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A COMPARISON OF MEASURED AND COMPUTED PLASMA
LOADING RESISTANCE IN THE TANDEM MIRROR
EXPERIMENT-UPGRADE (TMX-U) CENTRAL CELL

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A COMPARISON OF MEASURED AND COMPUTED PLASMA LOADING RESISTANCE IN THE TANDEM MIRROR EXPERIMENT-UPGRADE (TMX-U) CENTRAL CELL

ABSTRACT

The plasma loading resistance vs density plots computed with McVey's Code XANTENAL, agree well with experimental measurements in the TMX-U central cell. The agreement is much better for frequencies where $\omega/\omega_{ci} < 1$ than for $\omega/\omega_{ci} \geq 1$.

INTRODUCTION

The code, XANTENAL, is used as a tool to predict and optimize ion-cyclotron resonant frequency (ICRF) heating in TMX-U and Mirror Fusion Test Facility (MFTF-B). This is a preliminary evaluation of the accuracy of XANTENAL. We compare the measured change in antenna loading resistance as a function of plasma density with that predicted by XANTENAL.

Figure 1 shows the configuration of the ion-cyclotron resonant heating (ICRH) antenna used in this investigation. Figure 1a and b depicts the radial and axial perspectives of the antenna without its Faraday shield. Figure 1c shows the Faraday shield covering the antenna.

ANALYSIS OF EXPERIMENTAL DATA

During TMX-U operation, we use the network analyzer for two main purposes: (1) to provide the data needed to match the antenna impedance to the 50 Ω of the transmitter, either in vacuum or with plasma present; (2) to transform these data into the loading impedance of the antenna.

Other data that we need for the analysis in this report are the plasma density and radius. The line density is obtained from the microwave interferometer system (MIS); the plasma radius is obtained from the beam attenuation detectors (BAD); and the peak density is obtained from the radius

Figure 1. The ICRH antenna configuration. The 5-1/2-turn loop antenna has 6 conductors near the plasma and 5 near the wall. They are axially spaced 1/2 in. apart. The copper conductors are of 1/2-in.-square cross section, hollow, and water-cooled. The Faraday shield consists of 37 copper strips wrapped around the antenna spaced ~ 2 mm apart.

and line density. The combination and transformation of these data were accomplished by a computer program written in BASIC for the HP9816 desktop computer. This program was written as an interactive operating program that can either accept data from the keyboard and store it as a data file on disk, or can retrieve data files from disk and transform them into the desired plots. The data can be plotted on the CRT or the printer. We produced all of the plots in this report using this program. A copy of the program can be obtained from Ref. 1. Because this program was written as a series of general purpose independent subroutines, it would be easy to incorporate selected subroutines (e.g., the plot routine) into any HP9816 BASIC library.

In all of the frequency plots, f is in MHz, R_a (on the plots this is R_{Av}) is in ohms (Ω), and L is in μH . The solid line in the plots is resistance (horizontal displacement in network analyzer data), and the dashed line is reactance (vertical displacement in network analyzer data).

TRANSFORMATION OF NETWORK ANALYZER DATA

The experimental setup for obtaining the network analyzer data is shown in Fig. 2. These data are converted to the impedance looking into the tuning network via the transformation equations shown in Part 1 of Appendix A.

Figure 3 illustrates the circuit diagram of the antenna tuning network. The network is tuned by adjusting C_1 , C_2 , and C_3 , where the values of C_1 and C_2 are kept approximately equal. The major tuning criterion is to set the input impedance of the network as close to $(50 + j0) \Omega$ as possible throughout a given shot (to minimize reflected power). The network is retuned when the excitation frequency or the plasma density is changed.

Assuming all components in the tuning network are known, input impedance Z_{in} can be found as a function of Z_A (the antenna loading impedance) as shown in Part 2 of Appendix A:

$$Z_{in} = R_{in} + j X_{in} \quad Z_A = R_A + j X_A$$

$$R_{in} = \frac{R_A}{\omega^2 C_1^2 \left\{ R_A^2 + \left[X_A - \frac{1}{\omega} \left(\frac{1}{C_1} + \frac{1}{C_2} \right) \right]^2 \right\}} \quad (1)$$

Figure 2. Network analyzer experimental setup. Note that 100 W of transmitted power was used for all shots examined in this report. (Low power loading). XANTENAI does not account for the evolution of the plasma resulting from the power absorbed. The NA data was read off of a camera snapshot at 2 ms intervals and entered into the computer via a keyboard.

Figure 3. Circuit diagram of tuning network.

$$X_{in} = \frac{-X_A + \frac{1}{\omega} \left(\frac{1}{C_1} + \frac{1}{C_2} \right)}{\omega^2 C_1^2 \left\{ R_A^2 + \left[X_A - \frac{1}{\omega} \left(\frac{1}{C_1} + \frac{1}{C_2} \right) \right]^2 \right\}} - \frac{1}{\omega} \left(\frac{1}{C_1} + \frac{1}{C_3} \right) \quad (2)$$

where $\omega = 2\pi f$, f = excitation frequency.

It was determined from independent measurements¹ that the loading impedance of the antenna with no plasma (vacuum case) consisted of the resistance and inductance:

$$R_{AV} = 0.3 \, \Omega \quad L = 13.6 \, \mu H \quad (3)$$

thus

$$Z_{AV} = R_{AV} + j X_{AV} = R_{AV} + j\omega L \quad (4)$$

Because C_1 , C_2 , and C_3 stay constant throughout a shot, we can find their values from Eqs. (1) and (2) given (3) and (4) and the vacuum input impedance $Z_{inv} = R_{inv} + j X_{inv}$. However, because the three undetermined C 's are a function of the two measured quantities R_{inv} and X_{inv} , they can only be determined to within a one-variable function of one another. However, if we assume that $C_1 = C_2 = C$ (we will test the sensitivity to this assumption later), Eq. (1) can be solved directly for C :

$$C_1 = C_2 = C = \frac{2 X_{AV} \pm \left[\frac{R_{AV}}{R_{inv}} (X_{AV}^2 + R_{AV}^2 - 4 R_{AV} R_{inv}) \right]^{1/2}}{\omega (R_{AV}^2 + X_{AV}^2)} \quad (5)$$

We can now use Eq. (2) to solve for C_3 :

$$C_3 = \frac{-1}{\omega \left[X_{inv} + \frac{1}{\omega C} + \frac{X}{\omega^2 C^2 (R_{AV}^2 + X^2)} \right]} \quad (6)$$

where $X = X_{AV} - \frac{1}{\omega} \left(\frac{2}{C} \right)$.

Note the two possible values for C. In actuality, these two values occur very near one another (e.g., during 7/7/83 Shot #14 using Eq. (5), we obtain $C^+ = 220.4$ picofarad (pF) and $C^- = 196.9$ pF). However, we found that the larger value of C used in Eq. (6) gives a negative value for C_3 (i.e., it is an inductance for this case). For example C^- gives $C_3 = 25.34$ pF and C^+ gives $C_3 = -22.17$ pF (inductive). Thus, only C^- was used in the transformation equations.

