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I. Introduction

Heavy Ion (HI) physics has evolved with a class of detector problems and needs peculiar to this young and growing field of physics. Large solid angles and good accommodation for events with high multiplicities are a prerequisite in a large number of HI experiments. Usually it is desired to measure with high precision the momentum, charge, and mass of the particles. The high charge (Z) of the particles emphasizes such problems as space charge, recombination, plasma clearing times, and radiation effects. It is because of these various experimental requirements and problems that we have initiated a HI detector program in gaseous ion chambers at LBL. This program is the subject of my talk today.

I have divided my talk into three sections. The first is concerned with HI energy loss fluctuations. The remaining two sections discuss two detector alternatives we are considering: Bragg Curve Spectroscopy and Precision Relativistic DE/OX Measurements.
II. Heavy Ion Energy Loss Fluctuations

It is useful to be able to estimate the energy loss fluctuations in their slabs of materials such as targets, detectors, or windows in the preparation of most HI experiments. A commonly used approach in calculating the energy loss fluctuations is to use the energy independent theory of Bohr or an equivalent semiempirical approximation of Bohr's theory. Although this approach has some validity for relatively thick slabs of material and/or low energies, it is not valid for high energies and thin slabs of material. The purpose of this section is to provide a simple expression for estimating the energy loss fluctuations of heavy ions over a broad range of energies and thicknesses of materials.

Our approach is to use Vavilov's theory, replacing the projectile charge, $Z$, by an effective charge, $Z_{\text{eff}}$. The effective charge describes the equilibrium charge of the ions as the ion loses energy and exchanges charge with the medium.

$$\frac{Z_{\text{eff}}}{Z} = 1 - 1.032 \exp(-VR)$$

(1)

where

$$VR = \frac{\beta}{\alpha^2 0.89}$$

$\alpha$ is the fine structure constant and $\beta$ is the velocity relative to the velocity of light.
We then fit the energy loss fluctuations (FWHM) of the Vavilov distribution over a large range of energies and thicknesses of material with a semiempirical expression. Following the work of Seltzer and Berger, we define the energy loss fluctuations (FWHM) to be:

\[ W = \frac{\Delta_1 - \Delta_2}{\xi} \]  

(2)

where \( \Delta_1 \) and \( \Delta_2 \) are values of the energy loss at which the Vavilov distribution has fallen to half its peak value and

\[ \xi = 0.30058 \frac{m c^2}{2} z^2 \frac{Z_{med} S(g/cm^2)}{\Lambda_{med}}. \]

The Vavilov parameter, \( K \), is defined

\[ K = \frac{\xi}{\epsilon_{\text{max}}} \]  

(3)

where \( \epsilon_{\text{max}} \) is the maximum energy transfer to an electron

\[ \epsilon_{\text{max}} \sim \frac{2mc^2 \beta^2}{1 - \beta^2} \]

The semiempirical expression for the energy loss fluctuations (FWHM) is:
The expression yields the Landau limit for $K<1$

$$\Delta E(\text{FWHM}) = 4\xi$$

This expression is expected to underestimate the mean energy loss fluctuations for values of $K<10^{-3}$ where the approach of Ermilova et al. provides a better estimate when the mean energy loss is small compared to $K$ shell binding energies.

Our expression yields in the Gaussian limit of the Vavilov distribution, $K>1$.

$$\Delta E(\text{FWHM}) = 2.361 \left[ \frac{4}{2.87 K} \right]^{0.5}$$

This result is slightly less than that predicted by the Bohr theory. As an example we compare the present theory with the Bohr theory and the more precise calculation of W. K. Chu who uses a Hartree-Fock-Slater charge distribution and Bonderup and Hvelplund's theory in Table I. The comparison is made for 4 MeV and 2 MeV alpha particles in 1 cm Argon at STP.

It is interesting to note that our simplified approach follows approximately the velocity dependence of the more precise calculations of Chu. Our approach does, however, wash out any shell effects as were seen in Chu's calculations.
As a second example of the present theory we estimate the resolution of a 0.5 cm thick silicon detector in measuring the energy loss of 1.88 GeV/nucleon $^{55}$Fe ions to be 2.7% FWHM as compared to the experimental value of 3.0%.

III. Bragg Curve Spectroscopy

A gas ionization chamber is being developed for the purpose of measuring the charges, masses, and velocities of stopping heavy ions. These measurements are achieved by designing the detector and electronics such that the Bragg curves of the stopping ions can be retrieved. From the Bragg curves one then determines the range from the length of the track, the total energy from the integral of the specific ionization over the track, the dE/dx from the specific ionization at the beginning of the track, and the Bragg peak from the maximum of the specific ionization. This last signal measures the charge, Z, of the heavy ion unambiguously.

This type of heavy ion detector offers several advantages. Large solid angles are easily achieved. The detector is relatively insensitive to radiation effects. The resolution for identifying particles is intrinsically high because all the measurements are made in one medium, eliminating window or dead layer effects. The design goals are charge resolution of less than $10^{-2}$ at charge 100 and a mass resolution of less than 1 at mass 250.

The detector design is an ionization chamber with a Frisch grid to cathode distance longer than the range of the particles to be
detected (see Fig. 1). This distance in the prototype design is 28 cm. The particles enter normal to the cathode and parallel to the electric field. The electrons along the track are drifted through the grid and viewed as an anode current. The anode current as a function of time is proportional to the specific ionization along the track. The time duration of the anode pulse is proportional to the range and the integral of the current pulse is proportional to the energy of the stopped particle.

The first tests of this detector are being made using a transient digitizer (LeCroy) digitized the entire Bragg curve (current pulse). In Figure 2 we show the digitized Bragg curves of the 6.04 MeV and 8.76 MeV alpha particles from a ThC' and ThC" source.

In these first tests it is our intention to accumulate and store the digitized Bragg curves for off line computer analysis whereby various particle identification algorithms will be studied. For example, to achieve charge measurements we calculate the geometric mean of the specific ionization along the track.

$$\Delta E_{g,m.} = \left[ \prod_{i=1}^{N} \Delta E_i \omega_i \right]^{1/N}$$  \hspace{1cm} (7)

where $N$ spans the range of the track, $\Delta E_i$ is the measured value of the specific ionization at the $i^{th}$ position in the track and $\omega_i$ is a weighting factor that accounts for the detector filter response function.

The total energy is calculated from the arithmetic mean of the specific ionization.
The weighting factors, $\omega_i$, are required to achieve optimal energy resolution. The goal of the computer analysis of the digitized Bragg curves is a hardwired electronic algorithm, which will allow relatively high speed on-line particle identification for HI experiments. As an example, consider Eq. 7. A circuit that takes the integral of the logarithm of the current pulse is possible. A fast logarithmic converter circuit has been designed by V. Radeka and an exponential converter by F. S. Goulding. A weighting function may be applied with the use of an appropriate filter on the current signal.

Our future plans for Bragg curve spectroscopy include: studies of the energy loss fluctuations along the entire Bragg curve, studies of range straggling, studies of the equilibrium charge and charge exchange cross sections along the Bragg curve, determination of the detector filter response function, space charge studies, recombination effects, and optimization of particle identification algorithms for HI.

IV. Precision Relativistic $dE/dx$ Measurements

A Heavy Ion Superconducting Spectrometer (HISS) is being designed and constructed for the HI physics program at the Bevalac. The magnet will have an unusually large acceptance with the possibility of targets being placed within the magnetic field volume. The magnet will disperse HI reaction products according to their momentum to charge ratios. In order to achieve particle identification, both the
velocity and energy loss of the HI will be measured in addition to the momentum to charge ratio. I shall discuss here some of the design considerations of the dE/dx detector.

The main design goal is a charge resolution of less than 1 at charge 100. The approach is to use a thick (50 cm) xenon gaseous ionization chamber having an electron drift space between the cathode and grid of 49 cm (similar in design to the Bragg curve spectrometer) (see Fig. 1). The electrons along the track are drifted through the grid and viewed as an anode current. The anode current is then time digitized into a large number of samples, each of which is directly related to the specific ionization along the track. The time digitization of the current signal is to be made using Time Projection Chamber (TPC) electronics. This approach then becomes equivalent to having a large number of energy loss measurements for each particle. One gains in charge resolution using this technique at high energies and with their detectors as the fractional energy loss fluctuations become independent of detector thickness (Landau limit). See Eq. 4. The technique is well established in high energy physics as a means of achieving improved energy loss resolution of gas detectors operating in the relativistic rise region of specific ionization vs velocity.

In order to establish the precision of the measurements, we derive a noise equation and a figure of merit for this approach. We take as the total variance squared the sum of the squares of the variances that contribute independently.

We write the total variance for N samples as:
where $\sigma_v$ is due to Vavilov energy loss fluctuations, $\sigma_s$ is due to statistical fluctuations in the creation of electron-ion pairs and $\sigma_{el}$ is the electronic noise in the sample.

We define the mean energy loss per sample to be $\bar{\sigma}$ and for $N$ samples $\bar{\sigma}^2$. The variances per sample for a detector of thickness $L$ are then estimated. From Eqs. 2 and 4 we have:

$$\sigma^2 = \frac{1.65 \bar{\sigma}(L)/N}{\left[3 \frac{K(L)}{N} + 1\right]^{.5}}$$

(this approximate form over-estimates the variance)

$$\sigma_s = (\bar{\sigma}_C)^{.5}$$

$$\sigma_{el} = 424 \epsilon$$

where $\epsilon$ is the energy cost/ion pair.

In Figure 3 we show $\sigma_v/\bar{\sigma}(L)$ as a function of the number of samples for various values of $K(L)$. One gains the most for small values of $K(L)$. For 1.88 GeV/nuc. $^{56}$Fe ions and 50 cm xenon, $K = 1.1$.

As a figure of merit we calculate the total variance relative to the change in signal for a charge differences of one
Using the value of $H = 512$ and $L = 50$ cm for xenon at STP we find for particles in the energy range of 50 to 2000 MeV/amu and $Z_{proj}$ between 6 and 100, $F \leq 0.2$ which is consistent with our design goal of unit charge resolution for the heaviest ions that will be available from the upgraded Bevalac.

V. Summary

We have given a simple semiempirical expression for the energy loss fluctuations which spans the energy loss regions accounted by Landau, Vavilov, and Bohr. The new concept of Bragg Curve Spectroscopy (BCS) in an ion chamber is seen to yield signals giving the range, energy, $dE/dx$, and Bragg Peak. This last signal measures the projectile charge unambiguously. High resolution is expected in the BCS detector. Lastly, precision $dE/dx$ measurements of relativistic $HI$ are proposed following the similar well established techniques in high energy physics.

Acknowledgments


This work was supported by the Nuclear Science Division of the U. S. Department of Energy under contract No. W. 7405-ENG-48.
References


9. V. Radeka, Nucl. Instr. and Meth. 113 (1973) 401.

Table I. A comparison of the present theory with Bohr's theory and the more precise work of W. K. Chu on energy loss fluctuations.

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<td>19.6 keV</td>
<td>199.9 keV</td>
<td>22.4 keV</td>
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<td>2 MeV</td>
<td>18.6 keV</td>
<td>18.5 keV</td>
<td>22.4 keV</td>
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Fig. 1. Ionization chamber design.

Fig. 2. Digitized Bragg curves of the 6.04 MeV and 8.76 MeV alphaparticles from a ThC' and ThC" source.

Fig. 3. $\sigma/\xi$ as a function of the number of samples for various values of $K$. 
Fig. 1
EXPERIENCES WITH LARGE AREA
POSITION-SENSITIVE HEAVY ION DETECTORS

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Abstract:
A parallel plate avalanche counter with position read-out from a grid of sense wires via a delay-line method has been developed. Units with areas of 26x26 cm$^2$ and 91x91 cm$^2$ have been used successfully in a number of experiments. A time resolution of 240 ps FWHM and a x-y position resolution of 0.5 mm FWHM are obtained routinely.
Gas filled detectors have become more and more popular in heavy ion experiments, mainly due to their insensitivity to radiation damage and the large solid angles that are feasible at moderate cost. Large solid angles specifically are vital for a number of coincidence measurements in order to obtain reasonable coincidence efficiency or complete phase-space distributions. In this paper we present experiences with a type of detector that is especially suited for large area applications, i.e. the parallel plate avalanche counter with position read-out from sense wires via a delay-line method.

The basic design principle is shown in Fig. 1. Two foil electrodes form a conventional parallel plate avalanche detector with 3 mm gap, in addition a grid of sense wires with 2 mm spacing is introduced in the center of the gap and kept at the median potential, thus virtually not disturbing the homogeneous field of the gap. An avalanche arising from the primary ionization track due to the passage of an ion induces fast signals with 3 to 5 ns rise time (for our module sizes) on the electrodes and the nearest sense wires. The fast trigger signal from the foil electrode is used for timing. The sense wires are connected to a lumped delay-line and position information is derived from the arrival time of the delay-line signal at either end with respect to the fast trigger signal. As the signal rise times are kept larger than the discrete delays between sense wires, the delay-line actually performs an averaging process allowing
position interpolation between wires. For a detailed discussion see ref. 2. With constant fraction discriminator timing a position resolution of \( \leq 0.5 \text{ mm FWHM} \) is easily obtained for 2 mm wire spacing. The time resolution of the fast trigger signal is not impaired by the presence of the sense wires, one achieves 220 to 240 ps FWHM for heavy ions, as in other large area parallel plate avalanche counters. For x-y coordinate read-out, two independent units with crossed sense wires are spaced 65 mm apart, the space in between being used as a \( \Delta E \) ionization chamber (with moderate resolution).

The counters are operated at 5 Torr hexane. A separate 2 \( \mu \text{m mylar} \) foil, supported by steel wires with 2 cm spacing is used as the entrance window sealing against vacuum. Typical high voltages range from 450 to 600 V. For testing and calibration purposes, the counters may be operated at 600 to 650 V with nearly 100% efficiency for 5 MeV \( \alpha \)'s from a source. 300 ps time resolution is obtained for \( \alpha \)'s. A summary of detector specifications is compiled in Table I.

Due to our specific experimental needs we have designed our read-out electronics as to handle two-particle events. The six signals arising from a double event, i.e. two fast triggers, and two signals at each end of the delay-line, are schematically shown in Fig. 2. In order to record these signals, the first and second signal on each branch are routed to different time digitizer channels via electronic switches (kickers) without
destroying the timing information (see Fig. 3). The computer recognizes signals stemming from one particle by checking the sum of the left and right delay times, which is a constant for correlated signals. It turns out, that the time and position coordinate of both particles can be reconstructed even if one of the six signals is lost due to finite double pulse resolution of the electronics. The second particle is only missed if it is closer than 30 ns in time and 30 mm in position. (For a double x-y system, the particles must be a few ns apart to allow unambiguous x-y assignments). In case only single-particle events are wanted, the inspection of the sum of the left and right delay times may be used for pile-up rejection, giving a few ns resolving time.

During the last three years, we have mainly used detectors of two sizes: 26x26 cm$^2$ and 91x91 cm$^2$. The latter type is segmented and consists of 8 separate fast trigger elements and 4 delay-lines, each for the x- and y-coordinate. We use a demultiplexing system to reduce the original 64 timing channels to 16 channels plus a 16-bit pattern word, which still gives us full two-particle read-out capability. Fig. 4 gives an example of two-particle data taken with the 91x91 cm$^2$ detector. Due to the number of independent trigger elements, the above mentioned x-y ambiguity for near simultaneous particles applies only to a small fraction of events, and in many cases, this fraction is completely absent due to physical restrictions on the space-time correlation of double events. Two fission fragments in a three-
body event\(^1\) e.g. are always well separated on arrival at the
detector, either in time or in position. More details on the
read-out of the segmented counter are given in ref. 2.

The counters described here do not measure E or mass or charge
of the incident particles, so they are not very useful for
singles experiments. They prove to be powerful instruments for
the 'kinematic coincidence' technique, where the vector veloci-
ties of all outgoing reaction partners are measured, except may-
be for light evaporated particles. For up to 4 outgoing frag-
ments, this determines all kinematic variables completely, for
less than 4 they are even overdetermined. For the system 85 MeV
\(^{32}\text{S}^{27}\text{Al}\), with a 10 \(\mu\text{g/cm}^2\) target and 1 m flight path, we a-
chieved 0.35 amu mass resolution and 150 keV Q-value resolution
in the few-nucleon transfer region, a figure which can compare
even with magnetic spectrographs. Accuracy is usually limited
by the time-of-flight resolution, whereas the position measure-
ment is better than needed. This very consideration has prompted
the design of our 91x91 cm\(^2\) unit, which allows long flight paths
(up to ~1.3 m in the large scattering chamber at GSI, Darmstadt)
at large solid angles. In our investigation of the three-body
exit channel in the reactions of U and Pb beams on a variety of
targets\(^1\) we have combined a position-sensitive AE-E gas ioniza-
tion chamber\(^4\) with the 91x91 cm\(^2\) large area unit and exploited
the two-particle read-out feature. In these experiments we ty-
pically allowed rates of a few 10\(^n\) events/s in the large detector,
mainly elastically scattered beam.

For the large units it became necessary to compensate for the spatial variations of propagation delay which are due to the propagation of the signal on the electrodes and possibly to gap variations (0.1 mm gap variation results in ~300 ps drift time variation). Elastic scattering data with a pulsed beam are used to generate a map of time-corrections for the detector surface via a computer program. Using these local time corrections, the above reported time resolutions of 220 to 240 ps are achieved over the entire detector area.

Recently we have improved our large area ΔE-E gas ionization chamber by adding a 40×16 cm² parallel plate detector used in transmission. One position coordinate is measured in this detector, the other is obtained from drift time in the ionization chamber. We used ~50 μg/cm² stretched polypropylene foils resulting in a total thickness of ~200 μg/cm² including the two pressurized windows of the transmission counter. The improvement in timing allowed to determine the atomic number Z from ΔE and velocity v rather than from ΔE-E, giving Z-identification in the Pb-region for ions with energy as low as 1.7 MeV/u, for which the ΔE-E method fails (see Fig. 5). The above mentioned possibility of nanosecond pile-up rejection, which would be very difficult to achieve otherwise in an ionization chamber, has also been used.
In summary, the position-sensitive parallel plate avalanche counters with delay line read-out of sense wires make an instrument with excellent timing and position measurement characteristics for heavy ions. They are especially suited for coincidence measurements and have proven their practicality and usefulness in a number of experiments.

References


Table 1: Detector specifications

<table>
<thead>
<tr>
<th>Gap</th>
<th>3 mm</th>
<th>2 µm aluminized Mylar</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wires</td>
<td>20 µm ø</td>
<td>2 mm spacing</td>
</tr>
<tr>
<td>Delay line</td>
<td>2 ns/tap</td>
<td>50 Q impedance, commercial DIP</td>
</tr>
<tr>
<td>Gas</td>
<td>7 mbar (5 Torr) hexane</td>
<td></td>
</tr>
<tr>
<td>HV</td>
<td>450 to 650 V</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(~100% efficiency for 5 MeV α's at 600 V)</td>
<td></td>
</tr>
</tbody>
</table>

Two units stacked for x and y read-out

Resolution: Δx ≤ 0.5 mm FWHM
Δt ≤ 240 ps FWHM

Counts > 10⁶/s ³²S
> 10⁵/s Pb or U

Integral non-linearity < 1 mm
Differential non-linearity < 0.2 mm

Two-particle read-out if Δt > 30 ns or Δx > 30 mm
ΔE-resolution of PPAC 16 to 20% FWHM

(with charge sensitive ADC and 30 ns gate)
Figure Captions

Fig. 1 Position read-out of the parallel plate avalanche counter by means of a grid of sense wires connected to a delay-line

Fig. 2 Fast and delayed signals caused by two nearly simultaneous events. Two fast signals \( t_1, t_2 \) appear promptly, the delay-line signals have to propagate to the delay line ends. Reconstruction of the two events is possible even if one of the six signals is missing.

Fig. 3 Schematic block diagram of two particle read-out

Fig. 4 Spatial distribution of two fission fragments in coincidence with a recoil particle measured in another detector. The shadow of the support structure and the Faraday cup can readily be recognized.

Fig. 5 Z-resolution demonstrated with Hf, Pb and U. The top diagram shows \( \Delta E-E \), the bottom one \( \Delta E-v \), which gives better resolution for slow ions. (The low-\( \Delta E \) events are due to fission of Uranium).
Fig. 1
Fig. 2
Fig. 3
Fig. 4
Introduction

Today superconducting cyclotrons are clearly feasible and attracting widespread interest. It shows that simply tying two separately successful and established technologies together can be a breakthrough in itself. Actually it was here at Brookhaven in 1972, at the Magnet Technology Conference, that the first idea for our project was triggered. Shortly after that the first outline specifications of a superconducting cyclotron was drawn up. Our design has evolved but the 5 T field and the concept of using superconducting coils with saturated steel have survived. It took approximately eighteen months to put together a detailed proposal for a full scale magnet and rf system which was submitted and approved by June '74.

