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A Unified Creep-Plasticity Model for Structural Metals at High Temperature

D. N. Robinson

APPLIED TECHNOLOGY

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October 1978

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A UNIFIED CREEP-PLASTICITY MODEL FOR STRUCTURAL METALS AT HIGH TEMPERATURE*

D. N. Robinson

ABSTRACT

A constitutive theory for use in high-temperature structural analysis of breeder reactor components is presented. It is *unified* in the sense that plastic and creep strains are considered to arise from similar physical mechanisms. The theory is presented in multiaxial terms along with a discussion of its underlying assumptions, its relationship to other theories and the experimental observations upon which it is based and is intended to model. The results of an extensive parameter sensitivity study are reported which provide understanding of the influence of the various model parameters on the critical features of predicted response.

Keywords: viscoplasticity, constitutive equations, high-temperature plasticity, creep, structural analysis, rate-sensitivity, time dependency.

1. INTRODUCTION

Constitutive equations based on *classical* concepts of creep and plasticity¹ generally rest on the assumption that the inelastic strain can be decomposed into two distinct and additive contributions, one time-dependent (creep) and the other time-independent (plastic).[†] Experimental evidence collected on structural alloys at high temperature, however, does not generally support this assumption and instead points toward unified representations in which *creep* and *plasticity* are characterized as occurring simultaneously and interactively and time is an essential ingredient throughout.

*Work performed under DOE/RRT 189a OH048, High-Temperature Structural Design.

[†]There are provisions in Ref. 1 which take into account specific creep/plasticity interactions (cf. Sect. 4.3.5 of Ref. 1). Additional rules for accounting for such interactions are outlined in Refs. 2 and 3.

Several examples in which structural alloys have exhibited inherent time-dependency and substantial creep/plasticity interaction have been reported recently.⁴⁻⁸ References 4 through 6 identify such effects in the ferritic alloy 2 1/4 Cr-1 Mo steel. Contained specifically in those references is an example that caused considerable concern in connection with the design of the Clinch River Breeder Reactor steam generator. Reference 7 reports on experiments conducted on the austenitic alloy 316 stainless steel which show qualitatively similar features of creep/plasticity interaction as those found in 2 1/4 Cr-1 Mo steel.

Swindeman⁸ has reported on experiments recently completed on type 304 stainless steel at 600°C. Figure 1 is taken from that study and indicates a marked creep/plasticity interaction. The *solid* curve in the figure represents a stress-strain hysteresis loop established under constant strain-rate cycling through a strain range of 0.3%. The *dotted* curve denotes the locus of points to which the stress relaxes in one hour from various starting points around the hysteresis loop. The relaxation response is seen to vary greatly from point to point around the loop, even from points having the same starting stress. It may be concluded that the relaxation (long-time) response is measurably influenced by the inelastic state as reached through plastic (short-time) cycling. This behavior is not accounted for in classical formulations of constitutive equations such as that of Ref. 1, even when supplemented by special rules such as those described in Ref. 3.*

The consequences of a constitutive model *not* being able to accurately characterize material behavior such as that of Fig. 1, go beyond that of simply not predicting the correct structural deformation. Damage and life-time calculations (and hence the entire design process) are strongly dependent on the ability to accurately predict stresses and deformations. For example, consider calculations of *creep damage* using current ASME Code Case 1592 rules for the specimen of Fig. 1 cycled as shown with a hold time at some point around the loop. In general, this calculation based on the *actual* relaxation response as in Fig. 1 would not compare favorably with

*These rules were developed to account for related effects in 2 1/4 Cr-1 Mo steel.

Fig. 1. Relaxation strength at 1 hr for various strain positions around a hysteresis loop at 593°C (1100°F). Type 304 stainless steel at 0.3% strain range. Taken from Ref. 8.

the same calculation made using the *predicted* relaxation response based on the equations of Ref. 1.* In particular, if the hold time were taken at some point around the loop reached through some prior unloading, the two calculations might well show a large discrepancy; worse yet, that based on the prediction of Ref. 1 would be *nonconservative*.⁹

*This is not to say the interim constitutive equations of Ref. 1 ought to be promptly abandoned. The equations stated in Ref. 1, despite their shortcomings, are the *best* available at the present time, particularly in light of the wide spectrum of problems and conditions these equations are called upon to deal with in the high temperature design of structural components. To be sure, significant developments toward alternate constitutive formulations have been made recently, as reported herein and elsewhere in the literature, however, these are presently quite far from being readily implemented in large computer codes for performing structural analysis.

The present report deals with a constitutive theory for use with structural alloys at elevated temperature in which time is an essential ingredient throughout, and which can accommodate the simultaneous interaction of transient creep and rate-sensitive yielding. Although efforts have been made toward a general nonisothermal theory, the present development is limited to *isothermal* conditions and to temperatures of about 0.4 of the absolute melting temperature or greater. The model incorporates some aspects of the work of several authors, i.e., Onat,¹⁰ Rice,¹¹ Ponter and Leckie,¹² Lagnborg,¹³ and has been previously outlined, in part, through a series of publications by Robinson et al.¹⁴⁻²¹

The present model is phenomenological and continuum mechanical in nature, which we believe to be necessary inasmuch as its purpose is to form the basis of *macroscopic* structural analyses in support of design. Alternatively, if one starts from a strictly microstructural standpoint, there seems to be no real way of fighting through the forest of detail on that level to arrive at quantitative predictions of gross structural behavior as required by the design engineer. This is not to say that the present formulation has not been strongly guided by physical and microstructural studies and observations. As will be subsequently indicated in the following sections and has been previously pointed out in Refs. 14 through 21, the basis of the present model rests heavily on experimental observations, both microscopic and macroscopic.

A full statement of the constitutive model is presented in Sect. 2 including discussions concerning its relationship to other theories, its underlying assumptions and some of the experimental observations upon which it is based. Section 3 gives a brief geometrical interpretation of the theory, some of which has been previously presented in Ref. 19. In Sect. 4 the multiaxial theory is reduced to its most fundamental state, that of pure shear. A nondimensionalization of the equations for pure shear is then made, thereby defining dimensionless groups of parameters and reducing the total number of material parameters. Section 5 presents the results of a rather extensive parameter sensitivity study conducted on the nondimensionalized version of the model. This represents the

central effort of the present investigation and has a twofold objective, i.e.,

(1) to gain full understanding of the influence of the various material parameters (or parameters groups) on specific features of the response, e.g., transient and steady-state creep, relaxation, constant strain-rate monotonic and cyclic loading, etc., and

(2) to define the breadth of the model in the sense of its capability to represent the behavior of a variety of structural alloys through different choices of the material parameters.

Objective (1) above is aimed at facilitating the *fitting* of the model to a specific alloy and to aid in defining the necessary experimental base in order to quantify the material parameters. The required experimentation for implementation of the model has been discussed in a preliminary way in Ref. 14 and will be dealt with in detail in a subsequent report.

The final section of the report presents the conclusions drawn from the parameter study.

2. THE CONSTITUTIVE MODEL

A complete multiaxial statement* of the constitutive model is as follows:

$$2\mu\dot{\epsilon}_{ij} = \begin{cases} \frac{n-1}{F^2} \Sigma_{ij} ; & \left\{ \begin{array}{l} F > 0, S_{ij}\alpha_{ij} > 0 \text{ and } S_{ij}\Sigma_{ij} > 0 \\ \text{or} \\ F > 0 \text{ and } S_{ij}\alpha_{ij} \leq 0 \end{array} \right. \\ 0 ; & \left\{ \begin{array}{l} F \leq 0 \\ \text{or} \\ F > 0, S_{ij}\alpha_{ij} > 0 \text{ and } S_{ij}\Sigma_{ij} \leq 0 \end{array} \right. \end{cases} \quad (2.1)$$

$$\dot{\alpha}_{ij} = \begin{cases} \frac{2\mu H}{G^{\beta/2}} \dot{\epsilon}_{ij} - HG \frac{n-\beta-1}{2} \alpha_{ij} ; G > G_0 \text{ and } S_{ij}\alpha_{ij} > 0 \\ \frac{2\mu H}{G_0^{\beta/2}} \dot{\epsilon}_{ij} - RG_0 \frac{n-\beta-1}{2} \alpha_{ij} ; G \leq G_0 \text{ or } S_{ij}\alpha_{ij} \leq 0 , \end{cases} \quad (2.2)$$

$$\begin{aligned} \text{in which } F &= J_2/\kappa^2 - 1 & G &= T_2/\kappa^2 \\ J_2 &= \frac{1}{2}\Sigma_{ij}\Sigma_{ij} & T_2 &= \frac{1}{2}\alpha_{ij}\alpha_{ij} \end{aligned} \quad (2.3)$$

*The model is stated here for isothermal conditions and in the context of infinitesimal strain. Indicical notation is used with the convention that indices appearing in the same term twice are to be summed. A dot appearing over a symbol indicates the derivative with respect to time.

