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JAYCOR

11011 Torreyana Road
Post Office Box 85154
San Diego, California 92138

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BOUNDARY LAYER STUDIES
RELATED TO FUSION THEORY

FINAL REPORT

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ABSTRACT

The described work studied the boundary between closed and open field lines in EBT geometry, with emphasis on the microstability properties. These properties were established primarily for drift waves in the lower hybrid range of frequencies. The transport due to these modes was evaluated by a self-consistent treatment, using quasilinear models in a plasma diffusion code. The model was benchmarked against the EBT experimental results from ORNL and the sensitivity to transport model established. Viscosity was estimated to be negligible compared with anomalous transport. Drift wave turbulence gave a boundary layer size much more consistent with experiment than either collisional transport or Bohm diffusion.

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I. INTRODUCTION AND SUMMARY

Experimental measurements of electron density in the Elmo Bumpy Torus device EBT-S have indicated that the density in the core, or central region, of that device is quite uniform.¹ It is also known from direct measurement that a cooler, much less dense halo of plasma surrounds the core plasma.² The general picture that emerges from experimentation is that the radial plasma profiles in EBT are very flat in the core region and very sharp in a narrow boundary region where a transition to the halo plasma occurs. This strong disparity in plasma gradient scale lengths in the core and boundary regions suggests that quite different plasma transport processes are operative in the two regions.

Although the boundary region in EBT devices is quite narrow (~ 1 cm), the physical processes which occur in this region have a strong influence on the overall character of bumpy tori magnetic confinement. The primary reason for this is that the relativistic rings, which provide gross stability against the interchange instability of the toroidal core plasma, are resident in the boundary region. Current theoretical analyses, as reviewed by Spong,³ predict that the ring-core stability requires critical ranges of $\beta_{\text{core}} = 8\pi n_{\text{core}} T_{\text{core}} / B^2$ and $n_{\text{e}}^{\text{core}} / n_{\text{e}}^{\text{ring}}$, and both of these quantities will depend strongly on the structure of the boundary region and on the transport processes which determine that structure. In addition, the energy requirements for sustaining the relativistic ring are an extremely important consideration in determining whether the bumpy torus concept may be extrapolated to reactor regimes. A number of different physical processes--the characteristics of microwave power deposition, electron drag losses, synchrotron radiation losses, classical and anomalous

scattering losses--are all important and all are dependent to some degree on the plasma conditions in the boundary region. A striking example of this is seen in the recent experiments in EBT-S⁴ in which the application of fast wave ion cyclotron heating had the completely unexpected result of dramatically increasing the temperature of the relativistic electron rings. It is difficult to imagine a mechanism whereby the ICH is directly coupled to the rings. It is much more likely that the ICH is coupling to the core plasma as a surface wave which changes the plasma characteristics of the boundary region and only indirectly (although importantly) the ring properties themselves.

A first attempt at a theory of EBT radial structure which allowed for fundamentally different transport processes in the core and edge regions was that of Chiu and Guest.⁵ These authors used a two-point model for electron transport in which it was assumed that the transport in the core plasma was neoclassical while the transport in the edge plasma was assumed anomalous and modeled phenomenologically by Bohm transport. In the past year we have conducted a systematic analysis of the microinstability properties of the EBT boundary layer^{6,7,8} and have identified specific high frequency drift instabilities--the lower-hybrid-drift instability and the drift-cyclotron instability--as likely causes of turbulence and anomalous transport in the EBT boundary. Interestingly, neither the lower-hybrid-drift mode nor the drift-cyclotron mode would be unstable in the core plasma since the core plasma gradients and attendant diamagnetic drift velocities are far below the drift velocity thresholds for excitation of these modes. The identification of specific instabilities responsible for turbulence in the boundary region allows us to make substantial improvements on the pioneering effort of Chiu and Guest. In particular, we may utilize preexisting, well developed quasilinear and nonlinear theories of the lower-hybrid-drift mode to construct anomalous transport

coefficients which are much more sensitive to the parametric details of the boundary region than is the Bohm transport coefficient. Interestingly, we find that the magnitudes of the drift wave transport coefficients are much smaller than analogous Bohm transport coefficients.

In various sections below we summarize

- II. Relevant properties of the EBT boundary layer
- III. A local linear stability theory for a parametric environment typical of the EBT boundary layer
- IV. The properties of the localized lower-hybrid-drift instability in the EBT boundary
- V. The properties of the drift cyclotron instability in the EBT boundary
- VI. The nonlocal theory of the lower-hybrid-drift instability
- VII. The viscosity of the boundary layer
- VIII. Transport in the boundary due to drift wave turbulence
- IX. Boundary layer size determined from microturbulence, compared with sizes from classical and Bohm diffusion

These summaries are based on detailed reports which have been submitted previously to the DOE and will be referenced as appropriate.

In brief,

A. STABILITY RESULTS

We have examined the microstability properties of the EBT boundary layer. We discussed the physical properties of the boundary layer and determined reasonable ranges of values for the plasma parameters that characterize

microinstabilities. Three particular features of the boundary layer--strong plasma gradients, multispecie plasma, and the existence of a diamagnetic well--were conspicuous for their possible effect on microinstabilities. We developed an equilibrium model which includes many of the features of EBT boundary layer. On the basis of the strong plasma gradients, we identified high frequency drift waves as potentially operative instabilities and derived a local dispersion equation for these modes. One interesting result was the fact that the relativistic electrons, because of their extreme temperature, have no direct effect on the dispersive properties of high frequency drift waves. These electrons do, however, have a strong indirect effect by producing magnetic field gradients which, because of the response of the core plasma, have a stabilizing influence on these drift modes.

We analyzed the dispersive properties and stability characteristics of two particular high frequency drift waves: the lower-hybrid-drift instability and the drift-cyclotron instability. The lower-hybrid-drift was found to be unstable, with substantial growth rate, over a range of the density gradient scale lengths, densities, and temperature ratios characteristic of the EBT boundary layers. However, magnetic field gradients and ion and electron temperature gradients of the magnitudes typical of the boundary layer all act to reduce the growth rate of this instability. A cursory examination of the effect (slightly destabilizing) of a two-temperature ion distribution was also made.

The drift-cyclotron instability was also found to be unstable for weaker values of the plasma density gradient. We discussed the effects that density gradient and temperature ratio would have on the unstable spectrum of this mode. The effects of temperature gradients on the drift-cyclotron instability were also analyzed: ion temperature gradient in the same direction as

the density gradient being strongly stabilizing, while a similar electron temperature gradient only spreads and distorts the unstable spectrum.

This analysis must be viewed as a first step in understanding the microstability properties of the EBT boundary layer; there has been no other published work addressed specifically to this topic. Our intent has been to determine the general features of this complex stability problem; not to completely solve it. It is our hope that the results presented here aid in the identification of fluctuations in the EBT boundary layer. A laser scattering experiment designed to observe such fluctuations is expected to be performed.⁹

The development of a self-consistent theory of the structure of the EBT boundary layer, a theory important in making projections about the power balance and gross stability properties of future EBT devices, requires substantial further research of a supportive nature. Such a theory will probably take the form of a hybrid fluid-kinetic numerical model in which the gross features of the boundary are described by fluid theory, while the transport properties are described by a combination of anomalous and classical transport processes. The anomalous transport processes, in particular, are likely to be of paramount importance and much better linear and nonlinear theories of microinstabilities and microturbulence are required. From a stability theory point of view, it is important to relax the limitations of the local theory presented here and consider the global features of the boundary layer with a nonlocal theory.

B. NONLOCAL ANALYSIS

We have examined the nonlocal stability properties of the lower-hybrid-drift instability in model plasma profiles characteristic of the EBT boundary layer. We have reviewed the basic properties of this instability from the standpoint of local theory and have gone into some detail on how the nonlocal dispersion equation is obtained. We have constructed a set of model EBT plasma profiles and have systematically examined the structure of the lower-hybrid-drift eigenmodes and eigenfrequencies in these profiles, taking care to isolate specific geometrical and physical effects.

