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INFORMAL REPORT

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Heat Transfer in the Vacuum Chamber Wall

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In this note we consider some heat transfer problems associated with the incidence of synchrotron radiation upon the inner wall of the vacuum chamber. We assume power is supplied to the wall at P watts per azimuthal centimeter, so when the ribbon of radiation has a vertical height d , the incident power per unit area is P/d . Our goal is to determine the maximum temperature to be found on the inner surface of the chamber wall, when the wall has thickness L , and the outer surface of the wall is maintained at temperature T_w by water cooling. To proceed we study an idealized two-dimensional potential problem, where the two-dimensional cross-section is taken perpendicular to the electron beam. Rather than considering the actual geometry, we study the rectangular case sketched below.

The shaded region near point B is the heat source due to the synchrotron radiation and is of height d and depth l , so the power per unit volume is $\dot{q} = P/dl$. The wall, which in our case is made of aluminum, has conductivity k and thickness L . The right-hand boundary is fixed at the constant water temperature T_W . If the velocity of the water is sufficiently high, the water flow becomes turbulent. In this case the thermal contact between the water and the aluminum is poor, and the right-hand surface of the aluminum is not an isotherm. We describe the interface between the wall and the water by a film coefficient h defined as follows. Think of adding a very thin layer of a very poor conductor (thickness w , conductivity k') between the aluminum and the water. Now consider taking the limit

$$w \rightarrow 0, k' \rightarrow 0,$$

so as to keep the ratio,

$$k'/w = h, \text{ finite.}$$

The constant $h[\text{watts/cm}^2 - ^\circ\text{C}]$ is called the film coefficient.

To determine the temperature at the point B, we must solve the Poisson equation,

$$\nabla^2 \theta = -\dot{q}/k,$$

where $\theta = T - T_W$, and the conductivity k [watts/cm - $^\circ\text{C}$] is that of aluminum in the wall, and has the value k' in the strip of width w . The Dirichlet condition, $\theta = 0$, is imposed on the upper, lower, and right-hand boundaries while the Neumann condition, $\partial\theta/\partial x = 0$, is imposed on the left-hand boundary. A numerical solution to this problem has been obtained using the two-dimensional magnet program POISSON, which is designed to solve,

$$\nabla^2 A = -0.4\pi \mu J,$$

where A is the vector potential, J the current density, and μ the permeability. The heat transfer and magnetic problems are clearly isomorphic.

As discussed in Appendix B, an analytic solution can be obtained in the limit $H \rightarrow \infty$, $l \rightarrow 0$. One finds

$$\Delta T = T_B - T_W = \frac{P}{\pi k} \left[\ln\left(\frac{6.9L}{d}\right) + f(k/hL) \right] + O(d/L), \quad (1)$$

$$Q = -k \left. \frac{\partial T}{\partial x} \right|_A = \frac{P}{2L} \left[1 - g(k/hL) \right] + O(d/L^2), \quad (2)$$

where the functions $f(\lambda)$ and $g(\lambda)$ are sketched in Figs. 1 and 2, respectively.

These functions both vanish in the absence of turbulence ($h \rightarrow \infty$), i.e.

$f(0) = g(0) = 0$. Our numerical results agree with (1) and (2) to within a few percent.

First, let us consider the numerical calculations in the absence of turbulence ($h = \infty$), so the right-hand boundary of the aluminum is an isotherm. Our calculations were performed with

$$k = 2 \text{ watts/cm} \cdot ^\circ\text{C} \text{ (aluminum at } 100^\circ\text{C)}$$

$$P = 41.4 \text{ watts/cm}$$

$$d = 0.033 \text{ cm}$$

$$L = 0.2, 0.4, 0.8, 1.6 \text{ cm}$$

We chose $\lambda = 10^{-4}$ cm and $H = 16$ cm, so $\lambda \ll d \ll H$. In the table below we give the calculated temperature difference ΔT between the point B and the water, and the heat flux Q at the point A.

L	$\Delta T = T_B - T_W$	$Q = -k(\partial T / \partial x)_A$
0.2 cm	24.5 $^\circ\text{C}$	102 watts/cm ²
0.4	29.0	52
0.8	33.5	26
1.6	38.0	13

Here ΔT has been rounded off to the nearest 0.5°C . These results are described very well by Eqs. (1) and (2) with $h = \infty$.

In Fig. 3, we sketch the temperature distribution ($L = 0.4$ cm) in the wall, giving a few representative isotherms in the x - y plane. In Fig. 4 we plot the temperature gradient dT/dx along the line AB. One should note that although the gradient immediately outside the heat source at $x = 10^{-4}$ cm is large,

$$\left| \frac{dT}{dx} \right| = \frac{P}{kd} = 627^\circ\text{C/cm},$$

it falls off very rapidly.

Now let us consider the effect of turbulence by considering a non-infinite film coefficient. As a good approximation to a film coefficient,

$$h = 1.5 \text{ watts/cm}^2 - ^\circ\text{C},$$

we choose

$$w = 10^{-4} \text{ cm},$$

$$k' = 1.5 \times 10^{-4} \text{ watts/cm} - ^\circ\text{C}.$$

Some results of these calculations are given in the table below.