Here, $\dot{\epsilon}_{ij}$ denotes the inelastic strain-rate and Σ_{ij} the effective stress, i.e.,

$$\Sigma_{ij} = S_{ij} - \alpha_{ij} , \quad (2.4)$$

in which S_{ij} and α_{ij} are the *applied* stress and the *internal* (or *back*) stress, respectively. Both S_{ij} and α_{ij} are deviatoric quantities, i.e., S_{kk} and $\alpha_{kk} = 0$. α_{ij} plays the role of a second rank tensorial state variable. Equations (2.1) will be subsequently referred to as the *flow* law and Eqs. (2.2) as the *growth* law; the latter indicating the growth or rate of change of the state variable α_{ij} with deformation history.

The form of the growth law, i.e.,

$$\dot{\alpha}_{ij} = h\dot{\epsilon}_{ij} - r\alpha_{ij} , \quad (2.5)$$

is adopted from the now widely accepted Bailey-Orowan theory^{2,2} which presumes that high-temperature deformation of metals occurs under the action of two simultaneously competing mechanisms, a hardening process proceeding with accumulated deformation (characterized by the term $h\dot{\epsilon}_{ij}$) and a recovery or softening process proceeding with time (characterized by $-r\alpha_{ij}$). Steady state is thus taken to be the condition under which the two competing mechanisms balance and $\dot{\alpha}_{ij} = 0$.

In uniaxial terms, where s is the uniaxial component of internal stress, we can write, according to Eqs. (2.2),

$$h \sim \frac{1}{s^\beta} \quad (2.6)$$

and

$$r \sim s^{n-\beta-1} , \quad (2.7)$$

which is in agreement with the forms suggested by the experimental results of Mitra and McLean^{2,3} for the Bailey-Orowan hardening and recovery functions.

The dependence of $\dot{\epsilon}_{ij}$ on the effective stress, Eq. (2.4), is suggested through the definite *kinematic* nature exhibited by structural alloys at high temperature; upon reversal of stress these materials exhibit strong Bauschinger and related effects. These effects are assumed to be indicative of the development of an internal stress such that it enhances the remobilization of dislocations upon stress reversal.

The dependence of the functions F and G on the second principal invariants of the effective and internal stresses, Eqs. (2.3), reflects the assumption of full isotropy. The lack of dependence on the first principal invariant J_1 is well justified experimentally. The same is not necessarily true regarding the third invariant J_3 ; however, following classical developments, stress dependence on J_2 alone will be assumed here.

κ is a scalar state variable having the units of stress (MPa) and plays the role of a threshold stress such that when the second invariant of the effective stress falls below the value κ , i.e., when $\dot{F} < 0$, the inelastic strain rate vanishes.*

Changes in κ reflect hardening or softening in the sense that the threshold stress is raised or lowered. In the context of classical plasticity, changes in κ relate to *isotropic* hardening (or softening), whereas, in the same context, changes in the state variable α_{ij} basically give rise to *kinematic* hardening characteristics as already noted.

The characteristic *response times* associated with the scalar state variable κ and the tensorial state variable α_{ij} appear, in most materials, to be quite different. Changes in κ generally occur gradually in the course of deformation history, e.g., as one observes in the gradual isotropic hardening (or softening) associated with continued stress cycling. Changes in the internal stress α_{ij} , on the other hand, occur with a higher response rate and in cyclic stressing may change significantly in the course of each cycle. As the stress increases in one direction, α_{ij} may first change rapidly but at a diminishing rate as dislocations are generated and entangled and pile up at obstacles. At a stress reversal, remobilization of dislocations may occur and the internal structure, as reflected in α_{ij} , may change relatively rapidly.

In the subsequent development and calculations, gradual changes in κ are ignored and no growth law governing its rate of change is specifically written down. This is done mainly to allow a separate investigation into just how broad a range of behavioral features can be modeled on the basis of the single state variable α_{ij} . As will be shown, most of the important creep/plasticity interactions cited in the Introduction seem

* κ plays precisely the same role here as its counterpart in the well-known Bingham solid (cf. Bingham,²⁴ Prager²⁵).

to be accountable in terms of the single state variable α_{ij} , at least for loading histories which do not involve a large number of repeated cycles.

The inclusion of a growth law specifying $\dot{\alpha}$ is not a difficult matter and, in fact, can be accomplished in precisely the same way as in Robinson,^{2,26} thereby allowing long-term *isotropic* hardening or softening to occur.

The additional material parameters are μ , H , R , β , n , and G_0 . The parameter μ has units of viscosity (MPa·hr), H and R , the Bailey-Orowan hardening and recovery parameters, respectively, have the units (hr^{-1}) and the exponents n and β are unitless.

The constant G_0 (MPa²) plays two roles in Eqs. (2.2). Firstly, it represents the minimum value G is allowed to assume, thereby avoiding the singularity in the first term of Eqs. (2.2) at $G = 0$. This, of course, would not be necessary with the choice of another function which is not singular but has a suitably large value at $G = 0$. Secondly, G_0 influences the rate at which the inelastic state changes particularly *in those quadrants of state space occupied by the state point upon stress reversals* (cf. Onat¹⁰). It is believed that in addition to the sharply increased inelastic strain rate upon stress reversals, giving rise to Bauschinger and related effects, the inelastic state point likewise moves rapidly corresponding to a rapid readjustment of the internal stress α_{ij} . G_0 (as well as H and R) relates to the response time of the material in altering its internal structure.

Two additional quantities apart from F , G , and G_0 appear in the inequalities of Eqs. (2.1) and (2.2). The first of these, $S_{ij}\alpha_{ij}$, is that which indicates the occurrence of a stress reversal. That is to say, when the applied stress S_{ij} has a component in the direction opposite to that of the internal stress α_{ij} , i.e., when $S_{ij}\alpha_{ij} < 0$, a reversal is indicated and the appropriate equations are applicable. The second quantity $S_{ij}\Sigma_{ij}$ signifies the occurrence of unloading (i.e., unloading corresponds to $S_{ij}\Sigma_{ij} < 0$), and admits modeling of features such as *state recovery* in the absence of appreciable *anelastic strain recovery*, the well-known *hesitation period* in creep response following stress reduction, and the apparently related phenomenon observed in Fig. 1 regarding a *hesitation in the stress relaxation* response following partial unloading.

The flow law [the first of Eqs. (2.1)] and the growth law [the first of Eqs. (2.2)] are derivable from a single *flow potential*, i.e.,

$$\Omega = \Omega(\sigma_{ij}, \alpha_{ij}) = F(J_2) + G(T_2) , \quad (2.8)$$

in which

$$F = \frac{\kappa^2}{\mu(n+1)} F^{\frac{n+1}{2}}$$

and

$$G = \frac{R}{H} \frac{\kappa^2}{\mu(n+1)} G^{\frac{n+1}{2}} , \quad (2.9)$$

and F and G are defined as in Eqs. (2.3). That is to say,

$$\dot{\epsilon}_{ij} = \frac{\partial \Omega}{\partial \sigma_{ij}} = \frac{1}{2\mu} F^{\frac{n-1}{2}} \Sigma_{ij} \quad (2.10)$$

and

$$\dot{\alpha}_{ij} = -h \frac{\partial \Omega}{\partial \alpha_{ij}} = h \dot{\epsilon}_{ij} - r \alpha_{ij} , \quad (2.11)$$

in which

$$h = \frac{2\mu H}{G^{\beta/2}} \quad (2.12)$$

and

$$r = RG^{\frac{n-\beta-1}{2}} . \quad (2.13)$$

The concept of a flow potential from which the inelastic strain rate is derivable as in Eq. (2.10) was introduced by Rice.¹¹ Rice concluded that in a material model consisting of an inhomogeneous elastic system, such as a metallic polycrystal, which undergoes macroscopic inelastic deformation as a result of internal rearrangement through dislocation motion, a flow potential exists for each inelastic state and Eq. (2.10) holds.

Ponter and Leckie¹² have argued that for a constitutive theory based on the Bailey-Orowan model [Eq. (2.5)], the rate of change of the state variable may likewise be expressible in terms of the flow potential* as in Eq. (2.11).

Although the existence of a flow potential is not strictly necessary to the present development, e.g., the flow and growth laws can be stated as in Eqs. (2.1) and (2.2) without reference to their derivability from a potential function, its existence does, as pointed out by Ponter,²⁷ ensure desirable continuum and mathematical properties in structural analyses. One such property is uniqueness of solution. A second property is referred to by Ponter as a stability property, i.e., the existence of cyclic histories of stress which form the asymptotic state for large times within a body subjected to cyclic histories of loads and temperature. This, of course, is of importance in design applications where cyclic thermal and mechanical loading is predominant.**

Another consequence of the existence of a flow potential function concerns the direct application of the constitutive model in variational and bounding principles for obtaining approximate solutions or displacement and work bounds.

