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TRADE STUDIES OF PLASMA ELONGATION FOR NEXT-STEP TOKAMAKS

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ABSTRACT

The effect of elongation on minimum-cost devices is investigated for elongations ranging from 2 to 3. The analysis, carried out with the TETRA tokamak systems code, includes the effects of elongation on both physics (plasma beta limit) and engineering (poloidal field coil currents) issues. When ignition is required, the minimum cost occurs for elongations from 2.3 to 2.9, depending on the plasma energy confinement scaling used. Scalings that include favorable plasma current dependence and/or degradation with fusion power tend to have minimum cost at higher elongation (2.5-2.9); scalings that depend primarily on size result in lower elongation (~ 2.3) for minimum cost. For design concepts that include steady-state current-driven operation, minimum cost occurs at an elongation of 2.3.

1. INTRODUCTION

In previous tokamak design efforts,¹⁻³ the plasma elongation was specified as a fixed physics guideline, usually at the highest value, which is deemed possible based on experimental results and theoretical estimates. This approach led to assumptions of high elongation (up to 3.0), because plasma current scales favorably with elongation and higher plasma current generally improves plasma confinement. In addition, the Troyon beta limit, which also improves with plasma current, is usually used in design studies. Recent analysis of theoretical stability⁴ has resulted in a Troyon-like plasma beta limit that depends explicitly on the plasma elongation; at high separatrix elongations ($\kappa_z > 2.3$), this scaling predicts a degradation of the Troyon factor. Incorporation of the decrease in the Troyon beta limit at higher elongation opens the possibility of performing overall tokamak systems trade studies to determine the optimum elongation. We use the TETRA tokamak systems code⁵ to study the effect of elongation on next-step tokamak devices of minimum cost. This analysis includes the effects of elongation on both physics issues (i.e., the allowable beta) and engineering issues (e.g., the size of the toroidal field (TF) and poloidal field (PF) coils). The scaling of the PF coil currents is based on magnetohydrodynamic (MHD) equilibria, which are computed over the range of plasma elongation investigated here. We investigate the effect of elongation on minimum-cost devices for elongations ranging from 2 to 3.

2. METHOD

The TETRA tokamak systems code⁵ is used to perform the trade studies described here. This code contains models of the key tokamak subsystems, which are linked by a numerical optimizer. It can simultaneously satisfy different subsystem design constraints while minimizing (or maximizing) a prescribed figure of merit. The physics model is based on a profile-averaged global analysis. For this study, the coupling of elongation with the beta limit and the treatment of the PF coil system play important roles.

2.1 BETA DEPENDENCE ON ELONGATION

Following the results of ref. 4, we use a scaling for the Troyon beta limit

$$\beta_{\text{lim}} = C I_p / a B_t , \quad (1)$$

where I_p is the plasma current (in megamperes), a is the plasma minor radius (in meters), and B_t is the toroidal field on axis (in tesla). In general, the coefficient C depends on the plasma elongation, the plasma triangularity δ , and the edge safety factor q_ψ . Here we scale C with elongation for a constant δ and q_ψ . Also, since the results of ref. 4 are for limiter plasmas and we are modeling divertor plasmas, we use the divertor 95% flux surface values for elongation and triangularity when applying the numerical fits in ref. 4. To scale the 95% flux surface elongation, we use the approximation

$$\kappa_{95} = 0.9\kappa_z + 0.002 , \quad (2)$$

where κ_z (κ_{95}) is the separatrix (95% flux surface) elongation. This fit is obtained by comparing the values for κ_z and κ_{95} that are computed in the MHD equilibrium calculations used in this study (see Sect. 2.2). Over the range $2.0 < \kappa_z < 2.7$, for which Eq. (2) was derived, the κ_{95} values are predicted to within 1%. For this range of κ_z , the 95% flux surface triangularity δ_{95} goes from 0.41 to 0.24, even though the separatrix triangularity is fixed at 0.6. We pick an average value, $\delta_{95} = 0.33$, for this study. Finally, the systems code has no direct modeling of the q_ψ profile. In this study we use $q_\psi(0) = 1.05$ and $q_\psi(a) = 3.1$ when applying the fits of ref. 4. The plasma current is related to the safety factor and other plasma parameters in the systems code through a mean-field safety factor,⁶

$$\bar{q} = [B_t R / (2\pi \bar{B}_p)] \oint dl / R^2 , \quad (3)$$

where $\bar{B}_p$ is the mean poloidal field,

$$\bar{B}_p = \mu_0 I_p / \oint dl .$$

The $\bar{q}$ parameter is held fixed at 3.0, which corresponds to the $q_\psi(a) = 3.1$ value used in ref. 4 for the safety factor at the plasma edge. With these assumptions, C varies with κ_z as shown in Fig. 1. (Throughout the remainder of the report, "plasma elongation" refers to κ_z).

