu \epsilon_z - (1 - \nu) \alpha T \right] \\ \sigma_\theta &= K \left[C_1 + \left(C_2 / R^2 \right) (1 - 2\nu) + \nu \epsilon_z - (1 + \nu) \alpha T \right] \quad (2-15) \\ \sigma_z &= K \left[2\nu C_1 + (1 - \nu) \epsilon_z - (1 + \nu) \alpha T \right]\end{aligned}$$

The corresponding elastic strains for each zone are found by inserting equation 7 into equation 4.

$$\begin{aligned}\epsilon_r &= C_1 - C_2 / R^2 \\ \epsilon_\theta &= C_1 + C_2 / R^2\end{aligned} \quad (2-16)$$

To account for the effects of plasticity in composite cylinders, Hecker, et al., developed a method of elastic-plastic analysis.^{14,20} The underlying premise for this technique is to develop stress and strain relationships that include both the effects of elastic and plastic deformation; preferably the relationships are also compatible with treatment by Hooke's law. Currently the main source of plasticity data for a given material

is compiled in its axial-loading stress-strain curve. Hecotor has shown that quantities called the effective stress and effective strain, which include both the effects of elastic and plastic deformation, can be directly linked to the axial stress-strain curve. The effective stresses and strains are proportional to the second invariant of the stress deviator tensor and are a measure of the amount of deformation produced in an elastic and plastic material. They can also be derived from the strain energy of distortion theory, which relates the von-Mises-Hencky-Huber yield criterion for ductile materials to the effective stresses and strains.²⁰⁻²⁸

$$\sigma = \sqrt{\frac{1}{2}} \left[\left(\sigma_r - \sigma_\theta \right)^2 + \left(\sigma_\theta - \sigma_z \right)^2 + \left(\sigma_z - \sigma_r \right)^2 \right]^{1/2} \quad (2-17)$$

$$\bar{\epsilon} = \frac{\sqrt{2}}{2(1+M)} \left[\left(d_{\epsilon_r} - d_{\epsilon_\theta} \right)^2 + \left(d_{\epsilon_\theta} - d_{\epsilon_z} \right)^2 + \left(d_{\epsilon_z} - d_{\epsilon_r} \right)^2 \right]^{1/2} \quad (2-18)$$

The constant multipliers are chosen so that the effective stress and strain become equal to the applied stress and the largest principal strain during axial tension. For any three-dimensional stress field, the relationship between the effective stress and strain becomes equal to the uniaxial stress-strain curve with M acting as

the Poisson's ratio which changes with the effects of plasticity. Likewise, Young's modulus E , which is also modified to include plastic phenomenon, becomes equal to the instantaneous slope of the stress-strain curve P . For a given material, the modified Poisson's ratio M and Young's modulus E are determined from its stress-strain curve by

$$P = \frac{d\sigma_z}{d\epsilon_z}$$

$$M = \frac{1}{2} - \left(\frac{1}{2} - \nu \right) \frac{1}{E} \frac{d\sigma_z}{d\epsilon_z} \quad (2-19)$$

The input stress-strain curves are polynomials fitted by least square algorithms to available data for each composite material. With the new parameters M and P replacing the elastic constants ν and E , the stress-strain relationships (including both elastic and plastic effects) become

$$d\epsilon_r = \frac{1}{P} \left[d\sigma_r - M (d\sigma_\theta + d\sigma_z) \right]$$

$$d\epsilon_\theta = \frac{1}{P} \left[d\sigma_\theta - M (d\sigma_z + d\sigma_r) \right] \quad (2-20)$$

$$d\epsilon_z = \frac{1}{P} \left[d\sigma_z - M (d\sigma_r + d\sigma_\theta) \right]$$

Notice that the general form of equations 20 is equivalent to Hooke's law of equation 3, hence the transition from elastic to plastic behavior requires no separate analysis except the modification of E and ν to become P and M , since the structure of the stress-strain relations are identical for both regions.

The parameters P and M , which include both elastic and plastic effects, are dependent on the effective strain. However, the effective strain cannot be determined without the initial values of P and M . Therefore, an iterative technique is used to determine the effective strain and, consequently, the complete elastic and plastic solution to the cylindrical element composite. The methodology is tabulated below:

- 1.a. The initial guess for P and M is that they are elastic and, therefore, are equal to E and ν .
- 2.a. For thermal loading the initial stress-free high temperature T_0 ($T_0 \approx 1023$ K) is decremented by a small amount, $\Delta T \approx 1$ K. The first effective strain increment $d\bar{\epsilon}_n$ (equation 18) is calculated using the elasticity solutions from equations 16 in each radial zone of the composite. The total accumulative effective strain is given by

$$\bar{\epsilon}_{n\&} = \bar{\epsilon}_{n-1} + d\bar{\epsilon}_n \quad (2-21)$$

where l is the number corresponding to the loading step and n is the iteration number. By definition and compatibility with physical assumptions $\bar{\epsilon}_{-1} = 0$ at the stress-free temperature of T_0 .

- 3.a. The resulting total effective strain is used to calculate new values for P and M by equations 19 from the stress-strain curves of each material radial zone at each temperature. Due to the limited availability of stress-strain data, in general, room-temperature curves are used with elastic properties (E , ν , and α), expressed as functions of temperature. The current data used in this study are tabulated in the first appendix.
- 4.a. Using the new values of P and M , the elastic solutions of equations 15 and 16 are found again and a new value of $d\bar{\epsilon}_{n+1}$ is computed by means of equation 18.
- 5.a. Now if r_n corresponds to the correct location on the material stress-strain curves, then the correct values for P and M have been determined and hence $d\bar{\epsilon}_n$ represents the proper incremental strain. This is tested by comparing the error between $d\bar{\epsilon}_{n+1}$ and $d\bar{\epsilon}_n$ against a preset convergence criterion δ (good results were obtained with δ equal to 0.001).

- i. If $(d\bar{\epsilon}_{n+1} - d\bar{\epsilon}_n) / d\bar{\epsilon}_n < \delta$ for all composite regions, convergence has been obtained and $d\bar{\epsilon}_n$ represents the correct strain increment. The program is returned to decrement the temperature for the next thermal loading in step 2.
 - ii. If $(d\bar{\epsilon}_{n+1} - d\bar{\epsilon}_n) / d\bar{\epsilon}_n > \delta$ for any radial zone, then $d\bar{\epsilon}_{n+1}$ replaces $d\bar{\epsilon}_n$ for all rings and the code returns to step 3 to recalculate new values for P and M and a new d_{n+1} . This procedure is continued until condition 5.a.i is satisfied.
- 6.a. The thermal loading continues in temperature decrements until a liquid helium temperature of 4.2 K is achieved, with the code reporting the final three-dimensional stresses and strains and effective stress-strain results for all material zones within the cylindrical repeating element.

These efforts give the complete state of stress and strain on the composite components as the result of stresses induced by differential thermal contraction. The next stage of the program mathematically simulates the conditions inherent in a superconducting composite tensile test in which an applied axial load is placed on the conductor producing a corresponding axial z strain. The axial strain can be measured experimentally via

strain gages attached to the sample, what is unknown are the resulting changes in the radial and azimuthal strains in the Nb_3Sn zones. The code has the capacity to determine these by repeating steps one through six above, with slight modifications.

- 1.b. The initial guesses for P and M now assume the values remaining from the thermal loading solution (because the last reported total effective strain represents the residual strain states of the composite before axial loading).
- 2.b. With the temperature maintained at 4.2 K, the axial z strain is incremented in steps (to simulate a low-temperature tensile test) of $\epsilon_z = 0.0002$ percent. The elastic solutions of the tangential stress and strain components for equations 16 and 15 are found subject to the boundary conditions. However, now ϵ_z is known in equations 15, and the thermal expansion terms are set equal to zero. The total effective strain is still determined by equation 21 with $d\bar{\epsilon}_n$ being calculated after each axial loading increment.
- 3.b. - 5.b. Steps 3.b thru 5.b are the same as in the thermal-loading problem defined by steps 3.a. thru 5.a.

- 6.b. Axial loading continues until the thermally contracted composite has been stretched to an axial ϵ strain of 1.0 percent.

The program requires the following input data:

1. The radial positions and thicknesses of each composite material (niobium, Nb_3Sn , bronze, and copper) that makes up the cylindrical repeating element, determined by photomicrographs or the intended design of the composite cross section. In general, the tantalum anti-diffusion barriers are ignored, since they surround clusters of thousands of filaments; only the environment of an average filament is modeled.
2. Temperature dependent data on the Young's modulus, Poisson's ratio, thermal expansion coefficient, and stress-strain curves for each composite material. Poisson's ratio and stress-strain curves as a function of temperature are rare for most materials; therefore, room or liquid helium temperature axial stress-strain curves were used in conjunction with temperature dependent Young's modulus. This coarse approximation still yielded good qualitative and quantitative results. The input data, their source references, and the program listing are tabulated in Appendix 1.

The program first prints the residual stresses and strains resulting from the thermal cooldown from 1023 K to 4.2 K on each material zone of the composite. Thus the signs (compressive or tensile) and the magnitudes of the three-dimensional stress-strain state of the repeating composite cylindrical element are reported. These values are converted to effective stresses and strains by use of equations 17 and 18. Notice that the square root functions of the effective stress and strain equations lose information on signs of the deformation field; they simply report the magnitude of the material yielding or change of shape. Previous investigations have shown that critical superconducting properties such as J_c , T_c , and H_{c2} are dependent on the axial strain present on the conductor. For a conductor with a triaxial state of strain on the Nb_3Sn zones, these critical properties should vary as the magnitude of the elastic and plastic deformation, which is the effective strain.

Next, axial loading is applied to the composite: the code reports the resulting stress-strain fields for each zone at selected values of the axial z strain. The effective strains on the Nb_3Sn regions are inserted into appropriate scaling laws which relate the critical current density to the amount of strain on the Nb_3Sn . The details of these scaling laws are the subject of

Chapter 5, which develops a possible physical explanation to account for the reduction of the critical current with strain.

C. Analytic Results and Predictions of Program MAXIMSUPER

Currently two types of multifilamentary conductors have been modeled into the program; the internal niobium- and bronze-core filament composites. Conductors with niobium cores produced by the bronze-diffusion process have repeating fiber elements of niobium rods immersed in a bronze matrix with a reacted Nb_3Sn interface between the bronze and niobium. As shown in Fig. 13, clusters consisting of perhaps several thousand of these elements are surrounded by walls of tantalum metal, which act as diffusion barriers prohibiting the migration of tin from the bronze into the high purity second copper matrix. The code models only the immediate environment surrounding a typical filament, as described by Section A of this chapter. A second type of multifilamentary conductor consists of bronze-core filaments inside of niobium tubes with Nb_3Sn acting as the interface. In this case, the unreacted niobium acts as the diffusion barrier and eliminates the need for tantalum barriers. This new geometry also has the desirable property of placing the copper stabilizer closer to the Nb_3Sn superconductor, minimizing the

current transfer distance in the event of a superconducting to normal transition in the Nb_3Sn .

Several conductors of the two geometry types with various bronze-to-niobium volume ratios, Nb_3Sn layer thicknesses, and amounts of copper surrounding each filament have been modeled into the computer program. The first predominant observation was that the residual, thermally-induced ϵ_z strain, achieved by the cooldown from reaction temperature to liquid helium, resulted in ϵ_z values of approximately ~ -1.0 to -1.5 percent strain for all the conductors analyzed (see Appendix 2). The amount of residual axial z strain increases with the volume fraction of the bronze and/or copper present in the repeating composite element. Most of the residual strains occur between the reaction temperature (approximately 1023 K) and room temperature, which is expected, since the thermal contraction is proportional to the reference and ambient temperature difference. When axial tension is applied to the sample at 4.2 K, the effective strain on the Nb_3Sn zone (reported at the center of the layer thickness), calculated by equation 18, decreases to a minimum value and then increases with continued axial straining, as shown by the sample output listing of Appendix 2. The effective strain is a square root function that only reports the magnitude of the Nb_3Sn

change of shape. However, the program also gives the principle stresses and strains on the Nb_3Sn layers as a function of applied axial loading. These results show that the Nb_3Sn is always under compression (negative strains) from the radial and azimuthal strain components. In fact, the tangential strains on the Nb_3Sn become more compressive with increasing tensile strain.

Previous investigators have rearranged the existing strain-critical current data using the Rupp normalization method (see Figs. 4 and 5).²⁹⁻³⁰ The intrinsic strain is assumed to be zero at the peak critical current and is defined as $\epsilon_{\text{intr}} = \epsilon_z - \epsilon_{\text{icmax}}$. The final output page of the code also expresses the results using Rupp normalization by subtracting the axial ϵ_z strain, which yields the smallest effective strain, from the rest of the ϵ_z values. Also, notice that minus signs are appended to the effective Nb_3Sn strain magnitudes for axial ϵ_z values, in which the Nb_3Sn is thought to be under compression (before decreasing to the minimum value), while those thought to arise from tension have positive signs. The sign addition just visually indicates when the Nb_3Sn is under compression and tension to make the results commensurate with the experimental data, since the sign of the distortion is lost by the square root function used in calculating the effective

strain. Consequently, it is expected that higher effective strains on the Nb_3Sn result in lower critical currents I_c .

One of the most important attributes of the computer program is its ability to predict the axial strain component ϵ_z , which corresponds to the maximum I_c for a wide variety of conductor designs tested by different investigators. Fig. 15 shows a plot of the measured strain percentages of several conductor designs at peak I_c value (see also Fig. 4) versus the calculated strain percentage predicted by MAXIMSUPER. The code results are in agreement with the Luhman and Suenaga data (using composite monofilaments) which show that higher precompressive ϵ_z strains are obtained with increasing bronze-to-niobium ratios.⁵ In general, the program results show that the residual thermal strains on the Nb_3Sn zones are not only sensitive to the bronze-to-niobium ratio, but also to the layer thicknesses of the Nb_3Sn zones arriving from the diffusion reaction. This may also influence the necessary heat treatment (reaction time and temperature) which yields the highest value of critical current density J_c for a conductor (see Fig. 16). The higher elastic modulus for Nb_3Sn (as compared to niobium) results in less thermal strain in the Nb_3Sn for thicker reaction layers. Also included in Fig. 16 is data

from internal niobium-core filament conductors produced by some investigators from Fig. 4 and some recently measured samples of the internal bronze-core configuration. The good agreement shown in Fig. 15 indicates that the program is suitably adapted to encompass both conductor types.

One unique distinction is demonstrated by the MAXIMSUPER strain field predictions for the bronze-core filament geometry as compared to the niobium-core configuration. While the two conductor designs yield approximately the same z component strains ϵ_z , the radial ϵ_r and azimuthal ϵ_θ are much higher for the bronze-core filament design. To illustrate this effect, the code used the geometrical averaging of the three-dimensional strain fields to calculate the effective strain in equations 18 and 21 and plotted this value against the intrinsic ϵ_z strain component for a prototypic bronze- and niobium-core filament conductor in Fig. 17. Notice that the effective strains are much higher for the bronze-core sample because of the contributions to the effective strain (equation 18) from the larger tangential (ϵ_θ , ϵ_r) strain components. Fig. 17 illuminates an interesting anomaly: while earlier investigators have concluded that the peak critical currents of Figs. 4 and 5 occurred at approximately zero strain on

the conductor, it can be seen that the zero intrinsic ϵ_z strain values, which give the peak critical current, actually coincide with non-negligible radial and azimuthal strain components on the Nb_3Sn zones. Hence, the maximum current displayed at zero intrinsic z strain is the result of a minimum in the effective strain of the three-dimensional strain field, and the minimum value is not necessarily zero. This is particularly true for the internal bronze-core geometry, as shown in Fig. 17, in which the minimum effective strain on the Nb_3Sn zones is ~1 percent. In the case of axial loading, it is known that the tangential stresses arising on multicomponent, cylindrical composites result from the differences in the value of the Poisson ratios. Experimental and analytical studies performed on two- and three-component material composites have revealed that the intensity of the transverse stresses developed between two mechanically interacting materials is nearly in direct proportion to the differences in the value of their respective Poisson ratios.^{13,15,27} As an illustration, one can treat the elasticity equations 15 and 16 for the case of an axially loaded, two-component, cylindrical composite and produce two simultaneous linear equations which are solved to yield:²⁰

$$\sigma_r = K \left\{ \frac{a^2 \epsilon_z (1 - 2\nu) (1 - b^2/R^2)}{r} \right\} \quad (2-22)$$

$$\sigma_{\theta\theta} = K \left\{ \frac{a^2 \epsilon_z (1 - 2\nu) (1 - b^2/R^2)}{3} \right\} \quad (2-23)$$

$$\epsilon_z = \epsilon'_z + 2a^2 \epsilon'_z \left[\frac{K(1 - 2\nu)}{r} \right] \quad (2-24)$$

$$\epsilon'_r = \epsilon'_\theta = K' \epsilon'_z \left\{ \frac{a^2(1 - 2\nu) + b^2}{r^3} - (1 - \nu') \right\} \quad (2-25)$$

$$\epsilon'_z = \epsilon'_z + 2a^2 \epsilon'_z K' \left\{ \frac{a^2(a - 2\nu) + b^2}{r^3} - (1 - \nu') \right\} \quad (2-26)$$

$$\sigma_r = \frac{a^2 \epsilon_z}{r} \left[(1 - 2\nu) - \frac{b^2}{R^2} \right] - \nu' \epsilon_z \quad (2-27)$$

$$\sigma_{\theta\theta} = \frac{a^2 \epsilon_z}{r} \left[(1 - 2\nu) + \frac{b^2}{R^2} \right] - \nu' \epsilon_z \quad (2-28)$$

$$\epsilon'_r = \epsilon'_\theta = \epsilon'_z \quad (R = a) \quad (2-29)$$

$$\nu = \frac{1}{(1 - \nu')} \left\{ a^2(1 - 2\nu) \left(1 - \frac{K}{K'} \right) + b^2 \left[1 + (1 - 2\nu) \frac{K}{K'} \right] \right\} \quad (2-30)$$

This is the elastic solution for a composite with a core material of radius "a" surrounded by an annular ring of material with radius "b." The primes denote solutions and physical properties pertinent to the core region of

the double concentric cylinder. There are several conclusions that are apparent from these results:

1. All radial and azimuthal stresses vanish in both zones if the Poisson ratios of the two materials are equal.
2. The tangential stresses and strains scale nearly proportional to the differences of the value of the Poisson ratios.
3. The solutions are obviously not symmetrical with respect to the interchange of the two material locations (replacing the core material with the case material and vice versa yields different states of stress and strain).