A variational principle due to Kachanov²⁹ provides a simple illustration of how naturally the present model lends itself to such formulations. Suppose the deformation of a particular material consists of an elastic part

$$\epsilon_{ij}^e = \frac{\partial \Phi}{\partial \sigma_{ij}} \quad (2.14)$$

and, in accordance with the present model, an inelastic part

$$\dot{\epsilon}_{ij} = \frac{\partial \Omega}{\partial \sigma_{ij}} \quad (2.15)$$

*Ponter²⁷ has suggested that this is possible for other rheological models as well, e.g., that due to Besseling.²⁸

**Such stable asymptotic states are not ensured in structural analyses conducted according to the equations of Ref. 1 as discovered recently in connection with the CRBRP steam generator design. Experimental results, however, indicate that real materials do eventually reach a limit cycle.

The total strain-rate can then be written:

$$\dot{\epsilon}_{ij}^t = \frac{\partial}{\partial \sigma_{ij}} \left(\frac{d\Phi}{dt} + \Omega \right) . \quad (2.16)$$

Now, for a structure composed of this material which is acted on by known surface tractions T_i on part of the surface S_T and a prescribed velocity distribution v_i over the remaining part of the surface S_u , we may write for the variation of the stress field in the structure:

$$\int_{S_u} \delta T_i v_i dA - \delta \int_V \left(\frac{d\Phi}{dt} + \Omega \right) dV = 0 . \quad (2.17)$$

The δT_i on S_u are in equilibrium with the stress variation $\delta \sigma_{ij}$, which, in turn, is in internal equilibrium throughout the volume V . The variation in this case is taken only over the *stresses* and *not* their rates of change. Examples of the use of this variational principle in obtaining approximate solutions to structural problems are given by Rabotnov.³⁰

Variational and extremum principles, of course, provide the basis of the finite-element method of structural analysis by which most large-scale analyses are done. Direct application of both variational and bounding principles also form the basis of simplified and approximate methods of analysis which may be used in some cases to circumvent costly detailed analysis.

It is instructive to consider some special cases of the model as expressed in Eqs. (2.1) and (2.2). Excluding dependence on the state variable α_{ij} in Eqs. (2.1),* we recognize, as mentioned previously, a nonlinear form of the classical Bingham²⁴ model. When well above the threshold stress, i.e., when $F \gg 0$, the model reduces to that of a

*It is, of course, the particular dependence on α_{ij} that allows representation of many of the behavioral features of metals at elevated temperature.

nonlinear viscous material, i.e.,

$$2\mu \dot{\epsilon}_{ij} = f(J_2) S_{ij} . \quad (2.18)$$

In particular with $n = 1$, a *linear* viscous (Newtonian) material of viscosity μ results.

Another significant special case included in the present model is that corresponding to the combination of steady-state conditions ($\dot{\alpha}_{ij} = 0$) and large n . Under these conditions, we may write for the steady-state creep rate:

$$\dot{\epsilon}_{ij}^{(s)} \approx \left[\frac{1}{1 + (R/H)^{1/n}} \right] \frac{\partial G}{\partial \sigma_{ij}} , \quad (2.19)$$

in which G is the function given by the second of Eqs. (2.9) with G replaced by the argument

$$J_2^S / \kappa^2 = \frac{1}{2\kappa^2} S_{ij} S_{ji} . \quad (2.20)$$

In this case, G is seen to play the role of a plastic potential. The steady state inelastic strain rate is clearly directed normal to surfaces $G = \text{const.}$ in stress space. In particular, combining Eqs. (2.19), (2.9), and (2.20) we get

$$\dot{\epsilon}_{ij}^{(s)} = \lambda S_{ij} , \quad (2.21)$$

with

$$\lambda = \left[\frac{1}{1 + (R/H)^{1/n}} \right] \left(\frac{R/H}{2\mu} \right) \left(\frac{J_2^S}{\kappa^2} \right)^{\frac{n-1}{2}} . \quad (2.22)$$

Considering the case where both $R/H \rightarrow 0$ and $\mu \rightarrow 0$ uniformly, λ is indeterminate in Eq. (2.21). Under these conditions, Eq. (2.21) becomes identical

to the von Mises flow law of the theory of perfect plasticity. The surfaces $G = C = \text{const.}$ are equivalent to

$$J_2^S / \kappa^2 = \left[C \left(\frac{2\mu}{R/H} \right) \left(\frac{n+1}{2} \right) \right]^{\frac{2}{n+1}}, \quad (2.23)$$

which in the limit considered above, and as $n \rightarrow \text{large}$ goes to

$$J_2^S / \kappa^2 = 1 \quad (2.24)$$

or

$$\frac{1}{2} S_{ij} S_{ij} = \kappa^2, \quad (2.25)$$

the yield condition of von Mises. Thus, with large n , steady-state conditions, and with both the recovery mechanism (as measured by R/H) and the viscosity approaching zero, the present model reduces to the well-known von Mises theory of perfect plasticity. One might expect that in problems in which these conditions are met, or nearly met, the limit theorems of plasticity theory, as well as the techniques involved in slip-line analysis would be applicable.

Figure 2 is a flow chart of the constitutive model, given here as an alternative to the statement of the model as expressed in Eqs. (2.1) and (2.2).

Fig. 2. Flow chart for constitutive model expressed in Eqs. (2.1) through (2.3).

3. GEOMETRIC INTERPRETATION

Multiplying the first of Eqs. (2.1) by itself, we get

$$4\mu^2 I_2 = F^{n-1} J_2, \quad (3.2)$$

in which the quantity $I_2 = \frac{1}{2} \dot{\epsilon}_{ij} \dot{\epsilon}_{ji}$. I_2 will subsequently be referred to as the magnitude of the inelastic strain rate. Since $F = F(J_2)$, we write

$$I_2 = f(J_2), \quad (3.3)$$

indicating that surfaces of $J_2 = \text{constant}$ in a stress space also correspond to surfaces of $I_2 = \text{constant}$, i.e., surfaces of constant inelastic strain-rate magnitude.

As a result of the assumption of full isotropy, we may adopt a simple geometrical representation of the present model in the three-dimensional *principal* stress space $(\sigma_1, \sigma_2, \sigma_3)$, see Fig. 3. Shown in Fig. 3 for the undeformed state ($\alpha_{ij} = 0$) are surfaces of $J_2 = \text{constant}$ (and thus $I_2 = \text{constant}$). These appear as nested cylindrical surfaces whose generators lie parallel to the hydrostatic line $\sigma_1 = \sigma_2 = \sigma_3$. The cylinders in Fig. 3 are intended to represent surfaces having a constant ratio of inelastic strain rate magnitude. The increasingly close spacing as the strain rate becomes large reflects the nonlinearity of the model. The strain-rate vector corresponding to any applied stress point lies normal to the $J_2 = \text{constant}$ surface passing through that point. Note in Fig. 3 there is no component of $\dot{\epsilon}_{ij}$ along the hydrostatic line. Thus, inelastic response is volume preserving.

Figure 4 similarly represents surfaces of constant J_2 with the viewpoint taken along the line $\sigma_1 = \sigma_2 = \sigma_3$. Following the terminology of Hill,³¹ we call the plane of the page (i.e., $\sigma_1 + \sigma_2 + \sigma_3 = 0$) the π -plane and the projection of the $J_2 = \text{constant}$ cylinders appear as concentric circles in this plane. The projection of the applied stress vector σ_{ij} on the π -plane is, of course, S_{ij} the deviatoric stress.

Figure 4 depicts the situation of the deformed material $\alpha_{ij} \neq 0$. The current effective stress vector $\Sigma_{ij} = S_{ij} - \alpha_{ij}$ emanates from the translated center of the nested circles and the corresponding strain-rate vector $\dot{\epsilon}_{ij}$ is colinear with it. Stress points for which $J_2/\kappa^2 < 1$ [i.e.,

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Fig. 3. Surfaces of $J_2 = \text{constant}$ in principal stress space.

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Fig. 4. Projection of $J_2 = \text{constant}$ surfaces on π -plane showing position of surfaces following deformation.

Fig. 5. π -plane showing regions described by inequalities appearing in Eqs. (2.1) through (2.3).

$F < 0$ in Eq. (2.1)] lie inside the cross hatched region of Fig. 4 and produce zero inelastic strain rate.

Under a steady application of stress, the family of surfaces $J_2 = \text{constant}$ translate, finally coming to rest when steady state is reached and $\dot{\alpha}_{ij} = 0$. If, then, the applied stress were abruptly reduced to zero, the inelastic strain rate vanishes but $\dot{\alpha}_{ij}$ does not, allowing the surfaces to drift back toward the origin corresponding to *state recovery* in the material.

