SINGLE PASS COLLIDER MEMO

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TITLE: THE POSITRON FLUX CONCENTRATOR - AN UPDATE

HISTORY:

The type of concentrator that has been under study (on and off) for the last two years is shown in Fig. 1. The work was started by Dave Sherden and Co., and the object of the study was (and still is) to determine the correct parameters which will give the correct field in the central region which captures the maximum number of e^+ 's from the target.

A year ago H. Lepotet (who was visiting SLAC from France) and myself took over the work on the concentrator and I continued after Lepotet's departure.

It became clear to me, that the best approach to the problem is to construct and measure the performance of several concentrators so that one can relate the magnetic field obtained to the geometry, then any field shape needed may be easily realized. This procedure would also provide enough measurements allowing us to check any simplified theoretical model which we may develop. Such a model would be very useful in optimizing the design. The measured fields are also used in the transport programs (started by Sherden and Lepotet and continued by H. Lynch and K. Moffeit) to find the field shape and amplitude which give the maximum e^+ capture (see CN - 268 by K. Moffeit).

This note contains two sections. The first is a simplified model which seems to agree with most of the measurements I have carried out. The second describes the set up, a typical example of the field's shape and amplitude and the electrical requirements for the final concentrator.

A. THE TRANSFORMER MODEL:

Since the concentrator works like a transformer with an untuned secondary (The driving coil is the primary and the two halves of

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the concentrator constitute the secondary) I tried to find whether transformer rules can explain the experimental measurements. Following is a list of all the equations used in performing various checks.

Mutual Inductance (M):

1. $M = (\text{Flux linked with the Secondary}) / (\text{Primary current})$.
2. $M (\text{MAX}) = \sqrt{L_p L_s}$ L_p, L_s are the primary and secondary inductance respectively.
3. $M(\text{obtained}) = K \sqrt{L_p L_s}$ K is a coefficient of coupling.

The equations are from Terman- Radio Engineering. There you will also find a general expression for K for various types of coupling, which I cannot reconcile with 3 above.

Primary and Secondary Relations:

4. The presence of a coupled secondary adds to the primary a series impedance $Z_c = (\omega M)^2 / Z_s$; Z_s is the complex impedance of the secondary.

5. The secondary voltage is given by $V_s = -i\omega M I_p$ and

$$I_s = \frac{-i\omega M I_p}{Z_s}$$

Both these equations are also from Terman. Fig. 2 illustrates these two equations and the equivalent circuit of the concentrator.

Inductances:

The effective inductance of the primary is reduced by the presence of the secondary.

$$6. L_p(\text{effective}) = L_p(1 - K^2). \text{ Terman}$$

This equation was used only to calculate K after measuring $L_p, L_p(\text{effective})$

To these relations I will add two more expressions for the inductance of a solenoid with N turns, one is empirical, the other is easy to prove.

This assumes a sinusoidal primary current. The precise expression is: $- M dI_p / dt$

$$7. L(\mu H) = \frac{N^2 R^2}{9R + 10\lambda} \quad (\text{Wheelers rule})$$

Where the radius R and the length λ of the solenoid are in inches. This equation is supposed to take care of end effects and does give the right answer.

$$8. L(H) = \frac{N^2}{R_m} \quad \text{where } R_m, \text{ the magnetic reluctance is}$$

$$\text{given by } R_m = \frac{S}{\mu_0 A}, \quad S \text{ the length of flux path (meters),}$$

A the area transversed by flux (meters²). See Panofsky and Philips for this expression and a warning about its use.

I find that for a solenoid, if one takes S as the length of the solenoid, A the cross-sectional area the calculated inductance agrees with 7. The agreement is perfect if $S = \lambda + R$ (sol. radius). This expression was used in integrals when the area is changing as in the concentrator.

B. EXPERIMENTAL CONCENTRATORS:

J. Grant and myself learned how to construct concentrators of the type shown in fig. 1. Several were made with lengths ranging between 5 and 15 cms, and gradients g from .1 to .35. The number of turns ranged between 5 and 15. Generally a conservative turn separation was chosen between turns which were mostly made from 3/16" refrigeration tubing leading to ≈ 1 turn/cm for all the concentrators. One parameter was held constant namely the diameter D of the small opening at the target end, D was kept at .6 cm with some small variation after construction.

