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TOMOGRAPHICAL PROPERTIES OF UNIFORMLY REDUNDANT ARRAYS

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ABSTRACT

Recent work in coded aperture imaging has shown that the uniformly redundant array (URA) can image distant planar radioactive sources with no artifacts. This paper investigates the performance of two URA apertures when used in a close-up tomographic imaging system. It is shown that a URA based on m -sequences is superior to one based on quadratic residues. The m -sequence array not only produces less obvious artifacts in tomographic imaging, but is also more resilient to some described detrimental effects of close-up imaging. It is shown that in spite of these close-up effects, tomographic depth resolution increases as the source is moved closer to the detector.

Introduction

In 1961 Mertz and Young¹ introduced the concept of a coded or multiple pinhole aperture for use in x-ray imaging systems. The advantage offered by the coded aperture lay in its increased photon collection efficiency due to its large open area, which is perhaps 10,000 times that of a single pinhole. An additional advantage lay in the hope of obtaining three dimensional depth information. Some early work has been done in this regard by Barrett² and some more recent work using time-variant apertures has been done by Akcasu, et. al.³

In 1977 Fenimore and Cannon introduced the concept of using a notched uniformly redundant array (URA) in conjunction with a balanced correlation decoding method.⁴ Such an approach permits the artifact free encoding and decoding of distant planar sources.

In this paper we investigate the behaviour of two types of URA when used for close up imaging. Close-up imaging is necessary in order to obtain depth information for tomographical purposes. The properties of the two URA patterns are compared with a random array of equal open area. Our goal is to determine if a URA pattern exists which has the desirable defocus properties of the random array while maintaining artifact-free imaging properties for extended objects. The first URA pattern is a quadratic residue array, while the second is based on m -sequences. These two are shown with a random array in Fig. 1.

Fig. 1. From left to right: a uniformly array (URA) based on quadratic residues; a URA based on m -sequences, and a random array.

Basic Principles of Coded Aperture Imaging

The basic concept used in coded aperture imaging is quite simple. Consider imaging a point source. A single point of light casts a shadow of the aperture on the detector. The size of the shadow depends on the distance to the point while the location of the shadow depends on the lateral displacement of the point, see Fig. 2.

Fig. 2. The basic concept behind coded aperture imaging is shown here. Sources at different locations and distances cast aperture images on the detector at different locations and magnifications.

If there are k holes in the aperture, then k times as many photons are collected resulting in an improvement in the signal-to-noise ratio. The detector with a pattern of correlated elements is correlated with a pattern that is a function of the distance of the source from the aperture and the aperture magnification of the detector. The correlation is proportional to the distance of the source from the aperture.

If there are k holes in the aperture, then the field of view is increased. The detector with a pattern of correlated elements is correlated with a pattern that is a function of the distance of the source from the aperture and the aperture magnification of the detector. The correlation is proportional to the distance of the source from the aperture.

These correlations can be expressed in mathematical form. Let $P(x,y)$ be the number of photons received at the detector and $A_k(x,y)$ be the aperture magnification of the detector. Then the correlation is given by:

$$P(x,y) = \sum_k [O_k(x,y) * A_k(x,y)] \quad (1)$$

that is, the measured image is the sum of the correlations of each source plane with an aperture pattern of appropriate magnification. It will be shown later that this is strictly true only for point sources. Using only geometrical considerations, the amount m by which the aperture pattern is magnified by point sources in the k th plane is simply

$$m = \frac{z + f}{z} \quad (2)$$

where f is the aperture-detector separation.

To retrieve the k th plane information from $P(x,y)$, one correlates it with a properly magnified decoding array, $D_k(x,y)$:

$$\hat{O}_k(x,y) = P(x,y) * D_k(x,y) \quad (3)$$

If we momentarily assume a planar source distribution at a distance z from the detector and substitute the expression for P from Eq. (1), we then have for this k th plane,

$$\hat{O}_k(x,y) = O_k(x,y) * A_k(x,y) * D_k(x,y) \quad (4)$$

For the case in which A correlated with D is a delta function, we have,

$$\hat{O}_k(x,y) = O_k(x,y) * \delta(x,y) = O_k(x,y) \quad (5)$$

that is, the estimate of the source distribution is perfect. The virtue of the uniformly redundant array aperture is that A correlated with D is a delta function. This is achieved by mosaicking either A or D and employing balanced correlation as described in (5). Random arrays and Fresnel zone plates do not exhibit this perfect response.

we have computer simulated the above effects.

Computer Simulation of Oblique Effects

The simulated object consisted of a point source viewed from distances ranging from 3 to 24 inches. The encoded picture was therefore simply the shadow of the aperture appropriately magnified. The source was assumed to be always at distance z ; the obliquity and inverse square effects were functions of distance only. In each simulation, the source-aperture distance, z , and the aperture-detector distance, f , were kept equal to each other. Each encoded picture was decoded three times, corresponding to a plane through the source and planes 1, 2, and 3 on either side of it. As the tomographic depth resolution increases, the correlations from the true object plane to these neighboring planes should diminish. The performance of the three coded apertures in Fig. 1 were simulated and compared in this experiment.

Prior to the experiment, we expected the depth resolution to steadily increase as the simulated source was brought closer to the detector and then at some point to "fall off" as the close-up effects began to hamper the decoding procedure. We turned out not to be the case, however.

As the point source was brought closer and closer, the attenuation effects of the obliquity and inverse square factors increased and were eventually limited, at the closest distance, by the loss of the encoded image area to the detector. The obliquity and inverse square effects were not intended to be a significant factor to the extent that the image of the object was distorted. In fact, for our point source, we found that the depth resolution was not degraded by the obliquity and inverse square effects. Even though the obliquity and inverse square effect were present, the uniformity of the shadows cast on the detector, the balanced correlation properties of the arrays, and the correlation at zero distance were unaffected.

The performance of the coded aperture pattern was superior to others, however. The quadratic residue array was consistently poorer than the other two. The quadratic residue array had a more pronounced artifact factor to the extent that we could not expect the uniformity of the obliquity and inverse square nature of the array itself. In fact, the quadratic residue array was affected more by the obliquity and inverse square effects than the other two arrays.

The random array and the source array were not significantly affected by the obliquity and inverse square factors. The random array was degraded by the obliquity and inverse square factors of their random nature. The source array, however, produced nearly artifact-free images of the plane that passed through the source. This is by virtue of its being a CFA.

Visual inspection of the computer simulation, as well as supporting laboratory data, will accompany the oral presentation of this paper.

CONCLUSIONS

For images of point and planar sources, a uniformly redundant array (URA) is superior to a random array. For the random array, the CFA based on a quadratic residue array is superior to that based on quadratic residues. The quadratic residue array is superior to the random array when the source is extended in depth. The random array is superior to the quadratic residue array when the source is extended in depth. The quadratic residue array is superior to the random array when the source is extended in depth. The quadratic residue array is superior to the random array when the source is extended in depth.

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