Knowing Z_{in} and the capacitor values, we can derive Z_A as a function of Z_{in} as shown in Part 3 of Appendix A:

$$R_A = \frac{R_{in}}{\omega^2 C_1^2 \left\{ R_{in}^2 + \left[X_{in} + \frac{1}{\omega} \left(\frac{1}{C_1} + \frac{1}{C_3} \right) \right]^2 \right\}} \quad (7)$$

$$X_A = \frac{-X_{in} - \frac{1}{\omega} \left(\frac{1}{C_1} + \frac{1}{C_3} \right)}{\omega^2 C_1^2 \left\{ R_{in}^2 + \left[X_{in} + \frac{1}{\omega} \left(\frac{1}{C_1} + \frac{1}{C_3} \right) \right]^2 \right\}} + \frac{1}{\omega} \left(\frac{1}{C_1} + \frac{1}{C_2} \right) \quad (8)$$

The component of the plasma loading impedance resulting from the presence of the plasma (ΔZ_A) is simply given by

$$\Delta R_A = R_A - R_{Av} \quad (9)$$

$$\Delta X_A = X_A - X_{Av} \quad (10)$$

We can summarize the complete transformation of the network analyzer data as follows:

1. The uv vs t plot is normalized network analyzer data (see Fig. 4 and compare with line density in Fig. 5),
Eqs. (A1.1), (A1.2), and (A1.3);
2. The Z_{in} vs t plot is input impedance of tuning network (Fig. 6a),
Eqs. (3-8) (Z_{inv} is determined from network analyzer data at beginning of shot, $t \approx 0$);

Figure 4. The uv vs t plot for shot #16 on 7/7/83.

Figure 5. The fnd1 vs t plot for shot #16 on 7/7/83.

Figure 6a. The Z_{in} vs t plot for shot #16 on 7/7/83.

Figure 6b. The ΔZ_A vs t plot for shot #16 on 7/7/83.

3. The Z_A vs t plot is antenna plus plasma loading impedance, Eqs. (3), (4), (9), (10);

4. The ΔZ_A vs t plot is plasma loading impedance (Fig. 6b).

These transformations were tested by doing inverse transformations on the transformed plots.

SENSITIVITY ANALYSIS

In this section, we compare the sensitivity of the computed loading impedance with errors in the values of matching network capacitance and the antenna inductance and resistance. It was found that the loading resistance is sensitive mainly to changes in the antenna resistance, and the loading reactance is sensitive primarily to changes in capacitance and inductance.

First, we analyze the sensitivity of the plasma loading impedance ΔZ_A to the relative values of C_1 and C_2 . We use the same procedure (and equations) discussed above to obtain ΔZ_A , letting $C_1 = C(1 + \delta)$ and $C_2 = C(1 - \delta)$ where δ is a fractional deviation. The values for C and C_3 are solved in Part 4 of Appendix A using Eqs. (1) and (2) with the above assumption. The results are as follows:

$$C = \frac{2 X_{Av} - \left[\frac{R_{Av}}{R_{inv}} (1 - \delta)^2 (p_{Av}^2 + x_{Av}^2) - 4 R_{Av}^2 \right]^{1/2}}{\omega(1 - \delta^2) (R_A^2 + X_A^2)}, \quad (11)$$

$$C_3 = \frac{-1}{\omega \left[X_{inv} + \frac{1}{\omega C_1} + \frac{X}{\omega^2 C_1^2 (X^2 + R_{Av}^2)} \right]}, \quad (12)$$

where

$$X = X_{Av} - \frac{1}{\omega} \left(\frac{1}{C_1} + \frac{1}{C_2} \right),$$

$$C_1 = C(1 + \delta),$$

$$C_2 = C(1 - \delta). \quad (13)$$

Note that the smaller value of C was again used because the larger value makes C_3 inductive. The value of C is the average of C_1 and C_2 , and it is a function of δ , thus C_1 and C_2 must be derived from Eqs. (11) and (13). Figure 7 consists of plots showing the sensitivity of ΔR_A and ΔX_A to C_1 and C_2 . Note that the resistive portion of the plasma loading impedance is very insensitive to the exact values of C_1 and C_2 . Results are similar for all other shots examined in this report. Because it is only the resistive component that can be compared with the predictions of XANTENAL, we continue the analysis with the assumption that $C_1 = C_2$. (Though it is hard to distinguish the lines in places, the solid line is the plot for $\delta = 0$, the line dashed with equal spacing is for $\delta = +0.15$, and the line dashed with a solid:blank ratio of 2:1 is for $\delta = -0.15$. The bottom comment line gives the values of C and C_3 (in pF) for δ negative, zero, and positive, respectively. C_1 and C_2 must be found from Eq. (13) for each value of δ .

As our next step, we include in the transformation program a sensitivity analysis of ΔZ_A on R_{AV} and L , simply by letting $R_{AV} \rightarrow R_{AV}(1 + \delta_R)$ and $L \rightarrow L(1 + \delta_L)$ in Eqs. (4), (11), (12), and (7-10). The plots are similar to Fig. 7 for 0, and $\pm 15\%$ changes in R_{AV} and L are shown in Figs. 8 and 9. These results are similar for the other shots examined in this report. Note that changes in the resistive components affect both the plasma loading resistance and reactance. Changes in the reactive components primarily affect the plasma loading reactance.

PLASMA LINE-DENSITY DATA

A major problem in this study is that the diagnostics give information about the line density, whereas XANTENAL bases its predictions on the actual density profile $n(r)$. Thus, to make the comparison, an estimation of the density profile must be made.

The BAD data are used for this purpose. The neutral beam attenuation is measured across seven chords in the TMX-U central cell. This diagnostic gives the line density for each chord as a function of time. The best fit Gaussian profile

Figure 7. Sensitivity on $\delta C = 1.5 \Delta Z_A$ vs t plot for shot #16 on 7/7/83.

Figure 8. Sensitivity on $\delta R = 1.5 \Delta Z_A$ vs t plot for shot #16 on 7/7/83.

Figure 9. Sensitivity on $\delta L = 1.5 \Delta Z_A$ vs t plot for shot #16 on 7/7/83.

$$n(r) = n_c \exp \left[- \left(\frac{r - r_0}{r_p} \right)^2 \right] \quad (14)$$

is found at different times throughout each shot. The method used gives r_0 and r_p from a least-squares parabolic (explained in Part 1 of Appendix B) fit that is directly applied to the line density vs time BAD data. This method is based on the fact that the peak density n_{cc} on any chord through any radial Gaussian density profile is proportional to the line density along that chord; the constant of proportionality is the same for any chord. The constant is derived in Part 1 of Appendix C and is well known

$$n_{cc} = \frac{\int_c n(\vec{r}) dl}{\sqrt{\pi} r_p} \quad , \quad (15)$$

where c is any chord, $r_p = \text{const.}$ Thus r_p is always the same for both the density $n(\vec{r})$ and the line density. Shown in Table 1 is a compilation of the r_p and r_0 values obtained at different times for different shots, plus the BAD data used to determine these values. Part 1 of Appendix B shows a derivation and demonstration of the method used to find r_p and r_0 at each particular time. The time periods were selected as follows: three equally spaced times were chosen during the shot when the BAD data seemed satisfactory; in addition, the time when R_{in} was closest to 50Ω was chosen (i.e., when Z_{in} of tuning network most closely matched the characteristic impedance of the transmission line, the point of maximum power transfer. This time was determined from Z_{in} vs t data). Thus, four time periods were examined for each shot.

Note that the signal-to-noise ratio of the BAD data is usually poor. At times, data could not be read from the plots (of $\ln d$ vs t), to indicate these cases a hyphen (-) is used in Table 1.

We use the microwave interferometer data to determine the peak plasma density n_c as a function of time. These data, which are more accurate, give the average line density over a cross section; the center of which passed through $r = 0$. Because XANTENAL can only perform density scans for $r_p = \text{const.}$ it is necessary to select one or two particular values of r_p for which to make the comparison. To determine the n_c vs t plot (from Eq. (15) and MIS

Table 1. Beam attenuation detector (BAD) and related data.