Basically, we find that the lower-hybrid-drift instability is operative in a plasma environment characteristic of the EBT boundary layer. The unstable eigenmode may occupy a region of order several ρ_i in the boundary. The effect of the diamagnetic well, which seemed a promising device for suppressing this instability when viewed from the standpoint of local theory, was found to be much less pronounced when the effect was more correctly assessed with a nonlocal theory. The specific character of the instability was found, as might be expected, to depend strongly on the specific character of the equilibrium profiles. Extensive parametric surveys for different equilibrium profiles were not discussed, but the mathematical tools for performing such surveys were developed.

C. APPLICATION TO BOUNDARY STRUCTURE

The basic result of the contract work is that we now believe that the observed character of EBT plasma profiles--flat at the center, sharp at the edge--may be explained by the occurrence of different dominant transport processes in the two regions and present results to support this view. We have argued that high frequency drift instabilities are not only likely to exist

in the EBT boundary region, but may be expected to produce substantial amounts of anomalous transport. We have shown that while the magnitude of the transport coefficients associated with neoclassical transport in the central region and anomalous transport in the boundary are comparable, the mechanisms by which the transport occurs is quite different. This may have the consequence of resulting in substantially different plasma structures in the core and the boundary.

We have also suggested that the structure of the boundary region, although not directly observable at this time, may have important implications for the successful scaling of EBT devices to reactors and therefore is a subject worthy of further examination. We have focused on the essential elements of the different transport processes and have constructed a rudimentary model which combines these processes and predicts global plasma structure. We have found that the boundary width predicted by this model is consistent with available observations and have calculated how the boundary width should scale with important experimentally variable parameters.

There are many approximations, both explicit and implicit, which have been necessary in order to obtain a tractable form for analysis. While there are some obvious ways in which this procedure could be substantially improved, some of the approximations invoked are difficult to relax and their impact on answers is hard to assess. The utility of this model lies in the fact that although it appears mathematically cumbersome, it is actually quite tractable and various cases can be rapidly examined computationally with interactive graphics. The future, no doubt, belongs to the more sophisticated transport codes in which both neoclassical and anomalous transport coefficients will have to be used. Such codes have been successfully used in the study of high density

pinch plasmas for almost a decade. However, an essential ingredient of those analyses was the close comparison of transport codes and detailed experimental investigations of the plasma structure. It was only in this way that the anomalous transport processes could be "fine tuned." Until such time as such detailed comparisons are feasible for EBT plasmas, it is still of some utility to speculate on the plasma structure using such crude, but highly tractable, transport models as the one discussed in the present work.

II. PROPERTIES OF THE EBT BOUNDARY LAYER

In this section we discuss the physical properties of the EBT boundary layer with emphasis on identifying those properties which affect microinstabilities.

The traditional picture of the radial structure of the EBT plasma is that it consists of a toroidally confined plasma, surrounded by a relativistic electron ring, which is in turn surrounded by a tenuous, poorly confined surface plasma.² Our primary concern here is the boundary layer between the toroidally confined plasma and the tenuous surface plasma. No direct observations of the structure of this boundary layer have been made, but some of its properties may be deduced indirectly. For example, equilibrium considerations would suggest that the width of this boundary layer is of the same order as the width of the relativistic electron ring. The width of the ring, Δ_r , has been measured, primarily with skimmer probes, and found to be in the range, $\rho_{eH} < \Delta_r \lesssim 5 \rho_{eH}$, where ρ_{eH} is the Larmor orbit of the ring electrons.¹⁰ This indication of sharp gradients at the EBT boundary is substantiated by 1-D transport codes which model the radial structure of EBT.^{11,12} In these calculations, the use of neoclassical transport coefficients for the central plasma region leads to the prediction of flat profiles in the center bounded by sharp edge gradients. Direct (Thompson scattering) measurements of electron density and temperature have recently been made in the EBT-S plasma.¹ These measurements also indicate that the central plasma profiles are quite flat. Unfortunately, the density of the EBT-S core plasma is itself so low that the photon statistics of the Thompson scattering are only marginal and hence this diagnostic cannot be used to directly analyze the structure of the

lower density boundary layer. The preponderance of evidence points to a scale length, L , for the variation of density and temperature in the EBT boundary to lie in the range of $1 \text{ cm} \lesssim L \lesssim 5 \text{ cm}$.

In Figure 1 we illustrate some generic features of the radial profiles of an EBT plasma at the midplane between two adjacent mirror field coils. Diagrams of this type have been discussed previously.¹³ The toroidal magnetic field (Figure 1a) is seen to have a weak fall off with radius and a characteristic dip at the edge. This dip is a diamagnetic well dug by the relativistic electron ring. In addition to the toroidal magnetic field there is also a radial ambipolar electric field. The core plasma density (Figure 1b) is relatively flat inside the diamagnetic well and falls off rapidly to a much smaller level outside the diamagnetic well. The density of the ring electrons is much lower than the density of core plasma ($n_r/n_c \sim 0.10$), and is maximum at the center of the diamagnetic well. In the interior of the device, the core electrons are typically several times hotter than the core ions (Figure 1c). The exterior of the device is not extensively diagnosed but the temperatures are thought to be much cooler than the interior temperatures.² The profiles illustrated are a plausible approximation to the, as yet, unobserved experimental profiles. There are, of course, other physical effects which could add finer scale structure. For example, the region located near the bottom of the diamagnetic well could be especially susceptible to localized, magnetic curvature driven modes which might tend to flatten the density profile.¹⁴ Such a localized flattening, however, would steepen the density gradient in other parts of the boundary layer.

We may identify three features of the EBT boundary layer which would be expected to have strong effects on the microstability properties of the EBT boundary layer. The sharp plasma gradients suggest that a variety of

Figure 1. Generic features of the radial profiles of an EBT plasma.

drift type microinstabilities may be present. The multispecies character of the boundary layer plasma (core electrons, core ions, and ring electrons) introduces the possibility of various plasma component/plasma mode couplings with instability consequences. The diamagnetic well implies complex particle drifts with possibilities of stabilizing or destabilizing particle-wave resonances. In addition to these three features, there are two others of possible relevance: a background of high frequency radiation and the presence of a separatrix. The high frequency radiation is due to the intense electron-cyclotron resonance heating used to create the relativistic electron ring. It suggests both modifications of the electron response and the possibility of parametric processes. The separatrix is the boundary between field lines which are toroidally connected and those which intersect mirror field coils. The boundary layer in current EBT experiments spans the separatrix region. The direct loss of particles along field lines could lead to highly non-Maxwellian loss-cone type distributions with many microstability consequences. These two features are difficult to analyze on the basis of existing experimental information about the EBT boundary layer.

On the basis of the density gradients alone, we would expect a variety of drift modes.¹⁵ Roughly speaking, a criterion for which particular density gradient drift mode is most important can be based on how strong the density gradient is. If ρ_i is the ion Larmor radius and L_N is the density scale length, then we would expect the following modes to dominate in the following respective density gradient regimes: $\rho_i/L_N < (m_e/m_i)^{1/2}$, universal mode;¹⁶ $(m_e/m_i)^{1/2} < \rho_i/L_N < (m_e/m_i)^{1/4}$, drift-cyclotron mode;¹⁷ $(m_e/m_i)^{1/4} < \rho_i/L_N$, lower-hybrid-drift mode.^{18,19} This rule is not exact: for example, the universal mode can be unstable even in extremely inhomogeneous plasmas.^{20,21}

Plasma activity in the form of fluctuations has been observed in EBT in the frequency ranges characteristic of all three of these drift modes. The most extensive measurements have been made by Komori²² in the low frequency regime ($f < 500$ kHz). Less extensive measurements in the ion-cyclotron and higher frequency range have also been made and also give some indication of fluctuation activity.²³ As mentioned previously, preparations are underway to perform laser scattering experiments to observe high frequency fluctuations in EBT.²⁴ One of the purposes of this analysis is to provide information on the observable characteristics of high frequency drift modes in EBT.