L	$T_B - T_W$	$T_A - T_W$	$Q = -k(\partial T / \partial x)_A$
0.2 cm	43°C	21.5°C	32.4 watts/cm ²
0.4	41	14.5	22.1
0.8	41	9.3	14.4
1.6	42	5.7	8.9

For the case of $L = 0.4$ cm, we have sketched in Fig. 5 the temperature distribution in the aluminum wall, and in Fig. 6 we have plotted the temperature distribution along the vertical boundary between the aluminum and the film.

Acknowledgement

This investigation was suggested by G. K. Green. I wish to thank R. L. Gluckstern for his notes on the analytic solution discussed in Appendix 9, and J. Claus for introducing me to the program POISSON.

APPENDIX A: Cylindrical Geometry

Insight into the approximate magnitude of the results for the rectangular problem can be obtained by considering the cylindrical problem sketched below.

The outer cylinder radius is L , while the inner cylinder radius is chosen as d/π so the half-circumference is d . The Dirichlet condition $\theta = 0$ is specified on the outer cylinder, therefore

$$\theta = -c \ln(r/L).$$

The heat flux through the inner cylinder surface is fixed at q/d , hence

$$\theta = -\frac{P}{\pi k} \ln(r/L).$$

We then find

$$(\Delta T)_{\text{cyl.}} = \left(\frac{P}{\pi k}\right) \ln(\pi L/d) \quad (\text{A1})$$

and

$$Q_{\text{cyl.}} = -k(\partial\theta/\partial r)_{r=L} = \frac{P}{\pi L}. \quad (\text{A2})$$

One should compare Eqs. (A1 - A2) with Eqs. (1 - 2).

APPENDIX B: Rectangular Geometry, Step Function Source

This appendix is an extension of some notes by Dr. R. L. Gluckstern. To solve Poisson's equation in the limit $H \rightarrow \infty$, $L \rightarrow 0$, consider

$$\theta(x,y) = \int_0^{\infty} d\Omega \cos \Omega y A(\Omega) \sinh \Omega(L + w - x) \quad (B1)$$

$$(L \leq x \leq L + w)$$

$$\theta(x,y) = \int_0^{\infty} d\Omega \cos \Omega y [A(\Omega) \cosh \Omega(L - x) \sinh \Omega w + B(\Omega) \sinh \Omega(L - x)] \quad (B2)$$

$$(0 \leq x \leq L)$$

Continuity of $k \frac{\partial \theta}{\partial x}$ at $x = L$ implies

$$B(\Omega) = A(\Omega) \left(\frac{k'}{k}\right) \cos \Omega w \quad (B3)$$

An additional condition is given by

$$\left. \frac{\partial \theta}{\partial x} \right|_{x=0} = \begin{cases} -\frac{p}{kd}, & |y| \leq d/2 \\ 0, & |y| > d/2, \end{cases} \quad (B4)$$

which implies

$$\Omega A(\Omega) \left[\sinh \Omega L \sinh \Omega w + \frac{k'}{k} \cosh \Omega w \cosh \Omega L \right] \quad (B5)$$

$$= \left(\frac{2p}{\pi kd}\right) \frac{\sin(\frac{\Omega d}{2})}{\Omega}.$$

Putting these results together, we obtain Gluckstern's result

$$\theta(x,y) = \frac{2p}{\pi kd} \int_0^{\infty} d\Omega \cos \Omega y \frac{\sin(\Omega d/2)}{\Omega^2} \frac{k \cosh \Omega(L-x) \sinh \Omega w + k' \sinh \Omega(L-x) \cosh \Omega w}{k \sinh \Omega(L-x) \sinh \Omega w + k' \cosh \Omega(L-x) \cosh \Omega w} \quad (B6)$$

$$(0 \leq x \leq L).$$

Now, let us take the limit

$$k' \rightarrow 0, w \rightarrow 0, \text{ with } k'/w = h.$$

In this limit, we find

$$\Delta T = \vartheta(a, a) = \frac{2P}{\pi k d} \int_0^{\infty} d\Omega \frac{\sin(\Omega d/2)}{\Omega^2} \frac{k\Omega \cosh \Omega L + h \sinh \Omega L}{k\Omega \sinh \Omega L + h \cosh \Omega L}, \quad (87)$$

and

$$Q = -k \left. \frac{\partial \theta}{\partial x} \right|_{x=L} = \frac{2P}{\pi k d} \int_0^{\infty} d\Omega \frac{\sin(\Omega d/2)}{\Omega^2} \frac{kh\Omega}{k\Omega \sinh \Omega L + h \cosh \Omega L}. \quad (88)$$

