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STRANGEONIUM SPECTROSCOPY AT 11 GeV/c
AND
CHERENKOV RING IMAGING AT THE SLD*

Timothy Lawrence Bienz

Stanford Linear Accelerator Center
Stanford University
Stanford, California 94309

July 1990

Prepared for the Department of Energy
under contract number DE-AC03-76SF00515

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* Ph.D. thesis

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Abstract

This thesis is divided into two sections, which describe portions of the data acquisition system and online software for the Cherenkov Ring Imaging Detector (CRID) for the SLD, and analyses of several low cross section strangeonium channels in data from the LASS spectrometer.

The CRID section includes a description of the data acquisition system, determination of the preamplifier gain, and development of an online pulse finding algorithm based on deconvolution. Deconvolution uses knowledge of the preamplifier impulse response to aid in pulse finding. The algorithm is fast and shows good single pulse resolution and excellent double pulse resolution in preliminary tests.

The strangeonium analyses are based on data from a 4.1 event/nanobarn exposure of the LASS spectrometer in K^-p interactions at 11 GeV/c, and include studies of $\Lambda\eta\pi^+\pi^-$, $\Lambda K^+\bar{K}^-$, and $\Lambda\phi\phi$.

The $\Lambda\eta\pi^+\pi^-$ analysis includes a measurement of 8.9 ± 0.6 μbarns for the $\Lambda\eta'$ cross section. Production of $a_0(980)$, seen decaying to $\eta\pi^-$, in conjunction with $\Sigma(1385)$ is confirmed, and the first observation of $a_0(980)$ production in conjunction with Σ 's of higher mass is presented. Angular analysis of the $\eta\pi^-$ system indicates the possibility of an S-wave in the region of the $a_2(1320)$. The $\eta\pi^-$ and $K_S^0 K^-$ data from this experiment and similar data from a 4.2 GeV/c Bubble Chamber exposure were fit simultaneously using the coupled channel model of Flatté. The fit yields an $\eta\pi$ width of 120_{-31}^{+50} MeV/c², somewhat higher

than the 4.2 GeV/c analysis, and a coupling constant ratio $g_{KK}^2/g_{\eta\pi}^2$ of $2.4^{+1.1}_{-0.7}$, somewhat higher than the SU(3) value. The $K_S^0 K^-$ data were also fit to a $K\bar{K}$ molecule model. Both the Flatté and $K\bar{K}$ molecule models provide a satisfactory fit to the data.

The $\Lambda K^* \bar{K}^*$ analysis includes both the charged and neutral K^* . The total $\Lambda K^* \bar{K}^*$ cross section is measured to be 3.4 ± 0.2 μ barns. The $K^* \bar{K}^*$ mass spectrum shows a low mass enhancement, with a mass of 1.950 ± 0.015 GeV/c² and a width of 0.25 ± 0.05 GeV/c². A spin-parity analysis this region, assuming the dominance of a single J^{PC} , favors 1^{+-} , 2^{--} , or 2^{++} , and strongly disfavors 0^{-+} .

The $\Lambda \phi \phi$ cross section is measured to be 530 ± 80 nanobarns. Because of low statistics, a spin parity analysis is inconclusive, but disfavors $J^{PC}=0^{-+}$ if the cross section is dominated by a single J^{PC} .

Acknowledgments

While this thesis bears my name, it is the work of a number of people. David Leith, my advisor, was willing to offer me a position when I arrived at SLAC five years ago, and has always been willing to offer help and advice. Bill Dunwoodie spent countless hours providing help in the LASS analysis, and the physics behind the analysis. Dave Aston and Blair Ratcliff also provided assistance and advice when it was needed. I hope that I learned a fraction of the physics that this group exposed me to. Dick Bierce provided invaluable help in managing and cataloging the tape library and large series of batch jobs needed for my analysis. Dave Sherden and Owen Saxton patiently answered questions and helped solve problems with the SLD online software.

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Hai-Wen, my wife, deserves a significant portion of the credit for this thesis. She has been extraordinarily understanding and supportive throughout the long years it took to complete this work. Finally, my parents also deserve recognition. Throughout my life, they have allowed me to make my own decisions, and been supportive of the decisions that I have made. For all that they have given me, it is only appropriate that I dedicate this thesis to Hai-Wen and my parents.

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1. Introduction

With the time scales of experiments in high energy physics becoming longer and longer, it has become difficult for many graduate students to follow a single experiment from conception, through design and construction, data-taking, and analysis. Large experiments of today may require eight years for design and construction, followed by five or more years of running, and additional years of data analysis. These long time scales, coupled with the desire to give each student experience in both hardware and analysis, have made it more common for a student to work on two experiments that are in different phases; for example, one in the design or construction phase and another in the analysis phase. There are advantages to both the single- and multiple- experiment approaches. Restricting work to a single experiment allows a more complete understanding of the decisions made in the design of an experiment, the implications of those decisions, and the relationship between the physics done in an experiment and its design. On the other hand, working on multiple experiments provides a broader range of experience, and provides early exposure to a more diverse group of physicists and physics.

The work described in this thesis falls into the multiple-experiment category; it describes development work on the Cherenkov Ring Imaging Detector (CRID) for the SLD, and analysis of data from the LASS spectrometer. The CRID work, described in the first part of this thesis, details the architecture of the data acquisition system, portions of the preamplifier design, and the development of a pulse finding algorithm. The topics in this section have been selected from the areas in which I worked.

The LASS analysis, described in the second part of this thesis, contains analyses of several final states that contain strangeonium channels. The channels chosen for analysis are all low cross section hypercharge exchange channels. The particular channels chosen for analysis were picked for one or more reasons; current interest in the channel, lack of previous analyses, or the ability to look at a previously analyzed channel in a new momentum range. Of course, the visibility of the channel in the LASS spectrometer was also an important consideration. Channels with multiple missing neutral particles were not considered, because the spectrometer does not have the ability to reconstruct states with multiple missing neutrals in the final state. Finally, in choosing channels, consideration was given to the branching fractions of certain resonances (e.g. $K^*(892)$) into the various candidate channels.

2. Cherenkov Ring Imaging

The SLD detector^[1], shown in Figure 2.1 and Figure 2.2, has been designed to study the properties of the Z^0 particle, produced in 100 - 200 Hz collisions of 45 GeV/c electrons and 45 GeV/c positrons in the SLC^[2] at SLAC. The complete complement of detectors in the SLD includes (moving from the beamline outward) a CCD vertex detector, a drift chamber, a Cherenkov Ring Imaging Detector (CRID), a lead liquid argon calorimeter (LAC), and a warm iron calorimeter (WIC). All subsystems except the WIC are situated inside a 0.6 T solenoidal magnet. The WIC is situated outside the magnet, and utilizes the magnet's iron flux return. For some of the physics that the SLD intends to study, particularly for flavor tagging and heavy quark spectroscopy^[3], good particle identification is vital. Charged particle identification in SLD is provided mainly by the CRID. An example of the importance of the particle identification provided by the CRID is shown in Figure 2.3.

The remainder of this chapter discusses the overall design of the CRID, including the readout electronics and online software. Subsequent chapters discuss, in detail, the online pulse-finding algorithm and some features of the preamplifier. Before discussing any of this, however, it is important to review the Cherenkov effect itself.

2.1 Cherenkov Effect

When a charged particle passes through a medium with a velocity greater than the speed of light in that medium, it radiates photons. This effect was first observed by Cherenkov in 1934^[4], and explained by Frank and Tamm

Figure 2.1 Isometric view of the SLD detector.

in 1937^[5]. The photons are emitted in a cone, with the angle of emission related to the velocity by:^[6]

$$\cos \theta_C \approx \frac{1}{\beta n}$$

In this expression, θ_C is the angle of emission, β is the velocity of the particle, and n is the refractive index of the material through which the particle is passing. If a device is constructed such that the Cherenkov light is emitted in one relatively short path segment, and a photon detector is placed

Figure 2.2 Sectional view of the SLD detector.

perpendicular to the path in another location further downstream, a ring of Cherenkov photons will impinge on the detector, and the diameter of the ring can be used to determine the velocity of the particle. Combined with measurement of the particle's momentum (for example, from a drift chamber), it is possible to compute probabilities for various mass assignments, and hence various particle types. It is important to note that the angle of emission reaches an asymptotic value, so that as the velocity increases from its threshold value,

Figure 2.3 Number of events vs. $K\pi\pi$ mass for three particles with total charge ± 1 , from a sample of Monte Carlo Z^0 decays. (a) All three-track combinations, (b) CRID used, vertex detector not used, (c) vertex detector used, CRID not used, (d) CRID and vertex detector used. The combination of CRID and vertex detector provides a very clean D signal.

the angle is less and less sensitive to the particle's mass. This has implications for the design of the CRID, as will be seen later.

In order to design a detector, it is also important to know the number of photons expected from the Cherenkov effect, and the wavelength distribution of the emitted light. The number of photons produced can be calculated beginning from the expression for the energy radiated by the Cherenkov effect:^[7]

$$\frac{d^2E}{dx d\omega} = \frac{e^2\omega}{c^2} \left(1 - \frac{1}{\beta^2 n^2(\omega)} \right)$$

where E is the energy loss, x is the path of the particle, $n(\omega)$ is the frequency-dependent index of refraction, and β is the velocity of the particle. If the particle radiates over a distance L , the energy radiated is given by,

$$\frac{dE}{d\omega} = \frac{e^2\omega L}{c^2} \left(1 - \frac{1}{\beta^2 n^2(\omega)} \right)$$

The energy carried by each photon is $\hbar\omega$, so that the number of photons produced is

$$dN = \frac{dE}{\hbar\omega} = \frac{e^2 L}{\hbar c^2} \left(1 - \frac{1}{\beta^2 n^2(\omega)} \right) d\omega$$

hence, the total number of photons is

$$N = \frac{L\alpha}{c} \int \left(1 - \frac{1}{\beta^2 n^2(\omega)}\right) d\omega$$

where $\alpha = \frac{e^2}{\hbar c}$ is the fine structure constant.

In order to determine the number of photons that are seen, the efficiency of the photon detector must be taken into account. Real detectors are not completely efficient, and the efficiency is a function of wavelength. Calling the efficiency $\epsilon(\omega)$, the number of observed photons is given by

$$N = \frac{L\alpha}{c} \int \epsilon(\omega) \left(1 - \frac{1}{\beta^2 n^2(\omega)}\right) d\omega$$

Although this gives the expected number of photons, the actual number follows a Poisson distribution with N as the mean value of the distribution.

If we consider the situation in which the light is emitted in a region where n is approximately constant, i.e., away from an absorption band, we can rewrite the expression for the number of photons as

$$N = N_0 L \sin^2 \theta_C$$

where the frequently-used constant N_0 is given by

$$N_0 = \frac{\alpha}{c} \int \epsilon(\omega) d\omega$$

Knowledge of the number of photons expected is important in order to set a scale for the required efficiency of the CRID, and because the number of photons emitted can be used to assist in particle identification.

The wavelength distribution of the Cherenkov photons can be obtained by writing the expression for the number of photons in terms of λ instead of ω . Starting with the relation between frequency and wavelength,

$$\omega\lambda = \frac{2\pi c}{n(\lambda)}$$

and again considering regions where n varies slowly, so that n can be treated as a constant, it can be seen that the differentials are related through

$$|d\omega| = \frac{2\pi c}{n(\lambda)} \frac{|d\lambda|}{\lambda^2}$$

Substituting this into the expression for the number of photons emitted, one obtains the wavelength distribution of the radiated photons

$$N = 2\pi L\alpha \int_{(\beta n > 1)} \left(1 - \left(\frac{1}{\beta n(\lambda)}\right)^2\right) \frac{d\lambda}{n(\lambda)\lambda^2}$$

or

$$\frac{dN}{d\lambda} = \frac{2\pi L\alpha}{n(\lambda)\lambda^2} \left(1 - \left(\frac{1}{\beta n(\lambda)}\right)^2\right)$$

This shows that the wavelength distribution has a $1/\lambda^2$ shape, and so is peaked towards short wavelengths. This is important because it places many constraints on the materials and construction techniques that are used in the CRID. In particular, it requires that all of the material through which the photons pass be transparent to ultraviolet light.

The CRID uses information from the SLD drift chamber on the location and momentum of charged tracks, combined with measurements of the angular dimensions of Cherenkov light cones produced by the tracks and the number of Cherenkov photons, to derive particle identification probabilities. In addition to providing a measurement of a particle's momentum, the drift chamber tracking information is used to determine where to look in the CRID for a Cherenkov ring from a particular particle.

2.2 CRID

Traditional threshold Cherenkov counters look only for the presence or absence of light to help identify particles. The idea for a device that identifies particles by looking not only for the presence or absence of photons, but also at the distribution and number of photons produced by the particle is due to Roberts^[8], while the relatively recent interest in constructing such devices is due in large part to Ypsilantis and Seguinot^[9]. Several such devices have been built or are currently under construction, including devices for DELPHI at LEP, the OMEGA spectrometer at CERN, Fermilab, and the device described here.

The CRID was designed and built in two major sections; the barrel CRID and the endcap CRID. This discussion concerns only the barrel CRID, and

Figure 2.4 Sectional view of the barrel CRID.

while the endcap CRID is conceptually identical, it differs considerably in design. Figure 2.4 shows a cross-sectional view of the barrel CRID. The primary components of the CRID are the liquid radiator, time projection chamber (TPC) with proportional wire plane and readout electronics, gas radiator, and mirrors. Each of these will be discussed below.

2.2.1 Radiators

In the physics environment of the SLD, it is important to have good particle identification over a wide momentum range. Since the Z^0 decay products are much lighter than the Z^0 , particle momenta can be as high as

45 GeV/c (one half the center of mass energy). In order to provide good particle ID capabilities over a wide momentum range, the CRID utilizes two different radiators. A liquid, C_6F_{14} ^{†[10]}, with an index of refraction of 1.277 at a wavelength of 1900 Å, is used to cover the low momentum range and a gas radiator, C_5F_{12} [‡], with an index of refraction of 1.00173 at 1900 Å, is used to cover higher momenta. Because C_5F_{12} condenses to a liquid below 30° C, the entire CRID will be heated to 40° C. Figure 2.5 shows the momentum range over which particle identification is provided by each radiator. The combination of the two radiators is expected to provide π/K separation at better than the 3σ level from 230 MeV/c to 28.8 GeV/c, and K/p separation from 800 MeV/c to 46.2 GeV/c^[1].

The relative indices of refraction of the liquid and gas radiators were chosen so that there is no gap in coverage as a function of momentum. These particular materials were chosen for their indices of refraction, their compatibility with other materials used in the CRID, their ability to transmit the primarily short wavelength Cherenkov photons, and because they have low chromatic dispersion. Another consideration, particularly for the gas radiator, is safety. The radiator gas chosen is much safer than the flammable hydrocarbons which were otherwise acceptable alternatives. Because the gas radiator lies outside the radius of the TPC, the photons it produces travel away from the detector. The outer circumference of the CRID is covered with mirrors

† This material is known as FC-72 by 3M Corporation, St. Paul, Minnesota, or as PP1 by ISC Chemicals Ltd., Bristol, U.K.

‡ This material is known as FC-87 or PP-50 by its manufacturers.

Figure 2.5 Particle separation ability of the CRID at the middle of the barrel. Curves shown are saturated at 10σ . Dashed lines indicate where the heavier particle is below threshold. Dotted line in (b) is for doubled measurement errors.

to reflect these photons back towards the TPC, and to focus the photons so that they produce a ring at the detector's location.

The liquid radiator is contained as a 1.0 cm thick layer in trays situated near the inner radius of the CRID, inside the radius of the TPC. The gas radiator occupies a 45 cm deep region on the other side of the TPC. This configuration was chosen because the liquid produces rings that are very large and contain a relatively large number of photons for even a thin radiator, while the gas radiator produces small rings containing few photons. Using the chosen layout, particles have a long path in the gas, allowing more photon production, and the ring can be made larger. Because the liquid and gas radiators lie on opposite sides of the TPC, both sides of the TPC must be transparent to ultraviolet photons. The fact that the liquid and gas radiators lie on opposite sides of the TPC also has the advantage of potentially permitting a distinction between photons produced in the liquid and those produced in the gas. This will be described in the next subsection.

Because the liquid radiator is thin, the photons from it produce a good ring without further focussing. This proximity focussing is the only focussing used for the liquid radiator. The gas radiator, on the other hand, has a significant depth. Photons from it are both reflected and focussed into a ring by the mirrors placed at the outer circumference of the CRID.

2.2.2 Detector

A TPC with a proportional wire plane is used as the detector for the photons. A drawing of the TPC/detector combination is shown in Figure 2.6.

Figure 2.6 TPC and detector for barrel CRID.

The drift space inside each TPC is 1.268 m long, 30.7 cm wide, and the thickness tapers from 9.2 cm to 5.6 cm. Forty TPCs will be used in the CRID, arranged as twenty in each end of the barrel. The TPC's are filled with a gas mixture that is transparent to ultraviolet photons, has good electron lifetime, has a pulse height spectrum with a peak clearly separated from the noise, and includes a component which efficiently converts photons to single photoelectrons. Good electron lifetime is necessary to minimize losses as single electrons drift up to 1.3 m in the TPC. Pure ethane has been chosen for the TPC gas, with Tetrakis

di-methyl amino ethylene (TMAE) used as the gaseous photocathode. TMAE has a high quantum efficiency, reaching 50% below 1800 Å, and above 10% for wavelengths shorter than 2100 Å. The quantity of TMAE in the gas, determined by the temperature of the gas as it passes through TMAE-filled bubblers, is set so that the absorption length for photons is short compared to the thickness of the drift box. This assures that a high percentage of photons are converted before they exit the drift box. The concentration should not be too high however, or most photons would convert very near their entrance into the TPC; near the edges of the box, where electrostatic effects can alter their trajectories and cause losses. The present plan is to operate the bubblers at 28° C, providing a TMAE absorption length of 1.6 cm. Because the TMAE absorption length is short compared to the thickness of the TPC, photons preferentially convert near the surface where they enter the detector and so there is a spatial separation between the conversion points of photons from the gas and photons from the liquid radiator. This distinction can be potentially useful to the software that reconstructs the Cherenkov rings.

TMAE is highly reactive, necessitating careful choice and cleaning of materials that come into contact with it. In addition, commercially available TMAE must be carefully cleaned before using it, in order to remove contaminants which otherwise ruin electron lifetimes. Because the Cherenkov photons are preferentially emitted in the UV, the material used for the transparent faces of the drift box cannot be glass. Instead, sheets of fused silica (quartz) are used.

The TPC is tapered, widening from 5.6 cm thickness at the end far from the proportional wire plane to a thickness of 9.2 cm at the end adjacent to it. This taper prevents transverse diffusion from causing loss near the faces of the TPC. The taper is not symmetric, but instead, there is more taper on the side facing the gas radiator (the outer radius). This asymmetric taper is due to the radial component of the SLD magnetic field at the TPC location, and keeps photoelectrons from drifting into the TPC faces due to Lorentz forces^[11]. Metal traces are placed on the inner and outer surfaces of the quartz faces and sidewalls every 3.175 mm. These traces, whose potential is set by a resistor ladder, shape the field in the TPC. An additional wire field cage encloses the TPC, to further control the field shape in the TPC^[12].

At the end of the drift box, the single photoelectrons are detected by a proportional wire chamber. This chamber is composed of 93 7 μ m diameter carbon fibers on a 3.175 mm pitch. The diameter and composition of the fibers were chosen because they provide the correct resistance to minimize electronic noise in the system, as will be discussed later. The fibers are somewhat difficult to work with, however, since they have a breaking tension of 10.1 ± 2.1 grams, are difficult to see, and are somewhat difficult to attach. The fibers are oriented so that one end is near the TPC face where photons from the liquid radiator enter, and the other end is near the face where photons from the gas radiator enter. The fibers themselves are 10 cm long, slightly more than the width of the TPC at this point.

A mechanical blinding structure prevents photons produced in an avalanche from re-entering the TPC, photoionizing more TMAE, and drifting

onto other wires in the proportional chamber, an effect called photon feedback. In the CRID geometry, photon feedback has been measured to be approximately 1%. In contrast, an unblinded structure has been measured to have a photon feedback of 6-8%^[12].

The proportional chamber is operated at a nominal gas gain of 2×10^5 . The operating gas gain is determined by two opposing constraints. On one hand, the gain is pushed down by the need to minimize photon feedback and maximize the operating lifetime of the chamber. On the other hand, the desire to have a pulse height spectrum with a clear peak above the noise favors high gas gain.

2.3 Front-End Electronics

A schematic representation of the CRID electronics is shown in Figure 2.7. Both ends of each carbon fiber are read out into separate preamplifiers, which are located on boards just behind the cathode of the proportional wire plane. Because the charge produced in the avalanche is so small, sensitivity and noise are primary concerns. The design of the preamplifier will be discussed later. Both ends of each carbon fiber are read out in order to provide photon conversion depth information via charge-division. This conversion depth information allows the analysis to correct for parallax error, and provides information on whether a particular photon came from the liquid or gas radiator.

On each beam crossing, the output of each preamplifier is fed into a storage chain consisting of 512 samples stored in two analog sample-and-hold

Figure 2.7 Schematic representation of CRID front-end electronics. Each optical fiber, shown as a broad gray line, handles the data from 8 ADCs (two-thirds of a drift box). Each ADC handles the data from 16 preamplifiers (one end of each of 16 chamber fibers).

chips, each with the ability to hold 256 samples. These chips are referred to as Analog Memory Units (AMUs). Incoming signals can be digitized at intervals that are integer multiples of the 8.4 ns (119 MHz) SLC clock. A lower bound on the CRID sampling interval can be determined by the simple requirement that the 512 AMU samples must cover the 1.3 m length of the drift box. Using an approximate drift velocity of 5.5 cm/ μ s, the sampling interval can be no shorter than 46 ns. In reality, the CRID data will be digitized at intervals of 67.2 ns. The fact that the interval selected is longer than the minimum is because pulse resolution studies indicated that the sampling interval and preamplifier time constant should be equal, together with the fact that it was not possible to build 50 ns preamplifiers to tight enough tolerances.

When an event trigger occurs, the AMUs are read out, multiplexed to ADC's, digitized, and sent to processors for analysis. All of the electronics, up to and including digitization, is located on the CRID itself; the digital signals are sent via optical fibers to the outside of SLD, where the processors are located. The AMU's, ADC's, optical fibers, and processors are used not only by the CRID, but by all subsystems in SLD.

2.4 Online Processing

The CRID data acquisition system, shown in Figure 2.8, consists of three levels of processors; the Waveform Sampling Module (WSM), the Aleph Event Builder (AEB), and the SLD VAX. Each of these has specific functions, and the three levels are connected together using FASTBUS. The structure of the data acquisition system is essentially the same for all of the subsystems in SLD^[14]; variations arise only due to differences in the front-end electronics. For example, it is not appropriate to have a 512 sample readout for the calorimeters. Instead, four samples per calorimeter channel are taken; pedestal and signal at both high and low gain.

Before discussing these modules, it is helpful to get an idea of the amount of data produced by an event in SLD. The barrel CRID will be used as an example. For each event, each preamplifier produces 512 two-byte ADC words. There are two preamplifiers per wire, 93 wires per drift box, and 40 drift boxes in the barrel CRID. Multiplying this out, the barrel CRID produces approximately 7.6 megabytes of data for each event. At a nominal event rate of two Hertz, this is a rate of over 15 megabytes per second. If calibration data are included, the data size is substantially increased. As will be described shortly,

Figure 2.8 Structure of the CRID data acquisition system. Other subsystems in SLD have essentially identical structure. The broad grey bands represent optical fibers. The heavy black lines represent FASTBUS connections. Each WSM accepts four fibers (one per processor). In the CRID, each processor handles data from two-thirds of a drift box. The numbers in parentheses show the quantity of each component in the barrel CRID system.

each AMU bucket has eight calibration constants associated with it. This means that in the barrel CRID, there are (assuming two bytes per constant) nearly 61 megabytes of calibration constants for the AMU alone! In addition, there are other calibration constants, but the storage needed for them is much less than for the AMU constants. Since the goal of SLD is for the total data written to tape to be approximately 200 kilobytes per event, it is obvious that a significant amount of online processing is necessary. Even if each detector in SLD were able to write all the raw data to tape, there would be little hope of obtaining the computing power necessary to perform the data reduction offline. Therefore,

the SLD data acquisition system includes a large number of processors, and is designed to allow a high degree of parallel processing. In such a system, the processing software must be thoroughly debugged and understood, since there can be no reprocessing of data if a mistake in the software is discovered.

2.4.1 WSM

The Waveform Sampling Modules (WSMs) are the lowest level processors, and are responsible for correcting for the bucket-by-bucket characteristics of the AMU, zero-suppressing the data, and determining the three spatial coordinates and amplitude of each photoelectron pulse in the data. In addition, the processor in the WSM must distinguish between single photoelectron pulses and the comparatively huge pulses produced by the passage of charged particles through the CRID TPC. Information on the location and pulse height of each pulse in an event is formatted and passed on to the AEB upon request. In the CRID, each processor in a WSM is responsible for the data from 2/3 of a drift box. Thus, the 40 CRID drift boxes are serviced by 60 WSM processors. These are contained on 15 WSM boards.

The hardware of each of the four independent sections in a WSM consists of a custom chip, called the Digital Correction Unit (DCU), and a programmable processor. In addition, fiber optic receivers are located on the WSM board. The DCU has the responsibility of performing the bucket-by-bucket AMU corrections and zero suppressing the data.

Each bucket in the AMU is calibrated by measuring the output vs. input response using DC voltages. An eight-segment piecewise linear fit is made to

Figure 2.9 DCU zero-suppression constants. The dashed vertical lines represent samples. This example shows a leading count of 2 and a trailing count of 4. For this pulse, a total of 10 samples would be passed by the DCU.

the response of each bucket. The resulting number of constants is large enough, and software application of these constants on every event would be slow enough, that a custom chip, the DCU, was designed to carry out the task. The bucket-by-bucket calibration constants are stored in DCU memory, and applied by the DCU hardware.

Zero suppression is performed after the AMU calibration constants have been applied to the data, and is controlled by four programmable values; *leading threshold*, *trailing threshold*, *leading count*, and *trailing count*. The function of these four constants can be seen in Figure 2.9. Data samples entering the zero

suppression hardware are eliminated until the amplitude of a sample exceeds *leading threshold*. This sample and subsequent samples are passed. In addition, *leading count* samples before the triggering value are also passed. After the samples fall below the *trailing threshold* value, subsequent samples are again suppressed. However, *trailing count* subsequent samples are passed. The purpose of *leading count* and *trailing count* is to allow a pedestal measurement both before and after the pulse. The groups of samples passed by the zero suppression hardware are tagged so that it is possible to later determine where they occurred in the original data. Finally, the surviving samples are placed in a memory accessible to the programmable processor in the WSM.

The programmable processor, a Motorola 68020 operating at 16 MHz, is responsible for converting the raw samples into final measurements. In the case of the CRID, this means that three spatial coordinates and an amplitude must be determined for each pulse. In addition, a quality word is produced, indicating whether the pulse is a dE/dx pulse and whether it occurred shortly after an earlier pulse. One coordinate is given directly by the wire number on which the pulse occurred. The second coordinate is determined from the sample number in which the pulse occurred. The third coordinate is determined by the relative pulse height in the two ends of the wire on which the pulse occurred. Finally, the amplitude is determined by the sum of the pulse heights on both ends of the wire on which the pulse occurred. The algorithm for determining these numbers from the raw data will be described later. Here, it should be noted that a correction must be made for preamplifier gain, a correction which is not made by the DCU. This correction is based on the charge gain of the preamplifier, and so is a correction to the integrated area

under each pulse. In order to determine this correction, pulses of known charge are applied to the input of each preamplifier during calibration runs, and the pulse-finding algorithm is used to compute the output charge. These pulses are applied using calibration circuitry built into each preamplifier. The response of the preamplifier is parametrized as an eight segment, piecewise-linear curve. This calibration cannot be carried out until the AMU calibration has been completed. Compared to the AMU calibration, this calibration results in a small number of constants (8 per preamplifier, two preamplifiers per wire, 64 wires per WSM), and the correction only needs to be applied to found pulses. Because of this, the preamplifier gain correction is performed by software in the WSM.

After the WSM has analyzed all of the data in an event, it notifies its AEB that the data are ready for readout. Under normal operation, no raw data are sent up the analysis chain from the WSM; only pulse locations and amplitudes are available for recording and subsequent analysis.

2.4.2 AEB

The Aleph Event Builder (AEB), another programmable processor, was designed for the ALEPH experiment at LEP^[15]. Like the WSM, the AEB also uses a 16MHz Motorola 68020 processor. The SLD system uses one AEB per subsystem. Thus, the CRID has only one AEB. The purpose of the AEB is to collect data from its subsystem, format the data for tape, notify the VAX when it has assembled the data from an event, and upon request, transfer the data to the VAX. In addition, the AEB controls the WSMs for its subsystem.

2.4.3 SLD VAX

SLD currently uses a DEC VAX 8800 as its main online computer. The primary purpose of this computer, in data acquisition, is to collect events from the AEBs and log the event data to tape. In addition, the VAX passes commands to the AEBs, and through the AEBs to the WSMs in each subsystem. The VAX can also run a number of user processes for monitor and control of the experiment. In addition to the 8800, SLD uses a number of VAXstation 2000's as graphics workstations and several MicroVAXes for hardware monitoring. All of the VAXstations are configured into a VAXcluster with the 8800. The VAXstations are used for developing software, and for running control, monitoring, and analysis programs. The MicroVAXes are used for CAMAC-based slow monitoring and control functions. For example, a MicroVAX is partially used for temperature monitoring and control in the CRID.

2.5 VAX Software

The software used to monitor and control SLD is called the Solo Control Program (SCP), and is based on a program of the same name used for monitor and control of the SLC accelerator. The program can be run on VAXstations, VT100-type terminals, and on Commodore Amiga computers. The user interface is based on a matrix of virtual pushbuttons, one to four display areas, and a message area. The user initiates actions by 'pressing' various virtual pushbuttons on the screen. Selection and activation of a particular pushbutton is made using a mouse (on VAXstations and Amiga), or by using the keypad on VT100 type terminals. Software routines are attached to various buttons, and

can cause displays to be generated, options to be selected, or controls to be changed.

From a programmer's point of view, the SLD data acquisition software is organized into a number of cooperating processes, running on one or more machines. The user's SCP obtains data from the main data stream by making a request to the data logging program. A copy of the requested data is sent using the mailbox facilities of VMS. Other processes monitor and control the high voltage for the experiment, act as print servers and database servers, and allow CAMAC commands to be carried out, either on the local machine, or transparently to another machine. Additional processes run on the AEBs and WSMs for transmission and low-level analysis of the data stream.

The online code for SLD is written as a number of packages, implemented primarily as shareable images. These are used by all subsystems in SLD, for such things as database management, user interface support, interprocess communication, CAMAC, and display functions. Each subsystem in SLD also has its own programs, built on top of these packages. The use of shareable images has several advantages. It reduces the size of the user's programs on disk, since all users can share one disk copy of the shareable image. Program linking times are reduced, since shareable images are prelinked, so that only a few externally-visible symbols need to be linked into user programs from the shareable image. Finally, shareable images provide some degree of encapsulation. The person responsible for the shareable image is able to decide what entry points and data are visible to users. This helps improve the reliability and maintainability of the code, since users cannot

obtain access to unintended entry points, and cannot directly get at, and potentially damage, data in the shareable image.

A number of DEC's software engineering tools are extensively used for development and maintenance of the SLD code. The Code Management System (CMS) software is used to coordinate development between multiple programmers, and provides an archive of all versions of a file. This allows an easy way to back out changes if problems are found, and also allows easy line-by-line tracking of changes made to code. The Module Management System (MMS) allows a user to specify dependencies between pieces of a package. MMS 'knows' how to update a package based on these dependencies, built-in rules, and user-specified rules. Once the dependencies have been made known to MMS, it can bring a package up to date by checking all dependencies. Files that have been changed are used, together with rules, to update the files that depend on them. MMS automatically iterates this process until all files are up to date. As an example, if a user changes an *include* file, MMS can compile all routines that use the *include* file, update code libraries with the new versions of the object code, and link a new version of the executable program. The Language Sensitive Editor (LSE) is primarily used for fast syntax debugging, since source code can be compiled inside the editor, and the compilation errors are flagged in a way that makes it easy for a user to rapidly check and fix syntax errors. Other features of LSE, such as its ability to work with CMS and SCA, are less frequently used, although they can be extremely helpful at times. DEC's Source Code Analyzer (SCA) is used to trace the static structure of programs and to trace the usage of variables in the code. SCA relies on a database it creates at compile time. This database contains a list of all symbols in a package,

together with information on where and how they are used. There are direct ways, for example, to ask SCA to provide a list of all the places in a package where a certain variable is written to, or to ask for a list of all places in a package that a particular routine is called from. The Performance and Coverage Analyzer (PCA) is occasionally used to determine where performance bottlenecks may exist in the code. PCA watches as code executes, and records how often each statement is executed. Finally, the VMS debugger is used for general debugging purposes.

The disk directory structure of the online system is organized into major facilities such as CRID, LAC, Display, High Voltage Monitor and Control...etc. Each major facility may be broken down into minor facilities. As an example, the CRID is currently broken down into monitoring and data analysis. Each minor facility is further broken down into three parallel commonly used branches. These branches are development, preproduction, and production. Production code is meant to be stable, and reliable. Development code is unstable and unreliable. The preproduction area is used for staging development code that is believed to be production-ready, but has not yet been moved into production. This system has worked well for managing software development by a number of people.

2.6 Monitoring

There are two kinds of monitoring in SLD; monitoring based on the normal data stream from the detector, and monitoring obtained from other sources (e.g. CAMAC). The former will be called data monitoring, and the latter called slow monitoring. Slow monitoring for the CRID has been quite

well thought out, while data monitoring will undergo major development as operating experience is gained with the CRID.

For the CRID, the slow monitor quantities include temperature, heater ON/OFF, gas mixture, oxygen concentration, water vapor concentration, pressures, flow rates, UV transparency, electron lifetimes, and high voltage^[16]. Control is provided for valve positions, UV monochromator, electron lifetime monitor, heaters, and high voltage. All critical controls, e.g. valve positions, do not rely on software to provide protection from illegal states. Instead, hardware has been built to ensure the safety of critical control items.

Data monitoring includes drift velocity measurement and electrostatic distortion measurement. Both of these are accomplished using 19 fibers attached to each drift box. The fibers can be pulsed, using flashlamps, to provide fiducial markings in the data. Additional quantities, such as pulse height distributions, can also be determined from the data.