Some of the plasma parameters characteristic of current and proposed EBT experiments are shown in Table I.²⁵ On the basis of these measured parameters, we would expect some of the dimensionless parameters which characterized drift waves to fall in the following ranges: $0.1 \lesssim \omega_{pe}/\omega_{ce} \lesssim 1.0$, where $\omega_{pe} = (4\pi ne^2/m_e)^{1/2}$, $\omega_{ce} = eB/m_e c$, $T_{ec}/T_i \gtrsim 5$, $T_{er}/T_{ec} \sim 100$, where the subscripts ec, er, and i refer to core electrons, ring electrons and ions; $n_{er}/n_{ei} \gtrsim 0.1$; $0.1 \lesssim \rho_i/L \lesssim 1.0$, where $\rho_i = (2T_i/m_i)^{1/2}/\omega_{ci}$ and L is a typical scale length for variation of density, temperature or magnetic field. Very recent measurements²⁶ have complicated the situation concerning the measured ion temperature. It now appears that the ion distribution may be best modeled by a two- or three-temperature Maxwellian, with the bulk of ions being cool (~ 15 eV), but with substantial higher energy tails.

In what follows we examine linear, nonlinear, local, and nonlocal properties of high frequency drift modes for parameters relevant to EBT, deduce a boundary layer structure consistent with these modes, and compare the result both with experiment and with other models.

Table I. Nominal parametric values for some EBT devices.

	EBT-I	EBT-S	EBT-P
Magnetic Field (kg)	6.4	10	21
Core Density (10^{13} cm^{-3})	0.1	0.2	1.0
Core Electron Temperature (keV)	0.25	0.50	1.0
Ion Temperature (keV)	0.08	0.10	0.40
Ring Density (10^{13} cm^{-3})	0.03	0.04	0.2
Ring Temperature (keV)	150	500	500

III. LINEAR STABILITY THEORY IN THE EBT BOUNDARY

As a lowest order model which contains elements of the EBT boundary layer geometry essential to a drift-wave theory, we choose a slab-model plasma with linear gradients in density and magnetic field. Since the high frequency drift modes which we shall consider are localized to the boundary layer region, it is unlikely that either toroidal or cylindrical effects will greatly alter their dispersive character. We illustrate this geometry and the various particle drifts in Figure 2. We assume a strong magnetic field in the z (toroidal) direction with a weak linear variation in the x (radial) direction. We also assume a constant ambipolar electric field in the x direction. For the time being we include the effects of toroidal magnetic field curvature by modeling it as an artificial gravitational force in the radial direction. The relevant particle drifts are the diamagnetic drift, $v_{dj} = \frac{1}{2}(v_j^2/2\omega_{cj}) (\nabla n_j/n_j)$ with $v_j^2 = 2T_j/m_j$, $\omega_{cj} = q_j B/m_j c$, the gradient B drift, $v_{\nabla B j} = \frac{1}{2}(v_j^2/2\omega_{cj}) (\nabla B/B)$, the curvature B drift, $v_{gj} = g_j/\omega_{cj}$ (where $g_j = v_{\parallel j}^2/L_c$ is the artificial gravity), and the ExB drift in the ambipolar field, $v_E = cE_0/B$. From the figure, it is clear that we have chosen the reference point of our slab model to fall in a region of the boundary layer where $\nabla n \nabla B < 0$ (i.e., on the outside of the diamagnetic well).

The constants of particle motion in this slab geometry are the perpendicular energy, $\epsilon_{\perp j} = (\frac{1}{2}) m_j v_{\perp}^2 - [m_j g_j - q_j E] x$, the axial canonical momentum, $p_{\parallel j} = m_j v_{\parallel}$, and the canonical momentum, $p_{yj} = m_j (v_y + x \omega_{cj})$. For the core electrons (typically $\rho_{ec}/L \ll 1$), we choose the equilibrium distribution to have the form

Figure 2. Slab model of boundary layer illustrating particle drifts. Region depicted has $\nabla n \nabla B < 0$.

$$f_0(x, \underline{v}) \sim \exp\left[-\frac{\epsilon_{\perp}}{T} - \frac{mv_{\parallel}^2}{2T}\right] [1 + \epsilon p_y] \quad (1)$$

For the more strongly inhomogeneous ions and ring electrons, we choose the form

$$f_0(x, \underline{v}) \sim \exp\left[-(\epsilon_{\perp j} + mv_{\parallel}^2/2 - \alpha p_y)/T\right] \quad (2)$$

From the v_y moments of the normalized forms of the distributions, we have

$$\frac{v_{Di}}{v_i} = \left(\frac{1}{2} \frac{\rho_i}{L_N} - \frac{\rho_i}{L_C}\right) + \frac{v_E}{v_i} \quad (3)$$

$$\frac{v_{Dec}}{v_i} = -\frac{T_{ec}}{T_i} \left(\frac{1}{2} \frac{\rho_i}{L_N} - \frac{\rho_i}{L_C}\right) + \frac{v_E}{v_i} \quad (4)$$

$$\frac{v_{Der}}{v_i} = -\frac{T_{er}}{T_i} \left(\frac{1}{2} \frac{\rho_i}{L_N} - \frac{\rho_i}{L_C}\right) + \frac{v_E}{v_i} \quad (5)$$

where $L_N = (d \ln n/dx)^{-1}$ and L_C is the radius of curvature of a field line passing through $r = r_0$. From Ampere's law, we have

$$\frac{\rho_i}{L_B} = \frac{\rho_i}{L_{B0}} - \frac{\beta_{er}}{\Delta} \left(\frac{T_i}{T_{er}}\right) \left[\frac{v_{Di}}{v_i} + (1 - \Delta) \frac{v_{Dec}}{v_i} + \Delta \frac{v_{Der}}{v_i}\right] \quad (6)$$

where $\beta_{er} = 8\pi n T_{er}/B_0^2$, Δ is the relative fraction of ring to core electrons, $L_B = (d \ln B/dx)^{-1}$ and L_{B0} is the scale length for variation of B in the absence of the electron ring.

The nominal values of EBT boundary layer gradient scale lengths, $L_N \sim 1$ cm, $L_B \approx 2$ cm, $L_C \approx 20$ cm, $L_{Bo} \approx 40$ cm suggests the ordering,

$$L_N \lesssim L_B \ll L_C \lesssim L_{Bo} \quad (7)$$

and the nominal temperatures, $T_{er} \sim 500$ keV, $T_{ec} \sim 500$ eV, $T_i \sim 100$ eV, suggest the ordering

$$T_{er} \gg T_{ec} \gtrsim T_i \quad (8)$$

In this case we may approximate Eq. (6) by

$$\frac{\rho_i}{L_B} \approx -\frac{1}{2} \beta_{er} \frac{\rho_i}{L_N} \quad (9)$$

and $v_{Di}/v_i \approx \rho_i/2L_N$, $v_{Dec}/v_i = - (T_{ec}/T_i) (v_{Di}/v_i)$, $v_{Der}/v_i = - (T_{er}/T_i) (v_{Di}/v_i)$. This expression simply means that the dominant variation of the field is caused by the relativistic electron ring.

In addition to the assumptions implied by Eqs. (7) and (8), we have also neglected the effect of the ambipolar electric field. Its typical value, as deduced from the measured potential well depth $\phi \sim 400$ volts, is such that the relative contribution of the v_E term to the general drifts in Eqs. (3)-(5) is small. In other words, the constant ambipolar electric field makes a negligible contribution to the plasma equilibrium; the only effect of the constant ambipolar electric field on the stability analysis is to Doppler shift the drift-wave frequency.