By July of '78 the proposal had been turned into a working magnet and rf system and first operation was reported at the Bloomington Cyclotron Conference last year. The success was duly reported to the Atomic Energy of Canada Limited Board of Directors who strongly endorsed the proposed conversion to a working cyclotron, but in the climate of fiscal restraint at that time the conversion was not accepted by the Canadian Treasury Board for go ahead this fiscal year. We are, however, optimistic that funds will be released next fiscal year. This funding problem does not affect the magnet and rf tests which are continuing to make progress but the civil
engineering side has been slowed considerably. It is hoped that we could make up a large portion of the lost time if we were given approval late this year.

Facility Description

The proposed heavy ion research facility at Chalk River consists of a 13 MV MP tandem accelerator with a K=520 superconducting cyclotron operating as a post-accelerator. The cyclotron itself is described in some detail in references (1) (the magnet) and (2) (the rf system). Briefly, it is a four sector isochronous cyclotron with accelerating dees in each valley, two on each of two resonators on the cyclotron axis. Isochronism is provided by altering the current in two main magnet coils with fine tuning provided by trim rods.

Because CRNL already has the 13 MV tandem accelerator the philosophy since '72 has been to use it as an injector and it is of course excellent in just that role. The basic layout is shown in Fig. 1.

Two ion sources would be available to inject into the tandem through a low energy buncher. This buncher, which is a gridded re-entrant structure operating with both first and second harmonic frequencies, has been tested in air to full operating voltage. The beam is stripped in the tandem (generally gas stripping) and transported by a conventional magnet system to the cyclotron. The high energy buncher is not designed yet.

The beam transport system is necessarily convoluted to correctly match the six dimensional phase space of the beam at the cyclotron stripping foil and an elegant solution to this problem has been devised. The beam is stripped at the injection radius to a charge state approximately
three times greater and the beam is accelerated out to the extraction system and transported to several target areas.

The extraction system comprises both passive magnetic elements using saturated steel as well as an electrostatic deflector and active magnetic channel with superconducting windings. Detailed mechanical and electrical design of the electrostatic deflector is complete. Currently a short length of deflector, but with the complete high voltage feed section, is being prepared for in-vacuum high voltage tests. Iron and nickel compensation pieces have been designed to reduce the unwanted first harmonic fields generated in the acceleration region by the iron in the magnetic elements.

The cyclotron will deliver heavy ions ranging from 50 MeV/nucleon carbon to 10 MeV/nucleon uranium with an energy resolution of $\Delta E/E \lesssim 10^{-3}$ for an injected phase spread of 3°.

The beams typical of the machine are shown in Table 1. The intensities are typically of the order of 100 particle nA with a trend to higher current for lighter ions and lower current for heavier ions. For uranium, higher current can be obtained if foil rather than gas stripping in the tandem is used. The iodine beam is interesting because it clearly beats the trend to lower currents and we have selected it as our first ion to accelerate in the commissioning of the cyclotron. It has been chosen not only because of its high current but because it falls in the middle of our operating range of magnetic field and hence focusing properties. Also as a small political consideration, it would give us a 1 GeV beam.

Figure 2 shows where the cyclotron fits in the wider scheme of things. One of the advantages of having an existing tandem is that the
cyclotron can provide substantially more energetic beams than a stand-alone cyclotron such as MSU Phase 1. This of course is a two edged sword and we cannot hope to compete with MSU Phase II with its two superconducting cyclotrons. Having a bigger tandem of course is an advantage and the HHIRF tandem, even injected into ORIC, will provide quite high energy beams and injected into a superconducting cyclotron would compete with MSU II. In terms of time, MSU I will be on the air in 1980 while at CRNL we hope to have first beams in 1982. GANIL and MSU Phase II are expected to come on line 84/85 so maybe we will have a year or so to ourselves.

Cryostat Performance

Our first problem in the last year was to reduce the unacceptably high boil-off rate of liquid helium from the cryostat.

In Fig. 3 the boil-off rate in litres/h is plotted as a function of volume. The upper curve shows the initial boil-off where there were two problems - (1) the superficial losses were generally high indicated by the upward slope of the curve (boil off proportional to wetted surface) and (2) there was a large jump in boil-off rate at the midplane. To correct these problems the cryostat was disassembled and the superinsulation removed. Some modifications were made to the radiation shield near the midplane to correct the problem giving the large step in the boil-off curve. The superficial losses were reduced by using many more layers in the reinsulation than had been used initially. To do this we changed materials from the rather bulky dimpled mylar interleaving to the difficult-to-handle Dexter paper. The lower curve shows the large improvement in boil-off rate (by a factor of 2.6) to a filled boil-off rate of 17 litres/h. The distinct steps
in the boil-off at ~ 1/4 and 3/4 volume are attributed to the current leads which extend to those levels.

Currently the helium is transferred continuously from the storage dewar into the cryostat while simultaneously liquefying into the dewar with a net gain of up to 12 litres/h.

The Rf System

Both resonators have now been run up to 86 kV (design 100 kV) and the full frequency range of 30.8 to 61.6 MHz has been demonstrated. Full voltage has been achieved on a single resonator. The power required for 100 kV operation varies from 25 to 130 kW and is within the capability of the power amplifier.

The most serious problem remaining with the accelerating structure is the poor performance of the spring contacts on the sliding shorts. The difficulty is believed to be primarily the mechanical fit and possible movement of the dee stem in the tuner. A specially designed finger stock will allow larger deviations from symmetry than the commercially available material now used.

Magnet Performance

On the magnet itself we have been working hard to demonstrate several basic design parameters. Field stability is one of these and is shown in Fig. 4.

The field fluctuations are of the order of ± 6 x 10^{-6} or less and result in energy fluctuations of the order of 3.6 x 10^{-5}. The energy spread ΔE/E due to the bunch length of 3° is itself 3.4 x 10^{-4} so that the field fluctuation contribution is small.
It is important to minimize the harmonic content in the field, in particular the first harmonic. We can in fact compensate this harmonic by using the trim rods but of course want to keep their use to a minimum. Contributions to the first harmonic come from imperfections in the hills themselves, which were made to tight tolerances, the injection and extraction holes in the yoke wall, which were compensated by appropriate compensating yoke holes and from the coil itself, which can be moved radially. The coil proved to be easy to locate by examining the first harmonic at the outside of the field maps where \( \frac{dB}{dr} \) is large.

The measured first harmonic field is shown in Fig. 5 for average fields of 5 T and 1.2 T. Everywhere in the acceleration region the first harmonic is less than 0.4 mT which is easily within the tuning range of the rods.

The coil itself is movable axially as well as radially and misalignment would give rise to a radial component of field on the midplane. Such a component would deflect the beam vertically and, if large enough, would result in beam loss. The design alignment tolerance is \( \pm 0.3 \) mm. The coil is easily positioned to that accuracy, but the motion of the coil due to deflection of the magnet top end ring, from which it is suspended, is 0.12 mm. The coil will be placed at the central position so that the misalignment will be \( \pm 0.06 \) mm, within the tolerance required.

The reproducibility of the field, that is the difference between fields at the same currents but measured on different occasions, is \( \pm 0.2 \) mT at constant temperature. There are, however, rather large field changes as a function of temperature of the order of \( -0.5 \) mT/°C.
The problem of temperature variation should be controlled in the final location but in the non-airconditioned environment of the present magnet building we have been subject to the vagaries of Canadian spring weather.

The Trim Rods

The trim rods, of course, are one of the more novel features of our cyclotron and, naturally, reservations have been expressed about them. The rods have to be moved against large forces (up to 2.5 tonnes for the 60 mm rods) and must be moveable with considerable precision (± 0.013 mm for the outermost rod which is used for first harmonic generation). Also the drive motors for the rods are located above the cyclotron in a maximum stray field of ~ 100 mT. Although we had successfully made bench tests of the drive systems with test loads in externally generated fields, it was gratifying to actually operate a trim rod in the magnet while at maximum field. The rod was driven with a 0.1 HP dc motor through a reduction gear moving the rod at 8 mm/minute. The travel of the rod was measured using a precision linear potentiometer and was relocatable to within ± 0.013 mm, as required. The motor operates at a current 50% over that required for the same load with no stray field but with no noticeable heating.

Figure 6 shows the design layout for the trim rod drive.

There will be three motors for each hill, driving five, seven and one rod respectively. The single rod is used for setting up the first harmonic for resonant extraction. Pneumatically operated dog clutches are used to connect the motor to individual rods.
The trim rods' primary role is to isochronize the magnetic field. To look at the field adjustment that will be required of the rods, we have just completed a first set of field maps at a wide range of currents in the coils. With this as a data base, the required isochronous field for any given ion is selected from the set (interpolative least squares fit). The calculated fields from the trim rods (already demonstrated to be in good agreement with actual measured fields) are then used to find rod retraction necessary to complete the isochronization. This can be extremely good as seen in Fig. 7 where the isochronous fields for 3 MeV/nucleon uranium (the least relativistic ion) and 10 MeV/nucleon iodine (the commissioning ion) are shown.

The phase slip is small and axial betatron frequencies $v_z$ greater than 0.1 over the acceleration region. Two ions which are more difficult are 10 MeV/nucleon uranium and 50 MeV/nucleon carbon (Fig. 8). For uranium, which is at the maximum field, the range of coil current changes available for isochronism is small and in this case the trim rods are being worked rather hard. For carbon, isochronism can be easily met but at outside radii this results in a low $v_z$. So some isochronism has been sacrificed at these radii to increase $v_z$ at the expense of a larger phase slip. In both cases however, the fields would be quite adequate for acceleration to the extraction radius. This is gratifying since this has been obtained for the unmodified original flutter poles designed from calculations alone. Small changes in the flutter pole will probably be made during the next shut down to improve the isochronous field for 10 MeV/nucleon uranium.
Conclusion

The major parameters of the magnet have been successfully demonstrated and it will shortly be shut down for modifications to the cryostat to incorporate the rf and extraction systems. Remaining problems with the rf sliding short: we hope will be solved by improving the design of the sliding contact.

References


Table 1

Typical Cyclotron Extracted Beam Energies and Intensities

<table>
<thead>
<tr>
<th>Ion</th>
<th>Charge State Injected</th>
<th>Charge State Extracted</th>
<th>Energy MeV/u</th>
<th>Intensity (particle nA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{7}\text{Li})</td>
<td>1</td>
<td>3</td>
<td>50</td>
<td>640</td>
</tr>
<tr>
<td>(^{12}\text{C})</td>
<td>2</td>
<td>6</td>
<td>50</td>
<td>240</td>
</tr>
<tr>
<td>(^{37}\text{Cl})</td>
<td>5</td>
<td>15</td>
<td>33</td>
<td>80</td>
</tr>
<tr>
<td>(^{63}\text{Cu})</td>
<td>7</td>
<td>21</td>
<td>23</td>
<td>55</td>
</tr>
<tr>
<td>(^{127}\text{I})</td>
<td>7</td>
<td>23</td>
<td>10</td>
<td>200</td>
</tr>
<tr>
<td>(^{238}\text{U})</td>
<td>10</td>
<td>33</td>
<td>10</td>
<td>4*</td>
</tr>
</tbody>
</table>

* 36 with foil stripping.
Fig. 1 Layout for the Proposed Superconducting Cyclotron Facility

Fig. 2 Comparison of Some Heavy Ion Facilities

GANIL - Grand Accelerateur National d'Ion Lourds, Caen, France.
HHIRF - Holifield Heavy Ion Research Facility, Oak Ridge, Tennessee.
MSU - Michigan State University Heavy Ion Cyclotron Project, East Lansing, Michigan.
CRNL - Chalk River Superconducting Cyclotron Project, Chalk River, Ontario, Canada.

Fig. 3 Cryostat Performance. Upper curve shows initial boil-off and lower curve that after reinsulation.

Fig. 4 Measured Field Stability of the Cyclotron Magnet. The value $\pm 6 \times 10^{-6}$ corresponds to energy excursions of $\pm 3.6 \times 10^{-5}$. 
Fig. 5 Measured First Harmonic Field Component

Fig. 6 Design Detail of Trim Rod Drive System

Fig. 7 Average field $\langle B \rangle$, phase slip $\phi$ and axial betatron frequency $\nu_a$ for 10 MeV/u iodine and 3 MeV/u uranium.

Fig. 8 Average field $\langle B \rangle$, phase slip $\phi$, and axial betatron frequency $\nu_a$ for 50 MeV/u carbon and 10 MeV/u uranium.
Exotic new effects are anticipated in the coming studies of medium and high energy heavy-ion physics. These expectations might be reasonable since such interactions are virtually unexplored. However before we can expect to demonstrate the existence of exotic new phenomena, it is necessary to understand the basic gross features of nucleus-nucleus (N-N) collisions, e.g. elastic scattering and total reaction cross sections ($\sigma_R$). It is commonly assumed that $\sigma_R$ for N-N is constant and equal to the geometric limit. This paper will present experimental and theoretical evidence against this assumption. If these findings are verified by further experimental measurements, it is likely that our view of N-N collisions in this energy region will be seriously altered.

II. Nucleon-Nucleon and Nucleon-Nucleus Systems

The behaviour of the nucleon-nucleon (n-n) total cross-section ($\sigma_T^{nn}$) is well known. Fig. 1 displays $\sigma_T^{nn}(E)$ for incident (laboratory) energies up to 1 GeV. Notice the rapid decrease in $\sigma_T^{nn}$ with increasing energy up to about 300 MeV at...
which point "production causes the cross section to rise. This pronounced "dip" in $\sigma_{^T\text{NN}}(E)$ can be traced to the behavior of the scattering phase shifts (in particular the $^1S_0$), which is caused by the non-locality in the n-n interaction. The "dip" should be a characteristic feature of any heavier system which is dominated by the nucleon-nucleon interaction.

Indeed proton-nucleus total reaction cross-sections display an energy dependence, which tracks $\sigma_{^T\text{NN}}(E)$ - see Fig. 2 (from Ref. 1). Of course, low energy effects due to the Coulomb barrier and resonances are also observed. The fact that $\sigma_R$ climbs and then levels out in the energy region $E < 20$ MeV suggests that a geometric limit is reached. The fact that $\sigma_R(E)$ falls off sharply at higher energies means that this geometric limit is not maintained, i.e. some "transparency" occurs in the nucleon-nucleus interaction at these energies. The data may be parametrized with the formula:

$$\sigma_R = \pi(R + \delta)^2 [1 - \frac{2Ze^2}{(R+\delta)^2}] (1-T)$$

(1)

where $R$ is the effective nuclear radius, $\delta$ is the reduced wave length of the incident particle, $Z$ and $z$ refer to the target and projectile respectively, $E$ is the incident center-of-mass energy and $T$ is the transparency, which may be related to the mean-free path of the
Thus, transparency as defined in this equation represents the difference between the geometric cross section and the smaller values which are obtained at intermediate energies. Obviously, this transparency is directly related to the behavior of $\sigma_{T}^{nn}(E)$ - microscopic theoretical calculations of $\sigma_{R}^{nn}$ will be presented in Sec. IV.

Fig. 3 shows the application of eq. (1) to proton $\sigma_{R}$ data. The derived radii and T values are listed in Table I. The derived radii seem sufficiently large to justify the assumption that $T=0$ at the peak of $\sigma_{R}(E)$. The derived values of $T$ are in accord with the work of Renberg, et al.\textsuperscript{1} who also found that at a given incident energy, $T$ is smaller for proton-lead data ($T \sim 10\%$) than for lighter systems (e.g. proton + carbon yields $T \sim 35\%$).

III. Nucleus-Nucleus Systems

Very little $\sigma_{R}(E)$ data has been measured for N-N systems. However, the fact that composite projectiles and heavy-ions are known to be strongly absorbed (at low incident energies) has encouraged the assumption that $\sigma_{R}(E)$ simply levels off, above the Coulomb barrier, at the geometric limit and stays at that value at intermediate and high incident energies (See, for example, Ref. 2).
As we shall see, elastic scattering data is very useful in studying the energy dependence of the N-N interaction. It is well established that elastic scattering data can be reasonably well described with a diffraction model in which the minima of the angular distribution are given by the zeros of $J^2(kR \theta)$. Fig. 4 displays $α + ^{40}$Ca data at $E_{\text{cm}/\nu} = 32.2$ and 306.7 MeV incident energies. At both energies the first four minima are closely reproduced with this description, however, the radius required to fit the data changes from 6.40 fm at 32.2 MeV/n to about 4.9 fm at 306.7 MeV/n. This simple analysis suggests that the geometry seen in composite projectile-nucleus collisions changes as a function of incident energy. Thus the value of the "geometrical limit" for $a_R$ may be quite different between these two energies.

Quantitatively $a_R$ may be deduced from elastic scattering data using the optical model. Fortunately, a fairly large body of elastic scattering data does exist for composite projectile - nucleus systems so that $a_R(E)$ can be at least partially mapped out. Fig. 5 displays a compilation of the available data for $d + ^{12}$C, $p + ^{12}$C is also shown for comparison. Note that the three direct measurements of $a_R$ (shown with error bars) are in reasonable agreement with the points derived from elastic scattering data. It
is very clear that the d + \(^{12}\text{C}\) system displays a dramatic energy dependence. If we apply eq. 1 to the data, normalizing to the large number of data points in the 4-6 MeV/n region, we obtain the solid curve shown in Fig. 5 and the parameters given in Table I. This curve represents the T=0 limit. Another way of establishing the geometric value for \(\sigma_R\) is to use the electron scattering values for the r.m.s. radii to calculate \(\pi(R_d + R_{^{12}\text{C}})^2\) (where \(R = 5/3 R_{\text{res}}\)). This technique yields a value of 1082 mb. These data establish that \(\sigma_R\) is highly energy dependent (even more so than for protons). Thus the notion that \(\sigma_R\) simply reaches and maintains its geometric limit for N-N collisions appears to be completely incorrect.

The d + \(^{12}\text{C}\) data set contains a reasonable number of data points; it is impossible to map out \(\sigma_R(E)\) as well for other projectile-target combinations, however enough data exists to show that the same qualitative effects exist for a variety of cases. Fig. 6 displays proton and composite projectile \(\sigma_R(E)\) data for \(^{12}\text{C}, ^{16}\text{O}\) and \(^{40}\text{Ca}\) systems\(^5\). The curves are drawn using eq. 1 with parameters as listed in Table I. Fig. 7 compares d, \(^3\text{He}\), and \(\alpha\) data on a \(^{40}\text{Ca}\) target. Notice that in all cases, \(\sigma_R(E)\) increases with energy at low incident energy but does not stay constant at medium energies.
The heaviest system for which data can be gathered is the $^{12}\text{C} + ^{12}\text{C}$ case. Fig. 8 displays elastic scattering based $\sigma_R$ values at low energy along with two medium energy direct measurements. A curve drawn using eq. 1 is also shown with parameters listed in Table 1. It is clear that even for $^{12}\text{C}$ projectiles, $\sigma_R(E)$ does not simply stay constant at the geometric limit in the medium energy region. All of these systems, therefore, appear to display an $\sigma_R(E)$ behavior strongly reminiscent of $\sigma_T^{nm}(E)$. It seems obvious, therefore to attempt to fit these data with a microscopic model based on $\sigma_T^{nm}(E)$ as input.

IV. Glauber Calculations

Consider a charge-less, point projectile incident on a target nucleus at some impact parameter $b$. The probability of transmission through the nucleus is simply

$$T = \exp(-z/\lambda) \quad \lambda = (\rho \sigma_T^{nm})^{-1}$$

where $z$ is related geometrically to $b$ and $\rho$ is the nuclear density of the target. The total reaction cross section is then simply:

$$\sigma_R = \int 2\pi bdb(1-e^{-z/\lambda})$$

$$\int$$
This equation is not very useful for projectiles other than single nucleons. However, it is possible to generalize to the case of a finite projectile by writing:

\[
\sigma_R = 2\pi \int_{b_1}^{b_2} b db \left[ 1 - \exp(-\sigma_T^{\pi} \chi(b)) \right]
\]

\[
\chi(b) = \int dr_1^2 \int dz_1 \int dz_2 \frac{\rho(z_1 + z_2)}{\rho_1(z_1) \rho_2(z_2)}
\]

where \( z_1 \) and \( z_2 \) are internal coordinates. For the case of two uniform spheres of radii \( R_1 \) and \( R_2 \) we have:

\[
\chi(b) = 4\pi \rho \int_{0}^{2\pi} \int_{R_1}^{R_2} \left[ \frac{(R_2^2 - b^2 - r^2 + 2rb \cos \phi) \rho_1^2}{r^2} \right] dr d\phi
\]

These formulas may be integrated numerically. We have included a simple "straight line" Coulomb correction (which reduces \( \sigma_R \) at low energies). Parameter free predictions can be made since electron scattering data\(^9\) (for the \( \rho(r) \)) are used along with experimental values for \( \sigma_T^{\pi}(E) \). It should be noted that these formula are often referred to as the "optical limit" of Glauber theory\(^10\).