Data from the slow monitor system, along with monitor system control and configuration data, reside in databases. The database used by SLD, a four-level, hierarchical database, is a modified version of the SLC database. There are several reasons why a database mechanism was chosen. First, it allows monitor processes to run as batch jobs, operating regardless of whether or not anyone is logged on looking at the data. Secondly, by having a monitor data structure common to all SLD subsystems, a set of core functions can be made available for all of SLD. Individual subsystems can build on this core, instead of having to re-implement everything. Every subsystem needs to do

some simple readout, limit checking, and time history recording. These functions are made available by SLD. The generic SLD software knows how to read out certain devices. To monitor data from these devices, users need only specify how frequently the device is to be read out, the hardware location of the device, and limit values for the data. More complicated readout, limit checking and control is up to the individual subsystems to implement. Each database has a database server process associated with it. The database server allows access to a database from multiple machines.

When a device is read out, the monitored values are converted to the desired units, checked against limits, recorded in a time history, and placed into the database. Two sets of limits are provided; tight limits and loose limits. Tight limits are used to provide a way to watch for quantities that have started to drift, but have not yet gone bad. If a value is outside of the limits, the condition can be logged, printed, and reported to other processes. This is handled through the SLD SIGNAL error handling system. Other processes, including users' SCPs, can access the database to obtain the values of monitored quantities. In addition, users can send control requests directly to the monitor processes.

3. CRID Preamplifier

As with many of the elements of the CRID, the design of the preamplifier is driven largely by physics goals. The need for efficient single photoelectron detection places strong constraints on acceptable preamplifier designs. Particular attention must be paid to the noise introduced by the electronics. In addition, while the preamplifier is designed to respond to single photoelectrons, it must be able to withstand signals from charged particles passing through the TPCs. These dE/dx signals are up to 1000 times larger than the single photoelectron signals, and it is important that the preamplifier quickly recover after these saturating signals occur. Finally, because the electronics is located on the detector, where heating is a concern, they are powered only near the time of beam crossing, and powered down between beam crossings. For this reason, the preamplifier and associated electronics must stabilize quickly when they receive power.

After a brief discussion of the resolution required of the preamplifier, this chapter will concentrate on the noise performance of the preamplifier, and on determination of the gain. A more complete description of the preamplifier design can be found elsewhere^[17].

3.1 Resolution Requirements

In order to obtain the resolution needed to accurately identify particles, the overall resolution in the position of a hit must be on the order of 1 mm. There are a number of terms which contribute to the resolution^[11]; chromatic

error, multiple scattering, geometric error, momentum smearing, and measurement error.

Chromatic error arises from the variation of the index of refraction with wavelength. Over the range in which the TMAE/TPC system is sensitive (1700 - 2200 Å), this contributes an error of ± 5 mrad for the liquid radiator, and ± 3 mrad for the gas radiator.

Multiple scattering of charged particles occurs primarily in the outer wall of the drift chamber, the inner wall of the CRID, the quartz, liquid radiator, and the detector box. It produces a small error; $\pm 1.4/p$ mrad for the liquid radiator, and $\pm 0.4/p$ mrad for the gas radiator, where p is measured in GeV/c.

Geometric error arises from aberrations in the mirrors, and from the proximity focusing used for the liquid radiator. It contributes ± 7 mrad for the liquid radiator near the center of the drift box, but depends strongly on the direction of motion of the charged particle, and on the azimuthal angle of the Cherenkov photon. The error ranges from ± 3.5 mrad to ± 15 mrad for the liquid radiator. For the gas radiator, the contribution comes mainly from photons reflected from the edges of mirrors, and contributes ± 0.5 mrad.

Momentum smearing arises because charged particles follow curved trajectories, not straight lines, in magnetic fields. Because of this, the Cherenkov photons do not produce a unique cone. The error introduced by momentum smearing depends on the direction and momentum of the charged particle producing the photons. For the liquid radiator, momentum smearing

contributes an average error of $0.4/p$ mrad, while for the gas radiator, the average contribution is $15/p$ mrad.

Measurement errors are due to a combination of effects; diffusion of photoelectrons as they drift in the TPC, non-uniformities in the chambers, granularity in the readout system (wires and time), and noise in the electronics. The specification for the preamplifier's electronic noise is determined by the desire to equalize the detector resolution and conversion depth errors. Calculation of errors is complicated by the fact that the error due to the wire spacing is a tophat error while the errors in conversion depth and drift time are more nearly Gaussian. Ignoring diffusion and assuming that the conversion depth is measured to 1.4 mm, the errors for the liquid ring are ± 3 mrad due to detector resolution on wire number and drift distance, and ± 3 mrad due to the error in determining the conversion depth. Gas ring errors are approximately ± 1.7 mrad. The effect of transverse or longitudinal diffusion must be added in quadrature to these numbers, and longitudinal diffusion is expected to dominate the resolution in drift time. The amount of diffusion depends on the magnetic field and on the particular gas mixture used in the drift boxes, but is approximately $(0.15 \pm 0.05 \text{ mm}) \sqrt{\text{drift distance in cm}}$.

Combining the above errors in quadrature, the total uncertainty for a single photon produced in the liquid radiator is approximately ± 10 mrad. Assuming an average of 14 photons in a liquid ring, this should provide a measurement of the Cherenkov angle to around ± 3 mrad. For tracks above 5 GeV/c in the gas radiator, the error should be approximately ± 3 mrad per

photon, or better than ± 1 mrad for an entire ring. These calculations have been checked on data from the CRID preprototype^[18], and worked well.

3.2 Preamplifier Noise

As described above, the error arising from noise in the electronics has to produce no more than a 1.4 mm error in the determination of the photon conversion depth. For a 10 cm long wire, this implies that the charge division must have no more than a 1.4% error. The charge division resolution can be written as

$$\frac{\delta x}{L} = \frac{i_n}{i_0} \sqrt{1 - 2\frac{x}{L} + 2\left(\frac{x}{L}\right)^2}$$

where x is the position along the wire ($0 \leq x \leq L$), L is the length of the wire, i_n is the noise current, and i_0 is the signal current.

As can be seen, the resolution is best at the center of the wire, and degrades by a factor of $\sqrt{2}$ at the ends of the wire. In order to achieve 1.4% charge division resolution at the ends of the wire, the noise must be less than 1% of the signal. Since the gas gain has been set to provide an average signal of 2×10^5 for a signal photoelectron, the noise must be less than 2000 electrons. In addition, since a significant fraction of the signals will be substantially smaller than the average, it is important to try to reduce the noise below the nominal 2000 electron requirement.

Because the noise of a system is typically dominated by the noise in the first stage, it was important to choose a low-noise front end for the CRID preamplifier. For this reason, the front end of the CRID preamplifier input stage was based on a Field Effect Transistor (FET), a design commonly used for low noise input stages. For an FET stage, the noise mainly comes from four sources^[19]; thermal noise in the gate resistor, shot noise from the gate current, thermal noise in the FET channel, and $1/f$ noise in the FET channel. Additional noise can, of course, be picked up from sources outside the circuit. To minimize this pickup, the CRID preamplifiers are shielded in Faraday cages.

3.2.1 Thermal noise

Thermal noise, also called Johnson noise, arises due to Brownian motion of the charge carriers in a device. In a resistance, the average noise voltage generated in a frequency interval df can be expressed as

$$\overline{v^2} = 4kTRdf$$

where k is Boltzmann's constant, T is the absolute temperature, and R is the resistance (R_g for gate, R_{eq} for channel). The appropriate resistance to use for the gate resistance is the intrinsic resistance of the preamplifier in parallel with the resistance of the carbon fibers in the detector. For the channel, the appropriate resistance was fit based on noise measurements made on a prototype preamplifier.

3.2.2 Shot Noise

Shot noise arises because the current is carried by a series of charged particles, and is not a continuous flow. The average noise current in a frequency interval df can be written

$$\overline{i^2} = 2e\bar{i}df$$

where e is the charge of the electron, and $\bar{i}$ is the average current carried by the device.

This can be converted to a voltage by using the impedance of the device we are considering.

$$v = iZ$$

$$\overline{v^2} = \overline{i^2}|Z|^2$$

For the circuit under consideration, the gate current flows almost entirely through the input capacitance, so that the impedance can be approximated as

$$|Z| = \frac{1}{2\pi fC}$$

where C is the capacitance.

Substituting for the impedance and the shot noise current, we find

$$\overline{v^2} = \frac{2e\overline{I}_g}{4\pi^2 f^2 C^2} df$$

where $\overline{I}_g$ is the gate current, and the appropriate value for C is the sum of the intrinsic preamplifier capacitance and the capacitance of the proportional wire chamber used as a detector in the CRID.

3.2.3 1/f Noise

The final noise source, 1/f noise, is also known as flicker noise in vacuum tubes, and as modulation noise in transistors. Its origins are not well understood, but it is believed to arise from slow changes in the emission properties of the cathode in vacuum tubes, and from crystal imperfections and surface defects in transistors. In both cases, it can be expressed as

$$\overline{v^2} = A_f \frac{df}{f}$$

where A_f is an empirically determined constant.

3.2.4 Total Noise

Combining these expressions, we can obtain an expression for the noise of an FET stage*. Since the noise sources are incoherent, the amplitudes are added in quadrature.

where $b(f)$ is the frequency response of the preamplifier and i runs over the four noise sources listed above.

* The procedure used here follows that given by Delaney.

$$\overline{v_{tot}^2} = \int_0^{\infty} b(f) \sum_{i=1}^4 \overline{v_i^2} df$$

To carry out the integration, some assumption must be made about the frequency response of the amplifier. If the rise time of the amplifier is set equal to the fall time, a condition in which noise is minimized^[20], the frequency response is

$$b(f) = A^2 \left(\frac{2\pi f \tau}{1 + (2\pi f \tau)^2} \right)^2$$

where A is the passband gain and τ is the time constant.

The CRID preamplifier approximates equal rise and fall times with a $te^{-t/\tau}$ pulse shape.

Substituting and performing the integration, the result is

$$\overline{v_{tot}^2} = A^2 \left(\frac{kT}{2R_g C_{in}^2} \tau + \frac{eI_g}{4} \frac{\tau}{C_{in}^2} + \frac{kTR_{eq}}{2\tau} + \frac{A_f}{2} \right)$$

This can be minimized with respect to τ in order to determine the optimum time constant. For the CRID preamplifier, the optimum time constant is in the neighborhood of 50 ns. It has proven difficult to maintain tolerances for preamplifiers with a time constant shorter than 62 ns, so the real CRID preamplifiers have been built with a time constant of 67 ns. This particular value was chosen because it is equal to the sample interval to be used in the CRID data acquisition system, and simulations indicated that pulse-finding

resolution is best when the sample interval is equal to the time constant of the preamplifier.

The expression for noise can be rewritten to determine the equivalent noise charge (ENC), for which the design goal was 2000 electrons or less. To make the conversion, the relationship between charge and voltage, $Q = CV$, is used. Consider a charge placed on the input of the preamplifier. This is equivalent to placing a voltage step input of magnitude $V = Q/C_m$ on the input. The peak output voltage of the preamplifier will then be $Ae^{-1}Q/C_m$. In this expression, C is the capacitance, and the factor of e^{-1} comes in because of the particular form of the frequency response of the preamplifier that was chosen. Then, the ENC, Q , can be written as

$$Q^2 = \left(\frac{C_{in}^2}{Ae^{-1}} \right) \overline{v^2}$$

Substituting for $\overline{v^2}$,

$$Q^2 = 2.718 \left(\frac{kT}{2R_g} \tau + \frac{eI_g}{4} \tau + \frac{kTR_{eq}}{2} \frac{C_{in}^2}{\tau} + \frac{A_f}{2} C_{in}^2 \right)$$

It is this expression which relates the noise charge of the amplifier to the components used in the preamplifier, and to the fiber resistance. Substituting the values appropriate for the CRID preamplifier, and writing the answer in terms of electrons (by dividing by the electron's charge)^[17],

$$Q = \frac{2.718}{e} \left(\frac{kT}{2R_{int} R_{Ch}} \tau + \frac{el_g}{4} \tau + \frac{kTR_{EC}}{2} \frac{(C_{int} + C_{ch})^2}{\tau} + \frac{A_f}{2} (C_{int} + C_{ch})^2 \right)$$

Tests have been conducted on production preamplifiers connected to CRID proportional wire planes. For the noise tests, signals were injected into the front end of the preamplifier through a capacitor. In these tests, the system showed noise of approximately 1800 electrons, satisfying the original 2000 electron goal.

3.3 Preamplifier Gain

The preamplifier gain is determined by balancing two opposing factors. The desire to minimize ADC quantization noise drives the gain up, while the fact that the preamplifier must deal with a wide range of pulse heights drives the gain down. If the gain is made too low, most signals will have very small pulse heights. Then, the resolution of the ADC becomes a significant factor in the overall system noise, and carried to extremes, can dominate. On the other hand, if the gain is made too high, many single photoelectron pulses will saturate the ADCs, and be virtually useless. It is important, then, to have a good idea of what the single-photoelectron pulse height spectrum looks like at the preamplifier's input. This is a combination of the pulse height spectrum of the gas used in the TPCs, the geometry of the CRID, and the TMAE absorption length in the TPCs. The gas pulse height spectrum enters for obvious reasons. The geometry and TMAE absorption length enter because they determine the spatial distribution of photon conversions in the TPC. Because each end of the fiber is read out by a separate preamplifier (in order to do charge division), this

affects the pulse height seen by the two preamplifiers on opposite ends of the fibers.

In order to estimate the correct preamplifier gain, a Monte Carlo simulation was run. Input to the Monte Carlo included a pulse height distribution, information on the conversion depth in TMAE, the number of photons expected in liquid and gas rings, and the Cherenkov angles for the liquid and gas radiators.

The ethane pulse height spectrum was obtained using a LeCroy qVt, and was taken at the nominal operating voltage of the CRID detector. The low end of the pulse height spectrum, which exhibited a sharp rise below 28 counts due to noise, was replaced with a function, assuming an $A\alpha e^{-\alpha/\alpha_0}$ pulse height spectrum. The normalization (A) and α_0 were obtained from a fit to the remaining section of the pulse height spectrum. An example of the thrown pulse height spectrum is shown in Figure 3.1. The high end of the pulse height spectrum extended above the range of the qVt used to measure it. To compensate for the overflow pulses, two approximations were tried. In the first approximation, 3% of pulses were piled up at the qVt saturation value. In the other approach, the pulse height spectrum was artificially extended to higher pulse heights, using a slope derived from the actual pulse height spectrum. The actual pulse height spectrum probably lies between these two approximations.

The angle of photon entry into the TPC was taken to be 54° for all liquid radiator photons, and all gas radiator photons were assumed to have normal incidence. It was assumed that the average liquid ring contained four times as

Figure 3.1 Example of thrown ethane pulse height spectrum.

many photons as the average gas ring. Finally, the TMAE absorption length in the TPC was taken to be 1.6 cm.

A number of factors were not taken into account in the study, either because they were not known, or were specific to particular physics processes. For example, all of the tracks were assumed to travel through the liquid and gas radiators at normal incidence. In addition, the momentum distribution of particles was not simulated; it was assumed that all particles traveled at essentially infinite momentum. Finally, diffusion of the drifting electrons and the taper of the TPC were ignored.

Figure 3.2 Example of generated photon conversion depth distribution. Conversion depth is measured from the side adjacent to the liquid radiator.

The Monte Carlo generated and reconstructed individual photons. First, a pulse height was thrown using the ethane pulse height distribution (Figure 3.1). Next, a conversion depth in the TPC was generated, assuming an exponential distribution of conversion length inside the TPC, and using the photon angle in the TPC, a 1.6 cm TMAE absorption length, and an assumed 4:1 ratio of liquid-side photons to gas-side photons. An example of the conversion depth distribution is shown in Figure 3.2. Based on this conversion depth, the pulse height in each end of the fiber was calculated. Figure 3.3 shows an example of the preamplifier input pulse height distribution. Next, a $te^{-t/\tau}$

Figure 3.3 Example of preamplifier input pulse height distribution for preamplifier on liquid radiator side of TPC.

pulse shape was constructed, with the amplitude given by the preamplifier input pulse height just calculated. The pulse was digitized using the nominal sample interval of the CRID. The digitization was done with a simulated 10-bit ADC. In the real CRID a 12-bit ADC will be used, but calibration errors will probably mean that the values are not accurate to the full 12 bits. Gaussian noise was added to each sample, with the sigma of the noise distribution equal to 0.5% or 1% of the mean pulse height of the ethane distribution. The resulting data were passed to the deconvolution pulse finding algorithm. The amplitudes reconstructed by this algorithm were used to calculate the

conversion depth of the photon, and this result compared to the thrown conversion depth.

Two factors were considered in the resulting charge division resolution data. These were the percentage of photons falling in the ADC range, and the average conversion depth resolution on the pulses that were found. The results are shown in Figure 3.4, and indicate that the preamplifier gain should be set so that the mean ethane pulse height (2×10^5 electrons) falls at approximately 25% of the ADC range. Based on previous experience, this seemed somewhat low, and depended on the shape assumed for the high tail of the ethane pulse height distribution. The actual gain of the preamplifier, approximately $3 \mu\text{V}/\text{electron}$, places the mean pulse height at approximately 30% of the ADC range.

The charge division resolution obtained in this simulation is also surprisingly poor. There are three probable reasons for this. First, because of the TMAE absorption length and the liquid radiator photon angle, the photon conversion points are strongly biased towards the ends of the fiber, where resolution is poorest. In addition, the pulse height spectrum by definition contains a large number of pulses with amplitudes significantly below the average, where noise is much worse than the 1% nominal value. Finally, the noise added may be somewhat pessimistic. As can be seen in the figure, the resolution is very sensitive to the noise level assumed. For this study, it was assumed that the noise is a voltage noise, added independently to each sample. In reality, there will be some amount of voltage noise, but the majority of the

Figure 3.4 Pulse-finding efficiency and asymmetry resolution as a function of preamplifier gain for Monte Carlo data sample. The horizontal axis tells at what fraction of the full ADC range the peak preamplifier output occurs for an input of the mean pulse height (2×10^5 electrons).

nominal 1% noise may be a charge noise, which changes the pulse height, but affects all samples on a pulse more equally.

4. Pulse Finding

The general requirements for a good pulse finding algorithm are that it be efficient at finding single pulses, provide good rejection of fake signals and noise, and have good double pulse resolution. Because the CRID uses charge division to measure one coordinate, it is important that the algorithm accurately reconstruct pulse heights. In addition, the algorithm should also be fast and simple, if possible. A final constraint is imposed by the amount of data produced by the SLD. Since the data reduction must take place online, it is vital that the pulse finding algorithm be well understood. Once the data gets into the WSMs, the raw data are gone forever. There is no hope of reprocessing raw data at a later date.

This chapter discusses the development of a pulse finding algorithm suited to the CRID. It begins by discussing features of the CRID pulse shape. Following this, development of a pulse finding algorithm is described. Finally, tests of the algorithm on Monte Carlo and real data are described. At the time the work described here was carried out, the preamplifier time constant (τ) was 50 ns. Since that time, τ has been increased by 17 ns. However, the sampling interval has been increased proportionally, and so the results presented here should still be valid.

4.1 CRID Signal Properties

The photoelectron signals produced in the CRID drift boxes are quite narrow (characteristic avalanche time less than 1 ns)^[21] and their shape does not vary significantly from hit to hit. Because the photoelectron signals are very

narrow compared to the sampling interval of the CRID data acquisition system (67.2 ns/sample), and in order to minimize electronic noise, the CRID preamplifiers shape these fast pulses to much broader pulses. The impulse response of the preamplifier, $te^{-t/\tau}$, was chosen because it afforded low noise while using relatively few components. The ideal pulse shape, from a noise standpoint, is a cusp-like pulse with an e^{at} rising edge and an e^{-at} falling edge^[22]. Gaussian and triangular pulses are almost as good, but are also difficult or costly to build.

The time constant of the preamplifier, τ , was chosen to be equal to the sampling interval in order to minimize noise. The actual implementation of the preamplifier, shown in Figure 4.1, uses four sets of timing components which conspire to produce the $te^{-t/\tau}$ pulse shape.

4.2 Motivation for Considering Deconvolution

In many detectors, pulse shapes vary considerably from pulse to pulse, while pulse height remains relatively constant. The fact that the single-photoelectron CRID pulses all have the same shape, but vary in height, suggests that a different approach to pulse finding may be warranted. In fact, an algorithm based on deconvolution seems to be well-suited to the CRID. Deconvolution algorithms use knowledge of the pulse shape to help locate and separate pulses. In addition, it turns out that the $te^{-t/\tau}$ functional form of the preamplifier response makes the deconvolution equation simple.

As mentioned in the introduction, there are several properties which a pulse-finding algorithm should have; good efficiency and resolution for single

$$\tau_1 = R_1 C_1$$

$$\tau_2 = R_2 C_2$$

$$\tau_3 = \frac{R_2 R_3}{R_2 + R_3} C_2$$

$$\tau_4 = R_4 C_4$$

Figure 4.1 The upper drawing shows pulse shaping circuitry equivalent to that used in the CRID preamplifier. The lower drawing shows the Laplace transform (the frequency domain equivalent) of the circuitry. Nominal values for the time constants are at the time this study was conducted were: $\tau_1=800$ ns, $\tau_2=990$ ns, $\tau_3=\tau_4=50$ ns.

and double pulses, speed, and ease of implementation. Good resolution is needed in order to accurately determine the three spatial coordinates of photon conversion in the CRID drift boxes. Two of the coordinates are determined by the pulse amplitude and start time. The third spatial coordinate is simply given by number of the wire on which the signal is seen. The start time and amplitude can be more easily determined for narrow pulses than for the broadened preamplifier output pulses. In addition, it is easier to find two pulses near each other if the pulses are narrow, since pulse overlap is reduced.

Deconvolution provides a way to remove the pulse broadening which was deliberately introduced by the preamplifier. Deconvolved pulses are very narrow, because the signal which went into the preamplifier was very narrow. In addition, it will be shown that deconvolution can be implemented as a simple, fast algorithm that has good efficiency and excellent double-pulse resolution.

4.3 Derivation of Deconvolution Equation

The derivation of the deconvolution equation appropriate to the CRID makes use of the fact that convolution in the time domain becomes multiplication in the frequency domain. Therefore, the approach is to determine the frequency domain response of the preamplifier and use its reciprocal as the frequency-domain equation for implementing deconvolution. Laplace transforms are used in the calculations instead of Fourier transforms because it is traditional, they converge for a wider range of functions, and for functions which have both Fourier and Laplace transforms, both give identical

results. The approach used here is commonly used in filter and control system design.

As mentioned earlier, the impulse response, $g(t)$, of the CRID preamplifier is

$$g(t) = te^{-t/\tau}$$

The Laplace transform of this response is, with $a=1/\tau$,

$$G(s) = \mathcal{L}(g(t)) = \frac{1}{(s+a)^2}$$

$H(s)$, the deconvolution response needed to undo the effects of the preamplifier, is the reciprocal of this.

$$H(s) = \frac{1}{G(s)} = (s+a)^2$$

If we were dealing with an analog deconvolution circuit, this would be our design equation. Because the deconvolution is to be applied by a computer program to sampled data, we are dealing with a discrete-time system and so we must make a discrete approximation to $H(s)$. There are many approximations which can be used. We will choose a technique called pole-zero mapping, which maps the poles and zeros of $H(s)$ to analogous poles and zeros of a discrete-time system, $H(z)$. Explanations of the technique can be found in several signal-processing texts^[23]. The mapping is

$$z = e^{sT}$$

where z is the mapped location of the pole or zero, s is the original location of the pole or zero, and T is the sampling interval.

$H(s)$, the deconvolution equation we wish to map, has two zeros, both located at $s = -a$, and no poles. After mapping these, we obtain an expression for the discrete-time system

$$H(z) = z^2 - 2e^{-T/\tau}z + e^{-2T/\tau} = 1 - 2e^{-T/\tau}z^{-1} + e^{-2T/\tau}z^{-2}$$

where z^{-1} is a unit time delay.

The reason that it is possible to multiply $H(z)$ by z^2 and still claim that the resulting expression is equivalent to $H(z)$ is that multiplication by z^2 is just the introduction of a two sample time delay. In this application, such a delay does not matter.

The final step is to convert this equation into a difference equation, in much the same way that a Laplace transform can be converted back into a differential equation. The analogue of the Laplace transform is known as the z -transform, and is widely used in digital signal processing and controls. Discussions of the z -transform may be found in many texts^[23]. After applying the z -transform, we obtain a linear difference equation

$$h(k) = \mathcal{Z}^{-1}(H(z)) = x(k) - 2e^{-T/\tau} x(k-1) + e^{-2T/\tau} x(k-2)$$

where $\mathcal{Z}^{-1}$ is the inverse z-transform, $h(k)$ is the k^{th} deconvolved sample, $x(k)$ is the k^{th} preamplifier output sample, T is the time between samples, and τ is the preamplifier time constant.

Each deconvolved sample is a linear combination of three consecutive samples of the preamplifier output. This simple difference equation is applied to all of the data coming out of the preamplifier in order to perform the deconvolution.

4.4 Properties

Inspection of the deconvolution equation shows that all of the coefficients are constants, and so the computational cost of the deconvolution is only two multiplications and two additions per preamplifier output sample. Thus, the deconvolution is very fast. In addition, it can be seen that after the deconvolution has been applied, any $te^{-t/\tau}$ pulse will be reduced to at most two adjacent non-zero samples. In order to see this, first note that if we have zero as $x(k)$ we get zero as the deconvolution. Therefore, when we are on a pedestal, the deconvolution will produce zero. Next, let us determine what happens if all three samples used in the deconvolution are on a pulse. Copying from above, the deconvolution equation is

$$h(k) = x(k) - 2e^{-T/\tau} x(k-1) + e^{-2T/\tau} x(k-2)$$

Substituting the $te^{-t/\tau}$ pulse shape for $x(k)$, we obtain for $h(k)$

$$\begin{aligned}h(k) &= te^{-t/\tau} - 2e^{-T/\tau}(t-T)e^{-(t-T)/\tau} + e^{-2T/\tau}(t-2T)e^{-(t-2T)/\tau} \\&= e^{-t/\tau} (t - 2e^{-T/\tau}(t-T)e^{T/\tau} + e^{-2T/\tau}(t-2T)e^{2T/\tau}) \\&= e^{-t/\tau} (t - 2(t-T) + (t-2T)) \\&= e^{-t/\tau} (t - 2t + 2T + t - 2T) \\&= e^{-t/\tau} (0) \\&= 0\end{aligned}$$

Therefore, the deconvolution also produces zero whenever all three samples are on a pulse. The only time when it is possible for the deconvolution to produce non-zero data is when either one or two samples are on a pulse, and the third is still on the pedestal. The deconvolution has removed the pulse broadening, as it was supposed to, by reducing all single-photoelectron pulses to one or two non-zero samples. Since the quantities of interest are the amplitude and start time of the pulse, a method to determine them from the deconvolved data is needed. The start time is determined from the ratio of the two non-zero samples, and the amplitude is determined by the sum of the two samples, multiplied by a start-time-dependent factor. The amplitude correction is no greater than 15%. Both the amplitude correction factor and the ratio-to-time conversion factor are determined by generating pulses of known start time and amplitude, deconvolving them, and calculating the ratio and reconstructed amplitude. From this, a lookup table of ratio vs. start time and amplitude correction factor is constructed. Figure 4.2 shows plots of ratio and amplitude correction factor vs. pulse start time.

Figure 4.2 Ratio and amplitude correction factor vs. pulse start time. The upper plot shows the amplitude correction factor. The reconstructed pulse height is multiplied by this to obtain the actual pulse height. The lower plot shows the ratio of the first to the second nonzero deconvolved samples.

4.5 Performance on Monte Carlo Data

Tests of the deconvolution method were performed using simulated data. Pulses were generated with a $te^{-t/\tau}$ pulse shape, a known time constant, and a fixed amplitude. The peak amplitude of the pulses was chosen to be 73 ADC counts. The actual value was not critical, although it was chosen to be large enough that amplitude quantization would not dominate resolution, but not so large that the amplitude quantization was negligible. The pulses were sampled at the nominal CRID sampling rate. Next, a Gaussian noise of 1% of the peak pulse amplitude was added to each sample, to simulate some form of electronic noise. This amount of noise was chosen to approximate the noise for an 'average' pulse in the real CRID. Finally, the amplitudes of the samples were quantized, to simulate the effect of an ADC. Under these conditions, the amplitude resolution was 1.1%, which is adequate, and about as good as can be expected given the amount of noise added. The timing resolution was 0.05 samples, which is excellent. Because of the belief that double-pulse resolution would be dominated by the problems described in the next section, very little testing of double pulse resolution was done with simulated data. However, under ideal circumstances, where the pulse shape is $te^{-t/\tau}$ and the time constant is known, two pulses will be cleanly resolved if they are separated by two or more samples. Thus, under favorable conditions, where one pulse starts just before a sample is taken, the second can start just after the following sample, and the double pulse resolution is equal to the sample interval. Under unfavorable conditions, where one pulse starts just after a sample is taken, a second pulse will not be resolved unless it starts two sample intervals later. In some sense then, the average double pulse resolution is 1.5 times the sample interval. It should be noted that the double pulse resolution is independent of

the relative amplitudes of the two pulses, until noise or quantization becomes important. Because deconvolution has knowledge of the correct pulse shape, it can find double pulses which are invisible to the eye and to more traditional pulse-finding methods. Figure 4.3 shows an example of the double-pulse resolution potential of deconvolution. The sensitivity of the method to noise and to an incorrect preamplifier time constant was also measured. Results, shown in Figure 4.4 and Figure 4.5, indicated that performance degraded smoothly as noise increased, and that time constant variations of a few percent (the expected range) were acceptable.

4.6 Potential Problems

Because deconvolution relies on knowledge of the pulse shape, anything which alters the pulse shape can reduce the accuracy of deconvolution. There are three potential problems now known.

4.6.1 Chamber Positive Ion Response

Although the pulses coming out of the chamber have a very narrow peak, they have a long tail due to the drifting of positive ions in the chamber. The total positive ion drift time in a CRID chamber is approximately $30\mu\text{s}$ ^[24]. An initial estimate of the effect of this on the preamplifier pulse shape used a sum-of-exponentials approximation for the positive ion response. Based on this, it was decided that positive ions would not affect the pulse shape noticeably. It was later discovered that the sum-of-exponentials is not a good fit to the real data, and a more accurate model of the positive ion response was developed^[24]. Simulated pulses generated with the more accurate model were compared with real chamber pulses to make sure that the new model was

Figure 4.3 Example of double pulse resolution using deconvolution. The upper plot shows a signal composed of two pulses, differing in amplitude by a factor of 10. The first pulse starts at sample 2.5, the second at sample 5.5. No noise has been added to the signal. The lower plot shows the result of deconvolution. Two pulses are clearly visible. Each is composed of two adjacent nonzero samples.

Figure 4.4 Effect of noise on amplitude and time resolution. Gaussian noise was added to each sample and is expressed as a percentage of the peak pulse height.

Figure 4.5 Effect of error in preamplifier time constant (τ) on amplitude and timing resolution. The scale on the horizontal axis is the ratio of the preamplifier time constant to the sample interval (T). The sign of the timing error depends on whether the reconstructed pulse start time is earlier or later than the actual pulse start time.

adequate. Tests using simulated data with the newer model of positive ion response show that the effect of the positive ions can be well approximated as an increase in the effective preamplifier time constant. Figure 4.6 shows the effect of positive ion response on the preamplifier output pulse shape and on the deconvolved data.

4.6.2 Finite Frequency Response of Preamplifier

The ideal preamplifier pulse shape requires infinite frequency response in order to turn on sharply at the leading edge of the pulse. In reality, the preamplifier frequency response is finite. The primary effect of the finite frequency response is a rounding of the leading edge of the pulse. This results in incorrect ratios being found for pulses which start near a sampling instant. This, in turn, causes errors in determining both the start time and the amplitude (the amplitude is affected because the ratio is used to look up an amplitude correction factor). To study the effect of finite preamplifier frequency response, it was modeled as a single additional first order pole in the preamplifier's frequency response. While the actual preamplifier is much more complicated, it was believed that such a simple model would give qualitatively correct results. Figure 4.7 shows the effect of finite preamplifier frequency response on amplitude and timing resolution. In algorithms which implement the deconvolution method, pulse-rounding can be corrected for by using a smoothed pulse shape for generating the time vs. ratio lookup table^[25].

4.6.3 Mismatch of Timing Components in Preamplifier

In practice, the four sets of timing components in the CRID preamplifier will never be perfectly matched. Studies were performed to determine the

Figure 4.6 Effect of positive ion response of chamber on preamplifier output and deconvolved data. Upper plot shows preamplifier output with and without chamber positive ion response. Both the nominal preamplifier time constant and sampling interval are 50 ns. The lower plot shows the preamplifier output after deconvolution. A 50 ns time constant used in the deconvolution without ion response and a 68 ns time constant was used in the deconvolution with the ion response included.

Figure 4.7 Effect of finite preamplifier transistor time constant on amplitude and timing resolution. The numbers are based on a 50 ns preamplifier time constant and sampling interval. The effect of the transistor time constant is parametrized as a single additional first-order pole to the preamplifier transfer function. The sign of the timing error depends on whether the reconstructed start time is earlier or later than the actual pulse start time.

Figure 4.8 Effect of varying individual time constants in preamplifier. Each time constant was varied while the others were held at their nominal values. The sampling interval was 50 ns.

effect of errors in component matching on amplitude and start time resolution. The studies used simulated data generated using a model of the preamplifier correctly including four time constants. Each of the time constants was varied, singly and in groups, in the course of the study. The results of these tests, shown in Figure 4.8, indicate that for the several percent variations expected in the timing component values, resolution is not seriously degraded.

4.7 Ongoing Development

While it appears that deconvolution will work well for CRID pulse finding, there remain several issues that need to be investigated in order to

Figure 4.9 Pulse shape of final CRID preamplifier. The curve shows a fit to a $1/e^{-1/\tau}$ pulse shape.

adapt the algorithm to the real world and tune it. For the most part, these can only be investigated in the final CRID system. A few of the remaining issues are mentioned in this section.

A plot of the pulse shape from a CRID preamplifier is shown in Figure 4.9. While the pulse shape is relatively close to the desired shape, there are some differences, particularly in the tail, where the preamplifier undershoots the desired response. Some of this undershoot was deliberately introduced, in order to provide some compensation for the chamber's positive ion response,

mentioned earlier. Tests to determine the quality of the pulse shape produced by the chamber /preamplifier system have not yet been carried out.

Some rounding of the leading edge of the pulse shape can be seen, as expected. It is not yet certain that using a smoothed time vs. ratio lookup table is sufficient to compensate for this unavoidable aberration in the pulse shape.