Fig. 1. Troyon beta limit coefficient vs κ_z for $\delta = 0.6$ and edge $q = 3$.

2.2 SCALING OF PF COIL CURRENT WITH ELONGATION

TETRA requires as input a reference PF system and coil current distribution consistent with an MHD equilibrium. For a given plasma elongation and triangularity, the reference equilibrium is characterized by the reference-point plasma geometry, current, and profile parameters: $\mathbf{P} = (R, a, I_p, \beta_p, l_i)$. The poloidal magnetics module in TETRA computes the external field $B_z(r)$ of this equilibrium on the plasma midplane, $R - a < r < R + a$, from the reference coil coordinates $\mathbf{x}_j = (R_j, Z_j)$ and currents I_j . As TETRA searches for an optimal device/plasma design, this external field is scaled to the updated design point $\mathbf{P} = (R, a, I_p, \beta_p, l_i)$ by

$$B_z(r) = \frac{r f(\mathbf{P})}{r f(\mathbf{P})} B_z(r) , \quad (4)$$

where

$$f(\mathbf{P}) = I_p [\ln(8R/a) + \beta_p + l_i/2 - 3/2]$$

and

$$r = (r - R - a)(a/a) + R + a .$$

Also, coils are relocated to the current design point in the TETRA PF module as discussed below, and coil currents at the design point are determined by approximating the scaled external field (B_z) using a least-squares method with a regularization term.

To evaluate the impact of plasma elongation on the overall cost and performance of the International Thermonuclear Experimental Reactor (ITER), we computed a sequence of TETRA reference equilibria with HEQ,⁷ making certain assumptions about PF coil locations to preserve the continuity of the results. We attempted to isolate the dependence of coil currents on the plasma elongation by using HEQ in the fixed X-point mode to generate a family of equilibria with $R = 4.09$ m, $a = 1.44$ m, $B_t = 4.76$ T, $I_p = -16.21$ MA, $\delta_x = 0.6$, $\beta_p = 0.5$, $l_i = 0.6$, and flux linkage $\psi_{PF} = 90$ V·s for different elongations.

An example for $\kappa_z = 2.3$ is shown in Fig. 2. The PF system consists of five pairs of coils, labeled 1 through 5, with 1 a central solenoid. These coils are positioned for a vertical maintenance scheme. We assume $R_1 = R_2$, $Z_3 = Z_4$, and $R_4 = R_5$, so that coil centers lie on a rectangular contour surrounding the plasma [where

Fig. 2. Reference PF coil set used in the PF coil current scaling. Numbers next to coils identify coil groups.

R_i (Z_i) is the radial (vertical) position of coil i . This allows us to fix the distances between the plasma and coils for the reference equilibrium calculations, defining these distances by $D_1 = R - a - R_1$, $D_2 = R_5 - R - a$, and $D_3 = Z_3 - Z_x$. The height of the solenoid PF1 and the elevation of PF2 are scaled linearly with the X-point. No further optimization of coil positions was done for these calculations.

The dependence of coil currents on plasma elongation, subject to these assumptions, is shown in Fig. 3. The dots are the points calculated by HEQ, and the lines are the fits used here for the reference currents I_j in the PF coil current scaling.

In the systems code design point selection process, the radial location of the ohmic heating (OH) coil (PF1) is allowed to vary as a free parameter. The locations of the other PF coils are scaled as follows. Coil group 2 (see Fig. 2) is moved radially with the OH coil and is scaled to move vertically with the height of the TF coil. The height of coil group 3 is kept 0.25 m above the top of the TF coil, and its radial location is kept at the distance from the plasma inner edge used in the reference MHD runs. Coil group 4 is moved vertically with coil group 3, and its radial position is adjusted so that it is 0.25 m outside the outer leg of the TF coil at the midplane. Finally, the vertical position of coil group 5 is held fixed at 1.6 m, and its radial location is scaled with coil group 4.