The equations are applied to p,d and \( \alpha \) data on a \(^{12}\)C target in Fig. 9, 10, and 11. A rather impressive agreement with the experimental data is obtained at all energies, even down to 10.
MeV/n. The agreement below 100 MeV/n is surprising, since Glauber theory is usually thought to break down at such low energies\(^{11}\). We note, however, that we have included Coulomb effects (to first order), which has not been done in prior calculations and that previous, rather similar calculations by Bertini\(^{12}\), have successfully fit proton-nucleus \(c_R\) data down to 30 MeV. The enhanced transparency derived using eq. 1 (see Table I) falls naturally out of the Glauber calculations. Our calculations make it clear that the composite projectile-nucleus interaction is dominated, at medium energies, by the simple n-n interaction.

Fig. 12 displays our Glauber predictions for the \(^{12}C + ^{12}C\) system. The agreement with available experimental data suggests that heavy-ion collisions (or at least \(^{12}C + ^{12}C\)) may not be dominated by bulk (e.g. collective, hydrodynamic) effects, which would yield a geometric \(c_R\). Such bulk effects might become more important with heavier projectiles. Thus \(c_R\) (E) measurements may be capable of answering a fundamental question about heavy-ion collisions—i.e. to what extent are they dominated by n-n collisions as opposed to hydrodynamic behavior\(^{13}\). Furthermore, the fact that \(c_R\) falls below geometric in the medium energy region might allow the observation of exotic effects in n-n collisions, i.e. if \(c_R\) were geometric then exotic effects might be very hard to observe since there would be essentially no cross section available to such effects.
Eq. 4 contains the quantity:

\[ 1 - \exp(-X) \]

\[ X = -\sigma_T^{\text{nm}}(b). \]

This quantity may be thought of as one minus the transparency for a given impact parameter. Fig. 13 and 14 plot this quantity as a function of impact parameter for the $d + ^{12}\text{C}$ and $^{12}\text{C} + ^{12}\text{C}$ systems. At low incident energies (large $Q_{\text{cm}}$) there is no transparency until large distances are reached, thus the "radius of the system" is quite large justifying the notion that nucleus-nucleus collisions are strongly absorbing. However, at medium energies (small $Q_{\text{cm}}$) the transparency is non-negligible at much smaller impact parameters. This leaves a considerable radial region, which is "translucent" in N-N collisions. This region is not accessible in low energy experiments. The ability to probe the N-N system to markedly lower radii may have important implications for nuclear structure studies.

Our Glauber calculations allow us to realistically study density distributions in N-N collisions. For a given impact parameter ($b$) we may plot the (electron scattering) densities, displaced from each
other by \( b, \) and add them at the corresponding radii to obtain the resulting sum density along the impact parameter. The probability of having both ions intact and unexcited at given \( b \) may be estimated using plots as shown in Fig. 13-14. This procedure is illustrated for a \( ^{12}\text{C} \) with \( \sigma_{T}^{\text{NN}} \approx 30 \text{mb} \) \((E_{t}/n \approx 200-300 \text{ MeV})\) for two different impact parameters in Fig. 15 and 16. Notice that only low sum densities are achieved, at \( b \approx 3.5 \text{ fm} \). At \( b \approx 2.5 \text{ fm} \), densities are obtained which are significantly higher than normal. Since \( \exp(X) \) is still 6.5\%, there is a reasonable chance of reaching these conditions. It is not clear, however, that such low \( \sigma_{T}^{\text{NN}} \) values are conducive to exotic compressional effects, but our calculations, at least, serve as believable indicators of how/when higher than normal densities can be achieved.

V. Conclusions

Using the rather limited amount of available data, we find that \( \sigma_{R}(E) \) for composite projectiles displays an energy dependence, which is as strong or stronger than for protons. Specifically \( \sigma_{R} \) is not geometric in the energy region \( 50 < E_{t}/n < 600 \text{ MeV} \). These effects apparently stem from the strong energy dependence of \( \sigma_{T}^{\text{NN}} \). Glauber model calculations are capable of reproducing all the features of the data including
quantitative agreement in the energy region $E_{j/n}\geq 20$ MeV. These results emphasize the dominance of the $n-n$ interaction in all of the $N-N$ collisions for which we have data. These results are rather unexpected and are likely to have important implications for medium energy composite projectile and heavy-ion physics. Because of the present sparsity of data, our results are rather controversial and vast heaps of data are needed as soon as possible. Experiments are beginning by a LASL-LBL-U. Washington group at medium energies and by a LASL-LBL-U. Colorado group at low energies.

It gives me pleasure to acknowledge that this work was performed in collaboration with J. C. Peng. We have received valuable help from J. Ginocchio. This work was supported by the DOE-DMA.
References

### Table I

**Analysis Parameters**

<table>
<thead>
<tr>
<th>System</th>
<th>Radius in eq. 1 (fm)</th>
<th>Electron Scattering Radius (fm)</th>
<th>T (in minimum)</th>
<th>Geometric $\sigma_R(\text{ab})$</th>
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<tbody>
<tr>
<td>$p^+^{12}\text{C}$</td>
<td>2.98</td>
<td>3.16</td>
<td>36%</td>
<td>314</td>
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<tr>
<td>$p^+^{16}\text{O}$</td>
<td>3.39</td>
<td>3.49</td>
<td>32%</td>
<td>382</td>
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<td>$p^+^{40}\text{Ca}$</td>
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<td>4.45</td>
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<td>623</td>
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<tr>
<td>$\alpha^{12}\text{C}$</td>
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<td>5.87</td>
<td>64%</td>
<td>1082</td>
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<tr>
<td>$\alpha^{16}\text{O}$</td>
<td>4.97</td>
<td>5.36</td>
<td>52%</td>
<td>902</td>
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<tr>
<td>$\alpha^{40}\text{Ca}$</td>
<td>6.85</td>
<td>6.65</td>
<td>44%</td>
<td>1389</td>
</tr>
<tr>
<td>$^{12}\text{C}^{12}\text{C}$</td>
<td>6.82</td>
<td>6.33</td>
<td>36%</td>
<td>1257</td>
</tr>
</tbody>
</table>

* $(5/3)$ times $R_{rms}$ (from Ref. 9)

** $\pi R^2$ where $R$ is from column 3
FIGURE CAPTIONS

1) Nucleon-nucleon total cross sections as a function of incident lab. energy.

2) Examples of proton $\sigma_R(E)$

3) Proton $\sigma_R(E)$ data (dashed lines drawn to guide the eye) and fits (solid lines) using eq. 1 with $T=0$. The parameters of these fits are listed in Table 1.

4) Elastic scattering angular distributions for $^3\text{He}$ on Ca at $E_{cm}/n = 32.2$ and 306.7 MeV/n (note that the energy on the right side is incorrectly labelled) along with a diffraction model fit prediction for the minima positions.

5) $\sigma_R(E)$ for protons and deuterons on a carbon target. Deuteron points shown without error bars were taken from optical model predictions based on fits to elastic scattering data. The solid lines represent the use of eq. 1 (with $T=0$) with parameters as listed in Table 1 while the dashed lines are drawn to guide the eye. The horizontal axis should be $E_{cm}$/nucleon.
6) \( \sigma_R \) for (a) protons and (b) composite projectiles. At some energies more than one point is shown in (b); these points correspond to different sets of optical model parameters. The solid lines are calculations using Eq. (1) with \( T=0 \). The dashed lines are drawn to guide the eye.

7) \( \sigma_R \) for \( d, ^3\text{He} \) and \( \alpha \) particles on a \(^{40}\text{Ca} \) target.

8) \( \sigma_R \) for \(^{12}\text{C} + ^{12}\text{C} \). The low energy values are based on optical model fits to elastic scattering data.

9) Glauber model fits to \( \sigma_R(E) \) for \( p^{12}\text{C} \).

10) Glauber model fits to \( \sigma_R(E) \) for \( d^{12}\text{C} \).

11) Glauber model fits to \( \sigma_R(E) \) for \( ^{4}\text{C}^{12}\text{C} \).

12) Glauber model fits to \( \sigma_R(E) \) for \( ^{12}\text{C}+^{12}\text{C} \).

13) The quantity \( l\exp(x) \) vs. impact parameter for the \( d^{12}\text{C} \) system.

527
14) The quantity $1 - \exp(X)$ vs. impact parameter for the $^{12}\text{C} + ^{12}\text{C}$ system.

15) Densities for the $\alpha + ^{12}\text{C}$ system for an impact parameter of 3.5 fm. The $^{12}\text{C}$ ion is centered at $r=0$ while the $\alpha$ nucleus is centered at $r=3.5$ fm. (i.e. the impact parameter). The quantity $\exp(X)$ has a value of 37% at this $b$ (not $1 - \exp(X)$).

16) Same as Fig. 15 except $b=2.5$ fm.
Fig. 9. Proton reaction cross sections as a function of incident proton energy. The lines serve as guides for the eye.

FIG. 3
$a^{40}_{\text{Ca}}$
$E_{\text{c.m.}}/n \times 32.2\text{ MeV}$

ZERO'S OF

$J^2(xR^2)$
WITH $R = 6.40\text{ fm.}$

$\sigma (\text{mb/ sr})$

FIG. 4
FIG. 5

TARGET: $^{12}$C

DEUTERONS

PROTONS

$\sigma_R$ (mb)

$E_{\text{c.m.}}$ (MeV)

533
Fig. 8

$^{12}C + ^{12}C$

$(q\mu)^{12}_0$

$E_{c.m.}/n$
\( \alpha \cdot ^{12}\text{C} \)
\( \sigma_{1}^{\text{tot}} = 30 \text{ mb} \)
\( [(1 - \exp \chi) = 37\%] \)
\( b = 3.5 \text{ fm} \)

FIG. 15
$a + ^{12}\text{C}$

$\sigma_{1}^{aa} = 30 \text{ mb}$

$[(1 - \exp \chi) = 6.5\%]$  

$b = 2.5 \text{ fm}$

---

FIG. 16
Subthreshold Pion Production with Heavy Ions

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Abstract

An experiment which measured pion production from 125 to 400 MeV/nucleon heavy ion collisions is described. It is shown that Coulomb effects in the $\pi^-$ to $\pi^+$ ratio are very large and give some interesting insights into the reaction mechanism.
The threshold for pion production with free nucleon-nucleon collisions is \( = 290 \text{ MeV} \), and therefore production of pions in nucleus-nucleus collisions at beam energies less than 290 MeV/nucleon is referred to as subthreshold. There is also a threshold for production of pions by nucleon-nucleus collisions which depends on the choice of target but is typically \( = 150 \text{ MeV} \). If the projectile is a composite particle, then the Fermi motion of the nucleons in the projectile permits pion production even below the nucleon-nucleus threshold. In this talk the first systematic study\(^1\) of pion production which extends to below both of these thresholds will be described. In addition, since data were taken for both \( \pi^+ \) and \( \pi^- \), systematics of the charge-dependent effects in pion production are also observed for the first time.

Subthreshold pion production has been a subject of interest since soon after the discovery of mesons,\(^2\) but very little data have been published on the subject. There are proton data\(^3\) from Uppsala and Orsay, and a systematic study down to the energy conservation threshold for protons has been carried out recently at Indiana.\(^4\) There was also some deuteron and alpha work carried out in the late 1940's at Berkeley\(^5\) and recently at SREL,\(^6\) as well as \(^3\)He results from Maryland.\(^7\) Heavy ion results are restricted to 250 MeV/nucleon and above\(^8\) except for some emulsion work\(^9\) in which an anomalously high yield of pions was reported at bombarding energies from 100 to 280 MeV/nucleon. This work was refuted by two other experiments\(^10,11\) which saw no pions at all. The only reported measurement of \( \pi^-/\pi^+ \) ratios for composite projectiles near and below threshold is a 1953 \( \alpha \)-particle measurement from Berkeley.\(^12\)
A great deal of interest has centered on the energy region of 100 to 400 MeV/nucleon because it is in this region that exciting new effects are expected. For example, a region of double nuclear density at the interaction region of two heavy ions could induce a phase transition into Lee-Wick matter or a pion condensate. The yield of low energy pions is expected to be sensitive to these effects. The present experiment was designed to study pions with low energy in the center-of-mass frame.

The measurements were made with Ne (from 125 MeV/nucleon to 400 MeV/nucleon) and Ar beams (at 250 MeV/nucleon and at 400 MeV/nucleon) produced by the Bevalac at the Lawrence Berkeley Laboratory. Most of the measurements were made at 0°, but an angular distribution to 30° for both π⁺ and π⁻ was measured at a Ne bombarding energy of 250 MeV/nucleon. The targets of NaF, KCl, Cu, and U were 10 cm by 10 cm in size and 1-2 g/cm² thick, and the beam spot was about 1 cm in diameter on target. A 180° magnetic spectrometer (Fig. 1) was constructed from a large, flat-field dipole magnet. The target, spectrometer, and detectors were in air, with the beam exiting from vacuum about 2 m before the target. Pion yields at three different momenta were measured at each spectrometer field setting. The momenta were defined by lead collimator slits placed on the focal plane of the spectrometer, and the pions were detected by independent four-element plastic scintillator telescopes placed behind each slit. The collimator openings were chosen to give a range of pion momenta such that all pions stopped well within the third scintillator of the telescope. The relative solid angles and momentum acceptance of the three telescopes were measured with 80 MeV pions,
and the absolute value was determined by computer ray-tracing through the field measured on the median plane of the spectrograph. The absolute value is accurate to only a factor of two at present. The integrated beam intensity was determined with an ion chamber which was calibrated by individual particle counting at reduced beam intensities and by induced $^{14}$C activity.

Because the collimators narrowly limited the momentum range of particles reaching the telescopes, pions were the only particle type which could make a triple coincidence without triggering an anti-coincidence from the back elements. That the particle group stopping in the third scintillator was in fact pions was verified for $\pi^+$ by observing their total energy loss and the $\mu^+$ decay lifetime. The negative pions were characterized by the large range of pulse heights in the third scintillator which was due to stars from nuclear capture. To insure an equal efficiency for $\pi^+$ and $\pi^-$ only the energy loss in the first two detectors and the requirements of any signal in the third and no signal in the fourth element were used.

The data were corrected for $\pi$-decay in flight, absorption in the air, scintillators and absorbers, and multiple scattering. A complete set of corrected $0^\circ$ He data is given in Fig. 2. The angular distribution was measured in $7.5^\circ$ steps between $0^\circ$ and $30^\circ$ at 250 MeV/nucleon on NaF and U targets. It is quite isotropic, and the magnitude of the cross section and the pion energy distributions are in very good agreement with existing data at nearby angles.\textsuperscript{7,8} The target mass dependence of the data is approximately $A^{2/3}$ but differs somewhat at the lower pion energies.
This talk will present possible explanations for the magnitude of the cross section, its beam energy dependence, and the pion spectral shape. The large variations of the $\pi^-/\pi^+$ ratios will also be discussed below, but the angular distributions and the target-mass dependence will not be further treated.

The magnitudes of the pion production cross section can be compared with two simple models, a first-collision Fermi gas model and a thermal model. The Fermi gas model which is described in Ref. 15 was modified to take better account of the isobar dynamics in pion creation as well as momentum dependent absorption of the pions after creation. In the thermal fireball model, the cross section is given by

$$\frac{d\sigma}{d^3p} = \sigma_R \frac{<N_T+N_p>}{\rho_f(2\pi\hbar)^3} e^{-\sqrt{P^2+M_Y^2}/kT}$$

Here $<N_T+N_p>$ is the average number of particles in the overlap region (the value is 9.7 for mass 20 on mass 20), $\sigma_R$ is the heavy ion reaction cross section, and $\rho_f$ is the freeze-out density which is set, following Ref. 17, at $\rho_f = 0.12$ fm$^{-3}$. The temperature $T$ was determined by assuming that the incoming energy is thermalized in an equilibrium mixture of free nucleons and alpha particles. The temperature varies from 16.9 MeV at 125 MeV/nucleon to 55 MeV at 400 MeV/nucleon. As can be seen in Fig. 3 both models are consistent with the data at 125 MeV/nucleon, but at this low energy the thermal model is very sensitive to the temperature. The increase in cross section with bombarding energy is better described by the thermal model than the first-collision model which underpredicts the 400 MeV/nucleon cross section.
by more than an order of magnitude. However, at 400 MeV/nucleon the thermal model fails to account for the rise in cross section with pion energy.

The differences in \( \pi^+ \) and \( \pi^- \) cross sections can be qualitatively understood as consequences of the Coulomb field. In the quantum mechanical expression for the transition rate, \( W \),

\[
W = 2\pi\langle V \rangle^2 \frac{dn}{dE},
\]

and the role of the Coulomb field is to distort the pion wavefunction entering the matrix element \( <V> \), and to alter the energies of the nucleons and therefore the final density of states, \( \frac{dn}{dE} \). The Coulomb wave for \( \pi^- \) is enhanced near the nuclear charge, and the Coulomb wave for \( \pi^+ \) is reduced. The ratio of \( \pi^- / \pi^+ \) due to Coulomb distortion may be estimated as

\[
R_D = \frac{\Sigma(2L+1)F_L^2(-\eta, kR)}{\Sigma(2L+1)F_L^2(\eta, kR)}
\]

where \( F_L \) is the Coulomb wave defined in Ref. 18. A calculation with \( Z=20 \), \( R=4 \text{ fm} \), and with the pion momentum \( k \) computed in the center of mass frame agrees moderately well with the data 250 MeV/nucleon and below, but fails to account for the peak near the beam velocity at 400 MeV/nucleon. For a peripheral collision, it is more appropriate to calculate the Coulomb wavefunction in the coordinate system of the projectile, and choose \( Z \) to be the projectile charge. This is shown in Figs. 4 and 5 for all the Ne + NaF data. It is evident that a large part of the pion production is in a peripheral process.
The ratio $\pi^-/\pi^+$ is also affected by the density of final nuclear states. In the thermal model the exponential factor in the first equation above should include the chemical potential of the pion. The difference of neutron and proton chemical potentials equals the Coulomb energy $U_C$. This gives a ratio from density of final states, $R_{FS} = \exp(2U_C/kT)$. Assuming $U_C = 5$ MeV, we find at 125 MeV/nucleon that $R_{FS} = 0.6$ for the Fermi gas model, and $R_{FS} = 0.5$ for the thermal model. Combined with the wave distortion, we would expect an overall ratio of about unity, but the empirical ratio at 125 MeV/nucleon is $R = 0.6$. At higher energies, the temperature increases, and $R_{FS}$ approaches one.

In conclusion, the magnitude of subthreshold pion production is reasonably consistent with simple models. Important differences between $\pi^-$ and $\pi^+$ production have been observed and are qualitatively described by Coulomb effects due to the source moving with the projectile velocity. It remains to be seen whether a quantitative description can be made that is consistent with the differing features of the data at different energies. Other than the Coulomb field, there is no evidence of any collective fields affecting the pion production as has been suggested.\textsuperscript{19,20}

With respect to the topic of this conference, it can be seen in Fig. 5 that the very strong $\pi^-/\pi^+$ ratios observed at energies of 200 MeV/nucleon and above should also be present for pions of projectile velocity at energies below 200 MeV/nucleon. These pions fell below the acceptance of the present spectrometer, but it is not difficult to conceive of a similar but much smaller apparatus designed to study these pions, the energy of which will
be 10 to 35 MeV in the lab. Such an experiment will help resolve the question of the extent to which subthreshold pions are produced by a first collision, a thermal or some other mechanism at the energies being discussed at this conference. The velocity of the source of pions is clearly one of the basic quantities to be understood in heavy ion collisions, and because pions conveniently come with both positive and negative charges, we now have access to an unambiguous Coulomb effect which is directly related to this velocity.
References


Figure Captions

Fig. 1 A schematic layout of the 180° pion spectrometer used in the present experiment.

Fig. 2 The invariant cross section versus pion energy at 0° for various targets and Ne beam energies (in MeV/nucleon).

Fig. 3 Pion production cross sections for Ne + NaF (0°) as a function of pion rapidity, \( y = \frac{1}{2} \ln \left( \frac{1 + \beta}{1 - \beta} \right) \). The data are shown for \( \pi^+ \) (crosses) and \( \pi^- \) (dots) at bombarding energies for 125, 200, and 400 MeV/nucleon. The solid lines are the prediction of the thermal model at the three energies, and the dashed lines are the prediction of the first-change collision model at 125 and 200 MeV/nucleon. At 400 MeV/nucleon it underpredicts the cross section by too much to make a meaningful comparison.

Fig. 4 Ratio, \( R \), of \( \pi^- \) to \( \pi^+ \) cross sections in Ne + NaF (0°) as a function of pion energy in the projectile frame. The solid curve shows the Coulomb ratio \( R_D \) for a peripheral collision as described in the text.