Finally, the signal produced by the passage of a minimum ionizing particle must be handled in a reasonable manner by the pulse finding. The preamplifier's response to a simulated minimum ionizing particle is shown in Figure 4.10. It is obvious that this situation must be recognized and treated specially by the pulse finding. What is not certain at this time, however, is how and when to resume pulse finding after passage of a minimum ionizing particle. While deconvolution does not perform well on a rapidly moving baseline, it is difficult to believe that any algorithm can accurately reconstruct single photoelectron signals occurring in the first few hundred nanoseconds after passage of a minimum ionizing signal. It may turn out that the best alternative is to do no pulse finding until the baseline settles down, but studies are needed to determine at what point the pulse finding can be resumed.

Figure 4.10 Preamplifier recovery after a signal equivalent to that deposited by a minimum ionizing particle. For this pulse, 16 pC were deposited over a 200 ns interval, beginning at $t=0$.

5. Strangeonium Spectroscopy

There are a number of reasons for interest in strangeonium spectroscopy, the spectroscopy of $s\bar{s}$ mesons. First, as can be seen in Table 5.1, much less is known about the spectrum of strangeonium states than the spectrum of unflavored and strange mesons. Only three $s\bar{s}$ mesons, the ϕ , the $f_2'(1525)$ and the $\phi_3(1850)$, are relatively well-established at this time. Others, such as the $f_1(1420)$, are currently assigned to nonets, but the assignments are very uncertain.

Strangeonia can also give additional constraints on models of the interquark potential. The relatively simple potential models, for example the model of Godfrey and Isgur^[26], seem to work quite well, but additional tests of such models are still needed, and the discovery and classification of additional states can help to refine models. Mesons containing only u and d quarks probe the interquark potential at relatively long distances, where the confining potential dominates. At the other extreme, mesons containing the heavy c and b quarks probe the interquark potential at short distances, where the Coulomb potential is more important. Strangeonia can provide a probe of the interquark potential at intermediate distances, where both the confining and Coulomb forces are important. Thus, study of strangeonia can help to provide a bridge, and a sensitive test of models of the interquark potential.

Finally, in any study searching for exotic states (glueballs, hybrids, 4-quark states) it is important to know what the non-exotic states are, since these are the background in searches for exotics. Therefore, it is important to

$2S+1L_J$	J^{PC}	$u\bar{d}, u\bar{u}, d\bar{d}(I=1)$	$\bar{s}u, \bar{s}d$	$s\bar{s}$
1S_0	0^{-+}	π	K	
3S_1	1^{--}	ρ	$K^*(892)$	ϕ
1P_1	1^{+-}	$b_1(1235)$	K_{1B}	
3P_0	0^{++}	$a_0(980)$	$K^*_0(1430)$	
3P_1	1^{++}	$a_1(1260)$	K_{1A}	
3P_2	2^{++}	$a_2(1320)$	$K^*_2(1430)$	$f'_2(1525)$
1D_2	2^{-+}	$\pi_2(1670)$		
3D_1	1^{--}			
3D_2	2^{--}		$K_2(1770)$	
3D_3	3^{--}	$\rho_3(1690)$	$K^*_3(1780)$	$\phi_3(1850)$

Table 5.1 Summary of some of the known meson states. The η and η' have been left out of 1S_0 because the mixing of the pseudoscalar nonet makes neither of them strongly $s\bar{s}$.

know what the strangeonium spectrum is. In hadroproduction experiments like that discussed here, the production cross section is expected to be dominated by quark exchange processes, so that glueball production will be greatly suppressed relative to the total cross section. As a result, any glueball candidate should not be strongly produced in this experiment. For example, the $f_2(1720)$ is a glueball candidate. This object is seen strongly in radiative J/ψ in the Mark II^{[28][28]}, but is not seen at all in this experiment^[29]. This provides additional evidence that the $f_2(1720)$ may indeed not be a normal $q\bar{q}$ meson.

In kaon beam experiments, strangeonium states are best studied in hypercharge exchange channels. If analysis is restricted to peripheral processes so that an exchange picture is valid, the exchanged particle carries both strangeness and charge, converting the target proton to an uncharged, singly strange baryon (i.e. a Λ or Σ^0). The final state for these reactions thus consists of a Λ/Σ^0 and other particles. Figure 5.1 shows the quark line diagrams for two possible strangeonium production processes. In the forward Λ process, none of the initial state quarks carry over into the $s\bar{s}$ system. Such diagrams are known to be strongly suppressed. In addition, the forward Λ process involves baryon exchange, while the backward Λ process involves meson exchange. The cross section for baryon exchange processes drops more rapidly with increasing beam energy than for meson exchange processes. For both reasons, the hypercharge exchange (backward Λ) process is expected to dominate.

There are many hypercharge exchange states which can be studied in data from this experiment. Some of the channels have already been explored, including K^+K^- [30], $K_S^0 K^\pm \pi^\mp$ [31], and $K_S^0 K_S^0$. Others are currently being analyzed. This study will concentrate on a group of low cross section strangeonium channels, including $\Lambda\eta\pi^+\pi^-$, $\Lambda K^0 \bar{K}^0$, and $\Lambda\phi\phi$.

After a description of the LASS Spectrometer, including the reconstruction software and fitting software used in this experiment, the analysis will be presented. The most general selection (the inclusive Λ selection) will be discussed first, followed by discussions of each of the exclusive channels mentioned above.

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Figure 5.1 Example quark line diagrams for strangeonium production in a kaon beam. (a) shows a configuration referred to as *backward* Λ , in which the Λ follows the target proton. (b) shows a configuration referred to as *forward* Λ , in which the Λ follows the beam kaon.

6. The LASS Spectrometer

The Large Aperture Superconducting Solenoid (LASS) spectrometer was built and operated at SLAC through 1982. An overview of the spectrometer is shown in Figure 6.1, and a complete description has been published elsewhere^[32]. The spectrometer consisted of two main sections, a solenoidal spectrometer and a dipole spectrometer, which together provided almost 4π solid angle coverage. Two sections were used in order to obtain good momentum resolution, shown in Figure 6.2, and coverage for a wide range of particle momenta. Slow, wide-angle particles were well-reconstructed in the solenoid, while fast forward particles were reconstructed by the dipole section. The solenoidal section had a magnetic field of 22.4 kG, produced by a four-segment superconducting magnet with an inside diameter of 1.9 m and a length of 4.2 m. The dipole section used a conventional water-cooled aluminum magnet to produce a maximum field of 18.0 kG and a field integral of 30.1 kG-meters in a 1 m tall gap. The magnet pole pieces were 1.8 m wide and 1.1 m along the beam direction. For this experiment, a liquid hydrogen target was located in the solenoid.

Tracking was accomplished using a combination of magnetostrictive spark chambers located in the dipole region, cylindrical proportional wire chambers in the solenoidal section, and planar proportional wire chambers in both regions. Particle identification was provided by two multi-cell Cherenkov counters, a time-of-flight hodoscope, and dE/dx (using the cylindrical chambers). Each of these will be discussed in turn.

Figure 6.1 Overview of the LASS Spectrometer

The experiment described in this work, experiment E-135, took data with both K^- and K^+ beams during several runs in 1981 and 1982. A total of 1.4×10^8 triggers were taken, with about four times as many events taken using the K^- beam as the K^+ beam. The analyses described here use only the K^- data. Event reconstruction was performed at SLAC and Nagoya University in Japan, and required the equivalent of two CPU years on SLAC's two-CPU IBM 3081K. Reconstruction was finished in 1985. The resulting ~ 1000 data summary tapes (DST) have been used for a number studies of strange meson spectroscopy, strangeonium spectroscopy, and strangeness -2 and -3 baryons.

Figure 6.2 Momentum Resolution for the LASS Spectrometer, as used in experiment E-135. Resolution was determined from Monte Carlo data.

6.1 Beam

The beam used for experiment E-135 was an 11 GeV/c RF separated kaon beam, produced from the impact of 21 GeV/c electrons from the SLAC linac on a copper and beryllium production target. The electrons produced electromagnetic showers in the target. Secondary particles produced in these showers with an angle less than 17.45 ± 3.85 mrad were collected. The beamline following the production target first limited $\delta p/p$ to 2.5%, then removed remaining electrons with a 0.56 radiation length lead absorber. Next, one RF separator cleaned the beam. Different species of particles, which arrived in the RF separator at different times, received different accelerations, depending on the RF phase at the time of their arrival. Following the RF separator, a collimator was used to selectively pass the desired particle species (kaons, in this experiment).

The momentum of the beam was measured by an array of six overlapping scintillator paddles located at a momentum-dispersed focus. This array was able to measure momenta to 0.5% (full width). Further down the beamline, at a location 13.1 m upstream of the solenoid, another array of scintillator paddles measured the position of the beam. This array consisted of twelve 1/2" wide horizontal paddles and twelve 1/2" wide vertical paddles.

Two gas-filled threshold Cherenkov counters were next in the beamline. Both used parabolic focussing mirrors at the downstream end, and a photomultiplier read out the counter. The first of the Cherenkov counters, C_π , was filled with hydrogen at a pressure of 40 psia. Only pions produced light in this counter. The second counter, C_K , was filled with carbon dioxide at a

pressure of 75 psia, and produced light for both pions and kaons. Based on these devices, it was determined that the beam had a 70-80 K/ π ratio, and that the contamination of other particles in the beam was negligible.

Just upstream of the solenoid, the position of the beam was measured again, using a series of proportional chambers. This device consisted of 10 proportional wire planes, each containing sixty-four 20 μ m gold-tungsten sense wires, placed 1.016 mm apart. The chambers were divided into two distinct groups, approximately one meter apart. The wires in the upstream group of four planes were oriented horizontal, vertical, and $\pm 45^\circ$ to vertical, while the downstream set of six planes had two horizontal planes, two vertical planes, one plane oriented $+45^\circ$, and one plane oriented -45° . The pairs of horizontal and vertical planes in the downstream group were offset by 1/2 the wire spacing relative to each other. This package provided a measurement of the beam position to 300 μ m, and angle to 0.3 mrad.

Finally, three scintillators were located near the target. The first of these, known as the SE counter, was located 5m upstream of the target. This device counted both beam and halo particles, and provided an overall timing signal for the fast electronics. The 4"x4" SE counter was made especially thick (0.375") in order to produce a large signal with little time jitter. This counter also served to start timing for the time-of-flight system. The other two counters, SY and R, were located at the solenoid entrance, and served to separately identify halo and beam particles. SY was 2.54 cm diameter x 0.125" thick, and R was a 0.5" thick paddle with a 2.54 cm diameter hole in it.

The beam size in the target had an RMS of 0.5 cm in each dimension. The beam spills onto the target were 1.6 μ s long, and the beam typically ran at a 90 Hz repetition rate during data taking. The yield was approximately 4-5 kaons per pulse, and resulted in approximately 16 triggers per second, averaged over the running time of the experiment.

6.2 Target

The target for this experiment consisted of a cylinder filled with liquid hydrogen at approximately 20 K. At room temperature, the cylinder was 84 cm long and had a radius of 2.54 cm. The length shrank to 84 cm when the target was filled with liquid hydrogen. The liquid hydrogen was contained in a mylar bag, surrounded by mylar super-insulation, inside a 0.71 mm thick aluminum vacuum jacket. The vacuum jacket was attached by steel bracing to the upstream mirror plate of the solenoid. The temperature and pressure of the hydrogen were monitored over the course of the experiment. It was important to monitor both these quantities in order to obtain the density of the material in the target. This, in turn, was important in order to determine the interaction cross section. In order to escape the 2.54 cm target, protons must have a momentum of at least 225 MeV/c. This value holds only if the protons travel radially; any longitudinal motion increases the path length in the target, and hence the energy needed to escape the target.

6.3 Solenoid Region

An overview of the solenoid region of the LASS spectrometer is shown in Figure 6.3. The solenoid region included the cylindrical chambers, gap chambers, plug chambers, bore chambers, and trigger chambers, as well as

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Figure 6.3 Solenoid Region of LASS Spectrometer.

Cherenkov counter C₁ and the time of flight hodoscope. Each of these is discussed below.

6.3.1 Cylindrical Chambers

The target was surrounded by a group of six cylindrical proportional wire chambers, with radii of 6.05, 9.55, 12.99, 16.55, 29.41, and 49.02 cm. The chambers used 20 μ m gold-tungsten wires, sandwiched between layers of foil cathode strips. The foil strips were on an aluminum/mylar laminate and honeycomb paper core. On the inner four chambers, the cathode strips were placed at $\pm 10^\circ$ angles with respect to the wires, while on the outer two cylinders they were placed at $\pm 15^\circ$. The four inner chambers had active wire lengths of

100 cm and the outer two chambers had active wire lengths of 87 cm. The wires were anchored to fiberglass circuit boards, and a Lucite support ring was placed halfway down the cylinder.

Although the cylinders were not designed to measure dE/dx for particles passing through them, it nevertheless proved possible to obtain dE/dx particle identification information from them, as will be described later.

6.3.2 Planar Chambers

Each of the three sections of the solenoid magnet downstream of the target was instrumented with a group of chambers; the gap, bore, plug, and trigger chambers.

The gap chambers contained single wire planes spanning the entire inner diameter of the solenoid. Each gap chamber contained 758 anode wires at 2.032 mm intervals. The cathodes for these chambers were made of aluminum-mylar laminate, with 6.86 mm etched strips spaced 1.27 mm apart and oriented at $\pm 45^\circ$ to the wires. The cathode and anode were separated by 0.508 cm. The center 16.51 cm of these chambers was covered with a styrofoam-backed mylar barrier that served to deaden these otherwise very active regions. Tracks in these deadened regions were read out using the plug chambers.

The plug chambers were quite small, measuring only 26 cm x 28 cm. Three chambers were located near the gap chambers and two additional plug chambers were located downstream of the solenoid. The plug chambers were

constructed of 256 anode wires at 1.016 mm intervals. Each chamber contained three anode planes, oriented to read out x , y , and e coordinates, where e is defined to be 35° from the x coordinate. These chambers worked well in the high-rate (5-10 MHz instantaneous) region near the beam axis.

The bore chambers were used to provide measurements inside, as opposed to between, solenoid magnet segments. Each bore chamber was octagonal, to maximize coverage, and consisted of three planes, oriented to read out x , y , and e , where e here is oriented at 45° .

Finally, the trigger chambers were intended to be used to provide a high p_t trigger or to measure track multiplicities. In this experiment, they were only used to provide redundancy for tracking. Each trigger chamber consisted of an anode and segmented cathode planes. The anode was not used to provide signals in this chamber, only to produce a field. Because of this, the anode wires were spaced widely, at 4.064 mm. The upstream cathode had two concentric rings of pads on it. The inner ring contained 8 pads and the outer ring contained 128 pads. The downstream cathode was similarly divided into eight segments on the inner ring, but into only sixteen segments in the outer ring. The downstream cathode of the most downstream chamber was not read out.

6.3.3 Cherenkov Counter C_1

This threshold Cherenkov counter occupied a region just downstream of the solenoid and extended into the most downstream of the four solenoid coils.

The counter was filled with Freon 114 gas at atmospheric pressure. With this, the threshold for π 's to emit light was 2.6 GeV/c, while the K threshold was 9.2 GeV/c. The path length of a particle traveling in the counter was at least 180 cm. The counter was divided into 38 optically isolated cells, organized into four rings. The innermost ring, an 8.5 cm radius ring known as the D ring, was simply divided into two half circles. In addition, a helium bag deadened the central region of the D ring, where the beam passed through them. The outer three rings, with radii approximately 29.3 cm, 52.0 cm, and 83.0 cm, were each divided azimuthally into twelve 30° sections. The optical separation of cells was achieved using mylar sheets, double coated with aluminum. The counter segmentation did not extend all the way to the forward edge of the counter, but instead stopped approximately 40 cm from the upstream end. Because of this, it was possible for a particle to produce light in more than one cell, even if it passed through only one cell. At the downstream end of each cell, the light was reflected normal to the beam axis. The light then encountered an optical system consisting of a Fresnel lens, a light horn, and a photomultiplier tube. Because of interference with the floor, the three bottom cells in the A, B, and C rings had a somewhat different geometrical arrangement in their optical system. Unfortunately, the horns on these cells proved to be somewhat inefficient in collecting light from a particular region of the cell. As a result, the performance of these few cells was not as good as the performance of the other cells.

The collected light was read out using RCA 8850 Quantacon photomultipliers, which have exceptionally good single photon response and high quantum efficiency. The tubes were shielded in iron and μ -metal pipes, to

protect them from stray magnetic fields. In addition, a set of bucking coils are wound onto the inner magnetic shield, although these coils were not used.

Pulse height calibrations were carried out regularly, to ensure that the single photoelectron peak stayed approximately 150 counts above pedestal in the ADC. After the photomultiplier gains were initially set, there was little change, and photomultiplier voltages rarely needed to be adjusted.

The performance of the counter was studied using a large sample of pions from K^0 decays. These pions were used to determine the probability of detecting light in each cell as a function of momentum and the impact position of the track on the face of the cell. If a track passed within approximately 3 cm of a cell boundary, the adjacent cell was also checked for light. The response of each of the rings is shown in Figure 6.4. The curves in the figure correspond to simultaneous fits to the efficiency data in the three rings using an expression consisting of a constant background, an asymptotic efficiency value, and a threshold rise with a $\sin^2\theta_C$ form.

The resulting value of the pion threshold momentum is 2.61 GeV/c, as expected. The asymptotic efficiencies determined in this manner for the A, B, and C rings were 0.971 ± 0.006 , 0.987 ± 0.002 , and 0.998 ± 0.002 . In calculating these efficiencies, the inefficient regions of the bottom three cells of each ring have been omitted, as have tracks passing within 3 cm of cell boundaries. The excluded regions of the bottom three cells in the A, B, and C rings were approximately 50% efficient.

Figure 6.4 Momentum dependence of the efficiency of the three rings of Cherenkov counter C₁. The curves are described in the text.

Figure 6.5 Scintillator hodoscope used for time of flight measurements.

6.3.4 Time of Flight

The time of flight consisted of two main parts; the SE counter, located upstream of the target, and a segmented scintillator hodoscope, located immediately downstream of the Cherenkov counter C_1 . The hodoscope, shown in Figure 6.5, was circular and consisted of 24 wedges of scintillator. Each segment was 1 cm thick and covered a 15° angle. Four additional segments

(25-28) filled most of the central region of the counter, leaving a 3.8 cm radius hole in the center of the hodoscope. Readout of the wedges was accomplished using shielded photomultipliers, which were read into both ADCs and TDCs, while segments 25-28 were used as a hodoscope only, with no TDC or ADC readout. The TDCs were started by a signal from the SE counter, mentioned earlier. When a photomultiplier in the TOF counter sensed light, the associated TDC was stopped, and the pulse height was recorded in the ADC. The ADC information was used to correct the TDC information for "time walk". The resulting time measurement had an average resolution of approximately 0.5 ns, with individual paddles having resolutions between 0.3 ns and 0.6 ns. Particle identification information was derived from the TDC value by calculating a time of flight for electron, π , K, and p mass hypotheses, based on the track momentum and the point of origin of the track, and comparing the calculated and measured times. Results were expressed as a number of standard deviations from each mass hypotheses, and were referred to as TOFDEVs.

The TOF system exhibited approximately 3σ separation of e/π up to 350 MeV/c, π/K up to 1.1 GeV/c, and π/p up to 2.5 GeV/c. Timing resolutions were determined using high momentum (> 3 GeV/c) negative tracks produced at the primary vertex. These tracks were grouped according to trigger and operating conditions, and separate TOF constants were obtained for each group. Within each group, adjustments were made for small run-by-run drifts.

6.4 Dipole Region

The dipole region of the spectrometer, used to measure the momentum of particles within 50-100 mrad of the beam axis, contained magnetostrictive

Figure 6.6 Dipole region of the LASS spectrometer.

chambers, proportional chambers, two hodoscope arrays, and a Cherenkov counter. A view of this region of the spectrometer is shown in Figure 6.6.

6.4.1 Magnetostrictive Chambers

The magnetostrictive chambers provided most of the coordinates for tracking in the dipole. This type of chamber was chosen because it provided, at low cost, good resolution and high efficiency over a large area. The chambers operated as spark chambers, while readout was accomplished by means of a magnetostrictive wire that efficiently converts spark energy to a traveling acoustic pulse. Each chamber contained two modules, each of which consisted

of two planes of woven wire cloth separated by 1 cm. One of the planes was kept at high voltage, while the other plane was kept at ground. When a trigger occurred, a thyratron was used to generate a high-voltage pulse that caused a spark wherever ionization had been left by charged particle passage. The current produced in the wire cloth by the spark traveled to the ends of the wires, where it was converted to an acoustic signal by a magnetostrictive wand. Each wand was read out by a multi-hit TDC.

The first five chambers were 150 cm x 300 cm, while the downstream two chambers were 200 cm x 400 cm. The upstream module in each chamber read x and y , while the downstream module read at $\pm 30^\circ$. Polyurethane plugs were installed near the beam axis, to deaden this otherwise extremely high activity region. Leaving this area active would have seriously degraded chamber performance. The chambers located downstream of the dipole were displaced horizontally, so that the dipole-bent beam would still pass through the polyurethane plugs.

Each y plane was read out using a single magnetostrictive wand, as were the remaining planes in the first two chambers downstream of the dipole. All other chambers made use of two magnetostrictive wands, located on opposite sides of the chamber. Each wire plane had two fiducial wires placed along the side of the plane. These fiducial wires were electrically pulsed by the trigger. The resulting signals were used to calibrate the time scale for each event. The full pulse train from each wand was digitized, stored, and read out using a CAMAC module. The digitization had a 20 MHz clock frequency, resulting in approximately 0.27 mm resolution.

In order to compensate the upstream magnetostrictive chambers for the large stray magnetic fields, each wand was magnetically biased using a coil of wire wrapped around it.

6.4.2 Proportional Chambers

In addition to the two previously described plug chambers, the dipole region contained three proportional chambers, each of which consisted of a single anode plane of 512 vertical wires, with 4.23 mm wire spacing. The aluminum and polyester mesh cathodes of the JHUP and JHDN chambers had a 1.27 mm gap width, while the JHXY chamber used a segmented aluminum-mylar foil cathode of 128 2.62 mm horizontal strips, separated by 0.16 cm gaps. The JHUP and JHDN chambers measured only the x coordinate. The JHUP, JHDN, and JHXY chambers were used primarily to provide in-time corroboration and position information for dipole tracks. The efficiency of the y readout of JHXY was low enough (50%) so that the chamber was of limited use.

6.4.3 Scintillator Hodoscopes

Two scintillator hodoscopes, HA and HB, were located immediately downstream of the last magnetostrictive chambers. The HA hodoscope consisted of two rows of 21 83.82 cm long paddles. The center paddle in each row was offset vertically to create a hole through which the beam passed. The HB hodoscope was similar, except that each row contained 38 paddles. These hodoscopes trigger the spectrometer on events with dipole particles, and serve as in-time corroboration for tracking. A small circular counter, LP3, covered the

hole in the beam region and was used as an anti-coincidence counter, to ensure that there were no downstream beam tracks in otherwise good events.

6.4.4 Cherenkov Counter C₂

Threshold Cherenkov counter C₂ was the furthest downstream element in the spectrometer. It was used to provide π/K separation for fast dipole tracks. The counter was divided into eight segments, and was filled with 1.1 atmospheres of Freon 12 (CCl₂F₂) gas. Particles passing through C₂ entered through a 2 mm thick aluminum sheet and then passed through at least 175 cm of gas. Two rows of four mirrors, tilted 10° from the vertical, were used to reflect radiated photons into light pipes, then onto XP2041 photomultipliers. Four photomultipliers were located on the top of the counter and four were located on the bottom.

Pion threshold in the counter was 2.9 GeV/c and kaon threshold was 10.3 GeV/c. However, the effective thresholds were approximately 10% higher, due to geometry and the use of unshielded photomultipliers.

6.5 Trigger

In order to capture an unbiased physics sample for analysis, the event trigger requirements were quite minimal. The basic requirement was that a beam particle must enter the target and not emerge from it, and two or more charged particles must emerge from the target. In addition to this trigger, a few special purpose triggers were also used to monitor the spectrometer performance. The trigger can be divided into the beam trigger logic and the cluster logic, each of which is discussed below.

6.5.1 Beam Trigger Logic

The purpose of the beam logic was to identify and select beam particles, and to reject events with multiple beam particle candidates.

In order to be considered an acceptable beam for an event, a beam particle must be identified as a kaon and it must enter the target. In addition, there must not be a second beam within a ± 16 ns time window. The C_K and C_π Cherenkov counters, discussed earlier, were used to identify the particle as a kaon. A particle entered the target if signals were seen in the SE and SY paddles, and no signal was seen in the beam halo counter, R. Finally, signals from the θ - Φ hodoscope were used to ensure that no two beams occurred within 16 ns. The complete beam trigger can be written as:

$$\text{BEAMTRIG} = \theta\Phi \geq 1 \text{ and } \overline{\theta\Phi \geq 2} \text{ and SE and SY and } \bar{R} \text{ and } C_K \text{ and } \bar{C}_\pi$$

6.5.2 Cluster Logic

This section of the trigger logic determined when a suitable interaction had occurred in the target. To do so, it counted the number of hits on the anode wires of the two innermost cylinder chambers and the first two plug chambers. The cluster count from each anode plane was used to form the signals in Table 6.1. The central 32 wires of each plane of plug 1 were omitted in the formation of these signals, in order to further protect against multiple beams. In addition, only the center wires of plug 2 (64 wires for y and z coordinates, 80 wires for x) were included in the count, to allow events with a single forward track and a cylinder track to trigger the spectrometer. Each signal in Table 6.1 includes the requirement that cylinder 2 must have at least one hit, because cylinder 1 had

Signal	Definition
$PLG_1 \geq 1$	At least 2 of ($PLG_1 x \geq 1, PLG_1 y \geq 1, PLG_1 e \geq 1$)
$PLG_1 \geq 2$	At least 1 of ($PLG_1 x \geq 2, PLG_1 y \geq 2, PLG_1 e \geq 2$)
$PLG_2 \geq 1$	At least 2 of ($PLG_2 x \geq 1, PLG_2 y \geq 1, PLG_2 e \geq 1$)
$PLGS \geq 1$	$PLG_1 \geq 1$ or $PLG_2 \geq 1$
$CYL \geq 2$	$CYL_2 \geq 1$ and ($CYL_1 \geq 2$ or $CYL_2 \geq 2$)
$CYL \geq 3$	$CYL_2 \geq 1$ and ($CYL_1 \geq 3$ or $CYL_2 \geq 3$)
$\overline{CYL} \geq 3$	$\overline{CYL_1} \geq 3$ and $\overline{CYL_2} \geq 3$
$CYL = 2$	$CYL_2 \geq 2$ and $\overline{CYL} \geq 3$
$CYL = 1$ or 2	$CYL_2 \geq 1$ and $\overline{CYL} \geq 3$

Table 6.1 Cluster logic for event trigger. PLG and CYL denote plug and cylindrical chambers, respectively. Counts denote the number of hits in a chamber.

a high accidental rate. Finally, the $PLG_1 \geq 2$ signal was relaxed after approximately 25% of the data had been taken, in order to improve trigger efficiency. Prior to the change, the requirement for this signal was that at least two (instead of one) of the planes had to have at least two hits.

6.5.3 Complete Event Trigger

The complete set of triggers included physics triggers and scaled triggers. The physics triggers were $T0_1, T0_2, T0_3$, and $T0_4$. In addition, there were four scaled triggers, $T1, T2, T3$, and $T4$, used for calibration and diagnostic

purposes. The complete trigger logic is shown in Figure 6.7. Essentially, the triggers were as follows:

1. $T0_1$ - ≥ 1 forward and ≥ 2 cylinder tracks
2. $T0_2$ - Events with primarily transverse tracks
3. $T0_3$ - ≥ 1 forward and 1 or 2 transverse tracks
4. $T0_4$ - ≥ 2 forward tracks
5. $T1$ - Beam Decay Trigger. The three-prong decays of the beam (known as τ decays) served to study the dipole momentum resolution and to calibrate the P-hodoscope measurement of the beam momentum. This trigger rate was scaled down by a factor of 10.
6. $T2$ - Elastic Trigger. K^-p elastic scattering events were mainly used for studying track finding, and to help understand the dipole momentum resolution. This trigger rate was scaled down by a factor of 100.
7. $T3$ - Random Beam Trigger. Used to provide an unbiased event sample for Monte Carlo trigger efficiency studies. This trigger rate was scaled down by a factor of 1000.
8. $T4$ - Random Interaction Trigger. Used to provide an unbiased event sample for Monte Carlo trigger efficiency studies. This trigger rate was scaled down by a factor of 50.

Each time an event was taken, the spectrometer incurred deadtime, necessary for readout. In this spectrometer, the deadtime was dominated by the approximately 15 ms needed to clear the ionization from the spark chambers. This time increased for very high multiplicity events. On average,

Figure 6.7 Complete trigger logic for experiment E-135.

the detector was live approximately 60% of the time that beam was being delivered.

6.6 Data Acquisition

The spectrometer electronics were read out into a PDP-11/04 through five CAMAC branches. The data were then transferred through an IBM Series 7 computer to the SLAC central computer center, where it was written to tape using one IBM 370 in the SLAC TRIPLEX computer system*. The LASS data acquisition system was designed to handle approximately 50 events per second.

Although the CAMAC system consisted of five branches, only four were in use at any given time. Data from the detector electronics were read into device controllers for temporary storage until transferred to the PDP-11 using direct memory access (DMA). One CAMAC branch read the hodoscope, TOF, Cherenkov counter, and beamline electronics. A second branch read out the PWC anodes. Another branch handled the cathode foil signals, and a fourth branch read out the magnetostrictive chambers. The fifth branch was used to monitor the voltage settings of some counters and to record the contents of several scalars associated with the beam.

On each event trigger, the contents of the four controllers were copied in parallel to partitions in the PDP's memory. The PDP formatted the data before

* The SLAC TRIPLEX consisted of one 2MB IBM 360/91, two 3MB IBM 370/168, ten 6250 BPI tape drives, four IBM 5098 sensor based control units, and a number of disks, printers, and graphic display devices.

sending it off to the Series 7 system. The Series 7 sent the data over a 0.5 km coaxial cable to the TRIPLEX.

All the tape logging and online monitoring of the experiment was performed on the TRIPLEX. Tape logging was assigned the highest priority, while an analysis program ran on a sample of the events in the remaining time. Quantities such as missing mass, effective mass, track multiplicity, and chamber efficiencies could be histogrammed on an IBM 2250 display terminal, as could coordinates and found tracks for individual events.

6.7 Alignment

Because the gap chambers could be easily surveyed, they were used to define the coordinate system used for LASS. The wire positions in the chambers were well-known from bench measurements, and the z position of the chambers were surveyed.

The locations of the other chambers in the spectrometer were determined relative to the gap chambers by fitting straight tracks, obtained with the magnets turned off. Next, the cylindrical chambers were located by fitting tracks obtained with the magnets on. Tracks found in the planar chambers were projected back as helices. Allowance had to be made for the fact that the cylindrical chambers were not exactly round and were tilted and displaced with respect to the z axis of the coordinate system.

The beam chambers were aligned using events with a T3 trigger. To accomplish this, the plug and full-bore chamber tracks in such events were projected back into the beam chambers.

The alignment of the magnetostrictive chambers was unfortunately found to vary over a few hours. This was traced to instabilities in the wands, and the wand signals were watched carefully, by tracking fiducial signals during normal running. A new alignment was taken whenever significant variations were seen, typically once per week.

6.8 Calibration

Run-to-run variations were noticed in a number of ADC systems in the experiment. In particular, the TOF and Cherenkov ADCs exhibited pedestal fluctuations, while the cathode readouts had both pedestal and gain fluctuations. In order to correct for this, the first 100 events in each run were used to define run-by-run pedestals. In addition, dedicated calibration electronics were built into the cathode readouts. This allowed more frequent calibration.

Run-by-run monitoring was also performed for the momentum calibration. An absolute calibration was determined for the dipole, and the solenoid region and the P-hodoscope were calibrated relative to the dipole. In order to calibrate the dipole, a sample of $K_S^0 \rightarrow \pi\pi$ events were used, requiring one daughter pion to pass through the dipole. The K_S^0 mass was reconstructed from the observed daughters, and the result compared to the known K_S^0 mass. This resulted in determination of a scale factor of 1.007 for full field dipole

running, and 1.004 for half-field operation. K^-p elastic scattering events and τ events were also sources of good dipole tracks. Elastic events where the K^- passed through the dipole were a good way to compare the calibration of the P-hodoscope and the dipole at 11 GeV/c. The slower pions resulting from τ decays were a source of information regarding the momentum dependence of the dipole calibration.

A good measure of the run-to-run variations was obtained by summing the observed longitudinal momentum, and comparing that number to the known beam momentum. Small systematic shifts were observed in this quantity, and changes were correlated with the times at which the chambers were moved in order to accommodate an opposite polarity beam.

Information on dE/dx was obtained by examining the pulse height in the cylindrical chambers. The observed pulse height is related to the ionization left by a charged track by

$$I = f_{\theta} PH$$

In this expression, I is the ionization left by a charged track, θ is the angle of the track relative to the normal to the cylinder surface, f_{θ} is a path length correction, and PH is the observed pulse height. If the cylinder package was homogeneous, it would be expected that f_{θ} would have a $\cos\theta$ shape. In fact, the cylinder structure is not homogeneous, and f_{θ} has a more complicated shape. In particular, the tracks deposit more energy than expected as they become more and more forward. The complete explanation for this effect is not known, but

the effect was compensated for by using quadratic polynomial in $\cos\theta$. Another factor that determined the expected pulse height was the gas gain. A run-to-run calibration for each cylinder was carried out using fast negative tracks. Then the velocity (β) dependence was measured using protons from elastic scattering events. The quantity $(f_{\theta}PH)^{0.3}$ was observed to be nearly Gaussian. The β dependence of this quantity was parametrized as a quadratic. Using this, a likelihood function for π/p was constructed. The overall likelihood was determined as the product of the individual cylinder likelihoods.

6.9 Reconstruction Code

The event reconstruction code for this experiment passed through several stages: track finding, vertexing, and topology recognition. Each of these is discussed below.

6.9.1 Track Finding

The beam package measured the beam parameters. In many cases, there were several beam candidates per trigger, and the program must take care to use the correct one. As a start, anode clusters were formed and converted to spatial coordinates. Next, possible matchpoints between the upstream and downstream chambers were listed. In doing this, the time information for associated clusters was required to be consistent. The list of such matchpoints was used to generate the beam candidate list. Candidates with the best timing consistency, a satisfactory number of coordinate measurements, and good consistency with the allowed beam phase space were kept. Finally, information from the θ - Φ hodoscope and SY counter was used. In-time candidates were required to have in-time corroboration from these devices. Typically, there

were 1.5 beam tracks per event. Other found beam tracks were usually due to non-interacting kaons, which were also often seen in the downstream dipole chambers. In this case, the associated matchpoints in the dipole and the solenoid were not used for later track finding.