MAXIMSUPER models a superconducting composite filament with about four material components, each further segmented into several radial zones, in order to report the stress-strain conditions at various radial positions across the cylinder cross section. For the investigation reported in this dissertation, the superconducting filament was divided into typically ten to twelve annular zones. Although the Gaussian elimination method solves a much larger set of simultaneous linear equations for these composites, the resulting stress-strain state of each radial zone is congruent with the above conclusions concerning the simple two-zone cylindrical composite.

Therefore, it appears that the development of higher tangential (radial and azimuthal) strains, resulting from thermal contraction and axial loading, in the bronze-core composite geometry is simply due to the interchange of the locations of the bronze and niobium. The functional dependence of the resulting strains on the differences between the Poissons ratios of the bronze, niobium, and Nb_3Sn is not symmetrical with respect to changing the locations of composite materials.

The prediction of higher residual effective strains in the bronze-core geometry is experimentally verified in Fig. 18, which is a plot of the measured critical current density of several conductors at an applied magnetic field of $H = 12$ Tesla (and resistivity $\rho = 10^{-11} \text{ } \Omega \cdot \text{cm}$) as a function of the MAXIMSUPER predicted residual effective strain (before axial tensile loading). The critical current density data was obtained from the experimental results of various monofilament niobium-core and bronze-core geometry conductors (Chapter 3 contains the experimental methods). Photomicrographs of the conductor cross sections were examined to model each composite into the code. In general, Fig. 18 shows that one should expect the bronze-core configuration to be inferior to the niobium-core design, with a decrease in critical current density of factors from three to ten times.

One possible way to obviate the higher residual tangential strains inherent in the bronze-core geometry is to attempt to match the Poisson ratios of the bronze, niobium, and Nb_3Sn . Of the three components, the bronze is the first to undergo plastic deformation; consequently, its Poisson's ratio climbs from about 0.34 to about 0.50, the value obtained for a metal which yields under plastic flow. Beryllium has a Poisson's ratio of about 0.05 and is often used to both stiffen and increase the Poisson's ratio of companion metals in alloys. The addition of various amounts of beryllium to the bronze-core filament would produce a closer match of the Poisson's ratios, thus lowering the transverse stresses and strains on the Nb_3Sn layer. Recent work performed by Lunman and Suenaga has revealed that the effect of beryllium addition to the bronze matrix of niobium-core composites is to increase the amount of initial thermal prestrain in the axial z direction on the conductor.^{31,32} Some initial compressive axial z prestrain is a desirable factor in the design of conductors, because the composite is often subject to some tensile strain during fabrication and with coil energization (due to Lorentz forces). However, high residual transverse strains (ϵ_r , and ϵ_θ) are undesirable since they cannot be completely removed by axial straining, which only causes them to increase, as

shown by the code predictions in Fig. 17. Therefore, the continued study of the effects of metal alloying with bronze in the internal bronze-core configuration may provide a way to resolve the strain, and hence critical current density problems associated with this design.

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CHAPTER 3. CRITICAL CURRENT-STRAIN EXPERIMENTS

A. Introduction to the Chapter

Critical current versus strain measurements were performed on several conductors of the new internal bronze-core configuration. The purpose of these experiments was first to ascertain information on the effects of mechanical straining on the J_c performance of these multifilamentary superconducting composites and to compare these results with the theoretical predictions of the MAXIMuPER code as described in Chapter 2. This chapter documents the equipment, instrumentation, procedure, results, and conclusions of these experiments.

B. Equipment

Dr. D. Deis of Lawrence Livermore National Laboratory has designed and constructed a cryogenic tensile testing facility for evaluating the mechanical and superconducting properties of short-sample lengths (approximately 60 cm) of Nb_3Sn conductors. These conductors are prototypical of those which will be used for the fabrication of advanced, new generation, high-field ($B > 10$ T) magnets for the world magnetic fusion effort.

Photographs of the tensile-test apparatus are shown in Figs. 19 and 20. Basically, the device is a hydraulically controlled tensile machine capable of delivering a maximum load of 223 kN to the short-sample conductor, while applying a magnetic field perpendicular to the sample length. The conductor, tensile-pull grips, and superconducting magnet are immersed in a pool of liquid helium (at 4.2 K) within a "superinsulated" low-temperature cryostat. The crosshead (shown in Fig. 19) is raised and lowered by two hydraulic pistons, thus allowing the sample, magnet, and entire load structure to be placed into the cryostat after the sample is mounted or removed following a test. Force is applied to the sample by the main hydraulic load piston located at the top center of the crosshead, while the cryogenic structure takes the conductor tensile load into compression on its three stainless-steel cylinders. Support structures of the load frame and cryostat are constructed from nonmagnetic stainless steel to minimize distortion and interaction with the magnetic field.

Current is supplied to the sample by two copper contacts which are indium soldered at the conductor ends. Flexible braided copper leads connect the copper blocks to the main cryostat current leads to minimize external forces on the sample. The power supply generates a

maximum of 10 kA in the conductor, which produces I^2R heating in the current contacts and leads of approximately 5 W each. During operation, the entire cryostat, magnet, and sample have a liquid helium boil-off rate of about 60 l/hr.

Generally, the Lorentz force, resulting from a transverse field of 12 Tesla in the presence of a conductor carrying several thousand amps, would exert a force on the sample causing it to bow between the upper and lower tensile grips, which rigidly hold it in place. Side pole plates of epoxy G-10 prevent the sample from deforming under this force.

Electronic control of the tensile tester is maintained by a null detector servo-meter, which compares a programmed command input to a measured value and equalizes the two by sending an output response to the mechanical hydraulic system. The output response activates a servo-valve which regulates the amount and flow rate of hydraulic fluid to the pistons, which raise and lower the upper tensile grip. The null detector continuously checks the two signals and readjusts the system status until the measured response signal equals the programmed signal. The device can operate in two modes, under load control and under position control. During the load control mode, the task of the servo-detector is to have the system follow a

preprogrammed load. Therefore, the tensile grips are instructed to maintain a sequence of constant forces on the sample, regardless of the conductor elongation. The force exerted on the sample is reported to the servo-detector by a load cell located at the top of the test facility crosshead. Generally, for stress-strain measurements (and also for the strain-critical current experiments), the tensile tester is operated in the position control mode. The location of the moving upper tensile grip is reported to the servo-detector by a linear voltage transducer detector, LVDT, within an accuracy of several thousandths of an inch.

C. Instrumentation

Although the LVDT gives the position of the upper grip, the actual strain produced on the conductor is monitored by resistive strain gages affixed along the sample length. The strain gages and their associated instrumentation yield a resolution of strains as low as ± 0.001 percent, which for a typical 60-cm long sample corresponds to a length change sensitivity of ± 10 μ m. Figure 21 shows a typical conductor affixed with the resistive strain gages. The gages used were Micro-Measurements type WK-15-250BG-350 strain gages, which were bonded to the sample conductors with M-Bond 600

adhesive, according to the manufacturer's suggested technique. These instruments report longitudinal strain changes in the conductor by monitoring the change produced in the gage's resistivity, which depends on the thickness of the wire grid that forms the strain gage. The resistance of the nominally $350\ \Omega$ strain gages was measured using a four-arm Wheatstone bridge. The excitation current to the bridge was 10 mA constant current, which was found to produce minimal heating of the strain gage. Previous studies involving the effect of magnetic fields (magnetostriction), temperature compensation, apparent strain, and bridge excitation have been performed and reported elsewhere.¹

The voltage developed across the superconductor was measured using the four-point method: two leads supply current to the sample, and two additional leads between the current leads measure the voltage which occurs when the conductor undergoes the superconducting-to-normal transition. The resistance is simply the ratio of the voltage to sample current.² There are often problems associated with this measurement because a finite distance is required for the current, introduced at the conductor ends, to transfer from the copper and bronze matrix to the inner superconducting microfilaments. Therefore, the voltage taps have to be spaced sufficiently

away from the current connectors. A magnetic field gradient also tends to distort the uniform current distribution within the sample. This can be avoided by placing the voltage taps within a region of uniform field strength.^{3,4} Since the 12 Tesla magnet at the facility has a two-inch inner bore in which the field is constant, the voltage taps were spaced one inch apart in the center of each conductor. The strain gages, affixed in the center of each conductor were also within the high-field region of the magnet, making it necessary to correct the reported strain values with available magneto-resistance data.

The 12 Tesla magnet is a split-pole superconducting Noyon model purchased from Intermagnetics General Corporation. The split allows the sample conductor to be inserted into it with the magnetic field perpendicular to the sample length. Magneto-resistance probes mounted in the magnet bore measure the central field value. The magnet and conductor setup is illustrated in the schematic diagram of Fig. 22, while the actual experimental arrangement is shown in Fig. 23.

D. Experimental Procedures and Results

The procedure was to charge up the magnet to a field of 8 Tesla, measure the critical current, increment the

field to 10 and 12 Tesla, recording the critical current for each field value, then to place a tensile strain on the conductor and repeat the measurements. At each strain and field value, voltage versus current plots are produced by ramping a current into the sample from a power supply and measuring the voltage induced in the conductor as it transforms to the normal state. An example of such a V-I curve is shown in Fig. 24. Since the resistance of the superconducting state is zero, a standard resistance must be selected to define the transition to the normal state. The resistivity criterion of $1 \times 10^{-11} \text{ } \Omega\text{-cm}$ was chosen to represent the point of transition to normal conductivity. Data are taken from the V-I curves and used with

$$\rho = \frac{V_n}{IL} = 1 \times 10^{-11} \text{ } \Omega\text{-cm} \quad (3-1)$$

to produce plots of resistivity versus current, which are then extrapolated to yield the value of the conductor current and voltage, which equaled this value of resistivity (Fig. 25). In equation 1, A is the cross-sectional area of the niobium, niobium-tin, tantalum, and bronze (every component except the copper), while L is the distance between the attached voltage taps. For each strain increment, the critical current was measured at 8, 10, and 12 Tesla.

Four conductors of the internal bronze-core design were analyzed on the tensile testing facility. The details of the conductor fabrication are given in another report, but summary data displaying the major geometry, composition, fabrication, and resulting critical currents (with no applied strain on the conductors) are tabulated in Table 1.⁵⁻⁷ Typical photomicrographs of the conductor cross section are shown in Figs. 26 and 27. Plots of the critical current I_c normalized to the maximum I_c value as a function of applied axial strain are shown in Figs. 28 and 29. These curves are further reduced by the Rupp normalization mentioned in Chapter 2. The curves of Figs. 28 and 29 are further normalized with respect to the strain that produces the maximum critical current. The intrinsic strain is assumed to be zero at the peak critical current and is defined as $\epsilon_{\text{intr}} = \epsilon - \epsilon_{\text{imax}}$. These curves are shown in Fig. 30 with Rupp normalization. For purposes of comparison, the envelope curves of the internal niobium-core multifilamentary conductors are superimposed over the internal bronze-core data in this figure. In general, there is no difference between these curves and those obtained from the niobium-core filament conductors of Fig. 5. This indicates that the rate of critical current degradation with increasing axial strains is equivalent for both conductor geometries. However,

the initial critical current densities, J_c measured at 4.2 K, for the internal bronze-core geometry conductors are factors of three to ten times smaller than niobium-core designs with equivalent bronze-to-niobium ratios as shown in Fig. 18. These lower critical current densities are a direct verification of the analytic results of the MAXIMUSPER computer program, which predicts lower J_c values due to higher tangential strains produced on the Nb_3Sn zones, as the result of differential thermal contraction of the composite material components. Thus far, no variation of the heat treatment times and conditions has been shown to alleviate this effect. Several J_c values were measured by J. Zbasnik for conductors of this geometry with differing heat treatments.⁸ These results are tabulated in Table 2; in all samples, the J_c values are less than those obtained for the niobium-core configuration. For several conductors, the reported J_c values are lower by factors as large as 30 times.

Some corroborating data on this effect was independently obtained on these and other internal bronze samples by K. Aihara and M. Suenaga, who noticed a higher suppression of the transition temperatures T_c in these conductors.^{7,9-10} For a comparable bronze-to-niobium ratio, the T_c reduction in the internal bronze-core conductor is about 1.8 K, compared with about 0.2 K for

an internal niobium-core conductor. Similarly, the transition temperature degradation for the internal bronze samples, on a master plot of I_C reduction versus strain, would indicate a strain of greater than 1.0 percent in the Nb_3Sn layers (Fig. 6). This is in agreement with the code results of Fig. 18, which indicate predicted residual strains of -1.0 to -1.5 percent for the internal bronze-core configuration.

Additional information was ascertained on the stress-strain relations of these conductors. Several examples are shown in Figs. 31 and 32. In general, these curves seem to agree with those taken of the niobium-core samples; the stress-strain relationships are dependent on the volume fractions of the bronze, niobium, Nb_3Sn , and other composite material components. Beyond a certain tensile strain, "acoustical pings" could be heard, which were indicative of the fracturing of several Nb_3Sn filaments. These sounds were reproduced as spikes on the stress-strain curves. A comparison of the strain-critical current data revealed that the filament fracturing started at approximately 0.2 percent strain past the strain corresponding to the maximum in the critical current. This was verified by relieving the strain in a conductor (Fig. 29) and repeating the I_C measurements, which revealed that the strain degradation of the critical

current was reversible for strains less than 0.2 percent beyond the peak I_{Cmax} value. It is thought that although Nb_3Sn can withstand large precompressive strains of 41 percent, it fractures in tension for strains greater than 0.2 percent. These results may serve as an explanation for the asymmetry shown on the parabolic curves of I_C versus strain in Figs. 5 and 30. The fact that the critical current tends to decrease more rapidly when the conductor is under tension than compression may be due to three possible effects:

1. The development of cracks in the Nb_3Sn layers for tensile strains exceeding 0.2 percent at the peak intrinsic strain value.
2. There may be larger tangential strains induced in the composite matrix on the Nb_3Sn zones under tensile, rather than compressive, strains.
3. A possible asymmetry in the functional dependence of the I_C versus strain relationships for Nb_3Sn , I_C may be a function of both odd and even powers of ϵ .

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CHAPTER 4. LOW TEMPERATURE X-RAY DIFFRACTION EXPERIMENTS

A. Introduction to the Chapter

A possible clue to the anomalous degradation of the Nb_3Sn superconducting critical parameters J_c , H_{c2} , and T_c with increasing strain may be provided by the existence of the low-temperature phase transition reported for the material. Using ultrasonic attenuation and neutron scattering techniques, Testardi and others have observed that the elastic constants, and hence the velocity of sound, decrease in Nb_3Sn at low temperatures.¹⁻⁸ As noted by Kittel, the softening of an elastic constant is usually indicative of lattice instability and, therefore, can result in a phase transition.⁹

Generally, a lattice structure which is stable at higher temperatures becomes metastable as the temperature is reduced and finally transforms to a structure with less symmetry at the transformation temperature T_m . In the case of Nb_3Sn , T_m has been found to be approximately 50 K (T_m is used so as not to confuse the crystalline phase change with the superconducting transition temperature $T_c = 17.8$ K for Nb_3Sn).¹⁴

Investigators have measured the lattice constants during the transformation and have concluded that the phase change involves a cubic to tetragonal transition.^{6,10-12} Apparently, the tetragonal distortion readjusts not only the lattice c length, but also the a axis so as to maintain nearly constant (if not equal) lattice cell volume under the transition.

There is some question concerning the nature and classification of the phase change. Phase transitions in metallic systems can be divided into two main classes, nucleation-and-growth transformations and martensitic transformations. The former can proceed under isothermal conditions in that, given sufficient time, the amount of material transformed increases even if the temperature is held fixed, with thermal activation and diffusion acting as the driving force for the transformation. On the other hand, martensitic phase changes usually only occur while the temperature is changing and are dependent only slightly, if at all, on thermal agitation. Diffusion is not required; there is a change of the shape of each region that transforms, which can lead to the production of surface tilting and/or strain in the vicinity of the transformed volume in the material.

What appears to be confusing is the fact that some investigators have reported Nb_3Sn samples in which they

noted no observable transformation or single crystals which were only partially transformed.¹⁰⁻¹²

King has shown that this anomalous behavior can be attributed to the differences between the stoichiometry of the prepared specimens; variations in the ratio of niobium to tin lead to changes in the cubic lattice cell parameter.¹³ Thus, it appears as if only Nb_3Sn compounds whose unit cell sizes fall within a limited range of lattice parameters can actually undergo a phase transition. His results are shown in Fig. 33. This figure implies that if the cubic-to-tetragonal phase transformation is dependent on the lattice parameter, it may well also be a function of the strain on the material.

