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**APPLICATION OF WEIBULL CRITERION
TO FAILURE PREDICTION
IN COMPOSITES**

W. D. Cain
C. E. Knight, Jr.

April 1981

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TO FAILURE PREDICTION
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ABSTRACT

Fiber-reinforced composite materials are being widely used in engineered structures. This report examines how the general form of the Weibull Criterion, including the evaluation of the parameters and the scaling of the parameter values, can be used for the prediction of component failure.

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SUMMARY

The Weibull statistical failure theory, widely used for failure prediction in ceramic components, is applied to predicting failure of a generally loaded fiber composite material. Included is an examination of the general form of the Weibull criterion, evaluation of the parameters, and scaling of the parameter values. An integral form is presented which can be used in a finite-element code to predict structural failure. Composite-strand and ring-test data are shown to be representable by a two-parameter Weibull model.

INTRODUCTION

Fiber-reinforced composite materials are being more widely used in engineered structures. While the analysis of stress and deformation under loading is reasonably well developed, the same cannot be said for the prediction of component failure. As a class of structures, failure of continuous fiber-reinforced composites generally is controlled by the stresses aligned in the fiber direction. The ultimate failure to support load may be preceded by transverse cracking in the matrix due to tensile stresses normal to the fiber direction. In this study, made at the Oak Ridge Y-12 Plant,^(a) it is assumed that this does not cause ultimate failure; only the fiber direction stress is significant in the analysis. Furthermore, only tensile loadings have been investigated since most fiber composite structures are designed for tensile loading applications.

A statistical failure theory widely used for failure prediction in ceramic components is due to Weibull.¹ The theory has also been used in a few cases^{2,3} on fiber composite materials. This report examines the general form of the Weibull criterion, including the evaluation of the parameters and the scaling of the parameter values. An integral form is presented which may be used in a finite-element code to determine the predicted failure of a generally loaded structure.

(a) Operated by the Union Carbide Corporation, Nuclear Division, for the Department of Energy.

APPLICATION OF WEIBULL CRITERION TO FAILURE PREDICTION IN COMPOSITES

THEORETICAL DEVELOPMENT

According to the Weibull theory,¹ the probability of survival of a test specimen at a stress, σ , is given by:

$$P_s = \exp \left\{ - \left[\frac{\sigma - \sigma_U}{\sigma_O} \right]^m \right\} \quad (1)$$

where:

P_s is the probability of survival,

σ_U is the threshold stress below which the material does not fail,

σ_O is the scale parameter, and

m is the Weibull exponent.

In many applications of the Weibull criterion, the threshold stress is assumed to be zero. This reduces the form to a two-parameter distribution. In this formulation the threshold stress parameter is retained. Its effect is evaluated by statistical methods and by scaling requirements. The Weibull exponent, m , is a measure of the variability of the difference between failure stress and threshold stress ($\sigma_f - \sigma_U$) where σ_f is the failure stress for a given specimen. The higher values of m correspond to more tightly grouped failure stress distributions. The scale parameter, σ_O , sets the mean value of the distribution of $(\sigma - \sigma_U)$, and its value equals $(\sigma_p - \sigma_U)$ when σ_p is the stress level where the probability of survival is 0.368 for the population of test specimens.

Equation (1) applies to specimens of specific material volume. Based on a chain-model weakest-link hypothesis, specimens of twice the volume would have a survival probability of

$$\begin{aligned} P_{s2} &= P_{s1} \times P_{s1} \\ &= \exp \left\{ - \left[\frac{\sigma - \sigma_U}{\sigma_O} \right]^m \right\} \times \exp \left\{ - \left[\frac{\sigma - \sigma_U}{\sigma_O} \right]^m \right\} \\ &= \exp \left\{ - \left[\frac{\sigma - \sigma_U}{\sigma_O} \right]^m \times 2 \right\} \\ &= \exp \left\{ - \left[\frac{\sigma - \sigma_U}{\sigma_O} \right]^m \frac{V_2}{V_1} \right\} \end{aligned}$$

for the same stress, σ in both volumes. This may be generalized to any volume.

$$P_{s_{tot}} = \exp \left\{ - \left[\frac{\sigma - \sigma_u}{\sigma_o} \right]^m \frac{V_{tot}}{V_1} \right\} \quad (2)$$