In the solenoid region, track finding began by defining matchpoints in the gap chambers. Real tracks should follow helical paths in the magnetic field of the solenoid. Because of this, track finding began by selecting three matchpoints from different chambers, consistent with a helix. Other solenoid chambers were searched for corroborating coordinates. The regions searched for corroboration were determined by the uncertainties in the helix defined by the original matchpoints. Based on the total number of corroborating points found, a χ^2 fit to a helix was performed to the candidate points, and the track was kept if the result of the fit was sufficiently good (confidence level greater than 10^{-4}). If the result of the fit was not acceptable, coordinates with a χ^2 greater than 12 were dropped, and the fit recomputed. This process was repeated until either there were too few points left on the track or the fit converged with an acceptable confidence level. The fit included some allowance for multiple scattering, assuming that the scattering errors were uncorrelated and that the particle was a pion. This procedure was repeated using all combinations of planar chambers, and then a similar procedure was carried out for the cylindrical package.

Dipole tracks were found by first finding line segments downstream of the dipole. Pairs of x -coordinates from two similar coordinate planes were found, and the segment was considered a candidate if it was corroborated by

the associated spark gaps. Candidates were extrapolated into other downstream chambers, and a fit was performed if a satisfactory number of corroborating coordinates was found. If the least squares fit to a straight line had a confidence level greater than 10^{-4} , it was kept. Line segments upstream of the dipole were found in a similar manner, and the full set of upstream and downstream tracks were crossed through the dipole.

Upstream/downstream segment pairs that came close to meeting in the center of the dipole were tentatively matched, and a rough momentum was estimated based on the track bend angle in the dipole. Next, the downstream segment was swum back through the dipole using the rough momentum and the measured dipole field map. The x -slope of the measured upstream track segment and the swum downstream segment were compared. Based on this comparison, the momentum estimate was updated. This process was repeated until the measured and swum x -slopes agreed to 0.1 mrad. This usually only took two or three iterations. Finally, the x and y intercepts and the y -slope of the measured and final swum track were compared. If they did not match to within a few mrad and a few mm, the match was discarded. The momentum errors assigned to dipole tracks were based on the chamber and estimated multiple scattering errors.

The final step in track finding was the joining of the dipole to the solenoid track segments. This was accomplished by finding a solenoid track that matched the dipole track, and refitting the solenoid, using the dipole-measured momentum as a constraint. If the fit was successful, the fit results replaced the

previous fit results. Approximately 95% of the refits were successful. Failures most often resulted from particle decays in flight or large angle scattering.

6.9.2 Vertex Finding

An initial estimate of the primary vertex position in an event was made by taking the mean of the z coordinate of the points of closest approach of the tracks in an event to the beam. Only tracks that passed within 1.5 cm of the beam and were in the vicinity of the target were included. The result was kept as the vertex position if the fit was reasonable ($\Sigma d^2/\text{degree of freedom} < 1 \text{ cm}^2$). If not, the worst track was thrown out and a new vertex location calculated. This procedure rarely failed, except where there were two interactions in the target. To handle these rare cases, the tracks were split into an upstream group and a downstream group if the track dropping did not succeed. The group with the higher number of tracks was kept.

Secondary vertices were found by making a list of all oppositely charged track pairs, including those associated with the primary vertex. Combinations were kept if they had a $\Sigma d^2 < 0.8 \text{ cm}^2$ and had a pair mass near the Λ , $\bar{\Lambda}$, K^0 , or γ (zero) mass when the appropriate mass assignments were made. An additional list of $V^\pm$ candidates was built by combining Λ and $\bar{\Lambda}$ candidates with charged prongs and requiring the result to lie near the Ξ^- or Ω^- mass. In order to reduce the combinatorial background, $V^\pm$ candidates in the target were required to have a decay length of at least 1.0 cm.

Topology	Description
1	Single n -prong primary vertex
2	n -prong primary vertex with V^0 consistent with being a K^0
3	n -prong primary vertex with V^0 consistent with being a Λ or $\bar{\Lambda}$
4	n -prong primary vertex with V^0 consistent with being a γ
5	n -prong primary vertex with $V^\pm$
6	n -prong primary vertex with 2 V^0
7	n -prong primary vertex with V^0 and $V^\pm$
8	n -prong primary vertex with 3 V^0

Table 6.2 Event topologies used in this experiment

6.9.3 Topology Recognition

After vertex finding, the events were classified into various event topologies. Eight topologies, numbered 1 to 8, were used. The topologies and the associated numbers are shown in Table 6.2. Each event was allowed to have interpretations in more than one topology, but a unique interpretation was assigned in each topology. If there were multiple interpretations of an event in a single topology, the ambiguity was resolved differently depending on the topology. For topology 1, one or two tracks were allowed to be dropped from the vertex. Track dropping was allowed if the original Σd^2 per degree of freedom was poor. The track dropped was that one which had the largest impact on Σd^2 per degree of freedom. For topologies 2, 3, and 4, one track was

allowed to be dropped based on the same considerations as for topology 1. Remaining ambiguities in topologies 2, 3, and 4 were resolved by additional criteria. First, if only one V^0 was in the tight mass window, it was chosen. If not, the interpretation which used the most tracks was picked, unless an interpretation using fewer tracks had a significantly better Σd^2 per degree of freedom. In addition, for topology 3, the existence of a tight Λ interpretation automatically eliminated any $\bar{\Lambda}$ interpretations. Ambiguous interpretations in topologies 5, 6, 7, and 8 were resolved in a similar manner, and by maximizing the number of tight V^0 s.

6.10 MVFIT

MVFIT is a multi-vertex fitting program that can enforce both geometric and kinematic constraints on the tracks present in an event. MVFIT uses the measured coordinates, not the VRHUNT momentum vectors, as the data to be fit. This distinction is important, since it is possible for a program which fits vectors to end up with a solution that lies close to the measured vectors, yet is quite far from the coordinates, which are the measured data.

MVFIT automatically applies a geometric constraint that forces all tracks in an event to emanate from the primary or secondary vertices present in the current interpretation of the event. In addition, where secondary vertices are present, MVFIT automatically forces three-momentum conservation at the secondary vertices. MVFIT can optionally enforce additional kinematic constraints on an event. In particular, MVFIT can enforce three-momentum conservation at the primary vertex, energy balance at the secondary vertices, and energy balance at the primary vertex. Momentum and/or energy balance

at the primary vertex can be enforced either with or without a missing neutral track.

MVFiT describes the data in terms of the following parameters:

- Three-dimensional coordinates of the primary vertex.
- Polar and azimuthal angle of each track helix, at the primary vertex.
- Reciprocal of the momentum of each track at the primary vertex.
- Three-dimensional coordinates of each V^0 secondary vertex (the momentum of the V^0 is forced to be the sum of the three-momenta of the daughter tracks, and the V^0 momentum is forced to be parallel to the line joining the primary and secondary vertices).
- Path length of any V^- track, along with the parameters of the V^- track.

As a consequence of the parametrization, all tracks from the same vertex are forced to emanate from a common point.

The optional constraints are imposed through the method of Lagrange multipliers. Because of this, one additional parameter, the Lagrange multiplier, is added for each constraint equation.

MVFIT uses Chi-square minimization, and employs a modified Newton-Raphson approach^[33]. The equation for the function to be minimized is

$$\mathcal{L} = \sum \left(\frac{x_i - x_{pred}}{\sigma_i} \right)^2 + 2 \sum \lambda_j \mathcal{F}_j$$

where x_i are the measured coordinates, σ_i are the resolutions on the measurements, x_{pred} are the coordinates predicted using the present values of the fit parameters, λ_j are the Lagrange multipliers, and $\mathcal{F}_j$ are the constraint equations. This function is expanded to second order about a point specified by the current values of the free parameters. The expansion is inverted in order to determine the value of the parameters for which the gradient of $\mathcal{L}$ is expected to vanish. The parameters are then assigned these values, and the expansion is repeated. If the stepped values of the parameters do not seem to have brought $\mathcal{L}$ closer to a minimum, the step size is reduced. The fit is said to have converged when $\mathcal{L}$ changes very little (typically less than 0.1), any energy balance constraint at a secondary vertex is fulfilled at the level of 0.1 MeV, and any energy-momentum balance at the primary vertex is fulfilled at the level of 20 MeV/c. The primary vertex constraint is imposed on the sum over all the constraint equations.

Fits usually converge in less than five iterations. If the point dropping option has been enabled, once the fit has converged, the χ^2 contributions of all coordinates are determined. Any coordinate with a χ^2 contribution greater than 12 is dropped, and the fit is redone. This is repeated until either no more points have χ^2 contributions greater than 12, or more than five coordinates have been dropped. In the latter case, the fit is considered to have failed. MVFIT also handles situations in which the function never reaches a minimum, but oscillates around it. This is handled by cutting the step size, as described above.

Finally, MVFIT will not allow the position of a secondary vertex to move downstream of the measured coordinates of the decay daughters at that vertex. If the vertex position moves downstream of such coordinates, the coordinates are dropped, regardless of whether point dropping is enabled or not.

Energy loss in the liquid hydrogen target is modeled in the fit by using a Runge-Kutta stepping procedure^[33] to track each particle inside the target. Once the particle has left the target, its path is parametrized as a helix. If the Runge-Kutta stepping significantly modifies the particle's track, the helical section of the track will no longer pass close to the measured coordinates. In this case, the parameters describing the track are recomputed, and the Runge-Kutta stepping is applied again. This procedure is iterated until the track, after accounting for energy loss, passes through the measured coordinates.

Multiple scattering is difficult to account for correctly, since it introduces correlations between the predicted coordinates for a track. In MVFIT, it is treated approximately by expanding the uncertainty on a track's position as it passes through the various components of the spectrometer. The error increase is determined using known material distributions in the detector. The increase in uncertainty is folded into the uncertainty in the coordinates. Tracks also experience continuous multiple scattering in the liquid hydrogen target. This is accounted for by adding an additional term to $\mathcal{L}$.

The fits used in the analyses described in this work included

-
- Geometric constraints only (GeoFit)
 - Geometric constraints plus mass constraint on V^0 mass (mass-constrained fit)
 - Assume missing neutral at primary, assume mass for it, enforce energy balance (1-C Fit)
 - Enforce three-momentum conservation at primary vertex (3-C fit)
 - Enforce energy-momentum balance at primary vertex, assigning masses to each particle (4-C) fit

6.11 Monte Carlo

Although the precise manner in which the Monte Carlo is generated depends on the channel being studied, there are a number of features independent of the specific channel. The thrown phase space of the beam is based on a study of the events obtained with the T3 trigger. The generated beam is made to randomly interact in the target, accounting for nuclear absorption in liquid hydrogen. It is then tracked back out of the target, and corrected for energy loss and multiple scattering. In addition, the momentum is smeared by the resolution of the beam chambers. This process provides a good representation of the real beam. Additional tracks found in events with a T3 trigger are also tracked through the spectrometer, to create secondary beam hits in the downstream chambers. The hits are smeared by the resolution of the appropriate chambers.

Tracks produced by the interaction of the beam with a target proton are followed out of the target, correcting them for energy loss, multiple scattering, absorption, weak decays, and non-uniformities in the magnetic field. Multiple

scattering errors are added wherever there is material in the real spectrometer. Energy loss includes a determination of the effective magnetic field a particle sees over each step. Nuclear absorption and weak decays are modeled as exponential decays, using known lifetimes, decay branching fractions, and absorption lengths.

Tracks are followed through the mapped field of the spectrometer using a fourth-order Runge-Kutta stepping algorithm and are followed until they interact, decay, range out, or leave the spectrometer. A polynomial approximation to the measured spectrometer magnetic field is used.

The characteristics of the chambers, scintillators, and Cherenkov counters are modeled as a function of the angle of incidence and measured device efficiencies. In the chambers, cathode simulation is somewhat more difficult than anode, because the signal is more spread out, producing more hit overlap.

The performance of the overall Monte Carlo has been studied using real data from τ decays of the beam, elastic K^-p scattering, and n -prong interactions. The elastic scattering events served as a normalization check on the experiment, since the cross section for this process is well-known. Study of the τ decay events reproduced the known kaon mass very well.

7. Inclusive Λ Selection

The first stage of the analysis was, of necessity, the most general. It was desired to make a selection which would encompass all of the analyses planned in conjunction with this work, be as generally useful to others who may do analyses in the future, reduce the sample size as much as possible, and run as quickly as possible. Speed and reduction in size were particularly important since the software had to examine $\sim 1.13 \times 10^8$ K^- events on 961 Data Summary Tapes (DST's). The selection decided on to meet these four goals was an inclusive Λ selection.

As part of the E-135 reconstruction code, a list of V^0 candidates was compiled for each event, and saved as part of the event record. Flag bits indicated which hypotheses a particular V^0 candidate satisfied: loose Λ , tight Λ , loose K^0 , tight K^0 , loose γ , tight γ . The only features used in determining candidacy for a particular hypothesis were distance of closest approach of the two candidate daughter tracks and daughter pair effective mass. The distance of closest approach was parametrized as $d2v$, which was the sum of the squares of the distances of closest approach of the two daughter track helices to the decay vertex. Because the mass and $d2v$ could be examined quickly, and preliminary studies showed that a selection based on examining only these two quantities could be expected to result in at least a factor of five reduction in the number of events, only mass and $d2v$ cuts were used in the inclusive Λ selection. Candidates were required to have an effective mass within ± 20 MeV of the nominal Λ mass when the daughters were assigned the proton and π^-

masses, and the daughter tracks were required to have a $d2v$ of no more than 1cm^2 .

In practice, the selection was slightly more complicated. A routine (HDSLCT) was inserted so that it ran before the event was unpacked. This routine very quickly examined the event to see if it contained any loose Λ candidates. If not, the event was cut immediately. This routine provided a factor of 4.5 reduction in the number of events. If the event contained at least one loose Λ candidate, the event was unpacked, and various event topologies examined in more detail. During event reconstruction, events had been assigned to various event topologies, discussed in Section 6.9.3. One of the topologies, topology 3, consisted of events with a recognized Λ . Previous analyses of data in this experiment had shown that when an event had a topology 3 interpretation, the topology 3 Λ was usually correct. However, topology 3 imposed a minimum decay length requirement on the Λ , so that short decay length Λ 's do not appear in topology 3. These short Λ 's can only be found by looking through the V^0 candidate list, a pseudo-topology known as topology 9. For the inclusive Λ selection, events were first searched for a topology 3 interpretation. If present, the topology 3 Λ was used, and cuts made on it. If none was present, topology 9 Λ candidates were searched, and an event was kept if any Λ candidate passed the mass and $d2v$ cuts.

The $d2v$ for a sample of events[†] is shown in Figure 7.1. Figure 7.2 shows the mass plot for events with $d2v$ less than the cutoff, and Figure 7.3 shows the

[†] Approximately 0.5% of the experiment's data.

Figure 7.1 $d2V$ for sample of DST data.

Figure 7.2 Mass spectrum for a sample of Λ candidates with $d2V$ less than 1.0 cm^2 .

Figure 7.3 Mass spectrum for a sample of Λ candidates with d_2v greater than 1.0 cm^2 .

mass spectrum for events with $d2v$ greater than the cutoff. As can be seen from the figures, essentially no Λ signal was lost by making the distance cut. It can also be seen that the majority of the Λ signal was retained by the ± 20 MeV mass cut. Finally, the plots show that topology 3 included the vast majority of Λ 's.

This selection reduced the sample from 1.13×10^8 events to 1.67×10^7 events. Because events with a larger number of tracks have a greater likelihood of producing a chance Λ candidate, high multiplicity events are preferentially retained, and the reduction in the number of tapes is not as large as the reduction in the number of events. Nevertheless, the event sample size was reduced from 961 tapes to 245 tapes.

8. $\Lambda\eta\pi\pi$

8.1 Overview

Although $\Lambda\eta\pi\pi$ is not as obvious a strangeonium channel as some others (e.g. $\phi\phi$), it nonetheless is a strangeonium channel, since the 0^{-+} nonet to which the η belongs is not ideally mixed. Because of the non-ideal mixing, the η ends up with some admixture of $s\bar{s}$. Analysis of the $\Lambda\eta\pi\pi$ final state is complicated by the fact that it is a many-particle final state, so there are numerous possibilities for resonances in various subchannels. For example, known resonances which decay into $\eta\pi\pi$, $\eta\pi$, $\Lambda\pi$, $\Lambda\pi\pi$, and $\pi\pi$ can all contribute to this final state. This makes unraveling the structure and correlations in this final state difficult. Finally, $\Lambda\eta\pi\pi$ is an interesting channel to investigate since it is a potentially good channel for studying the $a_0(980)$ meson, a meson which is not well-understood, but which is known to decay to $\eta\pi$.

8.2 Selection of $\Lambda\eta\pi\pi$ sample

The $\Lambda\eta\pi\pi$ sample was selected from the inclusive Λ sample by adding requirements on the event topology, number of tracks, charge balance, and missing mass. In this experiment, the primary vertex of the $\Lambda\eta\pi\pi$ final state will contain a π^+ and a π^- from the η decay (the π^0 from the η decay is not seen, but reconstructed using the missing momentum), two additional charged pions, and a neutral Λ connector. In addition, there will be a secondary vertex where the Λ decays into $p\pi^-$.

In order to be retained in the $\Lambda\eta\pi\pi$ selection, events were required to either have an identified Λ and no other secondary vertices (topology 3), or a

single primary vertex and no secondary vertices (topology 1). Topology 1 was included because of the Λ decay length requirement for topology 3. By including topology 1 events, it was possible to pick up Λ 's with decay lengths shorter than the 1.0-1.5 cm cutoff for topology 3. In selecting topology 1 candidates, the V^0 candidate list for the event was searched for Λ candidates which satisfied the cuts used in the inclusive Λ selection. When such a combination was found, the Λ daughter tracks, which may or may not have been included as primary vertex charged prongs in the topology 1 interpretation, were flagged and the remaining tracks in the topology were tested to see if they passed the $\Lambda\eta\pi\pi$ cuts. Since it is possible that more than one V^0 candidate passed the inclusive Λ cuts, it is possible to have more than one Λ in a topology 1 event, and hence multiple $\Lambda\eta\pi\pi$ interpretations of a topology 1 event. As mentioned in the section on topology recognition, each event can have at most one topology 3 interpretation.

Events were required to have between two and five non- Λ , non- η charged prongs at the primary vertex. This allowed the selection to encompass $\Lambda\eta\pi\pi$ and $\Lambda\eta'\pi\pi$ (where $\eta' \rightarrow \eta\pi\pi$). In addition, this also allowed one extra charged prong at the primary vertex, so long as the event passed the charge balance and missing mass requirements when one of the tracks was dropped. The extra track was allowed since it was not known how many events might have a stray track pulled into the primary vertex by the topology recognition code. This number was expected to be small, but it was hoped that subsequent analysis could retain these few additional good events for analysis while rejecting the bad events. Single track dropping was allowed for both topology

1 and topology 3 events, and leads to the possibility of multiple $\Lambda\eta\pi\pi$ interpretations of both topology 1 and topology 3 events.

Candidate track combinations were required to have no more than $0.25 (\text{GeV}/c^2)^2$ missing mass squared when all of the charged primary tracks were assigned the $\pi^\pm$ mass and the V^0 was assigned the Λ mass. Finally, for an event to be considered as a candidate, it had to contain a $\pi^+\pi^-(\pi^0)$ track combination with mass less than $0.95 \text{ GeV}/c^2$. This cut was used to select η events, and also included ω events for possible later analysis by others. A total of 6.38×10^5 events passed these cuts. These events were written onto 11 tapes.

MVFIT was performed next. In each event, all track combinations which passed the $\Lambda\eta\pi\pi$ cuts listed above were fitted. Events which were $\Lambda\eta'\pi\pi$ candidates were not fitted or carried into later stages of the analysis. The fits tried were a GeoFit, a Λ mass-constrained fit, a 1-C Fit to $\Lambda\pi^+\pi^-(\pi^0)\pi^+\pi^-$, and a 4-C fit to $\Lambda\pi^+\pi^-\pi^+\pi^-$. The 4-C fit was used to help distinguish events with a real missing π^0 from those without a missing neutral. GeoFit was the first fit tried on each combination. If the GeoFit did not converge, no further fits were made to the combination. If the GeoFit converged, a Λ mass-constrained fit was attempted, starting from the GeoFit results. If the Λ mass-constrained fit did not converge, no further fits were attempted on the track combination. If the Λ mass-constrained fit converged, 4-C and 1-C fits were carried out, both starting from the results of the Λ mass-constrained fit. Point dropping was allowed in the GeoFit, but not in any of the other fits. After fitting, 2.78×10^5 events had at least a converged GeoFit and these events were written onto 5 tapes.

Cut	# events cut	# events remaining
Input MVFIT events	–	278,263
Require 1-C Fit	61,822	216,441
Require 1-C Confidence Level $>10^{-5}$	60,452	155,989
Require 4-C Confidence Level $<10^{-5}$	16,044	139,945
Require $\Sigma\text{Charge}=0$	429	139,516
Output events	–	139,516

Table 8.1 Summary of cuts used in the post-MVFIT $\Lambda\eta\pi\pi$ selection.

After MVFIT, another event selection was made. Events that had a 4-C fit confidence level greater than 10^{-5} were discarded, and only events with a 1-C fit confidence level greater than 10^{-5} were kept. This removed most of the 4-C contamination from the sample, while retaining almost all of the real 1-C events. Charge balance was again required, since occasionally the curvature of a high-momentum (stiff) track changed sign during fitting, causing its measured charge to change. Table 8.1 provides a summary of the cuts made in this selection.

Examination of the sample at this stage showed a large η' signal in the $\eta\pi\pi$ mass spectrum (Figure 8.1). Because the η' is very near threshold in $\eta\pi\pi$, it is a very clean signal and can be studied without additional cuts. In addition, the fact that the η' is near threshold causes it to produce very localized reflections in the $\pi\pi$ and $\eta\pi$ mass spectra. For these reasons, the η' was investigated separately from the remainder of the $\Lambda\eta\pi\pi$ sample.

Figure 8.1 Observed $\pi^+\pi^-\pi^+\pi^-(\pi^0)$ mass spectrum. The solid curve is the result of a fit using a lineshape derived from Monte Carlo data. The Monte Carlo included the $\pi^+\pi^-(\pi^0)$ and the $\pi^+\pi^-(\gamma)$ decay modes of the η , plus a quadratic background. Both the π^0 and the γ decay modes of the η are included since these final states are essentially indistinguishable in this experiment. The dashed curve shows the $\pi^+\pi^-(\gamma)$ contribution by itself.

8.3 Study of $\Lambda\eta'$ Sample

The observed $\eta\pi\pi$ mass distribution is shown in Figure 8.1. The figure also shows a fit to the data, using a lineshape derived from Monte Carlo data together with a quadratic background. The lineshape will be described shortly. The observed $t'(p \rightarrow \Lambda)$ distribution ($t' = t - t_{\min}$, where t is the square of the four-momentum transfer from the proton to the Λ and $t_{\min}$ is the minimum

Figure 8.2 Observed $t'(p \rightarrow \Lambda)$ distribution for $m_{\eta\pi\pi} < 1.0 \text{ GeV}/c^2$.

kinematically allowed t at the present $\eta\pi\pi$ mass) for events in the η' region ($m_{\pi^+\pi^-\pi^+\pi^-(\pi^0)} < 1.0 \text{ GeV}/c^2$) is shown in Figure 8.2.

The acceptance for this channel was calculated as a function of $t'(p \rightarrow \Lambda)$, and is plotted in Figure 8.3. To calculate the acceptance, approximately 40,000 Monte Carlo events of the reaction $K^-p \rightarrow \eta'\Lambda$, $\eta' \rightarrow \eta\pi\pi$, $\eta \rightarrow \pi^+\pi^-(\pi^0)$ were generated in two groups. The first group of 20,000 events was generated with a $t'(p \rightarrow \Lambda)$ slope of 6.9, to model the real data. The second group was generated with a $t'(p \rightarrow \Lambda)$ slope of 1.0, to fill in the high t' region. The acceptance was

Figure 8.3 Acceptance for the η' sample.

parametrized as a quartic polynomial for $-t' < 0.25 \text{ (GeV/c)}^2$, and as a constant for $-t' \geq 0.25 \text{ (GeV/c)}^2$. At $-t' = 0.25 \text{ (GeV/c)}^2$, the two parametrizations were matched by requiring the values to be the same and forcing the polynomial to have zero slope. The acceptance also corrects for the approximately six percent loss of events with a mass over 1.0 GeV/c^2 , the mass cutoff used for further analysis. The decrease in acceptance for $-t'(p \rightarrow \Lambda) < 0.25 \text{ (GeV/c)}^2$ is due to the fact that, for low t' and $\eta\pi\pi$ masses near threshold, protons from the decay of the Λ do not make it out of the target.

The lineshape used for determining the η' mass, width, and cross section is plotted on the $\eta\pi\pi$ mass spectrum in Figure 8.1. The lineshape is somewhat complicated. It includes a quadratic background, a contribution for the process $\eta' \rightarrow \eta\pi\pi$ with the subsequent decay $\eta \rightarrow \pi^+\pi^-(\pi^0)$, and a contribution for the process $\eta' \rightarrow \eta\pi\pi$ with a subsequent decay $\eta \rightarrow \pi^+\pi^-(\gamma)$. Allowance for the $\pi^+\pi^-(\gamma)$ decay mode of the η had to be included for several reasons. First, a π^0 and a γ cannot be distinguished in this channel. Both are observed only as missing momentum and the difference in $(\text{mass})^2$ is small enough that kinematic fitting will not distinguish the two given the resolution in missing mass. Second, the branching fraction for $\eta \rightarrow \pi^+\pi^-(\gamma)$ is significant, being approximately 20% of the branching fraction $\eta \rightarrow \pi^+\pi^-(\pi^0)$. Finally, at the η' , the systematic change in 5- π mass brought about by misidentifying a γ as a π^0 is $\sim 40 \text{ MeV}/c^2$, which is small enough that the signals overlap. The lineshape used to fit the $\pi^+\pi^-(\pi^0)$ decay mode consisted of two Gaussians, constrained to have the same mass, but allowed to have different widths and amplitudes. The Gaussians were allowed to have widths that varied linearly with mass, but the widths of both Gaussians were required to have the same mass dependence. The linear mass dependence of the width was necessary to provide a reasonable fit to the tails of the Monte Carlo distributions. A similar two-Gaussian shape was used for the $\pi^+\pi^-(\gamma)$ decay mode of the η . For the $\pi^+\pi^-(\gamma)$ fit, 20,000 additional Monte Carlo events of the reaction $K^-p \rightarrow \eta'\Lambda$ were generated with a $-t'(p \rightarrow \Lambda)$ slope of 6.9. In these events, the η was forced to decay to $\pi^+\pi^-(\gamma)$. These events were then analyzed as though they had a missing π^0 . For the overall fit shown in Figure 8.1, the shapes determined from the individual fits to $\eta \rightarrow \pi^+\pi^-(\gamma)$ and $\eta \rightarrow \pi^+\pi^-(\pi^0)$ were combined, fixing the ratio of the integrals of the two distributions to match the known branching fractions (4.91% for $\pi^+\pi^-(\gamma)$ vs. 23.7% for $\pi^+\pi^-(\pi^0)$) and fixing

the offset in the central values of the two contributions at the value determined from the Monte Carlo distributions. This required $\pi^+\pi^-(\gamma)$ to peak at a mass 42 MeV/c² higher than $\pi^+\pi^-(\pi^0)$. The free parameters in the overall fit were the amplitude and the mass of the η' signal, and the three parameters specifying the quadratic background. As can be seen in Figure 8.1, the total lineshape provides a reasonable fit to the data. The fit gives $m_{\eta'} = 0.955 \pm 0.001$ GeV/c² and a width consistent with the detector resolution for this channel (0.013 GeV/c²). The error on the mass is purely statistical. While the width is consistent with the 1988 Particle Data Group value^[34], 0.207 ± 0.020 MeV/c², the mass is 2.5 σ lower than the accepted value of 0.9575 ± 0.00024 GeV/c². The difference is not believed to be significant, since there are unknown acceptance effects that can easily cause a shift of a few MeV/c² in a structure near threshold, where the acceptance changes rapidly.

The cross section for $K^-p \rightarrow \eta' \Lambda$ was determined by considering only events below 1.0 GeV/c². This cut retained ~93% of the $\eta \rightarrow \pi^+\pi^-(\pi^0)$ signal, while eliminating approximately half of the $\eta \rightarrow \pi^+\pi^-(\gamma)$. More importantly, this cut reduced the background level in the η' sample significantly. Correcting for acceptance, for Λ , η , and η' visibilities, for background, and for the $\pi^+\pi^-(\gamma)$ decay mode of the η , the cross section obtained for $K^-p \rightarrow \eta' \Lambda$ from this experiment is 8.9 ± 0.6 μ barns. The error on this measurement is purely statistical. Figure 8.4 shows that this value agrees well with the results from other experiments.

The acceptance-corrected t' distribution is shown in Figure 8.5. An exponential fit to all of the data points gives a t' slope of 6.2 ± 0.3 , with a χ^2 per

Figure 8.4 Cross section measurements for the reaction $K^-p \rightarrow \eta'\Lambda$ from various experiments. The measurement from this experiment is shown as an open circle. The plot shows total cross section (forward and backward production combined).

degree of freedom of 1.7. If the four highest t' points are excluded, the fit yields a t' slope of 6.8 ± 0.4 , with a χ^2 per degree of freedom of 1.2. As can be seen from the plot, the fit tends to be quite good at low t' , but is systematically low at high t' . Similar deviations from exponential behavior have been observed in this channel at lower beam momentum^[35].

The data from two experiments at different energies can be used to estimate the parameters of the effective exchanged trajectory in a Regge

Figure 8.5 Acceptance-corrected t' distribution for $K^-p \rightarrow \eta'\Lambda$. The solid curve shows an exponential fit to all the data points, and gives a slope of 6.2 ± 0.3 , with a χ^2 per degree of freedom of 1.7. The dashed curve shows an exponential fit excluding the four highest t' data points, and gives a slope of 6.8 ± 0.4 , with a χ^2 per degree of freedom of 1.2.

exchange picture^[36]. The high-statistics 4.2 GeV/c Bubble Chamber experiment^[35] was chosen as the second experiment for this calculation. In the Regge picture, the differential cross section can be expressed as

$$\sigma(s, t) = F(t) \frac{s^{2\alpha_{eff}(t)}}{p_{lab}^2}$$

where s is the square of the center of mass energy, t is the square of the four-momentum transfer, p_{lab} is the lab momentum of the beam, $F(t)$ is the

residue function (typically an exponential), and $\alpha_{eff}(t)$ is the trajectory that is to be measured. This equation holds for any particular energy.

By comparing data from experiments at different energies, $\alpha_{eff}(t)$ can be measured. The ratio of the differential cross sections measured at the two energies is given by

$$\frac{\sigma_1(s, t)}{\sigma_2(s, t)} = \frac{F(t) \frac{s_1^{2\alpha_{eff}(t)}}{p_1^2}}{F(t) \frac{s_2^{2\alpha_{eff}(t)}}{p_2^2}}$$

where the subscripts 1 and 2 denote the two different energies. Rearranging, α_{eff} can be written as

$$\alpha_{eff}(t) = \frac{1}{2} \frac{\log \left(\frac{p_1^2 \sigma_1}{p_2^2 \sigma_2} \right)}{\log \left(\frac{s_1}{s_2} \right)}$$

For this calculation, the data from this experiment was binned identically to the 4.2 GeV/c data, and $\alpha_{eff}(t)$ was calculated on a bin-by-bin basis. Since Regge trajectories typically fit well as straight lines, $\alpha_{eff}(t)$ was parametrized as

$$\alpha_{eff}(t) = \alpha_0 + \alpha' t$$

Figure 8.6 α derived from comparison of η' cross section from this experiment and 4.2 GeV/c bubble chamber. The closed circles are the data. The open circles are the $\text{spin } 1, 2 \text{ and } 3 \text{ } K^*$'s. The line was fit to the data points and the $K^*(1430)$.

The fit was forced to pass through the $K^*(1430)$, a resonance that lies on the trajectory expected to contribute more strongly to $K^-p \rightarrow \eta' \Lambda$ ^[37]. The other trajectory, the $K^*(890)$ trajectory, is nearly degenerate with the $K^*(1430)$ trajectory. If the Regge picture is valid for the current data, a single line should pass through the data points and near all the known K^* orbital excitations. The results of the fit are shown in Figure 8.6, and the corresponding trajectory parameters are

$$\alpha_0 = 0.30 \pm 0.06$$

$$\alpha' = -0.83 \pm 0.03 \text{ (GeV/c)}^{-2}$$

$$\chi^2 / \text{degree of freedom} = 0.40 \text{ (12 degrees of freedom)}$$

As can be seen, the fit is quite good. It passes near the $K^*(890)$ and the $K^*(1780)$, particles that are on the other allowed exchange trajectory for this channel, and is consistent with the $K_4^*(2075)$ (not shown). The fact that a single line fits the K^* 's and the measured t dependence of the $K^- p \rightarrow \eta' \Lambda$ cross section indicates that a Regge picture provides a simple description of the s - and t -dependence of the cross section in terms of the exchange of two almost degenerate trajectories[†].

8.4 Selection of non- η' Sample

In order to look beyond the well-defined η' , it was necessary to further clean the sample. Additional cuts were added to remove misidentified particles, as well as to further reduce the number of events that did not have a real missing π^0 . In order to remove η' events, all events with a $5\text{-}\pi$ mass less than $1.1 \text{ GeV}/c^2$ were cut. To further remove 4-C contamination, the momentum of the missing π^0 was required to be above $0.2 \text{ GeV}/c$; the events removed by this requirement exhibit no η signal in the $\pi^+\pi^-(\pi^0)$ mass distribution.

Electron pairs were found to be a significant source of contamination. In order to remove them, cuts were made on oppositely charged track pairs. Any

[†] A fit with non-degenerate residue functions for the two trajectories yields a slightly improved description of the $11 \text{ GeV}/c$ t' distribution. The improvement was not statistically significant, however.

topology 9 loose γ candidate with non-overlapping daughter helices, pair mass less than $0.040 \text{ GeV}/c^2$, and pair opening angle less than 3.0 degrees was flagged as an electron pair. Any $\Lambda\eta\pi^+\pi^-$ candidate track combination which used at least one of the electron pair tracks was removed from consideration.