Strain induced transitions have already been reported on stainless steels and many other alloys, in which transformations have been excited at temperatures well above the martensitic start temperature T_m .¹⁴⁻¹⁸ Indeed, these results may be interpreted as if T_m itself is a function of the applied strain. This may be obtained analytically by writing the Gibbs free energy of the two phases as a sum of terms dependent on the temperature and strain. Minimizing this expression with respect to the temperature should yield the transformation temperature T_m . Since every martensitic transformation involves a distortion of a local region of the material which produces stresses on

the surrounding untransformed volume, it would be expected that all could be aided by the application of externally applied stresses.

In order to determine if the phase transition in Nb_3Sn is strain sensitive, it is necessary to study the transformation as a function of temperature under varying increments of strain. Phase transitions are normally observable by x-ray diffraction techniques; unfortunately Batterman and others have noted that T_m (for Nb_3Sn) occurs at approximately 50 K which necessitates the utilization of a low-temperature cryostat suitable for x-ray diffraction measurements. This chapter documents the equipment, instrumentation, experimental method, results and conclusions of the phase transition studies performed on Nb_3Sn .

B. Equipment

1. Low Temperature Cryostat--The cryostat used for the mounting, cooling, and x-ray analysis of the Nb_3Sn samples was a continuous flow Heli-tran LT-3-110 cryostat manufactured by Air Products, Inc. The continuous flow operation utilizes a flexible transfer line which transports liquid helium from a 50 l storage dewar to the cryostat. The continuous liquid helium stream cools a copper sample holder in the cryostat, which in turn cools

the attached sample by conduction. For a liquid helium flow rate of 0.7 l/hr., the net refrigeration capacity of the cryostat is 500 mW at 4.2 K, 3 W at 20 K, and 7 W at 50 K. The system uses a micrometer needle valve to permit control of the flow rate. The approximate cooldown time is 45 minutes from a warm start and about 15 minutes for subsequent cooldowns. Figures 34, 35, and 36 show the schematic views of the cryostat connections and system setup.

Radiation shields and vacuum shroud covers are interchangeable on the Heli-tran unit, with designs appropriate for a wide variety of low temperature applications, such as NMR, optical spectroscopy, x-ray diffraction, Mössbauer effect, etc. The vacuum shroud WMX-2A was selected because of its ease of mounting and compatible goniometer shaft to the Phillips x-ray diffractometer. The WMX-2A shroud has a one-half inch wide, 180 degree cutout window over which a sheet of beryllium has been attached to allow minimum absorption of the x-ray beam before it interacts with the sample. The cylindrical radiation shield also has a one-half inch by 180 degree cutout window (see Figs. 37 and 38).

The cryostat comes equipped with a 20 W noninductive heater, mounted in the tip of the base plate, which houses the sample holder. The temperature of the cryotip

is indicated by a chromel versus gold with 0.07 atomic percent iron doping thermocouple.

All materials used in the construction of the cryostat are non-magnetic, these include 304 stainless steel, OFHC copper, brass, and hard solder.

Several modifications were made in the as-manufactured equipment. These are tabulated below:

1. The liquid helium pickup tube of the flexible transfer line was found to be too short. A typical MVE 50 L storage dewar contains approximately 18 inches of liquid helium; the as-purchased transfer line could only pick up about 8 inches. This problem was eliminated by adding a 10-inch stainless-steel extender tube to the end of the transfer line.
2. The one-half inch by 180-degree cutout vacuum shroud beryllium window, which was originally affixed with "Fuller Resi-Weld Epoxy," was found to leak in vacuums of 1×10^{-5} Torr. The leak was sealed by coating the rim of the window with Torr-Seal epoxy.
3. A sample holder was constructed which would allow the material to be placed under both compressive and tensile strains, while at low temperatures, by means of a vacuum mechanical insert. The holder consisted of

an OFHC assembly which used an advancing Allen screw to separate two copper, wedge poles. The samples were tapped and mounted to the top of the wedge poles (see Fig. 39).

4. The Allen screw was advanced by a vacuum insert Allen driver which passes thru an O-ring seal in the shroud cover, directly opposite the beryllium window (see Fig. 38).

2. X-Ray Generator and Diffractometer--X-rays were produced by a Norelco generator operated at a power level of 50 kV at 30 mA. A wide angle diffractometer manufactured by Phillips Electronics, Inc., was used with copper K α radiation. Radiation incident on the crystal scintillation detector was diffracted by a graphite crystal monochromator to improve the signal-to-noise ratio.

The entire x-ray generator and diffractometer apparatus was operated under automated control by a computer data acquisition system assembled by Lawrence Livermore National Laboratory. The computer directs the angular scanning of the sample through a stepping motor and stores the digital count rate versus scan angle data on a magnetic disk. All computer hardware (dual floppy disk system, teletype, etc.) and software were received from the Digital Equipment Corporation. The software included

a data analysis routine (RT-11) which is used in conjunction with interactive graphics hardware (Nu. 4012 Tektronix terminal and Nu. 4631 hardcopy unit) to further process and display the stored data.

C. Instrumentation

1. Thermometry and Strain Gages--As mentioned in Section B-1, the Heli-tran cryostat comes with a calibrated chromel versus gold-iron thermocouple which is attached to the cold end just above the sample holder. Unfortunately, a separate temperature sensor affixed to the sample often revealed temperature differences of 10 K between the two locations. Therefore, the thermocouple was not used, since it could not be located closer to the sample. The thermometry used was Micro Measurement, Inc., cryogenic linear temperature sensors (CLTS). The CLTS is a small, surface-mounted thermometer gage formed from two thin, foil, sensory grids laminated into a glass-fiber-epoxy-resin matrix. The foils are manufactured from alloys of nickel and manganin which are processed for equal and opposite nonlinearities in their resistance versus temperature characteristics. The resulting composite gage displays a nearly linear change of resistance with temperature. A diagram depicting the CLTS construction and a typical calibration curve are shown in Figs. 40

and 41. Between room temperature and 4.2 K, the resistance changes approximately 70 Ω , yielding a linear sensitivity of 4.197 K/ Ω . Generally, a four-point resistance measurement enables the temperature to be determined within ± 0.04 K.

Two CLTS gages were used; one mounted to the sample holder reported its temperature to a Cryogenics Systems temperature controller. The temperature controller is a null detector that compares the measured temperature to a pre-selected value and directs output current to the Hei-tran cryostat base-tip heater until the two temperatures are equalized. The other CLTS sensor was affixed under the sample (which was the closest approach without being directly exposed to the incident x-ray beam) and reported the sample temperature via a four-point resistance measurement and the calibration curve of Fig. 41. Figure 39 is a photograph showing the sample, tensile sample holder, and temperature sensor locations for the experiment.

The strain gages used were the smaller Micro-Measurements type WK-09-25BT-350 resistive gages wired to form the active element of a four-arm Wheatstone bridge with three lead temperature and resistance compensation. These were mounted next to the Nb₃Sn ribbons (see next section) to avoid interception of the x-ray;

beam. The strain gage is shown next to the Nb_3Sn sample in Fig. 39.

D. Nb_3Sn Sample Preparation and Description

The niobium-tin samples used in this study were bronze-diffusion-processed Nb_3Sn tape, prepared by M. Suenaga and T. Luhman of Brookhaven National Laboratory. No direct analysis has been done to assay the stoichiometry of the formed Nb_3Sn layer, but transition temperature measurements of the tape revealed a small composite T_c of 17.5 K. Removal of the precompressive bronze layers would increment this value even closer to that of unstressed Nb_3Sn ($T_c = 17.8$ K).

The major advantage of using Nb_3Sn bronze-processed tape instead of multifilamentary conductors in this experiment is that the bronze can be completely acid-etched away exposing a flat continuous surface of Nb_3Sn to the incident x-ray beam. The flat thin ribbons also facilitate in the mounting and application of strains on the samples. Since both conductors are fabricated from the tin diffusion reaction and have comparable transition temperatures, their chemical stoichiometry and metallurgy should be equivalent. Extrapolation of these results to those which would have been obtained by using multifilamentary conductors is only complicated by the difference

in geometry between the two (flat ribbons versus cylindrical filaments):

To view the diffraction pattern of Nb_3Sn , it was first necessary to remove the outer bronze layers. This was done by etching the tape in 100 percent nitric acid. The resulting niobium plus Nb_3Sn foil, with a thickness of approximately 13 μm , was attached to an indium wafer with the same strain gage adhesive mentioned earlier. A resistive strain gage was also bonded to the indium, next to the Nb_3Sn ribbon. The indium wafer containing the Nb_3Sn foil and strain gage was bolted onto the two copper wedge poles of the tensile tester sample holder, as shown in Fig. 39.

The attractive properties of indium, used in this experiment, are its relatively good thermal conductivity and its large thermal contraction at low temperatures. The indium sheet places a residual compressive strain of -0.7 percent on the strain gage and Nb_3Sn foil when it is cooled from room temperature to 4.2 K. The purpose of the tensile testing sample holder is merely to relieve the precompression by expanding the wedge poles and the attached indium wafer. Generally, this would yield an effective straining range of -0.7 to 0.0 percent, or until the Nb_3Sn fractured at about +0.2 percent. However, the low temperature creep of the indium wafer

compounded the difficulty of obtaining measurements over this entire range. Instead, measurements were recorded at strains of -0.7, -0.5, and 0.4 percent, using the indium sample stage. An OFHC copper plate (which has a thermal contraction of -0.29 percent between room temperature and 4.2 K) was used to take additional data at -0.3 percent.

4. Experimental Procedure and Results

In order to observe a crystalline phase transformation, one generally studies the x-ray diffraction patterns before and after the transition. The change of crystal symmetry introduces more diffraction lines if the new structure has less symmetry. For a transition from cubic to tetragonal, the interplanar spacing d , changes from

$$\frac{1}{d_c^2} = \frac{h^2 + k^2 + l^2}{a^2} \quad \text{to} \quad (4-1)$$

$$\frac{1}{d_t^2} = \frac{h^2 + k^2}{a^2} + \frac{l^2}{c^2} \quad (4-2)$$

where c and t subscripts refer to the cubic and tetragonal symmetry; a, b, and c are lattice parameters of the unit cell; and h, k, and l are integer value indices of planes in the set (hkl). When equation 2 is combined with the

Bragg law, the angular locations for the diffracted lines are given by

$$2d \sin \theta = \lambda \sqrt{\frac{h^2 + k^2}{a^2} + \frac{l^2}{c^2}} \quad (4-3)$$

As the tetragonal deformation proceeds, c diverges from a and l increases rapidly. The diffracted line location of all planes is related to the set

$$h^2 + k^2 + l^2 = \text{constant},$$

and if the incident energy is fixed, the final diffracted energy remains the same, the result is the appearance of additional diffraction lines whose angular position is different from that of the initial hkl lines.

For the apparatus used in this study, the resolution of the tetragonal splitting of the peaks was limited because the transformation of Nb_3Sn involves a substantial reduction in the lattice parameter c as reported by King.¹² Thus, the structure of the final phase is only slightly tetragonal. To resolve the small resulting angular differences requires longer incident wavelength x-rays than those available to this experiment. Also, improved resolution could be obtained with a moveable sample holder capable of translating the Nb_3Sn in and about the focal plane of the x-ray beam. However, evidence for the transformation was obtained

using $\text{CuK}\alpha$ radiation by observing the (611, 5 $\bar{1}$ 2) diffraction line.

Figures 42 and 43 show the experimental setup of the diffractat, flexible transfer line, diffractometer, and the tetragonal straining insert. The resolution was adequate to resolve peaks defined by the $\text{CuK}\alpha_1$ line 1.54061 Å and $\text{CuK}\alpha_2$ line 1.5444 Å for this line, at room temperature, as shown in Fig. 44. With the onset of the tetragonal distortion, the family of "611" and "5 $\bar{1}$ 2" planes will now be indexed independently as $h_1k_1l_1$ and $h_2k_2l_2$ and therefore additional diffraction lines for the planes known as 611:

Therefore, what was once a single diffraction line for the cubic lattice splits into five lines each for the $\text{K}\alpha_1$ and $\text{K}\alpha_2$ radiation. Since the total integrated intensity remains the same, we would expect a broadening of the room temperature pattern with a corresponding decrease in the pattern peak intensity.

Figure 45 shows the Nb_3Sn diffraction profile after the sample was cooled to 8 K. Notice that the $\text{K}\alpha_1$ and

$K\alpha_2$ doublet is no longer resolvable, the peak intensity has lowered, and the angular width of the pattern has spread. The diffusing of the lines is in opposition to the effect of temperature on diffraction patterns, which tends to produce sharper images as the lattice thermal excitation decreases.²⁰

An attempt was made to determine the martensitic transformation temperature T_m by exciting the cryostat tip heater via the temperature controller, regulating the liquid helium flow valve, and repeating the diffraction scan at approximately 10 K increments. Figures 46 thru 55 indicate that the phase transition is active over a broad temperature range when the indium compressive strain on the Nb_3Sn is -0.65 percent. The predominant feature of these patterns is that the peak height decreases, while the peak FWHM increases as the temperature is lowered, until a certain minimum low temperature is achieved below which the transition is arrested. This can be seen in the plot of Fig. 56, which shows peak height (above background) as a function of temperature for an initial Nb_3Sn compressive strain of -0.65 percent at 8 K.

In general, the effects of inhomogeneous strains in a material is to broaden the lines of its diffraction pattern.²¹ In order to ascertain that the observed broadening of the (611, 532) lines at low temperatures was not

coupled with this effect, room temperature scans were performed at various strains. The peak height of the diffraction line was found to be constant for the small strain interval of 0 to 0.5 percent, indicating that the observed line broadening was more likely due to line splitting initiated by the phase transformation.

At this point the residual compressive strain exerted on the Nb_3Sn by the indium sample stage was relieved at 8 K by using the mechanical insert to spread the copper wedge poles. The x-ray scans over the (611, 532) lines were repeated at different values of the compressive strain and the peak maxima are displayed in Fig. 57. The last curve, at an initial strain of -0.29 percent was achieved by mounting the Nb_3Sn sample on an OFHC copper plate. The first observation obtained is that the higher strains tend to broaden the temperature range over which the transition occurs. At a strain of -0.641 percent, the transformation starts at approximately 100 K and finishes at 20 K; while at -0.29 percent, the range occurs between 40 and 50 K. The intermediate strain values also support this trend. A second result is that the change in peak height between 8 K and room temperature increases for larger strains. This may be indicative of the number volume of lattice cells in the material which actually undergo the cubic-to-tetragonal transformation. Fig. 58

shows a similar effect, reported by Vance, in which the elastic constants, as a function of temperature, were measured for different samples of $\text{Al}_5\text{V}_3\text{Si}$.²² Although diffraction patterns do not measure elastic moduli, possibly the common feature of the two different observations is that they both relate to volume-averaged quantities of the bulk material. The fraction of unit cells which have transformed acquire new lattice parameters and elastic constants corresponding to those of the tetragonal structure.

Assuming that the data support the occurrence of fractional transformations in the Nb_3Sn tape, it should be possible to normalize the plots of Fig. 57 by dividing each curve by its room-temperature peak height (since Nb_3Sn is known to be completely cubic at ambient temperatures). The resulting curves should be proportional to the volume fraction of cubic unit cells remaining in the material as a function of temperature. Figure 59 shows the normalized curves with the cubic volume fraction saturating to one at higher temperatures. Strain not only increases the temperature range over which the transformation takes place, but it is also interesting to notice that even below the cubic-to-tetragonal arresting temperature at which the phase transition stops, strain acts to

enhance the production of tetragonal unit cells in the material.

In order to investigate further the strain dependence of the phase transition at low temperatures, the cryostat was held at 8 K and diffraction profiles were taken of the (611, 532) line at different compressive strains. Figure 60 shows a plot of the peak height versus strain on the Nb_3Sn ribbon, while Fig. 61 repeats this plot with the peak heights normalized by the extrapolated zero strain intensity to display the cubic volume fraction as a function of strain at 8 K. Several observations concerning the strain dependence of the Nb_3Sn transformation can be made from the last three figures. Extrapolation to higher strains indicates that the Nb_3Sn transformation should be complete at a strain of approximately -1.8 percent at 8 K. Extrapolation to low strains indicates that Nb_3Sn may not undergo a phase transformation at zero strain. However, Nb_3Sn has a thermal contraction of -0.13 percent between 298 and 4.2 K, which may also augment the transformation.

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Chapter 4

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CHAPTER 5. CRITICAL CURRENT-STRAIN SCALING LAW AND CONCLUSIONS

A. Introduction to the Chapter

The purpose of this chapter is to formulate conclusions concerning the experimental and analytical results obtained in the previous sections of the current investigation. Thus far, the report has defined the strain sensitivity of the critical parameters of Nb_3Sn , discussed the development of an improved method of calculating the triaxial state of strain on the Nb_3Sn regions within a multifilamentary composite, and probed the underlying mechanism of the strain sensitivity, via low temperature x-ray diffraction, to find a linkage between the martensitic phase transformation and strain. The output strain values, as determined by the MAXIMSUPER program, are inserted into appropriate scaling laws that relate the functional dependence of the critical current density J_c to strains present on the superconductor. This chapter utilizes the experimental results of Chapters 3 and 4 to formulate the $J_c(\epsilon)$ scaling laws. The first step is to attempt to find an analytical expression which relates the volume fraction of niobium-tin transformed,

at low temperatures, to the amount of applied strain; then the effect of the two different lattice cell properties on H_{C2} and J_C are explored.