This method for scaling the PF coil currents was checked at the minimum-cost ASDEX H-mode point (see Sect. 3.1.2). The coil currents at the minimum-cost point found by TETRA were recomputed with HEQ. The recomputed coil currents were then used for the reference coil current set, and the cost minimization was redone. The cost at the new solution point was within 1% of the cost at the original point, and the major device parameters of the new point (R , A , I_p , and B_t) were within 2.3% of the original point.

2.3 OTHER DESIGN CONSTRAINTS

This analysis also includes a requirement for divertor maintenance. The separation between the TF coil outer legs at the divertor height must be large enough to permit horizontal removal of a complete divertor segment for a divertor divided into N segments (where N = number of TF coils). This criterion is satisfied by adjusting the gap between the outer shield and the TF coil outer leg. When this gap

Fig. 3. Reference currents used for PF coil current scaling. The points are those found by MHD equilibrium calculations.

is increased, the TF coil outer leg is pushed radially outward, and the separation between coils increases. The plasma elongation affects the position and size of the divertor section to be removed.

The physics constraints used in this study are similar to those used in recent next-step tokamak studies. The engineering constraints are similar to those used for the Tokamak Ignition/Burn Experimental Reactor (TIBER) study,² with a somewhat thicker shield and larger gaps. These constraints are summarized in Table 1. The primary physics performance requirements are an ignition margin limit, a beta limit, and an inductive volt-second condition. The beta is allowed to reach 80% of the limit. The ignition margin (IM) is defined as

$$IM = P_{\alpha} / (P_{rad} + P_{loss}) , \quad (5)$$

where P_{α} is the alpha particle power (see ref. 5), P_{rad} is the bremsstrahlung radiation power, and P_{loss} is the transport loss power,

$$P_{loss} = 1.5(n_e T_e + n_i T_i) \times \text{plasma volume} / \tau ,$$

with n_e (n_i) the average electron (ion) density, T_e (T_i) the density-weighted average electron (ion) temperature, and τ the specified global confinement scaling. The expressions used in this study for τ are summarized in the appendix. The required IMs used with the energy confinement scalings are picked to give roughly the same minimum cost.

Plasma vertical positioning control and stability are not analyzed here. As the plasma elongation increases, the vertical control problem becomes more difficult,⁸ requiring more control circuits and/or passive stabilization techniques. Therefore, the minimum-cost results presented here should be considered the best-case possibilities; additional costs will be associated with the need for increased vertical control at higher elongations.

3. RESULTS

Most baseline device parameters for next-step tokamak experiments are arrived at by finding a minimum-cost point that satisfies constraints similar to those listed

Table 1. Constraints used in trade studies of elongation

Physics constraints	
General	
Plasma beta β	$\leq 0.8 \times \text{Troyon limit}$
Plasma density n	$\leq \text{Greenwald limit}$
Plasma triangularity δ	0.6 (at the separatrix)
Plasma configuration	Double null divertor
Edge mean-field safety factor $\bar{q}$	3.0
Z_{eff}	1.8
n_{He}/n_e	0.05
Scrape-off, m	
Inner	0.10
Outer	0.12
Pulsed ignition phase	
Ignition margin	$\geq \text{input requirement}$
Plasma density profile	$[1 - (r/a)^2]^{0.5}$
Plasma temperature profile	$[1 - (r/a)^2]$
PF coil flux linkage to plasma	$\geq 1.6 I_p$
Steady-state phase	
Neutron wall load, MW/m ²	≥ 1
Q (fusion power/current drive power)	≥ 5
Plasma density profile	$[1 - (r/a)^2]$
Plasma temperature profile	$[1 - (r/a)^2]^{0.5}$
Engineering constraints	
Inner shield thickness (tungsten), m	0.55
Outer shield thickness (stainless steel), m	1.10
First wall thickness, m	0.02
Gap between OH coil and TF coils, m	0.03
Gap between TF coils and shield, m	
Inner	0.06
Outer	≥ 0.10
Number of TF coils	16
TF and PF coil type	Superconducting ^a
Ripple at plasma edge (peak to average), %	≤ 1.5
Superconductor	
Stability margin, J/m ³	$\geq 3 \times 10^5$
Material	Nb ₃ Sn
Cable conduit stress, MPa	≤ 600
Maximum quench hot-spot temperature, K	160
Maximum quench pressure	$< \text{allowable membrane stress}$