Finally, particle ID cuts were applied to the proton daughter from the Λ decay, the two charged pions from the η , and the π^+ that was not from the η decay. It should be noted that the fitting and kinematic cuts do an excellent job of cleaning up the sample, and particle ID cuts have only a small effect. Time-of-flight cuts applied to the proton from the Λ decay eliminated any combination in which the proton had a momentum $< 3.0 \text{ GeV}/c$, did not hit close to the boundary of a TOF paddle, was the only hit on the paddle, and had a time of flight more than $+4\sigma$ ($\text{TOFDEV} > +4.0$) away from the expected proton time-of-flight. This cut removed a small amount of pion contamination. The time of flight cuts applied to the two positively charged pions had the same momentum and cleanliness requirements as the proton cuts, and removed any combination in which the pion time-of-flight was more than -4σ ($\text{TOFDEV} < -4.0$) away from the expected pion time-of-flight. This removed a small amount of proton contamination. Cherenkov cuts were applied to the two charged prongs from the decay of the η . A candidate track combination was eliminated if the π^+ had a momentum $< 2.2 \text{ GeV}/c$, was the only hit in the Cherenkov cell, and had an ADC value > 100 counts. Cuts on the π^- were similar, except that the ADC cut was placed at 1000 counts instead of 100. The Cherenkov cuts served to remove remaining electron pairs.

Cut	Number of Combinations	
	Cut	Remaining
Input Events	–	139,516 Events
Require $m_{\pi^+\pi^-(\pi^0)}$ in (0.52-0.58) GeV/c ²	–	9083
Require $p_{\pi^0} > 0.2$ GeV/c	1044	8039
Electron pairs	987	7052
Time-of-Flight	225	6827
Cherenkov	515	6312
Require uniqueness	1265(528 Events)	5047
Require $m_{\eta\pi\pi} \geq 1.1$ GeV/c ²	158	4889
Output Events	–	4889 Events

Table 8.2 Summary of cuts used in the final $\Lambda\eta\pi\pi$ selection.

For all subsequent analysis, each event was required to have a unique interpretation. Each event was required to have a unique $\pi^+\pi^-(\pi^0)$ combination in the mass window 0.52–0.58 GeV/c². Table 8.2 shows a summary of the cuts made in this selection.

8.5 Examination of $\Lambda\eta\pi\pi$ Sample

As mentioned earlier, there can be resonant structure in several of the subsystems of the $\Lambda\eta\pi\pi$ final state. As a starting point for surveying the content of this final state, the sample was split into several subsamples; backward Λ , forward Λ , backward $\Sigma(1385)^+$, forward $\Sigma(1385)^+$, backward $\Sigma(1385)^-$, and forward $\Sigma(1385)^-$. The forward/backward designations refer to the direction of

the baryon in the overall CMS of the reaction. A $-t$ (not $-t'$) cut at 1.0 (GeV/c)^2 was applied to define each sample. In the resulting peripheral production subsamples, reactions with a backward baryon are expected to occur primarily via meson exchange, and reactions with a forward baryon are expected to occur via baryon exchange. Because the cross section for baryon exchange drops much faster with energy than the cross section for meson exchange, it was expected that most structure would be seen in the subsamples containing a backward baryon. In addition, backward $\Sigma(1385)^-$ production was not expected, since it would seem to require the exchange of a doubly-charged meson, needed to convert the charge +1 target proton to a charge -1 $\Sigma(1385)^-$. No such meson is known to exist.

In each of the following samples an η sideband subtraction has been made to obtain the net η mass spectrum. The region in $\pi^+\pi^-(\pi^0)$ mass $0.52 - 0.58 \text{ GeV/c}^2$ was used as the η signal. The regions in $\pi^+\pi^-(\pi^0)$ mass $0.48 - 0.51 \text{ GeV/c}^2$ and $0.59 - 0.62 \text{ GeV/c}^2$ were used as the η sidebands, leaving a small (0.01 GeV/c^2) gap between the η and sideband regions. A linear background was assumed, allowing the net η mass spectrum to be obtained by direct, unscaled subtraction of the sideband region from the η region. Table 8.3 shows the net number of events in each subsample.

None of the forward baryon subsamples contains a significant number of events. This result was expected because of the baryon exchange mechanism needed to peripherally produce such systems. In addition, none of the forward baryon subsamples exhibits any significant structure. These subsamples will not be considered further here.

Subsample	Net Number of Events	Estimated Cross Section(μb)
Forward Λ	51 ± 25	0.4 ± 0.2
Backward Λ	802 ± 59	5.9 ± 1.3
Forward $\Sigma(1385)^+$	11 ± 6	0.09 ± 0.05
Backward $\Sigma(1385)^+$	383 ± 28	3.2 ± 0.7
Forward $\Sigma(1385)^-$	12 ± 8	0.10 ± 0.07
Backward $\Sigma(1385)^-$	110 ± 15	0.9 ± 0.2

Table 8.3 Number of events in various $\Lambda\eta\pi^+\pi^-$ subsamples. An η sideband subtraction has been performed, as described in the text. A t cut has also been applied to each subsample, requiring the momentum transfer from beam or target proton to the listed baryon to be below $1.0 (\text{GeV}/c)^2$. Cross section estimates are for the range $-t < 1.0 (\text{GeV}/c)^2$ and are based on an acceptance of 0.25 ± 0.05 (obtained from $K^-p \rightarrow \eta^+\Lambda$ and $K^-p \rightarrow \eta\pi^-\Sigma(1385)^+$ Monte Carlo data). They take into account the branching ratios for $\Lambda \rightarrow p\pi^-$, $\Sigma(1385) \rightarrow \Lambda\pi$, and $\eta \rightarrow \pi^+\pi^-(\pi^0)$. In addition, a 15% correction has been applied to account for the tails of the η , present in the sideband region.

In the backward Λ subsample, there is quite strong η production (Figure 8.7). The $\eta\pi^+\pi^-$ mass spectrum (Figure 8.8) shows no obvious structure; there may be a small signal at the $f_1(1285)$ mass, discussed later. Figure 8.9 shows the $\eta\pi^+$ and $\eta\pi^-$ mass distributions. There are indications of a narrow peak at the $a_0(980)$ mass in the $\eta\pi^-$ mass spectrum. This will be discussed later. A broad enhancement can be seen around $1 \text{ GeV}/c^2$ in $\eta\pi^+$. Some $\rho(770)$ is present in $\pi^+\pi^-$ (Figure 8.10), along with a possible small, but intriguing, enhancement near the $f_0(975)$. No structure is apparent in the region near the $f_2(1270)$. The $\Lambda\pi^+$

Figure 8.7 Observed $\pi^+\pi^-(\pi^0)$ mass spectrum for the event sample with $-t(p \rightarrow \Lambda) < 1.0$ (GeV/c^2) , $m_{\eta\pi\pi} \geq 1.1 \text{ GeV}/c^2$. The sideband and signal regions are both shown. The open circles are unused points between the signal and sideband regions.

and $\Lambda\pi^-$ systems (Figure 8.11) both show $\Sigma(1385)$ production, and no obvious evidence for any of the higher mass Σ resonances which are known. The production of $\Sigma(1385)$ is much stronger in $\Lambda\pi^+$ than in $\Lambda\pi^-$. However, the clear evidence for any $\Sigma(1385)^-$ production in $\Lambda\pi^-$ was not expected since it would *a priori* seem to require the exchange of a doubly charged meson. It is possible, however, to produce $\Sigma(1385)^-$ in the decay of a higher mass Λ resonance. Examination of the data showed that $\Sigma(1385)^-$ production was all associated with low $\Lambda\pi\pi$ mass (Figure 8.12), where there are numerous *s*-channel hyperon

Figure 8.8 Observed $\eta\pi^+\pi^-$ mass spectrum for the event sample with $-t(p \rightarrow \Lambda) < 1.0 \text{ (GeV/c)}^2$, $m_{\eta\pi\pi} \geq 1.1 \text{ GeV/c}^2$.

resonances known. All of the $\Sigma(1385)^-$ seen appears to be the product of such decays. This is consistent with data from low energy kaon beam experiments^[38], where the cross sections for $\Sigma(1385)^-$ and $\Sigma(1385)^+$ are approximately equal due to the presence of numerous s -channel resonances in $\Lambda\pi\pi$. The $\Lambda\pi^+\pi^-$ mass spectrum (Figure 8.13) is consistent with the presence of several low mass $\Lambda\pi^+\pi^-$ resonances, including the $\Lambda(1670)$. Finally, Figure 8.13 also shows that $\Lambda\eta\pi^+$, $\Lambda\eta\pi^-$, and $\Lambda\eta$ are featureless.

Figure 8.9 Observed $\eta\pi^+$ (a) and $\eta\pi^-$ (b) mass spectra for the event sample with $-t(p \rightarrow \Lambda) < 1.0$ (GeV/c)², $m_{\eta\pi\pi} \geq 1.1$ GeV/c².

Figure 8.10 Observed $\pi^+\pi^-$ mass spectrum for the event sample with $-t(p \rightarrow \Lambda) < 1.0$ $(\text{GeV}/c)^2$, $m_{\eta\pi\pi} \geq 1.1 \text{ GeV}/c^2$.

Mass spectra against the $\Sigma(1385)^+$ show strong η production (Figure 8.14). In addition, there is some $\Sigma(1385)^-$ present in $\Lambda\pi^-$ (Figure 8.15) due to kinematic overlap. Figure 8.15 also shows the $\Lambda\pi^+\pi^-$ and $\Lambda\pi^+\eta$ mass distributions. The $\Lambda\pi^+\pi^-$ distribution shows some $\Lambda(1670)$, while $\Lambda\pi^+\eta$ is essentially featureless. The recoil $\eta\pi^-$ spectrum (Figure 8.16) shows a clear a_0 peak, but there is no clear evidence of $a_2(1320)$ production.

Mass spectra against the $\Sigma(1385)^-$ show relatively weak η production (Figure 8.17), as expected. The $\eta\pi^+$ distribution, shown along with the $\Lambda\pi^+$ and

Figure 8.11 Observed $\Lambda\pi^+$ (a) and $\Lambda\pi^-$ (b) mass spectra for the event sample with $-t(p \rightarrow \Lambda) < 1.0 \text{ (GeV/c)}^2$, $m_{\eta\pi\pi} \geq 1.1 \text{ GeV/c}^2$.

Figure 8.12 Observed $\Lambda\pi^-$ mass spectrum for $m_{\Lambda\pi^+\pi^-} > 2.1 \text{ GeV}/c^2$, $-t(p \rightarrow \Lambda) < 1.0 \text{ (GeV}/c^2)$, $m_{\eta\pi\pi} \geq 1.1 \text{ GeV}/c^2$. There is no evidence for $\Sigma(1385)^-$ associated with this range of $\Lambda\pi^+\pi^-$ mass.

$\Lambda\pi^+\pi^-$ distributions in Figure 8.18, is featureless. Due to kinematic overlap, $\Lambda\pi^+$ shows $\Sigma(1385)^+$. The $\Lambda\pi^+\pi^-$ spectrum shows production of $\Lambda(1670)$. The amount of $\Lambda(1670)$ production is sufficient to account for the observed $\Sigma(1385)^-$ signal. Finally, the $\Lambda\pi^-\eta$ spectrum (not shown) contains very few events below $3.0 \text{ GeV}/c^2$, and no structure in this mass region.

8.6 Examination of the $\Lambda\eta\rho$ Sample

A brief examination was made of the $\Lambda\eta\rho$ subsample. This sample is particularly interesting since $\eta\rho$ is pure isovector. Signal and background

Figure 8.13 Observed $\Lambda\pi^+\pi^-$ (a), $\Lambda\eta\pi^+$ (b), $\Lambda\eta$ (c), and $\Lambda\eta\pi^-$ (d) mass spectra for the event sample with $-t(p \rightarrow \Lambda) < 1.0 \text{ (GeV/c}^2\text{)}$, $m_{\eta\pi\pi} \geq 1.1 \text{ GeV/c}^2$.

regions were defined for the ρ . The ρ signal region was taken to be the mass range $0.70 \text{ GeV/c}^2 < m_{\pi\pi} < 0.85 \text{ GeV/c}^2$. The sidebands were taken to be 0.075 GeV/c^2 wide bands on either side of the signal region. A linear sideband subtraction was made for the ρ and for the η . The mass spectra for the resulting sample are shown in Figure 8.19. The net $\Lambda\eta$ and $\eta\rho$ mass spectra are structureless, while the net $\Lambda\rho$ mass spectrum shows some structure near 1.8 GeV/c^2 , associated with the ρ sidebands (hence the negative sign in the plot, which is ρ signal minus ρ sidebands).

Figure 8.14 Observed $\pi^+\pi^-(\pi^0)$ mass plot for $1.32 \text{ GeV}/c^2 \leq m_{\Lambda\pi^+} \leq 1.44 \text{ GeV}/c^2$, $-t(p \rightarrow \Lambda\pi^+) < 1.0 (\text{GeV}/c)^2$, $m_{\eta\pi\pi} \geq 1.1 \text{ GeV}/c^2$. The sideband and signal regions are both shown. The open circles are unused points between the signal and sideband regions.

8.7 Examination of the $\Lambda\eta\pi\pi$ Sample in the $f_0(970)$ Region

The region $0.95 \text{ GeV}/c^2 < m_{\pi\pi} < 1.05 \text{ GeV}/c^2$ shows some structure and was examined separately. No $\pi\pi$ sideband subtraction was performed on this sample, because of the proximity of the ρ signal in the $\pi\pi$ mass spectrum. An η sideband subtraction was performed as in previous samples. The $\eta\pi\pi$ mass spectrum for this sample is shown in Figure 8.20. No structure is evident. The $\Lambda\eta$ and $\Lambda\pi\pi$ mass spectra are also featureless, and are not shown.

Figure 8.15 Observed $\Lambda\pi^-$ (a), $\Lambda\pi^+\pi^-$ (b), and $\Lambda\pi^+\eta$ (c) mass spectra for the event sample with $1.32 \text{ GeV}/c^2 \leq m_{\Lambda\pi^+} \leq 1.44 \text{ GeV}/c^2$, $-t(p \rightarrow \Lambda\pi^+) < 1.0 \text{ (GeV}/c^2)$, $m_{\eta\pi\pi} \geq 1.1 \text{ GeV}/c^2$.

8.8 Study of the a_0 Meson

8.8.1 History of the a_0 Meson

The first claim of structure in the region near the a_0 meson came in 1965^[39] from a $\pi^-p \rightarrow p + X^-$ missing-mass experiment. This experiment claimed a 4.6 standard deviation effect in the missing mass, with $M_x = 962 \pm 5 \text{ MeV}/c^2$ and an estimated physical width $\leq 5 \text{ MeV}/c^2$. Over the following years, the region was quite controversial, with evidence for the a_0 appearing and disappearing^[40]. The first K^-p experiment to claim a resonance in the a_0 region was a bubble

Figure 8.16 Observed $\eta\pi^-$ mass plot for $1.32 \text{ GeV}/c^2 \leq m_{\Lambda\pi^+} \leq 1.44 \text{ GeV}/c^2$, $-\cos(\theta_{p \rightarrow \Lambda\pi^+}) < 1.0$, $m_{\eta\pi^-} \geq 1.1 \text{ GeV}/c^2$.

chamber experiment that published in 1968^[41]. The authors of this paper stated that it was unlikely that their resonance was the same as the resonance seen at $960 \text{ MeV}/c^2$, because of differences in the mass and width. This experiment claimed a signal of ~ 20 events in the $\eta\pi^-$ system peripherally produced against a $\Sigma(1385)^+$ in the reaction $K^-p \rightarrow \Lambda\pi^+\pi^-\eta$, although the paper noted that the $\eta\pi^-$ signal contains ~ 40 events before the $\Sigma(1385)$ cut was made. After fitting, a mass of $980 \pm 10 \text{ MeV}/c^2$ and a width of $80 \pm 30 \text{ MeV}/c^2$ were obtained for the resonance. Subsequent K^-p bubble chamber experiments^{[42][43][44]} also saw structure in the $\eta\pi^-$ system consistent with the 1968 experiment. The bubble

Figure 8.17 Observed $\pi^+\pi^-(\pi^0)$ mass plot for $1.32 \text{ GeV}/c^2 \leq m_{\Lambda\pi^-} \leq 1.44 \text{ GeV}/c^2$, $-t(p \rightarrow \Lambda\pi^-) < 1.0 (\text{GeV}/c)^2$, $m_{\eta\pi\pi} \leq 1.1 \text{ GeV}/c^2$. The sideband and signal regions are both shown. The open circles are unused points between the signal and sideband regions.

chamber experiments carried out at $3.1\text{--}3.6 \text{ GeV}/c^{[43]}$ and $4.2 \text{ GeV}/c^{[44]}$ are the only experiments carried out in kaon beams where a significant a_0 signal is seen associated with a charged mode decay of the η .

The nature of the $a_0(980)$ is still unclear. It is currently listed as the isovector member of the 0^{++} nonet, but it appears to be at the wrong mass to be the companion to the mostly $s\bar{s}$ $f_0(975)$. In turn, it would also be expected that the mostly $s\bar{s}$ isoscalar should be over $100 \text{ MeV}/c^2$ higher in mass than its

Figure 8.18 Observed $\Lambda\pi^+\pi^-$ (a), $\Lambda\pi^+$ (b), and $\eta\pi^+$ (c) mass spectra of the event sample with $1.32 \text{ GeV}/c^2 \leq m_{\Lambda\pi^-} \leq 1.44 \text{ GeV}/c^2$, $-(p \rightarrow \Lambda\pi^-) < 1.0 \text{ (GeV}/c^2)$, $m_{\eta\pi} \geq 1.1 \text{ GeV}/c^2$.

non-strange isoscalar partner, the $f_0(1400)$. Arguments have been made that the $a_0(980)$ and $f_0(975)$ are not normal meson, but $K\bar{K}$ molecules^{[45][46]}. Although the a_0 is observed to have a width of approximately $50 \text{ MeV}/c^2$ in its $\eta\pi$ decay mode, it also decays to $K\bar{K}$, which is observed as a several hundred MeV/c^2 wide threshold enhancement^[44]. Attempts have been made to explain this unusual behavior in terms of a very wide resonance^[47], which only appears narrow in $\eta\pi$ because of the opening of $K\bar{K}$ threshold (pulling down the high side in $\eta\pi$) and the requirement of analyticity (pulling down the low side in $\eta\pi$).

Figure 8.19 Observed $\Lambda\eta$ (a), $\Lambda\rho$ (b), and $\eta\rho$ (c) mass spectra for $\Lambda\eta$ sample.

Previous kaon beam experiments have indicated that the a_0 is produced in conjunction with a $\Sigma(1385)$. Finally, the spin of the a_0 has not been well-established in kaon beam experiments. Angular distributions have been examined, and the distributions are consistent with a spin-0 object, but statistics have been low enough that the distributions are also consistent with other interpretations. The best determination of the spin of the a_0 comes from a Crystal Ball experiment^[48], in which a_0 was observed in $\gamma\gamma$ interactions. Since any object produced in $\gamma\gamma$ interactions must have even spin, and the a_0 angular distributions are not consistent with spin-2, the spin of the a_0 must be zero.

Figure 8.20 Observed $\eta\pi^+\pi^-$ mass spectrum for the $f_0(975)$ region in $\pi^+\pi^-$. This sample requires $-t(p \rightarrow \Lambda) < 1.0 \text{ (GeV/c)}^2$, $m_{\eta\pi\pi} \geq 1.1 \text{ GeV/c}^2$, $0.95 \text{ GeV/c}^2 \leq m_{\pi\pi} \leq 1.05 \text{ GeV/c}^2$.

8.8.2 Examination of the Data

Events with $0.95 \text{ GeV/c}^2 \leq m_{\eta\pi\pi} \leq 1.0 \text{ GeV/c}^2$ and $-t(p \rightarrow \Lambda) \leq 2.0 \text{ (GeV/c)}^2$ were selected. This mass range should contain a significant fraction of the $a_0(980)$ events, and has the best signal to noise ratio for the $a_0(980)$. The $\Lambda\pi^+$ mass spectrum for this sample, shown in Figure 8.21, clearly contains a strong signal from the $\Sigma(1385)^+$ and additional structure in the range $1.6 \text{ GeV/c}^2 - 1.9 \text{ GeV/c}^2$, a region populated by numerous Σ resonances. This suggests that the a_0 is preferentially produced in conjunction with Σ 's. Such a production

Figure 8.21 Observed $\Lambda\pi^+$ mass spectrum for the event sample containing the a_0 . This group of events was selected from the $\Lambda\eta\pi^+\pi^-$ sample by adding the requirements $0.95 \text{ GeV}/c^2 \leq m_{\eta\pi^-} \leq 1.00 \text{ GeV}/c^2$, $-\mathbf{t}(\mathbf{p} \rightarrow \Lambda) \leq 2.0 (\text{GeV}/c)^2$. Structure is evident both at the $\Sigma(1385)$ and in the range $1.6 - 1.9 \text{ GeV}/c^2$, where many other Σ resonances exist. This pattern suggests that the a_0 is preferentially produced in conjunction with Σ resonances.

mechanism matches observations made by earlier experiments. Another known source of the a_0 meson is through decay of the $f_1(1285)$, which is known to decay to $a_0\pi$. Figure 8.22 shows the $\eta\pi^+\pi^-$ mass spectrum for events in the a_0 region. A small excess in the bin containing the $f_1(1285)$ is consistent with the presence of approximately six $f_1(1285) \rightarrow a_0\pi$ events. This number is consistent with the ~ 12 $f_1(1285)$ events seen in an earlier analysis of the $K_S^0 K^\pm \pi^\mp \Lambda$ final

Figure 8.22 Observed $\eta\pi^+\pi^-$ mass spectrum for the event sample containing the a_0 . This group of events was selected from the $\Lambda\eta\pi^+\pi^-$ sample by adding the requirements $0.95 \text{ GeV}/c^2 \leq m_{\eta\pi^-} \leq 1.00 \text{ GeV}/c^2$, $-\kappa(p \rightarrow \Lambda) \leq 2.0 \text{ (GeV}/c)^2$.

state in this experiment^[31]. A cross section can be calculated by assuming an acceptance of $25 \pm 5\%$ (estimated from $K^-p \rightarrow \eta'\Lambda$ Monte Carlo data and consistent with $K^-p \rightarrow \eta\pi^-\Sigma(1385)^+$ Monte Carlo data), correcting for the unseen decay modes of the Λ , $f_1(1285)$, and the η , the Clebsch-Gordan coefficient for $f_1(1285) \rightarrow a_0^-\pi^+$, and estimating that 45% of a_0 is in the bin $0.95 \sim 1.00 \text{ GeV}/c^2$. The value obtained through this calculation is $\sigma(K^-p \rightarrow f_1(1285)\Lambda) \text{BF}(a_0 \rightarrow \eta\pi) = 0.71 \pm 0.35 \mu\text{barns}$, where the error is a combination of the statistical error on six observed events, the uncertainty in the $f_1(1285) \rightarrow a_0^-\pi^+$ branching fraction, and

the uncertainty in the acceptance. Only the cross section times branching fraction can be calculated, since the branching fraction for $a_0 \rightarrow \eta\pi$ is not known.

Production of the a_0 was examined further by making various $\Lambda\pi^+$ mass cuts and by making various t cuts. Figure 8.23 shows the $\eta\pi^-$ mass plots for various $\Lambda\pi^+$ mass cuts and for $-t(p \rightarrow \Lambda) < 2.0 \text{ (GeV/c)}^2$. The data in the figure again indicate that, in K^-p interactions, a_0 production is correlated with Σ production. There is no evidence for a_0 production where there are no Σ 's, except for the possible existence of a few events corresponding to $f_1(1285)$ decay. In addition, there is no evidence for a_0 production for $-t(p \rightarrow \Lambda\pi^+) > 1.0 \text{ (GeV/c)}^2$, as can be seen in Figure 8.24 and Figure 8.25.

8.9 Acceptance Correction

For further investigation of the a_0 meson, analysis was restricted to the region in $\Lambda\pi^+$ mass around the $\Sigma(1385)^+$ ($1.34 \text{ GeV/c}^2 \leq m_{\Lambda\pi^+} \leq 1.44 \text{ GeV/c}^2$). This was done since a large fraction of the observed a_0 signal was in this region, and the $\Sigma(1385)$ is a single, well-defined $\Lambda\pi$ resonance, making Monte Carlo generation for acceptance calculation easier.

The acceptance calculation for this channel used event weighting. Each event was assigned a weight that was the product of two factors. One of the factors was calculated from the reconstructed $\eta\pi^-$ mass and $t'(p \rightarrow \Sigma(1385)^+)$. The other was calculated from reconstructed $\eta\pi^-$ mass and the Gottfried-Jackson[†]

[†] The Gottfried-Jackson angle is the angle, measured in the rest frame of the parent particle, between one of the decay daughters and the z -axis, where the z -axis is defined to be the beam direction.

Figure 8.23 Observed $\eta\pi^-$ mass spectra for various regions in $\Lambda\pi^+$ mass and $-t(p \rightarrow \Lambda) < 2.0$ (GeV/c) 2 . (a) $\Lambda\pi^+$ mass in $\Sigma(1385)$ region, defined as $m_{\Lambda\pi^+} < 1.5$ GeV/c 2 . (b) $\Lambda\pi^+$ mass in the upper Σ region, defined by $1.6 \text{ GeV/c}^2 \leq m_{\Lambda\pi^+} \leq 1.9 \text{ GeV/c}^2$. (c) $\Lambda\pi^+$ mass outside the regions in the other two plots. (a) and (b) show evidence for a_0 production in the Σ regions, while (c) shows that there is no evidence for a_0 production not associated with Σ .

Figure 8.24 Observed $\eta\pi^-$ mass spectra against $\Sigma(1385)^+$. The $\Sigma(1385)^+$ was defined by $m_{\Lambda\pi^+} \leq 1.50 \text{ GeV}/c^2$. The upper plot shows the range $-t(p \rightarrow \Lambda\pi^+) < 0.3 (\text{GeV}/c)^2$. The center plot shows the range $0.3 \leq -t < 1.0$. The lower plot shows the range $1.0 \leq -t < 2.0$. There is no evidence of a_0 production of $-t(p \rightarrow \Lambda\pi^+) > 1.0 (\text{GeV}/c)^2$.

Figure 8.25 Observed $\eta\pi^-$ mass spectra for $1.60 \leq m_{\Lambda\pi^+} \leq 1.90$. The upper plot shows the range $-t(p \rightarrow \Lambda\pi^+) < 0.3 (\text{GeV}/c^2)^2$. The center plot shows the range $0.3 \leq -t < 1.0$. The lower plot shows the range $1.0 \leq -t < 2.0$. There is no evidence of a_0 production for $-t(p \rightarrow \Lambda\pi^+) > 1.0 (\text{GeV}/c^2)^2$.

and Treiman-Yang[†] angles for the decay of the $\eta\pi^-$ system. Approximately 60,000 Monte Carlo events of the reaction $K^-p \rightarrow \eta\pi^- \Sigma(1385)^+$ were generated in order to characterize the acceptance. The events were generated in three $\eta\pi^-$ mass sections: $0.69 \text{ GeV}/c^2$ (threshold) - $1.20 \text{ GeV}/c^2$, $1.20 \text{ GeV}/c^2$ - $1.75 \text{ GeV}/c^2$, and $1.75 \text{ GeV}/c^2$ - $3.41 \text{ GeV}/c^2$ (the kinematic limit). Approximately equal numbers of events were generated in each mass region. The distribution generated in each region was flat in mass, Gottfried-Jackson angle, and Treiman-Yang angle. The resulting event density in mass was a reasonable approximation to the event density in the real data. Each mass region was generated with a t' slope which was derived from the $\Sigma(1385)^+ \eta\pi^-$ data in that region. These t' slopes were: 3.6 for the low mass region, 3.4 for the mid mass region, and 2.2 for the high mass region. After being passed through the same reconstruction, fitting, and event selection as the real data, angular and mass/ t' weights were determined as described in the following sections.

After the angular and mass/ t' weights had been determined, the reconstructed Monte Carlo events were weighted by the corrections. This was done as a check on the weighting, to make sure that the generated mass/ t' and angular distributions were recovered.

8.9.1 Mass and t' Acceptance

The data was divided into four $-t'$ bins; 0.0-0.2, 0.2-0.5, 0.5-1.0, and 1.0-2.0. The acceptance was plotted as a function of mass in each t' bin. Because the events were generated with different t' slopes and different mass densities

[†] The Treiman-Yang angle is the angle between the plane defined by the η and π^- , and the plane defined by the beam and the recoil (known as the production plane).

in each mass region, there were discontinuities in event densities at the region boundaries. Because of these discontinuities, reconstructed events would preferentially flow across the boundaries from high density regions into low density regions. In order to eliminate this potential problem, the events were re-weighted to be flat in mass and have the same t' slope. After reweighting, each t' bin was fitted with two straight line segments. A second segment was needed at low $\eta\pi$ mass because of the uniqueness requirement imposed on the real data. This requirement, that an event have a unique η in the mass range $0.52 - 0.58 \text{ GeV}/c^2$, preferentially removes low mass $\eta\pi$ combinations, where the π has little momentum relative to the η , and it is more likely that interchanging the two π present at the primary vertex in the event will make little difference in the reconstructed η mass. The two segments were joined at an $\eta\pi$ mass of $0.85 \text{ GeV}/c^2$, and were forced to have the same value at that mass. Bins with an $\eta\pi$ mass above $3.2 \text{ GeV}/c^2$ or below $0.7 \text{ GeV}/c^2$ were omitted from the fits, since they had large uncertainties, and very little real data are present in those bins. Figure 8.26 shows the acceptance for this reaction as a function of $\eta\pi$ mass and t' .

After each of the four t' bins was fit using two segments, the slope and intercept of those segments were separately parametrized as parabolas in t' , and fitted to the four points (four t' bins).

When the acceptance weight for a real event was to be calculated, the $\eta\pi$ mass and t' for the event were used to determine an acceptance slope and intercept from the parabolic fit. This slope and intercept were then used to determine the mass- t' weight based on the event's measured $\eta\pi$ mass.

Figure 8.26 Acceptance for the reaction $K^-p \rightarrow \eta\pi^-\Sigma(1385)^+$ as a function of mass and $t'(p \rightarrow \Sigma(1385)^+)$ (GeV/c^2). The lines show a fit consisting of two segments. The acceptance loss at low mass is due to the uniqueness requirement imposed on the η .

8.9.2 Angular Acceptance

Since we are investigating the properties of the $\eta\pi$ system, the angular quantities of interest are the spherical harmonic functions, $Y_l^m(\theta, \phi)$, of the decay of the $\eta\pi$ system. The appropriate angles to use are the Gottfried-Jackson and Treiman-Yang angles for the $\eta\pi$ system. It is natural, then, to expand the angular acceptance in terms of the spherical harmonics, defined by

$$Y_l^m(\theta, \phi) = \sqrt{\frac{2l+1}{4\pi} \frac{(l-m)!}{(l+m)!}} P_l^m(\cos\theta) e^{im\phi}$$

where the P_l^m are the associated Legendre functions. Here the normalization of the Y_l^m has been chosen so that

$$\int_{4\pi} Y_l^m(\theta, \phi) Y_{l'}^{m'}(\theta, \phi)^* d\Omega = \delta_{ll'} \delta_{mm'}$$

The angular acceptance is calculated by comparing generated and accepted angular distributions of Monte Carlo events. In particular,

$$a(\theta, \phi) = \frac{\frac{dA_f}{d\Omega}}{\frac{dA_t}{d\Omega}}$$

In this expression, a is the acceptance, A_t is the distribution of generated events, and A_f is the angular distribution of reconstructed events. Since the Monte Carlo was generated flat in the angles, the generated distribution is given by

$$\frac{dA_t}{d\Omega} = \frac{N_t}{4\pi}$$

where N_t is the number of generated events.

The observed distribution of events in the Monte Carlo data can be expressed as

$$\frac{dA_f(\Omega)}{d\Omega} = N_f \sum_{l=0}^{l_{max}} \sum_{m=-l}^l \alpha_{lm} Y_l^m(\theta, \phi)$$

where N_f is the number of reconstructed events, α_{lm} are constants, and l_{max} is the maximum value of interest, determined by examining the real data.

Although parity is conserved in strong interactions, the detector is not necessarily parity invariant. Because of this, it is not automatically true that parity invariance can be imposed to simplify the angular acceptance. Nevertheless, examination of data from this experiment has shown that the detector is, in fact, approximately parity invariant. Using parity invariance, the reflection property of the Y_l^m can be used to write Y_l^m as

$$Y_l^{-m}(\theta, \phi) = (-1)^m Y_l^m(\theta, \phi)^*$$

Then, $A_f(\Omega)$ can be rewritten as

$$\frac{dA_f(\Omega)}{d\Omega} = N_f \left\{ \sum_{l=0}^{l_{max}} \alpha_{l0} Y_l^0(\theta, \phi) + \sum_{l=1}^{l_{max}} \sum_{m=1}^l \alpha_{lm} 2 \operatorname{Re} Y_l^m(\theta, \phi) \right\}$$

This yields, for α_{lm}

$$\alpha_{lm} = \langle \text{Re } Y_l^m(\theta, \phi) \rangle$$

where the average is over the distribution of found events, A_f

Substituting the expressions for A_f and A_t into the acceptance expression, we obtain

$$a(\theta, \phi) = \frac{\frac{dA_f(\Omega)}{d\Omega}}{\frac{dA_t(\Omega)}{d\Omega}} = 4\pi \frac{N_f}{N_t} \left\{ \sum_{l=0}^{l_{\max}} \alpha_{l0} Y_l^0(\theta, \phi) + \sum_{l=1}^{l_{\max}} \sum_{m=1}^l \alpha_{lm} 2\text{Re } Y_l^m(\theta, \phi) \right\}$$

This is applied in each mass bin, where in each bin the average acceptance, given by given by the mass- t' parametrization above, is

$$\langle a(\theta, \phi) \rangle = \frac{1}{4\pi} \int a(\theta, \phi) d\Omega = \frac{N_f}{N_t}$$

The acceptance weight assigned to any event is then

$$w = \frac{1}{a(\theta, \phi)}$$

Figure 8.27 Normalized moments for Monte Carlo data. The moments are defined by following the η . The straight line fit to the moment, used for acceptance correction, is shown as a solid line. All moments not shown are consistent with zero.

A straight line was fit to each moment. Only those moments which had a fit inconsistent with zero were included in the calculation of $a(\theta, \phi)$. The results of the fits showed that only the Y_1^0 , Y_2^0 , Y_2^1 , and Y_2^2 moments differed systematically from zero, as shown in Figure 8.27.

Figure 8.28 Acceptance corrected unnormalized moments for the $\eta\pi^-$ system in the reaction $K^-p \rightarrow \eta\pi^- \Sigma(1385)^+$. The moments are defined by following the η .