Definition of terms:

- t = time from start pulse (ms).
 r_c = minimum radius of BAD neutral beam chord (cm). Note all chords are parallel.
 N = line density from BADs ($\times 10^{13} \text{ cm}^{-2}$).
 a, b, c = coefficients of parabola Eq. (B1.2).
 r_p = Gaussian plasma radius (cm).
 r_o = distance of location of N_{cc} from center $r = 0$ (cm). Note that this is a 1-D projection of r_o , limited by BAD locations.
 N_{cc} = peak line density estimated from the fit ($\times 10^{13} \text{ cm}^{-2}$).

Shot #14 of 7/7/83

$t = 20$		$t = 26$		$t = 40$		$t = 60$	
r_c	N	r_c	N	r_c	N	r_c	N
-19	5.0	-19	5.8	-19	6.8	-19	7.0
-14	2.3	-14	3.6	-14	6.9	-14	8.0
-9.7	3.4	-9.7	4.3	-9.7	6.6	-9.7	6.2
-4.9	2.2	-4.9	5.0	-4.9	8.1	-4.9	9.5
0	3.2	0	6.0	0	8.6	0	10.2
14.8	0.4	14.8	0.6	14.8	2.2	14.8	3.3
$a = -0.0021819$		$a = -0.0048685$		$a = -0.0024801$		$a = -0.001586$	
$b = -0.071770$		$b = -0.136055$		$b = -0.038757$		$b = -0.026629$	
$c = 30.64998$		$c = 30.64715$		$c = 31.88589$		$c = 32.04612$	
$r_p = 21.4$		$r_p = 14.3$		$r_p = 20.1$		$r_p = 21.5$	
$r_o = +16.4$		$r_o = -14.0$		$r_o = -7.81$		$r_o = -6.17$	
$N_{cc} = 3.69$		$N_{cc} = 5.28$		$N_{cc} = 8.20$		$N_{cc} = 8.98$	

Table 1. (Continued).

Shot #16 of 7/7/83

t = 30		t = 32		t = 40		t = 55	
r_c	N	r_c	N	r_c	N	r_c	N
-19	-	-19	0.5	-19	0.8	-19	0.85
-14	-	-14	-	-14	-	-14	-
-9.7	1.0	-9.7	1.5	-9.7	2.4	-9.7	2.3
-4.9	0.8	-4.9	1.1	-4.9	1.3	-4.9	2.0
0	1.7	0	2.5	0	3.2	0	3.9
14.8	0.7	14.8	0.8	14.8	1.0	14.8	1.15
a = -0.0035390		a = -0.0036434		a = -0.0032811		a = -0.0038076	
b = -0.0106288		b = -3.1059 x 10 ⁻⁵		b = -0.0069260		b = -0.0051921	
c = 30.21900		c = 30.5422		c = 30.78130		c = 31.01873	
$r_p = 16.81$		$r_p = 16.57$		$r_p = 17.46$		$r_p = 16.21$	
$r_o = 1.502$		$r_o = -4.26 \times 10^{-3}$		$r_o = -1.055$		$r_o = -0.682$	
$N_{cc} = 1.34$		$N_{cc} = 1.84$		$N_{cc} = 2.34$		$N_{cc} = 2.97$	

Table 1. (Continued).

Shot #31 of 7/7/83

t = 20		t = 40		t = 53		t = 55	
r_c	N	r_c	N	r_c	N	r_c	N
-19	-	-19	-	-19	-	-19	-
-14	-	-14	-	-14	-	-14	1.8
-9.7	-	-9.7	-	-9.7	-	-9.7	-
-4.9	5.2	-4.9	4.2	-4.9	4.0	-4.9	3.5
0	2.0	0	3.9	0	3.2	0	2.9
14.8	0.95	14.8	2.0	14.8	5.0	14.8	5.0
a = $\left\{ \begin{array}{l} (+) \\ \text{curvature} \end{array} \right.$		a = -0.0015453 [*] b = -0.022572 c = 30.62700		a = $\left\{ \begin{array}{l} (+) \\ \text{curvature} \end{array} \right.$		a = -0.00066113 b = 0.033653 c = 31.16906	
No data		$r_p = 25.4$ $r_o = -7.30$ $N_{cc} = 2.17$		No data		$r_p = 38.9$ $r_o = 25.5$ $N_{cc} = 5.28$	

* A least squares fit cannot be done because there are only three data points. For this case a,b,c, are found from Eq. (B2.1).

Table 1. (Continued).

Shot #34 of 7/7/83

t = 25		t = 42		t = 45		t = 65	
r_c	N	r_c	N	r_c	N	r_c	N
-19	5.4	-19	6.8	-19	7.4	-19	9.0
-14	4.0	-14	7.2	-14	7.8	-14	8.3
-9.7	5.7	-9.7	9.0	-9.7	9.7	-9.7	13.0
-4.9	4.3	-4.9	9.0	-4.9	9.0	-4.9	11.0
0	4.3	0	11.5	0	11.0	0	14.0
14.8	2.4	14.8	4.0	14.8	4.4	14.8	6.8
a = -0.00077952		a = -0.0023305		a = -0.0019850		a = -0.0017672	
b = -0.024386		b = -0.021319		b = -0.020741		b = -0.011801	
c = 31.35069		c = 32.1836		c = 32.18359		c = 32.43817	
$r_p = 35.8$		$r_p = 20.7$		$r_p = 22.4$		$r_p = 23.8$	
$r_o = -15.6$		$r_o = -4.57$		$r_o = -5.22$		$r_o = -3.34$	
$N_{cc} = 4.99$		$N_{cc} = 9.96$		$N_{cc} = 10.02$		$N_{cc} = 12.5$	

Table 1. (Continued).

Shot #36 of 7/7/83

t = 25		t = 37		t = 45		t = 65	
r_c	N	r_c	N	r_c	N	r_c	N
-19	4.4	-19	5.4	-19	8.1	-19	8.8
-14	1.7	-14	3.0	-14	5.7	-14	8.0
-9.7	5.4	-9.7	8.9	-9.7	14.0	-9.7	15.0
-4.9	2.0	-4.9	6.3	-4.9	8.0	-4.9	9.3
0	4.1	0	8.9	0	11.0	0	15.0
14.8	2.1	14.8	3.1	14.8	3.0	14.8	4.7
a = -0.00034129		a = -0.0024613		a = -0.0026463		a = -0.0026471	
b = -0.013169		b = -0.016351		b = -0.034138		b = -0.028804	
c = 31.00337		c = 31.8992		c = 32.14924		c = 32.38936	
$r_p = 54.1$		$r_p = 20.2$		$r_p = 19.4$		$r_p = 19.4$	
$r_o = -19.2$		$r_o = -3.32$		$r_o = -6.45$		$r_o = -5.44$	
$N_{cc} = 3.31$		$N_{cc} = 7.34$		$N_{cc} = 10.23$		$N_{cc} = 11.4$	

Table 1. (Continued).