The general dispersion equation for flute-like ($\underline{k} \cdot \underline{B} = 0$), electromagnetic oscillations is

$$\left| \begin{aligned} & \left(\frac{ck_y}{\omega} \right)^2 - \left[1 + \sum_j \frac{4\pi i \sigma_{xx}^j}{\omega} \right] - \sum_j \frac{4\pi i \sigma_{xy}^j}{\omega} \\ & - \sum_j \frac{4\pi i \sigma_{yx}^j}{\omega} - \left[1 + \sum_j \frac{4\pi i \sigma_{yy}^j}{\omega} \right] \end{aligned} \right| = 0 \quad (10)$$

where the σ_{ij} denote elements of the conductivity tensor and the summation is over the three species, core ions, core electrons, and ring electrons. The space-time dependence $\exp[-i\omega t + ik_y y]$ has been assumed.

The electron response (core and ring) to perturbations about the equilibrium in the frequency regime $\omega \ll \omega_{ce}$ is

$$\frac{4\pi i}{\omega} \sigma_{xx}^e = -2 \left(\frac{\omega_{pe}}{\omega} \right)^2 \left(1 - \frac{k_y v_{De}}{\omega} \right) F_{00}(a_e, \zeta_e) \quad (11)$$

$$\frac{4\pi i}{\omega} \sigma_{xx}^e = -\frac{4\pi i}{\omega} \sigma_{xx}^e = 2i \left(\frac{\omega_{pe}}{\omega} \right)^2 \left(\frac{\omega}{k_y v_e} \right) \left(1 - \frac{k_y v_{De}}{\omega} \right) F_{10}(a_e, \zeta_e) \quad (12)$$

$$\frac{4\pi i}{\omega} \sigma_{xx}^e = -2 \left(\frac{\omega_{pe}}{\omega} \right)^2 \left(\frac{\omega}{k_y v_e} \right)^2 \left(1 - \frac{k_y v_{De}}{\omega} \right) F_{10}(a_e, \zeta_e) \quad (13)$$

where the integrals

$$F_{\begin{Bmatrix} 00 \\ 10 \\ 11 \end{Bmatrix}}(a_e, \zeta_e) = \int_0^\infty dV_1^2 e^{-V_1^2} \left[\frac{\zeta_e}{\zeta_e - V_1^2} \right] \left\{ \begin{aligned} & J_0^2(a_e V_1) \\ & V_1 J_1(a_e V_1) J_0(a_e V_1) \\ & V_1^2 J_1^2(a_e V_1) \end{aligned} \right\} \quad (14)$$

allow for the effect of ∇B resonances of the wave with electrons. The various quantities used in Eqs. (11) through (14) are $\omega_{pe}^2 = 4\pi n e^2 / m_e$, $v_e = (2 T_e / m_e)^{1/2}$, $a_e = k_y \rho_e$, $\tau_e = \omega / k_y v_{Be}$. The subscripts appropriate to core and ring electrons have been suppressed in Eqs. (11) through (14) but, of course, must be included when these expressions are used in the dispersion equation. We omit details of the derivation of these formulae since they are well documented elsewhere.²⁷ The functions, $F(a_e, \tau_e)$ become the familiar Bessel functions of imaginary argument for $\nabla B \rightarrow 0$ ($\tau_e \rightarrow \infty$).

The form of the ion response differs, depending on whether we consider the frequency regime $\omega \simeq (\omega_{ce} \omega_{ci})^{1/2}$, typical of the lower-hybrid-drift instability, or the frequency regime $\omega \simeq \ell \omega_{ci}$, typical of the drift-cyclotron instability. In either case, if we restrict our attention to the short wavelength regime, $k_y \rho_{ec} \sim 1$, which characterizes maximum growth for both of these modes, then the only significant element of the dielectric tensor is the $4\pi i \sigma_{yy}^i / \omega$ element.

In the lower-hybrid frequency regime, expected when plasma gradients are strong, $(\rho_i / L_N) \gtrsim (m_e / m_i)^{1/2}$, we have

$$\frac{4\pi i}{\omega} \sigma_{yy}^i = (k_y \lambda_{Di})^{-2} \left[1 + \left(\frac{\omega - k_y v_{Di}}{k_y v_i} \right) Z \left(\frac{\omega - k_y v_{Di}}{k_y v_i} \right) \right] \quad (15)$$

In the drift-cyclotron frequency regime, expected for weaker gradients, $(m_e / m_i)^{1/2} < (\rho_i / L_N) < (m_e / m_i)^{1/2}$, we have ($\omega \simeq \ell \omega_{ci}$)

$$\frac{4\pi i}{\omega} \sigma_{yy}^i = (k_y \lambda_{Di})^{-2} \left[1 - \sum_{\ell} \left(\frac{\omega - k_y v_{Di}}{k_y v_i} \right) I_{\ell}(b_i) e^{-b_i} \right] \quad (16)$$

Forms for the ion response may be written which permit the continuous transition from one of these frequency regimes to the other.^{28,29}

If the relativistic electron ring contribution to the dispersion equation were not present, the lower-hybrid-drift instability or the drift-cyclotron instability would be present for gradients scales and plasma conditions typical of the EBT boundary layer.^{17-19,27-29} The important question arises: What is the effect (stabilizing or destabilizing) of the small fraction ($\sim 10\%$) of relativistic electrons on these modes? To answer this question we explicitly write the yy element of the dielectric tensor,

$$\begin{aligned} \sum_j \frac{4\pi i}{\omega} \sigma_{yy}^j &= 2 (1 - \Delta) \left(\frac{\omega_{pe}}{\omega} \right)^2 \left(\frac{\omega}{k_y v_{ec}} \right)^2 \left(1 - \frac{k_y v_{Dec}}{\omega} \right) F_{00}(a_{ec}, \zeta_{ec}) \\ &+ 2\Delta \left(\frac{\omega_{pe}}{\omega} \right)^2 \left(\frac{\omega}{k_y v_{er}} \right)^2 \left(1 - \frac{k_y v_{Der}}{\omega} \right) F_{00}(a_{er}, \zeta_{er}) \\ &+ \frac{4\pi i}{\omega} \sigma_{yy}^i, \end{aligned} \quad (17)$$

and compare the relative magnitudes of the two electron contributions,

$$R_{yy} = \left| \left(\frac{\Delta}{1-\Delta} \right) \left(\frac{T_{ec}}{T_{er}} \right) \left[\frac{\left(1 - \frac{k_y v_{Der}}{\omega} \right)}{\left(1 - \frac{k_y v_{Dec}}{\omega} \right)} \right] \frac{F_{00}(a_{er}, \zeta_{er})}{F_{00}(a_{ec}, \zeta_{ec})} \right| \quad (18)$$

For both the lower-hybrid-drift mode and the drift-cyclotron mode, the frequency is approximated by

$$\omega \sim k_y v_{di} \lesssim k_y v_{dec} \ll k_y v_{der} \quad (19)$$

and the wavelength (at maximum growth) by

$$k_{y^0} \rho_{ec} \sim 1 \quad (20)$$

Using these relations Eq. (18) reduces to

$$R_{yy} \sim \Delta \left(\frac{T_{ec}}{T_{er}} \right) \left| \frac{F_{00}(a_{er}, \zeta_{er})/\zeta_{er}}{F_{00}(a_{ec}, \zeta_{ec})/\zeta_{ec}} \right| \quad (21)$$

To proceed further, we must consider properties of the F-integrals: we explicitly examine

$$F_{00}(a, \zeta)/\zeta = \int_0^\infty dx e^{-x} \frac{J_0^2(a\sqrt{x})}{\zeta - x} \quad (22)$$

In terms of the parameters a, ζ which characterize this integral, the conditions (19) and (20) imply

$$a_{ec} \sim 1, \quad \zeta_{ec} \sim 1 \quad (23)$$

$$a_{er} \gg 1, \quad \zeta_{er} \ll 1 \quad (24)$$

We note immediately that for $\zeta \approx 1$, the resonant denominator contributes only a complex contribution of the same magnitude as that of the integrand. This can be rigorously established by making the bounding argument

$$\left| \int_0^\infty dx \frac{e^{-x}}{\zeta - x} \right| < \left| \int_0^\infty dx \frac{e^{-x} J_0^2(a\sqrt{x})}{\zeta - x} \right| \quad (25)$$

and relating the term on the left-hand side to the exponential integral.