Fig. 5 The invariant cross section at various beam energies in MeV/nucleon plotted versus kinetic energy in the projectile frame.
Fig. 1
Fig. 2

0° PION PRODUCTION

Ne + NaF

Ne + Cu

Eπ lab (MeV)

E^3,σ/′dp^3 (μb/sr - MeV²/C)

Ne + U

π⁺

π⁻
Ne + NaF $0^\circ$

- $125$ MeV/nucleon
- $150$
- $200$
- $250$
- $400$

$R = \frac{\pi^-}{\pi^+}$

$T_\pi$ (MeV) projectile frame

$R$ vs $T_\pi$ (MeV) projectile frame

Fig. 4
Fig. 5

$0^\circ$ PION PRODUCTION

$\text{Ne} + \text{NaF}$

$E \frac{d^3\sigma}{dp}$ ($\mu$b/sr-MeV$^2$/c)

$T_{\pi}$ (MeV) PROJECTILE FRAME

$\pi^+$

$\pi^-$
Projectile Rapidity Dependence in Target Fragmentation

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Abstract

The thick-target, thick-catcher technique has been used to determine mean kinetic properties of selected products of the fragmentation of Cu by $^1$H, $^4$He, and $^{12}$C ions (180-28000 MeV/amu). Momentum transfer, as inferred from F/B ratios, is observed to occur most efficiently for the lower velocity projectiles. Recoil properties of target fragments vary strongly with product mass, but show only a weak dependence on projectile type. The projectile's rapidity is shown to be a useful variable for quantitative inter-comparison of different reactions. These results indicate that $E_{\text{proj}}/A_{\text{proj}}$ is the dominant parameter which governs the mean recoil behavior of target fragments.

*Work performed at Brookhaven National Laboratory under the auspices of the U.S. Department of Energy and supported by its Office of Basic Energy Sciences.
One of the fundamental motivations for studying the interactions of heavy ions at intermediate energies is to delineate the features of the nuclear reaction mechanism(s). A systematic examination of heavy-ion reactions in the 10 to 200 MeV/amu energy range will connect our extensive knowledge of the low energy region (E < 10 MeV/amu) which is characterized by transfer, compound nuclear, and deeply inelastic scattering processes and our somewhat more limited studies of the fully relativistic energy region, (E > GeV/amu). There are some indications that asymptotic behavior has been observed in peripheral reactions and where there is currently strong interest in central collisions for the possibility that these may prove useful for inducing nuclear shock waves, producing states of abnormally high nuclear density, or as a means of defining the nuclear equation of state under conditions away from those (presently accessible) of near-equilibrium. It is convenient to systematize the nuclear reaction mechanism in the manner shown in Fig. 1 by using the variables of projectile energy and reaction impact parameter. One does not expect abrupt changes to occur in crossing the cross hatched boundaries of Fig. 1 but rather it is reasonable to assume that heavy-ion interactions in the intermediate energy region will reflect transitions from one or more of these domains to different ones. One expects that the nuclear physics in this energy range will be at least as interesting and varied as that at both low and high energy extremes and that it will challenge us to design experiments which are sensitive to the new phenomena and to have as goals the development of theoretical models to interpret them.

Research efforts of nuclear chemists at BNL have been directed toward these points for the last several years. It is a reflection of the successful acceleration of heavy ions to fully relativistic energies that we are approaching the intermediate energy region from the higher energy side.
rather than from the lower energy side! From the broad new areas of investigations which have developed from the use of these high energy heavy ion beams, I will restrict my remarks to a consideration of target fragmentation processes.

In a historical sense our studies of target fragmentation by high-energy heavy ions grew out of the long tradition in the Department of similar investigations with GeV protons. In particular, many of the experimental techniques and methods of interpretation which will be emphasized later in this talk had their origins in early experiments at proton synchrotrons where one frequently had to contend with high energy proton beams of very low intensity. In the quarter century since protons were first accelerated to GeV energies in the now-retired Cosmotron, a large number of target systems have been investigated at various laboratories at energies up to 400 GeV. These studies have involved radiochemical techniques, detection in mica and Lexan, and electronic measurements of fragments by dE/dX, E, and time of flight. There is, therefore, a sizable body of information in the literature on the interaction of GeV protons with complex nuclei. Building upon this base one can therefore inquire as to whether relativistic heavy ions differ from protons in their interactions with complex nuclei.

In the areas of target fragmentation, as opposed to projectile fragmentation where there is no analog in proton induced reactions, our studies have developed from a "first generation" series of experiments which concentrated on the systematics of production cross sections to a series of "second generation" studies of mean recoil properties of the target fragments. In the former case we were interested in testing the applicability to relativistic heavy-ion reactions of the hypothesis of limiting fragmentation and factorization and in the latter case we have attempted to extend these
tests in the area of fragment recoil behavior and to explore to what extent the kinetic properties of target fragments are complementary to those observed previously in projectile fragmentation. Our studies also bear upon a central question in studies of the nuclear reaction mechanism: whether the projectile's kinetic energy or its velocity (related to E/A) is the dominant parameter which characterizes the major features of the interaction.

As a means of orientation to the discussion which follows, it is useful to locate our studies of target fragmentation on the heavy-ion reaction "phase diagram" (Fig. 1). Our measurements concern target fragments which result from medium to large impact parameter. Table I lists the beams we have used at various laboratories in the investigations of reactions with Cu targets. Our measurements refer to the upper, right-hand portion of the phase diagram.

A variety of studies with targets ranging from Cu to U and using residual radioactivity techniques have shown that various relativistic heavy ions (4He, 12C, 20Ne, 40Ar), 0.4-2 GeV/amu, produce essentially the same distribution of final products as do protons of GeV energies.4-10 The relativistic heavy-ion reactions exhibit larger reaction cross sections than those for protons, but the distribution of products (except for very light ones) scales in proportion to this larger geometrical cross section.

These observations, while somewhat surprising at such relatively low energies, support the hypothesis3 of limiting fragmentation and factorization, which in their simplest application to nuclear reactions suggest that, at sufficiently high energy, all strongly interacting projectiles will yield similar product distributions. As an example of this, Fig. 2 shows ratios of product cross sections for the reactions of 80 GeV 40Ar and 28 GeV 1H.
with Cu. The cross-hatched band shows the region consistent with the ratio of total reaction cross sections and its associated error. The factoring of the individual product cross sections according to the ratio of the total reaction cross sections for 28 GeV $^1$H and 80 GeV $^{40}$Ar on Cu is a general feature for products which correspond to the removal of up to $\approx$40 nucleons from the target (i.e., down to $^{22}$Na). Still lighter products, e.g., $^7$He, appear to be significantly enhanced for the heavy-ion induced reaction over the proton induced one. Data of the type shown in Fig. 2 can be used to develop both charge dispersion and mass yield plots as shown in Figs. 3 and 4, respectively. The charge dispersion analysis shows that the data for $^{40}$Ar + Cu fall on the same curve as those for $^1$H + Cu and that the mass yield curves for 80 GeV $^{40}$Ar, 25 GeV $^{12}$C, and 28 GeV $^1$H reactions on Cu have essentially the same shape and slope for all three projectiles. The slope of the mass yield curve is related to excitation energy which is delivered to the target nucleus. Low excitation energy (rapidly decreasing cross section for increasing multinucleon removal, $\Delta A$) is correlated with large slope of the mass yield curve, while high excitation energy (less rapid decrease in cross section as $\Delta A$ increases) is correlated with smaller slope. Figure 5 shows this variation in slope for Cu spallation by protons and energetic heavy ions. At $\approx$1-2 GeV we see the onset of a saturation phenomenon in that the Cu target is no longer able to absorb additional excitation energy, and at even higher energies the fragmentation pattern is not changed by either different projectile types or their kinetic energy. Similar data are not yet available for heavier target systems. It would be interesting to know, for example, if heavier target systems show the same limiting slope behavior at $\approx$1-2 GeV, or whether this occurs at higher...
energy due to the ability of heavier targets to support more excitation as a result of their greater mass.

While limiting fragmentation behavior has also been observed\textsuperscript{3} in the Cu target systems with lower energy \textsuperscript{14}N (280 MeV/amu), it is clear that the strict factorization of the cross sections as shown in Fig. 2 will eventually break down at some lower projectile energy when deeply inelastic scattering, compound nucleus formation, or other processes become the dominant reaction mechanisms. It is therefore of considerable fundamental interest to explore the energy region below \approx 200–300 MeV/amu to see where the cross section factorization regime fails and to inquire whether this failure is observed at approximately the same projectile energy or velocity for all projectile-target combinations.

Models of the interaction of relativistic heavy ions (such as the abrasion-ablation model\textsuperscript{10}) suggest a monotonic relationship (on the average) between impact parameter and reaction product, with the simplest (smallest mass loss) being associated with the largest \( b \), and vice versa. Based on the procedure of Barshay et al.\textsuperscript{12,13} in which

\[
\frac{d\sigma}{db} = 2\pi b [1 - \exp(-T(b)T_{\text{NN}})] db,
\]

we have calculated the contributions of different ranges of impact parameters to \( \sigma_R \). In this equation, \( T(b) \) is the thickness function [as defined by Eq. (2.3) of Ref. 13] which includes all the geometrical properties of the colliding nuclei, and \( T_{\text{NN}} \) is the spin and isospin-averaged nucleon-nucleon cross section at 2 GeV/nucleon. This calculation predicts that impact parameters of \approx 8.2 fm make the largest contributions to \( \sigma_R \). For closer approaches, the nuclei are black to each other and the \( b \) term in the above equation is dominant. For larger \( b \), transparencies become significant,
rapidly reducing the contributions, yet some reactions at impact parameters in the 10-12-fm range are expected. We note that the most effective impact parameters are larger than the sum of the half-density radii of $^{40}$Ar and $^{63,65}$Cu, 3.39 and 4.23 fm, respectively, so that peripheral collisions are expected to play important roles. Impact parameters $\geq$8.3 fm contribute a cross section equal to the estimated yields of products with $A \geq 58$, the upturning region of the mass yield curve in Fig. 4. Furthermore, the $\approx$2160 $\omega$b deduced for target fragmentation residues having $A \geq 22$ are accounted for by collisions with $b \geq 5.25$ fm. Shown in Fig. 6 are realistic nuclear density distributions for Cu and $^{40}$Ar at this impact parameter.

There is little overlap of the central cores of the projectile and target even at this separation and in this sense target fragmentation is peripheral. Major parts of the Ar and Cu remain relatively undisturbed to ultimately yield the fragmentation products. Presumably events with smaller $b$ and greater overlap will result in increasingly violent interactions.

A more detailed view of heavy-ion reaction mechanisms in the intermediate-energy region can be obtained from the measurement of the energy and momentum transfer processes of these interactions. It is in these areas that our work has concentrated most recently. Because of the limitations imposed by low beam intensities, the thick target, thick catcher technique which was originally developed for high energy proton beams has been employed to survey the momentum transfer to target fragments from Cu for a variety of proton and heavy-ion projectiles over a broad energy range. Figures 7 and 8 show typical configurations of target stacks that are used in these experiments. Fragments which recoil from the Cu target are stopped in Mylar catchers. The quantities $F$ and $B$ denote the fractions of the activity of a particular nuclide formed in a Cu target of w mg/cm$^2$ thickness that were found
in the forward and backward catchers, respectively, of the stack perpendicular to the beam.

Before we try to interpret the results of such measurements in a detailed way which requires a model for the interaction, it is instructive to look merely at the ratio of $F/B$ for several products from Cu. We have chosen to use $^{24}\text{Na}$, $^{28}\text{Mg}$, $^{44}\text{Sc}$, $^{48}\text{V}$, $^{52}\text{Mo}$, and $^{58}\text{Co}$ as a convenient set of products, distributed in mass, which have both abundant $\gamma$-rays in their decays for identification purposes and long enough half-lives so their yields can be determined readily by assay of residual radioactivity following the target stack bombardment. These products, in a crude way, also sample a range of excitation energies in the heavy-ion interaction in that a product like $^{58}\text{Co}$ which is formed by the removal of a few nucleons from Cu represents only modest excitation of the target while $^{24}\text{Na}$ formed by the loss of $\approx 40$ nucleons from Cu results from the deposition of larger amounts of excitation energy into the target.

The ratios of $F/B$ for several different projectile-energy combinations are listed in Table II. We also include earlier data from Crespo et al.\textsuperscript{14} As expected, the ratios are greater than unity, indicating that more of the products are found in the forward hemisphere, ahead of the target, than behind it. For a particular product, a smooth trend of decreasing $F/B$ ratio is evident if the table is ordered by $E_{\text{proj}}/A_{\text{proj}}$ (related to the square of the projectile velocity) rather than by the total kinetic energy of the projectile. Since the $F/B$ ratio reflects the extent of momentum transfer from projectile to target, it is apparent that the projectile's velocity and not its type (proton, alpha, or $^{12}\text{C}$) is the governing factor in the momentum transfer process. For a particular projectile-energy combination the $F/B$ ratio generally increases with increasing product mass and then drops appreciably
for $^{58}\text{Co}$. The drop for the $^{58}\text{Co}$ data is presumed to be a reflection of changes in the range-energy relationship for low-velocity near-target products like $^{58}\text{Co}$. It is clearly of interest to extend these measurements to lower projectile velocities to explore more fully the processes which are responsible for the momentum transfers we see here. It would also be instructive to include both additional types of projectiles as well as other targets to get a more complete picture of these phenomena.

Additional information can be obtained from the present data by a model dependent analysis which assumes that the velocity distribution of a product can be resolved into two components (see Fig. 8), a forward directed $v_\parallel$ resulting from the initial projectile-target interaction, and a $V$ isotropic in the moving system, arising from the subsequent deexcitation of the prefragment to yield the observed products. This so-called two-step model is equivalent to the abrasion-ablation model proposed for high-energy, heavy-ion reactions. Details of how $v_\parallel$ and $V$ are derived from the experimental data can be found in the review article of Alexander and elsewhere. The mean range $R$ is related to $2v(F+B)$, and $v(F-B)$ is related to $n_\parallel R$, where $n_\parallel = v_\parallel /v$. It is also assumed that the mean range is related to $V$ through an exponential dependence, $R = V^N$.

Figures 9 and 10 show plots of $\beta_\parallel = v_\parallel /c$ and $V$, respectively, that were derived from our data. It is convenient and informative to plot these results as a function of the rapidity $\gamma$ ($-\tanh^{-1} \beta$) of the projectile. A number of interesting features are apparent from these figures. For a given projectile-energy combination $\beta_\parallel$ varies strongly with product mass, higher values of $\beta_\parallel$ being observed for the lighter products (e.g. $^{24}\text{Na}$), and smaller values of $\beta_\parallel$ for the near-target products (e.g. $^{58}\text{Co}$). The observed values of $\beta_\parallel$ are low and of the same order as the velocity retardation reported for
projectile fragments. We are therefore observing complementary behavior in the projectile and target fragmentation regimes which reflects the indistinguishability of the two interacting nuclei in their center of mass frame. This suggests that fragments even as light as $^{24}$Na arise from relatively unexcited parts of the target nucleus. One also notes that $\beta_{||}$ varies monotonically with respect to the number of nucleons removed from the target. For a given product $\beta_{||}$ increases rapidly for the slower moving projectiles, which indicates that the lower velocity projectiles transfer their momenta more effectively to the target system. The plots of $V$ (Fig. 10) give us some insight into the degree of excitation in the prefragments which are assumed to deexcite by light particle evaporation to yield the final products, shown in the figure. For a given projectile-energy combination we again see that the lighter products display the higher velocity behavior we saw in the $\beta_{||}$ results and that variation of $V$ with product mass is remarkably similar to Fig. 9. However, the variation of $V$ with projectile rapidity (velocity) is quite small with just a hint of the beginnings of an upturn for 720 MeV $^4$He (180 MeV/amu). The $\beta_{||}$ results suggest that heavy-ion projectiles at low rapidity ($E/A \approx 200-400$ MeV/amu) are effective in transferring momentum to excited prefragments by projecting them forward in the initial projectile-target interaction. The excitation of the prefragments, however, as inferred from $V$, does not seem to correlate well with either the type or energy of the incident projectile. Fully relativistic heavy ions, e.g. 25 GeV $^{12}$C, reveal only small differences when compared to high energy (28 GeV) protons, in accord with concepts of limiting, asymptotic behavior mentioned earlier.

The results from reactions with Cu target system cannot, in themselves, form a basis for constructing a detailed model of the interaction of heavy nuclei at intermediate to high energies. Clearly one needs to investigate
reactions with other targets in a similar way. Investigations with Au targets have been reported recently and other investigations with Ag, Ta, and U targets are in progress. Clearly, the measurements of product angular distributions and energy spectra will give us the comprehensive, detailed information which is crucial for testing, refining, or rejecting theoretical models of the mechanisms of these heavy-ion interactions. Unfortunately the low intensity of the presently available beams makes experiments of this type very difficult.

In summary, the use of heavy ions of intermediate energy holds the promise of providing a means for the systematic study of target fragmentation processes with the eventual goal of refining models of the mechanism(s) of the interaction of heavy nuclei in this bridging energy region. In particular a number of fundamental questions can be addressed:

1. At what projectile energy (or velocity) will the predictions of limiting fragmentation and factorization break down? What will be responsible for this breakdown? Will it occur at the same projectile energy (or velocity) for all projectile-target systems?

2. Why are the recoil properties of target fragments, such as momentum transfer, closely correlated with projectile rapidity (velocity) instead of the projectile's total energy? Will this occur in other projectile-target systems? Will this correlation continue at lower velocities?

3. Will projectile-target systems of large mass asymmetry (e.g. very light projectiles like τ-mesons or heavy-ion projectiles of mass greater than the target) display the same features of the systems already studied?
(4) Is the two-step (abrasion-ablation) model valid over a wide energy range? Will it break down at lower energies? What will be the cause of this and what will be the signature of the new processes which take over at these energies?

(5) What are the best measurements to perform in the next generation of experiments so that additional constraints can be applied to refining theoretical models of the interaction of heavy nuclei at intermediate energies?
19. N. T. Porile (private communication).
20. W. Loveland (private communication).
Table I

Proton and Heavy-Ion Beams used in the Study of the Spallation of Cu Targets

<table>
<thead>
<tr>
<th>Ion</th>
<th>E (MeV)</th>
<th>E/A (MeV/amu)</th>
<th>Laboratory</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^4$He</td>
<td>720</td>
<td>180</td>
<td>SREL</td>
</tr>
<tr>
<td>$^{14}$N</td>
<td>3900</td>
<td>280</td>
<td>PPA</td>
</tr>
<tr>
<td>$^{12}$C</td>
<td>4800</td>
<td>400</td>
<td>LBL</td>
</tr>
<tr>
<td>$^4$He</td>
<td>4000</td>
<td>1000</td>
<td>LBL</td>
</tr>
<tr>
<td>$^{40}$Ar</td>
<td>80000</td>
<td>2000</td>
<td>LBL</td>
</tr>
<tr>
<td>$^{12}$C</td>
<td>25200</td>
<td>2100</td>
<td>LBL</td>
</tr>
<tr>
<td>$^1$H</td>
<td>3900</td>
<td>3900</td>
<td>BNL</td>
</tr>
<tr>
<td>$^1$H</td>
<td>28000</td>
<td>28000</td>
<td>BNL</td>
</tr>
</tbody>
</table>
Table II
F/B Ratios of Selected Target Fragments from the Spallation of Copper by Various Projectiles

<table>
<thead>
<tr>
<th>Projectile</th>
<th>Energy (MeV)</th>
<th>Energy (MeV/A)</th>
<th>$^{24}$Na</th>
<th>$^{28}$Mg</th>
<th>$^{48}$Sc</th>
<th>$^{48}$V</th>
<th>$^{52}$Mn</th>
<th>$^{58}$Co</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^4$He</td>
<td>720</td>
<td>180</td>
<td>9.1 (25)</td>
<td>--</td>
<td>29.6 (20)</td>
<td>23.1 (33)</td>
<td>35.5 (35)</td>
<td>14.7 (35)</td>
</tr>
<tr>
<td>$^4$He$^*$</td>
<td>880</td>
<td>220</td>
<td>8.23 (29)</td>
<td>8.80 (66)</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>$^{12}$C</td>
<td>4800</td>
<td>400</td>
<td>8.62 (22)</td>
<td>8.41 (72)</td>
<td>12.8 (03)</td>
<td>12.3 (04)</td>
<td>11.4 (03)</td>
<td>6.42 (03)</td>
</tr>
<tr>
<td>$^1$H$^*$</td>
<td>700</td>
<td>700</td>
<td>3.37</td>
<td>4.13</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>$^4$He</td>
<td>4000</td>
<td>1000</td>
<td>4.25 (29)</td>
<td>2.83 (68)</td>
<td>4.86 (19)</td>
<td>4.31 (59)</td>
<td>4.05 (30)</td>
<td>2.96 (50)</td>
</tr>
<tr>
<td>$^{12}$C</td>
<td>25200</td>
<td>2100</td>
<td>2.63 (11)</td>
<td>2.69 (34)</td>
<td>3.37 (12)</td>
<td>3.21 (19)</td>
<td>3.21 (26)</td>
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<td>$^1$H$^*$</td>
<td>3000</td>
<td>3000</td>
<td>2.79</td>
<td>3.02</td>
<td>--</td>
<td>--</td>
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<tr>
<td>$^1$H</td>
<td>28000</td>
<td>28000</td>
<td>2.20 (05)</td>
<td>2.20 (11)</td>
<td>2.77 (06)</td>
<td>2.72 (06)</td>
<td>2.82 (06)</td>
<td>2.36 (10)</td>
</tr>
</tbody>
</table>

*Reference 14.
Figure Captions

1. A "Phase Diagram" for the Heavy Ion Reactions (from Reference 1). Reactions which are induced by heavy ions with energies of 10-200 MeV/A are expected to exhibit some or all of the features in the central part of the diagram.