8.10 Acceptance Corrected Data

Figure 8.28, Figure 8.29, and Figure 8.30 show the acceptance corrected unnormalized data moments for the $\eta\pi^-$ system. In these plots, the t_{lm} are related to the spherical harmonics Y_l^m by

Figure 8.29 Acceptance corrected unnormalized moments for the $\eta\pi^-$ system in the reaction $K^-p \rightarrow \eta\pi^- \Sigma(1385)^+$. The moments are defined by following the η .

$$t_{lm} = \sqrt{4\pi} \sum_i w_i Y_l^m(\Omega_i)$$

where w_i is the event weight and i runs over the number of events. In particular, the t_{00} distribution is the acceptance corrected mass spectrum, and shows a clear a_0 peak. Only t_{20} exhibits structure in the region of the $a_2(1320)$. Since the

Figure 8.30 Acceptance corrected unnormalized moments for the $\eta\pi^-$ system in the reaction $K^-p \rightarrow \eta\pi^- \Sigma(1385)^+$. The moments are defined by following the η .

a_2 is known to be a spin-2 object, in general there could be structure in moments through $l=4$. It is possible, however, for a spin-2 object not to be visible in any moment except t_{20} . In particular, in a model where the only wave present in the mass region near the a_2 is a D-wave and that the D-wave has $|D_+|^2 = |D_-|^2$ and $|D_0|^2 = 4/3 |D_+|^2$, then only the t_{20} moment will show structure (see Appendix A for the relation between moments and amplitudes). This would

imply that t_{00} would have 2.2 times as many events as t_{20} . In the data, however, t_{00} and t_{20} have approximately equal numbers of events in the a_2 region. This means that the model is inadequate. One way to account for the data is to also allow a resonant S-wave in the a_2 region, with approximately the same mass, width, and production phase as the a_2 . If $S \equiv 1/6 D_0$, S-D interference contributes much more to t_{20} than $|S|^2$ does to t_{00} , and the two moments can have equal numbers of events in the a_2 region. Such resonant S-wave behavior in this region has recently been observed in the $\eta\pi$ system^[49].

An attempt can be made to investigate the production mechanism for the a_0 by looking at the t dependence. From previous analyses of hypercharge exchange reactions in this experiment (e.g., ref. [30]), it is known that both natural and unnatural spin-parity production contribute for $-t < 0.3 \text{ (GeV/c)}^2$, and that natural spin-parity exchange dominates for $-t > 0.3 \text{ (GeV/c)}^2$. Moreover, while the a_0 can be produced only by unnatural parity exchange, the a_2 can be produced by both unnatural and natural parity exchange. The data from this experiment in the $\Sigma(1385)^+$ region was split at $-t = 0.3 \text{ (GeV/c)}^2$. The corresponding $\eta\pi$ mass spectra are shown in Figure 8.31, and indicate that the t dependence in the a_0 region is slightly steeper than in the a_2 region, although the effect is not dramatic. This again is qualitatively consistent with the simple model used to describe the behavior of the moments, in which only ~30% of the a_2 cross section would result from natural parity exchange.

8.11 Examination of $K_S^0 K^- \pi^+ \Lambda$ Event Sample

The a_0 meson is also known to decay to $K\bar{K}$. This decay mode was briefly investigated by using data from an analysis of the channel $K^- p \rightarrow K_S^0 K^- \pi^+ \Lambda$ in

Figure 8.31 Acceptance corrected $\eta\pi^-$ mass spectra for the reaction $K^-p \rightarrow \eta\pi^- \Sigma(1385)^+$. Corrections have been made for all unseen decay modes. The upper plot shows the spectrum for $-t(p \rightarrow \Sigma(1385)^+) < 1.0 \text{ (GeV/c)}^2$. Portions of this region are shown in the other two plots. The center plot shows the spectrum for $-t < 0.3$, a range where both natural and unnatural spin-parity exchange contribute. The lower plot shows the spectrum for $0.3 < -t \leq 1.0$, a range dominated by natural spin-parity exchange.

Figure 8.32 Observed $\Lambda\pi^*$ mass spectrum for events of the reaction $K^-p \rightarrow K_S^0 K^- \pi^* \Lambda$, $-t(p \rightarrow \Lambda\pi^*) < 1.0 \text{ (GeV/c)}^2$.

this experiment^[31]. Events were required to have a 4-C MVFIT confidence level greater than 10^{-5} and a $-t(p \rightarrow \Lambda\pi^*) < 1.0 \text{ (GeV/c)}^2$. The $\Lambda\pi^*$ mass spectrum for these events is shown in Figure 8.32. A clear $\Sigma(1385)^+$ is present in the data. Since a_0 production seems to be correlated with Σ production, the net $\Sigma(1385)$ sample was selected by performing a sideband subtraction in $\Lambda\pi^*$, using the range 1.34 - 1.44 GeV/c^2 as the signal range, and the range 1.44 - 1.54 GeV/c^2 as the sideband. Acceptance correction for the sample began with a constant acceptance of 0.18 from ref. [31], then included a factor of 0.6 to account for the tails of the $\Sigma(1385)$ and the sideband subtraction. Additional factors accounted

Figure 8.33 Acceptance-corrected $K_S^0 K^-$ mass spectrum for events of the reaction $K^- p \rightarrow K_S^0 K^- \Sigma(1385)^*$, $-t(p \rightarrow \Sigma(1385)^*) < 1.0 \text{ (GeV/c)}^2$. The first non-zero bin begins at threshold.

for the fact that only 50% of K^0 appears as K_S^0 , and for all unseen decay modes. Correction for the $\Sigma(1385)$ tails and sideband subtraction was obtained from the Monte Carlo data run for the $\eta\pi^-$ analysis. The acceptance-corrected $K_S^0 K^-$ mass spectrum is shown in Figure 8.33, and exhibits a threshold enhancement several hundred MeV/c^2 wide. This shape is qualitatively in agreement with that seen in the 4.2 GeV/c Bubble Chamber experiment^[44].

8.12 Flatté and $K\bar{K}$ Molecule Fits

The 4.2 GeV/c Bubble Chamber used the model of Flatté^[47] to fit simultaneously their $\eta\pi^-$ and $K_S^0 K^-$ mass spectra in the a_0 region. Although more complicated models exist^[50], the Flatté model will also be used here, in order to directly compare the results from this experiment to those from the 4.2 GeV/c experiment, and because limited statistics in this experiment likely make it insensitive to the details present in more complex models.

In the Flatté model, the a_0 is treated as a broad resonance lying very near $K\bar{K}$ threshold. The resonance appears narrow in the $\eta\pi$ channel because of the opening of the $K\bar{K}$ channel, pulling down the high side, and unitarity, bringing down the low side. The cross sections can be written as

$$\frac{d\sigma_i}{dm} = C m \left| \frac{\sqrt{\Gamma_i}}{m_R^2 - m^2 - i m_R (\Gamma_{\eta\pi} + \Gamma_{K\bar{K}})} \right|^2$$

$$\Gamma_{\eta\pi} = g_\eta^2 \left(\frac{q_\eta}{m} \right)$$

$$\Gamma_{K\bar{K}} = \begin{cases} g_K^2 \left(\frac{q_K}{m} \right) & \text{above threshold} \\ i g_K^2 \left(\frac{|q_K|}{m} \right) & \text{below threshold} \end{cases}$$

where the subscript i may be either $\eta\pi$ or $K\bar{K}$, q_η is the decay momentum of mass m into a $\pi\eta$ system, q_K is the decay momentum of mass m into a $K\bar{K}$

system (analytically continued below threshold), m_R is the a_0 mass, g_η^2 and g_K^2 are the squares of the coupling constants for the respective channels, and $\Gamma_{\eta\pi}$ and $\Gamma_{K\bar{K}}$ are the partial widths into the $\eta\pi$ and $K\bar{K}$ channels. The constants Γ_0 and m_R which appear in Flatté's numerator have been absorbed into the constant C here. In addition, the factor of m premultiplying the expression here was absorbed by Flatté into Γ_1 . Finally, the widths have been expressed as q/m , instead of the simple q used by Flatté.

Fitting their $K\bar{K}$ and $\eta\pi$ data to the Flatté model, the 4.2 GeV/c bubble chamber experiment obtained values of 0.974 ± 0.009 GeV/c² for m_R , and 72 ± 52 MeV/c² for $\Gamma_{\eta\pi}$. For their fits, the ratio g_K^2/g_η^2 was fixed at the octet-singlet SU(3) value of $3/2^\dagger$. In the current analysis, several fits were tried; a fit to only the 4.2 GeV/c $K\bar{K}$ and $\eta\pi$ data, a fit to only the $K\bar{K}$ and $\eta\pi$ data from this experiment, and two different simultaneous fit to the data from both experiments. The coupling constant ratio, the mass, and the $\eta\pi$ width were left free in all of the fits described here. In addition, a phase space background was allowed under each of the mass spectra. The phase space background was of the form

$$\frac{d\sigma_{BG}(m)}{dm} = Cq_1q_2(1 + Am + Bm^2)$$

where q_1 is the q for the decay of the center of mass system to a $\Sigma(1385)$ and a system of mass m , and q_2 is the q for the decay of a system of mass m to an $\eta\pi$

[†] If, instead of assuming no octet/singlet mixing for the η , the -19.5° mixing angle of the pseudoscalar nonet is used, the ratio is $\sim 3/4$.

Parameter	4.2 GeV/c	11 GeV/c	Simultaneous
m_R (GeV/c ²)	$0.975^{+0.012}_{-0.018}$	0.999 ± 0.035	$0.969^{+0.017}_{-0.020}$
$\Gamma_{\eta\pi}$ (MeV/c ²)	94^{+58}_{-35}	120^{+72}_{-53}	120^{+50}_{-31}
g_{KK}^2 / g_{η}^2	$1.6^{+1.0}_{-0.6}$	$3.2^{+3.3}_{-1.2}$	$2.4^{+1.1}_{-0.7}$

Table 8.4 Results of Flatté model fits to the data from this experiment and the 4.2 GeV/c bubble chamber experiment.

or KK system, as appropriate. Because of the narrowness of the Flatté model peak in the $\eta\pi$ spectra, the $\eta\pi$ curves were smeared by a Gaussian resolution of 20 MeV/c² for this experiment, and 7 MeV/c² for the 4.2 GeV/c experiment. The χ^2 for the fit was calculated using the integral of the signal plus background in each bin.

The results of the fits are summarized in Table 8.4, Figure 8.34 and Figure 8.35. The simultaneous fit has a χ^2 of 60 for 78 degrees of freedom. The 4.2 GeV/c numbers in the table are not identical to those published in the 4.2 GeV/c analysis because a different parametrization of the background has been used, and the Flatté parametrization has been changed slightly, as described above. Nevertheless, the 4.2 GeV/c numbers agree within errors. It is interesting to note that the ratio of the coupling constants in all of the fits is higher than either of the SU(3) predictions, although the effect is only 2.5σ at most.

Figure 8.34 Results of a simultaneous fit to the LASS and 4.2 GeV/c data using the Flatté model. The fit is described in the text. The curves show the phase space background and the total fit. The open circles in the $\eta\pi^-$ plot show the integral of the smeared curve in each data bin.

Figure 8.35 Results of a simultaneous fit to the LASS and $4.2 \text{ GeV}/c$ data using the Flatté model. The fit is described in the text. The curves show the phase space background and the total fit (the $K\bar{K}$ background is essentially zero). The open circles in the $\eta\pi^-$ plot show the integral of the smeared curve in each data bin.

Because the $a_2(1320)$ is known to decay to $\eta\pi$, another fit to all four spectra was attempted in which an $a_2(1320)$ signal was allowed in the LASS $\eta\pi$ spectrum. The a_2 was represented as D-wave relativistic Breit-Wigner, with the mass and width fixed at the particle data group values. The χ^2 for this fit was approximately the same as for the fit without the a_2 , and the ratio of the coupling constants agreed to within 1/3 of a standard deviation.

It is also possible to determine the energy dependence of the a_0 cross section. In order to accomplish this, the amplitude of the a_0 signal in this experiment, obtained with the a_0 mass, $\eta\pi$ width, and coupling constant ratio fixed at the 4.2 GeV/c values, was compared to the 4.2 GeV/c signal amplitude. The result, $\sigma_{11 \text{ GeV}}/\sigma_{4.2 \text{ GeV}} = 0.21 \pm 0.04$, implies an energy dependence of $p_{lab}^{-1.6 \pm 0.2}$, whereas an exponent of approximately 2.5 would be expected for the unnatural parity exchange believed to produce the a_0 .

Weinstein and Isgur^[45] recently published a model that attempts to explain the a_0 as a $K\bar{K}$ molecule. In their model, the shape of the $K\bar{K}$ mass spectrum is given by

$$\frac{d\sigma}{dm_{K\bar{K}}} = d(m_{K\bar{K}}) \frac{d\sigma^{\text{free}}}{dm_{K\bar{K}}}$$

They assume a square well for the $K\bar{K}$ potential, and if there is only one bound state, they obtain an expression for the function $d(m_{K\bar{K}})$

$$d(m_{K\bar{K}}) = \frac{m_{K\bar{K}} - m_{\text{threshold}} + V_0}{m_{K\bar{K}} - m_{\text{threshold}} + E_B(m_{K\bar{K}})}$$

Figure 8.36 Results of a fit to the $K\bar{K}$ data from this experiment using the $K\bar{K}$ molecule model of Weinstein and Isgur, described in the text.

$$f(m_{K\bar{K}}) = \frac{V_0}{E_B} \cos^2 \left(\left[\frac{m_{K\bar{K}} - m_{\text{threshold}}}{V_0} + 1 \right]^{1/2} \left[\frac{\pi}{2} + \left(\frac{E_B}{V_0} \right)^{1/2} \right] \right)$$

where E_B is the binding energy of the bound state and V_0 is the depth of the well. Following their paper, it is also assumed here that $(d\sigma^{\text{free}})/(dm_{K\bar{K}})$ is proportional to $K\bar{K}$ phase space. Weinstein and Isgur show a fit to the 4.2 GeV/c $K\bar{K}$ data using this model. A fit to the $K\bar{K}$ data from this experiment is shown in Figure 8.36. No background was used in this fit. The fit yields a χ^2 of 23 for

20 degrees of freedom, and the values of the parameters E_B and V_0 are very poorly defined, having values of $0.0047 \pm 0.0053 \text{ GeV}/c^2$ and $536 \pm 494 \text{ GeV}/c^2$, respectively. For comparison, the contribution of the LASS $K\bar{K}$ to the χ^2 for the simultaneous LASS and 4.2 GeV/c Flatté model fit is 22. Unfortunately, the $K\bar{K}$ molecule model currently does not predict the shape of the $\eta\pi$ mass spectrum or the relative $K\bar{K}/\eta\pi$ coupling, so no coupled-channel analysis can be carried out.

8.13 Conclusions

Study of the final state $\Lambda\eta\pi\pi$ encompassed a wide variety of processes and particles, including $\Lambda\eta'$, $\Lambda\eta\rho$, the $a_0(980)$, the $a_2(1320)$, and the region in $\pi\pi$ mass around the $f_0(975)$. The cross section for $K^-\bar{p} \rightarrow \eta'\Lambda$ at 11 GeV/c from this experiment is $8.9 \pm 0.6 \text{ } \mu\text{barns}$, in good agreement with other measurements at different energies. The mass and width of the η' agree with the Particle Data Group values. A comparison of the t dependence of the cross section from this experiment to data from the high statistics 4.2 GeV/c Bubble Chamber is in agreement with the Regge model, assuming a single exchanged trajectory.

Production of the $a_0(980)$ was observed to be correlated with Σ^* production, as in earlier experiments. This experiment provides the first observation of $a_0(980)$ production in conjunction with higher mass Σ^* 's, in the mass region 1.6–1.9 GeV/ c^2 . The angular distribution of the $a_0(980)$ decay is consistent with the claimed spin-0 nature of the $a_0(980)$. There is also evidence for a small number (~ 6) of $f_1(1285)$ events. This meson is known to decay to $a_0\pi$.

The data from this experiment in $K^-p \rightarrow \eta\pi^- \Sigma(1385)^+$ and $K^-p \rightarrow K_S^0 K^- \Sigma(1385)^+$ were compared with similar data from the 4.2 GeV/c bubble chamber using the model of Flatté. The results of fits to the data from both experiments indicate that the a_0 may couple more strongly to $K\bar{K}$ than is predicted for a $q\bar{q}$ state by SU(3), although the uncertainty is large. In addition, the a_0 cross changes somewhat more slowly as a function of lab momentum than would be expected for the unnatural parity exchange believed to produce the a_0 .

The $K\bar{K}$ data from this experiment were also fitted using the $K\bar{K}$ molecule model of Weinstein and Isgur. This model was also able to describe the $K\bar{K}$ shape well. Unfortunately, no predictions of the $\eta\pi$ spectrum or the relative $K\bar{K}$ and $\eta\pi$ couplings are currently available for this model, and the quality of the $K\bar{K}$ fit is essentially the same as for the Flatté model.

Analysis of the angular distribution of the $\eta\pi^-$ system in the region of the $a_2(1320)$ is consistent with the spin-2 nature of the a_2 , with certain restrictions on the relative sizes of the D_+ , D_- , and D_0 waves. The angular distributions also indicate the possibility of an S-wave in the a_2 region.

Examinations of $\Lambda\eta p$ and the $\pi\pi$ mass region near the $f_0(975)$ were inconclusive, due to limited statistics.

All of the studies undertaken of this final state were strongly limited by statistics. An increase of an order of magnitude in statistics would permit

greatly improved analyses in many areas, particularly the $a_0(980)$, $\Lambda\eta\rho$, and the $f_0(975)$ region.

9. $\Lambda K^+ K^- \pi^+ \pi^-$

9.1 Introduction

This is another many body final state, like $\Lambda \eta \pi \pi$. Because of this, it suffers from the same embarrassment of riches; there can be resonances present in many subsystems. This final state potentially contains several interesting subprocesses, including $\Lambda \phi \pi^+ \pi^-$, $\Lambda \phi \rho$, and $\Lambda K^* \bar{K}^*$. The data selection is somewhat simpler than for $\Lambda \eta \pi \pi$, because all final state particles are reconstructed in this channel; there are no missing neutral particles. After a description of the data selection, each of the subsamples mentioned above will be discussed.

9.2 Selection of $\Lambda K^+ K^- \pi^+ \pi^-$ Sample

The $\Lambda K^+ K^- \pi^+ \pi^-$ sample was selected from the inclusive Λ sample by adding requirements on the event topology, number of tracks, charge balance, missing momentum, and missing energy. Topologically, these events consist of a primary vertex with four charged prongs (not including the beam) and a neutral Λ connector, and a secondary vertex where the Λ decays into a proton and a pion. The topology requirements were identical to those used for the $\Lambda \eta \pi \pi$ selection; that is, events were required to have either a topology 3 or a topology 1 interpretation, and the topology 3 took precedence if the event had both topology 3 and topology 1 interpretations.

Candidate events were required to have four or five charged prongs (not counting the beam) at the primary vertex. The allowance for five charged prongs was made for the same reasons as in the $\Lambda \eta \pi \pi$ selection. In order for an

event with five charged prongs to be kept, the event had to meet the charge balance, missing energy, and missing momentum requirements when one track was dropped.

Candidate combinations were required to have a net zero charge at the primary vertex. In addition, they were required to have a missing longitudinal momentum of no more than 1.0 GeV/c, and a missing transverse momentum of no more than 0.2 GeV/c. The missing momentum cuts served to reduce contamination from events with a missing neutral particle and events that were incorrectly reconstructed. In addition, candidate combinations were required to have a missing mass squared of no more than $0.15 (\text{GeV}/c^2)^2$ when the tracks were called Λ , π^+ , π^- , K^+ , and K^- . A total of 309,004 events passed these cuts and were written to 5 tapes.

MVFIT was performed next. All track combinations which passed the $\Lambda K^+ K^- \pi^+ \pi^-$ cuts mentioned above were fit. The fits attempted were a GeoFit, a 3-C fit, and a 4-C fit. Point dropping was allowed only in the GeoFit, which was the first fit attempted. If the GeoFit converged, it was written to tape and a 3-C fit was tried next. The 3-C fit forced momentum balance and constrained the Λ mass to its known value, but did not impose energy balance (i.e., the primary vertex charged particles were not assigned masses). If the 3-C fit converged, it was written to tape and 4-C fits were attempted. Four 4-C fits, corresponding to the four possible $K^+ K^- \pi^+ \pi^-$ mass assignments for the four primary vertex charged prongs, were tried for each combination. Each fit which converged was written to tape. A total of 276,041 events had at least a converged GeoFit. These events were written onto 5 tapes.

Cut	Number of Combinations	
	Cut	Remaining
Input Events		276,041 Events
Input 4-C Fits		519,559
Require 4-C Fit Confidence Level $> 10^{-6}$	470,393	49,166
Require $\Sigma\text{Charge} = 0$	91	49,075
Cherenkov	4000	45,075
Time of Flight	5085	39,990
Output combinations		39,990
Output Events		30,569 Events

Table 9.1 Summary of cuts used in the post-MVFIT $\Lambda K K \pi \pi$ selection.

After MVFIT, another data selection was made. A summary of the cuts made in this selection is shown in Table 9.1. Candidate combinations were required to have a 4-C fit confidence level greater than 10^{-6} . In addition, charge balance was again imposed, since occasionally MVFIT would reverse the curvature, and hence the charge, of a stiff (high-momentum) track.

Particle ID cuts were also made. Combinations in which the primary vertex K^+ or K^- had an ADC value greater than 400 ADC counts in Cherenkov counter C_1 were removed. This cut removed pion contamination from the kaon sample. Time of flight cuts were made on the primary vertex K^+ , K^- , and π^+ , and on the proton produced by the decay of the Λ . Time of flight cuts were not made

on a track if there was more than one hit expected or observed in the paddle through which the particle passed. For the K^+ , proton contamination was removed by cutting any combination with a K^+ momentum less than 3.0 GeV/c in which the TOF was more than 3σ slower than a kaon, and within $\pm 3\sigma$ of the expected proton time. Pion contamination was removed by cutting any combination with a K^+ momentum less than 1.6 GeV/c in which the TOF was more than 2σ faster than a kaon, and within $\pm 3\sigma$ of the expected pion time. Pion contamination in the K^- was removed by cutting any combination with a K^- momentum less than 1.6 GeV/c in which the TOF was more than 2σ faster than a kaon, and within $\pm 3\sigma$ of the expected pion time. Proton and kaon contamination in the π^+ was removed by cutting any combination with a π^+ momentum less than 3.0 GeV/c where the TOF was more than 3σ slower than the expected pion time of flight and within $\pm 3\sigma$ of either the expected kaon or proton time of flight. Finally, $K^0 (\rightarrow \pi^+ \pi^-)$ contamination in the Λ was removed by cutting combinations in which the proton from the Λ decay had a momentum less than 3.0 GeV/c, a time of flight more than 3σ faster than the expected proton time of flight, and within $\pm 3\sigma$ of the expected pion time of flight.

In all subsamples, multiple interpretations of each event were allowed. If an event had multiple interpretations, each interpretation was weighted by the reciprocal of the number of interpretations the event had, so that each event had a total weight of one.

9.3 Examination of the $\Lambda K^+ K^- \pi^+ \pi^-$ Event Sample

At this stage, an initial examination of the data was made after splitting it into forward Λ and backward Λ subsamples. In each sample, a t (not t') cut was at 2.0 (GeV/c)^2 . For the forward sample, this cut was made on the t from the beam kaon to the Λ . For the backward sample, the cut was made on the t from the target proton to the Λ . A t cut was used rather than a t' cut in order to remove confusion between forward and backward samples at high meson system effective mass.

The forward Λ sample contains 4308 events, with an average of 1.3 combinations per event. The observed mass plots for the sample are shown in Figure 9.1, and reveal a number of structures. The $\pi\pi$ mass spectrum shows some ρ together with a small wide enhancement near the $f_2(1270)$, while the KK spectrum shows evidence for a small number of ϕ events. Some $K^*(890)$ is visible in both $K\pi$ spectra. Finally, both $\Lambda\pi$ spectra show some $\Sigma(1385)$, and a bump in the ΛK^- spectrum in the $1.8\text{--}1.85 \text{ GeV/c}^2$ bin is consistent with the $\Xi(1820)$. The remaining spectra (not shown) exhibit no significant structure. Cross sections can be estimated using the 4.1 event/nbarn sensitivity of the experiment, correction for the Λ branching fraction, and an average reconstruction efficiency of 0.33 ± 0.05 . The reconstruction efficiency was obtained from Monte Carlo data for the process $K^- p \rightarrow K^+ K^- \Lambda$, used later in this analysis. Although these Monte Carlo data were generated with a $t'(p \rightarrow \Lambda)$ slope of 2.0, the quoted errors on the reconstruction efficiency are large enough to account for the difference in t' slopes. While there is likely to be some pion contamination in the 'kaons', if it is assumed that all events in this subsample are $\Lambda K K \pi \pi$ events, the cross section for $K^- p \rightarrow \Lambda K^+ K^- \pi^+ \pi^-$ with

Figure 9.1 Observed mass spectra for the $\Lambda K^* K^- \pi^+ \pi^-$ event sample with $t(\text{beam} \rightarrow \Lambda) \leq 2.0 \text{ (GeV/c}^2\text{)}$.

$\sigma(\text{beam} \rightarrow \Lambda) < 2.0 \text{ (GeV/c)}^2$ is $5.0 \pm 0.8 \text{ } \mu\text{barns}$. The error on this cross section is simply the uncertainty in the reconstruction efficiency.

The backward Λ sample contains 14,840 events, with an average of 1.3 combinations per event. The observed mass plots for the sample are shown in Figure 9.2 and Figure 9.3. The $\pi\pi$ spectrum reveals a sizeable ρ signal, and perhaps a hint of $f_2(1270)$. Substantial ϕ and $f_2'(1525)$ signals are present in KK . Strong signals from $K^*(890)$ are present in both $K\pi$ mass spectra, and the $K^-\pi^+$ may show some $K^*(1430)$. Both $K\pi\pi$ spectra show broad enhancements between 1.25 and 1.50 GeV/c^2 , which may be due to $K_1(1270)$ and $K_1(1400)$. In the baryon systems, the $\Lambda\pi$ spectra show strong $\Sigma(1385)$ production, ΛK^+ shows a broad low mass enhancement possibly due to N^* production, and $\Lambda\pi\pi$ shows $\Lambda(1520)$ production, appearing as a narrow bump just above 1.5 GeV/c^2 . Mass spectra not shown do not exhibit any significant structure. Using the same reconstruction efficiency as for the forward Λ sample, and again assuming no pion contamination in the 'kaons', the cross section for $K^-\text{p} \rightarrow K^+K^-\pi^+\pi^-\Lambda$ with $\sigma(\text{p} \rightarrow \Lambda) < 2.0 \text{ (GeV/c)}^2$ is $17 \pm 2.6 \text{ } \mu\text{barns}$. The error on this cross section is simply the uncertainty in the reconstruction efficiency.

9.4 Study of $\Lambda\phi\pi\pi$ Sample

The $\Lambda\phi\pi^+\pi^-$ sample was selected from the $\Lambda K^+K^-\pi^+\pi^-$ sample by adding the requirement that the K^+K^- mass be in the range 1.01 - 1.0375 GeV/c^2 . A sideband region, 1.0375 - 1.065 GeV/c^2 , was also defined. Only an upper sideband was used since the low tail of the ϕ extends to very near the kinematic limit. A sideband subtraction was performed on all plots shown in this section. Since the ϕ signal and sideband region are the same width, the sideband

Figure 9.2 Observed mass spectra for the $\Lambda K^* K \pi^+ \pi^-$ event sample with $t(p \rightarrow \Lambda) \leq 2.0$ (GeV/c)².

Figure 9.3 Observed mass spectra for the $\Lambda K^* K^- \pi^+ \pi^-$ event sample with $t(p \rightarrow \Lambda) \leq 2.0 \text{ (GeV/c}^2\text{)}$.

subtraction was performed by simply subtracting the number events in the sideband region from the number in the ϕ signal region. Under the assumptions that the background changes slowly over the mass range used, and that the background is incoherent with the ϕ , this results in the net $\phi \pi^+ \pi^-$ mass spectra. As in earlier samples, this event sample was broken down into forward and backward samples. The forward sample required that $t(\text{Beam} \rightarrow K^* K^-) \leq 2.0 \text{ (GeV/c}^2\text{)}$, while the backward sample required that $t(p \rightarrow K^* K^-) \leq 2.0 \text{ (GeV/c}^2\text{)}$.

Figure 9.4 Signal and sideband regions used for the forward ϕ event sample.

The signal and sideband regions for the forward ϕ event sample can be seen in Figure 9.4, and the remaining mass spectra for this event sample are shown in Figure 9.5. The $\pi\pi$ mass spectrum exhibits fairly strong ρ production. Both $\Lambda\pi$ spectra show strong $\Sigma(1385)$ production, and $\Lambda\pi\pi$ contains an obvious narrow $\Lambda(1520)$ signal. The remaining mass spectra (not shown) do not exhibit significant structure. The sample contains 2905 events after sideband subtraction, and the cross section for $K^-\bar{p} \rightarrow \phi\pi\pi\Lambda$, using (as before) 0.33 ± 0.05 as the reconstruction efficiency, is $6.8 \pm 1.0 \mu\text{barns}$. The error on this cross section is simply the uncertainty in the reconstruction efficiency. Unlike the cross sections for forward and backward Λ , this measurement does not need to assume the

Figure 9.5 Observed mass spectra for the $\Lambda\phi\pi^+\pi^-$ event sample with $t(\text{beam} \rightarrow \phi) \leq 2.0 \text{ (GeV/c}^2\text{)}$. A sideband subtraction has been performed, as described in the text.

lack of pion contamination in the kaons, since the kaons are required to be in the ϕ , and a sideband subtraction was performed.

The plots for the backward ϕ sample are shown in Figure 9.6. There is only a small net ϕ signal, and very little structure in any of the other mass spectra. Only the $\Lambda\phi$ spectrum shows any structure, an intriguing enhancement at threshold. Using the net ϕ signal of 61 ± 20 events and again using 0.330 ± 0.05 as the reconstruction efficiency, the cross section for $K^-p \rightarrow \Lambda\pi\pi\phi$ is $0.14 \pm 0.05 \text{ } \mu\text{barns}$. The error on this cross section is a combination of the errors

Figure 9.6 Observed mass spectra for the $\Lambda\phi\pi^+\pi^-$ event sample with $t(p \rightarrow \phi) \leq 2.0 (\text{GeV}/c)^2$. A sideband subtraction has been performed, as described in the text. The K^+K^- mass spectrum shows only the ϕ signal region (filled symbols) and sideband (open symbols).

on the reconstruction efficiency and the statistical error on the number of events.

9.4.1 Study of $\Lambda\phi\rho$ Sample

Since the sideband-subtracted $\pi^+\pi^-$ mass plot shows a significant amount of ρ , it is worthwhile to look for a $\Lambda\phi\rho$ signal. Such a sample would be interesting because $\phi\rho$, like $\eta\rho$, is pure isovector. In addition, the background subtraction performed for this sample will be used as a model for later analyses of $K^+\bar{K}^-$ and $\phi\phi$.

The $\Lambda\phi\rho$ sample was selected from the $\Lambda\phi\pi^+\pi^-$ sample, and a background subtraction was performed, by defining regions in K^+K^- and $\pi^+\pi^-$ mass, near where the ϕ and ρ resonance bands intersect (see Figure 9.7). The basis for the sideband subtraction method is as follows. The region where the ϕ and ρ bands overlap contains contributions from several sources; $\phi\rho$, $\phi\pi\pi$, ρKK , and

Figure 9.7 Background subtraction regions used for $\Lambda\phi\phi$ sample. The background subtraction method is described in the text. Similar subtractions were used for $\Lambda K^* K^*$ and $\Lambda\phi\phi$ samples.

non-resonant $KK\pi\pi$. In order to obtain the net $\phi\rho$ contribution, the other three sources must be subtracted out. Referring to Figure 9.7, regions 1, 3, 7, and 9 contain only non-resonant $KK\pi\pi$ events. Regions 2 and 8 contain ρKK and non-resonant $KK\pi\pi$ events. Regions 4 and 6 contain $\phi\pi\pi$ events and non-resonant $KK\pi\pi$ events. Finally, region 5 contains contributions from $\phi\rho$, $\phi\pi\pi$, ρKK , and non-resonant $KK\pi\pi$ events. Under the assumption that the background varies linearly and that the various contributions are incoherent, and for convenience setting all nine regions to have the same area, the net background-subtracted sample is given by

$$n_{\phi\rho} = n_5 - \frac{(n_4 + n_6)}{2} - \frac{(n_2 + n_8)}{2} + \frac{(n_1 + n_3 + n_7 + n_9)}{4}$$

where n_i is the content of region i in Figure 9.7. The constant divisors in this expression normalize the total areas of the various zones. The reason that the contribution from the non-resonant zone (regions 1, 3, 7, and 9) is added, rather than subtracted, is that there is a non-resonant contribution in both the ϕ band and the ρ band. When the ϕ and ρ resonant bands are subtracted off, the non-resonant background is subtracted twice.

Using this background subtraction method, the net background-subtracted spectrum for any quantity can be obtained by accumulating separate histograms for that quantity in each of the four zones, and obtaining the net background-subtracted spectrum using the prescription above. One implication of this background subtraction method is that it is not possible to do event-by-event acceptance correction. Instead, acceptance

corrections must be applied bin-by-bin to the net spectrum, after background subtraction.

To obtain the ϕp sample, the background subtraction was performed using a p signal region of $0.60 - 0.95 \text{ GeV}/c^2$, and a ϕ signal region of $1.01 - 1.0375 \text{ GeV}/c^2$. In addition, the p sidebands were the ranges $0.425 - 0.60 \text{ GeV}/c^2$ and $0.950 - 1.125 \text{ GeV}/c^2$, and the ϕ sideband was the range $1.0375 - 1.065 \text{ GeV}/c^2$. The result of the subtraction, shown in Figure 9.8, show little evidence of structure. If an average reconstruction efficiency of 0.17 ± 0.05 is assumed, based on the background- subtracted $\Lambda K^* \bar{K}^*$ Monte Carlo (discussed below), the 212 ± 43 observed events correspond to a cross section for $K^- p \rightarrow \phi p \Lambda$ of $1.9 \pm 0.7 \text{ } \mu\text{barns}$, for $m_{pp} < 3.0 \text{ GeV}/c^2$ and $-t(\text{beam} \rightarrow \phi) < 2.0 \text{ (GeV}/c)^2$.