B. The influence of Strain on the Cubic-Tetragonal Transformation

The observation that the martensitic phase transition is continuous indicates that it may be amenable to treatment as a second order transformation. Following a derivation by Landau, one can write the Gibbs free energy of the double phase Nb_3Sn as a power series expansion of the tetragonal volume fraction¹

$$G(\epsilon, T, \eta) = G_0(\epsilon, T) + A(\epsilon, T)\eta^2 + C(\epsilon, T)\eta^4 + \dots \quad (5-1)$$

in which the coefficients G_0 , A , and C are functions of both strain and temperature. Cubic and first order terms were shown to vanish for second order transformations that resulted in changes in the parent phase crystal symmetry. The tetragonal volume fraction that minimizes this expression is determined by equating the derivative $\partial G / \partial \eta$ to zero, which yields

$$\eta_t^2 = - \frac{A(\epsilon, T)}{2C(\epsilon, T)} \quad (5-2)$$

Since Nb_3Sn is operated at low temperatures and small strains A can be expanded in a two variable Taylor series, about T and ϵ equal zero.

$$-A(\epsilon, T) = A_0(0,0) + \alpha\epsilon + \frac{1}{2}\beta\epsilon^2 + \gamma T + \frac{1}{2}\delta T^2 + \nu\epsilon T + \dots \quad (5-3)$$

At temperatures below 20 K, the tetragonal volume fraction appears to be relatively constant with temperature and nearly linear with strain, as shown in Figs. 59 and 61. Therefore, the series in equation 3 is truncated after the first four terms, and the denominator of equation 2 is approximated as a constant (set equal to unity). The tetragonal volume fraction now becomes

$$\eta_t = \sqrt{A_0 + \alpha\epsilon + \frac{1}{2}\beta\epsilon^2 + \gamma T} \quad (5-4)$$

Notice that if Nb_3Sn does not transform in the total absence of strains, then A_0 should be zero. However, even without external stresses, the material should undergo thermal contraction, making it possible to interpret the first and fourth terms as the thermal residual strain. Equation 4 can be written as

$$\eta_t = \sqrt{\alpha\epsilon + \frac{1}{2}\beta\epsilon^2 + \alpha\Delta T} \quad (5-5)$$

where $\alpha\Delta T$ is the Nb_3Sn thermal contraction between reaction temperature and 4.2 K. The corresponding cubic volume fraction is given by

$$\eta_c = 1 - \sqrt{\alpha\epsilon + \frac{1}{2}\beta\epsilon^2 + \alpha\Delta T} \quad (5-6)$$

C. The Effect of Strain on J_c and H_{c2}

In this section, an attempt will be made to derive the functional relationship between strain and the critical current for type II superconductors. The first step is to obtain an expression for J_c (ϵ); this is done with the aid of a derivation given by Kramer, which yielded that the pinning force on a flux line is:

$$F(h) = \frac{0.56}{(1 - a_0\sqrt{\epsilon})^2} \left(\frac{H_{c2}^{5/2}}{\kappa^2} \right) h^{1/2}(1 - h)^2 \quad (5-7)$$

for type II hard superconductors in which h is the reduced magnetic field strength, the ratio of the applied magnetic field to that of the upper critical field $h = H_a / H_{c2}$.²

The variables κ , a_0 , and ρ are the Ginzburg-Landau parameter, the flux line lattice spacing, and the pinning-site density. Luhman later modified this expression by noting that the flux line spacing is related to the quantum flux unit ϕ_0 by:³

$a_0 = (\phi_0 / H_d)^{1/2}$ in which

$$\phi_0 = \frac{hc}{2e} = 2.07 \times 10^{-7} \text{ gauss} \cdot \text{cm}^2 \quad (5-8)$$

Equation 7 now becomes

$$F(h) = \frac{0.56 H_a^{1/2} (H_{c2} - H_a)^2}{\left[1 - \left(\frac{\phi_0}{H_a} \right)^{1/2} \right]^2 H_a^2} \quad (5-9)$$

When the current in the superconductor reaches the critical current, the pinning force is equal to the Lorentz force:

$$F_L = J_c \times H_a \cdot c \quad (5-10)$$

Equations 9 and 10 yield the critical current density

$$J_c(h) = \frac{0.56 (H_{c2} - H_a)^2 c}{\left[1 - \left(\frac{\phi_0}{H_a} \right)^{1/2} \right]^2 H_a^{1/2} \kappa} \quad (5-11)$$

The major contributors to the pinning sites are the Nb_3Sn grain boundaries; since these are relatively insensitive to small strains ϵ , the pinning-site density, is a constant. Therefore, H_{c2} and κ are the only strain dependent parameters that determine the critical current density. Further insight concerning the physical roles of

H_{c2} and ξ can be obtained from the Ginzburg-Landau differential equation:

$$\alpha' |\psi|^2 + \frac{1}{2m^*} \left(\frac{\hbar}{i} \nabla - \frac{e^*}{c} \bar{A} \right)^2 \psi = 0 \quad (5-12)$$

in which the coefficients α' and β' are functions of temperature, $\bar{A}$ is the magnetic vector potential, $|\psi|^2$ is the density of superconducting electrons, and both m^* and e^* correspond to the mass and charge of the superconducting Cooper electron pairs.⁴ Near the critical current, the density of the superconducting electrons is small and equation 12 can be linearized to become

$$\left(\frac{\hbar}{i} \nabla - \frac{2e\bar{A}}{c} \right)^2 \psi = - \frac{2m^*\alpha'\psi}{\hbar^2} = \frac{1}{\xi^2} \psi \quad (5-13)$$

in which ξ is the coherence length. For a bulk sample in the presence of a uniform field H in the e_z direction, a possible gage choice is

$$\bar{A} = A_y \hat{e}_y = Hx \hat{e}_y \quad (5-14)$$

Inserting equation 14 into 13 yields

$$\left[-\nabla^2 + \frac{4\pi i}{\phi_0} Hx \frac{\partial}{\partial y} + \frac{2\pi H}{\phi_0} x^2 \right] \psi = \frac{1}{\xi^2} \psi \quad (5-15)$$

which has as eigenfunction solutions the Hermite polynomials as functions of x and is periodic in y and z .

$$\psi = e^{iK_y y} e^{iK_z z} f(x) \quad (5-16)$$

Substituting equation 16 into 15 produces

$$-f''(x) + \left(\frac{2\pi H}{\phi_0}\right)^2 \left(x - \frac{K_y \phi_0}{2\pi H}\right)^2 f = \left(\frac{1}{\xi^2} - K_z^2\right) f \quad (5-17)$$

The eigenvalues are those of a harmonic oscillator with cyclotron frequency

$$E_n = (n + 1/2) \hbar \omega_c = (n + 1/2) \hbar \frac{2eH}{m^*c} \quad (5-18)$$

which are equal to the energy cofactor of equation 17

$$(n + 1/2) \hbar \frac{2eH}{m^*c} = \frac{\hbar^2}{2m^*} \left(\frac{1}{\xi^2} - K_z^2\right) \quad (5-19)$$

Equation 19 defines the magnetic field able to nucleate inside the bulk superconductor. The highest field value, H_{c2} , occurs when $K_z = 0$ and at the ground state energy ($n = 0$).

$$H_{c2} = \frac{\hbar c}{2e\xi^2} = \frac{\phi_0}{2\pi\xi^2} \quad (5-20)$$

The corresponding solution at H_{c2} is

$$\psi(x, y) = \sum_n C_n \exp(iK_n y) \exp\left[-\frac{(x - x_n)^2}{2\xi^2}\right] \text{ with } (5-21)$$

$$x_n = \frac{K_n \tau_0}{2\pi H} \quad (5-22)$$

The solution given by equation 21, as noted by Abrikosov represents a two dimensional lattice structure of supercurrents as shown in Fig. 62.⁵ At the core of each current ring is the penetrated magnetic flux line quantum. For minimum energy density, the constants C_n have been determined to yield a lattice that is hexagonally closed-packed (HCP). The nearest neighbor distance between each supercurrent vortex a_0 is discussed by Tinkham and shown to be:⁴

$$a_0 = \left(\frac{4}{3}\right)^{1/4} \left(\frac{\xi_0}{H}\right)^{1/2} \geq \xi \quad (5-23)$$

Now the coherence length ξ is a material property: if the applied field strength sends so many flux lines into the superconductor that it is impossible to arrange them in a HCP structure, the supercurrents overlap, superconductivity is destroyed, and the material becomes normal. We can see from equation 23 that this occurs when H increases such as to make a_0 approach ξ .

Thus far the upper critical field has been shown to be a function of the coherence length in equation 20, and ξ in turn determines the spacial width of the

superconducting electron pair wave packet, as shown in equation 21. The goal of these efforts is to show that H_{C2} obtains its strain dependence through the coherence length, which is related to the elastic properties of the superconducting material. This relationship stems from the fact that ξ is a function of the Fermi velocity

$$\xi = \frac{\hbar v_F}{kT_c} \quad (5-24)$$

The superconducting electrons are bound into Cooper pairs via the electron-phonon-electron coupling. Essentially one electron, transversing through the crystal, leaves a distortion of the lattice in its wake, another electron is attracted to the surplus positive charge of the deformation and hence behaves as if it is coupled to the first electron (Fig. 63). Obviously, the lattice distortion cannot propagate faster than the material sonic velocity, hence if the electrons are accelerated to higher speeds, the pair bonding is destroyed. The maximum kinetic energy allowable to the Cooper electron pair is simply equal to the condensation energy gap at 0 K

$$\frac{1}{2} M v_s^2 = \Delta(0) = 1.764 kT_c \quad (5-25)$$

Only electrons close to the Fermi surface participate in the supercurrent, so equations 24 and 25 can be combined to yield

$$\xi = \frac{\hbar}{mV_s} \quad (5-26)$$

which relates the coherence length to the maximum electron velocity, i.e., the material sonic velocity. The longitudinal and transverse velocities are usually equal to particular summations of the elastic constants, with the sum dependent on the crystallographic direction of the traveling wave. An isotropic average velocity can be expressed as

$$\langle V_s \rangle = \left(\frac{\sum_{ij} a_{ij} C_{ij}}{\rho} \right)^{1/2} \quad (5-27)$$

where the values of C_{ij} are material elastic moduli and ρ is the density. Notice that a stiffer material with high moduli is more favorable to producing a smaller coherence length. Hence the upper critical field H_{c2} is also related to the elastic constants via equations 20, 26, and 27.

$$H_{c2} = \frac{\pi \phi_0 M^2 V_s^2}{2 \hbar^2} = \frac{\pi \phi_0 M^2}{2 \hbar^2} \left(\sum_{ij} \frac{a_{ij} C_{ij}}{\rho} \right) \quad (5-28)$$

The upper critical field is, therefore, a linear function of the elastic constants. Combining equations 24, 26, and 27 yields an expression for the critical temperature T_c .

$$T_c = \frac{MV^2}{2K} = \frac{M}{2K} \left(\sum_{ij} a_{ij} c_{ij} \right) \quad (5-29)$$

which is also linear with the material stiffness.

The phase transition in Nb_3Sn involves a softening of the lattice at low temperatures, which leads to structural instability; this in turn causes the cubic-to-tetragonal transformation. The results of the last chapter indicate that the transformation is partial with only a certain volume fraction η_t of the lattice cells undergoing the transition. Furthermore, the fraction changed can be incremented by the application of external strains. If the elastic moduli measurements obtained by Vieland are indicative of those arising from samples of varying lattice parameters and hence internal strains, then the elastic constants may be volume averages of the two crystal structures present (see Fig. 58). The values of C_{ij} of expressions 28 and 29 are related to strain by equations 5 and 6.

$$\langle C_{ij} \rangle = C_{ijt} \eta_t(\epsilon) + C_{ijc} \eta_c(\epsilon) \quad (5-30)$$

in which the average moduli are determined by a "rule of

mixtures" calculation of the volume fraction of cubic and tetragonal cells and their corresponding moduli.

Guided by these considerations H_{c2} for the transforming Nb_3Sn may also be written as a strain dependent volume average

$$H_{c2}(\epsilon) = H_{c2c} V_c(\epsilon) + H_{c2t} V_t(\epsilon) \quad (5-31)$$

There is additional experimental evidence that supports the model of two coexisting phases in Nb_3Sn each having a contribution to its superconducting properties. Matthias and Dayan have observed double transition temperatures in some samples of Nb_3Sn , and Forer has shown that the tetragonal structure has a lower upper critical field H_{c2} .⁶⁻⁸ This proposed theory predicts lower critical parameters for the tetragonal structure due to its softer lattice.

A return to the critical current density scaling law of equation 11 shows that the Ginzburg-Landau order parameter κ must also be strain dependent because of the relationship

$$\kappa = \lambda / \xi(\kappa) = \frac{\pi \lambda m V_S}{h} = \frac{\pi \lambda m}{h} \sqrt{\frac{\sum_{ij} a_{ij} c_{ij}}{f}}, \quad (5-32)$$

in which λ is the penetration depth. The critical current versus strain scaling law used in the MAXIMSUPER

program combines equation 11 with the strain sensitive functions of κ and H_{c2}

$$J_c(\epsilon) = \frac{0.56 \left(H_{c2}(\epsilon) - H_a \right)^2}{\left[1 - \left(\frac{\phi_0}{H_a} \right)^{1/2} \right]^2 H_a} \kappa(\epsilon) \quad \text{with} \quad (5-33)$$

$$H_{c2}(\epsilon) = H_{c2t} \sqrt{\alpha\epsilon + \frac{1}{2} \beta\epsilon^2 + \alpha\Delta T} \\ + H_{c2c} \left(1 - \sqrt{\alpha\epsilon + \frac{1}{2} \beta\epsilon^2 + \alpha\Delta T} \right) \quad (5-34)$$

$$\kappa(\epsilon) = \kappa_t \sqrt{\alpha\epsilon + \frac{1}{2} \beta\epsilon^2 + \alpha\Delta T} \\ + \kappa_c \left(1 - \sqrt{\alpha\epsilon + \frac{1}{2} \beta\epsilon^2 + \alpha\Delta T} \right) \quad (5-35)$$

The normalized critical current of a conductor in a tensile test is then equation 33, divided by $J_c(\epsilon_m)$, the current density occurring with the smallest strain on the Nb_3Sn during the test;

$$\frac{I_c(\epsilon)}{I_c(\epsilon_m)} = \frac{\kappa^2(\epsilon_m)}{\kappa^2(\epsilon)} \left(\frac{H_{c2}(\epsilon) - H_a}{H_{c2}(\epsilon_m) - H_a} \right)^2 \quad (5-36)$$

with the unknown constants in equations 34, 35, and 36 determined by fitting the formulas to the universal

degradation curves of Fig. 5 by means of a least squares data fitting program (See Appendix 2).⁹

D. Conclusions and Suggestions for Future Study

The primary objective of this study was to develop an improved method of calculating the stresses and strains in the Nb_3Sn zones in a multifilamentary composite during the course of a tensile test. It is possible to simulate completely the test results with a mathematical model, if the analysis also includes a mechanism that couples the strain state of the Nb_3Sn with the degradation of its critical parameters, the most important being the critical current density J_c . The relationship of J_c to strain required additional experimentation into the nature of the low temperature martensitic phase transformation; this was accomplished by x-ray diffraction techniques. The major conclusions and summary of findings revealed by this work are tabulated below:

1. The author has completed the development of a computer program MAXIMSUPER that calculates the three-dimensional strain field present in the Nb_3Sn regions in a multifilamentary superconducting composite arising from both the residual thermal contraction of the composite materials from reaction temperature down to

4.2 K and those additional strains resulting from the axial loading of the conductor during an uniaxial tensile test.

2. A preliminary application of the program revealed a major difference in the triaxial strain states of two conductor designs: concentric material filaments, in which the bronze acts as the core, and niobium-core filaments. The code predicted higher tangential strains (radial and azimuthal) for the bronze-core geometry.
3. Experimental measurements of the critical parameters J_c and T_c for the two conductor designs indicated a more pronounced suppression of J_c and T_c for the bronze-core multifilamentary conductors, thereby verifying the analytic predictions of the computer program.
4. In order to relate the influence of strain on the critical parameters, additional experiments were performed to ascertain the dependence of the martensitic phase transformation on strain. The results were consistent with a partial transformation in which Nb_3Sn was composed of both the cubic and tetragonal phases at low temperatures. The application of strains to the compound enhanced the volume fraction of tetragonal lattice cells.

5. A theoretical model was developed to show that critical parameters J_c , H_{c2} , and T_c are related to the superconducting material sonic velocities and hence are dependent on the elastic moduli of the cubic and tetragonal phases present in Nb_3Sn .
6. It was postulated that the tetragonal phase has lower values of its critical parameters than the cubic because of its softer moduli. Therefore, the strain sensitivity of J_c , H_{c2} , and T_c is due to the strain-assisted production of tetragonal lattice cells at low temperatures.
7. Expressions were derived that relate the critical parameters to the strain incremented volume fraction of tetragonal phase. These functions comprise the J_c -strain scaling laws utilized in the computer program.