The values of the parameters σ_u , σ_o , and m are determined from the data for a set of test specimens of given material volume. Then for volume V_1 ,

$$P_{s1} = \exp \left\{ - \left[\frac{\sigma_1 - \sigma_{u1}}{\sigma_{o1}} \right]^{m_1} \frac{V_1}{V_1} \right\} ,$$

and for volume V_2 ,

$$P_{s2} = \exp \left\{ - \left[\frac{\sigma_2 - \sigma_{u2}}{\sigma_{o2}} \right]^{m_2} \frac{V_2}{V_2} \right\} ,$$

setting the probabilities of survival equal,

$$\left[\frac{\sigma_1 V_1^{\frac{1}{m_1}} - \sigma_{u1} V_1^{\frac{1}{m_1}}}{\sigma_{o1} V_1^{\frac{1}{m_1}}} \right]^{m_1} = \left[\frac{\sigma_2 V_2^{\frac{1}{m_2}} - \sigma_{u2} V_2^{\frac{1}{m_2}}}{\sigma_{o2} V_2^{\frac{1}{m_2}}} \right]^{m_2} \quad (3)$$

and, if the material variability is constant,

$$m_1 = m_2.$$

Thus, the stress $\times$ (volume) $^{\frac{1}{m}}$ product may be conveniently used as a distribution parameter. Correspondence of the terms indicates that the stress parameter values will scale with the specimen volume. These scaling relationships are given by:

$$\sigma_2 = \sigma_1 \left(\frac{V_1}{V_2} \right)^{\frac{1}{m}} , \quad (4)$$

$$\sigma_{u2} = \sigma_{u1} \left(\frac{V_1}{V_2} \right)^{\frac{1}{m}} , \text{ and} \quad (5)$$

$$\sigma_{o2} = \sigma_{o1} \left(\frac{V_1}{V_2} \right)^{\frac{1}{m}} . \quad (6)$$

The average failure stress, σ_2 , in a specimen of volume, V_2 , is calculated by equation (4) given the average failure stress of σ_1 , in volume, V_1 . The parameters, σ_u and σ_o , used in equations (1) and (2) for a material volume, V_2 , are calculated by equations (5) and (6) given the parameter values in volume, V_2 .

Equation (1) may be modified to predict the probability of survival of specimens of volume, V_2 , using test data from specimens of volume, V_1 .

$$Ps_2 = \exp \left\{ - \left[\frac{\sigma_2 - \sigma_{u1} \left(\frac{V_1}{V_2} \right)^{\frac{1}{m}}}{\sigma_{o1}} \right]^m \frac{V_2}{V_1} \right\} \quad (7)$$

The theoretical development up to this point has assumed a uniform stress throughout the volume. The extension to nonuniform states of stress is straightforward and makes the theory applicable to real structures.

For a nonuniformly stressed structure of total volume, V_3 , a differential element, dV_3 , has a stress σ_3 . The probability of survival, dPs_3 , of that element is given by:

$$dPs_3 = \exp \left\{ - \left[\frac{\sigma_3 - \sigma_{u3}}{\sigma_{o3}} \right]^m \frac{dV_3}{V_3} \right\}$$

or, since σ_{u3} and σ_{o3} are usually unknown,

$$dPs_3 = \exp \left\{ - \left[\frac{\sigma_3 - \sigma_{u1} \left(\frac{V_1}{V_3} \right)^{\frac{1}{m}}}{\sigma_{o1}} \right]^m \frac{dV_3}{V_1} \right\}.$$

The total probability of survival, Ps_3 , for the structure of volume, V_3 , is given by the product of all the dPs_3 's for the volume elements. The cumulative product of all the exponential terms becomes the summation of all the exponent values. This summation of exponents becomes the integral of the exponent terms as $dV_3 \rightarrow 0$. This results in

$$Ps_3 = \exp \left\{ \int_{V_3} - \left[\frac{\sigma_3 - \sigma_{u1} \left(\frac{V_1}{V_3} \right)^{\frac{1}{m}}}{\sigma_{o1}} \right]^m \frac{dV_3}{V_1} \right\}. \quad (8)$$

Equation (8) may be summed very simply over the elements in a finite-element model of the structure to determine its overall probability of survival at any given loading condition.

EVALUATION OF PARAMETERS

The accuracy of the Weibull criterion in predicting failure depends greatly upon the values of the parameters in the distribution. Computer programs exist for computing values of m and σ_0 , assuming $\sigma_U = 0$; i.e., a two-parameter Weibull model. One program was found in the Oak Ridge area for the evaluation of all three parameters. However, as this program used a heuristic factor to weight the value of σ_U toward 0.8 times the lowest strength data point on all the data sets, other approaches were tried.

An optimization code based on the sequential simplex search method was used to evaluate the parameters. Both maximum likelihood and least squares estimates for the parameters were found. Also, an experimental approach based on data sets for different material volumes was used. A discussion of the procedures used follows.