9.5 Study of the $\Lambda K^* \bar{K}^*$ Sample

9.5.1 Introduction

$\Lambda K^* \bar{K}^*$ is one of a group of states which contain two vector mesons. Other states in this group include $\omega\omega$, pp , and $\phi\phi$. Analyses of these states is complicated by the fact that the vector mesons are not stable, but have finite widths. In the case of $\phi\phi$ and $\omega\omega$, this is not too much of a problem, since both the ϕ and the ω are relatively narrow, with full widths of only a few MeV/c^2 . Both the p and the K^* , however, have full widths of many MeV/c^2 . The width of the K^* , for example, is $51.3 \pm 0.8 \text{ MeV}/c^2$ ^[34].

Figure 9.8 Background-subtracted $\phi\pi$ mass spectrum.

Several groups have recently studied the $K\bar{K}^*$ system in different production modes. Mark III has studied $K^0\bar{K}^{*0}$ in radiative J/ψ decay^{[51][52]} and has approximately 800 events in their net $K\bar{K}^*$ mass spectrum, to which they fit four resonances, including the η_c . Qualitatively, their spectrum has a broad enhancement near threshold, then falls off until the η_c is reached. Their angular analysis indicates that almost the entire mass spectrum is dominated by pseudoscalar states. The ARGUS collaboration has observed both $K^0\bar{K}^{*0}$ and $K^{*+}K^-$ in $\gamma\gamma$ interactions. They observed approximately 40 events in $K^0\bar{K}^{*0}$ ^[53] and approximately 27 events in $K^{*+}K^-$ ^[54]. The shapes of their $K\bar{K}^*$ mass

distributions are in agreement with the shape of the Mark III mass spectrum. Both the TPC/Two-Gamma experiment^[55] and the TASSO collaboration^[56] have searched for $K^0\bar{K}^0$ in $\gamma\gamma$ interactions, but have limited statistics and have only been able to place upper limits on cross sections. The WA76 Collaboration have seen $K^0\bar{K}^0$ production in both π^+p and pp interactions^[57]. They have a sample containing approximately 80 events in the two production modes combined, and are unable to separate the production modes. Like the Mark III and ARGUS spectra, the WA76 spectrum has a peak near threshold. The WA76 spectrum has no significant structure elsewhere.

Attempts have been made to explain the shape of the $K^0\bar{K}^0$ spectrum seen in $\gamma\gamma$ interactions as the result of the exchange of a single kaon Regge trajectory^[58]. Such a model appears to reproduce the shape of the ARGUS mass spectra. Attempts have also been made to explain the $\gamma\gamma$ shape in terms of $Q^2\bar{Q}^2$ states^[59].

9.5.2 Examination of the Data

The selection and background subtraction for the $K^0\bar{K}^0$ sample were made in a manner similar to that for the $\Lambda\phi$ event sample. The signal region was defined to be a box extending $\pm 75 \text{ MeV}/c^2$ from the nominal neutral $K^*(892)$ mass of $896 \text{ MeV}/c^2$. The sidebands were defined as $150 \text{ MeV}/c^2$ wide bands on both sides of the K^* signal region. The requirement $-t(\text{beam} \rightarrow K^0\bar{K}^0) \leq 2.0 \text{ (GeV}/c^2)$ was also imposed. After the background subtraction, the remaining sample contained 580 ± 41 events below $3.0 \text{ GeV}/c^2$. A similar selection was attempted for $-t(p \rightarrow K^0\bar{K}^0) \leq 2.0 \text{ (GeV}/c^2)$, but this selection resulted in no net $K^0\bar{K}^0$ signal.

Figure 9.9 Observed K^*K^* mass spectrum, before background subtraction.

Before showing the background-subtracted mass spectrum, it is useful to first show the mass spectrum for the K^*K^* overlap region before background subtraction, in order to see the effects of the background subtraction process. This plot is shown in Figure 9.9. The observed background-subtracted K^*K^* mass spectrum is shown in Figure 9.10. The mass spectrum shows a broad enhancement near threshold, and no other significant structure. The observed background-subtracted dihedral and helicity cosine distributions for the K^*K^* mass region containing the enhancement (threshold to 2.2 GeV/c²) are shown

Figure 9.10 Observed $K^* \bar{K}$ mass spectrum for the $A K^* \bar{K} \pi^+ \pi^-$ final state. A background subtraction has been made, as described in the text.

in Figure 9.11, and are consistent with being flat. The observed background-subtracted t' distribution for the same mass region is shown in Figure 9.12.

9.5.3 Acceptance Correction

In order to study the acceptance, 40000 Monte Carlo events of the reaction $K^- p \rightarrow K^* \bar{K} \Lambda$ were generated. The events were generated with a flat $K^* \bar{K}$ mass distribution and with a $t'(p \rightarrow \Lambda)$ slope of 2.0. The t' slope was chosen to approximately match the data. In addition, the K^* and $\bar{K}^*$ were generated

Figure 9.11 Observed background-subtracted dihderal angle (a) and helicity cosine (b) distributions for the K^*K^- event sample with K^*K^- mass below $2.2\text{GeV}/c^2$. The helicity plot is the sum of the K^+ and K^- data, which are consistent within statistics.

Figure 9.12 Observed background-subtracted t' distribution for the K^*K^* event sample with K^*K^* mass below $2.2\text{GeV}/c^2$. The line shows a fit through all seven points, and has a t' slope of 2.0 ± 0.2 .

with non-relativistic Breit-Wigner lineshapes, and with the nominal neutral $K^*(892)$ mass and width. The Monte Carlo data were passed through the same data selection and background subtraction code as the real data. For comparison, the acceptance was also calculated after reweighting the K^* lineshape of the Monte Carlo data to give each K^* a P-wave relativistic Breit-Wigner lineshape appropriate for the spin-1 $K^*(892)$, with an energy-dependent width given by^[60]

$$\Gamma(m) = \Gamma_0 \left(\frac{q}{q_0} \right)^3 \frac{m_0}{m}$$

Figure 9.13 Comparison of the mass dependence of the $K^* \bar{K}$ acceptance for non-relativistic and relativistic Breit-Wigner K^* lineshapes. The form of the relativistic Breit-Wigner used is described in the text.

where $\Gamma(m)$ is the width at mass m , Γ_0 is the nominal width at the resonance mass, and m_0 is the resonance mass. A comparison of the mass dependence of the acceptance for both the original and the reweighted Monte Carlo data, shown in Figure 9.13, indicates that the lineshape has a non-negligible effect on the acceptance. Because of this, subsequent acceptance studies were done using the reweighted Monte Carlo data. It should be noted that the relative shapes of the spin-0 and spin-1 acceptance curves in Figure 9.13 can be understood. The

net effect of the spin-1 weighting is to tilt the resonance lineshape to higher masses. For low $K^*\bar{K}^*$ masses, the high end of the K^* resonance lineshape is cut off by phase space. In this case, the fact that the spin-1 lineshape has less of a low-mass tail than the spin-0 means that more of the spin-0 will leak into the lower sideband region and be subtracted off, reducing the acceptance for spin-0 relative to spin-1. For high $K^*\bar{K}^*$ masses, there is no phase space cutoff of the K^* lineshape, and the fact that the spin-1 has more of a high-mass tail than the spin-0 means that more of the spin-1 lineshape than spin-0 will leak into the upper sideband, reducing the spin-1 acceptance relative to the spin-0.

After reweighting, the t' , dihedral angle, and helicity cosine acceptance was examined in various $K^*\bar{K}^*$ mass slices. The acceptance was found to be flat in all three quantities. As a result, the acceptance was parametrized only as a function of $K^*\bar{K}^*$ mass.

It is not possible to correct for acceptance on an event-by-event basis, because the $K^*\bar{K}^*$ mass spectrum is obtained through the rather complicated sideband subtraction process described earlier (section 9.4.1). Instead, the net $K^*\bar{K}^*$ mass spectrum is calculated after all events have been processed, and the acceptance correction is applied, bin-by-bin, to this net $K^*\bar{K}^*$ spectrum.

The acceptance corrected $K^*\bar{K}^*$ mass spectrum is shown in Figure 9.14 and, for comparison, the WA76, Mark III, and Argus spectra are shown in Figure 9.15. The curve drawn on Figure 9.14 is an S-wave relativistic Breit-Wigner, fit to the 9 bins beginning at $1.75 \text{ GeV}/c^2$. The fit has a χ^2 of 2.77

Figure 9.14 Acceptance corrected $K^* \bar{K}$ mass spectrum from $\Lambda K^* K^- \pi^+ \pi^-$ final state. Corrections have been made for Clebsch-Gordan coefficients and for all unseen decay modes. The curve shows a fit using an S-wave relativistic Breit-Wigner, described in the text.

for 6 degrees of freedom, a mass of $1.950 \pm 0.015 \text{ GeV}/c^2$, and a width of $0.25 \pm 0.05 \text{ GeV}/c^2$.

While the overall shape of the acceptance corrected mass spectrum is not significantly different from the uncorrected spectrum, relatively little importance should be placed on the first few low mass points. The acceptance is very low in this region, and the acceptance is strongly dependent on the details of the K^* lineshape. The third point ($1.7\text{-}1.75 \text{ GeV}/c^2$), in fact, has a value

Figure 9.15 $K^* \bar{K}^*$ mass spectra from Argus, Mark III, and WA76.

of -331.33 ± 154.07 after acceptance correction and is not included on the plot. After acceptance correction, there are 3934 ± 274 events in the $K^* \bar{K}^*$ mass spectrum below $3.0 \text{ GeV}/c^2$, ignoring the single large negative point near threshold. Using the 4.1 event/nbarn sensitivity of the experiment, and correcting for all branching fractions and Clebsch-Gordan coefficients, this corresponds to a total cross section of $3.4 \pm 0.2 \text{ } \mu\text{barns}$ for the process

$K^+p \rightarrow \Lambda K^+ K^*$ at 11 GeV/c, with $-t(\text{beam} \rightarrow K^+ K^*) \leq 2.0 \text{ (GeV/c)}^2$ and $K^+ K^*$ mass below $3.0 \text{ GeV}/c^2$.

9.5.4 Angular Analysis

It is possible to examine various angular distributions in order to obtain information regarding the spin-parity of the structure observed in the mass spectrum. Such a spin-parity analysis has been carried out by the Mark III. The details of the spin parity analysis are given in Appendix B. Here, it should be noted that the analysis assumes the dominance of a single spin-parity, and further assumes that the background is incoherent. The analysis in this experiment is, of necessity, somewhat broader than the Mark III analysis. In the Mark III radiative J/Ψ decay analysis, the J^{PC} of the J/Ψ and the recoil photon are both known. Because C-parity is conserved in strong interactions, this fixes the C-parity of any $K^+ K^*$ resonance produced. Any $K^+ K^*$ resonance must have positive C-parity, since the photon and J/Ψ both have negative C-parity. In contrast, a $K^+ K^*$ resonance produced in this experiment can have either positive or negative C-parity.

Two angles are generally used to describe the angular distributions; the dihedral angle (χ) and the helicity angle (θ). The dihedral angle is the angle between the decay planes of the K^+ and K^* , while the helicity angle is the angle between the K^+ (or K^*) line of flight and the line of flight of the K^+ (or K^-) decay daughter of the K^+ (K^*), measured in the K^+ (K^*) rest frame. The event distributions in these angles are parametrized as

$$\frac{dI}{d\chi} = \frac{N}{2\pi} \{1 + \beta \cos 2\chi\}$$

$$\frac{dI}{d\cos\theta} = \frac{N}{2} \{1 + \gamma P_2(\cos\theta)\}$$

where N is the total number of events, P_2 is the appropriate Legendre polynomial, and β and γ are coefficients whose values depend on the J^{PC} of the object. Table 9.2 contains a summary of the allowed values of these coefficients for various J^{PC} .

In order to investigate the J^{PC} of the threshold enhancement, a χ^2 was determined for each J^{PC} by simultaneously fitting the observed dihedral angle and helicity distributions. The free parameters in these fits were the overall normalization and, depending on the spin-parity assumption, the values of the appropriate helicity amplitudes. For these fits, the observed helicity distribution was binned into 10 bins and the dihedral angle distribution into 8 bins, yielding a total of 18 points to be fit.[†] In addition, the fit function was integrated inside each bin, instead of simply using the value of the function at the center of the bin. The results of this angular analysis are shown in Table 9.3, Figure 9.16, Figure 9.17, and Figure 9.18. Unfortunately, these results in Table 9.3 indicate that the statistics are not sufficient to make a definite spin-parity determination. Nevertheless, certain observations can be made. The structure is most consistent with being 1^{+-} , 2^{-+} , or 2^{++} and is least consistent with 0^{-+} , the favored spin-parity of the Mark III object.

[†] The dihedral angle distribution shown in Figure 9.11 has been folded around $\chi=90^\circ$, relying on parity invariance. For the fit, the distribution has been left unfolded and extends to 180° . This does not affect the fit.

J^{PC}	β	γ	$\chi(\beta=0)$	$\beta(\gamma=0)$
0^{++}	$\frac{2 a_{11} ^2}{2 a_{11} ^2 + a_{00} ^2}$	$\frac{2(a_{00} ^2 - a_{11} ^2)}{2 a_{11} ^2 + a_{00} ^2}$	2	2/3
0^{-+}	-1	-1	Not Allowed	Not Allowed
1^{++}	0	1/2	1/2	Not Allowed
1^{+-}	$\frac{- a_{11} ^2}{ a_{11} ^2 + 2 a_{10} ^2}$	$\frac{ a_{10} ^2 - a_{11} ^2}{ a_{11} ^2 + 2 a_{10} ^2}$	1/2	-1/3
1^{-+}	0	1/2	1/2	Not Allowed
1^{--}	$\frac{2 a_{11} ^2}{2 a_{11} ^2 + 4 a_{10} ^2 + a_{00} ^2}$	$\frac{2(a_{10} ^2 + a_{00} ^2 - a_{11} ^2)}{2 a_{11} ^2 + 4 a_{10} ^2 + a_{00} ^2}$	5/4	$\frac{1}{3} < \beta < \frac{2}{3}$
2^{-+}	-2/5	-1/10	Not Allowed	Not Allowed
2^{--}	0	-7/10	-7/10	Not Allowed
2^{+-}	0	1/2	1/2	Not Allowed
2^{++}	$\frac{2 a_{11} ^2}{2 a_{11} ^2 + 4 a_{10} ^2 + 2 a_{1-1} ^2 + a_{00} ^2}$	$\frac{2(a_{00} ^2 + a_{10} ^2 - a_{1-1} ^2 - a_{11} ^2)}{2 a_{11} ^2 + 4 a_{10} ^2 + 2 a_{1-1} ^2 + a_{00} ^2}$	0	0

Table 9.2 Coefficients describing angular distributions for the decay of a resonance, with quantum numbers J^{PC} , into two vector particles, each of which subsequently decays to two pseudoscalars. The relationship between the angular distributions and the coefficients β , γ is given in the text. For the vector-vector system, only orbital angular momentum up to D wave is considered in this table. The a_{ij} are helicity amplitudes for the vector-vector system, and are defined in Appendix B. For $J^{PC} = 2^{++}$, both β and γ can be zero if $|a_{11}|^2 = 0$ and $|a_{1-1}|^2 = |a_{00}|^2 + |a_{10}|^2$.

J^{PC}	$ a_{11} ^2$	$ a_{10} ^2$	$ a_{1-1} ^2$	χ^2/DOF	Confidence Level
0^{++}	0.34 ± 0.03			1.55	0.073
0^{-+}				2.86	6.9×10^{-4}
1^{++}				1.61	0.053
1^{+-}	0.38 ± 0.09			0.63	0.86
1^{-+}				1.61	0.053
1^{--}	0.15 ± 0.12	0.18 ± 0.02		0.98	0.47
2^{-+}				0.59	0.90
2^{--}				1.16	0.29
2^{+-}				1.61	0.053
2^{++}	0.00 ± 0.08	0.08 ± 0.02	0.28 ± 0.02	0.59	0.88

Table 9.3 Results of spin-parity analysis of K^*K^* system in $\Lambda K^*K^*\pi^+\pi^-$ final state in this experiment.

This agrees with what one would claim by simply looking at Figure 9.17 and Figure 9.18. Despite the differences in angular structure between the Mark III and these data, the mass spectra look very similar. Since the production mode

Figure 9.16 β and γ for various spin-parity assumptions. The row marked *independent* shows the results of independent fits to the helicity and dihedral angle distributions.

in this experiment is quite different from Mark III, it is certainly possible that different structures are being seen by the two experiments.

It is possible that the K^+K^- enhancement is not resonant, but is a reflection from structure in some other subsystem of the $\Lambda K^+K^-\pi^+\pi^-$ final state. As a check

Figure 9.17 Dihedral angle distribution for $K^* \bar{K}^*$ event sample and fits for various spin-parity assumptions.

Figure 9.18 Helicity distribution for K^+K^- event sample and fits for various spin-parity assumptions.

Figure 9.19 Gottfried-Jackson cosine vs. K^+K^- mass for K^+K^- overlap region. The angle is defined by following the K^- .

on this possibility, a plot of the Gottfried-Jackson cosine[†] vs. K^+K^- mass was made (Figure 9.19). In such a plot, resonant structures tend to extend through all angles, while kinematic reflections tend to bunch up at ± 1 . As can be seen in the figure, the threshold enhancement extends through all angles, as would be expected from a resonance. In addition, there is another structure extending to high K^+K^- mass and peaked near $+1$. The peaking up at $+1$ means that the K^-

[†] The Gottfried-Jackson angle is the angle, measured in the rest frame of the parent particle, between one of the decay daughters and the z-axis, defined to be the beam direction.

tends to follow the beam. Investigation of this structure shows that it is correlated with low ΛK^+ mass, and is likely a reflection of N^* production. It should be noted that the plot contains only events from the central region used in background subtraction. No background subtraction has been performed, but as seen earlier, the background subtraction does not make a large difference in the region of the enhancement. In addition, while it is possible to make a background-subtracted scatterplot by binning the data and assigning gray levels to the bin contents, the relatively low statistics in this sample would require coarse binning.

9.5.5 Examination of the $\Lambda K_S^0 K_S^0 \pi^+ \pi^-$ Data

A completely separate analysis of $\Lambda K^+ K^-$ as seen in the $\Lambda K_S^0 K_S^0 \pi^+ \pi^-$ final state in this experiment has been carried out^[61]. This analysis provides data on $K^+ K^-$, and it is interesting to compare the results to those obtained in the $\Lambda K^+ K^-$ just described. A number of cuts were made to defined the $\Lambda K_S^0 K_S^0 \pi^+ \pi^-$ sample and, because of the difference in event topologies between this sample and the $\Lambda K^+ K^- \pi^+ \pi^-$ sample, the cuts made are quite different than those made on the $\Lambda K^+ K^- \pi^+ \pi^-$ sample. In particular, the fact that all three V^0 decays must be seen as separate vertices significantly helps in removing background, and no particle ID cuts are required. For this sample, a confidence level of at least a 10^{-3} was required for the GeoFit, and no more than six points were allowed to be dropped from any track during MVFIT. In addition, K^0 candidates were required to have a helicity no higher than 0.85 in the GeoFit results. Finally, a confidence level of at least 10^{-5} and a Λ decay length of at least 0.5 mm were required in the 4-C fit. A background subtraction was carried out for this sample. The same background and signal regions were used as for the $K^+ K^-$

Figure 9.20 Acceptance corrected $K^* \bar{K}^*$ mass spectrum for the $\Lambda K_S^0 K_S^0 \pi^+ \pi^-$ final state. Corrections have been made for all Clebsch-Gordan coefficients and unseen decay modes. A background subtraction has been made, as described in the text.

analysis in the $\Lambda K^+ K^- \pi^+ \pi^-$ final state. A total of 45 ± 12 events remained after the background subtraction.

Because of the relatively low statistics in the $\Lambda K_S^0 K_S^0 \pi^+ \pi^-$ final state, Monte Carlo data was only used to determine an average acceptance of 0.088 for the channel. The acceptance-corrected $K^* \bar{K}^*$ mass spectrum is shown in Figure 9.20. Although the statistics are significantly lower than for the $\Lambda K^+ K^- \pi^+ \pi^-$ final state, the mass spectra have the same characteristics; an

Figure 9.21 Dihedral angle distribution for the low mass $K^{*+}K^{-}$ event sample.

enhancement near threshold, and no other significant structure. Using the 4.1 event/nbarn sensitivity of this experiment, and correcting for all branching fractions, Clebsch-Gordan coefficients, and for the 50% of each K^0 which appears as K_S^0 , the cross section for $K^-p \rightarrow \Lambda K^{*+} K^-$, below $3.0 \text{ GeV}/c^2$, is $3.7 \pm 1.0 \text{ } \mu\text{barns}$. This is in reasonable agreement with the value obtained from the $\Lambda K^{*+} K^- \pi^+ \pi^-$ analysis.

The dihedral angle (Figure 9.21) and helicity (not shown) distributions are also consistent with those obtained for the $\Lambda K^{*+} K^- \pi^+ \pi^-$ final state.

9.6 Conclusions

As expected, a study of $\Lambda K^+ K^- K^+ K^-$ reveals a number of resonances, including ϕ , ρ , $\Sigma(1385)$, $\Lambda(1520)$, and $K^*(890)$, in subsystems of this final state. The cross section for $K^- p \rightarrow K^+ K^- \pi^+ \pi^- \Lambda$ with $-t(p \rightarrow \Lambda) < 2.0 \text{ (GeV/c)}^2$ is measured to be $17 \pm 2.6 \text{ } \mu\text{barns}$, under the assumption that there is no pion contamination of the 'kaons'. Under similar assumptions, the cross section for $K^- p \rightarrow \Lambda K^+ K^- \pi^+ \pi^-$ with $-t(\text{beam} \rightarrow \Lambda) < 2.0 \text{ (GeV/c)}^2$ is measured to be $5.0 \pm 0.8 \text{ } \mu\text{barns}$.

Examination of the $\phi\pi\pi$ event sample shows a cross section of $6.8 \pm 1.0 \text{ } \mu\text{barns}$ for the process $K^- p \rightarrow \phi\pi\pi\Lambda$ with $-t(\text{beam} \rightarrow \phi) < 2.0 \text{ (GeV/c)}^2$, and $0.14 \pm 0.05 \text{ } \mu\text{barns}$ for the process $K^- p \rightarrow \Lambda\pi\pi\phi$ with $-t(p \rightarrow \phi) < 2.0 \text{ (GeV/c)}^2$.

An analysis of $\Lambda\phi\phi$ provides a cross section of $1.9 \pm 0.7 \text{ } \mu\text{barns}$, for $m_{\phi\phi} < 3.0 \text{ GeV/c}^2$ and $-t(\text{beam} \rightarrow \phi) < 2.0 \text{ (GeV/c)}^2$. No obvious structure is present in the $\phi\phi$ mass spectrum.

Analyses of the $K^+ K^-$ system in both neutral and charged K^* decay modes show a prominent peak near threshold in the $K^+ K^-$ mass spectrum, and no other significant structure. The shapes of both the charged and neutral K^* mass spectra are qualitatively in agreement with those seen in other experiments. An S-wave relativistic Breit-Wigner fit to the neutral K^* spectrum finds a mass of $1.950 \pm 0.015 \text{ GeV/c}^2$ and a width of $0.25 \pm 0.05 \text{ GeV/c}^2$. The $K^{*0} \bar{K}^0$ sample exhibits a $t'(\text{beam} \rightarrow K^+ K^-)$ slope of 2.0 ± 0.2 in the region of the threshold enhancement. Although statistics are limited, if the enhancement is assumed to be a single resonant object, angular analysis shows that it is least consistent

with being having $J^{PC}=0^{-+}$ and is most consistent with 1^{+-} , 2^{-+} , or 2^{++} . This angular structure differs from the 0^{-+} structure claimed in the same mass region by Mark III in an analysis of the neutral K^* mode produced in radiative J/ψ decay. Examination of the structure of the enhancement in the Gottfried-Jackson angle shows that it populates all angles, as would be expected of a resonance, but not of a kinematic reflection. Angular analysis of the charged K^* decay mode in this experiment was not carried out, due to the limited statistics, but the shapes of the angular distributions for charged and neutral K^* decay modes are consistent. Significantly better statistics are needed in order to ascertain the spin-parity of the threshold enhancement. Finally, the total K^+K^- cross section below $3.0 \text{ GeV}/c^2$, as measured in the neutral and charged K^* decay modes, agree, with values of $3.4 \pm 0.2 \text{ } \mu\text{barns}$ and $3.7 \pm 1.0 \text{ } \mu\text{barns}$, respectively.

10. $\Lambda\phi\phi$

10.1 Introduction

The $\phi\phi$ system is of particular interest since it is the system in which Lindenbaum, et al., have claimed the existence of between one and three 2^{++} glueballs^[62]. The primary basis for their claim is the relatively high cross section for $\pi p \rightarrow \phi\phi n$ relative to $K^- p \rightarrow \phi\phi\Lambda$. The latter process has been examined in the 8.25 GeV/c bubble chamber^[63] and at 18.5 GeV/c in the Ω' spectrometer^[64]. The bubble chamber experiment observed 18 events, giving a cross section of $(0.7 \pm 0.2)\mu\text{barns}$ for $K^- p \rightarrow \phi\phi\Lambda$ at 8.25 GeV/c. The Ω' spectrometer, which was unable to distinguish Λ from Σ^0 , observed 70 $\phi\phi\Lambda/\Sigma^0$ events, giving a cross section of $(190 \pm 30)\text{nbarns}$ for $K^- p \rightarrow \phi\phi\Lambda/\Sigma^0$ at 18.5 GeV/c. The cross section for $\pi p \rightarrow \phi\phi n$ is $(40 \pm 10)\text{nbarns}$ at 16 GeV/c, as measured in the Ω Spectrometer^[66], and $(23 \pm 2)\text{nbarns}$ at 23 GeV/c^[65]. Thus, $\phi\phi$ production in a pion beam is only down by a factor of six relative to a kaon beam. In contrast, single ϕ production is down by approximately a factor of 60^[67]. Finally, in contrast to the 2^{++} structures claimed by Lindenbaum, et al., a recent Mark III analysis^[69] claims a pseudoscalar state near threshold.

10.2 Selection of the $\Lambda\phi\phi$ Event Sample

A $\Lambda K^+ K^- K^+ K^-$ event sample was selected from the inclusive Λ sample by adding cuts on event topology, number of tracks, charge balance, missing energy, and missing momentum. The topology requirements were similar to those used for the event samples discussed in earlier chapters. An event was required to have either a topology 3 or a topology 1 interpretation, and one extra primary vertex charged track was permitted, if the event balanced charge

when the extra track was dropped. Since events in this final state have no missing neutral particles, momentum and energy balance requirements were fairly tight, allowing no more than 0.2 GeV/c missing transverse momentum, 1.0 GeV/c missing longitudinal momentum, and $0.25 (\text{GeV}/c^2)^2$ missing mass squared when the primary vertex tracks were assigned the Λ and kaon masses. No K^+K^- mass requirement was added at this level to select the ϕ , despite the large reduction in sample size that such a requirement would have provided. This decision was made in order to obtain an event sample that could be used by others for a more general KKKK analysis. A total of 258,981 events passed the cuts, and were written onto 4 tapes. As in the event selections described previously, each event could have more than one track combination which passed the cuts.

MVFIT was performed next. Fitting was attempted on each track combination that passed the cuts described above. Prior to calling MVFIT, a track combination was required to have at least one K^+K^- pair with a mass below $1.10 \text{ GeV}/c^2$, in order to preferentially retain combinations containing a ϕ . The decision not to fit the entire KKKK event sample was made since MVFIT is fairly costly in CPU time, and it was known that the current analysis would require a ϕ . A GeoFit was tried first, with point dropping allowed. If the GeoFit converged, a 3-C fit was attempted, starting from the GeoFit results. For the 3-C fit, point dropping was not allowed, the Λ mass was constrained to its nominal value, and momentum balance was imposed. If the 3-C fit converged, a number of 4-C fits with different particle mass assignments were attempted, each starting from the results of the 3-C fit, and with point dropping disabled. A total of six 4-C events were attempted for each track combination; one with all four

primary vertex charged prongs assigned the $K^\pm$ mass, four with two prongs assigned the $K^\pm$ mass and two the $\pi^\pm$ mass, and one in which all four prongs were assigned the $\pi^\pm$ mass. The fits using $\pi^\pm$ mass hypotheses were attempted in order to provide some information on how certain the kaon identification was, if such information was needed later to clean up the sample. A total of 31,084 events had at least a converged GeoFit, and were written to one tape.

The cuts used to define the final sample were quite simple. In part, this was due to the fact that the ϕ lies very near kinematic threshold, so that there is relatively little background under it. A preliminary examination of the $\phi\phi$ sample showed that the simple requirement that both K^+K^- pairs have masses near the ϕ would result in a relatively clean sample by itself. Nevertheless, a few cuts were added to clean the sample. These cuts are summarized in Table 10.1. A confidence level cut was imposed, requiring that a candidate combination have a non-zero 4-C $\Delta K^+K^-K^+K^-$ confidence level. Charge balance was imposed, to remove the few events in which a high momentum track changed curvature, and hence charge, during fitting. A K^+K^- mass cut removed combinations in which the fitted K^+K^- pair mass had moved above the ϕ cutoff of 1.10 GeV/ c^2 . Finally, Cherenkov consistency cuts were added for all four primary vertex charged prongs. These cuts eliminated any candidate combination in which any of the four kaons produced light (>400 ADC counts) as it passed through C_1 . Despite the appearance that the Cherenkov cuts eliminated a noticeable percentage of candidate combinations, only a few combinations were removed from the $\phi\phi$ sample by this cut.

Cut	Number of Combinations	
	Cut	Remaining
Input Events		31,084
Require 4-C Fit Confidence Level >0		5857
Require $\Sigma\text{Charge}=0$	23	5854
Require K^+K^- Pair Mass <1.10 GeV/c ²	560	5237
Cherenkov	1324	3913
Output Combinations		3913
Output events		3891

Table 10.1 Summary of cuts used to define the final $\Lambda\phi K^+K^-$ event sample.

10.3 Examination of the Data

Figure 10.1 shows the mass spectrum for the two prongs to which the 1.10 GeV/c² mass cut was applied, for convenience called the ' ϕ ' prongs. The mass spectrum for the other two kaon prongs is shown in Figure 10.2, and shows a quite clean ϕ signal, but no noticeable $f_2'(1525)$ signal. The kaon pair mass scatterplot is shown in Figure 10.3, and exhibits no structure except at the $\phi\phi$ intersection, indicating the presence of $\Lambda\phi\phi$ events. The observed $\phi\phi$ mass spectrum is shown in Figure 10.4. It exhibits a threshold enhancement, followed by a broad bump, extending from 2.2 to 2.9 GeV/c².

The net $\phi\phi$ mass spectrum was obtained by using the same two dimensional background subtraction technique previously used for $\phi\rho$ and

Figure 10.1 Observed mass spectrum for 'φ' prongs in the 'φKK' event sample.

K^+K^- (see section 9.4.1). Because the ϕ lies close to threshold, only upper sideband regions were used. For this analysis, the ϕ signal region was the range 1.00–1.04 GeV/c^2 and the sideband region extended from 1.04–1.08 GeV/c^2 . The $\phi\phi$ region is outlined in Figure 10.3. With only an upper sideband used, care had to be taken to correctly weight the background so as not to overestimate the background under the ϕ . This is not a problem when both upper and lower sidebands are used and the background is assumed to vary linearly, as was the case in the ϕp and K^+K^- analyses.

Figure 10.2 Observed mass spectrum for K^+K^- prongs in the ' ϕKK ' event sample. The upper plot shows the overall spectrum and the lower plot shows the ϕ region on a finer scale.

Figure 10.3 Kaon pair mass scatterplot for the $\phi\phi$ region. The rectangle encloses the range used as the $\phi\phi$ signal region during sideband subtraction, described in the text.

In order to determine the appropriate background weighting, the K^+K^- mass spectrum for the events in the ϕ signal region (1.00-1.04 GeV/c^2) was examined. This spectrum is shown in Figure 10.5. The data were fit with a curve composed of a two body phase space background of the form q/m and a Gaussian. A Gaussian was used instead of a Breit-Wigner because the $\sim 4.4 \text{ MeV}/c^2$ width of the ϕ was less than the typical 6-10 MeV/c^2 resolution of the LASS spectrometer for a 4-C process. In order to force the Gaussian to go to zero at threshold, an energy-dependent width of the form

Figure 10.4 Observed $\phi\phi$ mass spectrum, before background subtraction.

$$\sigma(m) = \sigma \frac{(q/m)}{(q_0/m_0)}$$

was used. In this expression, q_0 and m_0 are evaluated at the ϕ mass, which was a fit parameter. The fit yielded a value of 1.019 ± 0.001 GeV/c 2 for the ϕ mass, in good agreement with the accepted value of 1.01941 ± 0.00001 . In addition, the fit indicated that the average background under the ϕ was approximately 70% of the background level in the upper sideband region. For this reason, sideband events were weighted by 0.7 for all background subtracted plots.

Figure 10.5 Observed K^*K^- mass spectrum for events with the other K^*K^- pair in the ϕ region (1.00-1.04 GeV/c^2). The curves show a fit to the data using a Gaussian plus a two body phase space background, described in the text.

The net $\phi\phi$ mass spectrum is shown in Figure 10.6. Although the statistics were degraded by the background subtraction, the shape of the mass spectrum did not change significantly. As a check on the sensitivity of the spectrum to the background location, the background region was moved up by 10 MeV/c^2 , to the range 1.05–1.09 GeV/c^2 . The net $\phi\phi$ mass spectrum (not shown) was not qualitatively altered by this change.

The dihedral angle and helicity distributions for the mass range between threshold and 3.0 GeV/c^2 are shown in Figure 10.7. As in the $K\bar{K}$ analysis, the

Figure 10.6 Observed $\phi\phi$ mass spectrum. A background subtraction has been carried out, as described in the text.

dihedral angle distribution has been folded, under the assumption of parity invariance, and the helicity distributions for both ϕ decays have been added together. Although the statistics in this sample are quite limited, an angular analysis identical to that used in the $K\bar{K}^*$ analysis was attempted. As before, the angular analysis assumed an incoherent background and the dominance of a single J^{PC} state. Because the two ϕ 's are identical particles, only positive C-parity combinations are possible. The results of this analysis are shown in Table 10.2, and indicate that if a single J^{PC} dominates, 0^{-+} is disfavored. Unfortunately, because of the low statistics no further distinctions can be made.

Figure 10.7 Observed background-subtracted dihedral angle (a) and helicity cosine (b) distributions for the $\phi\phi$ event sample. The helicity was defined by following the K^+ .

J^{PC}	$ a_{11} ^2$	$ a_{10} ^2$	$ a_{1-1} ^2$	χ^2/DOF
0^{++}	0.26 ± 0.04	—	—	1.23
0^{-+}	—	—	—	3.75
1^{++}	—	—	—	1.00
1^{-+}	—	—	—	1.00
2^{-+}	—	—	—	1.39
2^{++}	$0.0^{+0.6}_{-0.0}$	0.21 ± 0.04	0.06 ± 0.04	1.28

Table 10.2 Results of $\phi\phi$ angular analysis. For definition of helicity amplitudes, see section 9.5.4.