Thus, we have

$$\left| F_{00}(a, \zeta) \right| \sim \int_0^{\infty} dx e^{-x} J_0^2(a\sqrt{x}) = I_0\left(\frac{a^2}{2}\right) e^{-\left(\frac{a^2}{2}\right)} \quad (26)$$

Equation (21) then reduces to

$$R_{yy} \approx \Delta \left(\frac{T_{ec}}{T_{er}} \right) \frac{\left(\frac{1}{a_{er}} \right)}{I_0\left(\frac{a_{ec}^2}{2}\right) e^{-\left(\frac{a_{ec}^2}{2}\right)}}, \quad (27)$$

where we have used $I_0\left(\frac{a^2}{2}\right) e^{-\frac{a^2}{2}} \simeq \frac{1}{a\sqrt{\pi}}$; $a \gg 1$, to simplify the $F_{00}(a_{er}, \zeta_{er})$ term. Since $I_0\left(\frac{a^2}{2}\right) e^{-\frac{a^2}{2}} \sim O(1)$ for $a \sim O(1)$ we finally have

$$R_{yy} \sim \Delta \left(\frac{T_{ec}}{T_{er}} \right) \left(\frac{1}{a_{er}} \right) \ll 1 \quad (28)$$

In an analogous fashion we can also establish the orderings

$$R_{xy} \sim \Delta \left(\frac{T_{ec}}{T_{er}} \right)^{\frac{1}{2}} \left(\frac{1}{a_{er}} \right)$$

$$R_{xx} \sim \Delta \left(\frac{1}{a_{er}} \right) \quad (29)$$

for the relative magnitudes of the electron contributions of the other elements of the dielectric tensor. In all cases we see that the relative contribution of the ring electrons is small by at least (Δ/a_{er}) when compared with the core electron contribution.

In addition, since the beta of the core plasma is very small in current experiments, the electromagnetic contributions of the core electrons may easily be shown to be small with respect to the electrostatic contribution.²⁷ Thus, the dispersion equation for high frequency drift waves in a parametric environment typical of the EBT boundary layer is well described by the electrostatic part of the general dispersion equation (10) with the relativistic electrons making a negligible contribution. Explicitly, the dispersion equation is

$$1 + 2 \left(\frac{\omega_{pe}}{\omega} \right)^2 \left(\frac{\omega}{k_y v_{ec}} \right)^2 \left(1 - \frac{k_y v_{Dec}}{\omega} \right) F_{00}(a_{ec}, \zeta_{ec}) + \frac{4\pi i}{\omega} \sigma_{yy}^i \approx 0 \quad (30)$$

Although the relativistic electrons do not make a significant direct contribution to the dispersion equation for high frequency drift waves, they make a substantial indirect contribution. From the equilibrium condition (9) and, indeed, from the existence of the diamagnetic well in the toroidal field which is dug by the relativistic electrons, we see that the relativistic electrons lead to strong magnetic field gradients. The field gradients result in substantial gradient B drifts of the core electrons. The resonance of the lower-hybrid-drift mode with the core electron gradient B drift can be strongly stabilizing on this mode in regions where $\nabla B \nabla n < 0$.²⁷ The same is true for the drift-cyclotron mode.³⁰

IV. PROPERTIES OF THE LOCALIZED LOWER-HYBRID-DRIFT (LHD) INSTABILITY IN AN EBT BOUNDARY LAYER

A number of analyses^{18,19,27,28,31-37} have been made of the dispersion and stability properties of the LHD mode. Few, if any, of these calculations are applicable to the parameter range (low density, hot electrons) characteristic of the EBT boundary. Therefore, we proceed with a numerical analysis of Eq. (30) extended to include temperature gradients. The results are shown in Figures 3-9, which are self explanatory. These results have been discussed in depth in our various reports (e.g., "Microstability Properties of the EBT Boundary Layer," Phys. Fluids, in press).

Figure 3. Effect of plasma density on dispersive properties of the lower-hybrid-drift instability.

$$\omega_n = \omega_{pi} \gamma [1 + (\omega_{pe}/\omega_{ce})^2]^{\frac{1}{2}}$$

Figure 4. Effect of temperature ratio on dispersive properties of the lower-hybrid-drift instability.

Figure 5. Effect of plasma density and density gradient scale length on the maximal growing lower-hybrid-drift instability (and associated real frequency).

Figure 6. Effect of relativistic electron ring beta (and associated magnetic field gradient) on maximum lower-hybrid-drift growth rate.

Figure 7. Effect of ion temperature gradient on the maximum lower-hybrid-drift growth rate.

Figure 8. Effect of electron temperature gradient on the unstable lower-hybrid-drift spectrum.

Figure 9. Effect of two temperature ion distribution function on the maximum lower-hybrid-drift growth rate.

V. PROPERTIES OF THE LOCALIZED DRIFT ION CYCLOTRON INSTABILITY IN AN EBT BOUNDARY LAYER.

The drift cyclotron wave has also been the subject of previous analyses²⁹ in other parameter ranges than the EBT boundary. As in Section IV, we solve numerically the dispersion relation, Eq. (30), modified to include temperature gradients. The results are shown in Figures 10-13, discussed in detail in our reports.

Figure 10. Effect of density gradient scale length on the dispersive properties of the drift-cyclotron instability. Temperature ratio is $T_{ec}/T_i = 1$.

Figure 11. Effect of density gradient scale length on the dispersive properties of the drift-cyclotron instability. Temperature ratio is $T_{ec}/T_i = 4$.

Figure 12. Effect of ion temperature gradient on the drift-cyclotron instability.

Figure 13. Effect of electron temperature gradient on the drift-cyclotron instability.

VI. NONLOCAL ANALYSIS OF THE LOWER-HYBRID-DRIFT INSTABILITY

In the local analysis presented above, consideration was restricted to the region of the boundary layer where the stabilizing ∇B resonance would be greatest, i.e., where $\nabla B \nabla n$ was most negative. In reality, drift modes have a radial extent in the boundary layer and see different values of ∇B . Indeed, they are resident in the diamagnetic well. To obtain a more realistic estimate of the effect of this diamagnetic well in drift modes, we must use nonlocal theory.

By standard methods we derive the nonlocal dispersion equation,

$$\begin{aligned} & \left\{ - (k_y \lambda_{Di})^2 + \frac{\zeta_i}{2} [-2 \zeta_i + (1 - 2\zeta_i^2) Z(\zeta_i)] \right. \\ & \quad \left. - \frac{1}{2} \left(\frac{T_i}{T_{ec}} \right) \left(\frac{\omega - k_y v_{Dec}}{\omega} \right) a_{ec} \frac{\partial F_{00}}{\partial a_{ec}} \right\} \left(\frac{1}{k_y^2} \frac{\partial^2 \phi}{\partial x^2} \right) \\ & + \left\{ (k_y \lambda_{Di})^2 + [1 + \zeta_i Z(\zeta_i)] \right. \\ & \quad \left. + \left(\frac{T_i}{T_{ec}} \right) \left[1 + \left(\frac{\omega - k_y v_{Dec}}{\omega} \right) F_{00} \right] \right\} \phi = 0 \end{aligned} \quad (31)$$

where $\zeta_i = (\omega - k_y v_{Dic})/k_y v_i$. In writing (31) we have taken the ions to be unmagnetized and hence, have restricted our nonlocal analysis to the lower-hybrid-drift mode.