We shall consider the leading terms in the asymptotic region $d \ll L$. Using the integral representation

$$\ln \alpha = \int_0^{\infty} \frac{dz}{z} (e^{-z} - e^{-\alpha z}), \quad (89)$$

we find

$$\Delta T = \frac{P}{\pi k} \left[\ln\left(\frac{L}{d}\right) + c + f(k/hL) \right] + O\left(\frac{d}{L}\right), \quad (90)$$

where

$$c = \int_0^{\infty} \frac{dz}{z} \left(\frac{\sin z}{z} - 1 + \tanh 2z \right), \quad (91)$$

and

$$f(\lambda) = \int_0^{\infty} \frac{dz}{\cosh^2 z} \frac{\lambda}{1+\lambda z \tanh z}. \quad (92)$$

Numerical evaluation shows $c \approx 1.93$, and we plot $f(\lambda)$ in Fig. 1. Similarly we find

$$Q = \frac{P}{2L} (1 - g(\lambda)) + O\left(\frac{d}{L^2}\right), \quad (93)$$

where

$$g(\lambda) = \frac{2}{\pi} \int_0^{\infty} \frac{dz}{\cosh^2 z} \frac{\lambda z \sinh z}{1+\lambda z \tanh z}. \quad (94)$$

The function $g(\lambda)$ is graphed in Fig. 2. It follows immediately from (94) that as $\lambda \rightarrow \infty$, $g(\lambda) \rightarrow 1$.

APPENDIX C: Rectangular Geometry, General Source Distribution

Instead of the step function of Eq. (B4), let us consider a general distribution defined by

$$\left. \frac{\partial \theta}{\partial x} \right|_{x=0} = - \frac{p}{\pi k} \int_0^{\infty} d\Omega F(\Omega \sigma) \cos \Omega y, \quad (C1)$$

where $F(\Omega)$ is normalized to satisfy

$$F(0) = 1. \quad (C2)$$

One then finds that Eqs. (B7) and (B8) become

$$\Delta T = \frac{p}{\pi k} \int_0^{\infty} \frac{d\Omega}{\Omega} F(\Omega \sigma) \frac{k\Omega \cosh \Omega L + h \sinh \Omega L}{k\Omega \sinh \Omega L + h \cosh \Omega L}, \quad (C3)$$

and

$$Q = \frac{p}{\pi k} \int_0^{\infty} \frac{d\Omega}{\Omega} F(\Omega \sigma) \frac{kh\Omega}{k\Omega \sinh \Omega L + h \cosh \Omega L}. \quad (C4)$$

In the asymptotic region $\sigma \ll L$, one obtains

$$\Delta T = \frac{p}{\pi k} \left[2n\left(\frac{L}{2\sigma}\right) + c' + f(k/hL) \right] + O\left(\frac{\sigma}{L}\right) \quad (C5)$$

where

$$c' = \int_0^{\infty} \frac{dn}{n} [F(n) - 1 + \tanh 2n], \quad (C6)$$

and $f(\lambda)$ is defined in Eq. (B12). For a Gaussian distribution,

$$F(\Omega \sigma) = e^{-\Omega^2 \sigma^2 / 2}, \quad (C7)$$

we find $c' \approx 1.57$ compared with $c \approx 1.93$ for the step function of Appendix B. This shows that for $d = 2\sigma$,

$$(\Delta T)_{\text{step}} - (\Delta T)_{\text{Gaussian}} = 0.36 p/\pi k. \quad (C8)$$

If $p = 41.4$ watts/cm and $k = 2$ watts/cm $^{\circ}\text{C}$, this difference is 2.4°C .

Similarly, from (C4) we find

$$Q = \frac{P}{2L} [1 - g(k/hL)] + O\left(\frac{\sigma}{L^2}\right), \quad (C9)$$

where $g(\lambda)$ is defined in (B14). We note that to the order we are considering, Q is independent of the nature of the distribution $F(\Omega)$.

FIGURE CAPTIONS

Fig. 1 $\theta(\lambda)$ vs λ , where $\theta T = \frac{Q}{\pi k L} \left[\ln\left(\frac{5.2L}{\lambda}\right) + f(k/hL) \right]$.

Fig. 2 $g(\lambda)$ vs λ , where $Q = \frac{P}{2L}(1 - g(k/hL))$.

Fig. 3 Isotherms in the aluminum wall in absence of turbulence ($\tau = \infty$). The vertical line $x = L = 0.4$ is an isotherm.

Fig. 4 Temperature gradient ($-dT/dx$) along the line AB.

Fig. 5 Isotherms in the aluminum wall when there is turbulence, film coefficient $h = 1.5 \text{ watts/cm}^2 \cdot ^\circ\text{C}$.

Fig. 6 Temperature distribution along the vertical interface between the aluminum and water for $h = 1.5 \text{ watts/cm}^2 \cdot ^\circ\text{C}$.

$g(\lambda)$

0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8

Figure 2

$\lambda = 10$

Figure 5

57(°C)

Figure 6

15

10

5

$$L = 0.4 \text{ cm}$$

$$W = 1.0 \text{ cm}$$

$$d = 0.032 \text{ cm}$$

$$S = 10^{-6} \text{ cm}$$

$$10 = 10^{-6} \text{ cm}$$

$$R = 2 \text{ cm} / \text{cm}$$

$$V = 1.5 \text{ cm}^3 / \text{cm}^3$$

0

1

2

3

4

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