A basic problem in mathematics and engineering involves finding an n -dimensional vector $X = (x_1, x_2, \dots, x_n)$ such that the value of a function $F(X)$ of this vector is a minimum. In 1962, Spendley, Hext, and Himsworth⁴ introduced an ingenious geometric concept for determining an optimum by evaluating the function to be optimized at a set of points forming a simplex in the factor space; then, by continually forming new simplices by reflecting the worst point about the hyperplane of the remaining points, a new, better point is found. Nelder and Mead⁵ developed an algorithm which extends this concept so that the simplex adapts itself to local landscape using reflected, expanded, and contracted points to locate a minimum. The biggest advantage of this method is that no derivatives are required. In fact, an algebraic expression for a function is not necessary; all that is required is a method of function evaluation. The only requirement is that the function be continuous in the area being searched. Another attractive feature of this method is the ease with which constraints on the parameters of the function can be implemented.

The method described and exemplified here is the technique developed by Nelder and Mead.⁵ The searching technique is iterative and entirely geometric in nature.

A simplex is a geometric figure which is a line segment in one dimension, a triangle in two dimensions, a tetrahedron in three dimensions, and a polyhedron with $n + 1$ linearly independent vertices in n -dimensions. In our context, the number of dimensions is the same as the number of independent parameters of the function F to be minimized.

The sequential simplex search technique searches for a minimum of the function F by comparing the function values at the $n + 1$ vertices of a general (but not necessarily regular) simplex followed by a replacement of the vertex where the function has its worst value. Consider the minimization of a function $F(X)$ where X is n -dimensional. Let $X_0, X_1, \dots, X_n$ be $n + 1$ points in n -dimensional space defining the current simplex. Let $F(X_i)$ be the value of the function of X_i and define h as the suffix so that

$F(X_h) = \text{worst } [F(X_i)]$, and ℓ as the suffix such that $F(X_\ell) = \text{best } [F(X_i)]$. Remove X_h from the set of $n + 1$ points and define the centroid of the remaining points by

$$\bar{X} = \sum_{\substack{i=0 \\ i \neq h}}^n \frac{X_i}{n} \quad (9)$$

At each stage in the searching process, X_h is replaced by a new point. Four distinct operations are possible for each replacement: reflection, expansion, contraction, and shrinkage. A positive constant is associated with the reflection, expansion, and contraction operations.

The replacement operation used is determined in the following manner. Define:

$$X_R = (1 + \alpha) \bar{X} - \alpha X_h, \quad (10)$$

where:

α is a positive number and called the reflection coefficient.

After computing $F(X_R)$, several possibilities arise:

- (1) If $F(X_R) < F(X_\ell)$, then $F(X_R)$ is a new minimum and a test is made to see if it is advisable to move further in the same direction in which X_R was encountered. Now define $X_E = \gamma X_R + (1 - \gamma) \bar{X}$ where γ is a positive number greater than unity and is called the expansion coefficient. If $F(X_E) < F(X_\ell)$, then define a new set of vertices by replacing X_h with X_E . This operation is called an expansion.
- (2) If $F(X_E) \geq F(X_\ell)$, the expansion process has failed and a new set of vertices is obtained by replacing X_h with X_R , a reflecting operation.
- (3) If $F(X_R) > F(X_i)$ for all $i \neq h$, then refine X_h to be either X_h or X_R depending on which point the value of F is best. Compute

$$X_C = \beta X_h + (1 - \beta) \bar{X} \quad (11)$$

where:

$0 < \beta < 1$ and β is called the contraction coefficient.

If $F(X_C) \leq F(X_h)$, then X_h is replaced by X_C to form a new set of vertices, a contracting operation. If $F(X_C) > F(X_h)$, the contraction has failed; then each X_i is replaced by

$(X_i + X_l)/2$. This amounts to halving the simplex size about the X_l vertex, a shrinking operation.

- (4) If $F(X_l) \leq F(X_R) \leq F(X_H)$, then replace X_H by X_R to form a new set of vertices. This is also a reflecting operation. The iteration process continues until the best value of the function has been reached.

Although the sequential simplex search method involves numerous tests and iteration cycles, it has proven to be effective in the solution of certain nonlinear systems where other methods have failed. Restraints on the parameters are easily applied through the use of penalty functions or artificial barriers.

The maximum likelihood estimates (MLE) of the parameters are found by maximizing the function given by Mann, et al.⁶

$$L = n! \left(\frac{m}{\sigma_0} \right)^n \prod_{i=1}^n \left(\frac{\sigma_i - \sigma_u}{\sigma_0} \right)^{(m-1)} \cdot \exp \left[- \sum_{i=1}^n \left(\frac{\sigma_i - \sigma_u}{\sigma_0} \right)^m \right] \quad (12)$$

where:

- L is the maximum likelihood function,
- n is the number of data points,
- i is the data point index,
- σ_i is the ranked failure stress at index i, and
- m, σ_u, σ_0 are the Weibull parameters.

The least squares estimates (LSE) are given by minimizing the sum of the squares of the difference between the probability of survival, calculated using current parameter values and the failure stress levels, and the theoretical probability of survival based on the rank of the failure stress point.