Improved focusing techniques, longer wavelength x-ray radiation, and diffraction pattern scans of higher ($h00$) planes should yield enhanced resolution and more dramatic splitting of the diffraction lines during the transformation. The apparent strain enhancement of the transition in Nb_3Sn should also be investigated in the other Al5 superconductors. This includes the germanium compounds, such as V_3Ge and Nb_3Ge , which, according to the results

of Testardi and others, experience a hardening of the elastic constants at low temperature.¹⁰⁻¹⁴ The theory presented in this report predicts that the critical parameters of these compounds should increase with strain.

Calculations of the strain state in the Nb_3Sn layer, using codes such as MAXIMSUPER, require more complete data on the mechanical properties of the composite components for additional accuracy. This includes temperature dependent properties, such as stress-strain curves, Poisson ratios, elastic moduli, and thermal expansion coefficients at both high and low temperatures. Thus far, good agreement between the MAXIMSUPER predictions and experimental results indicate that the code may be suitable for superconducting composite design applications.

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Chapter 5

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Fig. 1. Construction of the MFTF magnet. It is currently the world's largest superconducting magnet; capable of storing 409 MJ at 7.68 T maximum field.

Fig. 2. The MFTF superconducting Nb-Ti conductor. The composite consists of Nb-Ti filaments in a copper matrix surrounded by an external copper stabilizer.

Fig. 3. The unit cell of the A-15 crystal structure for A_3B compounds. Currently, the highest known T_c superconducting compounds are in this group. The smaller A atoms form orthogonal chains.

Fig. 4. Critical current versus strain curves. A comparison of the results of critical current-strain measurements on monolithic conductors by different investigators shows that widely varying data has been reported (D. Deis, Aug. 1977).

Fig. 5. Rupp normalized critical current-strain curves. The critical current obtained at fixed magnetic field for various strains can be normalized to the peak current and intrinsic strain. These results are from Fig. 4 (D. Deis, Aug. 1977).

Fig. 6. The transition temperature as a function of strain. The effect of strain on the T_c of various monofilament conductors with differing bronze-to-niobium ratios, Bz:Nb = 1.1 to 44 (T. Luhman, Oct. 1978).

Fig. 7. The upper critical field as a function of strain. The effect of strain on the upper critical field of various conductors plotted using Rupp normalization (J. Ekin, 1980).

Fig. 8. Cross section of a multifilamentary conductor. A cluster of Nb/Nb₃Sn multifilaments in a bronze matrix surrounded by tantalum diffusion barriers to prevent tin from contaminating the OFHC copper stabilizer. The five micron diameter filaments consist of niobium cores with an Nb₃Sn outer ring.

Fig. 9. Cross section of another multifilamentary conductor design. These clusters show niobium tubes in a copper matrix with bronze cores (internal bronze geometry).

Fig. 10. Modeling of a cylindrical composite element. This figure shows a representation of how a hexagonal fiber composite filament and its local environment is mathematically modeled as a cylindrical element (L. Ebert, June 1969).

Fig. 11. Internal niobium-core filaments shown distorted from the manufacturing process. The dark cavities are voids formed during the diffusion reaction in which the tin is depleted from the bronze matrix.

Etched 500X

Etched 500X

Fig. 12. 1500X photomicrograph of an internal bronze-core conductor design. Several filaments are shown here. The inner dark rings are Nb₃Sn layers (approximately 1 micron thick). The outer rings are unreacted niobium surrounded by high purity copper.

Fig. 13 Several clusters of internal niobium-core filaments surrounded by tantalum barriers.

50X

100X

Fig. 14. Modeling of the internal bronze- and niobium-core conductor designs. The schematic shows the internal niobium- and bronze-core filament geometries and the tangential strain components present on the Nb_3Sn zones. These conductor designs are mathematically modeled into the MAXIMSUPER computer program.

Fig. 15. Measured versus predicted ϵ_{icmax} plot. A plot of the measured strain values that yielded peak critical currents (ϵ_{icmax}) versus the MAXIMSUPER predictions for a variety of niobium- and bronze-core composites. The triangles are monofilaments with the bronze-to-niobium ratios written next to each point. The circles and squares are internal niobium- and bronze-core multifilamentary conductors (R. W. Hill, 1979).

Fig. 16. Critical current density as a function of Nb_3Sn reaction time at several magnetic field strengths for various internal bronze-core conductors (R. M. Scanlan, 1979).

Fig. 17. Plot of effective strain versus intrinsic z strain. The plot shows the effective strain on the Nb₃Sn layers versus the intrinsic z strain from the MAXIMSUPER code. The S-6a and A-14 are typical bronze- and niobium-core conductor designs (R. W. Hoard, 1979).

Fig. 18. Plot of measured critical current density of several conductors at $H = 12$ T as a function of the code predicted residual effective strain. The sample designations are the same as those in Fig. 17.

Fig. 19. The low temperature tensile tester showing the sample area raised above the cryostat.

Fig. 20. Inside the cryostat: A view of the upper tensile grip, conductor, and copper braid current lead.

Fig. 21. Sample conductor fitted with test instrumentation. Attached to the short sample conductor are strain gages and voltage taps. The side pieces are epoxy G-10 structural poles to prevent the sample from bending under the action of Lorentz forces.

Fig. 22. Tensile tester schematic. This schematic shows the active components of the tensile tester sample area. The figure shows the split pole 12 T magnet, sample, current leads, voltage taps, and the location of the tensile grips.

Cryogenic apparatus for 4.2k mechanical
testing of Nb_3Sn conductors

Fig. 23. Inside the cryostat: Showing the complete assembly for the strain-critical current experiments.

Fig. 24. Typical voltage versus current data plots obtained during the tensile test experiments.

Fig. 25. Typical extrapolation plots used to determine the critical current at a resistivity of $10^{-11} \Omega - \text{cm}$.

Fig. 26. Cross section of an internal bronze-core conductor.

132

100X

200X

Fig. 27. Cross section of an internal niobium-core conductor.

Fig. 28. Internal bronze-core conductor I_C versus strain curve No. 1. In this figure, the normalized critical current is plotted as a function of the applied axial strain for a conductor at 8, 10, and 12 T. The dashed line represents the reduction of I_C after the strain is lowered from 1.1 percent at 12 T.

Fig. 29. Internal bronze-core conductor I_C versus strain curve No. 2. In this figure, the I_C -strain characteristics are plotted for a second conductor at $H = 10$ and 12 T. Once again, the dashed lines show that the conductors current-carrying characteristics are irreversibly degraded at strains higher than 0.2 percent of the strain that yields the maximum critical current, i_{cmax} .

Fig. 30. Rupp normalized critical current data for bronze core conductors plotted against the intrinsic strain. The square, circular, and triangular symbols represent data points taken at $H = 8, 10$, and 12 T, respectively. The numbers are conductor identification names from Table 1. The solid envelope curves are the 8 and 12 T lines from Fig. 5.

Fig. 31. Stress-strain curve from internal bronze-core conductor No. 1. The vertical ticks occurred simultaneously with the emission of "acoustical pings," thought to be indicative of Nb₃Sn filament breakage.

Fig. 32. Stress-strain curve for an internal bronze-core conductor No. 2.

Fig. 33. Martensitic-phase-transformation diagram. This figure shows the relationship between the lattice parameter (or stoichiometry) and the superconducting and martensitic transformation temperatures T_s , A_s , A_t , and A_f . The data was obtained from observing the reverse (tetragonal-to-cubic) transformation in samples heated from 6.5 K, where A_s and A_f refer to the start and finish of the reverse transformation and M_s is designated as the normal cubic-to-tetragonal start temperature (H. W. King, 1969).

1. Helium dewar
2. Helium cylinder
3. Pressure regulator
4. Mercury manometer, 0 to 20" Hg or pressure gauge on helium cylinder
5. Vacuum shroud
6. Vacuum shroud pump
7. Vacuum pump for operation below 4.2° K
8. Accessory flow control panel
9. Accessory temperature controller, manual or automatic

Fig. 34. Typical heli-tran cryostat set-up.

Fig. 35. Schematic of flexible transfer line to storage Dewar connections.

Fig. 36. Schematic of heli-tran cryostat cold end and connections.

Fig. 37. Inside the heli-tran LI-3-110A cryostat, the vacuum shroud cover has been removed to show the cut-out radiation shield and sample.

Fig. 38. The assembled cryostat mounted in the diffractometer. The 1/2 inch by 180 degree beryllium window can be seen on the vacuum shroud cover. The mechanical vacuum insert for straining the sample is also shown below the beryllium window.

Fig. 39. Tensile test sample holder for x-ray diffraction. This figure shows the sample holder attached to the tip of the cryostat cold end. The strain gage and Nb_3Sn ribbon are bonded to the top of an indium wafer. The wafer is secured (by the four screws shown) to two movable copper wedge poles. These poles spread apart when a set screw (in the outer side of the holder) is advanced by the mechanical vacuum insert shown in Fig. 38. Two CLTS thermometers are also shown; one is under the indium stage, the other is mounted on the end of the sample holder.

Fig. 40. Enlarged drawing of CLTS temperature sensor.

Fig. 41. Calibration curve for the CLTS temperature sensor. Two CLTS sensors were immersed in baths of liquid helium, nitrogen, and ice to produce this curve.

Fig. 42. Another view of the cryostat and diffractometer.

Fig. 43. Overall setup for the x-ray diffraction experiment. This figure shows the low temperature x-ray diffraction cryostat, diffractometer, automated data acquisition system, temperature control, and strain gage instrumentation.

Fig. 44. Room temperature diffraction profile of the (611, 532) lines of Nb_3Sn . The two peaks are due to the $K_{\alpha 1}$ and $K_{\alpha 2}$ wavelengths.

Fig. 45. Diffraction profile of the (611, 532) lines of Nb_3Sn at 8 K. Notice that the $K_{\alpha 1}$ and $K_{\alpha 2}$ peak separation is no longer resolvable.

Fig. 46. Diffraction profile of the (611, 532) lines at 174 K. Figs. 46 through 55 show diffractometer traces of the (611, 532) line for Nb₃Sn at various temperatures.

Fig. 47. Diffraction profile of the (611, 532) lines at 152 K.

Fig. 48. Diffraction profile of the (611, 532) lines at 138 K.

Fig. 49. Diffraction profile of the (611, 532) lines at 124 K.

Fig. 50. Diffraction profile of the (611, 532) lines at 98 K.

Fig. 51. Diffraction profile of the (611, 532) lines at 80 K.

Fig. 52. Diffraction profile of the (611, 532) lines at 53 K.

Fig. 53. Diffraction profile of the (111, 532) lines at 43 K.

Fig. 54. Diffraction profile of the (611, 532) lines at 33 K.

Fig. 55. Diffraction profile of the (611, 532) lines at 19 K. The sample studied in the series of Figs. 46 through 55 had a precompressive strain of -0.65 percent at 19 K. This strain was due to the thermal contraction of the indium sample stage between 19 K and room temperature.

Fig. 56. Plot of the peak height (above background) as a function of temperature. The initial compressive strain at 8 K is -0.65 percent.

Fig. 5. Plot of the peak height (above background) as a function of temperature and strain. The various strain values refer to the initial strains at 8 K. The initial peak height, at 8 K, is a function of the alignment of each sample used in the experiment. Fig. 59 shows that only the change in peak height between low to high temperatures is important.

Fig. 58. The temperature dependence of Young's modulus for three different V_3Si samples. Compare these curves to those of Fig. 57.

Fig. 59. Plot of the normalized peak height as a function of temperature and strain. These curves were made from Fig. 57 by assuming that at high temperatures the cubic volume fraction is equal to one and normalizing the four curves to their respective maximum heights. Once again the strain values are those measured at 8 K.

Fig. 60. Peak height as a function of strain at 8 K. One Nb_3Sn sample was held at 8 K, and the peak height of its (611, 532) diffraction profile was measured as a function of strain.

Fig. 61. Normalized peak height as a function of strain at 8 K. The plot of Fig. 60 was normalized by the extrapolated peak height at zero strain. This figure indicates that the Nb_3Sn should be completely transformed to the tetragonal phase at a strain of approximately -1.8 percent.

Fig. 62. Triangular lattice structure of supercurrents near H_{c2} . The numbered contours indicate the relative value of the superconducting electron density (A. M. Campbell, et. al., March 1972).

Fig. 63. The electron-phonon-electron interaction. The electrostatic attraction between the first electron and lattice ions leave an inward distortion of the ions in the wake of the electron. The distortion simulates a region of higher positive-charge density, which attracts a second electron.

Table 1

Internal Bronze-Core Conductor Fabrication Parameters and Critical Current Properties

	Conductor	Conductor	Conductor	Conductor
<u>Item</u>	<u>S-1</u>	<u>S-4</u>	<u>S-6a</u>	<u>S-6b</u>
Size (mm)	5.13 x 1.68	5.13 x 1.68	7.61 x 7.62	7.62 x 7.62
Number Filaments	703	2,257	30,625	30,625
Filament Diameter	55 μ m	26 μ m	35.2 μ m	35.2 μ m
Bronze Composition	13% Sn 87% Cu	---	---	---
Composition After Reaction %				
Cu	63%	71%	48.7%	48.7%
Nb + Bz	32.6%	22.7%	42%	38.6%
Nb ₃ Sn	4.4%	6.3%	9.3%	12.7%
Heat Treatment	750 ^o - 36 hrs.	---	---	---
I_c ($\rho_r = 10^{-11} \Omega \cdot \text{cm}$)				
At H = 8 T	807 A	1,070 A	9,100 A	---
H = 10 T	406 A	640 A	4,150 A	7,625 A
H = 12 T	139 A	273 A	1,950 A	2,600 A

TABLE 2.

Critical Current Properties for Several Internal Bronze Core Conductors
at Various Heat Treatments and Filament Diameters

Filament Diameter mm	Heat Treatment	I_c (12T)	Nb+Bz Area (cm ²)	Nb ₃ Sn Area (cm ²)	% Nb ₃ Sn/100	J_c (Nb ₃ Sn)
13	750C - 36 hr.	110	0.020	4.0×10^{-3}	0.20	2.75×10^4
26	750C - 36 hr.	675	0.074	2.2×10^{-2}	0.30	3.10×10^4
26	750C - 36 hr.	215	0.023	3.4×10^{-3}	0.15	6.30×10^4
42	750C - 36 hr.	165	0.023	3.7×10^{-3}	0.16	4.50×10^4
55	750C - 36 hr.	85	0.030	3.7×10^{-3}	0.12	2.20×10^4
13	650C - 120 hr.	200	0.020	5.2×10^{-3}	0.26	3.80×10^4
26	650C - 120 hr.	400	0.076	1.4×10^{-2}	0.18	2.80×10^4
26	650C - 150 hr.	80	0.023	3.6×10^{-3}	0.16	2.20×10^4
42	650C - 150 hr.	65	0.023	2.6×10^{-3}	0.11	2.50×10^4
55	650C - 150 hr.	55	0.030	2.3×10^{-3}	0.08	2.40×10^4
26	700C - 100 hr.	130	0.023	3.6×10^{-3}	0.16	3.60×10^4
42	700C - 100 hr.	100	0.023	3.6×10^{-3}	0.16	2.80×10^4
55	700C - 100 hr.	70	0.030	---	---	---

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APPENDIX 1
COMPUTER PROGRAM LISTING

And what of my life
Spent studying fields,
The interconnecting fabrics
That weave reality's hills,
A parable that's true:
All with sight cannot see.
So, I looked (again) upon my own,
To view them interwoven with you.

CHAT 167D 10:50:35 A08/27/80
 PROGRAM MAXIMSUPER (OUTPUT,TAPE6=OUTPUT)
 CODE ANALYSIS
 CALL CHANGE(SH+TEST)
 OPTIMIZE

C ** PROGRAM MAXIMUM SUPERCONDUCTOR CALCULATES THE CRITICAL **
 C ** CURRENT FOR VARIOUS MULTIFILAMENT COMPOSITE STRAIN DEPENDENT**
 C ** SUPERCONDUCTORS ARRANGED IN CYLINDRICAL GEOMETRY. **
 C ** THIS IS DONE BY FIRST CALCULATING THE RESIDUAL STRAIN DUE TO**
 C ** THERMAL CONTRACTION, THEN THE STRAINS ARE FOUND WHEN THE **
 C ** COMPOSITE IS PUT UNDER AXIAL LOADING. FINALLY THE CRITICAL **
 C ** CURRENT OF THE COMPOSITE IS CALCULATED AS A FUNCTION OF THE **
 C ** EFFECTIVE STRAIN ON THE NB3SN ZONES. **

C ** CREATION DATE : JULY 3, 1979 BY RONALD WARREN HOARD **
 C ** MAGNETIC FUSION ENGINEERING DEPARTMENT **
 C ** LAWRENCE LIVERMORE NATIONAL LABORATORY, LVM., CALIFORNIA **

COMMON /KON/A(23,23),B(23,1),C(23,1),AP(23,23),BP(23,1)
 COMMON /WAR/R(11),N1(12),DEFE(11),DEFS(11),EFE(11)
 COMMON /HDA/E(12),ALPHA(12),EPS(3,11),DSIG(3,11,100),
 1DEPS(3,11,100)
 REAL NU(12),K(12),ML(12),PL(12)
 COMMON /ENG/NU,K,ML,PL,NRING,NZONE,L
 COMMON /ILL/AL(22,22),BL(22,1),CL(22,1),ALP(22,22),BLP(22,1)
 REAL JCNORM(100)
 COMMON /HFE/HAS(5),IMIN
 INTEGER V
 DIMENSION DEFEN(100,11),LABEL(12),OUTVAL(20)
 DIMENSION WKAREA(25),TEFE(11),SIG(3,11),TSIGE(11)
 DIMENSION OUTEPZ(100),OUTEPN(100).