Please note that these concentrators were "somewhat crude" and did not fit together very precisely, so their performance constitutes a lower limit.

All the work was done at low voltage < 500 Volts. The experimental setup is shown in figure 3. The object of the pulsing circuit is to store some of

the energy back into the capacitance. Unfortunately this circuit is usable at low voltage only. The currents and voltages needed for the final concentrator are too high for a practical use of available SCR's (see later).

For most of the concentrators studied the inductance of the conc. coil w/o concentrator was measured in two ways: with a precision bridge and by deduction from the current pulse width and the known capacity used. In the second method the inductance of leads + calibration coil was also measured and subtracted to obtain the concentrator inductance alone.

The effect of the target close to the concentrator input and the accelerator disc close to the output was simulated with copper plates at distances 2 → 3 mm each end.

Some other tests were conducted, for example using insulator thicknesses between coil and conc. 1½ mm to 3mm. We may be able to use thinner insulators depending on voltages used in final setup. The transformer model used here can be used to determine the effect of this thickness on the effective inductance and coupling (see Appendix for sample calculations).

Another check was to shield the coil with a continuous cylinder of cu and with a split cylinder with the two halves insulated. The result of the first was to reduce the max field by 50% when the separation between coil and shield was = 4mm. This result is alarming since the beam pipe constitutes such a shield. The beam pipe must have a large diameter to reduce this loss. The split shield had no effect. These two results can be explained by Lens's Law and give the following rule of thumb:

All eddy currents generated by the magnetic field of conc. coil must be INSIDE THE COIL. Currents in materials outside the coil although would reduce the effective inductance they will also reduce B_z in the cavity in spite of the increase of current (at same voltage)

due to the decrease in inductance.

Results and Model Checks.

1. The coil inductances ranged between 5 and 10 μH without concentrator and between 2.5 and 3.5 μH with concentrator in place.
2. The current pulse width on the average was 20 μs at the base.
3. The effect of the target simulation did not change the field shape but reduced the max field by 10% on the average.
4. The maximum field ranged between 2.5 KG/KA and 4.9 KG/KA depending on gradient g and N .

The concentrator that gave the maximum capture (using K. Moffeit and H. Lynch's transport studies) has the following parameters:

$z = 8 \rightarrow 10 \text{ cms.}$
 $N \text{ (Practical)} = 12 \text{ turns}$
Max field at input = 70 KG

The field shape matched one of the concentrators tested. This gave a max field of 4.18 KG/KA reducing to 3.8 KG/KA due to the target. The insulator thickness between coil and concentrator was 3 mm (very conservative) and we can learn to drop it to .5 mm or less by clever insulation methods. The model indicates that going from 3 to 1 mm reduces the effective inductance by = 30% and improves the mutual inductance by = 15%.

Conservatively, the primary current required is:

$$\frac{70}{3.8} = 19.5 \text{ KA. This is a very conservative upper}$$

limit on current required. Fig. 4 shows the field shape for this concentrator as an example.

An interesting empirical law governing the max field was found. For any set of concentrators with the same parameters except for the gradient g , the max field at input was found to be $\propto \sqrt{g}$. The rule is illustrated (Fig. 5) by a set of 3 identical concentrators except for g .

To check on the transformer model the following calculations were done:

a. Using the definition of M (rule 1) and the equivalent circuit of primary, the reduction of the inductance due to the concentrator was calculated and hence L_{eff} . The agreement with L_{eff} measurement was very good for all concentrators. A sample calculation is shown in Appendix A1.

b. From measurement of L_{eff} and L (coil) of sample concentrator and using rule 6) the value of K was calculated to be $\approx .76$

The model gave a value of .73. If the over estimate of L_2 mentioned previously is allowed for $.73 \pm .02$ (very good agreement).

c. Using rule 3) and $K = .76$, M (TOT) was calculated to be .31 μH . using rule 1) M (TOT) was derived

$$M \text{ (TOT)} = \frac{2 \alpha L_1}{N} \quad \text{where } \alpha \text{ is the ratio of average cross-sectional area of one half of concentrator (the central cavity not included) to the cross-sectional area of coil. For the sample concentrator } \alpha = .31. \text{ This gave } M = .3 \mu H$$

d. Finally, we come to the question of the Eddy current distribution on the surface of the concentrator. The transformer model is used to do that in Appendix A.2. H. Lynch will be given this distribution to find whether it reproduces the field shape. One point, though, troubles me in this check. As it is used by Lynch and was used by Lepotet and myself implies that the field in the cavity is produced by the currents on the inside surface of the plug only. I have not convinced myself one way or the other.