Figure 5 gives a geometrical interpretation of the remaining inequalities in Eqs. (2.1) and (2.2). With the current state represented by the vector α_{ij} , we see that the locus of stress points for which $S_{ij}\alpha_{ij} = 0$ provides a demarcation between forward stressing ($S_{ij}\alpha_{ij} > 0$) and reversed stressing ($S_{ij}\alpha_{ij} < 0$). Through these inequalities, the model allows for, not only enhanced inelastic strain rate, but also accelerated readjustment of the state under reversed stressing. The *rate* of such adjustment is determined by the choice of the material parameters.

Also shown in Fig. 5 is a cross-hatched region representing points for which $S_{ij}\Sigma_{ij} < 0$. Once having deformed the material, establishing

the state point α_{ij} , reduction of applied stress into the region $S_{ij}\Sigma_{ij} < 0$ causes the inelastic strain rate to vanish. However, $\dot{\alpha}_{ij} \neq 0$ in this situation and the state point readjusts until the reduced stress point is no longer contained in $S_{ij}\Sigma_{ij} < 0$ at which point $\dot{\epsilon}_{ij} \neq 0$. This corresponds to the well established *hesitation period*^{32,33} observed in creep tests under stress reductions.

4. SIMPLE SHEAR — NONDIMENSIONALIZATION

In order to examine in detail the material behavior represented by Eqs. (2.1) through (2.3), we now reduce these equations to the case of *pure shear*. Then, we have for the applied and internal stress:

$$\sigma_{ij} \equiv S_{ij} = \begin{bmatrix} 0 & S & 0 \\ S & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (4.1)$$

and

$$\alpha_{ij} = \begin{bmatrix} 0 & A & 0 \\ A & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad (4.2)$$

respectively.

It is convenient for computational purposes to nondimensionalize Eqs. (2.1) through (2.3) by introducing the following dimensionless variables and parameter groups. The applied and internal stress components are normalized relative to the material parameter κ , thus giving the dimensionless stresses,

$$\tau = S/\kappa \quad (4.3)$$

and

$$\alpha = A/\kappa. \quad (4.4)$$

The time is nondimensionalized as

$$T = Ht, \quad (4.5)$$

where t and T are the real and nondimensional time, respectively. Two dimensionless parameter groups are taken as

$$R = R/H \quad (4.6)$$

and

$$B = H\mu/\kappa. \quad (4.7)$$

The first, R , is a measure of the relative importance of the recovery and hardening terms which appear in the growth law Eqs. (2.2). The second, B , may be considered a measure of the relative importance of *viscous* to *plastic* dissipation.* Using Eqs. (4.1) through (4.7), the constitutive model expressed in Eqs. (2.1) and (2.2) becomes:

$$B\dot{\gamma}^0 = \begin{cases} \frac{n-1}{F^2} (\tau - \alpha) ; & \begin{cases} F > 0, \tau\alpha > 0 \text{ and } \tau(\tau - \alpha) > 0 \\ \text{or} \\ F > 0 \text{ and } \tau\alpha \leq 0 \end{cases} \\ 0 & ; \begin{cases} F \leq 0 \\ \text{or} \\ F > 0, \tau\alpha > 0 \text{ and } \tau(\tau - \alpha) \leq 0 \end{cases} \end{cases} \quad (4.8)$$

$$\dot{\alpha}^0 = \begin{cases} \frac{B\dot{\gamma}^0}{|\alpha|^\beta} - R \alpha |\alpha|^{n-\beta-1} ; & |\alpha| > \alpha_0 \text{ and } \tau\alpha > 0 \\ \frac{B\dot{\gamma}^0}{\alpha_0^\beta} - R \alpha \alpha_0^{n-\beta-1} ; & |\alpha| \leq \alpha_0 \text{ or } \tau\alpha \leq 0 \end{cases} \quad (4.9)$$

in which

$$F = (\tau - \alpha)^2 - 1, \quad \alpha_0 = G_0^1/2 > 0 \text{ and } \gamma \quad (4.10)$$

denotes the inelastic shear strain.

The derivative with respect to nondimensional time T is denoted in Eqs. (4.8) and (4.9) by an open circle appearing over a quantity [i.e., $(^0) \equiv d/dT$]. Clearly, $(^0) = 1/H (\dot{})$.

As a further indication of the special cases built into the present model, let us consider the case in which $F \gg 0$, $\tau > 0$, and $\alpha > 0$. Let

*This is directly related to the *reciprocal* of the Bingham number as defined by Prager.²⁵

us also, for convenience, take $n = 4$ and $\beta = 1$. Then, we may write from Eqs. (4.8) and (4.9),

$$\dot{\gamma}^0 = \frac{1}{B} (\tau - \alpha)^4 \quad (4.11)$$

and

$$\alpha \dot{\alpha}^0 = B\dot{\gamma}^0 - R\alpha^4 \quad (4.12)$$

If we further identify the internal stress α in Eq. (4.12) as:

$$\alpha = K(\rho)^{1/2}, \quad (4.13)$$

i.e., taking α as being proportional to the square root of the average dislocation density ρ , we may then write Eq. (4.12) as

$$\dot{\rho}^0 = U\dot{\gamma}^0 - V\rho^2, \quad (4.14)$$

in which U and V represent combinations of the constants R and B . Further writing

$$U = 1/bL, \quad (4.15)$$

where b is the Burger's vector and L is the average distance a dislocation moves before being completely immobilized, and

$$V = 2MT, \quad (4.16)$$

where T is the average dislocation-line tension and M the dislocation mobility, we may express Eq. (4.14) as:

$$\dot{\rho}^0 = \frac{\dot{\gamma}^0}{bL} - 2MT\rho^2. \quad (4.17)$$

The form of this equation may be recognized as that due to Lagneborg¹³ which represents the rate of change of dislocation density as a balance between strain-induced dislocation generation, on the one hand, and climb-induced dislocation elimination on the other.

Several calculations have been performed using Eqs. (4.8) and (4.9) under a variety of loading conditions and combinations of the major non-dimensional parameters B , R , n , and β . These are reported in detail in the next section.

5. RESULTS OF THE PARAMETER STUDY

The nondimensionalized Eqs. (4.8) through (4.10) have been integrated under several combinations of material parameters and loading histories. The integrations were carried out on a Tektronix 4051 computer making use of a modified fourth-order Runge-Kutta code. An upper error bound of 10^{-5} was imposed in the calculations with the step size on nondimensional time T automatically varying to achieve this accuracy.

The parameter α_0 ($= \sqrt{G_0}$), corresponding to the minimum attainable value of $|\alpha|$, was taken to be 0.001 throughout the calculations. Since in many cases $|\alpha|$ reached values of ten or more, this represents a range of internal stress of about 10^4 . The value of $\alpha_0 = 0.001$ establishes a maximum inelastic strain rate $\dot{\epsilon}_{ij}$ and growth rate $\dot{\alpha}_{ij}$, here presumed to be those of the undeformed state. Accordingly, $|\alpha| = 0.001$ was also used as the initial value of α in calculations originating from the virgin state. Virtually no difference in the response was found in any case in which the initial value of α was taken to be a factor of ten above or below $|\alpha| = 0.001$.

The results are presented under five separate headings corresponding mainly to the type of loading history involved. The categories are (1) Creep, (2) Relaxation, (3) Monotonic constant strain-rate, (4) Cyclic constant strain-rate, and, finally, (5) General, which contains results concerning fundamental features of the model and some discussion of how these relate to general observations on the representation of material behavior by E. T. Onat.¹⁰

(1) Creep

Figure 6 shows two families of curves γ vs T and α vs T which represent the response of the model under constant shear stress; shear stress values of $\tau = 4, 6, 8$, and 10 are considered with the material parameters*

*These values were chosen as being representative of those used in Ref. 18 for modeling 2 1/4 Cr-1 Mo steel creep curves. The parameters in this study are, for the most part, varied around these representative values.

Fig. 6. γ vs T and α vs T for the constant stresses $\tau = 4, 6, 8$, and 10 with $B = 3000$, $R = 0.1$, $n = 4$ and $\beta = 1$.

taken as $B = 3000$, $R = 0.1$, $n = 4$, and $\beta = 1$. The creep curves γ vs T (inelastic shear strain vs dimensionless time) show classical behavior in that they exhibit a primary creep period followed by steady state. The corresponding α vs T curves (internal stress vs dimensionless time) show an increase in internal stress from zero ($\alpha = 0.001$) with an eventual saturation at steady state. Once into a steady-state condition the internal stress (and hence, the internal dislocation structure) remains constant.

One feature evident in the creep response is that the duration of primary creep varies inversely as the stress magnitude, i.e., low stresses give a long duration of primary creep. This is in keeping with experimental observations made on most of the structural alloys of interest in fast breeder reactor design.