C ** N1 = 1,2,3,... IF N1=1 ZONE 1 IS NIOBIUM **
 C ** IF N1=2 ZONE 1 IS BRONZE **
 C ** IF N1=3 ZONE 1 IS NB3SN **
 C ** IF N1=4 ZONE 1 IS VACUUM **
 C ** IF N1=5 ZONE 1 IS COPPER **

C ** THE NUMBER OF MATERIAL ZONES SPECIFIED MUST BE EQUAL TO **
 C ** NZONE + 1 WITH THE LAST OUTER ZONE = VACUUM IN ORDER TO **
 C ** SATISFY THE VANISHING RADIAL STRESS BOUNDARY CONDITION. **
 C ** A RADIUS MUST BE ASSIGNED TO EACH MATERIAL ZONE EXCEPT: **
 C ** N1 = VACUUM = 4. **

DATA N1/2,2,3,3,1,1,5,5,4/,LN83SN/4/
 DATA R/10,RE-6,21,8E-6,21,2E-6,21,6E-6,22,3E-6,
 125,8E-6,27,5E-6,28,5E-6,25,5E-6/
 DATA LABEL1/12HNIOBIUM /,LABEL2/12HBRONZE /
 DATA LABEL3/12HNB3SN /,LABEL4/12HVACUUM /
 DATA LABEL5/12HCOPPER /
 DATA OUTVAL/-1,E-2,-9,E-3,-8,E-3,-7,E-3,-6,E-3,-5,E-3,-4,E-3,
 1-2,E-3,-1,E-3,0,0,1,E-3,2,E-3,3,E-3,4,E-3,5,E-3,6,E-3,7,E-3,
 28,E-3,9,E-3,1,8E-2/,NOUTPT/20/,DEPZ/0.0002/
 DATA HA/4,0E4,8,0E4,10,0E4,12,0E4,15,0E4,20,0E4/
 DATA NRING/8/,TO/1024,2/,DT/-0.5/,GAMMA/0.001/,NM/10/
 NZ=NRING+1

```

C
C      ** ECHO INPUT DATA TO THE OUTPUT PRINTER      **
C
      NZONE=NRING+1
      NMATL=NZONE+1
      NUNPTS=1
      DO 500 I=1,NMATL
        IF(NI(I).EQ.1) LABEL(I)=LABL1
        IF(NI(I).EQ.2) LABEL(I)=LABL2
        IF(NI(I).EQ.3) LABEL(I)=LABL3
        IF(NI(I).EQ.4) LABEL(I)=LABL4
        IF(NI(I).EQ.5) LABEL(I)=LABL5
500  CONTINUE
      WRITE (6,501)
501  FORMAT (///2X,"THE ZONES ARE MADE OF THE FOLLOWING MATERIALS:"/
12X,"LISTED IN THE ORDER OF EACH CORRESPONDING ZONE NUMBER"/)
      WRITE (6,502) (LABEL(I),I=1,NM)
502  FORMAT (2X,6A12)
      WRITE (6,503)
503  FORMAT (///2X,"THE CORRESPONDING ZONE RADII ARE:"/)
      WRITE (6,504) (R(I),I=1,NZ)
504  FORMAT (11(E10.2))

      D
C      +++++ THERMAL LOADING REGION      +++++
C
      NEQN=2*NRING+3
      L=1
      T=70
      CALL CONST(T)
      CALL ELAST1(DT)
      CALL ELAST2(DT,NEQN,IER)
      IF(IER.EQ.129) GO TO 700

C      ** PRINT THE INITIAL OUTPUT VALUES OF A,B,C,DEPS, AND DEFE      **
C
      WRITE (6,1)
1  FORMAT (///5X,"THE INITIAL A MATRIX IS:"/)
      WRITE (6,2) ((A(I,J),J=1,NEQN),I=1,NEQN)
2  FORMAT (11(E10.2)/)
      WRITE (6,3)
3  FORMAT (///5X,"THE INITIAL B VECTOR IS:"/)
      WRITE (6,2) (B(I,1),I=1,NEQN)
      WRITE (6,4)
4  FORMAT (///5X,"THE INITIAL C VECTOR IS:"/)
      WRITE (6,2) (C(I,1),I=1,NEQN)
      WRITE (6,5)
5  FORMAT (///5X,"THE INITIAL DIFFERENTIAL STRAINS ARE:"/)
      WRITE (6,6) NZ,((DEPS(I,J,1),J=1,NZ),I=1,3)
6  FORMAT (*1E10.2)/)
      WRITE (6,7)
7  FORMAT (///5X,"THE EFFECTIVE STRAINS FOR EACH REGION ARE:"/)
      WRITE (6,6) NZ,(DEFE(I),I=1,NZ)

C      ** DECREMENT THE INITIAL TEMPERATURE AND COMPUTE NEW STRAINS      **
C
100  T=T+DT
      L=L+1

```

```

CALL CONST(T)
CALL ELAST1(DT)
CALL ELAST2(DT,NEON,IER)
IF(IER.EQ.129) GO TO 700
DO 20 I=1,NZONE
  DEFEN(L,I)=DEFE(I)
20 CONTINUE
250 L=L+1
  CALL CONST(T)
  CALL PLASTC(ML,PL)
  CALL ELAST1(DT)
  CALL ELAST2(DT,NEON,IER)
  IF(IER.EQ.129) GO TO 700
  DO 30 I=1,NZONE
    DEFEN(L,I)=DEFE(I)
30 CONTINUE
  DO 40 I=1,NZONE
    DNP1=DEFEN(L,I)
    DN=DEFEN(L-1,I)
    DELTA1=ABS(DNP1-DN)
    DELTA2=ABS(GAMMA*DN)
    IF(DELTA1.GT.DELTA2) GO TO 300
40 CONTINUE
    DO 150 I=1,NZONE
      EPS(1,I)=EPS(1,I)+DEPS(1,I,L-1)
      EPS(2,I)=EPS(2,I)+DEPS(2,I,L-1)
      EPS(3,I)=EPS(3,I)+DEPS(3,I,L-1)
      SIG(1,I)=SIG(1,I)+DSIG(1,I,L-1)
      SIG(2,I)=SIG(2,I)+DSIG(2,I,L-1)
      SIG(3,I)=SIG(3,I)+DSIG(3,I,L-1)
      TSIGE(I)=0.70711*SORT((SIG(1,I)-SIG(2,I))**2+
1(SIG(2,I)-SIG(3,I))**2+(SIG(3,I)-SIG(1,I))**2)
      TEFF(I)=(0.70711/(1+NU(I))*SORT((EPS(1,I)-EPS(2,I))**2+
1(EPS(2,I)-EPS(3,I))**2+(EPS(3,I)-EPS(1,I))**2)
150 CONTINUE
      GO TO 50
300 DO 200 I=1,NZONE
      EFE(I)=EFE(I)+DEFEN(L,I)
200 CONTINUE
      GO TO 250
50 IF(T.GT.4.2) GO TO 100
  OUTEP2(I)=EPS(3,I)
  OUTEPN(I)=TEFE(LNBSGN)

C      ** PRINT THE RESIDUAL STRAIN NEAR 4.2 DEG. KELVIN      **
C
C      WRITE (6,8) T
8  FORMAT (///5X,"THE RESIDUAL STRAINS AT T="E12.3,2X,"ARE:///<)
  WRITE (6,5) NZ,((EPS(I,J),J=1,NZ),I=1,3)
  WRITE (6,9) T
9  FORMAT (///5X,"THE EFFECTIVE STRAINS AT T="E12.3,2X,"ARE:///<)
  WRITE (6,6) NZ,((TEFE(I),I=1,NZ)
  WRITE (6,10) T
10 FORMAT (///5X,"THE RESIDUAL STRESSES AT T="E12.3,2X,"ARE:///<)
  WRITE (6,6) NZ,((SIG(I,J),J=1,NZ),I=1,3)
  WRITE (6,19) T
19 FORMAT (///5X,"THE EFFECTIVE STRESSES AT T="E12.3,2X,"ARE:///<)

```

```

      WRITE (6,6) NZ, (TSIG(1), I=1, NZ)
      WRITE (6,505) T
505  FORMAT (///SX, "AXIAL LOADING RESULTS AT T="E12.3, 2X, "DEG. K"///)
C
C      ***** AXIAL LOADING REGION *****
C
C      ** APPLY AXIAL LOADING AT 4.2 DEG. K AND COMPUTE NEW STRAINS **
C
      NEQN=2*NRING+2
400  DO 11 I=1, NZONE
      EPS(3, I)=EPS(3, I)+DEPZ
      TEFE(1)=(0.70711/(1+NU(1)))*SQRT((EPS(1, I)-EPS(2, I))**2+
      1(EPS(2, I)-EPS(3, I))**2+(EPS(3, I)-EPS(1, I))**2)
      EFE(1)=TEFE(1)
11  CONTINUE
      L=1
      CALL CONST(T)
      CALL ELAST3(DEPZ)
      CALL ELAST4(DEPZ, NEQN, IER)
      IF (IER.EQ.129) GO TO 700
      DO 12 I=1, NZONE
      DEFEN(L, I)=EFE(1)
12  CONTINUE
350  L=L+1
      CALL CONST(T)
      CALL PLASTC(M, PL)
      CALL ELAST3(DEPZ)
      CALL ELAST4(DEPZ, NEQN, IER)
      IF (IER.EQ.129) GO TO 700
      DO 14 I=1, NZONE
      DEFEN(L, I)=EFE(1)
14  CONTINUE
      DO 15 I=1, NZONE
      DNP1=DEFEN(L, I)
      DN=DEFEN(L-1, I)
      DELTA1=ABS(DNP1-DN)
      DELTA2=ABS(GAMMA*DN)
      IF (DELTA1.GT.DELTA2) GO TO 600
15  CONTINUE
      DO 16 I=1, NZONE
      EPS(1, I)=EPS(1, I)+DEPS(1, I, L-1)
      EPS(2, I)=EPS(2, I)+DEPS(2, I, L-1)
      SIG(1, I)=SIG(1, I)+DSIG(1, I, L-1)
      SIG(2, I)=SIG(2, I)+DSIG(2, I, L-1)
      SIG(3, I)=SIG(3, I)+DSIG(3, I, L-1)
      TSIG(1)=(0.70711)*SQRT((SIG(1, I)-SIG(2, I))**2+
      1(SIG(2, I)-SIG(3, I))**2+(SIG(3, I)-SIG(1, I))**2)
      TEFE(1)=(0.70711/(1+NU(1)))*SQRT((EPS(1, I)-EPS(2, I))**2+
      1(EPS(2, I)-EPS(3, I))**2+(EPS(3, I)-EPS(1, I))**2)
16  CONTINUE
      GO TO 60
600  DO 17 I=1, NZONE
      EFE(1)=TEFE(1)+DEFEN(L, I)
17  CONTINUE
      GO TO 350

```

C

```

C      ** PRINT THE OUTPUT STRESSES AND STRAINS FOR CERTAIN Z STRAINS **
C
50 DO 18 I=1,NCOUTP
   TEST=ABS(EPS(3,1))-OUTVAL(1)
   IF(TEST.LE.2.E-4) GO TO 550
18 CONTINUE
   IF(EPS(3,1).LT.1.E-2) GO TO 400
550 NUMPTS=NUMPTS+1
   OUTEPZ(NUMPTS)=EPS(3,1)
   OUTEPN(NUMPTS)=TEFE(LN33SN,
   WRITE (6,506) EPS(3,1)
506 FORMAT (///5X,"THE STRAINS FOR AN AXIAL STRAIN OF"E12.3,2X,"*://)
   WRITE (6,6) NZ,((EPS(I,J),J=1,NZ),I=1,3)
   WRITE (6,508) EPS(3,1)
508 FORMAT (///5X,"THE EFFECTIVE STRAINS AT EPSZ="E12.3,2X,"*://)
   WRITE (6,6) NZ,(TEFE(I),I=1,NZ)
   WRITE (6,507) EPS(3,1)
507 FORMAT (///5X,"THE STRESSES FOR AN AXIAL STRAIN OF"E12.3,2X,"*://)
   WRITE (6,6) NZ,((SIG(I,J),J=1,NZ),I=1,3)
   WRITE (6,21) EPS(3,1)
21 FORMAT (///5X,"THE EFFECTIVE STRESSES AT EPSZ="E12.3,2X,"*://)
   WRITE (6,6) NZ,(TSIG(I),I=1,NZ)
   IF(EPS(3,1).LT.1.E-2) GO TO 400
C
C      ***** CRITICAL CURRENT CALCULATIONAL REGION *****
C
   PD 25 IHA=1.6
   CALL ICHNORM(OUTEPZ,OUTEPN,NUMPTS,I,JCNORM,IHA,CRICUR)
   WRITE (6,560)
560 FORMAT (1H1,5X," OUTPUT VALUES OF STRAINS AND NORMALIZED CRITICAL
1 CURRENTS"///)
   WRITE (6,501)
   WRITE (6,502) (LABFL(I),I=1,NM)
   WRITE (6,503)
   WRITE (6,504) (R(I),I=1,NZ)
   WRITE (6,566) HA(IHA)
566 FORMAT (///5X,"APPLIED MAGNETIC FIELD IN GAUSS = ",E12.3)
   WRITE (6,567) CRICUR
567 FORMAT (//5X,"NB3SN CRITICAL CURRENT DENSITY AT MINIMUM STRAIN ="
1,E12.3," AIPS/CM**2")
   WRITE (6,561)
561 FORMAT (///5X,"AXIAL STRAIN",5X,"NB3SN STRAIN",5X,"ICNORMALIZED"/
1//)
   DO 563 I=1,NUMPTS
   WRITE (6,562) OUTEPZ(I),OUTEPN(I),JCNORM(I)
562 FORMAT (5X,E12.3,5X,E12.3,5X,E12.3/)
563 CONTINUE
C
C      **CONVERT Z STRAINS AND CRITICAL CURRENTS TO RUPP NORMALIZATION**
C
   IF(IHA.GT.1) GO TO 25
   NUMPM1=NUMPTS-1
   NCOUNT=0
   DO 22 I=1,NUMPM1
   IF(ABS(OUTEPN(I)).LT.ABS(OUTEPN(I+1))) GO TO 23
   GO TO 22
23 NCOUNT=NCOUNT+1

```



```

      IF(NCOUNT.EQ.1) KINDEX=1
22  CONTINUE
      IF(JCNORM(KINDEX).EQ.0.0) GO TO 601
      ADJUST=1.0/JCNORM(KINDEX)
      GO TO 602
601  ADJUST=1.0
602  RUPPNM=OUTEPZ(KINDEX)
      DO 24 J=1,NUMPTS
      OUTEPZ(J)=OUTEPZ(J)-RUPPNM
      JCNORM(J)=JCNORM(J)*ADJUST
24  CONTINUE
      WRITE (6,564)
564  FORMAT (1H1,5X," OUTPUT VALUES OF STRAINS AND NORMALIZED CRITICAL
      : CURRENTS CONVERTED TO RUPP NORMALIZATION"////)
      WRITE (6,501)
      WRITE (6,502) (LABEL(I),I=1,NM)
      WRITE (6,503)
      WRITE (6,504) (R(I),I=1,NZ)
      WRITE (6,506) HACTHA)
      WRITE (6,507) CRICUR
      WRITE (6,561)
      DO 565 I=1,NUMPTS
      WRITE (6,562) OUTEPZ(I),OUTEPN(I),JCNORM(I)
565  CONTINUE
25  CONTINUE
      GO TO 800
700  WRITE (6,13)
      13  FORMAT (5X,"** THE A MATRIX IS SINGULAR: PROGRAM TERMINATION **")
800  STOP
      END

```



```

      NU(J)=0.35
C      ** THIS VALUE OF POISSON'S RATIO IS GUESSED FOR Nb3Sn      **
      K(J)=E(J)/((1+NU(J))*(1-2*NU(J)))
      GO TO 100
4     ALPHA(J)=0.0
      E(J)=0.0
      NU(J)=0.0
      K(J)=0.0
      GO TO 100
5     ALPHA(J)=1.57E-6 + 1.267E-7*T - 2.553E-10*(T**2)
      IF(T.GT.320.0) ALPHA(J)=1.67E-5+5.26E-9*(T-273.0)+
      12.73E-12*((T-273.0)**2)
C      ** THERMAL CONTRACTION DATA FROM BATTILLE HANDBOOK (1977) AND **
C      ** FROM D. S. EASTON ET. AL. (1979)      **
      DEGF=1.8*(T-273.0)+32.0
      E(J)=1.1032E11 - 6.895E07*(DEGF)
      IF(E(J).GT.1.15E11) E(J)=1.15E11
      IF(E(J).LT.0.0) E(J)=0.0
      NU(J)=0.34 + 1.102E-4*(T-298.0)
C      ** POISSON'S RATIO FOR COPPER AS A FUNCTION OF TEMPERATURE IS **
C      ** FROM HECKER'S THESIS (1968) WHICH ALSO HAS YOUNG'S MODULUS **
      K(J)=E(J)/((1+NU(J))*(1-2*NU(J)))
100  CONTINUE
      RETURN
      END

```

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```

C
C *****
C
      SUBROUTINE ELAST1(DT)
      COMMON /RGN/A(23,23),B(23,1),C(23,1),AP(23,23),BP(23,1)
      COMMON /UAR/R(11),N1(12),DEFE(11),DEFS(11),EFE(11)
      COMMON /HQA/E(12),ALPHA(12),EPS(3,11),DSIG(3,11,100),
1 DEFS(3,11,100)
      REAL NU(12),K(12),ML(12),PL(12)
      COMMON /ENG/NU,K,ML,PL,NRING,NZONE,L
      INTEGER V