Shot #10 of 7/8/83

t = 30		t = 35		t = 40		t = 50	
r_c	N	r_c	N	r_c	N	r_c	N
-19	1.4	-19	1.0	-19	0.9	-19	-
-14	-	-14	-	-14	-	-14	-
-9.7	0.8	-9.7	1.0	-9.7	1.7	-9.7	1.2
-4.9	2.6	-4.9	2.0	-4.9	3.0	-4.9	2.5
0	1.2	0	1.6	0	2.3	0	3.0
14.8	-	14.8	-	14.8	-	14.8	-
a = +0.0007657		a = +0.00038		a = -0.0035065		a = -0.011928*	
(+) curvature		(+) curvature		b = -0.011568		b = -0.021239	
				c = 30.84556		c = 31.03222	
No data		No data		$r_p = 16.9$		$r_p = 9.52$	
				$r_o = -1.65$		$r_o = -0.890$	
				$N_{cc} = 2.51$		$N_{cc} = 3.03$	

* A least squares fit cannot be done since there are only three data points. For this case a,b,c, are found from Eq. (B2.1).

data}, we take ΔR_A vs t data and eliminate time between them (by simple matching) to obtain the ΔR_A vs n_c plot. An example plot of the MIS data and the completely reduced experimental data ΔZ_A vs n_c (both for Shot #16 of 7/7/83) are shown respectively in Figs. 5 and 10. In Fig. 10, the dashed line is reactance.

COMPARISON OF EXPERIMENTAL DATA WITH XANTENA1

SETUP FOR XANTENA1 RUNS

XANTENA1*, written by Brian McVey at TRW (Ref. 2), uses a method similar to that discussed in Chapter 5 of Stix³ to determine the low-frequency electric fields in a plasma cylinder. The electric field $\vec{E}$ for a particular antenna configuration is found using a Fourier integral, with respect to $k_z (=2\pi/\lambda_z)$, of an analytic expression for $\vec{E}$ for a periodic current distribution. The current density $\vec{j}$ is found from $\vec{E}$ and the plasma conductivity. The rf magnetic field $\vec{B}$ is found using one of Maxwell's curl equations (see Ref. 2 for further details).

Figure 11a is an example of an input file for XANTENA1. A full explanation of all the parameters is given in Ref. 4. The underlined parameters in Fig. 11a are the only parameters that are varied from run to run in this investigation.[†] The independent variable indvar defines the parameter to be varied (e.g., $\vec{E}$ vs indvar). Indvar is the peak density n_c in this investigation. The scanning of indvar is determined by ivarlow

*We ran a W. F. Cummins modified version (XANTEN11) of XANTENA1 for all shots examined in this report. The only difference between the two codes is that XANTEN11 takes into account radial variations in the collision frequency, whereas XANTENA1 does not.

[†]Parameters that appear to need changes, such as colfreq (the collision frequency) and wnor (ω/ω_{c1}), are actually ignored by XANTENA1 and calculated internally; they are remnants of earlier versions of the code.

Figure 10. The ΔZ_A vs n_c plot for shot #16 on 7/7/83.

adt0716/n1 20 lines

```

1  indvar=6  ivarlow=.125e12  ivarhi=1.50e12  ivarinc=.125e12  iplot=1  ifield=3
2  ipart=1  ispectr=0  integkz=2
3  kmincm=1.e-5  kmaxcm=10.  kstepcm=1.e-7  eps=1.e-4  m=10  nptskz=20
4  naxial=100  nazim=5
5  $
6  acm=34.1  ccm=52.5  lengcm=600.  rhocm=34.  phideg=0.  zcm=0.0
7  wnor=.840  freq=2.48e6  kzocm=.01
8  ncoils=2
9  icoil= 6 6
10 cur= (100.,0.) (-100.,0.)
11 bcm= 38. 48.
12 widcm= 15. 15.
13 angdeg=90. 90.
14 $
15 bfld=3874.  ntot=.782e12  npart=2
16 mass=2. 5.46e-4  charge= 1 -1  conc= 100 100  tpar=20. 30.  tper=20. 30.
17 colfreq= 1.483e4 3.81e5
18 iprof=3  nstrat=20  wdenem=16.8  wtempcm=28. 28.
19 $
20 $

```

Figure 11a. Example input file for XANTEN1 for shot #16 of 7/7/83.

Figure 11b. Model of antenna configuration used in XANTEN1.

(initial value of indvar), invarinc (the step increment), and ivarhi (maximum value of indvar). The antenna excitation frequency is freq; the Gaussian plasma radius, determined from the values in Table 1, is wdenom.

The program XANTENAI does not model azimuthal variations of the input parameters except for the antenna configuration. The antenna configuration is modeled by a superposition of two uniform partial-turn current sheets as shown in Fig. 11b. The value of antenna current is modeled at 100 A. The vacuum chamber radius ccm and the limiting plasma radius acm (the radius at which the plasma density drops abruptly to zero) are modeled as shown in Fig. 1a.

Another shortcoming of XANTENAI is that it cannot be used to model axial variations in the dc magnetic field. Thus, $\vec{B} = B_0 \hat{z}$ is modeled as constant in z of the magnitude of the actual magnetic field at $r = 0$ and $z = 130$ cm, the nominal position of the antenna. The value (3.874 kG) is obtained from a general magnetic field code. The code's input currents are set to the conditions of BB-3, the magnetic field configuration, which is used for the examined shots.

The same temperature profile is used for all runs. A crude estimate of the actual ion and electron temperatures for a few of the shots is determined from Langmuir probe data. Ballpark estimates of ion and electron temperatures are $T_{ic} = 20$ eV and $T_{ec} = 30$ eV. No real estimate of the temperature profile can be made, therefore, 28.0 cm is chosen as reasonable when compared with the density radii. There was not enough available computer time, in the limited time the author had to complete this project, to study the sensitivity of the results on temperature--a critical, unanswered question.

Another major deficiency in XANTENAI is that the radial variations in the input parameters n , T_i , and T_e are not smooth, but are modeled as stepwise-continuous, i.e., an onion-skin model is used. This is because the code is a simple extension of a previous version of the code that modeled an isotropic cylindrical plasma. We chose 20 as the number of stratifications (equally spaced) for all runs investigated here. From our previous use of the code, we determined that any number over 10 produces consistent results.

OUTPUT OF XANTENAI RUNS

The output of XANTENAI consists of $\vec{E}$, $\vec{B}$, and some power calculations as a function of n_c at a particular radius, chosen as 34.0 cm (parameter rhocm in Fig. 11a), which is 1 mm inside the radius where the plasma density drops to zero. The power calculations are of concern in this investigation. These consist of the radial power flow (W) and the power absorbed by each respective species (W/cm²). The radial power flow represents the power flowing into a cylinder of radius rhocm = 34.0 cm. (XANTENAI finds it by integrating the Poynting vector over the surface of a cylinder of radius rhocm.) The radial power flow thus represents the total power radiated by the modeled antenna P_{AM} . (The vacuum chamber wall is assumed a perfect conductor.) Figure 12 shows an example of P_{AM} vs n_c data determined by XANTENAI for Shot #16 of 7/7/83. XANTENAI outputs only real power calculations, the reactive component is not found. Thus, only the resistive component of the antenna loading impedance can be determined, using the following.

$$R_A(\text{ohms}) = .0072 P_{AM}(\text{watts}), \quad (16)$$

where

P_{AM} = power radiated by model antenna ,

R_A = loading resistance of real antenna .

Equation (16) is derived from the ac power law and from the current of the model antenna in XANTENAI as shown in Part 2 of Appendix C. We can use Eq. (16) to transform the data from XANTENAI (Fig. 12) into data that can be compared with the reduced experimental data.

Before plotting the superimposed experimental and predicted R_A vs n_c data, we divide the MIS line-density data into three distinct regions: startup, mid-shot, and final decay, so that these regions can be distinguished from each other in the final plot. The reason for this division of the data is that operating conditions are different for each region during the shot; conditions are fairly stable only during mid-shot. Figures 13-18a are the MIS line-density plots for all shots examined. The two markers that appear on each plot represent the data point that is included in the next region. For

Figure 12. The $P(W)$ vs n_c plot for shot #16 on 7/7/83.

example, mid-shot occurs from the first marker on the plot up to, but not including, the second. Figures 13-18b are also the superposed plots of the experimental ΔR_A vs n_c and R_A vs n_c plots predicted by XANTENAL. The experimental data is shown as discrete data points: 0's represent startup, 1's mid-shot, and 2's final decay. The reduced XANTENAL data is shown as a solid line.