10.4 Acceptance Correction

As explained in the description of the $K\bar{K}^*$ analysis (section 9.5.3), the background subtraction method does not permit an event-by-event acceptance correction. Instead, acceptance must be corrected for on a bin-by-bin basis.

Because of the limited statistics, the acceptance was only examined as a function of mass. In order to determine the acceptance, 20,000 events of the reaction $K^-p \rightarrow \phi\phi\Lambda$ were generated, with a flat distribution in $\phi\phi$ mass and a $t'(p \rightarrow \Lambda)$ slope of 2. The generated events were passed through the same selection, fitting, final cuts, and background subtraction as the real data. The acceptance is plotted as a function of mass in Figure 10.8.

Figure 10.8 Mass dependence of the $\phi\phi$ acceptance.

The acceptance corrected $\phi\phi$ mass distribution, including correction for all unseen decay modes, is shown in Figure 10.9. For comparison, the mass spectra from several other experiments are shown in Figure 10.10. While the mass spectra from WA76, MPS, and the current analysis all have broad enhancements, they differ somewhat in location and width. An attempt to fit the spectrum from this analysis with the three resonances claimed by Lindenbaum, et al., ($m_1=2.011$ GeV/c², $\Gamma_1=0.202$ GeV/c², $m_2=2.297$, $\Gamma_2=0.149$, $m_3=2.339$, $\Gamma_3=0.319$) resulted in a poor fit, having a χ^2 per degree of freedom of 20. For this fit attempt, the three resonances were all parametrized as having an S-wave Breit-Wigner shape. An additional attempt to fit the threshold region

Figure 10.9 Acceptance corrected $\phi\phi$ mass spectrum. Corrections have been made for all unseen decay modes.

using an S-wave Breit-Wigner with the parameters extracted from the $K\bar{K}^*$ analysis described in Section 9.5.3 was also unsuccessful.

Based on the data from this experiment, the total cross section for the process $K^-p \rightarrow \phi\phi\Lambda$ at 11 GeV/c is 530 ± 80 nanobarns after correction for all unseen decay modes. Figure 10.11 shows the variation of cross section with beam momentum, using data from this experiment, the Ω' spectrometer, and the 8.25 GeV/c bubble chamber. A fit to the cross section data yields a momentum dependence of $p_{\text{lab}}^{-1.75 \pm 0.31}$.

Figure 10.10 $\phi\phi$ mass spectra from various experiments.

Figure 10.11 Cross section for the process $K^-p \rightarrow \phi\phi\Lambda$, measured in various experiments. The open circle is the measurement from this experiment. The line shows a fit to the data, with a momentum dependence of $p_{\text{lab}}^{-1.75 \pm 0.31}$.

10.5 Conclusions

While the event sample contains a small, but clear, $\phi\phi$ signal, there is no evidence for an $f_2'(1525)$ signal against the ϕ . Examination of the $\phi\phi$ spectrum shows what appears to be a threshold enhancement, together with a broad, flat enhancement covering the range 2.3-2.6 GeV/c². The total cross section for the process $K^-p \rightarrow \phi\phi\Lambda$ at 11 GeV/c is 530 ± 80 nanobarns after correction for all unseen decay modes. Using measurements from various kaon beam experiments, the variation of the cross section with lab momentum was determined to be $p_{\text{lab}}^{-1.75 \pm 0.31}$.

Because of the low statistics, angular analysis is inconclusive, but $J^{PC} = 0^{-+}$ is disfavored if the $\phi\phi$ spectrum is dominated by a single J^{PC} .

Appendix A. Moment-Amplitude Relations

Shown below are the relations between observed data moments, through $L=6$, and the underlying amplitudes^[68], through F-wave. A factor of $\sqrt{2L+1}$ is implicit in each of the amplitudes. Because the strong interaction conserves parity, each expression consists only of bilinear products where both terms have natural parity or both terms have unnatural parity.

$$t_{00} = |S|^2 + |P_0|^2 + |P_-|^2 + |P_+|^2 + |D_0|^2 + |D_-|^2 + |D_+|^2 + |F_0|^2 + |F_-|^2 + |F_+|^2$$

$$\begin{aligned} t_{10} = & 2|S||P_0| \cos(\phi_S - \phi_{P_0}) + \frac{4}{\sqrt{5}}|P_0||D_0| \cos(\phi_{P_0} - \phi_{D_0}) \\ & + \frac{2\sqrt{3}}{\sqrt{5}}(|P_+||D_+| \cos(\phi_{P_+} - \phi_{D_+}) + |P_-||D_-| \cos(\phi_{P_-} - \phi_{D_-})) \\ & + \frac{6\sqrt{3}}{\sqrt{35}}|D_0||F_0| \cos(\phi_{D_0} - \phi_{F_0}) \\ & + \frac{4\sqrt{6}}{\sqrt{35}}(|D_+||F_+| \cos(\phi_{D_+} - \phi_{F_+}) + |D_-||F_-| \cos(\phi_{D_-} - \phi_{F_-})) \end{aligned}$$

$$\begin{aligned} t_{11} = & \sqrt{2}|S||P_-| \cos(\phi_S - \phi_{P_-}) + \frac{\sqrt{6}}{\sqrt{5}}|P_0||D_-| \cos(\phi_{P_0} - \phi_{D_-}) \\ & - \frac{\sqrt{2}}{\sqrt{5}}|P_-||D_0| \cos(\phi_{P_-} - \phi_{D_0}) + \frac{6}{\sqrt{35}}|D_0||F_-| \cos(\phi_{D_0} - \phi_{F_-}) \\ & - \frac{3\sqrt{2}}{\sqrt{35}}|D_-||F_0| \cos(\phi_{D_-} - \phi_{F_0}) \end{aligned}$$

$$\begin{aligned}
t_{20} = & 2|S||D_0| \cos(\phi_s - \phi_{D_0}) + \frac{2}{\sqrt{5}}|P_0|^2 - \frac{1}{\sqrt{5}}(|P_+|^2 + |P_-|^2) \\
& + \frac{2\sqrt{5}}{7}|D_0|^2 + \frac{\sqrt{5}}{7}(|D_+|^2 + |D_-|^2) + \frac{4}{3\sqrt{5}}|F_0|^2 + \frac{1}{\sqrt{5}}(|F_+|^2 + |F_-|^2) \\
& + \frac{6\sqrt{3}}{\sqrt{35}}|P_0||F_0| \cos(\phi_{P_0} - \phi_{F_0}) \\
& + \frac{6\sqrt{2}}{\sqrt{35}}(|P_+||F_+| \cos(\phi_{P_+} - \phi_{F_+}) + |P_-||F_-| \cos(\phi_{P_-} - \phi_{F_-}))
\end{aligned}$$

$$\begin{aligned}
t_{21} = & \frac{\sqrt{6}}{\sqrt{5}}|P_0||P_-| \cos(\phi_{P_0} - \phi_{P_-}) + \frac{\sqrt{10}}{7}|D_0||D_-| \cos(\phi_{D_0} - \phi_{D_-}) \\
& + \sqrt{2}|S||D_-| \cos(\phi_s - \phi_{D_-}) + \frac{4\sqrt{3}}{\sqrt{35}}|P_0||F_-| \cos(\phi_{P_0} - \phi_{F_-}) \\
& + \frac{2\sqrt{5}}{15}|F_0||F_-| \cos(\phi_{F_0} - \phi_{F_-}) - \frac{3\sqrt{2}}{\sqrt{35}}|P_-||F_0| \cos(\phi_{P_-} - \phi_{F_0})
\end{aligned}$$

$$\begin{aligned}
t_{22} = & -\frac{\sqrt{3}}{\sqrt{10}}(|P_+|^2 - |P_-|^2) - \frac{\sqrt{15}}{7\sqrt{2}}(|D_+|^2 - |D_-|^2) - \frac{\sqrt{2}}{\sqrt{15}}(|F_+|^2 - |F_-|^2) \\
& + \frac{\sqrt{3}}{\sqrt{35}}(|P_+||F_+| \cos(\phi_{P_+} - \phi_{F_+}) - |P_-||F_-| \cos(\phi_{P_-} - \phi_{F_-}))
\end{aligned}$$

$$\begin{aligned}
t_{30} = & \frac{6\sqrt{3}}{\sqrt{35}} |P_0| |D_0| \cos(\phi_{P_0} - \phi_{D_0}) + 2|S| |F_0| \cos(\phi_S - \phi_{F_0}) \\
& - \frac{6}{\sqrt{35}} (|P_+| |D_+| \cos(\phi_{P_+} - \phi_{D_+}) + |P_-| |D_-| \cos(\phi_{P_-} - \phi_{D_-})) \\
& + \frac{2\sqrt{10}}{15} (|D_+| |F_+| \cos(\phi_{D_+} - \phi_{F_+}) + |D_-| |F_-| \cos(\phi_{D_-} - \phi_{F_-})) \\
& + \frac{8\sqrt{5}}{15} |D_0| |F_0| \cos(\phi_{D_0} - \phi_{F_0})
\end{aligned}$$

$$\begin{aligned}
t_{31} = & \frac{4\sqrt{3}}{\sqrt{35}} |P_0| |D_-| \cos(\phi_{P_0} - \phi_{D_-}) + \frac{6}{\sqrt{35}} |P_-| |D_0| \cos(\phi_{P_-} - \phi_{D_0}) \\
& + \sqrt{2} |S| |F_-| \cos(\phi_S - \phi_{F_-}) + \frac{2\sqrt{5}}{15} |D_-| |F_0| \cos(\phi_{D_-} - \phi_{F_0}) \\
& + \frac{\sqrt{2}}{\sqrt{5}} |D_0| |F_-| \cos(\phi_{D_0} - \phi_{F_-})
\end{aligned}$$

$$\begin{aligned}
t_{32} = & -\frac{\sqrt{6}}{\sqrt{7}} (|P_+| |D_+| \cos(\phi_{P_+} - \phi_{D_+}) - |P_-| |D_-| \cos(\phi_{P_-} - \phi_{D_-})) \\
& - \frac{1}{\sqrt{3}} (|D_+| |F_+| \cos(\phi_{D_+} - \phi_{F_+}) - |D_-| |F_-| \cos(\phi_{D_-} - \phi_{F_-}))
\end{aligned}$$

$$\begin{aligned}
t_{40} = & -\frac{4}{7}(|D_+|^2 + |D_-|^2) + \frac{6}{7}|D_0|^2 + \frac{1}{11}(|F_+|^2 + |F_-|^2) + \frac{6}{11}|F_0|^2 \\
& - \frac{2\sqrt{2}}{\sqrt{7}}(|P_+||F_+|\cos(\phi_{P_+} - \phi_{F_+}) + |P_-||F_-|\cos(\phi_{P_-} - \phi_{F_-})) \\
& + \frac{8}{\sqrt{21}}|P_0||F_0|\cos(\phi_{P_0} - \phi_{F_0})
\end{aligned}$$

$$\begin{aligned}
t_{41} = & \frac{2\sqrt{15}}{7}|D_0||D_-|\cos(\phi_{D_0} - \phi_{D_-}) + \frac{5\sqrt{2}}{\sqrt{35}}|P_0||F_-|\cos(\phi_{P_0} - \phi_{F_-}) \\
& + \frac{2\sqrt{5}}{\sqrt{21}}|P_-||F_0|\cos(\phi_{P_-} - \phi_{F_0}) + \frac{\sqrt{30}}{11}|F_0||F_-|\cos(\phi_{F_0} - \phi_{F_-})
\end{aligned}$$

$$\begin{aligned}
t_{42} = & -\frac{\sqrt{10}}{7}(|D_+|^2 - |D_-|^2) - \frac{\sqrt{10}}{11}(|F_+|^2 - |F_-|^2) \\
& - \frac{\sqrt{5}}{\sqrt{7}}(|P_+||F_+|\cos(\phi_{P_+} - \phi_{F_+}) - |P_-||F_-|\cos(\phi_{P_-} - \phi_{F_-}))
\end{aligned}$$

$$t_{50} = \frac{20\sqrt{5}}{3\sqrt{77}}|D_0||F_0|\cos(\phi_{D_0} - \phi_{F_0})$$

$$- \frac{10\sqrt{10}}{3\sqrt{77}}(|D_+||F_+|\cos(\phi_{D_+} - \phi_{F_+}) + |D_-||F_-|\cos(\phi_{D_-} - \phi_{F_-}))$$

$$t_{51} = \frac{10}{\sqrt{77}}|D_0||F_-|\cos(\phi_{D_0} - \phi_{F_-}) + \frac{20\sqrt{2}}{3\sqrt{77}}|D_-||F_0|\cos(\phi_{D_-} - \phi_{F_0})$$

$$t_{52} = -\frac{5}{\sqrt{33}}(|D_+||F_+|\cos(\phi_{D_+} - \phi_{F_+}) - |D_-||F_-|\cos(\phi_{D_-} - \phi_{F_-}))$$

$$t_{60} = -\frac{25}{11\sqrt{13}}(|F_+|^2 + |F_-|^2) + \frac{100}{33\sqrt{13}}|F_0|^2$$

$$t_{61} = \frac{50\sqrt{7}}{33\sqrt{13}}|F_0||F_-|\cos(\phi_{F_0} - \phi_{F_-})$$

$$t_{62} = -\frac{5\sqrt{35}}{11\sqrt{39}}(|F_+|^2 - |F_-|^2)$$

Appendix B. Angular Correlations in Vector-Vector Decays

This appendix describes the angular correlations for the decay of a spin-0, 1, or 2 particle to two vector particles, followed by the subsequent decay of each of the vector particles into two pseudoscalars[†]. Since each of three decays in this chain is characterized by two angles, the total system is described in terms of six angles. For many purposes, however, a number of these angles are integrated over, and distributions of lower dimension are used. The two most commonly used distributions are, in fact, one dimensional distributions, obtained by integrating over five of the available angles. The first of these commonly used distributions is the helicity angle distribution of the decay products of the vector particles. The second commonly used distribution is the distribution of what is known as the dihedral angle. Both of these angles are defined below. Examination of the helicity and dihedral angle distributions is sufficient to establish the spin and parity of the parent particle, and in some cases is sufficient to determine the L-S structure of the decay final state. Higher dimensional correlations provide additional information on production amplitudes.

The following work is carried out in the helicity basis, which is more convenient for these calculations than the L-S basis because the helicity operator is invariant under rotations and Lorentz boosts along the direction of motion. This relieves one from concerns of transforming spins from Lorentz frame to Lorentz frame. A similar analysis has been carried out by the Mark III

[†] Calculations for spin-1 and spin-2 carried out by W. Dunwoodie.

group, for the restricted case of positive C-parity^[70]. Richman^[71] provides a good overview of the helicity formalism, and the description presented below generally follows his paper.

The coordinate system is defined as follows. For concreteness in defining the axes, assume the reaction

For the parent state, the axes are defined as

$\hat{z}$: Unit vector along beam direction in the rest frame of the $K\bar{K}$ system

$\hat{y}$: Unit vector along $\hat{\Lambda} \times \hat{p}$ in the center of mass system

$\hat{\Lambda}$ is a unit vector in the direction of the Λ momentum

$\hat{p}$ is a unit vector in the direction of the target proton momentum.

$\hat{x} : \hat{y} \times \hat{z}$

For the K^* , the axes are defined by

$\hat{z}_1 : \widehat{K^*}$ in the rest frame of the $K\bar{K}$ system

$\hat{y}_1$: Unit vector along $\hat{z} \times \hat{z}_1$

$\hat{x}_1 : \hat{y}_1 \times \hat{z}_1$

and the angles θ_1, ϕ_1 are the polar angles of the K^- in the K^* rest frame, with respect to these axes.

For the $\bar{K}^*$, the axes are defined by

$\hat{z}_2 : \widehat{K^*} \text{ in the rest frame of the } K^* \bar{K}^* \text{ system} = -\hat{z}_1$

$\hat{y}_2 : \text{Unit vector along } \hat{z} \times \hat{z}_2 = -\hat{y}_1$

$\hat{x}_2 : \hat{y}_2 \times \hat{z}_2 = +\hat{x}_1$

and the angles θ_2, ϕ_2 are the polar angles of the K^* in the K^* rest frame, with respect to these axes.

In this coordinate system, the frequently used dihedral angle χ is given by

$$\chi = \phi_1 + \phi_2$$

The two body decay of a particle can be characterized by two angles, typically taken to be the direction angles of one of the decay daughters relative to some z-axis. In addition, for convenience, we will work in the rest frame of the parent particle. In this frame, the momenta of the two decay daughters are equal and opposite, and the momentum is suppressed in the following equations. The amplitude for the decay daughters to be in a particular helicity state can be written (ignoring constant factors that do not affect the angular distribution)

$$A_M^J = \langle \theta, \phi, \lambda_1, \lambda_2 | U | J M \rangle$$

In this expression, λ_1 and λ_2 are the helicities of the two daughter particles, J and M are the angular momentum and z-axis projection of the parent particle, and U is the interaction operator for the reaction. Inserting a complete set of states, the amplitude becomes

$$\begin{aligned}
A_M^J &= \sum_{j,m} \langle \theta, \phi, \lambda_1, \lambda_2 | j, m, \lambda_1, \lambda_2 \rangle \langle j, m, \lambda_1, \lambda_2 | U | JM \rangle \\
&= \sum_{j,m} \langle \theta, \phi, \lambda_1, \lambda_2 | j, m, \lambda_1, \lambda_2 \rangle \delta_{mM} \delta_{jj} a_{\lambda_1 \lambda_2} \\
&= \langle \theta, \phi, \lambda_1, \lambda_2 | J, M, \lambda_1, \lambda_2 \rangle a_{\lambda_1 \lambda_2}
\end{aligned}$$

where we have introduced the symbol $a_{\lambda_1 \lambda_2} = \langle j, m, \lambda_1, \lambda_2 | U | JM \rangle$. The first part of this expression can be simplified using^[72]

$$\langle \theta, \phi, \lambda_1, \lambda_2 | J, M, \lambda_1', \lambda_2' \rangle = \left(\frac{2J+1}{4\pi} \right)^{1/2} \delta_{\lambda_1 \lambda_1'} \delta_{\lambda_2 \lambda_2'} (\mathcal{D}_{M\lambda}^J(\phi, \theta, 0))^*$$

where $\lambda \equiv \lambda_1 - \lambda_2$ and $\mathcal{D}_{M\lambda}^J(\phi, \theta, 0)$ is the rotation operator for spin J . Note that a number of texts use $-\phi$ as the third argument of the $\mathcal{D}$ -function. Because a direction in 3-dimensional space can be described in terms of two angles, the choice of the third angle has no significance. The choice of zero is convenient for calculations, while $-\phi$ has some mathematically desirable properties in the $\theta \rightarrow 0$ limit^[73]. Using this expression, the amplitude can be written as

$$A_M^J = \left(\frac{2J+1}{4\pi} \right)^{1/2} (\mathcal{D}_{M\lambda}^J(\phi, \theta, 0))^* a_{\lambda_1 \lambda_2}$$

Typically, experiments do not measure helicities, and so the amplitude must be summed over helicities λ_1 and λ_2 . The angular distribution is given by the intensity, which is expressed in terms of the density matrix, ρ , describing

the production process, and products of the amplitudes, A_M^J . Thus, the angular distribution for a two body decay of a body of spin J can be written

$$\frac{d\sigma}{d\Omega} = \sum_{\lambda_1, \lambda_2} \left[\sum_{M, M'} A_M^J \rho_{MM'} (A_M^J)^* \right]$$

Sequential decays can be built up from the expression for the decay amplitude given above. In particular, for the two body decay of a parent into two daughters, each of which subsequently undergoes a two body decay itself, we can write the amplitude as

$$\begin{aligned} A_M^J = & \sum_{\lambda_1 \lambda_2} \left(\frac{2J+1}{4\pi} \right)^{1/2} (D_{M\lambda}^J(\phi, \theta, 0))^* a_{\lambda_1 \lambda_2} \times \\ & \left(\frac{2J_1+1}{4\pi} \right)^{1/2} (D_{\lambda_1 \lambda'}^{J_1}(\phi_1, \theta_1, 0))^* a_{\lambda_3 \lambda_4} \times \\ & \left(\frac{2J_2+1}{4\pi} \right)^{1/2} (D_{\lambda_2 \lambda''}^{J_2}(\phi_2, \theta_2, 0))^* a_{\lambda_5 \lambda_6} \end{aligned}$$

In this expression, unsubscripted quantities are used for the parent, quantities with a subscript of 1 and 2 are for the daughters of the parent decay, and subscripts 3 through 6 are for the second generation decay daughters. In addition, $\lambda \equiv \lambda_1 - \lambda_2$, $\lambda' \equiv \lambda_3 - \lambda_4$, and $\lambda'' \equiv \lambda_5 - \lambda_6$. The angular distribution is given in terms of the density matrix and the A_M^J as before.

For the specific case of the decay of the parent to two spin-1 particles and the subsequent decay of each spin-1 particle to two spin-0 particles, we have $J_1 = J_2 = 1, \lambda_3 = \lambda_4 = \lambda_5 = \lambda_6 = 0$. Using this information, and suppressing constant factors of the form $((2J + 1)/4\pi)^{1/2}$, the angular distribution can be written

$$\begin{aligned} \frac{d\sigma}{d\Omega d\Omega_1 d\Omega_2} = \sum_{M, M'} \left[\left\{ \sum_{\lambda_1 \lambda_2} (\mathcal{D}_{M\lambda}^1(\phi, \theta, 0))^* a_{\lambda_1 \lambda_2} (\mathcal{D}_{\lambda_1 0}^1(\phi_1, \theta_1, 0))^* \times \right. \right. \\ \left. \left. (\mathcal{D}_{\lambda_2 0}^1(\phi_2, \theta_2, 0))^* \right\} \rho_{MM'} \left\{ \sum_{\lambda'_1 \lambda'_2} (\mathcal{D}_{M'\lambda'}^1(\phi, \theta, 0))^* \times \right. \right. \\ \left. \left. a_{\lambda'_1 \lambda'_2} (\mathcal{D}_{\lambda'_1 0}^1(\phi_1, \theta_1, 0))^* \left(\mathcal{D}_{\lambda'_2 0}^1(\phi_2, \theta_2, 0) \right)^* \right\} \right] \end{aligned}$$

This is the basic expression for the angular correlation. It can be simplified, however, by using properties of the $\mathcal{D}$ -functions, invoking parity conservation, and in some cases, by using identical particle symmetry. We begin with the $\mathcal{D}$ -functions. The $\mathcal{D}$ -functions can be written as^[74]

$$\mathcal{D}_{MM}^J(\alpha, \beta, \gamma) = e^{-iM\alpha} d_{MM}^J(\beta) e^{-iM'\gamma}$$

and the $d_{MM}^J(\beta)$ can be calculated from the Wigner formula^[75]

$$d_{m'm}^j(\beta) = \sum_n \left[\frac{(-1)^n [(j+m)! (j-m)! (j+m')! (j-m')!]^{1/2}}{(j-m'-n)! (j+m-n)! (n+m'-m)! n!} \right. \\ \left. \times \left(\cos \frac{\beta}{2} \right)^{2j+m-m'-2n} \left(-\sin \frac{\beta}{2} \right)^{m'-m+2n} \right]$$

where the sum extends over all integers where the arguments of the factorials have non-negative arguments.

Substituting for $\mathcal{D}_{MM'}^J$, we obtain for the angular distribution

$$\frac{d\sigma}{d\Omega d\Omega_1 d\Omega_2} = \sum_{MM'} \left[\left(\sum_{\lambda_1 \lambda_2} \left[e^{iM\phi} d_{M\lambda}^J(\theta) a_{\lambda_1 \lambda_2} \right] \left[e^{i\lambda_1 \phi_1} d_{\lambda_1 0}^1(\theta_1) \right] \times \right. \right. \\ \left. \left[e^{i\lambda_2 \phi_2} d_{\lambda_2 0}^1(\theta_2) \right] \right) \rho_{MM'} \left(\sum_{\lambda_1' \lambda_2'} \left[e^{iM'\phi} d_{M'\lambda'}^J(\theta) a_{\lambda_1' \lambda_2'} \right] \times \right. \\ \left. \left[e^{i\lambda_1' \phi_1} d_{\lambda_1' 0}^1(\theta_1) \right] \left[e^{i\lambda_2' \phi_2} d_{\lambda_2' 0}^1(\theta_2) \right] \right)^* \right]$$

This expression can be further simplified by using relationships between the various $a_{\lambda_1 \lambda_2}$, brought about by the requirements of parity invariance of the strong interaction and the identical nature of the vector particles^{[72][76]}. Note that the latter is appropriate in $\phi\phi$, but not in $K\bar{K}$. Parity conservation introduces the requirement that

$$a_{-\lambda_1 - \lambda_2} = \eta (-1)^J a_{\lambda_1 \lambda_2}$$

where η and J are the intrinsic parity and angular momentum of the parent,

respectively. In general, parity conservation reduces the number of independent amplitudes by approximately a factor of two. Notice that, in particular, parity conservation implies that $a_{00} = 0$ when $\eta(-1)^J = -1$.

For the case of identical particles, there is the additional requirement that

$$a_{\lambda_1 \lambda_2} = (-1)^J a_{\lambda_2 \lambda_1}$$

Finally, although the calculations are carried out in the helicity basis, it is often desirable to use the angular distributions to obtain information on the L-S structure of a resonance instead of the helicity structure. In order to accomplish this, a transformation from the helicity basis to the L-S basis must be known. That transformation, given by Jacob and Wick^[77], is

$$\langle JM; LS | JM; \lambda_1 \lambda_2 \rangle = \left(\frac{2L+1}{2J+1} \right)^{1/2} \langle LS 0 \lambda | LS J \lambda \rangle \langle s_1 s_2 \lambda_1 - \lambda_2 | s_1 s_2 S (\lambda_1 - \lambda_2) \rangle$$

where the last two terms on the right hand side are Clebsch-Gordan coefficients of the form $\langle j_1 j_2 m_1 m_2 | j_1 j_2 J M \rangle$. Table A.1 shows the relationship between helicity amplitudes and L-S amplitudes through $J=2$, and for orbital angular momenta through D-wave.

The angular distributions for various spin-parity combinations are summarized in section 9.5.4 of the text. As an example of the calculations, we calculate the helicity and dihedral angle distributions for the case of a $J^P = 0^-$

J^{PC}	a_{11}	a_{10}	a_{00}	a_{-10}
0^{++}	$\frac{a_S^0}{\sqrt{3}} + \frac{a_D^2}{\sqrt{6}}$		$-\frac{a_S^0}{\sqrt{3}} + \frac{\sqrt{2}a_D^2}{\sqrt{3}}$	
0^{-+}				
1^{++}				
1^{+-}	$\frac{a_S^1}{\sqrt{6}} - \frac{a_D^1}{\sqrt{3}}$	$\frac{a_S^1}{\sqrt{6}} + \frac{a_D^1}{2\sqrt{3}}$		
1^{-+}				
1^{--}	$-\frac{a_P^2}{\sqrt{15}} + \frac{a_P^0}{\sqrt{3}}$	$-\frac{3a_P^2}{2\sqrt{15}}$	$-\frac{2a_P^2}{\sqrt{15}} - \frac{a_P^0}{\sqrt{3}}$	
2^{-+}				
2^{--}				
2^{+-}				
2^{++}	$\frac{a_S^2}{\sqrt{30}} + \frac{a_D^0}{\sqrt{3}} - \frac{a_D^2}{\sqrt{21}}$	$\frac{a_S^2}{\sqrt{10}} - \frac{a_D^2}{\sqrt{28}}$	$\frac{\sqrt{2}a_S^2}{\sqrt{15}} - \frac{a_D^0}{\sqrt{3}} - \frac{2a_D^2}{\sqrt{21}}$	$\frac{a_S^2}{\sqrt{5}} + \frac{\sqrt{2}a_D^2}{\sqrt{7}}$

Table A.1 Relationship between L-S and helicity amplitudes needed for vector-vector analysis. Only spins 0,1, and 2, and orbital angular momenta through D-wave are considered. The helicity amplitudes are written in the form a_L^S , where S is the total spin and L is the orbital angular momentum.

parent, decaying to K^*K^* , with the subsequent decay of each K^* to $K\pi$. The amplitude for this process is

$$A_0^0 = \sum_{\lambda_1 \lambda_2} a_{\lambda_1 \lambda_2} d_{0\lambda}^0(\theta) e^{i\lambda_1 \phi_1} d_{\lambda_1 0}^1(\theta_1) e^{i\lambda_2 \phi_2} d_{\lambda_2 0}^1(\theta_2)$$

But $d_{0\lambda}^0 \neq 0$ only for $\lambda = 0$. Since $\lambda \equiv \lambda_1 - \lambda_2$, this implies $\lambda_1 = \lambda_2$. In addition, $d_{00}^0 = 1$. Using these facts, the amplitude can be written as

$$\begin{aligned} A_0^0 &= a_{00} d_{00}^1(\theta_1) d_{00}^1(\theta_2) + a_{11} d_{10}^1(\theta_1) d_{10}^1(\theta_2) e^{i(\phi_1 + \phi_2)} \\ &+ a_{-1 -1} d_{-10}^1(\theta_1) d_{-10}^1(\theta_2) e^{-i(\phi_1 + \phi_2)} \end{aligned}$$

In order to simplify this expression further, we can use parity conservation. For negative parity, the relations to be used are

$$a_{-1 -1} = -a_{11}$$

$$a_{00} = 0$$

Using these, the amplitude becomes

$$A_0^0 = a_{11} \left[d_{10}^1(\theta_1) d_{10}^1(\theta_2) e^{i(\phi_1 + \phi_2)} - d_{-10}^1(\theta_1) d_{-10}^1(\theta_2) e^{-i(\phi_1 + \phi_2)} \right]$$

The explicit forms of the d-functions needed may be calculated from the Wigner formula, or looked up in the Particle Data Group table. They are

$$d_{10}^1(\theta) = -\frac{\sin\theta}{\sqrt{2}}$$

$$d_{-10}^1(\theta) = +\frac{\sin\theta}{\sqrt{2}}$$

Substituting, and using the definition $\chi \equiv \phi_1 + \phi_2$, the amplitude becomes

$$\begin{aligned} A_0^0 &= a_{11} \left[\left(-\frac{\sin\theta_1}{\sqrt{2}} \right) \left(-\frac{\sin\theta_2}{\sqrt{2}} \right) e^{i(\phi_1 + \phi_2)} - \left(\frac{\sin\theta_1}{\sqrt{2}} \right) \left(\frac{\sin\theta_2}{\sqrt{2}} \right) e^{-i(\phi_1 + \phi_2)} \right] \\ &= a_{11} \frac{\sin\theta_1 \sin\theta_2}{2} \{ e^{i\chi} - e^{-i\chi} \} \\ &= ia_{11} \sin\theta_1 \sin\theta_2 \sin\chi \end{aligned}$$

Next, we calculate the intensity. Because we are considering spin-0, we must have $M=0$, and so the density matrix has only one element, ρ_{00} . Then, the intensity is given by

$$\begin{aligned} \frac{dI}{d\phi_1 d\chi d\cos\theta_1 d\cos\theta_2} &= |A_0^0|^2 \\ &= |ia_{11} \sin\theta_1 \sin\theta_2 \sin\chi|^2 \\ &= |a_{11}|^2 \sin^2\theta_1 \sin^2\theta_2 \sin^2\chi \end{aligned}$$

Now, to obtain the helicity angle distribution, integrate the intensity over all but θ_1 (or, equivalently, θ_2), to obtain

$$\begin{aligned}
 \frac{dI}{d\cos\theta_1} &= \int |a_{11}|^2 \sin^2\theta_1 \sin^2\theta_2 \sin^2\chi \, d\cos\theta_2 \, d\phi_1 \, d\chi \\
 &= |a_{11}|^2 \sin^2\theta_1 \int_{-1}^1 (1-x_2^2) dx_2 \int_0^{2\pi} d\phi_1 \int_0^{2\pi} \sin^2\chi \, d\chi \\
 &\sim \sin^2\theta_1
 \end{aligned}$$

Finally, the dihedral angle distribution is obtained by integrating the intensity over all angles except χ , giving

$$\begin{aligned}
 \frac{dI}{d\chi} &= \int |a_{11}|^2 \sin^2\theta_1 \sin^2\theta_2 \sin^2\chi \, d\cos\theta_1 \, d\cos\theta_2 \, d\phi_1 \\
 &= |a_{11}|^2 \sin^2\chi \int_{-1}^1 (1-x_1^2) dx_1 \int_{-1}^1 (1-x_2^2) dx_2 \int_0^{2\pi} d\phi_1 \\
 &\sim \sin^2\chi
 \end{aligned}$$

Notes on the Calculations

Although the calculations for spins 1 and 2 are conceptually identical, they become rapidly more tedious as the spin of the parent meson increases. As has been seen above, calculation for spin-0 can be carried out on one or two

pages, while the spin-2 calculation takes over 20 pages. Because the calculations are tedious but relatively straightforward, they may seem ideally suited for solution by symbolic math software such as Mathematica^[78]. Such an approach was indeed attempted, as a check on the hand calculations. Several difficulties were encountered, however, and the attempt was dropped after two weeks.

The first, relatively minor, difficulty was that the current version of Mathematica, version 1.2, has no built-in knowledge of either the d - or the $\mathcal{D}$ -functions. Definitions for these functions had to be added by hand, and tested.

The second, somewhat more serious, difficulty was that the current version of Mathematica has no inherent understanding of real numbers. Because of this, terms such as $(\cos x)^* \cos x$, which simplify to $\cos^2 x$ when x is real, were instead left as (in Mathematica's notation) `Conjugate[Cos[x]]Cos[x]`, greatly expanding printout size, and in many cases blocking further simplification. After some searching, a package that 'taught' Mathematica about real numbers was located and used.

The final, and eventually fatal, problem was that the current version of Mathematica has no concept of symmetry. In this particular application, symmetry can be used to greatly simplify the calculations. Specifically, the integrands that arise are products of powers of sines and cosines, integrated over symmetric limits. The fact that the integral of an odd function over symmetric limits is identically zero can be used to rapidly remove a very large

percentage of the terms in the integrand, without actually doing any integration. Unfortunately, Mathematica has no way to make use of such a fact. Because of this, Mathematica must actually integrate every term, including the large percentage that will be identically zero by symmetry. As a test, an attempt was made to have Mathematica integrate *one* such term. Mathematica was still calculating after several minutes on an HP9000 series computer, while a human was done after several seconds. Wolfram Research was contacted, but did not provide any suggestions on how to significantly speed the calculations.

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