2. Cross section ratios vs product mass for the reactions of 80-GeV $^{40}$Ar and 28-GeV $^1$H with Cu. The cross hatched band shows the region consistent with the ratio of total reaction cross sections and its associated error.

3. Comparison of charge dispersion curves for the spallation of Cu by 80-GeV $^{40}$Ar and 28-GeV $^1$H. Points were obtained by the procedure described in Reference 6.

4. Mass yield curves for the spallation of Cu by 80-GeV $^{40}$Ar, 25-GeV $^{12}$C (Reference 6), and 28-GeV $^1$H (Reference 6). The vertical scale for $^{12}$C is arbitrary. Points were obtained using the procedure of Reference 6 by adding computer generated values for non-observed products to the measured cross sections. These are filled when >50% of the total was observed, open when >20% observed. Errors include 20% uncertainties assumed in the calculated values.

5. Slope of the Cu spallation mass-yield curve as a function of the kinetic energy of the incident projectile. Filled circles are for protons, open for heavy ions as indicated.

6. Nuclear density distributions for Cu and $^{40}$Ar showing the extent of overlap when the separation of centers is 5.25 fm.
7. Pictorial view of a typical thick-target/thick-catcher irradiation geometry. The beam enters from the left and first transverses a target at 90° to the beam. The beam then passes through a similar target stack which is inclined at 20° to the beam direction.

8. Schematic view of the thick-target/thick catcher assembly. The lower portion of the figure shows the resolution of the velocity vector of a target fragment into an initial \( v_\parallel \) resulting from the projectile-target interaction and a \( V \) (assumed to be isotropic) resulting from the deexcitation of excited prefragments which eventually yield the observed products.

9. Dependence of \( S_\parallel (= \frac{v_\parallel}{c}) \), the first (abrasion) step velocity, as a function of projectile rapidity for several target fragments from Cu.

10. Dependence of \( V \) the second (ablation) step velocity, as a function of projectile rapidity for several target fragments from Cu.
Fig. 2
B,F BACKWARD AND FORWARD CATCHERS
T TARGET (w mg/cm^2)
B1 BLANK CATCHER

\[ \langle \text{RANGE} \rangle = 2w(F+B) \]
\[ \langle \eta_\parallel R \rangle = w(F-B); \quad \eta_\parallel = v_\parallel / V \]
\[ R \propto V^N \]
Fig. 9
\[ V(\text{MeV/amu})^{1/2} \]

**Fig. 10**

- 4.8-GeV \(_{12}^2\text{C}\)
- 25-GeV \(_{12}^2\text{C}\)
- 28-GeV \(_{1}^1\text{H}\)
- 24\text{Na} + 28\text{Mg +}
- 44\text{Sc}
- 48\text{V}
- 52\text{Mn}
- 56\text{Co}
- 0.72-GeV \(^{4}\text{He}\)
ABSTRACT

The main features of the tandem-linac system for heavy-ion acceleration are reviewed and illustrated in terms of the technology and performance of the superconducting heavy-ion energy booster at Argonne. This technology is compared briefly with the corresponding technologies of the superconducting linac at Stony Brook and the room-temperature linac at Heidelberg. The performance possibilities for the near-term future are illustrated in terms of the proposed extension of the Argonne booster to form ATLAS.

I. INTRODUCTION

I have been asked to give a brief review of the characteristics of linacs used as energy boosters for heavy-
ion beams from tandem electrostatic accelerators. Three linacs of this kind need to be considered: (1) the superconducting linac at Argonne, now a useful (but incomplete) machine, (2) the room-temperature linac at Heidelberg, also in use and incomplete, and (3) the recently funded superconducting linac at Stony Brook, being built in collaboration with a group from Cal Tech. Although these three projects make use of rather different technologies, they all have the same basic objectives — to extend the energy range of the tandem without much loss of beam intensity or beam quality. That is, the characteristics of these tandem-linac systems are all aimed at the needs of precision high-resolution nuclear-structure physics of the kind that could be carried out with a very large tandem, if one could afford to buy such a tandem.

II. BASIC FEATURES OF A TANDEM-LINAC ACCELERATOR SYSTEM

Before starting to discuss the characteristics of any particular tandem-linac system, let us make sure that the basic acceleration process is understood. Figure 1 is a schematic of the main elements in a tandem-linac system. The tandem is operated in the usual way, with a negative-ion source near ground potential and a stripper in the high-voltage terminal.
Before injection into the linac, the ion beam from the tandem is usually stripped a second time in order to increase the charge state. However, if the linac has enough accelerating power to provide the desired output-beam energy at the original charge state, then the second stripper is not used, thus increasing the beam intensity. Several schemes for three-stage stripping are feasible, but there is rather little interest in them because of the loss in beam intensity.

In order to preserve the beam quality of the tandem, the linac requires an injected beam that is bunched into very narrow pulses (< 100 ps) synchronized to the rf frequency of the linac. Also, because of the low intensity of a doubly-stripped tandem beam, the bunching process must be carried out without much loss of intensity. These demanding objectives (narrow pulse and high efficiency) are achieved by two-stage bunching: (1) a harmonic buncher before the tandem puts most of the dc beam from the source into pulses that are about 1 ns wide at the tandem output, and then (2) a more powerful post-tandem resonator compresses the tandem pulse to the desired 100-ps width. A beam chopper removes unbunched particles.

A new and essential feature of the bunching system is the need to correct continuously for the effect of uncontrolled variations in the transit time of the beam through the tandem. This is achieved by sensing continuously the rf phase with
which the beam bunch arrives at the post-tandem buncher and by using this information to control the phase of the pre-
tandem buncher. Since beam bunches may not be intense enough to be detected individually and nondestructively, the phase detector needs to be able to sense the integrated effect of many pulses.

A distinctive feature of the linacs of interest for this paper is that they all consist of an array of many short, independently-phased fixed-frequency rf resonators; the rf frequencies involved in the several linacs range from 97 to 152 MHz. Because each resonator has only a few (two or three) accelerating gaps, it can accelerate effectively over a fairly wide range of ion velocity, as seen in Fig. 2. The velocity profile of the system of resonators is established by adjusting the phase of each resonator to match the phase of the beam incident on it. The output beam energy can be varied easily and rapidly by varying the phase and/or accelerating field of the last resonator.

Normally, the linac is operated in a phase-focussing mode in which a beam-energy excursion tends to be corrected by means of its interaction with the phase. The degree to which the incident-beam quality is preserved depends in a complicated way on this phase-focussing effect and on the extent to which the accelerating field varies linearly with time over the width of the beam pulse. This linearity requirement, which
is rather extreme, is another distinctive feature of the new heavy-ion boosters and is the reason that the incident beam pulse should be exceptionally narrow.

Radial excursions of the beam are controlled by focusing elements located frequently along the linac. These transverse lenses are needed to counterbalance the defocusing effect of the resonators but especially to control the unavoidable tendency of a beam with non-zero emittance to diverge. In order to minimize non-linear effects in both transverse and longitudinal (energy-time) phase space, the beam diameter should be much smaller than the diameter of the drift-tube aperture.

Because of the rather small size of the beam within the linac, easily achieved because of the good emittance of a tandem beam, the beam transmission of the linac is essentially 100 percent. Thus, the intensity of the beam out of the linac depends primarily on the performance of the ion source and tandem and on the stripping process. For double stripping, the output-beam intensity (particles per sec) can be in the range 2 to 4% of the intensity injected into the tandem, for very heavy and lighter ions, respectively.

Although the essential quality of the incident beam can be preserved in the linac, the acceleration process is such that the output beam may have a rather large energy spread. Also, even if the beam pulse is very narrow at the linac
output, it may be greatly broadened by the time it reaches a down-stream experiment. Thus, in order to benefit from the inherent beam quality (small product $\Delta E \Delta t$), one must debunch the beam (increase $\Delta t$ and decrease $\Delta E$) if a small energy spread is desired and rebunch the beam if a narrow pulse is desired by the experiment. A single resonator well downstream from the linac can perform either of these functions, and such a debuncher/rebuncher should be considered to be an integral part of a tandem-linac accelerator system.

III. BRIEF DESCRIPTION OF ANL SUPERCONDUCTING LINAC

The linac of a tandem-linac system was described above in terms of rather general ideas related to performance. In order to give a taste of the hardware required to achieve this performance in practice, let me describe briefly the Argonne superconducting booster$^{1-4}$. The layout of the accelerator system is shown in Fig. 3. The tandem is an upgraded FN tandem, the booster is located in a former target room, and the beam from the booster goes into a small new experimental area*. The bunching system between the tandem

*Figure 3 does not show the proposed ATLAS system, which involves an extension of the linac and the construction of a large new target area (see Fig. 12).
and the linac has the general characteristics described in section II and is described in detail in Ref. 5 and 6.

A schematic representation of the booster as it is expected to be in late 1980 is shown in Fig. 4. The heart of the system is the split-ring resonator, a three-gap structure made of superconducting niobium. Superconducting solenoids at frequent intervals confine the radial excursions of the beam. The basic accelerating section of the linac consists of a linear array of these resonators and solenoids within a cryostat that can be isolated from the others both with respect to vacuum and cryogenics.

The four sections of the booster make use of resonators that have two different lengths. One type is 35.6 cm long and is optimized for a projectile velocity $\beta = v/c = 0.105$ (sections C and D). A second type is 20.3 cm long and is optimized for $\beta = 0.060$.

Each resonator consists of an inner drift-tube assembly made of pure niobium and a housing made of sheet niobium that is explosively bonded to copper, as shown in Fig. 5. The rf power dissipation into liquid helium is typically 4 watts per resonator. The inner assembly is cooled by 4.8 K liquid helium within the hollow loading tubes and drift tubes, and heat generated in the housing is conducted to a helium-cooled heat sink through the copper backing of the bonded niobium.
RF power is fed to the resonating drift-tube assembly from a 150-watt solid-state rf amplifier by means of capacitive coupling from a 3/8-in diameter superconducting probe. Fast tuning is achieved by means of a high-power voltage-controlled reactance (VCX), which is used to lock the rf phase of each resonator to the phase of a master oscillator.

The performance characteristics of the high-β resonators are given by Fig. 6. The design aim is an average accelerating field of 4.25 MV/m for a power loss of 4 watts, which implies a voltage gain of 1.5 MV (i.e., 1.5 MeV per charge) from each unit. Note that the performance of individual resonators is at this goal. The resonators in the booster are initially being operated at a somewhat lower field, in the range 3.0 to 3.5 MV/m, and will gradually be pushed up to the design goal when several limitations have been removed. The accelerating field of the 20-cm units is about the same (for the same power dissipation) as the field in the larger units but, of course, the integrated voltage gain of the shorter unit is smaller.

The resonators are cooled to a temperature of about 4.8 K by means of flowing two-phase helium in a closed circulating system. The driving pressure for the flow is the refrigerator itself, which (with three compressors) supplies nominally 95 watts of cooling and a flow rate of 7 gm/s at 4.6 K.
The superconducting solenoids used to limit the transverse excursions of the beam are hybrid magnets consisting of a superconducting coil and a soft iron return yoke and shield. The measured peak field is 7.6 Tesla; and the length of the coil is chosen to give a focussing power \( P_g = \int B^2 dz \) that is strong enough not only to counterbalance the defocusing action of the resonators but also is strong enough to allow the beam to be focussed to a waist between each pair of solenoids. Flowing liquid helium cools the solenoids in the same way as the resonators.

All of the cryostats for the booster are end-loading units, and except for section A, all are of the same size. An assembly drawing of a cryostat with resonators in place is shown in Fig. 7, and an impression of an accelerator section during assembly is given by Fig. 8.

In each cryostat, the array of resonators is surrounded by a nitrogen-cooled heat shield and, outside of it, a vacuum wall (see Figs. 4 and 7). Even though the interior of the resonator is open to the outer vacuum region, including the warm outer vacuum wall, the pressure inside the resonators is extremely low (\(<10^{-8}\) Torr) during operation because of cryopumping on the outer surfaces of the resonators.

Each cryostat can be isolated from the others and removed from the beam line without disturbing the cooling or vacuum of the tanks remaining on line. Once off line, the whole inner assembly of an accelerator section can be rolled out
the end of the cryostat, and all disassembly is then done in
the open. When a section is ready to be put into service, it
can be cooled down off line, completely tested, and finally
moved on line while still cold. While the maintenance of a
section is carried out off line, the sections remaining on
line can be used for acceleration.

Both the booster and the bunching system are controlled
with the assistance of an 11/34 model PDP computer, which
interacts with CAMAC crates by means of serial instructions.
In general terms, hard-wired feedback circuitry is used to
control resonator phase and amplitude on a fast time scale,
whereas the computer sets the reference values and monitors
and controls phase and amplitude on a slow time scale.
Similarly, the computer sets and monitors the solenoid fields.
For other parameters, such as temperature and vacuum pressure,
the computer provides only monitoring. And finally, the
computer is used to record and analyze beam diagnostic
information, and this makes it possible to tune the linac
rapidly.

The beam from the linac passes into a small new target
room that houses a large new scattering chamber, an existing
spectrograph, and various specialized reaction chambers. A
debunching/rebunching resonator on the main beam line mani-
pulates the phase ellipse of the output beam to meet the needs
of the experimenter.
IV. STATUS OF ANL BOOSTER PERFORMANCE

From the point of view of most members of this audience, the most important news about the superconducting linac is, I suppose, that it is now a working reality for heavy-ion acceleration. The four-section ANL booster described by Fig. 4 is some eighteen months from completion but, because of its modular characteristics, the two completed sections are already being used to provide useful beams for nuclear-physics research. The first beam-acceleration tests, with two resonators, were made in June 1978, and this small beginning has by now progressed to the almost routine operation of the accelerator system shown in Fig. 9. In total, some 1800 hours of beam time have been logged.

In the most recent series of runs, carried out with eight resonators throughout the month of June 1979, four different nuclear-physics experiments were performed. Three ion species were accelerated, as summarized by Table I. The maximum energies achieved imply that, on average, the resonators provide 1.2 MV of accelerating potential, yielding a total of 9.3 MV. Equivalently, the average accelerating field within the resonators is 3.3 MV/m. The energy performance of the tandem-linac system as a whole may be summarized by the statement that it is equivalent to that of a 15-MV tandem with two stripper for ions with $A \geq 40$. 
As many of you are aware, one of the most difficult problems connected with the use of superconducting resonators has been to control the influence of mechanical vibrations on the resonant rf frequency and hence on the rf phase. This problem has recently been solved in the ANL resonators by simultaneously using two control techniques: (1) negative phase feedback with stored energy from a voltage-controlled reactance is used to lock the resonator phase to the master oscillator, and (2) amplitude modulation and electromechanical coupling is used to dampening mechanical motion. During the June 1979 runs, all eight resonators were in phase lock about 99% of the time, and almost all out-of-lock time was generated by occasional malfunctions by two of the eight resonators.

As a result of the recent breakthrough in phase control, the superconducting booster now runs with great reliability and has been operated for many long periods of time (~24 hours) without human intervention. Throughout the set of runs in June 1979, the system was operated by the beam users much of the time, with linac-development personnel available on an on-call basis. Even the procedures involved in changing beam energy are simple enough that the user can carry them out after a few minutes of instruction.

A very attractive feature of the independently-phased linac is that, because of its modular design and because of the flexibility provided by independent phase, almost any resonator configuration can accelerate a beam, and hence
the linac can provide useful beams long before the system is completed. The past and projected performance of the ANL booster at several stages of completion are summarized by Fig. 10. The present eight-resonator system is useful mainly for ions with $A < 40$, since the high-$\beta$ resonators now in use cannot effectively accelerate the slow-moving ions of greater mass. The next step, scheduled for completion in October, will be to add four more resonators and thus increase the beam energy substantially. And finally, the mass range will be extended, first by putting four low-$\beta$ resonators in section A (early 1980) and then by adding section B, which ultimately will have seven low-$\beta$ resonators.

V. COMPARISON OF LINAC POST-ACCELERATORS

Time limitations do not permit me to give descriptions of the Stony Brook-Cal Tech and the Heidelberg linacs except in the bare outline given by Fig. 11 and Tables II and III. The planned sizes of the systems are summarized in Fig. 11, where the length drawn for each linac is proportional to its accelerating voltage and where the status of construction and funding is indicated. Note that, because of its more powerful injector, the Heidelberg linac provides a substantially higher beam-output energy, for a given linac voltage, than do the other two systems.

Tables II and III provide the basis for a comparison of the technologies and the costs of the three boosters. The
main thrust of the ANL design has been to develop the ultimate in resonator and linac performance. The best available rf superconductor (niobium) was chosen so as to minimize rf losses on the helium-cooled surfaces, and the rf frequency was made as low as feasible so as to minimize the difficulty of beam bunching. Both decisions have resulted in severe developmental and fabrication problems. Now that the technology is well developed, however, one need consider only construction and operating costs, and operational effectiveness.

The individual ANL resonators are very costly but, because of the large energy gain provided by each unit, the overall system cost is competitive with other designs. The small number of units required to form a complete booster is expected to be advantageous from the point of view of operational ease and reliability. The low rf frequency and large drift tubes help preserve the incident beam quality.

The recently funded Stony Brook linac\textsuperscript{10,11} will also consist of superconducting split-ring resonators\textsuperscript{12}, but the superconductor is lead plated on a copper base, a technology developed at Cal Tech. As in the Argonne design, two sizes of resonators will be used: 16 units with $\beta = 0.055$ and 21 units with $\beta \approx 0.10$.

The lead-plated resonators cost much less than the ANL niobium structure, but the higher surface resistivity and the low critical magnetic field of lead are drawbacks that have important impacts on the resonator design. In particular,
the properties of lead result in a design with (1) a relatively high rf frequency (152 MHz), (2) very small drift tubes, and (3) a simple circular form for the loading arms that support the drift tubes. These characteristics make resonator fabrication, helium distribution, and phase control considerably easier than for the ANL design. On the other hand, more helium refrigeration is required, and the smaller resonator size and lower accelerating fields measured to date combine to give an energy gain that is less than half that of the ANL resonators. Thus, although I do not have complete information about the projected costs of the Stony Brook linac, it appears that the overall construction costs per MV of acceleration are roughly the same for the two superconducting systems. The extent to which the higher rf frequency and the smaller drift tubes of the Stony Brook resonators are disadvantageous for beam quality remains to be seen from operational experience.

Although the technology involved in the Heidelberg room-temperature linac is entirely different from that of the superconducting linacs, the basic concepts are similar. The Heidelberg linac$^{13,14}$ consists of an array of independently-phased resonators of the spiral type, a two-gap structure with a single small-diameter drift tube. Because of the rather broad velocity acceptance of a two-gap accelerating structure (see Fig. 2), only one size of resonator is needed for a 10-MV booster of ions in the lower half of the periodic table.
The dominating cost of the room temperature linac is the cost for the rf-power transmitters. Consequently, the main emphasis of the Heidelberg design has been to minimize overall costs by using a 20-kw transmitter (a standard size in the communications industry) to drive each resonator and by optimizing resonator performance for this particular power level. The result is a system in which the accelerating field is relatively small and energy gain per resonator is small for continuous-wave operation, but the overall linac cost is lower than it would be if the accelerating field (and hence the power dissipation) were substantially greater.

From our present prospective, the main drawbacks of the Heidelberg linac are its construction and operating costs, which are both substantially greater than for an equivalent superconducting linac. On the other hand, the fact that all components are obtainable commercially is an immense asset for most laboratories. The operational disadvantage of having very many independently controlled units (because of the low energy gain per resonator) tends to be counterbalanced by the easy accessibility of all components in a room-temperature system; also, many nuclear physicists seem to regard high-power rf technology as being easier to master than superconducting rf technology, but this may be a matter of taste.