Also depicted on some of the plots are isolated data points that represent XANTENAL predictions for other input conditions. The conditions under which these runs were made are indicated on each figure. These data points were obtained from radial scans (indvar = r), at the point where r matched that (rhocm) of the density scan data. Thus, only one data point was obtained per radial run. These runs were made so that the investigator could examine the radial power flow curves to see effects such as halo heating. The points are included on the plots to indicate the sensitivity of the results to variations in plasma parameters.

DISCUSSION

From the final data comparisons in Figs. 13-18, we conclude that the predictions of XANTENAL are very good considering the model used by XANTENAL.

We observe that the matching is better for $\omega/\omega_{ci} < 1$ than otherwise. Note that the slow wave is dominant for $\omega/\omega_{ci} < 1$ and the fast wave for $\omega/\omega_{ci} > 1$. In Figs. 13-15, the matching is very good throughout the shot. From the four data points using radial XANTENAL runs (which represent a sensitivity analysis), it appears that the deviations of experimental data from the predictions can be attributed to fluctuations in r_p and temperature. In three of the radial runs the temperature profile is changed. The fluctuations are large considering that the center temperatures are held fixed. As mentioned earlier, a major unanswered question in this investigation is the sensitivity of XANTENAL's predictions to the values of temperature. As seen from the BAD data in Table 1 for Shot #14, the plasma is quite unstable at the beginning of the shot (note r_o). We should emphasize that XANTENAL does not model azimuthal variations. In addition, note the fluctuating

Figure 13a. The $fnd1$ vs t plot for shot #14 on 1/7/83.

Figure 13b. The ΔR_A vs n_c plot for shot #14 on 7/7/83.

Isolated data points are:

- + : $r_p = 20.0$ cm;
- ⊕ : $r_p = 15.0$ cm, r_{temp} (ions and electrons) = 20.0 cm;
- x : $r_p = 15.0$ cm, r_{temp} (ions and electrons) = 28.0 cm;
- ⊗ : $r_p = 15.0$ cm, r_{temp} (ions and electrons) = 200.0 cm.

Note: all other parameters are the same as the density scan.

Figure 14a. The $fnd1$ vs t plot for shot #16 on 7/7/83.

Figure 14b. The ΔR_A vs n_c plot for shot #16 on 7/7/83.

Isolated data point is + : $r_p = 16.6$ cm.

Note: all other parameters are the same as the density scan.

Figure 15a. The J_{nd1} vs t plot for shot #31 on 7/7/83.

Figure 15b. The ΔR_A vs n_c plot for shot #31 on 7/7/83.

Figure 16a. The $fnd1$ vs t plot for shot #34 on 7/7/83.

Figure 16b. The ΔR_A vs n_c plot--run #1 for shot #34 on 7/7/83.

Isolated data point is +: $r_p = 20.7$ cm.

Note: all other parameters are the same as the density scan.

Figure 16c. The ΔR_A vs n_c plot--run #2 for shot #34 on 7/7/83.

Isolated data point is +: $r_p = 20.7 \text{ cm}$.

Note: all other parameters are the same as the density scan.

Figure 17a. The $\int nd1$ vs t plot for shot #36 on 7/7/83.

Figure 17b. The ΔR_A vs n_c plot--run #1 for shot #36 on 7/7/83.

Isolated data point is +: $r_p = 20.2$ cm.

Note: all other parameters are the same as the density scan.

Figure 17c. The ΔR_A vs n_c plot--run #2 for shot #36 on 7/7/93.

Figure 18a. The snd1 vs t plot for shot #10 on 7/7/83.
 Note: plenum pressure was increased to reach higher density in this shot.

Figure 18b. The ΔR_A vs n_c plot for shot #10 on 7/7/83.

behavior of the predicted ΔR_A . We speculate that this is due to the numerics of the code, and is not a real phenomenon.

Figures 16 and 17 are very close to $\omega/\omega_{ci} = 1$, which is a critical ICRH point. XANTENAL agrees well with the experiment for densities above $n = 10^{12} \text{ cm}^{-3}$, but does not agree at lower densities. We made the second run (Fig. 16c) to determine whether or not predictions would match better at lower densities if a larger r_p was chosen; the BAD data indicates a larger r_p earlier in time (see Table 1). The predictions do match better at lower densities, however, some other effect is dominant in the code below $n = 1 \times 10^{11} \text{ cm}^{-3}$, giving a predicted loading resistance that is too large.

Figure 17 is just above $\omega/\omega_{ci} = 1$. Note that now an effect is present that seems to predict ΔR_A too small at low densities, but again the agreement is good above $n = 10^{12} \text{ cm}^{-3}$. A second run was made to determine whether predictions would match better at lower densities if a larger r_p was chosen; the BAD data indicates a larger r_p earlier in time just like the above case. In this instance, predictions show no change.

Figure 18 shows a poor match. The prediction of XANTENAL is consistent with its previous predictions with $\omega/\omega_{ci} > 1$. The experimental data also seem to follow a pattern of increasing more rapidly at lower densities (as frequency increases). The predictions and experiment seem to be diverging in this case.

The status of other systems appear sufficiently constant from shot to shot that the effects of these systems do not explain any discrepancies. Note that the fluctuation pattern in the predicted data stops for $\omega/\omega_{ci} > 1$. This may result from the stratification model.

One of the major modeling errors in XANTENAL is its constant axial dc magnetic field.* This is illustrated by the sketch in Fig. 19 of the points

*XANTENAL accounts for radial variations in the magnetic field by calculating

$$B(r) = \frac{n(r)kT(r)}{\frac{B^2}{2\mu_0}},$$

while adjusting $B(r)$ accordingly. For these runs B is very small, so that $B(r)$ is essentially constant.

Figure 19. Locations of axial $\omega/\omega_{ci} = 1$ points relative to ICRH antenna.

along the z-axis at which $\omega/\omega_{ci} = 1$ because of the axial variation in B. The method for finding these points is outlined in Appendix C, Part 3.

From Fig. 19, we observe that it is remarkable that XANTENAI's predictions were so close. The $\omega/\omega_{ci} = 1$ points represent ion-cyclotron resonance points. Reflections and absorptions of waves occur abruptly in the vicinity of these points; these are effects not modeled by XANTENAI. The exact locations of these points may be slightly different or may even vary during a shot because of fluctuations in the dc magnetic field. This major assumption of the constancy of $B(z)$ is likely the dominant reason for the discrepancies. For shots 14, 16, 31, 34, the slow wave is dominant because $\omega/\omega_{ci} < 1$ at the antenna. In addition, the $\omega/\omega_{ci} = 1$ points depicted in Fig. 19 represents locations of resonant absorption of this wave. The code XANTENAI's predictions for these cases are based on the excitation of this slow wave and its propagation and eventual dissipation as $z \rightarrow \pm \infty$. The code predicts perfect absorption of the slow wave if any dissipative mechanism is present. Thus, if the code accurately models the excitation of the wave near the antenna, one would expect good agreement between XANTENAI and the experiment for these cases because the wave is absorbed by some means in both cases. In fact, this is where best agreement occurred. For shots 36 and 10, on the other hand, the fast wave is dominant (the slow wave cannot exist for $\omega/\omega_{ci} > 1$) and the $\omega/\omega_{ci} = 1$ points depicted in Fig. 19 represent locations of at most, mode conversion. Theoretically the fast wave is not affected by the $\omega/\omega_{ci} = 1$ points [because it is right hand circularly polarized (RHP)], but mode conversion into a slow wave at this point has been observed.⁵ The code XANTENAI certainly cannot model this phenomenon. XANTENAI models the plasma with the Boltzmann equation using the particle conserving Krook collision model, therefore, all waves that it predicts are lumped together. The electrostatic ion-cyclotron waves are also present, but contribute little (mostly to halo heating). Because the attenuation coefficients vary rapidly with frequency for these waves (see p. 232 of Stix³), XANTENAI's predictions for these waves are poor due to its constant $\vec{B}$ field. Because of XANTENAI's reliance on its predictions of purely fast and electrostatic ion-cyclotron waves vs the real situation of rapidly varying attenuation coefficients for the electrostatic waves (thus affecting the rf fields) and possible mode