C
C      ** THIS SUBROUTINE CALCULATES THE ELASTIC MATRIX COMPONENTS      **
C      ** WHICH YIELD A SYSTEM OF LINEAR EQUATIONS                      **
C
C      ** DEFINE THE B VECTOR COEFFICIENTS FROM DISPLACEMENT EQUATION **
C
      NEQN=2*NRING+3
      DO 1 J=1,NEQN
        B(J,1)=0.0
1 CONTINUE

C
C      ** DEFINE THE B VECTOR COEFFICIENTS FROM THE STRESS EQUATION      **
C
C
      NRP2=NRING+2
      DO 2 I=0,NRING
        J=NRP2+I
        B(J,1)=DT*(K(M+1)*ALPHA(M+1)*(1+NU(M+1))-K(M+2)*ALPHA(M+2)*
1 (1+NU(M+2)))
2 CONTINUE

C
C      ** DEFINE THE B VECTOR COEFFICIENTS FROM THE FORCE EQUATION      **
C
C
      NZF=NRP2+NZONE
      B(NZF,1)=K(1)*(1+NU(1))*ALPHA(1)*DT*(R(1)**2)
      DO 3 N=1,NRING
        B(NZF,1)=B(NZF,1)+K(N+1)*(1+NU(N+1))*ALPHA(N+1)*DT*
1 (R(N+1)**2-R(N)**2)
3 CONTINUE

C
C      ** DEFINE THE A MATRIX COEFFICIENTS FROM DISPLACEMENT EQUATION **
C
C
      DO 4 I=1,NEQN
        DO 5 J=1,NEQN
          A(I,J)=0.0
5 CONTINUE
4 CONTINUE
      NRM1=NRING-1
      A(1,2)=1.0
      DO 6 I=0,NRM1
        J=2*I+1
        M=2+I
        V=1+I
        A(M,J)=R(V)
        A(M,J+1)=1/R(V)

```

```

      IF (I.EQ.0) A(M,J+1)=0.0
      A(M,J+2)=-R(V)
      A(M,J+3)=-1/(R(V))
6 CONTINUE

```

```

C
C      ** DEFINE THE A MATRIX COEFFICIENTS FROM THE STRESS EQUATION **
C

```

```

      N=M+1
      J1=2*NRING+1
      DO 7 I=0, NRING
        J=2*I+1
        M=N+1
        V=1+J
        A(M,J)=K(V)
        A(M,J+1)=-K(V)*(1-2*NU(V))/R(V)**2
        IF (I.EQ.0) A(M,J+1)=0.0
        IF (J.GE.J1) GO TO 8
        A(M,J+2)=-K(V+1)
        A(M,J+3)=K(V+1)*(1-2*NU(V+1))/R(V)**2
8      A(M,NEQN)=K(V)*NU(V)-K(V+1)*NU(V+1)
7 CONTINUE

```

```

C
C      ** DEFINE THE A MATRIX COEFFICIENTS FROM THE FORCE EQUATION **
C

```

```

      I=NEQN
      A(I,NEQN)=K(1)*(1-NU(1))*R(1)**2
      A(1,1)=K(1)*2*NU(1)*R(1)**2
      DO 9 J=1, NRING
        M=2*J+1
        A(I,NEQN)=A(I,NEQN)+(R(J+1)**2-R(J)**2)*K(J+1)*(1-NU(J+1))
        A(1,M)=(R(J+1)**2-R(J)**2)*K(J+1)*2*NU(J+1)
9 CONTINUE
      RETURN
      END

```

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C
C
C
      SUBROUTINE ELAST2(DT,NEQN,IER)
      COMMON /RDN/A(23,23),B(23,1),C(23,1),AP(23,23),BP(23,1)
      COMMON /UAR/R(11),NI(12),DEFE(11),DEFS(11),EFE(11)
      COMMON /HQA/E(12),ALPHA(12),EPS(3,11),DSIG(3,11,100),
1DEFS(3,11,100)
      REAL NU(12),K(12),ML(12),PL(12)
      COMMON /ENG/NU,K,ML,PL,NRING,NZONE,L
      DIMENSION UKAREA(25)

C
C
C
      ** THIS SUBROUTINE PERFORMS THE ELASTIC SOLUTION          **
      ** IT CALLS UPON AN IMSL ROUTINE TO SOLVE A SET OF LINEAR EONS.**
      ** THE EQUATIONS ARE: A*C = B WHERE C IS AN UNKNOWN VECTOR **

      DO 1 I=1,NEQN
      DO 2 J=1,NFQN
      AP(I,J)=A(I,J)
2 CONTINUE
      BP(I,1)=B(I,1)
1 CONTINUE
      J=NEQN
      CALL LEQTF1(AP,1,J,23,BP,0,UKAREA,IER)
      DO 3 I=1,NFQN
      E(I,1)=BP(I,1)
3 CONTINUE

C
C
C
      ** NOW DEFINE THE DIFFERENTIAL STRESSES AND STRAINS      **

      DO 4 J=0,NRING
      J1=2*J+1
      J2=2*J+2
      J3=NEQN
      DEPS(1,J+1,L)=C(J1,1)-C(J2,1)/(R(J+1)**2)
      DEPS(2,J+1,L)=C(J1,1)+C(J2,1)/(R(J+1)**2)
      DEPS(3,J+1,L)=C(NEQN,1)
      M=J+1
      DSIG(1,M,L)=K(M)*(C(J1,1)-C(J2,1)/(R(M)**2))*(1-2*NU(M))+NU(M)*
      IC(J3,1)-(1+NU(M))*ALPHA(M)*DT)
      DSIG(2,M,L)=K(M)*(C(J1,1)+C(J2,1)/(R(M)**2))*(1-2*NU(M))+NU(M)*
      IC(J3,1)-(1+NU(M))*ALPHA(M)*DT)
      DSIG(3,M,L)=K(M)*(2*NU(M)*C(J1,1)+C(1-NU(M))*C(J3,1)-(1+NU(M))
      1*ALPHA(M)*DT)
4 CONTINUE

C
C
C
      ** NOW CALCULATE THE DIFFERENTIAL EFFECTIVE STRESS AND STRAIN **

      DO 5 I=0,NRING
      M=I+1
      DEFS(M)=0.70711*SQRT((DSIG(1,M,L)-DSIG(2,M,L))**2+
      1(DSIG(2,M,L)-DSIG(3,M,L))**2+(DSIG(3,M,L)-DSIG(1,M,L))**2)
      DEFE(M)=(0.70711/(1+NU(M)))*SQRT((DEPS(1,M,L)-DEPS(2,M,L))**2+
      1(DEPS(2,M,L)-DEPS(3,M,L))**2+(DEPS(3,M,L)-DEPS(1,M,L))**2)
5 CONTINUE
      RETURN
      END

```

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```

C
C *****
C
SUBROUTINE PLASTC(ML,PL)
COMMON /R0N/A(23,23),B(23,1),C(23,1),AP(23,23),BP(23,1)
COMMON /UAR/R(11),NI(12),DEFE(11),DEFS(11),EFE(11)
COMMON /H0P/E(12),ALPHA(12),EPS(3,11),DSIG(3,11,100),
1DEPS(3,11,100)
REAL NU(12),K(12),ML(12),PL(12)
COMMON /ENG/NU,K,ML,PL,NRING,NZONE,L
C
C ** THIS SUBROUTINE MODIFIES THE VALUES OF POISSON'S RATIO **
C ** AND YOUNG'S MODULUS TO ACCOUNT FOR PLASTICITY EFFECTS **
C ** NI = 1,2,3,... IF NI=1 ZONE 1 IS NIOBIUM **
C ** IF NI=2 ZONE 1 IS BRONZE **
C ** IF NI=3 ZONE 1 IS NB3SN **
C ** IF NI=4 ZONE 1 IS VACUUM **
C ** IF NI=5 ZONE 1 IS COPPER **
C
DO 100 I=0,NZONE
J=I+1
IF(NI(J).EQ.1) GO TO 1
IF(NI(J).EQ.2) GO TO 2
IF(NI(J).EQ.3) GO TO 3
IF(NI(J).EQ.4) GO TO 4
IF(NI(J).EQ.5) GO TO 5
1 PL(J)=E(J)
IF(EFE(J).GT.2.0E-3) PL(J)=2.3E10
IF(EFE(J).GT.6.0E-3) PL(J)=1.6E9
C
C ** FROM THE ROOM TEMPERATURE STRESS-STRAIN CURVE OF NIOBIUM **
C ** REPORTED BY G. ZIEGLER (1978) **
ML(J)=0.5-(0.5-NU(J))*PL(J)/E(J)
NU(J)=ML(J)
K(J)=PL(J)/((1+ML(J))*(1-2*ML(J)))
GO TO 100
2 PL(J)=1.24E11
IF(EFE(J).GT.4.0E-3) PL(J)=2.29E09
IF(PL(J).GT.E(J)) PL(J)=E(J)
C
C ** THIS IS THE LHE TEMPERATURE STRESS-STRAIN CURVE FOR BRONZE **
C ** FROM D. S. EASTON ET. AL. (1979) **
ML(J)=0.5-(0.5-NU(J))*PL(J)/E(J)
NU(J)=ML(J)
K(J)=PL(J)/((1+ML(J))*(1-2*ML(J)))
GO TO 100
3 PL(J)=E(J)
C
C ** NB3SN IS ASSUMED TO BEHAVE AS A PERFECT ELASTIC MATERIAL **
ML(J)=0.5-(0.5-NU(J))*PL(J)/E(J)
NU(J)=ML(J)
K(J)=PL(J)/((1+ML(J))*(1-2*ML(J)))
GO TO 100
4 PL(J)=0.0
ML(J)=0.0
NU(J)=ML(J)
K(J)=0.0
GO TO 100
5 PL(J)=1.15E11

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```
IF(EFE(J).GT.2.5E-3) PL(J)=4.4E10
IF(EFE(J).GT.4.0E-3) PL(J)=1.25E9
C  ** THIS IS THE LHE TEMPERATURE STRESS-STRAIN CURVE FOR COPPER **
C  ** MEASURED BY D. DEIS (1977) **
ML(J)=0.5-(0.5-NU(J))*PL(J)/E(J)
NU(J)=ML(J)
K(J)=PL(J)/((1+ML(J))*(1-2*ML(J)))
100 CONTINUE
RETURN
END
```


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```

C
C *****
C
SUBROUTINE ELAST3(DEPZ)
COMMON /RON/A(23,23),B(23,1),C(23,1),AP(23,23),BP(23,1)
COMMON /WAR/R(11),NI(12),DEFE(11),DEFS(11),EFE(11)
COMMON /HOA/E(12),ALPHA(12),EPS(3,11),DSIG(3,11,100),
1DEPS(3,11,100)
REAL NU(12),K(12),ML(12),PL(12)
COMMON /ENG/NU,K,ML,PL,NRING,NZONE,L
COMMON /ALL/AL(22,22),BL(22,1),CL(22,1),ALP(22,22),BLP(22,1)
INTEGER V

C
C ** THIS SUBROUTINE CALCULATES THE ELASTIC MATRIX COMPONENTS **
C ** FOR AXIAL LOADING OF THE COMPOSITE AT LIQUID HELIUM TEMP. **
C ** THE INITIAL LOADING IS THE RESIDUAL Z STRAIN AT 4.2 DEG. K **
C ** THE SUBROUTINE YIELDS A SYSTEM OF LINEAR EQUATIONS **
C
C ** DEFINE THE B VECTOR COEFFICIENTS FROM DISPLACEMENT - STRESS**
C
NEQN=2*NRING+2
DO 1 I=1,NEQN
  BL(I,1)=0.0
1 CONTINUE

C
C ** DEFINE THE B VECTOR COEFFICIENTS FROM THE FORCE EQUATION **
C
NZF=NZONE+1
DO 2 M=0,NRING
  J=NZF+M
  BL(J,1)=DEPZ*(K(I+2)*NU(M+2)-K(M+1)*NU(M+1))
2 CONTINUE

C
C ** DEFINE THE A MATRIX COEFFICIENTS FROM DISPLACEMENT EQUATION **
C
DO 4 I=1,NEQN
  DO 5 J=1,NEQN
    AL(I,J)=0.0
5 CONTINUE
4 CONTINUE
NRM1=NRING-1
AL(1,2)=1.0
DO 6 I=0,NRM1
  J=2*I+1
  M=2+I
  V=1+I
  AL(M,J)=R(V)
  AL(M,J+1)=1/R(V)
  IF(1.EQ.0) AL(M,J+1)=0.0
  AL(M,J+2)=-R(V)
  AL(M,J+3)=-1/R(V)
6 CONTINUE

C
C ** DEFINE THE A MATRIX COEFFICIENTS FROM THE STRESS EQUATION **
C
N=M+1

```

```

J1=2*NRING+1
DO 7 I=0, NRING
  J=2*I+1
  M=N+1
  V=I+1
  AL(M, J) = K(V)
  AL(M, J+1) = -K(V)*(1-2*NU(V))/ (R(V)**2)
  IF (I.EQ.0) AL(M, J+1) = 0.0
  IF (J.GE. J1) GO TO 7
  AL(M, J+2) = -K(V+1)
  AL(M, J+3) = +K(V+1)*(1-2*NU(V+1))/ (R(V)**2)
7 CONTINUE
RETURN
END

```

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```

C
C
C
*****
SUBROUTINE ELAST4(DEPZ,NEON,IER)
COMMON /RON/A(23,23),B(23,1),C(23,1),AP(23,23),BP(23,1)
COMMON /WAR/R(11),NI(12),DEFE(11),DEFS(11),EFE(11)
COMMON /HOA/E(12),ALPHA(12),EPS(3,11),DSIG(3,11,100),
1 IDEPS(3,11,100)
REAL NU(12),K(12),ML(12),PL(12)
COMMON /ENG/NU,K,ML,PL,NRING,NONE,L
COMMON /ALL/AL(22,22),BL(22,1),CL(22,1),ALP(22,22),BLP(22,1)
DIMENSION WKAREA(25)

C
C
C
** THIS SUBROUTINE PERFORMS THE ELASTIC SOLUTION **
C
C
C
** IT CALLS UPON AN IMSL ROUTINE TO SOLVE A SET OF LINEAR EQNS.**
C
C
C
** THE EQUATIONS ARE: A*C = B WHERE C IS AN UNKNOWN VECTOR **

DO 1 I=1,NEON
DO 2 J=1,NEON
  ALP(I,J)=AL(I,J)
2 CONTINUE
  BLP(I,1)=BL(I,1)
1 CONTINUE
  J=NEON
  CALL LEUTIF(ALP,1,J,22,BLP,0,WKAREA,IER)
DO 3 I=1,NEON
  CL(I,1)=BLP(I,1)
3 CONTINUE

C
C
C
** NOW DEFINE THE DIFFERENTIAL STRESSES AND STRAINS **

DO 4 J=0,NRING
  J1=2*J+1
  J2=2*J+2
  DEPS(1,J+1,L)=CL(J1,1)-CL(J2,1)/(R(J+1)**2)
  DEPS(2,J+1,L)=CL(J1,1)+CL(J2,1)/(R(J+1)**2)
  DEPS(3,J+1,L)=DEPZ
  M=J+1
  DSIG(1,M,L)=K(M)*(CL(J1,1)-(CL(J2,1)/(R(M)**2)))*(1-2*NU(M))+
1 NU(M)*DEPZ
  DSIG(2,M,L)=K(M)*(CL(J1,1)+(CL(J2,1)/(R(M)**2)))*(1-2*NU(M))+
1 NU(M)*DEPZ
  DSIG(3,M,L)=K(M)*(2*NU(M)*CL(J1,1)+(1-NU(M))*DEPZ)
4 CONTINUE

C
C
C
** NOW CALCULATE THE DIFFERENTIAL EFFECTIVE STRESS AND STRAIN **

DO 5 I=0,NRING
  M=I+1
  DEFS(M)=0.70711*SQRT((DSIG(1,M,L)-DSIG(2,M,L))**2+
1 (DSIG(2,M,L)-DSIG(3,M,L))**2+(DSIG(3,M,L)-DSIG(1,M,L))**2)
  DEFE(M)=(0.70711/(1+NU(M)))*SQRT((DEPS(1,M,L)-DEPS(2,M,L))**2+
1 (DEPS(2,M,L)-DEPS(3,M,L))**2+(DEPS(3,M,L)-DEPS(1,M,L))**2)
5 CONTINUE
RETURN
END

```