^aDetails of the superconductor analysis are given by J. R. Miller et al., *High Current Density Magnets for INTOR and TIBER*, UCRL-95759, Lawrence Livermore Natl. Lab., Livermore, Calif., December 1986.

in Table 1. Usually the plasma elongation is also specified. Here we find minimum-cost design points that satisfy the specified constraints (in Table 1) for a range of elongations, thereby obtaining both the elongation corresponding to the minimum-cost device and the sensitivity of the cost to the choice of elongation. The parameters listed in Table 2 are allowed to vary as free parameters. Sensitivities to elongation are performed for "pulsed ignition phase" rules, which require an IM based on some energy confinement scaling, and for "steady-state phase" rules, which imply a current-driven mode with a minimum wall load and minimum Q (fusion power/current drive power) limits. The constraints used in these phases are summarized in Table 1.

Table 2. Quantities allowed to vary as free parameters

Plasma parameters	
	Temperature
	Density
	Major radius
	Aspect ratio
	Field on axis
Radial build quantities	
OH solenoid	Inner radius
	Outer radius
TF coil	Thickness
	External case thickness
Gap between outer shield and outer TF coil leg	
Magnet quantities	
TF coil conductor	Helium fraction
	Copper fraction
	Conduit jacket thickness
	Current density
OH coil conductor	Helium fraction
	Copper fraction
	Conduit jacket thickness
	Current density

3.1 PULSED IGNITION PHASE

3.1.1 Kaye-Goldston Scaling

Figure 4 shows direct cost vs elongation over the range $2.0 < \kappa_z < 3.0$ when Kaye-Goldston energy confinement time scaling⁹ is used. An IM of 1.0 is required, and an H factor of 1.3 is used in the energy confinement time scaling, with the other pulsed ignition phase constraints listed in Table 1. Other major device parameters are indicated in Fig. 4. There is a sharp decrease in cost as the elongation increases from 2.0 to about 2.4. For intermediate elongations ($2.4 < \kappa_z < 2.9$) the cost decreases slightly with elongation, and above $\kappa_z = 2.9$ the cost begins to increase. The major radius follows a similar pattern.

The cost increases with elongation at high κ_z because of (a) decreasing Troyon factor (and β_{lim}) for κ_z above 2.4 (see Fig. 1), (b) full utilization of beta for $\kappa_z > 2.75$, and (c) increasing PF coil costs. The power degradation term in the Kaye-Goldston confinement scaling prevents the minimum-cost design from reaching the beta limit and reduces the cost sensitivity to the beta limit. In fact, for elongations less than 2.75, the beta value is less than the limit, as is commonly found for confinement scalings with power degradation terms. At $\kappa_z = 2.75$, the Troyon factor $C = 2.68$, which represents a 20% drop from its peak at $\kappa_z = 2.2$. Above $\kappa_z = 2.75$, C drops rapidly (Fig. 1), causing the total cost to increase. The PF coil costs begin to increase above $\kappa_z = 2.45$ (as does their fraction of the total direct cost), but cost reductions due to the smaller device size and lower fusion power overwhelm this effect until $\kappa_z = 2.9$.

We also note that, for Kaye-Goldston scaling, higher elongation has a direct beneficial effect on energy confinement. In the confinement scaling, there is a $\kappa_z^{0.28}$ term as well as an $I_p^{1.24}$ term. For $\kappa_z > 2.0$,

$$I_p \propto a^2 B_t \kappa_z^2 / (Rq) \quad (6)$$

With this expression, the Kaye-Goldston confinement time τ_{KG} scales approximately as

$$\tau_{KG} \propto \kappa_z^{2.76} n_e^{0.26} R^{0.41} B_t^{1.15} a^{1.99} A_i^{0.5} / (P_a^{0.58} q^{1.24}) \quad (7)$$

Fig. 4. Effect of elongation, for the pulsed-phase operation with a Kaye-Goldston IM of 1.0 and $H = 1.3$, on minimum direct cost, major radius, beta limit and actual beta value, and PF coil (plus power handling equipment) cost as a fraction of total direct cost.

where A_i is the ion mass. Thus, there is a strong enhancement in confinement time with elongation for this scaling law. This effect, along with the power degradation term P_α , tends to push the minimum-cost point to higher elongations than those found for other confinement scalings (see Sects. 3.1.2-3.1.4).