A clear operational advantage of the room-temperature device is that the maximum beam energy can be extended
substantially by operating the resonators in a pulsed mode. For example, the accelerating voltage of the Heidelberg resonators increase from 0.33 to 0.60 MV when they are pulsed with a 25% duty factor to a power level of 80 kW. Superconducting resonators can also be operated in a pulsed mode, but this capability is of little interest because, at the customary operating field, power loss increases so very rapidly with increasing field (see Fig. 6). On the other hand, the superconducting devices have a greater potential for future improvements in CW operation.

VI. THE ATLAS PROPOSAL

All of the tandem-linac systems described above are the first of their kind, prototypes of what is likely to be a steady stream of future accelerators. The most immediate of these future projects is the Argonne Tandem-Linac Accelerator System (ATLAS)\(^4\), which has been favorably reviewed for funding in FY1981. Here I will mention only those features of ATLAS that illustrate some interesting aspects of the tandem-linac accelerator concept.

The overall layout of ATLAS is shown by Fig. 12. The present tandem and the 4-section booster described earlier will continue to be used in their original configuration. ATLAS involves the addition of three more linac sections and the construction of a large new target area.
The mass-energy performance of ATLAS is given by the upper curve of Fig. 13, where it is assumed that a gas stripper is used in the tandem terminal for $A > 50$. To some extent, the shape of the performance curve (which depends on resonator characteristics) is a matter of choice, especially with regard to the location of the mass cutoff. For ATLAS, it has been decided to emphasize the acceleration of ions in the lower half of the periodic table, and consequently the new resonators will be designed for the relatively high velocity $\beta \gtrsim 0.135$. On the other hand, because of the modular nature of the cryostats, it would be feasible at a later time to extend the mass range by adding more low-$\beta$ sections and by moving the various sections to the locations required to give the desired ion-velocity profile. The dashed extension of the upper curve illustrates the performance that could be achieved for an addition expenditure of about $1.2$ million.

One of the most interesting aspects of ATLAS is the fact that it will be able to provide two beams without loss of the effective beam current to either. This is possible because, when the second stripper is at the entrance of the booster (as it will be), the booster accelerates all charge states above some critical value to about the same energy and with the same beam quality. Thus, at the 40° bend in the ATLAS linac one can form two beams, one of which is directed into Target Area II and the other into the second stage of linac
acceleration. The maximum energies of the two beams are given in Fig. 14 by the curves labelled Area II and Area III. Because of the flexibility provided by independent phasing of the resonators, the energies of the beams going into Areas II and III can be independently varied.

As remarked earlier, one of the attractions of the superconducting approach to heavy-ion linacs is that there is a potential for substantial improvements in the technology, which is still in an early stage of development. An example of the unplanned benefits that can result from such advances in the technology is provided by a comparison of the curves labelled "original proposal" and "booster" in Fig. 13. In our case, the big unplanned technical advance was the conception of a new accelerating structure (the split ring) and the development of new techniques for resonator fabrication and control. There is every reason to believe that similar pleasant surprises remain in store for the future.

ACKNOWLEDGEMENTS

This paper would not have been possible, of course, without the hard, innovative work of the authors of references 1 through 9, my associates in the superconducting-linac project at Argonne. I am also grateful to P. Paul of Stony Brook and H. Ingwersen of Heidelberg for up-to-date information about the status of their respective projects.
REFERENCES


4. The most complete description of some aspects of the ANL superconducting-linac technology is given in the proposed document for ATLAS (February 1978) and in the Addendum to that proposal (December 1978). Copies may be obtained from the author of the present paper.


Table I. Ion beams accelerated by the ANL superconducting linac during June 1979.

<table>
<thead>
<tr>
<th>Ion</th>
<th>Tandem Energy (MeV)</th>
<th>Max. Linac Energy (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{16}\text{O}$</td>
<td>61</td>
<td>128</td>
</tr>
<tr>
<td>$^{28}\text{Si}$</td>
<td>76</td>
<td>166</td>
</tr>
<tr>
<td>$^{32}\text{S}$</td>
<td>85</td>
<td>204</td>
</tr>
</tbody>
</table>

Resonator Performance:

- $E_a = 3.3$ MV/m
- $\Delta V_R = 1.16$ MV
- $\Xi \Delta V_R = 9.3$ MV
Table II. Comparison of the technologies involved in tandem-linac heavy-ion accelerators for resonators with \( s \leq 0.10 \).

<table>
<thead>
<tr>
<th>Resonator</th>
<th>Argonne</th>
<th>Stony Brook</th>
<th>Heidelberg</th>
</tr>
</thead>
<tbody>
<tr>
<td>rf frequency</td>
<td>97</td>
<td>152</td>
<td>108</td>
</tr>
<tr>
<td>Conductor</td>
<td>SC Niobium</td>
<td>SC Lead</td>
<td>Copper</td>
</tr>
<tr>
<td>Acceleration (MV) per resonator</td>
<td>1.5</td>
<td>0.7</td>
<td>0.35</td>
</tr>
<tr>
<td>Design Emphasis</td>
<td>High-performance resonators</td>
<td>Cryogenic and resonator simplicity</td>
<td>Minimum rf power</td>
</tr>
<tr>
<td>Primary Problems</td>
<td>Resonator cost</td>
<td>High rf frequency</td>
<td>Cost of rf equipment</td>
</tr>
<tr>
<td></td>
<td>Need for flowing LHe</td>
<td>Large No. of units</td>
<td>Cost of power</td>
</tr>
</tbody>
</table>
Table III. Comparison of costs for heavy-ion linacs. The number of asterisks associated with each item gives a rough measure of the relative cost of the item. The quantity "cost per MV" is intended to be the cost for reproducing the hardware of an existing design, including everything except the building.

<table>
<thead>
<tr>
<th></th>
<th>Argonne</th>
<th>Stony Brook</th>
<th>Heidelberg</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resonators</td>
<td>****</td>
<td>**</td>
<td>**</td>
</tr>
<tr>
<td>RF Power and Controls</td>
<td>**</td>
<td>***</td>
<td>****</td>
</tr>
<tr>
<td>Cryogenics</td>
<td>***</td>
<td>****</td>
<td></td>
</tr>
<tr>
<td>Vacuum</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>Cost per MV</td>
<td>$125,000&lt;sup&gt;a&lt;/sup&gt;</td>
<td>500,000 DM&lt;sup&gt;b&lt;/sup&gt;</td>
<td></td>
</tr>
</tbody>
</table>

<sup>a</sup>In 1979 dollars.

<sup>b</sup>Cost reported by E. Jaeschke at the Symposium on Post Accelerators, held at Munich in September, 1978.
Figure 1. Schematic representation of a tandem-linac accelerator system, as used for the acceleration of $^{32}\text{S}$ ions.
Figure 2. Transit-time factor $F_t$ of 2-gap and 3-gap accelerating structures. The transit-time factor gives the relative accelerating voltage as a function of ion velocity.
Figure 3. Layout of the 4-section Argonne tandem-linac accelerator system now being assembled.
Figure 4. Schematic representation of the superconducting-linac heavy-ion energy booster at Argonne.
Figure 5. The Argonne high-$s$ superconducting resonator.
Figure 6. Performance characteristic of the Argonne high-$\delta$ resonator.
Figure 7. End view of a beam-line cryostat with a resonator in place.
Figure 8. Accelerator section C at a stage of assembly when four of six high-$\beta$ resonators have been installed.
Figure 9. Configuration of the 8-resonator booster used during June 1979.
Figure 10. Energy performance of the booster at several stages of completion.
Figure 11. Schematic of tandem-linac systems now being built.
Figure 13. Beam energies available from various parts of ATLAS.
I. Introduction:

The problem of the usefulness of a magnetic spectrometer for a heavy ion accelerator like Ganil is a delicate one which has been under study for about two years \(^1\). The following report gives a brief summary of the stage of reflection as it stands today.

II. Scientific motivations for considering a magnetic spectrometer:

Physicists have manifested their interest in the following scientific domains.

- Elastic scattering and nucleus-nucleus interaction: study of refractive phenomena and rainbow effect, information on potentials of interaction, hope for studying nuclear force dependence on nuclear matter density at very high energy.
- Inelastic scattering: study of high order multipolarity of giant collective modes. Connexion between these modes and fission.
- Transfer reactions and spectroscopy: Transfer of large number of nucleons, in high spin state configurations, with strong alignment. Possibility of studying new mechanisms, validity of sudden approximation versus adiabatical approximation.
- Study of exotic nuclei
- Quasi molecular states
- Study of orbiting phenomena in heavy systems
- Study of fragments at rest: study of temporary stretched tri-nuclear configurations of molecule type (C - A - C').

- Study of strongly damped collisions: study of many aspects of fusion-evaporation, fusion-fission, deep inelastic collisions, preequilibrium processes, prompt emission of particles, correlations between light and heavy fragments.

III. Experimental requirements of the apparatus:
The goals defined above lead to severe experimental constraints and can be achieved within the following limits for the spectrometer characteristics:

- an energy resolution $\Delta E/E = 2 \times 10^{-4}$

- a relative angular precision of 0.2° in angular distribution measurements

- the necessity of measuring at angles very close to 0°

- the necessity of measuring low cross sections (~1 µb/sr) eventually in presence of a fairly high background

- the necessity of identifying isotopes in mass and atomic number, implying clear separation between A and A + 1 up to A ~ 150 at least.

IV. Description of the spectrometer:
The experimental requirements listed above can be satisfied with the spectrometric ensemble shown on fig. 1. It is of the so called "energy loss spectrometer" type and consists of an analyser upstream from the target and the spectrometer downstream. The spectrometer involves a dipole, two quadrupoles $Q_1$ and $Q_2$ and a sextupole. The $Q_2$ magnet is of Panofsky type and all other magnets are C shaped to allow beam dumping far from the target at extreme forward angles. The quadrupole $Q_1$ focuses in the vertical direction and $Q_2$ in the horizontal one, $Q_2$ is movable along the central trajectory allows variable dispersion and kinematic corrections and reduces the focal surface displacement. The exit face of the dipole is inclined to 25° with respect to the central trajectory and concave. This insures vertical focusing increases the horizontal dispersion and reduces the angle between the focal plane and the normal to central trajectory to 0°. Mass identification does not require energy dispersion on the target nor kinematic corrections. In this case the analyser is not necessary and one could use a velocity filter for beam rejection at extreme forward.
angle measurement (see fig. 1). This spectrometric ensemble is placed on a carriage movable on air cushion. Some geometrical parameters are reported on fig. 1 some of the spectrometer characteristics are presented in table I.

<table>
<thead>
<tr>
<th>Table I</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Energy resolution</strong></td>
</tr>
<tr>
<td><strong>dispersion for $\delta p/p = 10^{-4}$ and $\theta/\delta p/p = 0$</strong></td>
</tr>
<tr>
<td><strong>(variable from 0.5 to 1.1)</strong></td>
</tr>
<tr>
<td><strong>horizontal magnification</strong></td>
</tr>
<tr>
<td><strong>vertical magnification</strong></td>
</tr>
<tr>
<td><strong>solid angle ($\pm 40$ mr horizontal, $\pm 30$ mr vertical)</strong></td>
</tr>
<tr>
<td><strong>image size on target</strong></td>
</tr>
<tr>
<td><strong>straight focal surface length</strong></td>
</tr>
<tr>
<td><strong>orientation with respect to normal to central trajectory</strong></td>
</tr>
<tr>
<td><strong>analyzed energy range</strong></td>
</tr>
<tr>
<td><strong>maximum field of dipole</strong></td>
</tr>
<tr>
<td><strong>$B_p$</strong></td>
</tr>
<tr>
<td><strong>Weight of iron</strong></td>
</tr>
</tbody>
</table>

V. The detection system:

We have seen that the spectrometer can be used for high energy resolution measurement or mass (and $Z$) identification. In the first case one needs two counters giving the position for trajectory reconstruction and subsequent kinematic corrections and eventually an energy counter. In the second case in addition to position sensitive counter one needs start and stop detectors for time of-flight measurement (t.o.f.) and energy counter for subsequent mass determination and eventually $\Delta E$ counter for $Z$ determination.
Energy counter: gas ionisation chambers are excellent detectors, easy to make to large size, cheap, suffering no radiation damage, easy to adapt to experimental conditions by gas pressure variation and offering a $\Delta E/E$ resolution less than 1%. However, very deep counters are required to stop ions such as $E_{Ae^{-}} < C_{Ba} > 50$ MeV/A (see fig. 2). In those cases, additional plastic scintillator or Si solid state counters seem to be the alternative in spite of well known inconveniences. Liquid Argon counters presently under development might be promising in the future.

Timing counter: they are channel plates with carbon foil and parallel plate counters. The first give good time resolution ($\approx 100$ ps) but are expensive, small in size and sensitive to radiations, the second are cheap, may have large size and satisfactory time resolution (from 150 to 350 ps depending on size) but their thickness may introduce straggling problems.

Position counter: parallel plate counters are at present the most suitable. Spatial resolution of 1 mm is possible, achieve for large size counters.

Fig. 3 shows the limits for mass identification with 20% overlap between two neighbouring masses for a path of $L = 4$ m long, $\Delta L/L = 10^{-3}$ and $\Delta(Bp)/Bp = 10^{-3}$ the dashed lines correspond to time resolution $\Delta T = 200$ ps, the dotted lines to $\Delta T = 400$ ps and the dotted-dashed lines to $\Delta T = 600$ ps assuming $\Delta E/E = 1\%$ and 1.5%. The full line corresponds to maximum Ganil energies.

Two different forms of operation should be considered: measurements at angles beyond $\theta \geq 4^\circ$ (see fig. 4A) and below this limit (see fig. 4B). In the first case where the start counter can be placed at $\approx 75$ cm from the target one has a t.o.f. path of $\approx 10$ m allowing separation of all masses up to U even with a $\Delta T \approx 600$ ps. One can use channel plates or parallel plates ($\approx 6$ cm x 5 cm) as start and parallel plates ($\approx 75$ x 8 cm) as stop detector. This configuration requires counters of quite reasonable size and performance. $Q_2$ can be used to reduce stop and E counter sizes. Reconstitution of trajectories through the spectrometer is required. The second configuration demands that all counters be placed beyond the dipole (and $Q_2$ must be turned off for mass identification) reducing the t.o.f. to only 4 m long. Parallel plate counters ($\approx 70$ x 8 cm) must be used for position and timing measurements. Some of the requisite performances for the counters are summarized on table II.
Table II

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Counter</th>
<th>Horizontal localisation accuracy</th>
<th>Vertical localisation accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>start</td>
<td>1 nm</td>
<td>not needed</td>
</tr>
<tr>
<td>( \theta \geq 4^\circ )</td>
<td>stop</td>
<td>( &lt; 10 \text{ mm} )</td>
<td>not needed</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1 mm for high resolution</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>5 mm if ( Q_z ) is on</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>start</td>
<td>5 mm</td>
<td>4 mm</td>
</tr>
<tr>
<td>( \theta \leq 4^\circ )</td>
<td>stop</td>
<td>( &lt; 10 \text{ mm} )</td>
<td>4 mm</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1 mm for high resolution</td>
<td></td>
</tr>
</tbody>
</table>

VI. General concluding remarks:

As shown on fig. 3, the mass separation of nuclei with maximum GANIL energy in configuration B \((\theta < 4^\circ \text{ and probably often used})\) is complete up to \(A \sim 60\). If one admits \(20 \%\) overlap between two neighbouring masses one can reach \(A \sim 120\), improvements on time resolution (\(\leq 200 \text{ ps}\)) should in principle allow separation up to uranium. However, energy and angular straggling may be a severe limitation to performance cited above, particularly for heavy nuclei with low energy.

The determination of the reaction angle of the analysed ion depends on trajectory reconstructions, which can be affected by multiple scattering in the target and in the first position sensitive detector, and on the beam emittance. Usually angles can be estimated with an accuracy of \(0.1^\circ\). The horizontal and vertical angular magnification being \(\sim 1 \text{ and } \sim 1/5\) respectively this leads to \(\Delta \theta = 0.1^\circ \text{ and } \Delta \phi = 0.5^\circ\). This uncertainty on \(\phi\) creates a problem for \(\theta < 4^\circ\). Also, strong kinematic corrections lead to a large horizontal dimension of the beam on the target. Beside the difficulty of beam dumping, those two effects make high resolution measurement at \(\theta \geq 4^\circ\) with full horizontal emittance (\(\sim 5 \text{ mm mrad}\) only possible for \(K \leq 0.3\). Higher \(K\) coefficients and good \(\theta\) determination imply horizontal emittance reduction. For \(\theta < 4^\circ\) horizontal and vertical
emittance must be severely limited (0.5 m and 1.5 m-mm-mrad for horizontal and vertical emittance respectively) in order to keep good energy resolution. Of course, target inhomogeneities may also contribute to energy resolution degradation.

The spectrometer is equipped with a complicated shaped vacuum chamber attached to the scattering chamber (see fig. 5). The whole ensemble rotates on a vertical axis at the center of the scattering chamber by means of a sliding seal. The long radius small angle sector is to allow beam dumping at extreme forward angles. The next sector can house cumbersome counters like t.o.f. set up and the beam dump can also be attached to this sector. The circular scattering chamber is very roomy and offers space for several large counters.

The target room housing the spectrometer will allow a rotation from about -45° to 110°.

Notes

1) Many people have been collaborating on this study but J. Gastebois and M. Langevin have certainly been taking an essential part in it.
2) For start counter the reported horizontal accuracy allows \( \Delta L/L = 10^{-3} \) and \( \Delta \theta \leq 0.5° \) and the vertical accuracy to \( \Delta \phi \leq 0.5° \).

For stop counter the horizontal accuracy allows \( \Delta (B_0)/B_0 = 10^{-3} \) and \( \Delta \theta \leq 0.5° \) and the vertical accuracy leads to \( \Delta \phi \leq 0.5° \).
Fig. 1: General lay out of the spectrometer

Fig. 2: Range versus energy for some ions in isobutane at atmospheric pressure

Fig. 3: Mass identification limits with the spectrometer

Fig. 4

Fig. 5: Lay out of the spectrometer vacuum chamber
A K = 800 High Resolution Heavy Ion Spectrograph

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Cyclotron Laboratory and Physics Department
Michigan State University, E. Lansing, MI 48824.

This is a progress report on design considerations for a large heavy ion spectrograph to be used with MSU Phase II. A preliminary sketch indicating the position of the K = 800 spectrograph on the floor plan of the new heavy ion laboratory is shown in Fig. 1. Although the spectrograph design presented here is rather detailed, it is only intended to be suggestive and a focal point for discussion by the sponsors group and prospective users of the facility. We first outline the need for such a spectrograph and the choice of parameters such as energy and angular resolution, and then discuss the choice of optical mode for the system. Finally, a preliminary design of a particular beam transport-spectrograph combination is presented.

The collaborators in this work at MSU are Leigh Harwood and Edwin Kashy. Additional valuable input has been provided by K.L. Brown (SLAC), H.A. Enge (MIT), K. Halbach (LHII), S. Martin (Jülich), C. Morris (LASL), P. Roussel (Orsay), and H.A. Thiessen (LASL). The two existing large spectrographs which have been the seeds of our considerations are "Big Karl" at Jülich and the "High Resolution Spectrometer" (HRS) at LAMPF.

MSU Phase II will provide unique high quality heavy ion beams such as 200 MeV/amu $^{40}$Ca. Because of the high energies of these beams ($8$ GeV in this case), a large spectrograph is
essential even for some of the simplest studies of their interaction. Qualitatively new phenomena such as phase transitions in nuclear matter may, for example, be observable through elastic scattering studies. Such measurements sound trivial, but at these energies require energy resolution of $\frac{1}{10^4}$ (800 keV for the example above) and angular resolution of $\approx 1$-$2$ mr. In contrast with low energy heavy ion experience, target energy loss and angular scattering effects would not preclude the possibility of such resolutions even with target thickness $> 1$ mg/cm$^2$. At these high beam energies a range of $10\% \Delta E/E$ (800 MeV in this example) seems adequate. The solid angle should be as large as is economically feasible and achievable within the primary constraints listed above. A tentative goal of 5 msr seems a reasonable compromise between the 10-15 msr of the current generation of smaller spectrographs and the 3 msr of the slightly larger "HRS". This value of 5 msr would certainly be adequate for a great range of elastic and quasi-elastic reaction studies. The last important specification is for a detector system capable of individual isotope identification for heavy ions of the masses and energies to be studied ($A = 40$, $E = 50$-$200$ MeV/amu). The design goals for the spectrograph are summarized in Table I.

The spectrograph itself is only one of many components in the total cyclotron-spectrograph system. The main components of the system are listed in Table II. The first component is the accelerator or cyclotrons, in this case. At the exit of the
cyclotron the beam has a non-zero energy spread (≈ 0.1%), some spatial dispersion, and some angular divergence which can be represented by a six dimensional phase space ellipse. A phase space matching network consisting of a magnetic dipole and several quadrupoles is used to create horizontal and vertical waists with no energy dispersions at a point which is the object for the rest of the beam transport system.