The failure to reproduce the shape of the cavity field which troubled us might not be due to a failure to calculate this current distribution. A related question also is unanswered. Consider the currents on the surface of $\frac{1}{2}$ plug, does this current flow in planes perpendicular to the Z axis? Or is the distribution different on the outside and inside surface. This question is crucial to the transformer model calculation, and to the axial field calculation.

APPENDIX

A.1 Calculation of L (effective)

Using rule 1 (Definition of M) then:

$$M(\text{each half}) = \frac{\alpha L_1}{N} \quad ; \quad L_1 \text{ is the coil's}$$

unmodified inductance, N the no. of turns, and α is the ratio of the average cross-sectional area of one half of the concentrator plug to the cross-sectional area of the coil.

Using rule 4 (also see Fig. 2)

$$\begin{aligned} Z(\text{effective}) &= Z_p + \frac{(\omega M)^2}{Z_s} = r_1 + i\omega L_1 + \frac{\omega^2 \alpha^2 L_1^2}{N^2 (r_s + i\omega L_s)} \\ &= r_1 + i\omega L_1 + \frac{\omega^2 \alpha^2 L_1^2 (r_s - i\omega L_s)}{N^2 (r_s^2 + \omega^2 L_s^2)} \end{aligned}$$

Separating the inductive component and noting that $r_s \ll \omega L_s$ we obtain

$$L(\text{effective}) = L_1 - \frac{\alpha^2 L_1^2}{N^2 L_s}$$

Hence the primary inductance is reduced by the second term due to one half of concentrator, the total reduction is $2\alpha^2 L_1^2 / N^2 L_s$

For the sample concentrator illustrated here:

$$L_1 = 6 \mu\text{H} \text{ (measured and calculated).}$$

$$L_s = .015 \mu\text{H} \text{ (using rule 8 integrated along the length).}$$

$$N = 12$$

$$\alpha = .31 \text{ (Geometry).}$$

Substituting:

$$L(\text{effective}) = 6 - 3.2 = 2.8 \mu\text{H}.$$

$$L(\text{effective}) \text{ from Bridge measurement} = 2.51 \mu\text{H}$$

Note that rule 8 as mentioned over estimates L_2 . The over estimate here amounts to 1.08. If this correction is applied

$$L(\text{effective}) \rightarrow 6 - 3.46 = 2.54 \mu\text{H}$$

Such an agreement was found to be true for several different concentrators.

A.2. RELATION BETWEEN PRIMARY AND SECONDARY CURRENT:

Since the two halves of the concentrator are identical they have equal currents. But each current flows only half a circle around the central cavity. So the magnitude of the total secondary current producing the field is equal to the magnitude of the current in either half.

$$\text{Using } M(\text{one half}) = \frac{\alpha L_p}{N} \quad \text{and rule 5.}$$

$$\begin{aligned} I_s &= \frac{-i\omega M I_p}{r_s + i\omega L_s} \quad \text{usual notation.} \\ &= \frac{-i\alpha\omega L_p}{N(r_s + i\omega L_s)} \times I_p \quad (\alpha \text{ as before}) \end{aligned}$$

Normalizing and separating real and imaginary.

$$I_s = - \frac{\omega L_p \alpha}{N(r_s^2 + \omega^2 L_s^2)} (\omega L_s + i r_s) \times I_p$$

which can be written as:

$$I_s = - \frac{\omega L_p a}{N \sqrt{r_s^2 + \omega^2 L_s^2}} e^{i\theta} \times I_p$$

$$\text{Where } \theta = \tan^{-1} \frac{r_s}{\omega L_s} = \tan^{-1} 1/Q$$

The -ve sign indicates that the secondary current is in the opposite direction from the primary as expected.