It is worthwhile at this point to remind the reader that, although the results are presented here in a dimensionless form (to reduce the total number of parameters and facilitate the parameter study), physically dimensioned quantities may be readily extracted. For example, examining the creep curve corresponding to $\tau = 10$ in Fig. 6, we see that primary

creep terminates at a dimensionless time of about $T = 0.1$. With the hardening parameter H taken to be $H = 10^{-3} \cdot \text{hr}^{-1}$, the real time, in hours, corresponding to $T = 0.1$ is:

$$t = \frac{T}{H} = 100 \text{ hr} , \quad (5.1)$$

that is to say, the primary creep duration is 100 hr. With $H = 10^{-4} \cdot \text{hr}^{-1}$, $T = 0.1$ corresponds to 1000 hr, etc.

Now, if we estimate the steady-state creep rate for $\tau = 10$ in Fig. 6, we get:

$$\left(\frac{d\gamma}{dT} \right)_{ss} \approx 0.006 . \quad (5.2)$$

Again, with $H = 10^{-3} \cdot \text{hr}^{-1}$, the *actual* steady-state creep rate is:

$$\left(\frac{d\gamma}{dt} \right)_{ss} \approx \frac{0.006}{10^{-3}} = 6 \text{ microstrain/hr} . \quad (5.3)$$

If, furthermore, we take $\kappa = 10$ MPa, then the *actual* shear stress S corresponding to the dimensionless stress $\tau = 10$ is given by:

$$\tau = \frac{S}{\kappa} = 10 , \quad (5.4)$$

that is,

$$S = 100 \text{ MPa} . \quad (5.5)$$

Since, in this case, $R = R/H = 0.1$ and $H = 10^{-3} \cdot \text{hr}^{-1}$, we have for the recovery parameter

$$R = 10^{-4} \cdot \text{hr}^{-1} . \quad (5.6)$$

With $B = 3000$ and H and κ as given above, we get:

$$B = \frac{H\mu}{\kappa} = 3000 \quad (5.7)$$

or

$$\mu = 3 \times 10^7 \text{ MPa} \cdot \text{hr} . \quad (5.8)$$

Figure 7 shows the results of varying the parameter B through the range 1000 to 5000 with $R = 0.1$, $n = 4$, and $\beta = 1$, under a constant stress of $\tau = 10$. The major effect is seen to be a proportional scaling of the γ vs T curves with greater creep rates corresponding to smaller values of B . The α vs T curve is seen to be identical in each case, meaning the duration of primary creep is also the same for all B values.

Figure 8 indicates the effect of varying the parameter R with fixed values of $B = 3000$, $n = 4$, and $\beta = 1$, again for the constant stress $\tau = 10$. Here, the result is clearly not just a scaling of the γ vs T curves, each curve has quite a different shape. In this case, larger creep rates are associated with larger R values, and the duration of transient creep is seen to be very sensitive to changes in R ; smaller R giving longer primary creep duration.

Figure 9 shows the effect of changing the exponents n and β . Here, they are changed to $n = 5$ and $\beta = 2$ from $n = 4$ and $\beta = 1$ as in Fig. 6. This choice allows for greater nonlinearity ($n = 5$) in the flow law Eqs. (4.8) and also ($\beta = 2$) in the hardening term of the growth law Eqs. (4.9), while at the same time retaining the same value of the exponent $(n-\beta-1) = 2$ in the recovery term of Eqs. (4.9). Comparing the γ vs T response here with that of Fig. 6, we observe the effect of the greater nonlinearity in the creep rates while, at the same time, we observe only a small influence on α vs T .

Figure 10 illustrates the response of the model under conditions of reversed stress in creep. The stress history is one of constantly applied stress of magnitude $\tau = 5$ for $0 < T < 0.1$, abruptly changed to $\tau = -5$ for $T > 0.1$. The material parameters are taken as noted, i.e., $B = 3000$, $R = 0.1$, $n = 4$, and $\beta = 1$.

The γ vs T curve of Fig. 10 shows typical transient creep in $0 < T < 0.1$, approaching but not quite attaining a steady state condition before the stress is reversed at $T = 0.1$. Correspondingly, α vs T in $0 < T < 0.1$ shows α increasing from zero and approaching saturation at around $\alpha = 3$.

With the abrupt change of stress, the strain rate reverses immediately, attaining a magnitude nearly equal to that of the virgin state. The internal stress α readjusts rapidly as the applied stress is reversed, tracing out in $T > 0.1$ virtually the mirror image of the α vs T curve for $0 < T < 0.1$.

Fig. 7. γ vs T and α vs T for the constant stress $\tau = 10$ with $B = 1000, 2000, 3000, \text{ and } 5000$, $R = 0.1$, $n = 4$ and $\beta = 1$.

Fig. 8. γ vs T and α vs T for the constant stress $\tau = 10$ with $B = 3000$, $R = 0.001, 0.01, 0.1, \text{ and } 1.0$, $n = 4$, and $\beta = 1$.

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Fig. 9. γ vs T and α vs T for constant stresses $\tau = 6, 8$, and 10 with $B = 3000$, $R = 0.1$, $n = 5$, and $\beta = 2$.

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Fig. 10. γ vs T and α vs T with constant stress $\tau = 5$ for $0 < T < 0.1$ and $\tau = -5$ for $0.1 < T < 0.2$ with $B = 3000$, $R = 0.1$, $n = 4$, and $\beta = 1$.

The *kinematic-like* behavior displayed in Fig. 10 is well documented for structural alloys at high temperature. In fact, it is precisely this feature of response that led to the adoption of the auxiliary hardening rules for stress reversals specified in Sect. 4.4.4.1 of Ref. 1. Those rules provide the fundamentally isotropic hardening creep law of Ref. 1 with limited kinematic hardening features. There is no need for such rules with the present model since it is inherently kinematic.

Figure 11 illustrates reversed stressing with an unsymmetrical stress cycle. The changes in stress are abrupt, as before, and vary according to $\tau = 5$ in $0 < T < 0.1$, $\tau = -3.5$ in $0.1 < T < 0.2$ and $\tau = 5$ for $T > 0.2$. The γ vs T response is similar to that of Fig. 10 but the strain does not return to zero in response to the reduced stress in the reversed direction, and a bias toward $\gamma > 0$ results as loading is continued. Because of the nonlinearity in stress dependence, such as bias, even for small stress differences in the forward and reversed cycles, may be quite marked. A tendency for the strain-time response curve to drift in this way is often observed in experiments conducted under alternating uniaxial tension and compression.

Fig. 11. γ vs T and α vs T with constant stress $\tau = 5$ for $0 < T < 0.1$, $\tau = -3.5$ for $0.1 < T < 0.2$ and $\tau = 5$ for $0.2 < T < 0.3$ with $B = 3000$, $R = 0.1$, $n = 4$, and $\beta = 1$.

Figure 12 illustrates an important feature of the response of the present model. Here a shear stress of $\tau = 10$ is applied over $0 < T < 0.1$; the stress is reduced to zero for $0.1 < T < 0.3$, after which the original stress $\tau = 10$ is reapplied for $T > 0.3$. The material parameters are, once again, taken as $B = 3000$, $R = 1$, $n = 4$, and $\beta = 1$.

The important features of the strain response in Fig. 12 are:

1. no anelastic strain recovery upon stress reduction,* and
2. the occurrence of state (static) recovery** during the period at zero stress.

The latter is evidenced by the reduction in the state variable α in $0.1 < T < 0.3$, and the accompanying increased strain rate upon reloading.

There is convincing experimental evidence that this is a good idealization of the behavior of 2 1/4 Cr-1 Mo steel at temperatures in the range 500 to 600°C.¹⁹ Although anelastic recovery is not entirely absent in that material at such temperatures, it generally amounts to a small percentage of the total accumulated creep strain (no more than ~25% of the *elastic* strain) in tests similar to that of Fig. 12. State recovery, on the other hand, does appear to be significant in such tests on 2 1/4 Cr-1 Mo steel.

Figure 13 illustrates one of the possible consequences of ignoring recovery of state in materials where it may be present, specifically in situations with alternating cycles of stressing and unstressing. The dotted curve of Fig. 13 shows the response where α does not recover with stress reduction but instead remains at the saturated (steady state) value. Clearly, after many such cycles this model might significantly underpredict the accumulated creep strain.

It is further suspected that important structural phenomena, such as plastic ratchetting, may be influenced by the occurrence of state recovery

*The reader is reminded that the actual material response would, of course, include an immediate elastic strain which is not included in Fig. 12.

**This is commonly referred to as static recovery³⁴ in the metallurgical literature to distinguish it from dynamic recovery which occurs under accompanying inelastic strain. The term state recovery is preferred here since, although inelastic strain is absent, the internal structure is clearly not static.