If the spectrograph is a long distance from the accelerator it is often necessary to next have an achromatic beam transport module. Karl Brown has shown that a module which contains 4 dipoles, 8 quadrupoles, and 8 sextupoles can be made equivalent to the unity matrix with zero second order geometric and chromatic aberrations. With small beam energy spread it is possible to ignore the chromatic aberrations and construct a system in which the second order geometric aberrations cancel by symmetry. An example of such a module is indicated in Fig. 2. The dipole magnets of the module can also serve as switching magnets. At the midpoint of this module there is momentum dispersion adequate to measure or limit the accelerator momentum spread. No additional beam defining slits are necessary from this point to the spectrograph so that slit edge scattering can be kept to a minimum.

Between the achromatic beam transport module or modules and a beam dispersive system it is desirable to have another phase space matching section consisting, for example, of a quadrupole quartet. This allows independent adjustment of horizontal and
vertical spot sizes and their corresponding divergences. The choices depend on the details of the phase space ellipses received from the accelerator and the requirements of the experiment, such as the relative importance of energy resolution vs. angular resolution.

Because the accelerator has a non-zero energy spread (≈ 0.1%) which is much larger than the desired experimental energy resolution (≈ 0.01%) it is necessary to operate the system in the dispersion-matching or energy-loss mode. In this mode the resolving power (D/M) of the beam dispersive or "beam analyzing" section is as important or more important than that of the spectrograph. An energy-loss system is shown in Fig. 3. In this example the dispersions are in the vertical plane and the beam dispersive section includes the two 45° bends on the left side of the figure. A phase space matching system consisting of a quadrupole quintet images the beam on the target and the QQDD spectrograph follows the target. The present example also includes a phase space rotator located in the first drift section of the beam dispersion system, labeled TWISTER (DRIFT) in Fig. 3. This quadrupole quintet interchanges the horizontal and vertical phase spaces when turned on, and is optically equivalent to a drift otherwise. (The quadrupoles are rotated 45° from the normal orientation.) Hence, the phase space rotation is optional and the choice is dictated by the particular experimental requirements. Typical phase space ellipse areas for the MSU superconducting cyclotrons are likely

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to be 1 mm-mr horizontally and 5 mm-mr vertically. With spectrograph dispersion in the vertical plane, better energy resolution is possible if the 1 mm-mr phase space is rotated into this plane. On the other hand, if angular resolution (measured horizontally) is the dominant experimental consideration, then it is best not to energize the TWISTER.

A very important component of any heavy ion spectrograph system is the detection apparatus. The present paper, however, is primarily concerned with the conceptual design of the spectrograph and the associated beam transport system and not with the details of particle identification. Rapid progress continues to be made in this area with the Bragg-curve-spectrometer concept discussed at this conference by C. Gruhn offering particularly exciting possibilities for high energy heavy ion identification. The position resolution requirements of the present spectrograph design are discussed below.

The discussion above of the beam TWISTER and phase space considerations implicitly assumed a vertically dispersing system such as the example illustrated in Fig. 3. In this example, the beam is dispersed vertically on the target, the scattering is in the horizontal plane, and the spectrograph disperses vertically, i.e. a "VHV" system. The large energy-loss spectrographs at Bates and LAMPF are examples of systems which operate in this mode. (At both of these laboratories the beam is dispersed horizontally and then the phase space is rotated 90° to yield vertical dispersion at the target. With vertical beam
dispersion as indicated in Fig. 3 the phase space rotation is optional and is dictated by other considerations.) Most recently constructed spectrographs, however, operate in the "HHH" mode with the scattering and the dispersion both in the horizontal plane. The various Q3D spectrographs as well as the "Big Karl" spectrograph at Jülich are examples of this mode.

The choice of spectrograph mode, i.e. vertical (VHV) or horizontal (HHH) involves detailed consideration of reaction kinematics and angular resolution requirements and how they interact with the requirements for good energy resolution. The essential difference is that in the VHV mode the scattering and dispersion measurements are essentially decoupled with the former being in the horizontal plane and the latter in the vertical, while in the HHH mode the measurements are inherently coupled since both are in the same plane.

In the VHV mode the spectrograph optics are parallel-to-point in the axial direction and point-to-point in the radial or dispersive direction. Hence the detector must be two-dimensional with momentum and scattering angle information being read out in orthogonal directions. A schematic view of the focal plane of such a system is indicated in Fig. 4. The scale and the reaction lines indicated are for the spectrograph to be described below and indicated at the right hand side of Fig. 3. Kinematics effects manifest themselves as tilted lines on the detector so that no focal plane motion or quadrupole
tuning is required as the kinematic parameter changes with scattering angles. The focal plane momentum calibration also varies with kinematic compensation in the HHH mode, but is invariant in the VHV mode. It is also possible to observe sharp lines from several kinematically different reactions simultaneously.

In practice, the quickest way to tune the beam line and spectrograph parameters of an energy-loss system is via direct observation of the beam on a scintillator in the focal plane of the spectrograph. This technique has been developed and proven very useful with the $k = 50$ cyclotron and Enge split-pole spectrograph at MSU. It is currently also in use at the Princeton Q3D, ORSAY, and at the "Big Karl" spectrograph in Jülich. This technique is particularly appropriate for the VHV mode because in this mode the tuning is angle independent, i.e. there is no reoptimization required as a function of scattering angle.

An additional advantage of the VHV mode is that attempts to achieve good angular resolution are independent of tuning or optimization of energy resolution. Here it is the angular divergence of the beam before the scattering which is relevant. In the VHV mode the angle after the target is measured via the 2-dimensional detector as indicated in Fig. 4, whereas in the HHH mode this angle can be measured via two one-dimensional focal-plane detectors and trajectory reconstruction. However, the angular divergence of the beam on target cannot be determined
by measurements after the scattering process. The angular divergence of the beam is given by the horizontal phase space area at the target divided by the horizontal spot size on target. In the HHH mode there is always a compromise between a larger spot size to achieve good angular resolution and a narrow spot to achieve good energy line widths. In the VHV mode the problems are decoupled so that it is possible to use wide spots with small divergence in the horizontal (scattering angle) direction while simultaneously using narrow spots with larger divergences in the vertical (dispersive) direction.

For example, with the 5 mm-mr phase space rotated to be horizontal and the 1 mm-mr vertical, a horizontal divergence of 1 mr with a spot size of 5 mm could be used with a vertical spot size of 0.25 mm and divergence of 4 mr. Thus, the push towards better energy resolution does not imply reduced angular resolution.

The main disadvantage of the VHV mode and advantage of the HHH mode is related to the kinematics effects due to the angular divergence of the beam in the horizontal plane. A kinematic parameter $k = (1/p) \frac{dp}{d\theta} = 0.2$ implies 2 parts in $10^4$ momentum change per mr scattering angle change. Hence, a beam divergence of 1 mr in the VHV mode implies an energy resolution limit of 4 parts in $10^4$, and there is no way to compensate for this other than by increasing the beam spot width to reduce the divergence. In this kinematic parameter, corresponding to $^{90}\text{Zr}(^{40}\text{Ca},^{40}\text{Ca})$ at 20° as indicated at the bottom.
of Fig. 4 in the VHV mode, it is best to not rotate the phase space. The beam spot could be focused for 5 mr x 1 mm vertically and 0.25 mr x 4 mm horizontally to yield equal contributions of 1 part in 10⁴ energy resolutions in both directions, or 1.4 parts in 10⁴ total energy resolution at k = 0.2.

The HHH mode has the potential for slightly better energy resolution in this case because the kinematic effect of the beam divergence can be compensated for by focusing the beam either in front of or behind the target to introduce the proper position-angle correlation on target. For a spectrograph resolving power D/M = 20 this implies a spot size increase of 4 mm per mr of beam divergence at k = 0.2. Assuming the tuning is perfect, an energy resolution of 1 part in 10⁴ is achievable in this mode. In practice, however, the calculated tuning changes are not perfect⁴ and the two modes would probably yield comparable resolutions.

In summary, it seems that simplicity of the tuning and the invariance of the focal plane calibration in the VHV mode are strong points in its favor. This simplicity is particularly important in a system which is to be used by a large number of different research groups. The two-dimensional focal plane detector of the VHV mode is a slight complication, but the HHH mode requires a minimum of two one-dimensional detectors and trajectory reconstruction to extract the scattering angle in comparable detail.

Hence, we have chosen for a more detailed design study the VHV energy-loss system shown in Fig. 3. The QQDD
The spectrograph shown here is a combination of the QQQDQ "Big Karl" system at Jülich and the QDD "HRS" system at Los Alamos. The design is not complete at this time but optics calculations with second order corrections do indicate that the principal design goals of Table I can be achieved with such a system. Detailed ray tracing calculations will be carried out to determine the extent of higher order aberrations. The design of the superconducting quadrupoles and dipoles with superconducting windings will be initiated after a first order design is agreed upon. The present second order solution involves sextupole components in both quadrupoles as well as on the first, second, and fourth dipole edges. There is an axial waist between the two dipoles so that residual radial second and higher order aberrations can be tuned with an active multipole element at this location with minimum coupling to axial aberrations. The relatively large total bend angle of 161° was chosen partially to help keep the height of the system reasonable. Plots of the radial and axial beam envelopes through the QQQD spectrograph are shown in Fig. 5. The parameters of the spectrograph are listed in Table III.

This material is based upon work supported by the National Science Foundation under Grant No. Phy-7822696.
References


2. P. Poussel, private communication.


4. E. Kashy and P. Roussel, Preliminary results on kinematic Tuning at Orsay, private communication.
Table I. Design Goals for the K = 800 Spectrograph.

<table>
<thead>
<tr>
<th></th>
<th>Magnetic Rigidity: 4 T-m (K = 800, 200 MeV/A ⁴⁰Ca)</th>
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<tr>
<td>2</td>
<td>Energy Resolution: 1 in 10⁴ (800 keV at 8 GeV)</td>
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<td>3</td>
<td>Solid Angle: 5 msr</td>
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<td>4</td>
<td>Energy Range: 10% (100 MeV at 1 GeV)</td>
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<td>5</td>
<td>Angular Resolution: 2 mr</td>
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<td>6</td>
<td>Angular Range: -20° to 160°</td>
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Table II. Components of Cyclotron-Spectrograph System.

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<td>3</td>
<td>Beam transport-achromatic</td>
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<tr>
<td>4</td>
<td>Phase space matching</td>
</tr>
<tr>
<td>5</td>
<td>Beam dispersion (phase space rotation)</td>
</tr>
<tr>
<td>6</td>
<td>Phase space matching</td>
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<tr>
<td>7</td>
<td>Spectrograph</td>
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<td>8</td>
<td>Detectors</td>
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Table III. Parameters of the K = 800 Spectrograph

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<td>Solid angle:</td>
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<td>Axial dispersion:</td>
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<td>Radial magnification:</td>
<td>$M = 0.33$</td>
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<td>Focal plane tilt:</td>
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<td>Magnetic rigidity:</td>
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<tr>
<td>Dipole gap:</td>
<td>$d = 6$ cm</td>
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<td>#2 4.0 m long x 75 cm wide (86° bend)</td>
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<td>#2 100 tons</td>
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<td>Quadrupole sizes:</td>
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Figure Captions

Figure 1. Schematic plan view of the research facilities of the coupled superconducting cyclotrons at Michigan State University. The $K = 800$ spectrograph is indicated in the lower left hand corner.

Figure 2. An achromatic beam transport module with intermediate dispersion and switching magnets.

Figure 3. An elevation view of a possible energy loss spectrograph system in the VHＶ mode.

Figure 4. Schematic view of the two dimensional focal plane of the proposed $K = 800$ spectrograph. The kinematics effects for elastic scattering of $^{40}$Ca beams from $^{90}$Zr and $^{208}$Pb targets are indicated.

Figure 5. Plots of the radial and axial beam envelopes of the proposed $K = 800$ spectrograph.
Figure 2

Achromatic Beam Transport Module (K=800)
K=800 Spectrograph  Focal Plane Detector

40 cm tall (radial)
7.4 cm/%  \( \frac{\Delta p}{p} \)
\( M_x = \frac{1}{3} \)

\( ^{208}\text{Pb}(^{40}\text{Ca}, ^{40}\text{Ca}) \)

10 cm wide (axial)
0.7 mm/mr

\( ^{90}\text{Zr}(^{40}\text{Ca}, ^{40}\text{Ca}) \)

Figure 4
A Reaction Product Mass Spectrograph
for Intermediate Energies

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One of the major areas of use for any accelerator in the 10—200 MeV/nucleon energy range will be in production of new nuclei far from β-stability or in the "super-heavy" mass region. A device which can measure the mass of these nuclei directly as they are emitted from the target would be extremely useful. During the past several months several designs for such a reaction product mass spectrograph (RPMS) to be used with the MSU superconducting cyclotrons have been studied. The following gives the details of the current plans of a system which appears potentially useful over a wide (1—200 MeV/nucleon) energy range. My collaborators at MSU are Jerry Nolen and Ed Kashy. H.A. Enge (MIT), Mike Nitschke (Berkeley), and Richard Pardo (MSU, ANL) have also been very helpful in the design.
Basic concept

Many types of so-called "double focussing mass spectrometers" have been designed and built during the past several decades. These devices consisted of combinations of electric and magnetic dispersive elements arranged to provide simultaneous focussing in energy and angle for a given mass. These devices, however, were intended for use with particles from an ion source and therefore were not designed for the solid angles and rigidities necessary for the direct focussing of heavy-ion reaction products.

More recently a heavy-ion device has been constructed by an MIT-Brookhaven collaboration. The Energy-Mass Spectrometer (EMS) consists of a crossed field Wien filter followed by an Enge split pole spectrograph. The velocity dispersion of the Wien filter is in the vertical direction, and the momentum dispersion of the split-pole spectrograph is in the horizontal direction. The result is not a mass focal plane but a two dimensional focal plane where particles of constant m/q (mass/charge) form tilted lines. The two-dimensional detectors necessary for this design have proved to be a problem. The emphasis of this design is to retain momentum information while spacially separating the particles of different m/q. At present, a modified version of the EMS is used, without the split-pole,
as a 0° beam eliminator and has proved very useful for fusion studies. Devices similar to the EMS have been proposed for the Super-HILAC at Berkeley\textsuperscript{1,2} and for the 25 MV tandem facility at Oak Ridge,\textsuperscript{1} the difference being the Wien filter has separated magnetic and electric field regions. The disadvantage we see in the design, aside from the field gradient problems mentioned below and the previously mentioned two-dimensional detector problems, is that the reaction products are not focussed to a point. If one wishes to pass the reaction products of a given mass on to a counter of some type behind the focal plane, e.g. a surface barrier detector to look at decays, then the products need to be localized to as small an area as possible. With the two dimensional focal plane, however, the products are dispersed along a tilted line which varies in size according to the momentum range of the products. If one designed a high dispersion version of this device, then a mass line could easily be a few centimeters long, requiring detectors of several centimeters in size. The larger the detector, the larger the background problem becomes.

We feel a good alternative is to return to a mass spectrometer design in which all particles of a given mass (and charge state) are focussed to a point on a
mass focal plane. This can be accomplished by appropriate combinations of devices that disperse according to energy, velocity, and momentum to yield a composite that disperses according to mass. The devices available are electric septums, Wien filters, and magnetic dipoles. If $x$ is the displacement of a ray away from the central ray, then one obtains for the three devices

- **Electric septum:**
  $$x = a \frac{\Delta p}{E} = a \frac{\Delta m}{m} + 2 \frac{\Delta v}{v} = a\left(\frac{\Delta p}{p} - \frac{\Delta m}{m}\right)$$

- **Wien filter:**
  $$y = b \frac{\Delta v}{v} = b\left(\frac{\Delta p}{p} - \frac{\Delta m}{m}\right)$$

- **Magnetic dipole:**
  $$x = c \frac{\Delta p}{p} = c\left(\frac{\Delta m}{m} + \frac{\Delta v}{v}\right)$$

where $a$, $b$, and $c$ are constants reflecting the sizes and field strengths of the devices. If, for example, one combines an electric septum with a magnetic dipole with $c = -2a$, then

$$x = a\left(2 \frac{\Delta p}{p} - \frac{\Delta m}{m}\right) - 2a \frac{\Delta p}{p} = -a \frac{\Delta m}{m}$$

and the dispersion depends only on mass.

From monetary considerations, it would be best if the same elements could be used to build a mass spectrograph useful over the full energy range of the MSU Phase II facility. This constraint rules out electric septums since the limit on attainable field gradients ($\sim 80$ kV/cm with a 10 cm gap) would require a 50 m radius of curvature for the 200 MeV/nucleon particles available from Phase II. A 5 m length is necessary to achieve a reasonable mass.
resolution (= 1%) with reasonable solid angle (= 1 msr) at 200 MeV/A. Because the curvature of the septum is determined by the maximum energy particles and the maximum electric field, this device would not be optimized for lower energies, where larger resolving powers are possible. One would thus need separate septums for low energies (= 20 MeV/nucleon) and high energies (= 150 MeV/nucleon). Construction of either of the two septums would be a costly, time consuming project in itself. We have chosen instead to use one of the Mark VI 15-foot velocity separators used previously in the high energy program at the Bevatron. One of these separators has been obtained and is at MSU. The advantage of this device is that since the electrostatic plates are straight, it is effective over the entire energy range of interest (1 MeV/A to 200 MeV/A) and its dispersive capabilities match the experiments of interest at the different energies, as will be discussed below.

It should be noted that our device is similar in design to the Recoil Separator (RS) proposed at Daresbury. There are two differences. First, their device is designed for the region below 5 MeV/nucleon. Secondly, there is the major problem in their design of having the primary beam strike the electrostatic plates in the
Wien filter. This could cause problems if one is attempting to maintain a 600 kV voltage over a 7 cm gap as they propose. Production of secondary particles could also be a problem from the beam striking the plates. As described below, we propose to stop the beam on a Faraday cup before it enters the RPMS. The same solution could be used at Daresbury.

In comparing various recoil spectrometers a figure of merit is the total electric field volume integral:

\[ I_\text{E} = EdL \]

where \( E \) is the electric field in kV/cm, \( d \) is the gap separation in cm, and \( L \) is the length of the plates in meters. If the particles of interest approximately fill the gap then this integral gives a rough measure of the product of mass resolving power times solid angle achievable with the device at a given particle energy. For the Berkeley Mark VI device, \( I_\text{E} \approx 3000 \text{ kV-m} \). This large value is necessary for the instrument to be useful at the high particle energies considered here. The proposed Daresbury device may achieve \( I_\text{E} \approx 600 \text{ kV-m} \). The HILAC-Oak Ridge device has \( I_\text{E} \approx 480 \).

Physical Layout

The design currently under consideration is shown in Fig. 1. The design might appear excessive with its 12
multipoles, but these multipoles are necessary to make possible the correction of the large number of second-order aberrations.

For Phase I operation (up to 80 MeV/A) overall length is 18.5 m. The layout will be larger for Phase II (up to 200 MeV/A) because of the quadrupole focusing limitations. Since such a long device would be exceedingly difficult to move from angle to angle, we intend to change the angle of incidence of the beam rather than moving the spectrometer. This can be accomplished for a span of 30° by using a pair of small superconducting dipoles. A single larger angle, such as 60°, might be obtained by inflection from the adjacent beam line.

The multipoles are arranged as four triplets. They will be superconducting because of the large field gradients needed for the high energy beams. Present cost estimates indicate they are also less expensive than conventional elements. Each triplet is identical in construction, thereby simplifying design and construction details. One interesting engineering detail is the positioning of the last triplet and focal plane. In order to match the dispersion of the Wien filter at different momenta, the dispersion of the dipole must be variable. This requires a variable deflection angle for the dipole magnet.
Thus, the last triplet must be able to move with respect to the remainder of the spectrometer. This would normally be a minor problem. Since the Mark VI Wien filter disperses vertically, the magnet must disperse vertically and the triplet and focal plane detector must move vertically via a sliding seal connection to the dipole. We have as yet not done a detailed engineering study, but the initial study indicates the present cryostat design is satisfactory.

Each multipole will be a combined quadrupole, sextupole, and octupole, with each 2n-pole component being independently variable. As stated above, the multipoles will be superconducting. We are considering the Panofsky design for the quadrupole and octupole components since this design provides high quality fields without elaborate construction procedures. The variable octupole components are produced by the same coils as produce the variable quadrupole components, an added benefit of the Panofsky design. The specific design for the sextupoles has not been decided at this time. We should be able to correct most of the second-order aberrations and several of the third order aberrations, our aim being line widths of about 1 mm.
Optics, Calculations, and Characteristics

The optics calculations have been done with the computer code TRANSPORT. This is a first and second order matrix multiplication program. All of the standard magnetic devices (quadrupoles, dipoles, etc.) are included in the program; Wien filters are not. The matrices for the Wien filter are calculated from the equations given by Ioaniviciu. These matrices are then input into TRANSPORT where necessary.