```

LPSN=ABS(OUTEPN(1))
IF (I.EQ.1MIN) CRICUR=2CURRE
FRACT=3.08642E+3*(EPSN**2)
FRACT=SQRT(FRACT)
HC2EP=A1*FRACT + A2*(1.0 - FRACT)
GLPEP=A3*SQRT(FRACT) + A4*(1.0 - SQRT(FRACT))
TFAC=0.56*(HC2EP - HA(IHA))*2
BFAC1=(1.0 - SQRT((FLUX*FLLDEN)/HA(IHA)))*2
BFAC2=SQRT(HA(IHA))*GLPEP**2
FDCUR=TFAC*A5/(BFAC1*BFAC2)
IF (HA(IHA).GE.HC2M) GO TO 2
IF (HA(IHA).GE.HC2LP) GO TO 2
FNDUR(1)=FDCUR/CIFRE
GO TO 1
2 JURNRM(1)=0.0
1 CONTINUE
3 RETURN
END

```

This section contains a sample output of an internal-bronze-core geometry conductor modeled on the MAXIMUPER computer program. The following comments and output listings, together with comment cards in the program, are intended to assist the reader and serve as a brief user's guide.

First, the program echoes the input data: the materials comprising each zone (NI) and the corresponding zone radii in meters (R). Then, the code prints the triaxial stresses and strains and resulting effective stresses and strains in each of the specified zones arising from the thermal contraction between 1024 K to liquid helium temperature. Axial loading is applied to the model conductor at 4.2 K, and the resulting stress-strain states are printed at various values of the axial ϵ strain.

After the axial strain has increased to 1.0 percent, the program prints a summary sheet of the stress-strain states and critical current densities for the Nb_3Sn zone in which the J_c values are to be calculated (usually assigned to the center of the Nb_3Sn annular ring layer). The listing includes a tabulation of the axial strain versus the effective strain on the central Nb_3Sn zone and the resulting normalized critical current density degradation. The effective strains on the Nb_3Sn are

scanned to determine the lowest value, and J_c is calculated at this minimum strain. All other J_c calculations at various axial strains occurring before the minimum are affixed with minus signs (for visual effects) to indicate that the Nb_3Sn is initially under compression before the applied loading relieves the precompression and places it in tension.

The first entry in the axial strain column represents the total axial thermal contraction arising from the cool-down between 1024 to 4.2 K. For most conductors modeled, it varied between -1.0 to -1.5 percent. In the laboratory, this value is never measured, since the gages are affixed at room temperature and zeroed at liquid helium temperature. In order to convert the program results to predict laboratory experiments, the axial strain, which corresponds to the minimum Nb_3Sn effective strain, is subtracted from every entry in the axial strain column. This normalizes the program output into the Rupp format indicated in figures 5, 28, 29, and 30. Therefore, the zero offset axial strain (intrinsic strain) and normalized critical current density columns predict the Rupp normalization curves as a function of applied magnetic fields for each conductor modeled.

THE ZONES ARE MADE OF THE FOLLOWING MATERIALS:
LISTED IN THE ORDER OF EACH CORRESPONDING ZONE NUMBER

BRONZE	BRONZE	NB3SN	NB3SN	NB3SN	NIOBIUM
NIOBIUM	COPPER	COPPER	VACUUM		

THE CORRESPONDING ZONE RADII ARE:

1.00E-05 2.10E-05 2.12E-05 2.16E-05 2.23E-05 2.50E-05 2.75E-05 2.85E-05 2.95E-05

THE RESIDUAL STRAINS AT T= 4.200E+00 ARE:

-1.79E-02 -1.79E-02 3.50E-03 3.12E-03 2.40E-03 7.36E-03 5.31E-03 -2.21E-02 -2.17E-02
-1.79E-02 -1.79E-02 -1.77E-02 -1.73E-02 -1.67E-02 -1.40E-02 -1.21E-02 -1.25E-02 -1.28E-02
-1.47E-02 -1.47E-02 -1.47E-02 -1.47E-02 -1.47E-02 -1.47E-02 -1.47E-02 -1.47E-02 -1.47E-02

THE EFFECTIVE STRAINS AT T= 4.200E+00 ARE:

2.45E-03 2.45E-03 1.47E-02 1.43E-02 1.35E-02 1.45E-02 1.26E-02 5.84E-03 5.39E-03

THE RESIDUAL STRESSES AT T= 4.200E+00 ARE:

1.55E+08 1.55E+08 1.35E+08 9.73E+07 3.57E+07 -1.70E+06 -3.00E+07 -1.50E+07 8.11E-06
1.55E+08 1.55E+08 -1.93E+09 -1.90E+09 -1.83E+09 -2.93E+09 -3.00E+00 4.10E+08 4.21E+08
2.77E+08 2.77E+08 -1.63E+09 -1.63E+09 -1.63E+09 -2.62E+08 -3.04E+08 3.72E+08 3.89E+08

THE EFFECTIVE STRESSES AT T= 4.200E+00 ARE:

1.22E+08 1.22E+08 1.94E+09 1.88E+09 1.78E+09 2.77E+08 2.75E+08 4.07E+08 4.06E+08

THE STRAINS FOR AN AXIAL STRAIN OF $-9.911\text{E-}03$:

-2.00E-02	-2.00E-02	2.07E-03	1.67E-03	1.01E-03	4.72E-03	2.70E-03	-2.47E-02	-2.42E-02
-2.00E-02	-2.00E-02	-1.98E-02	-1.94E-02	-1.88E-02	-1.61E-02	-1.47E-02	-1.47E-02	-1.50E-02
-9.91E-03	-9.91E-03	-9.91E-03	-9.91E-03	-9.91E-03	-9.91E-03	-9.91E-03	-9.91E-03	-9.91E-03

THE EFFECTIVE STRAINS AT EPSZ= $-9.911\text{E-}03$:

6.76E-03	6.76E-03	1.41E-03	1.35E-02	1.27E-02	1.24E-02	1.02E-02	8.73E-03	8.39E-03
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THE STRESSES FOR AN AXIAL STRAIN OF $-9.911\text{E-}03$:

1.56E+08	1.56E+08	1.36E+08	9.79E+07	3.56E+07	-1.77E+06	-3.08E+07	-1.50E+07	9.56E-06
1.56E+08	1.56E+08	-1.96E+09	-1.92E+09	-1.86E+09	-2.93E+08	-3.07E+08	4.10E+08	4.22E+08
4.59E+08	4.59E+08	-1.40E+09	-1.40E+09	-1.40E+09	-2.54E+08	-2.96E+08	3.78E+08	3.95E+08

THE EFFECTIVE STRESSES AT EPSZ= $-9.911\text{E-}03$:

3.03E+08	3.03E+08	1.86E+09	1.81E+09	1.71E+09	2.74E+08	2.71E+08	4.10E+08	4.09E+08
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THE STRAINS FOR AN AXIAL STRAIN OF $7.889E-03$:

-2.80E-02 -2.88E-02 -2.71E-03 -3.19E-03 -3.96E-03 -4.40E-03 -6.37E-03 -3.37E-02 -3.33E-02
 -2.88E-02 -2.88E-02 -2.86E-02 -2.61E-02 -2.73E-02 -2.47E-02 -2.30E-02 -2.34E-02 -2.37E-02
 $7.89E-03$ $7.89E-03$ $7.89E-03$ $7.89E-03$ $7.89E-03$ $7.89E-03$ $7.89E-03$ $7.89E-03$ $7.89E-03$

THE EFFECTIVE STRAINS AT $EPSZ = 7.889E-03$:

2.45E-02 2.45E-02 2.41E-02 2.37E-02 2.30E-02 1.90E-02 1.79E-02 2.51E-02 2.49E-02

THE STRESSES FOR AN AXIAL STRAIN OF $7.889E-03$:

1.55E+08 1.65E+08 1.43E+08 1.02E+08 3.55E+07 -1.84E+06 -3.08E+07 -1.51E+07 1.73E-05
 1.65E+08 1.65E+08 -2.10E+09 -2.06E+09 -1.99E+09 -2.92E+08 -3.07E+08 4.10E+08 4.22E+08
 5.08E+06 5.08E+08 -5.59E+08 -5.59E+08 -5.59E+08 -2.25E+08 -2.68E+08 4.01E+06 4.17E+08

THE EFFECTIVE STRESSES AT $EPSZ = 7.889E-03$:

3.44E+08 3.44E+08 1.99E+09 1.92E+09 1.80E+09 2.63E+08 2.59E+08 4.21E+08 4.20E+08

APPLIED MAGNETIC FIELD IN GAUSS = 4.000E+04

NB3SN CRITICAL CURRENT DENSITY AT MINIMUM STRAIN = 1.942E+06 AMPS/CN**2

AXIAL STRAIN	NB3SN STRAIN	ICNORMALIZED
-1.471E-02	-1.427E-02	9.240E-01
-1.011E-02	-1.353E-02	1.000E+00
-9.911E-03	1.355E-02	9.995E-01
-9.111E-03	1.364E-02	9.893E-01
-8.911E-03	1.366E-02	9.862E-01
-8.111E-03	1.381E-02	9.707E-01
-7.911E-03	1.386E-02	9.661E-01
-7.111E-03	1.407E-02	9.447E-01
-6.911E-03	1.413E-02	9.386E-01
-6.111E-03	1.439E-02	9.118E-01
-5.911E-03	1.446E-02	9.045E-01
-5.111E-03	1.478E-02	8.729E-01
-4.911E-03	1.486E-02	8.644E-01
-4.111E-03	1.523E-02	8.286E-01
-3.911E-03	1.532E-02	8.191E-01
-2.111E-03	1.629E-02	7.273E-01
-1.911E-03	1.640E-02	7.165E-01
-1.111E-03	1.698E-02	6.721E-01
-9.113E-04	1.701E-02	6.608E-01
-1.113E-04	1.752E-02	6.150E-01
8.865E-05	1.765E-02	6.034E-01
8.897E-04	1.820E-02	5.567E-01
1.009E-03	1.833E-02	5.450E-01
1.899E-03	1.890E-02	4.981E-01
2.009E-03	1.905E-02	4.864E-01
2.899E-03	1.964E-02	4.399E-01
3.009E-03	1.979E-02	4.284E-01
3.899E-03	2.040E-02	3.829E-01
4.009E-03	2.055E-02	3.717E-01
4.899E-03	2.118E-02	3.277E-01

APPLIED MAGNETIC FIELD IN GAUSS = 4.00E+04
 IRISON CRITICAL CURRENT DENSITY AT MINIMUM STRAIN = 1.942E+06 AMP/CM²

PIXEL STRAIN IRISON STRAIN ICHORMALIZED

-4.60E-03	-1.427E-02	9.240E-01
0.	-1.353E-02	1.008E+00
2.00E-04	1.355E-02	9.965E-01
1.00E-03	1.364E-02	9.893E-01
1.200E-03	1.366E-02	9.862E-01
2.000E-03	1.381E-02	9.707E-01
2.200E-03	1.386E-02	9.661E-01
3.000E-03	1.407E-02	9.447E-01
3.000E-03	1.413E-02	9.386E-01
4.000E-03	1.429E-02	9.113E-01
4.200E-03	1.446E-02	9.045E-01
5.000E-03	1.478E-02	8.729E-01
5.200E-03	1.486E-02	8.643E-01
6.000E-03	1.523E-02	8.253E-01
6.000E-03	1.532E-02	8.191E-01
8.000E-03	1.628E-02	7.273E-01
8.200E-03	1.640E-02	7.165E-01
9.000E-03	1.688E-02	6.721E-01
9.200E-03	1.701E-02	6.608E-01
1.000E-02	1.752E-02	6.150E-01
1.020E-02	1.765E-02	6.034E-01
1.100E-02	1.820E-02	5.567E-01
1.120E-02	1.833E-02	5.450E-01
1.200E-02	1.890E-02	4.901E-01
1.220E-02	1.905E-02	4.864E-01
1.300E-02	1.964E-02	4.399E-01
1.320E-02	1.979E-02	4.284E-01
1.400E-02	2.040E-02	3.829E-01
1.420E-02	2.055E-02	3.717E-01
1.500E-02	2.113E-02	3.277E-01

APPLIED MAGNETIC FIELD IN GAUSS = 8.000E+04

NB3SN CRITICAL CURRENT DENSITY AT MINIMUM STRAIN = 3.096E+05 AMPS/CM**2

AXIAL STRAIN	NB3SN STRAIN	NORMALIZED
4.600E-03	-1.427E-02	8.649E-01
0.	-1.354E-02	1.000E+00
2.000E-04	1.355E-02	9.973E-01
1.000E-03	1.364E-02	9.806E-01
1.200E-03	1.366E-02	9.751E-01
2.000E-03	1.381E-02	9.473E-01
2.200E-03	1.386E-02	9.390E-01
3.000E-03	1.407E-02	9.011E-01
3.200E-03	1.413E-02	8.905E-01
4.000E-03	1.429E-02	8.436E-01
4.200E-03	1.445E-02	8.309E-01
5.000E-03	1.468E-02	7.766E-01
5.200E-03	1.486E-02	7.622E-01
6.000E-03	1.523E-02	7.021E-01
6.200E-03	1.532E-02	6.865E-01
8.000E-03	1.628E-02	5.395E-01
8.200E-03	1.640E-02	5.228E-01
9.000E-03	1.688E-02	4.559E-01
9.200E-03	1.701E-02	4.392E-01
1.000E-02	1.752E-02	3.736E-01
1.020E-02	1.765E-02	3.575E-01
1.100E-02	1.820E-02	2.947E-01
1.120E-02	1.833E-02	2.796E-01
1.200E-02	1.890E-02	2.214E-01
1.220E-02	1.905E-02	2.076E-01
1.300E-02	1.964E-02	1.556E-01
1.320E-02	1.979E-02	1.435E-01
1.400E-02	2.040E-02	9.919E-02
1.420E-02	2.055E-02	8.918E-02
1.500E-02	2.118E-02	5.386E-02

GRAPHED MAGNETIC FIELD IN GAUSS = 1.000E+05
 X-RAY CRITICAL CURRENT DENSITY AT MINIMUM STRAIN = 1.000E+05 BR/SQCM*2

GRAPHED STRAIN	MINUM STRAIN	ICNORMALIZED
0.000E+00	-1.427E-02	7.901E-01
1.000E-04	1.353E-02	1.000E+00
1.000E-03	1.304E-02	9.957E-01
1.000E-02	1.366E-02	9.694E-01
1.000E-01	1.381E-02	9.606E-01
1.000E+00	1.354E-02	9.170E-01
1.000E+01	1.356E-02	9.044E-01
1.000E+02	1.437E-02	8.455E-01
1.000E+03	1.413E-02	8.292E-01
1.000E+04	1.439E-02	7.931E-01
1.000E+05	1.446E-02	7.390E-01
1.000E+06	1.478E-02	6.564E-01
1.000E+07	1.486E-02	6.172E-01
1.000E+08	1.522E-02	5.521E-01
1.000E+09	1.532E-02	5.104E-01
1.000E+10	1.628E-02	3.149E-01
1.000E+11	1.640E-02	3.149E-01
1.000E+12	1.688E-02	2.356E-01
1.000E+13	1.701E-02	2.168E-01
1.000E+14	1.752E-02	1.476E-01
1.000E+15	1.765E-02	1.319E-01
1.000E+16	1.820E-02	7.626E-02
1.000E+17	1.823E-02	6.441E-02
1.000E+18	1.890E-02	1.621E-02
1.000E+19	1.905E-02	1.191E-02
1.000E+20	1.964E-02	1.742E-03
1.000E+21	1.979E-02	2.504E-04
1.000E+22	2.040E-02	0.
1.000E+23	2.05E-02	0.
1.000E+24	2.118E-02	0.

APPLIED MAGNETIC FIELD IN GAUSS = 1.22E+05

NEEDN CRITICAL CURRENT DENSITY AT MINIMUM STRAIN = 1.65E+04 AMP'S/CM**2

APPL STRAIN NEEDN STRAIN IC(NORMALIZED

4.00E-03	-1.427E-02	5.636E-01
0.	-1.354E-02	1.000E+00
2.00E-04	1.355E-02	9.983E-01
1.00E-03	1.404E-02	9.319E-01
1.00E-03	1.404E-02	9.126E-01
2.00E-03	1.391E-02	9.180E-01
2.00E-03	1.390E-02	7.917E-01
3.00E-03	1.407E-02	6.714E-01
5.00E-03	1.414E-02	6.390E-01
4.00E-03	1.435E-02	5.039E-01
4.00E-03	1.434E-02	4.695E-01
5.00E-03	1.470E-02	3.336E-01
5.00E-03	1.491E-02	3.008E-01
6.00E-03	1.507E-02	1.795E-01
6.00E-03	1.537E-02	1.525E-01
8.00E-03	1.620E-02	3.244E-03
8.00E-03	1.640E-02	2.649E-04
9.00E-03	1.686E-02	0.
9.00E-03	1.701E-02	0.
1.00E-02	1.752E-02	0.
1.00E-02	1.765E-02	0.
1.10E-02	1.820E-02	0.
1.12E-02	1.833E-02	0.
1.00E-02	1.890E-02	0.
1.20E-02	1.905E-02	0.
1.30E-02	1.964E-02	0.
1.32E-02	1.979E-02	0.
1.40E-02	2.040E-02	0.
1.42E-02	2.055E-02	0.
1.50E-02	2.118E-02	0.

A comparison of the MAXIMSUPER program predictions with experimental data for the strain degradation of I_c . The above plot is from Fig. 5, with the squares representing the 4 and 12 Tesla predictions of equation 5-36.

VITA

Ronald Warren Hoard, the son of Robert Glen Hoard and Golda (Taylor) Hoard, was born in Chicago, Illinois, on July 13, 1953.

His secondary schools were Hyde Park and James H. Bowen High Schools, both in the city of Chicago. After graduation he attended Cornell University in Ithaca, New York, and received two degrees from the Department of Applied and Engineering Physics:

1. Bachelor of Science in Engineering Physics,
June 1975
2. Masters of Engineering in Engineering Physics,
June 1976

He then enrolled in the Department of Nuclear Engineering at the University of Washington in Seattle, Washington, where the present work was completed in conjunction with the Lawrence Livermore National Laboratory in Livermore, California.