3.1.2 ASDEX H-Mode Scaling

Figure 5 shows the dependence on elongation of the minimum direct cost when ASDEX H-mode scaling¹⁰ is used with an IM requirement of 2.5. This value was chosen so that the costs of the resulting designs would be similar to those in the preceding case. Otherwise, the constraints of Sect. 3.1.1 are used. The minimum cost is at about $\kappa_z = 2.5$, with small cost increases up to $\kappa_z = 2.9$. There is no power degradation term in this energy confinement scaling, and the plasma beta is always at the limit for minimum-cost cases. The minimum-cost point occurs where the beta limit coefficient begins to drop with increasing elongation, as reflected by the drop in the beta values. Above this elongation level, the PF coil fraction of the total cost also begins to increase.

The ASDEX energy confinement scaling varies as $I_p \cdot R$. As noted above, the plasma current scales approximately as κ^2 for the range of elongations of interest here. This favorable scaling with elongation does mitigate the detrimental beta degradation above $\kappa_z = 2.5$ and results in the rather shallow minimum-cost curve seen in Fig. 5.

3.1.3 Shimomura Scaling

An elongation scan for minimum cost using Shimomura scaling¹¹ with an IM requirement of 1.5 is shown in Fig. 6. The dependence of the minimum cost on elongation is stronger than in the previous examples, with the minimum-cost point occurring at $\kappa_z = 2.3$. About half of the cost increase above $\kappa_z = 2.3$ is due to increases in the cost of the PF coils and the PF power equipment. Most of the remaining cost increases are due to increases in the reactor size. Although the major and minor plasma radii are approximately constant above $\kappa_z = 2.3$, the vertical height increases with elongation, thus increasing the reactor size. The

Fig. 5. Effect of elongation, for pulsed-phase operation with an ASDEX H-mode IM of 2.5, on minimum direct cost, major radius, beta (same as the beta limit), and PF coil (plus power handling equipment) cost as a fraction of total direct cost.

Fig. 6. Effect of elongation, for pulsed-phase operation with a Shimomura IM of 1.5, on minimum direct cost, major radius, beta (same as the beta limit), and PF coil (plus power handling equipment) cost as a fraction of total direct cost.

Shimomura confinement scaling has only a weak elongation dependence ($\propto \kappa^{0.5}$), which contributes to the stronger cost increase at high elongations.

3.1.4 Mereshkin-Mukhovatov Scaling

Figure 7 shows the dependence of direct cost dependence on elongation for Mereshkin-Mukhovatov scaling¹² with IM = 1.0. This confinement scaling contains no elongation, plasma current, or power degradation terms, and the resulting minimum-cost points are all at the beta limit. The minimum-cost point occurs at about $\kappa_z = 2.3$ and shows the strongest cost increase at higher elongations for the confinement scalings studied here. The plasma beta increases with elongation up to $\kappa_z = 2.75$ due to the rise in plasma current. Above this elongation, the degradation of the beta limit coefficient overwhelms the plasma current increase, and there is a decrease in the allowable beta. Above $\kappa_z = 2.3$, the cost of the PF coil and associated power supply as a fraction of the total cost begins to increase. This accounts for about 63% of the total cost increase, with the remainder attributable to increases in the device size caused by the increasing plasma height.

3.2 STEADY-STATE CURRENT-DRIVEN PHASE

Figure 8 shows the sensitivity of cost to elongation with the steady-state current-driven phase constraints. This formulation has no IM condition but does require an engineering $Q \geq 5$ and an average neutron wall load ≥ 1 MW/m². We use a combination of lower hybrid and neutral beam current drive; the lower hybrid heating drives the current near the plasma edge, and the neutral beams drive the current in the central region. Also, some allowance is made for bootstrap current. These choices are similar to the method used in the TIBER study. The minimum cost occurs at about $\kappa_z = 2.3$, with a sharp cost increase occurring at higher elongation. The beta limit is always met in the steady-state phase, as are the wall load and Q limits. Most of the cost increases above $\kappa_z = 2.3$ occur in the PF coil system. Increases in injection power costs are the second leading contributor to the rise in cost at high elongations. The injection power costs rise because of the increase in plasma current, which increases in order to maintain the allowable beta level.