To proceed, we choose some representative profiles for the density and field profiles in the EBT boundary layer,

$$n(x) = n_0 \frac{[1 + \delta + \tanh(x/\lambda)]}{(2 + \delta)} , \quad (32)$$

$$B(x) = B_0 \left[1 - \beta_A \operatorname{sech}^2 \frac{x - x_M}{\lambda} \right] . \quad (33)$$

These profiles and their logarithmic derivatives are illustrated in Figure 14. The maximum in $d \ln n / dx$ suggests a preferred localization point for lower-hybrid-drift eigenmodes within the boundary layer. In Figure 15 we illustrate a sequence of eigenmodes ("radial") within the boundary layer for a reference case of $B = \text{constant}$, $\frac{\omega_{pe}}{\omega_{ce}} = 10$, and $\frac{T_{ec}}{T_i} = 1$. The unit of distance is $x_L = v_i / \omega_{lh} = \rho_{ec} (T_i / T_e)^{1/2}$. It is clear that the eigenmodes span a significant portion of the boundary layer - typically the mode width is $\sim 40 x_L \sim \rho_i$. In addition, the growth rate of the higher order modes is not much different from the fundamental. In Figure 16 we repeat the calculation for a lower density case, $\omega_{pe} / \omega_{ce} = 1$, which is more representative of EBT. Interestingly, we see that the density dependence of λ_D^2 comes in to play in localizing the modes in this case with the result that the eigenmodes are located to the inside of $(1/n \, dn/dx)_{\max}$.

In a nonlocal analysis, many of the effects of interest are coupled by the equilibrium requirements and it is often difficult to focus on a single effect. For example, there are two different effects if we introduce a diamagnetic well into the calculation illustrated in Figure 14. First we increase the growth rate because reducing the magnetic field strength increases the diamagnetic current (if $1/n \, \partial n / \partial x$ is held constant). Secondly, we introduce ∇B , the stabilizing effects of which being our focus of attention. Therefore, to isolate this second effect we show the effect of a diamagnetic well

Figure 14. Model profiles of density and magnetic field for the EBT boundary layer.

Figure 15. A sequence of lower-hybrid-drift eigenmodes high density case ($\omega_{pe}/\omega_{ce} = 10$).

Figure 16. A sequence of lower-hybrid-drift eigenmodes
low density case ($\omega_{pe}/\omega_{ci} = 1$).

on the eigenmode illustrated in Figure 14a in two stages. In Figure 17 the well depth is increased but ∇B effects are artificially suppressed. We thus see an increase of the growth rate as β_A increases. In Figure 18 we also include the ∇B effects and see that the eigenmode is distorted (because ∇B effects operate differently depending on the sign of $\nabla B \nabla n$) and that the increase in growth rate is relatively smaller. In Figure 19 we isolate this stabilizing effect by showing the difference between the local theory maximum growth rate (calculated at $\frac{1}{n} \frac{\partial n}{\partial x} \Big|_{\max}$) and the nonlocal maximum growth rates for the cases illustrated in Figure 17 and Figure 18.

In essence, the magnetic field gradients associated with the diamagnetic well in the EBT boundary layer are stabilizing on the lower-hybrid-drift mode but (as might be expected) not as much so as a local analysis would indicate. These results represent a first step toward understanding global profile effects on high frequency drift modes in EBT-like boundary layers. Quantitative results, valid for existing or proposed EBT devices, will require careful parametric surveys of stability properties which we defer to later work. A more detailed presentation of the nonlocal analysis is contained in a separate report.⁸

Figure 17. Effect of diamagnetic well on fundamental lower-hybrid-drift eigenmode (∇B effects artificially suppressed).

Figure 18. Effect of diamagnetic well on fundamental lower-hybrid-drift eigenmode (∇B effects included).

Figure 19. Comparison of local and nonlocal maximum growth rates of lower-hybrid-drift illustrating the stabilizing influence of ∇B effects.

VII. THE VISCOSITY OF THE BOUNDARY LAYER

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The classical theory for viscosity in a boundary layer is worked out in detail,³⁸ and contributes to the diffusion equation through the off diagonal element of the pressure tensor,

$$nm \frac{d\vec{V}}{dt} = ne \left(\vec{E} + \frac{\vec{V} \times \vec{B}}{c} \right) - \nabla p - \nabla \cdot \vec{\pi} \quad (34)$$

where

$$\pi_{ij} \equiv nm \langle (v_i - \vec{V}_i)(v_j - \vec{V}_j) \rangle - p \delta_{ij}$$

Viscosity effects are contained in π . In the absence of a magnetic field, the viscous stress is

$$\nabla \cdot \pi_{\alpha\beta} = -\nabla \cdot \eta_0 \left[\frac{\partial V_\alpha}{\partial X_\beta} + \frac{\partial V_\beta}{\partial X_\alpha} - \frac{2}{3} \delta_{\alpha\beta} \nabla \cdot \vec{V} \right] \quad (35)$$

where $\eta_0 \simeq nT\tau$, with τ the mean free time, is the viscosity coefficient. The magnetic field complicates the situation, both in altering the stress tensor and in altering the flow velocities, V_α , which are then generally of higher order in $1/\omega_c\tau$. A further complication comes in the calculation of the η 's, which in the classical formulae assume that the width of boundary layers is greater than a mean free path $V\tau$. For a bumpy cylinder, the poloidal drift velocity varies radially, giving a viscous stress³⁸

$$\nabla \cdot \pi_{xy} \sim \frac{\partial}{\partial x} \eta_{\perp} \frac{\partial V_y}{\partial x}$$

where

$$\eta_{\perp} \sim \frac{\eta_0}{(\omega_c \tau)^2}$$

$$\eta_0^i = \left(\frac{m_i}{m_e} \right)^{\frac{1}{2}} \eta_0^e$$

$$\eta_0^e = 0.73 \text{ nT} \tau_e$$

$$\tau_e = \frac{3\sqrt{m_e} T_e^{3/2}}{4\sqrt{e} \lambda_e^4 n}$$

It is useful to compare the transport due to classical viscosity with that due to both classical and anomalous resistivity, for typical EBT parameters

$$F_{Vis}/F_{Res} \simeq \frac{m_i}{m_e} \frac{v_{ii}}{v_{ei}} \simeq 40 \quad (36)$$

$$F_{Vis}/F_{Anomalous} \simeq 40 \times 10^{-4} \simeq F_{Vis}/F_{Neoclassical} \quad (37)$$

The neoclassical diffusion is also large compared with viscous effects because of the enhancement factor, $a^4/R^2 \rho_L^2$, and the proportionality to v_{ee} rather than v_{ei} .

VIII. TRANSPORT WITH BOUNDARY LAYER DUE TO DRIFT WAVE TURBULENCE

The theory of neoclassical transport in EBT is well developed. Not only have comprehensive calculations of the transport coefficients been made, but these coefficients have been incorporated into sophisticated transport codes which model the global evolution of EBT plasma profiles. It is not our intent here to assess the status of neoclassical transport in EBT, but rather to simply take some of the well established neoclassical formulae and incorporate them within a theory which focuses on the role of anomalous transport in the boundary region. Some references on neoclassical transport in EBT are Kovrizhnykh,³⁹ Jaeger and Hedrick,⁴⁰ and Hazeltine, et al.⁴¹

The neoclassical (Kovrizhnykh³⁹) coefficient for electron density transport is

$$D_N^{Kov} \approx \frac{1}{24} \left(\frac{\bar{v}_{tor}}{\Omega_{prec}} \right)^2 \nu^{eff} ; \quad \nu^{eff} \ll \Omega_{prec} \quad (38)$$

where $\bar{v}_{tor} = (\bar{v}_e^2 / \omega_{ce}) (1/R_{tor})$; $\Omega_{prec} = (\bar{v}_e^2 / \omega_{ce}) (1/2R_{curv}) - cE_r/B$, $\nu^{eff} = 2\nu_{ei}$, $\bar{v}_e = (2T_e/m_e)^{1/2}$, $\omega_{ce} = eB/m_e c$, and $\nu_{ei} = 4\pi n e^4 \ln \Lambda / m_e^2 \bar{v}_e^3$. Here, R_{tor} is the scale length for the radial fall off in toroidal magnetic field, R_{curv} is the scale length for field curvature, E_r is the ambipolar electric field, and ν^{eff} is the effective collision frequency. The physical interpretation of (38) is straightforward: the electrons precess rapidly in the poloidal direction due to their drift motion in curved magnetic fields caused by the bumpiness of the toroidal field; they execute many transits before suffering a collision which moves them to a different precessing orbit; the approximate distance they move

from one precessing orbit to another depends on their slower vertical drift motion in the nonuniform toroidal field. The rapid precessing motion is also enhanced by the $E \times B$ drift motion due to the ambipolar electric field. Basically, the electrons diffuse by taking large spatial steps (~ 1 cm) at the relatively low frequency ($\sim 10^3 \text{ s}^{-1}$) at which classical collisions of electrons with ions and other electrons occur. We illustrate this process in Figure 20.