The magnitude of the primary current I_p can be obtained by solution of the primary equivalent circuit (see Fig. 2).

The complete solution of the problem can also be obtained by solving the coupled equations of the system. It is tedious to uncouple them. However H. Lepotet solved the equations numerically; B. Tomlin by one method, and myself by solving the differential equation of the primary using its equivalent circuit then solving the secondary coupled differential equation substituting the primary current obtained. All three solutions agreed. The analytic expression I obtained is complicated and will not be given here. If you are interested in the rise and fall of primary and secondary currents and the phases involved, please see me.

A.3. SECONDARY CURRENT DISTRIBUTION:

This is an attempt at using the transformer method to derive the Eddy current distribution on the surface of the concentrator plug. The distribution will be given as a ratio of I_z to I_o , where I_o is the current at the plug end with the minimum size hole. The planes of the currents are assumed perpendicular to the Z axis (see remark before).

Consider two sections perpendicular to the z axis one at the small hole end and one at z . The thickness of each section is small. Now calculate M and I for each section using the same method as for the total secondary current

With section thickness = λ , and all parameters at the small hole denoted by a subscript 1 and at z by a subscript 2.

$$M_1 = \frac{a_1 L_p}{N} ; \quad M_2 = \frac{a_2 L_p}{N}$$

$$I_1 = \frac{-\omega M_1 I_p}{Z_1} ; \quad I_2 = \frac{-\omega M_2 I_p}{Z_2}$$

Z_1, Z_2 are the respective complex impedences.

$$Z_1 = r_1 + i\omega L_1 \quad Z_2 = r_2 + i\omega L_2$$

and a_1, a_2 are as usual

$$a_1 = \frac{A_1}{A_0} , \quad a_2 = \frac{A_2}{A_0} , \quad A_0 \text{ is the coil area.}$$

The differences in the two currents are due to the differences in the areas, inductances, and resistances. We have

$$\frac{I_2}{I_1} = \frac{A_2}{A_1} \cdot \frac{r_1 + i\omega L_1}{r_2 + i\omega L_2} ;$$

r_1, r_2 are the resistances due to skin depth.

$$r_1 = S_1 \rho / 2\delta \quad ; \quad r_2 = S_2 \rho / 2\delta$$

Where S is the circumference of the cross section, ρ the resistivity of the plug material (copper), and δ is the skin depth at the test frequency.

Substituting and using rule 8 for the inductance we obtain:

$$\frac{I_2}{I_1} = \frac{A_2}{A_1} \times \frac{S_1 \rho / \delta + i \omega \mu_0 A_1}{S_2 \rho / \delta + i \omega \mu_0 A_2} \quad (\text{complex ratio})$$

This can be written as

$$\frac{I_2}{I_1} = \frac{A_2}{A_1} \sqrt{\frac{(S_1 \rho / \delta)^2 + \omega^2 \mu_0^2 A_1^2}{(S_2 \rho / \delta)^2 + \omega^2 \mu_0^2 A_2^2}} e^{i\theta_1 - \theta_2}$$

Where:

$$\theta_1 = \tan^{-1} (\omega \mu_0 A_1) / (S_1 \rho / \delta)$$

$$\theta_2 = \tan^{-1} (\omega \mu_0 A_2) / (S_2 \rho / \delta)$$

Note that ω has cancelled, and that there seems to be a phase difference between the two currents which is not surprising.

With the help of the figure here:

$$A_1 = \frac{\pi}{2} (R_o^2 - b_1^2)$$

$$A_z = \frac{\pi}{2} (R_o^2 - b_z^2)$$

$$= \frac{\pi}{2} \{ R_o^2 - (b_1 + gz)^2 \}$$

Typical cross-section

$$S_1 = \pi (R_o + b_1) + 2 (R_o - b_1)$$

$$S_z = \pi (R_o + b_z) + 2 (R_o - b_z)$$

$$= \pi (R_o + b_1 + gZ) + 2 (R_o - b_1 - gZ)$$

Now we collect all the equations in one place and give some relevant values which apply to the sample concentrator