Fig. 7. Effect of elongation, for pulsed-phase operation with a Merezhkin-Mukhovatov IM of 1.0, on minimum direct cost, major radius, beta (same as the beta limit), and PF coil (plus power handling equipment) cost as a fraction of total direct cost.

Fig. 8. Effect of elongation, for steady-state current-driven operation, on minimum direct cost, major radius, beta (same as the beta limit), and PF coil (plus power handling equipment) and injection system costs as fractions of total direct cost.

3.3 DISCUSSION

Figure 9 shows the plasma current for the pulsed-phase and steady-state minimum-cost cases vs elongation. As indicated, all of the cases have minimum plasma current at the elongation that provides minimum cost. For low elongations, the plasma current can decrease as κ_z increases while maintaining a relatively constant β_{lim} level since the plasma minor radius decreases with κ_z (see Fig. 10). For large elongations, the plasma minor radius is relatively flat, and the plasma current increases to compensate for the drop in the β_{lim} coefficient. Generally the energy confinement scalings with a direct favorable plasma current dependence (Kaye-Goldston and ASDEX H-mode) have higher plasma current levels (15–24 MA) than the other scalings (around 10 MA). Also, the steady-state current-driven case favors lower plasma currents.

The aspect ratios of the minimum-cost designs are displayed in Fig. 11. For the pulsed-phase cases with the Shimomura and Mereshkin-Mukhovatov scalings and for the steady-state example, the aspect ratio is largest (around 4) near the minimum-cost point. For the Kaye-Goldston and ASDEX H-mode examples, which have explicit plasma current dependence, the aspect ratio increases monotonically

Fig. 9. Plasma currents vs elongation. The minimum-cost points are indicated. KG = Kaye-Goldston, AH = ASDEX H-mode, SH = Shimomura, MM = Mereshkin-Mukhovatov, SS = steady-state current-drive.

Fig. 10. Plasma minor radius vs elongation. The minimum-cost points are indicated. KG = Kaye-Goldston, AH = ASDEX H-mode, SH = Shimomura, MM = Merezhkin-Mukhovatov, SS = steady-state current-drive.

Fig. 11. Plasma aspect ratio vs elongation. The minimum-cost points are indicated. KG = Kaye-Goldston, AH = ASDEX H-mode, SH = Shimomura, MM = Merezhkin-Mukhovatov, SS = steady-state current-drive.

over the entire range of elongation examined here. The aspect ratio level for these two cases is lower (2.5–3.5) than for the Shimomura, Merezhkin-Mukhovatov, and steady-state cases (3–4.5).

Levels of B_t are displayed in Fig. 12. Their pattern of variation is similar to that seen for aspect ratio, with the Kaye-Goldston and ASDEX H-mode examples having a continually increasing field level and the other cases having peak field values near the minimum-cost points. The overall variation of B_t is lower (10–20%) than that observed for the plasma minor radius and plasma current (30–50%). Thus, the allowable beta is more strongly influenced by variations in minor radius and plasma current.

The peak field at the TF coil winding is relatively constant, with B_t between 10.5 and 12 T for all pulsed-phase points examined here. For the steady-state example, the peak TF coil field is slightly higher, varying from 12.1 to 12.8 T. The peak fields in the OH coils have much larger ranges of variation. For the Kaye-Goldston and ASDEX H-mode examples, which have large plasma current levels, the peak fields in the central solenoid vary between 9 and 12 T. For the other cases (which have lower plasma current levels), the OH coil is not pushed to its limits (see Table 1), and the peak field varies between 4 and 7 T.

Fig. 12. Field on axis vs elongation. The minimum-cost points are indicated. KG = Kaye-Goldston, AH = ASDEX H-mode, SH = Shimomura, MM = Merezhkin-Mukhovatov, SS = steady-state current-drive.

One assumption used here is that the plasma beta must be $< 80\%$ of the limit expressed in Eq. (1). Figure 13 shows the direct cost and major radius of the minimum-cost ASDEX H-mode pulsed-phase example for the 80% beta limit and for a case in which beta is allowed to reach 100% of the beta limit. Reductions in cost (5–7%) and major radius at 100% of the beta limit remain approximately constant for all elongations. Similar changes are observed for the other device parameters. This study indicates that the elongation for which the minimum cost occurs is not sensitive to the absolute value of the beta limit.