Fig. 12. γ vs T and α vs T for constant stress $\tau = 10$ for $0 < T < 0.1$, $\tau = 0$ for $0.1 < T < 0.3$ and $\tau = 10$ for $0.3 < T < 0.5$ with $B = 3000$, $R = 0.1$, $n = 4$, and $\beta = 1$.

Fig. 13. γ vs T and α vs T for constant stress $\tau = 10$ for $0 < T < 0.5$, $\tau = 0$ for $0.5 < T < 1.0$, $\tau = 10$ for $1.0 < T < 1.5$, $\tau = 0$ for $1.5 < T < 2.0$ and $\tau = 10$ for $T > 2.0$ with $B = 3000$, $R = 0.5$, $n = 4$, and $\beta = 1$.

in some thermal and loading histories. The tendency of a material at elevated temperature to return to a *softer state* seems almost certain to inhibit a structure composed of that material from achieving the particular residual stress state necessary for shakedown. Thus, ratchetting or alternating plastic straining may be enhanced.

Figure 14 shows the response to a loading sequence similar to that of Fig. 12; here, $\tau = 6$ for $0 < T < 0.2$, $\tau = 0$ for $0.2 < T < 0.5$ and $\tau = 6$ for $T > 0.5$. The material parameters, however, are taken to be quite different from those of Fig. 12, i.e., here $B = 10,000$, $R = 0.001$, $n = 5$, and $\beta = 3$. The features of the response are likewise very different from those of Fig. 12. Here, both anelastic strain recovery and state recovery are absent and the response appears to be much like that predicted by the strain-hardening creep model of Ref. 1. This serves to illustrate the ability of the present model to represent a variety of seemingly very different types of material behavior through different choices of the material parameters. Material behavior quite different from that of Ref. 1 is demonstrated through one choice of parameters, and behavior much like that of Ref. 1 through another choice.

Another manifestation of the same phenomenon illustrated in Fig. 12 is that pertaining to transients in the creep rate produced by *partial* stress reductions. Numerous experiments have been conducted at elevated temperature in which a specimen is crept into the steady-state regime, whereupon a fraction of the stress is removed and the strain monitored in time. Investigators³²⁻³⁵ have frequently observed, particularly in pure metals and class II alloys, that the creep rate immediately following the stress reduction is essentially zero and eventually increases to a new steady-state value. This is commonly referred to as a *hesitation period*. In some cases,³⁶ there has been direct microstructural observation which shows that the hesitation period is accompanied by transients in the dislocation structure, i.e., in the dislocation density and cell size. That is to say, the internal structure is readjusting in time to the changed conditions.

Figure 15 gives the predicted response of the present model under conditions of the kind described above. Steady state is first reached corresponding to a stress $\tau = 10$, thereupon the stress is suddenly reduced

Fig. 14. γ vs T and α vs T for constant stress $\tau = 6$ for $0 < T < 0.2$, $\tau = 0$ for $0.2 < T < 0.5$, and $\tau = 6$ for $T > 0.5$ with $B = 10,000$, $R = 0.001$, $n = 5$, and $\beta = 3$.

Fig. 15. γ vs T and α vs T for period following stress reductions of $\Delta\tau = 2, 3, 4$, and 5 from an original stress of $\tau = 10$ with $B = 3000$, $R = 0.1$, $n = 4$, and $\beta = 1$.

by the respective amounts $\Delta\tau = 2, 3, 4$, and 5 . The γ vs T curve represents the inelastic strain-time response following the indicated stress reduction. Similarly, the α vs T curve represents the readjustment occurring in the internal state prior to a new steady state being reached for the reduced stress $\tau - \Delta\tau$.

The stress drop labeled $\Delta\tau = 4$ in Fig. 15, for example, shows a delay time of about $\Delta T = 0.1$ and appears to regain new steady-state conditions at about $T \approx 0.35$. The transient readjustment in the internal stress α ranges from $\alpha \approx 6.3$ corresponding to steady state for $\tau = 10$ to $\alpha \approx 3.8$ corresponding to steady state for the reduced stress $\tau = 6$.

For stress reductions that are sufficiently small, e.g., $\Delta\tau = 2$ in Fig. 15, there is no discernible hesitation period. In any case, however, the internal stress readjusts in time to a new steady-state value corresponding to the reduced stress. The rate at which this adjustment occurs, and hence the length of the hesitation period, may be thought of as a *characteristic response time* of the material. One might expect that this characteristic time would be controlled in the model by those parameters effecting the balance of hardening and recovery rates such as R or β . This is indeed so, as illustrated in Fig. 16 where the parameter R has been increased by a factor of ten to $R = 1$.

Fig. 16. γ vs T and α vs T for period following stress reductions of $\Delta\tau = 3, 4, 5$, and 6 from an original stress of $\tau = 10$ with $B = 3000$, $R = 1.0$, $n = 4$, and $\beta = 1$.

In Fig. 16 we see that, other things being equal, the larger value of the parameter R has significantly increased the response rate, the hesitation period being sharply decreased for the same $\Delta\tau$. Although not shown here, the exponent β , as expected, also has a strong influence on the characteristic response time. This again serves to illustrate the versatility of the model in representing a rather broad range of behavior.

(2) Relaxation

The relaxation response of the model for $B = 1000, 5000$, and $10,000$ with $R = 0.1$, $n = 4$, and $\beta = 1$ is shown in Fig. 17. It is, of course, necessary in this case to include elasticity which introduces another parameter g , the nondimensional shear modulus. g is obtained by normalizing the actual elastic shear modulus with respect to κ ; in Fig. 17, $g = 5000$. The starting stress in the calculations of Fig. 17 is $\tau_0 = 5$.

The stress relaxation response shows high initial relaxation rates, higher for smaller values of B . The rates diminish in time and the stress relaxes to a level that is lower for smaller B values. The reader is reminded that the real time scale can be stretched or contracted depending on the choice of the hardening parameter H . It appears the model is capable of capturing the very high initial stress relaxation rates observed in experiments.

Two additional plots are shown in Fig. 17, one α vs T indicating the internal stress vs time and the second τ vs α , the *state space*.

On first glance at the α vs T plot, it appears that the internal stress saturates much in the same way as in creep. Further inspection shows, however, that α reaches a maximum as the stress relaxes in time and thereafter diminishes. This phenomenon is evident in the α vs T curve corresponding to $B = 10,000$, but is even more evident in the state space τ vs α . There are shown the trajectories of the state point (α, τ) during relaxation. As discussed by Onat¹⁰ the relaxation trajectories *must* exhibit these maxima during relaxation in order that there be a true steady state under constant stress (creep) conditions. The point (α, τ) at which the slope is vertical on any of these curves gives the steady-state value of α for steady-state creep at that stress τ . For example, the point S in Fig. 17c can be reached through creep at a stress of about

Fig. 17. (a) τ vs T , (b) α vs T , and (c) τ vs α showing stress relaxation response from $\tau = 5$ with $B = 1000$, 5000 , and $10,000$, $R = 0.1$, $n = 4$, $\beta = 1$ and $g = 5000$. Figure 17(c) indicates state paths OS (creep) and PS (relaxation).

$\tau = 3.75$ (the state point moves along OS). When the state point S is reached through creep, it remains stationary and steady-state creep occurs. If, on the other hand, the point S is reached during relaxation along state path PS , the state point does not remain stationary. Instead, α then

begins to decrease as the stress continues to relax. Thus, according to this model there is *no steady state associated with relaxation*.*

More will be said about this interpretation below under the section entitled (5) General. There it will be shown that stress relaxation may be considered a special case of a constant (total) strain rate deformation corresponding to which unique *fixed points* exist in the state space.

Figure 18 gives the effect on stress relaxation of the parameter R . The parameter values here are $B = 3000$, $n = 4$, $\beta = 1$, and $g = 5000$, with $R = 0.01, 0.1, 1.0$, and 10 . As in Fig. 17, the starting stress is taken as $\tau_0 = 5$. We see that in the initial stage of stress relaxation, i.e., in this case for T less than about 0.025 , the response is not substantially effected by the parameter R . For $T > 0.025$, however, the response is significantly influenced by R , larger values of R corresponding to a greater amount of stress relaxation for a given time. The maximum in the internal stress during relaxation also occurs at a shorter time for larger values of the parameter R as is evident in Fig. 18b. This maximum is also well defined for the larger R values in the state space of Fig. 18c.

The question of whether the state variable (in a theory in which there is but *one* measure of the inelastic state) does first increase, reach a maximum, and then decrease during stress relaxation is of considerable theoretical and practical interest. Already pointed out is the connection between such behavior and steady-state creep as discussed by Onat.¹⁰ We shall touch upon this issue again in the subsection entitled General of this report.

(3) Monotonic-constant strain rate

Two plots are shown in Fig. 19, one of stress vs total strain (τ vs Γ)** and the other the state space (τ vs α); in each case curves are shown for a range of dimensionless strain rates T . Note that the total strain rate T is obtained by taking the derivative of strain Γ with respect to *dimensionless* time, so that $\dot{\Gamma} = TH$ is the *actual* strain rate. The material

*This is in accord with the observations of E. Hart^{37,38} that no steady state exists, since his experiments involve mainly relaxation tests.

