MASTER

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M. Month and G. Parzen

May 3, 1978

ACCELERATOR DEPARTMENT

BROOKHAVEN NATIONAL LABORATORY
ASSOCIATED UNIVERSITIES, INC.

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ABSTRACT

The space charge detuning of a dc beam due to electrons trapped is shown to be essentially energy independent. This leads to a curvature in the ν vs p relation. Left uncorrected, this would result in a "brick-wall" instability. It is shown that the strength of octupole field required to correct the curvature increases as the third power of the energy. Thus, large octupole correction coils are needed. The possibility of using stronger sextupole corrections to reduce the octupole requirement is suggested. However, this would mean that the working line would not shrink with energy, and ISABELLE would be required to operate with a tune spread ≥ 0.02 even at 400 GeV.

The space-charge detuning of a dc beam, due to trapped electrons can lead to a brick-wall instability at high energies, which is difficult to correct with octupole correction coils. This effect was pointed out by G. Parzen¹ in connection with the ISABELLE accelerator. This report considers this effect more generally and presents the underlying principles and equations.

At high energy, most space-charge phenomena tend to disappear. Thus, correction of transverse coherent instabilities tends to become less significant. However, there is one space-charge mechanism that does not diminish with increasing energy - the space-charge detuning due to trapped electrons in a dc beam.

The vertical detuning at high energy at the beam center is given by

$$\Delta v_v(o) = \frac{k N r_p \eta}{\pi \gamma C} \int \frac{\beta_v(s)}{b(a+b)} ds$$

where N is the number of protons in the ring,

r_p is the classical proton radius ($= 1.54 \times 10^{-18}$ m),

γ is the energy in units of proton rest mass,

β_v is the usual betatron amplitude function,

a, b are horizontal and vertical beam $\frac{1}{2}$ -sizes,

C is the ring circumference,

η is the ratio of trapped electrons to protons, and

k is a form factor determined by the transverse distribution of the proton beam: $k = 3/2$ for a parabolic distribution, $k = 3$ for a Gaussian distribution.

This expression can be simplified to the approximate form,

$$\Delta v_v(o) = \frac{k N r_p \eta}{2 \pi E} \quad (1)$$

where E is the normalized emittance ($E = \text{Area}/\pi$). We assume for simplicity, $E_h = E_v$. Since E is an energy invariant, then assuming no dilution during acceleration, the space-charge detuning due to neutralizing electrons is independent of energy.

1. G. Parzen, BNL Report ISA 78-5 (1978), for further references concerning the brick-wall instability and tune shifts.

Again, for simplicity, we assume that the detuning falls to zero at the beam edge and is parabolic. Thus, the detuning can be expressed by

$$\Delta v_v = \Delta v_v(o) \left[1 - r^2/r_o^2 \right]$$

where r_o is the beam radius, and r is the radial distance from the center. With beam radius in terms of momentum, this can also be expressed by

$$\Delta v_v = \Delta v_v(o) \left[1 - \delta_p^2/\delta_+^2 \right]$$

with $\delta_p = \Delta p/p$ the position in momentum from the beam center, and δ_+ is the $\frac{1}{2}$ -beam momentum spread.

Let us assume that only one dimension is involved. Then we can correct the detuning term with quadrupole and octupole terms. Our concern here is the latter, since, if left uncorrected, the curvature introduced in the v vs p relation would result in a so-called brick-wall instability. If we have n octupole correction elements, each of length ℓ and strength B_3 (kG/m^3), then to correct the detuning we require:

$$\frac{n}{4\pi} \frac{B_3 \ell}{p} \left(3 \beta x_p^2 \right) = \frac{\Delta v(o)}{\delta_+^2},$$

where p is the momentum or magnetic rigidity ($p = B_o \rho$), and β , x_p are β -function and dispersion function values at the octupole position. Thus, the required octupole strength is given by

$$B_3 \ell = \frac{4\pi \Delta v(o)}{3n\beta x_p^2} \left(\frac{p}{\delta_+^2} \right).$$

Although, in practice, some dilution of Δp is to be expected during the acceleration of the beam, it is also expected to be small compared to the energy change. Thus, we can take Δp to be roughly independent of energy. If we take $\delta_+(o)$ to be the momentum spread at the injection momentum, p_o , then we have,

$$B_3 \ell = \frac{4\pi \Delta v(o) p_o}{3n\beta x_p^2 \delta_+^2(o)} (p/p_o)^3. \quad (2)$$

Thus, since $\Delta v(o)$ is independent of energy, we see that the required octupole correction increases as the third power of the energy.