Fig. 13. Cost (top) and major radius (bottom) for pulsed-phase operation with ASDEX H-mode scaling when beta is allowed to reach 80% and 100% of the Troyon beta limit.

4. CONCLUSIONS

Trade studies to determine the effect of plasma elongation on the cost of next-step tokamaks were performed. The effects of beta limit (Troyon factor) degradation and increasing PF coil system complexity with increasing elongation are accounted for. The optimal plasma (separatrix) elongation is about 2.3 for a plasma triangularity of 0.6 and an edge safety factor of 3. The elongation at which the minimum cost occurs depends on the energy confinement scaling assumption when pulsed-phase ignited scenarios are studied. Scalings with power degradation terms (which do not push the beta limit at lower elongations) and scalings with favorable plasma current variations tend to have minimum costs at somewhat higher elongations (2.5–2.9), but even for these cases the cost sensitivity above elongations of 2.3 is small. Ignited pulsed-phase cases using confinement scalings that depend primarily on size, as well as steady-state current-drive operation cases, have minimum costs at elongations near 2.3, with severe cost penalties arising at higher elongations.

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NOTATION

a	plasma minor radius, m
A	plasma aspect ratio
A_i	average mass of the plasma ions, amu
$\bar{B}_p$	mean poloidal field, T
B_t	magnetic field on axis, T
B_z	scaled external field
C	Troyon factor
f_d	triangularity factor
I_p	plasma current, MA
l_i	dimensionless internal plasma inductance
n_e	volume-averaged plasma electron density, 10^{20} m^{-3}
n_i	volume-averaged plasma ion density, 10^{20} m^{-3}
P_α	fusion alpha power, MW
P_{loss}	transport power loss
P_{rad}	bremsstrahlung radiation power
$\bar{q}$	mean-field safety factor at the plasma edge
q_c	cylindrical safety factor
q_ψ	edge safety factor
Q	energy gain (fusion power/current drive power)
R	plasma major radius, m
T_e	volume-averaged plasma electron temperature, keV
T_i	volume-averaged plasma ion temperature, keV
β	plasma beta
β_{lim}	Troyon beta limit
β_p	poloidal beta
δ	plasma triangularity
δ_z	plasma triangularity at the separatrix
δ_{95}	plasma triangularity at the 95% flux surface
κ_z	plasma elongation at the separatrix
κ_{95}	plasma elongation at the 95% flux surface
τ	energy confinement time
ψ_{PF}	flux linkage of the PF coil system to the plasma, V·s

Appendix

EQUATIONS OF THE ENERGY CONFINEMENT SCALING LAWS USED IN THIS STUDY

Kaye-Goldston H-mode

$$\tau_{KG} = \frac{0.055 H \kappa_z^{0.28} I_p^{1.24} n_e^{0.26} R^{1.65} (A_1/1.5)^{0.5}}{B_t^{0.09} a^{0.49} P_\alpha^{0.58}} \quad (s) \quad , \quad (A.1)$$

where H is the H-mode multiplying factor, which is set to 1.3 in this study. This confinement time is combined via an inverse quadrature with the neo-Alcator scaling τ_{NA} ,

$$\tau = \frac{1}{[(1/\tau_{KG}^2) + (1/\tau_{NA}^2)]^{0.5}} \quad (s) \quad , \quad (A.2)$$

where

$$\tau_{NA} = 0.07 n_e R^2 a q_* \quad (s) \quad ,$$

q_* is the cylindrical safety factor,

$$q_* = 5 B a^2 (1.0 + \kappa_{95}^2 f_d) / (I_p R) \quad ,$$

and f_d is a triangularity factor,

$$f_d = 1 + 2\delta_{95}^2 - 1.2\delta_{95}^3 \quad .$$

ASDEX H-mode

$$\tau = 0.1 I_p R \quad (s) \quad (A.3)$$

Shimomura

$$\tau = 0.045 R a B_t \kappa_z^{0.5} A_1^{0.5} \quad (s) \quad (A.4)$$

Merezhkin-Mukhovatov

$$\tau = \frac{0.014 \bar{n}_e R^3 A_i^{0.5}}{T_i^{1.5} A^{0.25}} \quad (\text{A.5})$$

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