$$\frac{|I_z|}{|I_1|} = \frac{A_z}{A_1} \times \sqrt{\frac{(S_1 \rho / \delta)^2 + \omega^2 \nu_o^2 A_1^2}{(S_z \rho / \delta)^2 + \omega^2 \nu_o^2 A_z^2}} \quad \text{-----I}$$

$$A_1 = \frac{\pi}{2} (R_o^2 - b_1^2), \quad A_z = \frac{\pi}{2} (R_o^2 - b_z^2) \quad \text{-----II}$$

$$\left. \begin{aligned} S_1 &= \pi (R_o + b_1) + 2 (R_o - b_1) \\ S_z &= \pi (R_o + b_z) + 2 (R_o - b_z) \end{aligned} \right\} \quad \text{-----III}$$

$$b_2 = (b_1 + g_2)$$

$$g = .23, \quad b_1 = .33 \text{ cm}$$

}-----IV

$$\omega, f = 157 \times 10^3 \text{ rad}, \quad 50 \times 10^3 \text{ c/s} \text{ -----V}$$

$$\rho_{\text{Copper}} = 1.724 \times 10^{-6} \text{ ohm-cm}$$

$$\delta = \frac{6.62}{\sqrt{f}} \text{ cm}$$

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r_0 = Radius at TARGET END
 R = OUTER RADIUS OF CONCENTRATOR PLUG
 l = LENGTH
 r_z = RADIUS AT Z

$$\left\{ \begin{aligned} g &= \text{GRADIENT} = \tan\left(\frac{1}{2} \text{ CONE ANGLE}\right) \\ &= (R_l - r_0)/l \end{aligned} \right.$$

R_0 = COIL RADIUS.

FIG 1 : SCHEMATIC OF CONCENTRATOR.

← TYPICAL CROSS-SECTION OF $\frac{1}{2}$ CONCENTRATOR PLUG

USING EQUATIONS IN TEXT :

1) EQUIVALENT PRIMARY CIRCUIT IS

$$Z_c = \frac{(\omega M)^2}{Z_s} / \text{HALF}$$

$$Z_s = r_s + i\omega L_s$$

2) SECONDARY CURRENT IS

$$(-i\omega M) / Z_s$$

M is MUTUAL INDUCTANCE TO ONE HALF

FIG 2 : EQUIVALENT CIRCUIT OF CONCENTRATOR.

NOTE THAT MAGNITUDE OF CURRENT IS THE SAME IN EACH HALF BUT EACH HALF SUPPLIES HALF THE CURRENT PRODUCING THE CAVITY FIELD.

FIG. 3 SCHEMATIC OF SETUP :

$$C = 10 \mu f$$

$$\Delta t \approx 40 \mu.s$$

{ CALIB. COIL YIELDS 2.9 KG/KA
ON AXIS AT MID.POINT (CALCULATED)

{ PEARSON TRANSFORMER MEASURES
CURRENT. THE SAME CURRENT ALWAYS
PASSES THROUGH CONC. COIL & CALIB.
COIL

PARAMETERS:

$$l = 10 \text{ cm}, N = 12, g = 0.23$$

$$B_0 \text{ (NO TARGET SIMUL)} = 4.13 \text{ KG/KA}$$

$$B_0 \text{ (WITH " ")} = 3.77 \text{ "}$$

$$L(\text{COIL}) = 5.9 \text{ } \mu\text{H}$$

$$L(\text{EFF.}) = 2.5 \text{ "}$$

$$\text{INSULATOR BETWEEN COIL \& PLUG} = 3 \text{ mm}$$

FIG. 4. SAMPLE CONCENTRATION FIELD SHAPE ON AXIS.

FIG 5_a: $(B_{\text{MAX}})^2$ VS g (GRADIENT)

$B/B(\text{MAX})$
ON Z AXIS

3 CONCENTRATORS A, B, C ; $l = 15 \text{ CMS}$, $N = 15$

-o-	A :	$g = 0.106$	$B(\text{MAX}) = 3.0 \text{ KG/KG}$
-x-	B :	$g = 0.189$	$B(\text{MAX}) = 4.3 \text{ "}$
-•-	C :	$g = 0.233$	$B(\text{MAX}) = 4.9 \text{ "}$

FIG. 5_b