**The total shear strain Γ is the sum of the inelastic shear strain γ and the elastic shear strain γ_e .

Fig. 18. (a) τ vs T , (b) α vs T , and (c) τ vs α showing stress relaxation from $\tau = 5$ with $B = 3000$, $R = 0.01, 0.1, 1.0$, and 10 , $n = 4$, $\beta = 1$, and $g = 5000$.

parameters are taken as shown, i.e., $B = 3000$, $n = 4$, $\beta = 1$, $R = 0.1$, and $g = 10^4$. The strain-rate spans one order of magnitude, $T = 1$ to $T = 10$.

The stress-strain response shows substantial rate sensitivity under the parameters chosen with curve shapes typical of materials such as

Fig. 19. τ vs Γ and τ vs α for constant strain rates $T = 1, 2, 4, 6, 8$, and 10 with $B = 3000$, $R = 0.1$, $n = 4$, $\beta = 1$, and $g = 10,000$.

2 1/4 Cr-1 Mo steel. The yield stress is changed significantly, almost 50%, with the 10 to 1 change in strain rate.

The state paths τ - α show similar shape to the respective τ - Γ curves except that the initial slope is vertical, the state variable α , of course, remaining constant with elastic deformation.

Figure 20 shows the effect on the constant strain-rate response of changing the parameter R . Here, $R = 1.0$ with all other parameters the same as in Fig. 19. Very little effect is observed for the tenfold increase in R , at least up to 0.5% strain as shown.

In the stress-strain response shown in Fig. 21, all parameters have been taken as in Fig. 19, except B , which has been changed from 3000 to 1000. Here, we show τ vs Γ curves corresponding only to the extreme values of $T = 1$ and $T = 10$. Considerable change is seen due to the change in B , yield occurs at a lower stress, and strain hardening is less pronounced corresponding to the lower B value. The change in yield associated with the order of magnitude change in T , however, is still approximately 50% as in Fig. 19.

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Fig. 20. τ vs Γ for constant strain rates $T = 1, 2, 4, 6, 8$, and 10 with $B = 3000$, $R = 1.0$, $n = 4$, $\beta = 1$, and $g = 10,000$.

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Fig. 21. τ vs Γ for constant strain rates $T = 1$ and 10 with $B = 1000$, $R = 0.1$, $n = 4$, $\beta = 1$, and $g = 10,000$.

Comparison of Fig. 22 and Fig. 19 shows the effect of the exponent n . With $n = 5$ as in Fig. 22, the yield stress is reduced and exhibits substantially less rate sensitivity ($\sim 25\%$ for $T = 1$ to 10) than that in Fig. 19 with $n = 4$.

In Fig. 23, two parameters are changed from those of Fig. 19, n and β ; $n = 5$ and $\beta = 3$ in Fig. 23 as opposed to $n = 4$ and $\beta = 1$ in Fig. 19. The rate sensitivity is similar to that of Fig. 22, substantially less than in Fig. 19, but the shape of the stress-strain curve is quite flat, qualitatively resembling type 304 stainless steel.

The choice of material parameters in Fig. 24 reverts to those of Fig. 19 with $T = 1$, only here partial unloading occurs at $T = -1$ after about 0.2% strain; reloading then resumes at $T = 1$. One important feature in Fig. 24 is that upon unloading the stress-strain response does not immediately follow an *elastic* unloading line corresponding to the elastic modulus. This is often observed in experimental results (e.g., see Phillips³⁹). As seen in the state space (τ vs α) of Fig. 24, inelastic deformation continues to occur shortly after the reversal of strain rate, i.e., α continues to increase. The amount of α increase depends upon the magnitude of the strain rate; for a reversed strain rate which is *high enough*, essentially elastic unloading occurs immediately. The shape of the reloading curve depends on the reloading rate and also on the rate at which prior unloading occurred. This is entirely in keeping with the experimental observations of Phillips³⁹ and Phillips and Tang.⁴⁰

The effect of changing the strain rate during monotonic loading is shown in Fig. 25. With the material parameters again taken as in Fig. 19, the strain rate is changed from $T = 1$ to $T = 10$ after about 0.2% strain. The τ - T response curve appears typical of experimental results for such loading. The curve in the state space for $T = 10$ eventually merges with that which would be traced out if the strain rate were $T = 10$ from the start.

(4) Cyclic constant strain rate

Figure 26 shows one and one-half cycles of fixed strain range cycling at $\pm 0.5\%$ strain and a constant strain-rate magnitude of $|T| = 1$. The material parameters are taken to be $B = 3000$, $R = 0.1$, $n = 4$, $\beta = 1$, and

Fig. 22. τ vs Γ for constant strain rates $T = 1$ and 10 with $B = 3000$, $R = 0.1$, $n = 5$, $\beta = 1$, and $g = 10,000$.

Fig. 23. τ vs Γ for constant strain rates $T = 1$ and 10 with $B = 3000$, $R = 0.1$, $n = 5$, $\beta = 3$, and $g = 10,000$.

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Fig. 24. τ vs Γ and τ vs α for constant strain rates $T = 1, -1$, and 1 with $B = 3000$, $R = 0.1$, $n = 4$, $\beta = 1$, and $g = 10,000$, showing partial unloading followed by resumed loading at constant strain rate magnitude.

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Fig. 25. τ vs Γ and τ vs α for constant strain rates $T = 1$ in $0 < \Gamma < 0.002$ and $T = 10$ in $\Gamma > 0.002$ with $B = 3000$, $R = 0.1$, $n = 4$, $\beta = 1$, and $g = 10,000$.

Fig. 26. τ vs Γ for 1 1/2 cycles at constant strain rate $\dot{\Gamma} = 1$ through $-0.005 < \Gamma < 0.005$ with $B = 3000$, $R = 0.1$, $n = 4$, $\beta = 1$, and $g = 10,000$.

$g = 10^4$. Here, for the first time, is a situation in which the loading path carries the state point into all quadrants of the state space, thus bringing into play all the inequalities appearing in the governing Eqs. (4.8) through (4.10). With the scalar state variable κ held constant, no *isotropic* hardening occurs and both the stress-strain response and the state path quickly become locked into respective limit cycles as shown in Fig. 27.

It is instructive to consider the limit cycle in the state space $(\tau-\alpha)$ in some detail. As the state point moves in the first quadrant along $a \rightarrow u$ in Fig. 27, we have $F > 0$, $\tau\alpha > 0$, $\tau(\tau-\alpha) > 0$, and $|\alpha| > \alpha_0$, so that according to Eqs. (4.8) and (4.9), $\dot{\gamma} > 0$ and $\dot{\alpha} > 0$. Thus even during unloading to a , the strain and the state variable may increase as in Fig. 24, however not so noticeably in the present case. As unloading is continued and the state point moves from a to b , we have either $F \leq 0$ or $\tau(\tau-\alpha) \leq 0$ at each point, which in either case requires that $\dot{\gamma} = 0$ so that the response is elastic. Also from a to b , $\dot{\alpha} < 0$; however, the small decrease in α is

Fig. 27. τ vs Γ and τ vs α showing limit cycles for constant strain rate $\dot{\Gamma} = 1$ through $-0.005 < \Gamma < 0.005$ with $B = 3000$, $R = 0.1$, $n = 4$, $\beta = 1$, and $g = 10,000$.

hardly detectable in this particular example. At a lower strain rate, α would show a more measurable decrease from a to b . At point b , a stress reversal occurs and the state point enters the fourth quadrant in the state space. Now, $\tau\alpha < 0$, $F > 0$, and $|\alpha| > \alpha_0$ so according to Eqs. (4.8) and (4.9), $\dot{\gamma} < 0$ and $\dot{\alpha} < 0$, with $|\dot{\alpha}|$ being quite large as the state point moves across the fourth quadrant from b to c . At c the state point moves into the third quadrant and the behavior is precisely the same as that from o to c , only with opposite sign.

The hysteresis loop of Fig. 27 is shown again in Fig. 28a with four points indicated around the loop, points 1 and 4 at a stress level of $\tau = 6$ and points 2 and 3 at a stress level of $\tau = 10$. The curves in Fig. 28b indicate stress relaxation from each of the four points. The reader is reminded that it is *dimensionless* stress vs *dimensionless* time (τ vs T) which is plotted in Fig. 28b and that the real time scale is stretched or contracted depending on choice of the parameter H . For the present purpose, however, it is not the real time scale that is of interest but, instead, comparisons of the relaxation curves with one another.