The physical picture for anomalous diffusion is quite different and is roughly described by the following scenario. Associated with the strong plasma gradients in the EBT boundary, there exist strong diamagnetic plasma currents which flow in the poloidal direction. High frequency drift waves with rapid growth rates ($\gamma \sim \omega > \omega_{ci}$) are excited by the diamagnetic current and propagate in the direction of the ion diamagnetic current (i.e., opposite to the electron flow). For any of several possible reasons, the instability saturates with a typical amplitude of the saturated electrostatic wave being $e\phi/T_i \sim 0.1$. The saturated wave spectrum is turbulent or hashy and has typical wavelengths on the order of $\rho_e = \bar{v}_e/\omega_{ce}$. The original electron motion is strongly impeded by interactions with the drift wave turbulence and one can speak of an effective collision frequency for electrons. This effective collision frequency is orders of magnitude larger than classical electron collisions ($\nu_{\text{eff}}^{\text{LHD}} \sim 10^7 \text{ s}^{-1}$). On the other hand, the electrons no longer have time to execute the large-excursion orbit which typifies their motion in the relatively collisionless core region. In the boundary region the electron motion is typified by gyration about field lines (the frequency of which is still much faster than the turbulent collision frequency) and so the effective collisions with turbulence scatter electrons from one field line to another yielding an effective diffusive step size of order ρ_e (~ 0.01 cm). This is also shown in Figure 20.

Figure 20. Schematic representations of the neoclassical and anomalous transport processes.

An approximate expression for the diffusion coefficient for density diffusion due to the lower-hybrid-drift instability may be written as

$$D_N^{\text{LHD}} \cong \frac{\beta}{2} \left(\frac{c}{\omega_{pe}} \right)^2 \nu_{\text{eff}}^{\text{LHD}} \approx \rho_e^2 \nu_{\text{eff}}^{\text{LHD}} \quad (39)$$

[see Eq. (41) of Reference 42 and associated discussion for the details of the derivation of this equation]. An approximate upper bound for the effective collision frequency of electrons with lower-hybrid-drift turbulence is

$$\nu_{\text{eff}}^{\text{LHD}} \approx \frac{1}{8} \sqrt{\frac{\pi}{2}} \left(\frac{\rho_i}{L_N} \right)^2 \omega_{\text{LH}} \quad (40)$$

where $\omega_{\text{LH}} = \omega_{pi} / \left(1 + (\omega_{pe}/\omega_{ce})^2 \right)^{\frac{1}{2}}$. It is important to note that the form of $\nu_{\text{eff}}^{\text{LHD}}$ is such as to self limit the narrowness of the boundary region in which the instability exists. By that we mean that the sharper the density gradient, the more virulent the instability and the larger the anomalous diffusion rate. Consequently, the larger the diffusion rate, the greater the tendency of the density profile to flatten. This represents a significant improvement over the use of a phenomenological anomalous diffusion coefficient like that which characterizes Bohm diffusion,

$$D_{\text{Bohm}} = \frac{1}{16} \frac{c T_e}{e B} \quad , \quad (41)$$

where the limiting of the boundary region width is accomplished by the extremely large value of the coefficient.

For parameters typical of EBT experiments, a characteristic value of the anomalous diffusion coefficient is $D_N^{LHD} \sim 10^3 \text{ cm}^2/\text{s}$, some two orders of magnitude less than D_{Bohm} . In fact, the magnitude of D_N^{LHD} is of the same order as D_N^{Neo} although the physical mechanisms for diffusion, as we have discussed, are quite different. Although the magnitude of the diffusion rates in the core and boundary regions are approximately the same, it is still important to treat the regions separately. In steady-state operation, the plasma structure, i.e., the relative magnitudes of density and temperature gradients, depends on the relationship between the coefficients D_N , D_T (density diffusion due to the density gradient and the temperature gradient, respectively), and K_N , K_T (energy diffusion). The relative magnitudes of these coefficients will be completely different for neoclassical transport and anomalous transport.

IX. BOUNDARY LAYER SIZE AND STRUCTURE

We have attempted to assess the influence of LHD turbulence in determining the boundary layer thickness by developing a highly simplified model of the EBT profiles and comparing LHD results with other diffusion effects or models.

The basic equations for electron density and energy transport in an EBT environment are

$$\frac{\partial n}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r\Gamma) = n_e n_o \langle \sigma v \rangle_{ion} \quad (42)$$

$$\frac{\partial}{\partial t} \left(\frac{3}{2} nT \right) + \frac{1}{r} \frac{\partial}{\partial r} (rQ) + e\Gamma E_r = - n_e n_o E_I \langle \sigma v \rangle_{ion} + P_\mu \quad (43)$$

where

$$\Gamma = - D_n \frac{\partial n}{\partial r} - D_T \frac{\partial T}{\partial r} + n \mu_n E_r \quad (44)$$

$$Q = - K_n \frac{\partial n}{\partial r} - K_T \frac{\partial T}{\partial r} + n \mu_T E_r \quad (45)$$

Here we are including only ionizing encounters between plasma particles and neutrals; charge exchange processes are neglected. We also neglect direct energy exchange between electrons and ions, because it is small compared to the dominant energy source which is the microwave power P_μ . We include the energy input from ionization but, in practice, it is also very small. We proceed by specifying the density and temperature profiles to have the "stick" form shown in Figure 21. This form clearly distinguishes a core region and a boundary region. We will take the values of density and temperature at the

Figure 21. "Stick" model for EBT transport.

center and on the edge to be specified empirically and parameterize the profiles by three parameters: r_k , the position of the "knee," and n_k and T_k , the values of density and temperature at the "knee." We shall seek steady state solutions of the transport equations in terms of these three parameters.

Our first two equations are obtained by averaging Eqs. (42) and (43) over the core region

$$\begin{aligned} \langle \dot{n} \rangle_{\text{core}}(r_k, n_k, T_k) = \int_0^{r_k} r dr \left\{ \frac{1}{r} \frac{\partial}{\partial r} (r \Gamma^{\text{Neo}}) \right. \\ \left. - n_e n_o \langle \sigma v \rangle_{\text{ion}} \right\} = 0 \end{aligned} \quad (46)$$

$$\begin{aligned} \langle \dot{T} \rangle_{\text{core}}(r_k, n_k, T_k) = \int_0^{r_k} r dr \left\{ \frac{1}{r} \frac{\partial}{\partial r} (r Q^{\text{Neo}}) + e \Gamma^{\text{Neo}} E_r \right. \\ \left. + n_e n_o E_I \langle \sigma v \rangle_{\text{ion}} - P_\mu \right\} = 0 \end{aligned} \quad (47)$$

For the neoclassical transport coefficients we choose

$$D_n = \frac{v_{\text{tor}}^2}{24} \frac{v_e}{v_e^2 + \Omega_{\text{prec}}^2} \quad (48)$$

$$D_T = \frac{2n}{T} D_n \quad ; \quad K_n = \frac{7}{2} T D_n \quad ; \quad K_T = \frac{101}{2} n D_n \quad (49)$$

$$\mu_n = -\frac{e}{T} D_n \quad ; \quad \mu_T = -\frac{e}{T} K_n \quad (50)$$

with v_{tor} and v_e defined following Eq. (38).