conversion at the resonance point, we observe poorer agreement for those cases, which is expected. This is meant to be a preliminary discussion of a very important matter that merits further study.

Two effects, which have not been included in this work, are the attenuation of the rf magnetic field by the Faraday shield and the uncertainties in the antenna resistance. The Faraday shield has been measured to attenuate the rf magnetic field by a factor of 0.68. The loading resistance, inferred from McVey's code, should then be reduced by the square of the attenuation, a factor of 0.46. The antenna resistance, at high frequencies, varies inversely with the square root of the frequency. In addition, the antenna resistance is uncertain, probably too high by as much as a factor of two; because the techniques used for measuring it at the time of this work were later found to be incorrect. As shown in Fig. 8, the experimental loading resistance and reactance are proportional to the assumed antenna resistance. Therefore, the measured loading resistance will probably be reduced by a factor of up to two. If both the predicted and measured resistance are reduced by these changes, the agreement between the code predictions and the measurements may remain intact.

CONCLUSIONS

The most significant result of this work is our finding that McVey's Code XANTENAL correctly predicts the scaling of the loading resistance with the density for data in which ω/ω_{ci} is less than one at the antenna. The agreement in the magnitude of the loading resistance may be fortuitous, as discussed above. For data with ω/ω_{ci} comparable to or slightly greater than one, the predicted scaling of the loading resistance with the density did not agree with experiment; however at the higher densities above $1 \times 10^{12} \text{ cm}^{-3}$, the predicted magnitude is near the experimental value. We note that the code predictions change rapidly near the cyclotron frequency, so we believe that the errors are primarily because the code does not include the axial variations of the magnetic field.

Finally, during this study, we developed the techniques for determining the plasma loading impedance from network analyzer data and wrote a convenient BASIC plot-package. Both of these are implemented using the HP9816 desktop computer.

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APPENDIX A

PART 1

Smith Chart Transformations

u = normalized horizontal displacement

v = normalized vertical displacement

r = normalized resistance

x = normalized reactance

Transformation equations

$$r = \frac{1 - (u^2 + v^2)}{(1 - u)^2 + v^2} \quad x = \frac{2v}{(1 - u)^2 + v^2} \quad (A1.1)$$

Inverse transform (note $r \rightarrow -u$, $x \rightarrow v$)

$$u = \frac{r^2 + x^2 - 1}{(1 + r)^2 + x^2} \quad v = \frac{2x}{(1 + r)^2 + x^2} \quad (A1.2)$$

If Z_0 = characteristic impedance of line,

$$R = Z_0 r \quad X = Z_0 x, \quad (A1.3)$$

where

$Z = R + jX$ = Impedance looking down the line.

PART 2

Z_A to Z_{in} Transformation

From Fig. 3, we write the KVL equations and solve for Z_{in} as a function of Z_A , assuming that C_1 , C_2 , C_3 are known:

$$V_2 + V_3 = V_1,$$

$$-V_3 + V_4 + V_5 = 0,$$

$$\left(\frac{1}{j\omega C_3}\right) I_1 + \left(\frac{1}{j\omega C_1}\right) (I_1 - I_2) = V_1,$$

$$\left(\frac{1}{j\omega C_1}\right) (-I_1 + I_2) + Z_A I_2 + \frac{1}{j\omega C_2} I_2 = 0,$$

$$\begin{bmatrix} \frac{1}{j\omega C_3} + \frac{1}{j\omega C_1} & -\frac{1}{j\omega C_1} \\ -\frac{1}{j\omega C_1} & \frac{1}{j\omega C_1} + Z_A + \frac{1}{j\omega C_2} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ 0 \end{bmatrix},$$

$$\underline{A} \underline{I} = \underline{V} ,$$

$$\underline{I} = \underline{A}^{-1} \underline{V} ,$$

$$\underline{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} ,$$

$$\underline{A}^{-1} = \frac{1}{|\underline{A}|} \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix} ,$$

$$I_1 = \frac{a_{22}}{|\underline{A}|} V_1 ,$$

$$Z_{in} = \frac{V_1}{I_1} = \frac{|\underline{A}|}{a_{22}} = a_{11} - \frac{a_{12} a_{21}}{a_{22}} ,$$

$$a_{11} = \frac{-j}{\omega} \left(\frac{1}{C_1} + \frac{1}{C_3} \right) ,$$

$$a_{12} = a_{21} = j \frac{1}{\omega C_1} ,$$

$$a_{22} = Z_A - j \frac{1}{\omega} \left(\frac{1}{C_1} + \frac{1}{C_2} \right) ,$$

let

$$Z_A = R_A + jX_A ,$$

then

$$a_{22} = R_A + j \left[X_A - \frac{1}{\omega} \left(\frac{1}{C_1} + \frac{1}{C_2} \right) \right] ,$$

let

$$a_{22} = R + jX ,$$

then

$$Z_{in} = a_{11} - \frac{a_{12} a_{21}}{R + jX} = a_{11} - \frac{a_{12} a_{21}}{R^2 + X^2} (R - jX) ,$$

$$= -j \frac{1}{\omega} \left(\frac{1}{C_1} + \frac{1}{C_3} \right) - \frac{R - jX}{(-\omega^2 C_1^2)(R^2 + X^2)} ,$$

$$Z_{in} = \frac{R}{\omega^2 C_1^2 (R^2 + X^2)} - j \left[\frac{X}{\omega^2 C_1^2 (R^2 + X^2)} + \frac{1}{\omega} \left(\frac{1}{C_1} + \frac{1}{C_3} \right) \right] ,$$

$$\text{where } R = R_A \text{ and } X = X_A - \frac{1}{\omega} \left(\frac{1}{C_1} + \frac{1}{C_2} \right) ,$$

let

$$Z_{in} = R_{in} + jX_{in} ,$$

then

$$R_{in} = \frac{R_A}{\omega^2 C_1^2 \left\{ R_A^2 + \left[X_A - \frac{1}{\omega} \left(\frac{1}{C_1} + \frac{1}{C_2} \right) \right]^2 \right\}} \quad (A2.1a)$$

$$X_{in} = \frac{-X_A + \frac{1}{\omega} \left(\frac{1}{C_1} + \frac{1}{C_2} \right)}{\omega^2 C_1^2 \left\{ R_A^2 + \left[X_A - \frac{1}{\omega} \left(\frac{1}{C_1} + \frac{1}{C_2} \right) \right]^2 \right\}} - \frac{1}{\omega} \left(\frac{1}{C_1} + \frac{1}{C_3} \right) . \quad (A2.1b)$$