Fig. 28. (a) τ vs Γ , (b) τ vs T , and (c) τ vs α showing relaxation response, (b) and (c), from points labeled 1, 2, 3, and 4 on the limit cycle hysteresis loop (a). Here, $B = 3000$, $R = 0.1$, $n = 4$, $\beta = 1$, and $g = 10,000$.

Comparing the curves labeled 2 and 3 in Fig. 28b, corresponding to stress relaxation from points 2 and 3 in Fig. 28a, we see that the response is markedly different. Relaxation from point 2 involves much higher initial relaxation rates and in any fixed time period shows more relaxation than the corresponding curve for relaxation from point 3.

A similar but more exaggerated result is seen comparing the stress relaxation from points 1 and 4, each with a starting stress of $\tau = 6$. The curve labeled 4 in Fig. 28b, which involves substantial unloading prior to relaxation, shows *no* stress relaxation until well beyond $T = 0.1$. Curve 1, on the other hand, shows a classical relaxation response. This is in good qualitative agreement with the experimental results by Swindeman⁸ shown in Fig. 1. That set of results was taken from type 304 stainless steel at $\sim 600^\circ\text{C}$; however, similar results have also been obtained by Swindeman on 2 1/4 Cr-1 Mo steel. As indicated in the Introduction, failure to account for creep/plasticity interactions of this type can, under the present ASME code rules, lead to overpredictions of component lifetimes in design.

The qualitatively different relaxation responses shown in Fig. 28b are not surprising when viewed in the context of the state space as in Fig. 28c. There we see the limit state path as in Fig 27 plus the state paths associated with each of the relaxation curves in Fig. 28b. Evidently, the state points from which each of the stress relaxations initiate are quite distinct. Consequently, as one might expect, the state trajectories and the stress-time relaxation responses are likewise quite different.

(5) General

Figure 29 shows a result of considerable theoretical interest. The figure represents constant strain-rate extension at a rate of $\dot{T} = 0.1$ with the material parameters chosen as indicated. Points *a*, *b*, *d*, and *e* in the state space (τ vs α) of Fig. 29 represent four different initial inelastic states from which constant rate extension is begun. Note that, regardless of the *initial state*, each state path eventually terminates at the *same point c*. As discussed by Onat,¹⁰ point *c* is termed the *fixed point* of the state space. There exists a fixed point in the state space corresponding to each constant strain rate.

The stress-strain (τ vs Γ) response associated with each state path is also indicated in Fig. 29. Each τ - Γ curve is seen to be quite different depending on the initial inelastic state. In fact, the stress initially *decreases* under an *extensional* strain rate from the state point *a*. The state point moves relatively *rapidly* through the second and fourth

Fig. 29. τ vs Γ and τ vs α for constant strain rate $T = 0.1$ deformation from various initial states labeled a , b , d , and e in state space τ vs α . Point c is a *fixed point* corresponding to $T = 0.1$. Here, $B = 5000$, $R = 1.0$, $n = 4$, $\beta = 1$, and $g = 5000$.

quadrants and into the first and third quadrants, respectively, as seen by considering the state curves beginning at a or e in Fig. 29. The state point can be brought to point a or e only under rather special circumstances, i.e., by considerable inelastic deformation followed by reversed stressing. The rapid movement of the state point through the second or fourth quadrants thus corresponds to readjustment of the internal dislocation structure following the reversed stressing.

As mentioned under subsection (2), relaxation can be thought of as a special case of constant strain-rate deformation, i.e., with the total strain rate equal to zero. Figure 30 shows the stress relaxation from various beginning states labeled a , b , c , d , e , and f with the same material parameter choices as in Fig. 29. Each state path appears to be heading toward the *origin* of the state space, which would be precisely the case if the threshold stress κ were zero, i.e., if inelastic strain could occur at any stress, however small. In fact, the state paths a , b , and c move toward the fixed point $(\alpha, \tau) = (0, 1)$ and the paths d , e , and f move toward $(\alpha, \tau) = (0, -1)$. With the *dead band* created by the

Fig. 30. τ vs T and τ vs α showing stress relaxation from initial states labeled a , b , c , d , e , and f in state space τ vs α . Here, $B = 5000$, $R = 1.0$, $n = 4$, $\beta = 1$, and $g = 5000$.

nonzero threshold stress κ , the fixed point under relaxation is not unique. Instead there is one fixed point for positive starting stresses and one for negative ones. Practically speaking, the fixed point in relaxation is never reached since the rate at which the state point moves becomes increasingly smaller. Theoretically, however, the concept of a fixed point in state space is helpful in predicting the vastly different responses when beginning from different inelastic states. The results indicated in subsection (4) showing the relaxation response from various points around an established hysteresis loop are, of course, a special case of the results shown in Fig. 30.

Another interesting and important result is shown in Figs. 31 and 32. Figure 31 characterizes a set of tests in which several *identical* specimens are first deformed under a relatively high strain rate $\dot{T} = 30$ and then allowed to relax from a range of stress levels. The material parameters are chosen to represent a material that has a rather high stress nonlinearity ($n = 5$) and a rather weak recovery mechanism relative to that of hardening ($R = 0.01$). This is intended to be representative of the *low* end of the *temperature* range over which the present model may be applicable.

Fig. 31. τ vs T and τ vs α showing stress relaxation from various stresses following constant strain rate deformation at $T = 30$ with $B = 10,000$, $R = 0.01$, $n = 5$, $\beta = 1$, and $g = 10,000$.

The relaxation trajectories are shown in Fig. 31 in terms of τ vs T and in the state space. The state paths, for the most part, are nearly vertical, indicating that the state variable α does not change drastically during relaxation, at least compared to its change during initial load-up. This is just the observation made by Hart^{37,38} and co-workers in their relaxation tests performed to characterize their *state variable* (the *hardness state* σ^*). Thus, under the present choice of parameters, the present theory seems to be in agreement with the constitutive theory of Hart.

Figure 32 shows another representation of the same results shown in Fig. 31. Here, the relaxation response is presented not as τ vs T as in Fig. 31, but instead as $\log \tau$ vs $\log \dot{\gamma}$, i.e., *log stress vs log strain rate*. The curves in Fig. 32 show the same features as many of those reported by Hart et al. The curves are concave upward and can be translated, just as Hart argues, along a constant direction in that space to form a single *master curve*. Hart makes use of this property to quantify critical material parameters in his constitutive model. Similar construction in the

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Fig. 32. Log stress ($\log \tau$) vs log inelastic strain rate ($\log \dot{\gamma}$) corresponding to stress relaxation responses shown in Fig. 31.

case of the present theory might provide values of *some* of the necessary parameters as well.

One conclusion from the results shown in Figs. 31 and 32 is that relaxation experiments coupled with monotonic constant strain rate tests in themselves cannot provide all of the information needed to fully characterize the behavior of any given material. This is evident in the state space of Fig. 31, where we see that only a small portion of the state space is spanned by the relaxation tests described. Even many more such tests cannot hope to cover all regions of the state space, which is, no doubt, necessary to fully characterize a material for general loading histories.

6. SUMMARY AND CONCLUSIONS

A constitutive theory has been presented for use in high-temperature structural analysis. The theory is *unified* in the sense that plastic and creep strain is considered as arising from basically the same physical mechanisms. The model is inherently time-dependent and is found to display many of the features, often characterized as creep/plasticity interactions, found in experimental observations on structural alloys of interest.

The constitutive model has been stated in a full multiaxial form, its basic assumptions have been stated, many of the experimental observations upon which it rests have been cited, and its relationship to some other classical and modern theories has been discussed.

A reduction of the multiaxial constitutive equations has been made to that of pure shear, after which the equations were nondimensionalized thereby defining dimensionless groups of parameters and reducing the total number of material parameters.

An extensive set of calculations were made, using the reduced equations, under several combinations of the dimensionless parameters and loading histories. This parameter study represents the main effort of this study and the major conclusions drawn from it are as follows:

1. The present theory allows modeling of most of the features involving time dependency and creep/plasticity interaction observed in structural alloys at high temperatures. These features include creep, relaxation, recovery, rate dependent monotonic and cyclic yielding, creep and relaxation hesitation following stress reduction, etc.
2. Through various choices of the material parameters, the model seems capable of representing the behavior of a range of materials such as 2 1/4 Cr-1 Mo steel, on the one hand, and type 304 stainless steel on the other.
3. The model contains many of the characteristics discussed by E. T. Onat¹⁰ as being necessary for a rational representation of inelastic, hereditary behavior of metals. Many features as discussed by Onat

regarding the geometry of the *state space*, e.g., fixed points, maxima of relaxation trajectories, etc., are present in this model.

4. The present model appears to contain the well-known constitutive theory of Hart^{37,38} as a special case. That is to say, with the choice of material parameters representative of the lower end of the temperature range where the present model may be applicable ($T/T_m \leq 0.4$), this model predicts behavior with the same features as those reported in the experiments of Hart et al. and used to quantify material constants in their theoretical model.

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