Since the optimization code allows fixing of any of the variables, equation (4) was used to compute a value of m for input to the code. Data sets from two different material volumes were generated. In equation (4) the stress values, σ_1 and σ_2 , must be corresponding points on the strength distribution curve so the average failure stress values were chosen. Knowing the specimen volumes, the value of m is easily computed. Using the computed m , the code searches for optimum values of σ_u and σ_0 to satisfy the governing function.

RESULTS

Tensile strength data was collected for a fiber composite material of Kevlar 49^(b) fiber and DER332/T403^(c) epoxy resin with a fiber volume fraction of 0.70. The tensile test specimens used were a 10-in.-long epoxy impregnated strand of the winding material and a ring nominally 0.250 in. wide X 0.120 in. thick X 6.000 in. inside diameter. Four data sets of strand tensile strengths of 40 to 50 specimens each were available from an earlier test program.⁷ A data set of hydrostatically burst test rings of about 40 specimens was generated.

The separate data sets were analyzed by the simplex search code using both a least squares function and a maximum likelihood function independently. The strand data sets were also pooled and analyzed. These results are summarized in part A of table 1. The MLEs are determined by maximizing the logarithm of the MLE function. The LSEs are determined by minimizing the sum of the square of the deviations from the theoretical positions in the probability of survival equation.

Table 1
SIMPLEX SEARCH CODE EVALUATION OF WEIBULL PARAMETERS

Data Set	m		σ_U , ksi		σ_O , ksi		Function Value	
	MLE	LSE	MLE	LSE	MLE	LSE	ln(MLE)	LSE
Part A								
Strands Number 1	19.79	4.5	84.6	425.5	462.7	120.4	-0.857×10^2	0.0141
Strands Number 2	9.55	2.53	0.0	487.5	565.2	63.0	-0.103×10^3	0.319
Strands Number 3	12.1	2.87	83.4	398.3	447.2	129.1	-0.108×10^3	0.0232
Strands Number 4	13.4	27.0	231.9	0.0	329.3	562.2	-0.791×10^2	0.0425
Strands Pooled	15.7	17.3	107.2	33.5	441.2	515.7	-0.118×10^3	0.0160
Rings	19.6	2.90	8.3	227.4	268.1	47.5	-0.540×10^2	0.0272
Part B(1)								
Strands Pooled	20.66 ⁽¹⁾	20.66 ⁽¹⁾	0.0	0.0	549.1	548.8	-0.119×10^3	0.0262
Rings	20.66 ⁽¹⁾	20.66 ⁽¹⁾	0.0	0.0	276.6	275.4	-0.540×10^2	0.0406
Part C(2)								
Strands Pooled	19.6	18.42	0.0	0.0	548.6	549.2	-0.118×10^3	0.0160
Rings	20.2	18.72	0.0	0.0	276.4	275.8	-0.540×10^2	0.0359

(1) The m values are calculated for scaling equation (4), $m = \ln\left(\frac{V_1}{V_2}\right) / \ln\left(\frac{\sigma_2}{\sigma_1}\right)$.

(2) Two-parameter Weibull theory ($\sigma_U = \sigma$).

(b) E. I. du Pont de Nemours & Co., Inc.

(c) DER332, Dow Chemical Co.; T403, Jefferson Chemical Co.

These results show a wide range of parameter values determined for the different data sets. Based on the function values for this range, the functions for this three-parameter Weibull model appear to be slowly changing and not very sharply defined at any peak value. In checking the scaling equations (4-6) with the calculated parameter values, only the higher values of m gave reasonable agreement for scaling between the strand and ring data. Based on this observation, the scaling equation (4) was used to calculate a value of m which would give a proper scaling factor between the material volume in the strand specimen and the material volume in the ring specimen.

The parameter m in the simplex code was fixed at a value of 20.66 [based on equation (4) and average failure stresses]. The other parameters were then evaluated for the pooled strand data and the ring data. These are summarized in table 1, part B. The value of σ_U was 0.0 in all cases and the MLE and LSE for σ_0 were in agreement. The maximized function values for the MLE are almost identical to those in part A data although the parameter values are varied. The minimized function values for the LSE are also close.

By fixing the value of σ_U at 0.0 in the code, the parameters for the more common two-parameter Weibull theory are evaluated. These are given in part C of table 1. These data agree reasonably well with the data in part B. Again, the function-optimized value is close to its corresponding value in parts A and B.

CONCLUSIONS

These material test data are shown to be representable by a two-parameter Weibull model. The Weibull exponent, m , may be found by the scaling equation (4) or by the simplex search code by setting $\sigma_U = 0.0$. The value determined by the scaling equation would seem to be better since it calculates a value which directly accounts for material volume or size effect on tensile strength. In any given material system with tensile test data for two different material volumes, use of the scaling equation to calculate m will allow the simplex search code to evaluate σ_U .

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