PART 3

Z_{in} to Z_A Transformation

From Fig. 3 we write the KVL equations and solve for Z_A as a function of Z_{in} , assuming C_1, C_2, C_3 are known:

$$-V_1 + V_2 + V_3 = 0$$

$$-V_3 + V_5 = -V_4$$

$$-Z_{in}I_1 + \frac{1}{j\omega C_3} I_1 + \frac{1}{j\omega C_1} (I_1 - I_2) = 0 ,$$

$$\frac{1}{j\omega C_1} (I_2 - I_1) + \frac{1}{j\omega C_2} I_2 = -V_4 ,$$

$$\begin{bmatrix} -Z_{in} + \frac{1}{j\omega} \left(\frac{1}{C_1} + \frac{1}{C_3} \right) & -\frac{1}{j\omega C_1} \\ -\frac{1}{j\omega C_1} & \frac{1}{j\omega} \left(\frac{1}{C_1} + \frac{1}{C_2} \right) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -V_4 \end{bmatrix} ,$$

$$\underline{B} \underline{I} = \underline{V} ,$$

$$\underline{I} = \underline{B}^{-1} \underline{V} ,$$

$$\underline{B} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} ,$$

$$\underline{B}^{-1} = \frac{1}{|\underline{B}|} \begin{bmatrix} b_{22} & -b_{21} \\ -b_{12} & b_{11} \end{bmatrix} ,$$

$$I_2 = \frac{b_{11}}{|\underline{B}|} (-V_4) ,$$

$$Z_A = \frac{V_4}{I_2} = \frac{-|\underline{B}|}{b_{11}} = -b_{22} + \frac{b_{12} b_{21}}{b_{11}} ,$$

$$b_{11} = -Z_{in} + \frac{1}{j\omega} \left(\frac{1}{C_1} + \frac{1}{C_3} \right) ,$$

let

$$Z_{in} = R_{in} + j X_{in} ,$$

then

$$b_{11} = -R_{in} - j \left[X_{in} + \frac{1}{\omega} \left(\frac{1}{C_1} + \frac{1}{C_3} \right) \right] ,$$

$$b_{12} = b_{21} = j \frac{1}{\omega C_1} ,$$

$$b_{22} = -j \frac{1}{\omega} \left(\frac{1}{C_1} + \frac{1}{C_2} \right) ,$$

let

$$b_{11} = R + jX ,$$

then

$$\begin{aligned} Z_A &= j \frac{1}{\omega} \left(\frac{1}{C_1} + \frac{1}{C_2} \right) - \frac{1}{\omega^2 C_1^2} \frac{1}{R + jX} \\ &= j \frac{1}{\omega} \left(\frac{1}{C_1} + \frac{1}{C_2} \right) - \frac{R - jX}{\omega^2 C_1^2 (R^2 + X^2)} , \end{aligned}$$

$$Z_A = \frac{-R}{\omega^2 C_1^2 (R^2 + X^2)} + j \left[\frac{X}{\omega^2 C_1^2 (R^2 + X^2)} + \frac{1}{\omega} \left(\frac{1}{C_1} + \frac{1}{C_2} \right) \right] ,$$

let

$$Z_A = R_A + j X_A ,$$

then

$$R_A = \frac{R_{in}}{\omega^2 C_1^2 \{ R_{in}^2 + \left[X_{in} + \frac{1}{\omega} \left(\frac{1}{C_1} + \frac{1}{C_3} \right) \right]^2 \}} , \quad (A3.1a)$$

$$X_A = \frac{-X_{in} - \frac{1}{\omega} \left(\frac{1}{C_1} + \frac{1}{C_3} \right)}{\omega^2 C_1^2 \left\{ R_{in}^2 + \left[X_{in} + \frac{1}{\omega} \left(\frac{1}{C_1} + \frac{1}{C_3} \right) \right]^2 \right\}} + \frac{1}{\omega} \left(\frac{1}{C_1} + \frac{1}{C_2} \right) \quad (A3.1b)$$

Note the simple differences in Eqs. (A2.1) and (A3.1):

$$R_{in} \leftrightarrow R_A \quad X_{in} \leftrightarrow X_A \quad \frac{1}{\omega} \left(\frac{1}{C_1} + \frac{1}{C_2} \right) \leftrightarrow -\frac{1}{\omega} \left(\frac{1}{C_1} + \frac{1}{C_3} \right)$$

PART 4

Derivation of Capacitor Sensitivity Equations

Taking the vacuum case of Eq. (A2.1a) and assuming

$$C_1 = C(1 + \delta) \quad C_2 = C(1 - \delta) \quad (A4.1)$$

we can solve for C:

$$R_{inv} = \frac{R_{Av}}{\omega^2 C^2 (1 + \delta)^2 \left\{ R_{Av}^2 + \left[X_{Av} - \frac{1}{\omega C} \left(\frac{1}{1 + \delta} + \frac{1}{1 - \delta} \right) \right]^2 \right\}}$$

$$\left\{ R_{Av}^2 + X_{Av}^2 - 2X_{Av} \frac{1}{\omega C} \left[\frac{2}{(1 + \delta)(1 - \delta)} \right] + \frac{1}{\omega^2 C^2} \left[\frac{2}{(1 + \delta)(1 - \delta)} \right]^2 \right\}$$

$$C^2 \left[\omega^2 (1 + \delta)^2 (R_{Av}^2 + X_{Av}^2) \right] - C \left[4 \omega X_{Av} \left(\frac{1 + \delta}{1 - \delta} \right) \right] + \frac{4}{(1 - \delta)^2} - \frac{R_{Av}}{R_{inv}} = 0$$

$$C = \frac{4 X_{Av} \omega \frac{1 + \delta}{1 - \delta} \pm \left\{ 16 \omega^2 X_{Av}^2 \frac{(1 + \delta)^2}{(1 - \delta)^2} - 4 \omega^2 (1 + \delta)^2 (R_{Av}^2 + X_{Av}^2) \left[\frac{4}{(1 - \delta)^2} - \frac{R_{Av}}{R_{inv}} \right] \right\}^{1/2}}{2 \omega^2 (1 + \delta)^2 (R_{Av}^2 + X_{Av}^2)}$$

$$C = \frac{2 x_{Av} \pm \left[-R_{Av}^2(4) - (R_{Av}^2 + x_{Av}^2) \left(-\frac{R_{Av}}{R_{inv}} \right) (1 - \delta)^2 \right]^{1/2}}{\omega(1 + \delta) (R_{Av}^2 + x_{Av}^2) (1 - \delta)},$$

$$C = \frac{2 x_{Av} \pm \left[\frac{R_{Av}}{R_{inv}} (1 - \delta)^2 (R_{Av}^2 + x_{Av}^2) - 4 R_{Av}^2 \right]^{1/2}}{\omega(1 - \delta^2) (R_{Av}^2 + x_{Av}^2)} \quad (A4.2)$$