We specify the ambipolar electric field in the same fashion as Chiu and Guest,⁵ assuming

$$-\frac{e \phi_{\text{ambi}}}{T} = K \text{ (constant)} \quad (51)$$

from which

$$E_r = K \frac{\partial T}{\partial r} \quad (52)$$

We choose the value $K = 1$.

The neutral density is specified by

$$n_0(r) = n_0(a) \frac{\cosh \left[\int_0^r \frac{d\hat{r}}{\lambda(\hat{r})} \right]}{\cosh \left[\int_0^a \frac{d\hat{r}}{\lambda(\hat{r})} \right]} \quad (53)$$

where

$$\lambda^{-1}(r) = \frac{n_0 \langle \sigma v \rangle_{\text{ion}}}{v_0} \quad (54)$$

with v_0 the thermal velocity of incoming neutrals. The details of the derivation of Eq. (53) is contained elsewhere [J510-81-059, Appendix I]. Because of the complexity of this neutral model, we numerically perform the required integrations in (46), (47), and (53). In addition, we neglect all second derivatives implied in the expressions for the fluxes.

The third basic equation is obtained by requiring particle balance in the boundary

$$\begin{aligned} \langle \dot{n}_e \rangle_{\text{boun}}(r_k, n_k, T_k) = \int_{r_k}^a r dr \left\{ \frac{1}{r} \frac{\partial}{\partial r} (r \Gamma^{\text{LHD}}) \right. \\ \left. - n_e n_o \langle \sigma v \rangle_{\text{ion}} \right\} = 0 \end{aligned} \quad (55)$$

where

$$\Gamma^{\text{LHD}} \cong - D_n \frac{\partial n}{\partial r} \quad (56)$$

with D_n given by (39) and (40). In this first application of the model, we have taken the simplest meaningful expressions for lower-hybrid-drift wave turbulence. Formulae for the other transport coefficients, D_T and μ_n , may be derived for parameters characteristic of the EBT boundary, but we simply neglect these complicating factors at this stage. Our first goal is to establish that a physically reasonable solution for the boundary region exists. In addition, we do not consider at this time the complication of requiring energy balance in the boundary. This can easily be done but would require a fourth parameter to characterize the stick profiles. The obvious choice would be to take different "knee" radii for the density and temperature profiles.

Generally speaking, the construction of transport coefficients for the boundary is just as complex, perhaps more so, as it is for the core. One point is that the ion physics would enter much more directly in the physics of the drift-wave turbulence. In the present model, the ions only participate through the ambipolar electric field.

To apply the "stick" transport model, we choose a set of parameters typical of EBT-I operation. The choice is dictated by the fact that these parameters are well documented and were chosen as the nominal set on which the JAYCOR and Oak Ridge National Laboratory neoclassical transport codes were benchmarked.⁴³

$$\begin{aligned}
 a &= 10 \text{ cm} & ; & \quad rR_{\text{curv}} = 150 \text{ cm}^2 & ; & \quad R_{\text{tor}} = 150 \text{ cm} \\
 B &= 6400 \text{ G} & ; & \quad T_i = 50 \text{ eV} \\
 n_{a(\text{neut})} &= 8.8 \times 10^8 \text{ cm}^{-3} & ; & \quad T_{a(\text{neut})} = 0.5 \text{ eV} & ; & \quad E_I = 13.5 \text{ eV} \\
 T_{eo} &= 400 \text{ eV} & ; & \quad T_{ea} = 40 \text{ eV} \\
 n_{eo} &= 2 \times 10^{12} \text{ cm}^{-3} & ; & \quad n_{ea} = 2 \times 10^{11} \text{ cm}^{-3}
 \end{aligned}$$

$$P_T = 4\pi R_{\text{tor}}^2 \int_0^a r dr P_\mu(r) = 6 \text{ kW (microwave deposition assumed uniform)}$$

This set of parameters, together with the nonlinear integral Eqs. (46), (47), (55), and required auxiliary equations, uniquely determines the density and temperature profile parameters r_k , n_k , and T_k . The solution, obtained graphically, is

$$\begin{aligned}
 r_k &= 9.20 \text{ cm} \\
 n_k &= 1.75 \times 10^{12} \text{ cm}^{-3} \\
 T_k &= 394 \text{ eV}
 \end{aligned} \tag{57}$$

These solutions imply a boundary width,

$$\Delta = a - r_k \approx 1 \text{ cm} , \quad (58)$$

which is consistent with experimental inferences. The boundary width must currently be inferred because the relative low density of present EBT devices makes direct Thompson scattering measurements of the density in the boundary region unfeasible.

Two features of the character of the solution warrant remarks. The temperature profile in the core region is much flatter than the density profile. This is a consequence of the fact that the Kovrizhnykh transport coefficient K_T is much larger than the other neoclassical coefficients. Secondly, the width of the boundary region is sharply defined by the form of the anomalous diffusion coefficient which scales as

$$D^{\text{LHD}} \sim \left(\frac{1}{n} \frac{\partial n}{\partial r} \right)^2 .$$

Tests of this model with Bohm diffusion lead to much broader boundary widths.

It is useful to consider how the boundary region width implied by anomalous transport scales with the important experimentally variable parameters.

There are two experimental parameters which may be readily varied in EBT experiments. The first is the amount of microwave power which is beamed into the plasma. The second is the density of neutrals at the plasma edge. This is controlled by gas puffing. It is interesting to see how the width of

the boundary region scales with variations in these parameters. Since the variation of either of these parameters has substantial effects on the gross properties of an EBT plasma, as well as on the boundary, we must attempt to isolate the gross effects from the boundary effects.

We shall effect the isolation from gross plasma effects by assuming that the density and temperature (at both the plasma center and the plasma edge) vary according to the scaling implied by a neoclassical point model.⁴⁴

$$n_{o,a} = K_1 p_\mu^{3/5} (n_{neut})^{-1/5} \quad (59)$$

$$T_{o,a} = K_2 p_\mu^{2/5} (n_{neut})^{-4/5} \quad (60)$$

An alternate point model formulation, which is consistent with these scalings and also compared favorably with empirical scaling, has been recently given by McBride.⁴⁵

Assuming the scalings implied by Eqs. (59) and (60), with all other parameters the same as for the case detailed previously, we focus on the effect of varying p_μ and n_{neut} . In Figure 22 we hold the neutral density fixed at $n_{neut} = 8.8 \times 10^8 \text{ cm}^{-3}$ and vary the applied microwave power. We see that the effect of an increase in the microwave power is to increase the boundary region width, albeit by a small amount.

In Figure 23 we hold the applied microwave power fixed at $P_T = 6 \text{ kW}$ and vary the neutral density. In this case the boundary region width also increases but by a more substantial amount. This variation of Δ , although substantial in a relative sense, may be very difficult to observe in practice. This is illustrated in Figures 24 and 25 where the evolution of the profiles

Figure 22. Variation of boundary region width with total applied microwave power (neutral density held fixed).

Figure 23. Variation of boundary region width with neutral density (microwave power held fixed).

Figures 24 and 25. Temperature and density profile changes under an increase in neutral density (see Figure 23).

under the change in neutral density is shown. The change in gross plasma parameters is obviously much larger than the change in the boundary region.

The boundary width is deeply involved with the hot electron ring structure and start-up, the stability of the core-ring interchange modes, and the power balance for the device, but these considerations are far beyond the scope of the present contract.

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