To get an order-of-magnitude feeling of what this implies, we use formulas (1) and (2). If we take, for ISABELLE, $k = 3$ (Gaussian beam), $N = 6.4 \times 10^{14}$ (current, $I = 8$ A), $E = 15$ μ rad-m and $\eta = 2 \times 10^{-4}$, then this gives $\Delta v(o) = 6.3 \times 10^{-3}$. As we will see, this is a very large number indeed.

To correct this entirely with octupole correction coils, we use (2) with the ISABELLE parameters $\beta = 65$ m, $x_p = 1.4$ m (for vertical correction, the coils will be located at a vertical $\beta_{\max}$), $n = 54$ octupole coils, $\delta_+(o) = 0.5 \times 10^{-2}$, and $p_o = 980.7$ kG-m (corresponding to $p_o = 29.4$ GeV/c). Therefore,

$$B_3 \ell = 144 \left(\frac{p}{p_o} \right)^3 (\text{kG/m}^2) .$$

If the coils are in the quadrupoles, $\ell = 1.5$ m, and the octupole field required is

$$B_3 = 96 \left(\frac{p}{p_o} \right)^3 (\text{kG/m}^3) ,$$

or

$$B_3 = 1 \times 10^{-4} (p/p_o)^3 (\text{kG/cm}^3) ,$$

It is customary to describe the correction fields in terms of "b" coefficients with b_n related to B_n by $B_n = B_o b_n$ where B_o is the bending field strength. Since $B_o \propto p$, then, using $B_o = 3.7$ kG at injection, we can write b_3 as a function of energy.

$$b_3 = 2.6 \times 10^{-5} (p/p_o)^2 (\text{cm}^{-3}) ,$$

that is the b_3 required for correction increases quadratically with energy. At the highest field, $p/p_o = 13.6$, leading to,

$$b_3 (\text{high energy}) = 5 \times 10^{-3} (\text{cm}^{-3}) .$$

This number is two orders of magnitude higher than the amount of b_3 provided for in the ISABELLE correction system design. If we have a factor of two increase in the nominal beam size (i.e., through momentum dilution),

this value decreases by a factor of 4 to $b_3 \approx 1.3 \times 10^{-3} \text{ cm}^{-3}$, which is 25 times as large as the amount of b_3 given in the design parameters.

It should be pointed out that the above computation, by and large, uses pessimistic parameters. Thus, better vacuum conditions could make η smaller. Furthermore, the beam may not have a Gaussian distribution. Also, we have neglected any beam size arising from dispersion. On the other hand, we have not folded in the complication resulting from correction in two planes simultaneously. Another point that should be mentioned is that "full" correction of the octupole term is not strictly required. We could stretch the working line with sextupoles. With a central detuning of $\sim 6 \times 10^{-3}$, a working line length of about 0.025 in tune would undoubtedly be sufficient to obviate the need for octupole correction. However, to obtain such a large tune spread at high energy requires a large linear chromaticity, ξ . Since δ_+ at high energy is reduced from 0.5% to about 0.07% (assuming a factor of 2 dilution) then the required ξ is:

$$\xi = 0.025 / (2 \times 10^{-4}) = 17.9.$$

This means that a large sextupole correction would be needed (as compared with $\xi \approx 2.5$ at injection). Thus, if sufficient octupole is not provided, a reevaluation of the sextupole correction must be performed to see if a chromaticity of the order of $\xi = +20$ can be obtained.

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