Taking the vacuum case of Eqs. (A2.1b) with (A4.2) and (A4.1), we can solve for C_3 :

$$x_{inv} = \frac{-X}{\omega^2 C_1^2 (R_{Av}^2 + x^2)} - \frac{1}{\omega} \left(\frac{1}{C_1} + \frac{1}{C_3} \right),$$

where

$$X = x_{Av} - \frac{1}{\omega} \left(\frac{1}{C_1} + \frac{1}{C_2} \right),$$

$$\frac{1}{\omega C_3} = - \left[\frac{X}{\omega^2 C_1^2 (x^2 + R_{Av}^2)} + \frac{1}{\omega C_1} \right] - x_{inv},$$

$$C_3 = \frac{-1}{\omega \left[x_{inv} + \frac{1}{\omega C_1} + \frac{X}{\omega^2 C_1^2 (x^2 + R_{Av}^2)} \right]}, \quad (A4.3)$$

where

$$X = x_{Av} - \frac{1}{\omega} \left(\frac{1}{C_1} + \frac{1}{C_2} \right).$$

APPENDIX B

PART 1

Method for Finding r_p and r_o

Readily available for this study was an n th-order polynomial least squares fitting routine. For $n = 2$, we obtain a best-fit parabola to a set of data points. We must fit Eq. (14):

$$N = N_{cc} \exp - \left(\frac{r - r_o}{r_p} \right)^2, \quad (B1.1)$$

$$\ln N = \ln A - \frac{r^2}{r_p^2} + \frac{2 r r_o}{r_p^2} - \frac{r_o^2}{r_p^2},$$

$$\ln N = \left(-\frac{1}{r_p^2} \right) r^2 + \left(\frac{2r_o}{r_p^2} \right) r - \left[\ln A - \left(\frac{r_o}{r_p} \right)^2 \right],$$

$$y = a x^2 + b x + c. \quad (B1.2)$$

Thus, if we find the best fit parabola, Eq. (B1.2), where

$y = \ln N$, $N = f_c n d l$, c is a chord,

$x = r$,

then

$$r_p = (-a)^{-1/2} \quad (B1.3a)$$

$$r_o = -b/2a \quad (B1.3b)$$

$$N_{cc} = \exp(c - b^2/4a) = \text{peak line density}. \quad (B1.3c)$$

Note that fitting to $\ln N$ rather than N tends to emphasize (weight) the importance of smaller-value data points. This is not of great importance here because the value of the data points range over only about one order of magnitude (which makes the weighting difference on the order of $\ln 10 = 3$).

Also, remember that the BAD data was noisy.

Next we use Eq. (B1.3) in an example:

We take the BAD data from $t = 20$ ms of Shot #14 of 7/7/83 in Table 1.

In the least squares fitting routine, we enter 6 data points. Their x -values are given in the r_c column, and the y values as $\ln(5.0 \times 10^{13}) = 31.54304$, $\ln(2.3 \times 10^{13}) = 30.76652$, etc. This fitting routine gives the coefficients a , b , c in Table 1. Using Eq. (B1.3), we obtain r_p , r_0 , and N_{cc} .

PART 2

Method for Finding r_p and r_0 for only Three Data Points

When there are only three data points, the least squares fitting routine cannot work; therefore, a parabola is set through the data points by the method shown below. Note that the values of r_p and r_0 obtained by this method are very prone to error and treated as such.

Again using Eq. (B1.2), but now for three particular points p_1 , p_2 , p_3 :

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} x_1^2 & x_1 & 1 \\ x_2^2 & x_2 & 1 \\ x_3^2 & x_3 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix},$$

therefore,

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} x_1^2 & x_1 & 1 \\ x_2^2 & x_2 & 1 \\ x_3^2 & x_3 & 1 \end{bmatrix}^{-1} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}. \quad (B2.1)$$

For the example of shot #31 of 7/7/83 at $t = 40$ ms

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} (-4.9)^2 & -4.9 & 1 \\ 0 & 0 & 1 \\ (14.8)^2 & 14.8 & 1 \end{bmatrix}^{-1} \begin{bmatrix} \ln(4.2 \times 10^{13}) \\ \ln(3.9 \times 10^{13}) \\ \ln(2.0 \times 10^{13}) \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 24.01 & -4.9 & 1 \\ 0 & 0 & 1 \\ 219.04 & 14.8 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 31.369 \\ 31.295 \\ 30.627 \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \frac{1}{1428.64} \begin{bmatrix} -14.8 & 19.7 & -4.9 \\ 219.04 & 195.03 & -24.01 \\ 0 & 1428.64 & 0 \end{bmatrix} \begin{bmatrix} 31.369 \\ 31.295 \\ 30.627 \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} -0.0015453 \\ -0.0225723 \\ 30.627 \end{bmatrix} .$$

APPENDIX C

PART 1

Derivation of Eq. (16)

For any chord through the plasma, we may rewrite Eq. (14) as:

$$n(\vec{r}) = n_c \exp\left(-\frac{|\vec{r} - \vec{r}_0|^2}{r_p^2}\right)$$

let

$$\vec{r} = x\hat{x} + y\hat{y}, \quad \vec{r}_0 = x_0\hat{x} + y_0\hat{y},$$

then

$$|\vec{r} - \vec{r}_0|^2 = (x - x_0)^2 + (y - y_0)^2 = (x - x_0)^2 + (y - y_0)^2.$$

Integrate over the arbitrary chord, the x-axis ($y = 0$),

$$\text{line density} = N \equiv \int_C n(\vec{r}) dl,$$

$$n(\vec{r}) = n_c \exp\left[-\frac{(x - x_0)^2 + y_0^2}{r_p^2}\right] = n_c \exp\left[-\left(\frac{y_0}{r_p}\right)^2\right] \exp\left[-\left(\frac{x - x_0}{r_p}\right)^2\right],$$

but

$$n_{cc} \equiv n_c \exp\left[-\left(\frac{y_0}{r_p}\right)^2\right] = \text{peak density along chord},$$

then

$$N = \int_{-\infty}^{\infty} n_{cc} \exp\left[-\left(\frac{x - x_0}{r_p}\right)^2\right] dx = n_{cc} r_p \sqrt{\pi},$$

thus we obtain Eq. (16),

$$n_{cc} = \frac{\int_C n(\vec{r}) dl}{\sqrt{\pi} r_p} \quad (C1.1)$$

PART 2

Derivation of Eq. (16)

Define:

- P_{AM} = power radiated by model antenna (in XANTEN1),
- I_M = peak current in model antenna,
- R_{AM} = loading resistance of model antenna,
- R_A = loading resistance of real antenna,
- I = peak current in real antenna,

now we can write:

$$P_{AM} = \frac{1}{2} I_M^2 R_{AM} \quad (\text{ac power law}) ,$$

$$R_{AM} = \frac{2 P_{AM}}{I_M^2} ,$$

but the real antenna has six turns*, the modeled one (note that the modeled antenna is a current sheet), therefore,

$$I = \frac{1}{6} I_M = \text{current on each turn of real antenna, producing the same fields as the model.}$$

* Actually the antenna has 5.5, but the extra .5 turn is closely modeled by an image current in the chamber wall because of the unpaired sixth conductor near the plasma.

If we wish to predict R_A from the model,

$$R_A = \frac{2 P_{AM}}{I^2} = \frac{2(36) P_{AM}}{I_M^2}$$

because $I_M \approx 100$ A in every run of XANTENAI:

$$R_A = .0072 P_{am} \quad (C2.1)$$

PART 3

Method for Finding the $\omega/\omega_{ci} = 1$ Points in Fig. 19

The term ω is fixed and B varies. We must find the location where B is such that $\omega/\omega_{ci} = 1$:

$$\omega_A = 2\pi f_A = \omega_{ci} = \frac{eB}{m_i},$$

$$B = 2\pi \frac{m_i}{e} f_A.$$

For deuterium

$$B(\text{kG}) = 1.312 f_A (\text{MHz}). \quad (C3.1)$$

From the values of B calculated from Eq. (C3.1) and the antenna frequencies, we find the z positions by matching these calculated B values with those from the magnetic field computer code mentioned on p. 24. Note that the field varied by 3.7% over the axial length of the ICRH antenna (14 cm). These positions are depicted in Fig. 19.

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