The first order optics have been designed to facilitate correction of second order aberrations. Some of the rays are illustrated in Fig. 1. Note that there is a horizontal cross-over after the first triplet following the Wien filter. A velocity focal plane exists at this point. Thus, a crude velocity selection can be made by introducing a set of slits which pass the desired range of velocities and prevent others from continuing to the mass focal plane.

At present, second order calculations continue. A 2 mm wide line width has already been attained for 3 msr solid angle. As the calculations continue, details of the optics may change. The present first order characteristics are given in Table I. Fig. 2 shows the mass resolving power over a range of energies. At 200 MeV/
nucleon, D/M is still about 1 mm/%. With a 1 mm line width, one could resolve masses 100 and 101, for example.

Experiments

The primary purpose of this device is to study nuclei far from the valley of β-stability. Three different reaction processes will be used: fusion, deep-inelastic transfer, and fragmentation. The mass spectrometer will be run in slightly different modes for each mechanism. Each is described below.

1. Fragmentation

At the higher energies (≥ 50 MeV/nucleon) fragmentation is the dominant reaction mechanism. Here, fragments of the projectile are sheered off by the target nucleus leaving some other fragment of the projectile to fly on "untouched" because the reaction happens too fast for the individual nucleons to react. The fragments have roughly the beam velocity and are kinematically concentrated near 0°. For fragmentation studies, the beam cannot be separated from the reaction products by the velocity separator. Both will pass through the velocity separator identically, but the neutron-rich fragments which are lower in Z than the beam can be
cleanly separated from the beam via their higher magnetic rigidities. In two recent Bevalac experiments a simple QQD spectrometer was used to identify many new particle-stable nuclei as fragmentation products. With the higher beam intensities of the MSU facility and true mass focal plane of the spectrograph it should be possible to greatly extend this work. Decay studies of the newly produced isotopes should also be possible in some cases.

2. Deep-inelastic transfer

These reactions occur at lower energies than fragmentation. The target and projectile temporarily "stick" together and exchange particles and energy through the overlap region. This statistical exchange process gives finite probabilities for producing new nuclei. A group at Saclay recently reported three new nuclei observed in deep-inelastic transfer. Deep-inelastic transfer has its maximum cross-section away from zero degrees. Thus, the beam can be stopped in a Faraday cup and will not enter the mass spectrometer. As stated previously, the reaction angle will be changed by inflecting the beam at a different angle rather than by moving the spectrometer. A range of 0° to 30° plus selected larger angles will be possible.
3. Fusion

At beam energies near the Coulomb barrier, neutron deficient nuclei can be produced by fusion reactions. As fusion cross-sections are concentrated near 0°, it is necessary for the spectrograph to have 0° within its acceptance. However, there is a factor of about 2 difference between the beam velocity and that of the fusion products. With the high dispersion planned for the velocity separator, the beam would be deflected into the electrostatic plates, a very undesirable consequence. The present plan is to place a beam stop at the entrance to the first quadrupole. At this position, the stop needs to occlude only a small (10%) amount of the accepted solid angle and fusion recoil cone. The remainder (halo) of the beam will be less than 10⁻³ of the central beam and can be deflected onto the electrostatic plates with little effect. Also, at these low energies the dispersion (1 cm/%) is sufficient to resolve mass peaks over the entire present nuclide chart and beyond. In particular, we hope to be able to resolve exotic nuclei with Z > 100.
Summary

The present design for a RPMS at MSU appears useful over the entire energy range available on the Phase II MSU facility. It consists of a 5 m long Wien filter followed by a magnetic dipole, with sufficient multipole focussing elements to provide the optics desired. A one dimensional mass focal plane has been chosen as the optimum design. Dispersion ranges from 1 cm/% at 10 MeV/nucleon to 1 mm/% at 200 MeV/nucleon for \( A = 20 \) particles. Present results of design calculations show that 2 mm line widths can be fairly easily obtained. Second order aberration studies continue at this time.

Acknowledgments

This material is based upon work supported by the National Science Foundation under Grant No. Phy-7822696.
References

1. H.A. Enge, private communication.
2. M. Nitschke, private communication.
3. A.N. James et al., Technical memorandum DL/NSF/TM 38, Daresbury Laboratory.
Table I. RPMS Specifications (as of 7/11/79).

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<th>Specification</th>
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<td>Mass dispersion/magnification (D/M)</td>
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<td>Multipoles:</td>
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<td>Wien filter:</td>
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<td>Second order line width (at 1 mser and ± 5% velocity and mass range):</td>
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Figure Captions

1. Block diagram of RPMS with the $x/9$, $x/6\rho$, and $y/4$ rays indicated below. "M" indicates a multipole focussing element, "ExB" the Wien filter, and "D" the magnetic dipole.

2. Dispersion possible with the Mark VI Wien filter. These calculations are for the magnetic field limit of 470 Gauss. The magnification of the system is unity for all calculations and the solid angle is roughly constant over the energy range.
Figure 1
Figure 2

Graph showing the relationship between $E/A$ (MeV) and $D$ (cm/%) for $D(Q=A/2)$ and $D(^{238}U)$. The graph also shows the relationship between $E$ and $E(KV/cm)$.
Heavy-Ion Direct-Reaction Mechanisms
- the Spectroscopic Approach

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Outline

1. Viewpoint; Direct-Reaction Spectroscopy with Heavy Ions
2. Quasi-Elastic Reactions Induced by Light and Heavy Ions; Contrasts and Inherited Prejudices
3. Averaged Heavy-Ion Direct-Reaction Theories
4. The Structural Elements of Direct-Reaction Theories
5. The Heavy-Ion Optical Potential; the "Sensitive Radius" and other Curiosities
6. Interpolation and Extrapolation in Angular Momentum
7. Treatment of Long-Range Coulomb Coupling
8. Iterative Techniques for Coupled-Channel Problems
9. Summary and Conclusions

1. Viewpoint: Direct-Reaction Spectroscopy for Heavy Ions

The approach to heavy-ion reactions that I am going to discuss is that which claims direct descent from the reaction models developed for light-ion spectroscopy, with whose aid we have learned a fair amount of what we now know about nuclei. For want of a better label, I shall refer to this approach as conventional microscopic direct-reaction theory. Theories of this sort cannot hope to give a comprehensive view of what happens in heavy-ion collisions without involving an essential element of averaging. It is my contention that two theoretical approaches of the requisite sort have been formulated — statistical multi-step and giant-resonance multi-channel — and that neither has been implemented numerically to anything like the extent that is now possible.

To set the stage for this discussion, consider Fig. 1 (pirated from G. F. Bertsch's talk at this conference). It is a sketch of the energy-dependence and qualitative content of the total reaction cross-section in a collision between two unspecified but not-too-heavy ions. At the low-energy end ($E/A \leq 10$ MeV per nucleon), the cross section is dominated by compound-nucleus formation (complete fusion), with a small quasi-elastic component. At higher energies, more complicated, strongly-damped, but still direct processes predominate; complete fusion disappears from contention as the compound-nucleus lifetime diminishes. The quasi-elastic component persists with little change in magnitude with increasing energy. The key question for conventional microscopic theories is whether they can account for the onset of strongly-damped processes and their competition at low bombarding energies with quasi-elastic reactions.

There have of course been numerous studies along the lines outlined above; yet the key physical questions still lack definitive answers. I believe that the conventional microscopic direct-reaction approach has an important role to play in the search for a detailed understanding of heavy-ion processes. Much
would be lost were the field abandoned entirely to alternative approaches, since these alternatives have weaknesses and limitations that are often complemented by the strengths of conventional direct-reaction approaches. Time-dependent Hartree-Fock calculations, for example, are firmly rooted in many-body theory, provide a qualitative view of the time-evolution of heavy-ion collisions but are difficult to relate to specific cross sections for specific processes. Conventional microscopic direct-reaction calculations, in contrast, have no trouble identifying cross sections but are less soundly based in many-body theory and do not readily yield a qualitative, panoramic view of the processes whose cross-sections they calculate. The two approaches are complementary.

Direct reactions induced by low-energy heavy ions (E/A ≤ 10 MeV/A) have been extensively studied over the past decade. These studies have gradually built an understanding of the detailed structure and systematic features of the structural elements — optical potentials, S-matrix elements, transition form factors and so forth — common to all direct-reaction theories. We will discuss those systematic features and show how they can be used to carry out heavy-ion direct-reaction calculations with greater efficiency.

2. Quasi-Elastic Reactions Induced by Light and Heavy Ions; Contrasts and Inherited Prejudices.

Low-energy heavy-ion spectroscopy traces its ancestry to the sort of light-ion spectroscopy carried on at Van de Graaf and low-energy cyclotrons since the mid 1950's. Heavy-ion studies of this kind have yielded many nuggets of information about nuclear level structure and have provided us with useful information about the pertinent matrix elements or transition form factors. Their light-ion ancestry, however, has undoubtedly poisoned the field with baneful prejudices. Furthermore, the majority of heavy-ion direct-reaction calculations have been carried out with the aid of techniques, even of computer
programs, taken over with little modification from light-ion studies. This results in wasteful computations and gives an unduly gloomy impression of the practical limits to conventional direct-reaction calculations for heavy ions.

I will now summarize some of the main points of contrast between the low-energy spectroscopy of light and heavy ions, gleaned from some recent review talks and review articles. For the purposes of this comparison, my comments on heavy-ion spectroscopy will refer to reactions induced by ions of mass-number $A$ no greater than 40 and energies of 10 MeV per nucleon or less.

1) Nuclear-structure studies with light-ion-induced reactions such as $(d,p)$, $(p,d)$, $(p,p')$ etc. yield the clearest spectroscopic information at proton energies above 10 MeV, deuteron energies above 15 MeV, i.e. at energies of 10 MeV/ per nucleon or higher. This is distinctly above the energy-range of most low-energy heavy-ion studies. Proton- and deuteron-induced reactions at energies much lower than 10 MeV per nucleon yield results rich in peculiar energy-dependences characteristic of compound-nuclear backgrounds, rich in "anomalous" transitions and poor in information about nuclear level-structure. Extension of heavy-ion quasi-elastic studies into the energy-range $10 \leq E/A \leq 30$ MeV of the light-ion spectroscopy may resolve some of the inconsistencies and peculiarities encountered at lower energies.

2) The optical model for nucleon-induced reactions is much more soundly-based in many-body theory than its heavy-ion counterpart; its phenomenological parameterization is physically more reasonable and reproduces a wider range of data much more accurately. Indeed the lack of a theoretically-based parameterization of the absorptive part of the optical potential is probably the greatest single flaw in heavy-ion direct-reaction physics.

3) Coupled-channel effects in light-ion direct reactions (for $E/A \leq 30$ MeV) typically involve sizeable effects from a few specific channels. In heavy-ion reactions, on the other hand, we seem to be dealing with an accumulation of individually-smaller effects from many channels. It is of course hard to make
and support this statement quantitatively. It is however clear that much greater mileage has been achieved in light-ion studies without explicit introduction of channel-coupling effects. This has resulted in the accretion of a huge body of information about parentage relations between nuclear states, embodied in spectroscopic factors, while learning little of value about the radial structure of the corresponding overlap functions or form factors.  

Heavy-ion direct reactions force us into a multi-channel framework wherein not only are the pertinent reaction calculations much harder but also the results are acutely sensitive to aspects of the form factors about which light-ion studies have left us in almost total ignorance. Whether such sensitivity survives the necessary averaging processes remains to be seen.

3. Averaged Heavy-Ion Direct Reaction Theories.

When we study heavy-ion induced direct reactions we wish to ask such questions as the following. How is the cross-section divided between elastic and inelastic scattering, one-nucleon transfer, two- and four-nucleon transfer and so forth? What are the angular distributions and how do they vary with bombarding and excitation energy? How much of the cross-section goes to highly-excited states of the colliding nuclei (strongly-damped processes)? In addressing such questions it is neither possible nor of interest to treat each pertinent final state of target and projectile separately. An element of averaging over final states (or excitation energy) is essential.

One crucial point - too often ignored - must be kept in mind in assessing theoretical descriptions of heavy-ion direct reactions. A proper discussion of mass-, charge- and energy-transfer in such processes will require that the sharing of flux between elastic scattering, inelastic scattering, one-, two- and four-nucleon transfer, be correctly reproduced. No microscopic theory, for light or heavy ions, has yet been able to achieve such an ambitious goal. In particular, two-nucleon transfer cross sections are often seriously underestimated.
and studies of alpha-transfer seldom claim to address anything other than relative transition-rates. Thus when simplifying approximations are proposed that have been hallowed by custom in physical situations where only relative cross-sections for given mass-transfer are required, let the buyer beware!

Two sorts of averaged direct-reaction theory have been proposed. Both involve replacement of individual states of the colliding nuclei by giant resonances or "excitons".

Class I. Statistical Multi-Step Theories

In a recent paper, Feshbach, Kerman and Koonin discuss the calculation of energy-averaged cross-sections for nuclear reactions. They consider both compound nucleus and direct contributions; I shall discuss only direct processes although the relevance of their treatment of CN reactions to the calculation of fusion cross sections is obvious. The multi-step direct cross-section is obtained by folding, with respect to momenta of continuum particles, products of direct-transition probabilities equal in number to the number of steps. The final cross section is obtained by summation over all numbers of steps.

This theory has been implemented only for nucleon-induced reactions. For heavy-ion reactions what is needed is identification of the key excitons, study of their form factors and great virtuosity in computing (or parameterizing) the large number of DWBA-like transition amplitudes involved. It is here that the accumulated knowledge of the structure and simplifying features of heavy-ion transition amplitudes (to be discussed later in this talk) will play an essential role.

Work related to that under discussion has been reported by Agassai, Ko and Weidenmüller, whose major objective is the microscopic derivation of a transport theory of strongly-damped processes, and by Udagava and Tamura. The latter authors have not yet published a complete account of their theory and its assumptions and averaging procedures; it is clearly similar in spirit to that
of Feshbach et. al. Udagawa and Tamura have applied their theory to light-ion reactions and to the continuous spectra in heavy-ion-induced two-nucleon transfer reactions; it is not clear whether the parameterized forms they use for the key direct-reaction matrix elements will adequately reproduce the relative strengths of one-, two and more-nucleon transfers.

Class II. Giant-Resonance Coupled-Channel Theories

To discuss the sort of physical question mentioned at the beginning of this section, many final nuclear states, covering a wide range of excitation energies in a variety of different residual nuclei, play a significant role. Far too many channels must be coupled for a multi-channel theory without an element of averaging to have any chance of success.

Averaging can be introduced by replacing individual final states by excitations or giant resonances. It is possible that such giant-resonance coupled-channel models, including a few inelastic giant multipole resonances, a few single-nucleon, two-nucleon and possibly four-nucleon excitations in each nucleus, can account for the main features of heavy-ion direct reactions at not-too-high energy (E/A ≤ 30 MeV/A). The necessary calculations have not yet been attempted directly, although they are now within the bounds of possibility.

Broglia, Dasso, Pollarollo and Winther\textsuperscript{14} have carried out semi-classical studies of a multi-channel giant-resonance model of the kind under discussion. Comparison of their results with what emerges from fully quantum-mechanical calculations will be of particular interest and importance here.

Semi-classical approximations tend to deteriorate as they encounter larger internal excitation energies and larger mass-transfers.

4. The Structural Elements of Direct-Reaction Theories

Direct-reaction theories are cooked with varying culinary techniques from a few basic ingredients. These common structural elements include optical potentials, transition form factors, S- or T-matrix elements and Coulomb functions.
We now turn to a discussion of the simplifying features of the structural elements of heavy-ion direct reactions.

First, let us dispose of matters of notation. In a collision between two ions (Target and Projectile) a channel is a definite pair of internal states \((x_T,x_P)\), \((\alpha_T,\alpha_P)\) of the collision partners. A basic state \(\alpha\) in a given channel is specified in the channel-spin representation by the quantum numbers

\[
\alpha = \left( x_T, x_P, \alpha_T, \alpha_P \right) \tag{1}
\]

where \(\alpha\) (vector sum of \(J_T\) and \(J_P\)) is the channel spin, \(1\) the relative orbital angular momentum, \(J_0\) the total angular momentum and parity. The coupling scheme is indicated in Fig. 2.

An 'effective' Schrödinger equation is solved in a truncated model space spanned by the basis states \(\alpha\) (Eq. (1)) of a 'few' channels. In \(r\)-space, a set of coupled radial equations is obtained for each value of \(J_0\); dropping the common label \((J_0)\), we have

\[
L_{\alpha}(r) \cdot R_{\alpha}(r) = \sum_{\beta} U_{\alpha\beta}(r) R_{\beta}(r) \tag{2}
\]

where

\[
L_{\alpha} = \frac{\text{d}^2}{\text{d}r^2} - \frac{\sigma_1 + 1}{r^2} - U_{\alpha}(r) \tag{3}
\]

for elastic and inelastic scattering. If rearrangement channels are involved (transfer reactions), the simple coupling potential \(U_{\alpha\beta}(r)\) is replaced by a non-local operator;

\[
\sum_{\beta} U_{\alpha\beta}(r) R_{\beta}(r) \rightarrow \int K_{\alpha\beta}(r,r') R_{\beta}(r') \text{d}r' \tag{4}
\]
In a coupled-channel theory, these radial equations are to be solved subject to
the condition of regularity at the origin and with the asymptotic form

\[ R_\alpha(r) + \frac{i}{r} \left[ \delta_{\alpha\alpha_0} I_\alpha(r) - \frac{k_\alpha}{k_{\alpha_0}} S_{\alpha\alpha_0} O_\alpha(r) \right] \]  

(5)

where \( \alpha_0 \) is a basis state in the channel containing incoming waves, \( I_\alpha, O_\alpha \) are
incoming and outgoing Coulomb functions and \( S_{\alpha\alpha_0} \) is an S-matrix element.

The structural elements of direct-reaction theories are thus the
optical potentials \( U_{\alpha\alpha}(r) \) (and their associated elastic-scattering solutions or
distorted waves), the inelastic coupling potentials \( U_{\alpha\beta}(r) \), the transfer kernels
\( K_{\alpha\beta}(r, r') \), the Coulomb functions \( I_\alpha, O_\alpha \) and the S-matrix elements \( S_{\alpha\alpha_0} \). The role
of these structural elements in a coupled-channel theory is clear from Eqs. (2)
to (5) above; in DWBA theories the essential quantities are matrix elements
obtained by weighting coupling potentials or transfer kernels with products of
single-channel distorted waves and integrating over \( r \) (or over \( r \) and \( r' \)).

In both sorts of averaged direct-reaction theory discussed in Sec. 3,
the first step is to devise coupling potentials and transfer kernels that are
appropriate for giant-resonance excitations (excitons) and are at the same time
sufficiently simple to permit the predictions of the theory to be worked out. In
the case of multi-step theories, the working-out process requires the efficient
evaluation of multitudes of integrals over products of DWBA matrix elements,
folded in with level-densities. The multi-channel giant-resonance theories demand
the development of techniques for the rapid solution of very large systems of
coupled radial equations. In either case, the computations required are so com-
plex, that it will be essential for further progress to exploit to the full any
simplifying features that the structural elements of the theory may possess.
5. The Heavy-Ion Optical Potential; the "Sensitive Radius" and other Curiosities.

Of all the structural elements of direct-reaction theories, the optical potential is the most basic. It is an effective interaction acting within the truncated model space spanned by the chosen active reaction channels. It depends explicitly on the choice of model space and can be expected to be different in single-channel and multi-channel descriptions of the same reactions.

Consider first the single-channel, optical-model description of elastic scattering. Let \( V \) denote the sum of two-body interactions between nucleons of the target \( T \) and the projectile \( p \), while \( H \) is the complete Hamiltonian of the system. Effective-interaction theory\(^{15} \) specifies the structure of the elastic-channel optical potential between \( T \) and \( p \); taking matrix elements with respect to channel-basis states \( \alpha \) (see Sec. 4), we have

\[
V_{\text{opt}} = V_{00} + \sum_{\alpha \beta} V_{0\alpha} \left[ \frac{1}{E_{\text{c.m.}}} \right]_{\alpha \beta} V_{\beta 0}
\]

or

\[
V_{\text{opt}} = V_F + \Delta V
\]

where the subscript 0 refers to the elastic channel, and no explicit account has been taken of antisymmetry between nucleons of \( T \) and \( p \).

\( V_{00} \) is the folded potential for the target-projectile system; it is purely real and, in the region of partial-overlap crucial to heavy-ion direct interactions, it is reasonably-well understood. All effects of coupling to other channels, including effects of absorption from the elastic channel, together with exchange effects, are contained in \( \Delta V \), which has so far defied quantitative study for heavy-ion systems. It is likely to be predominantly imaginary, strongly dependent on \( \varepsilon \) and non-local. Otherwise we know little about it; its radial dependence in particular is obscure and is most unlikely to resemble the radial form of \( V_F \).
In the absence of theoretical guidance, the heavy-ion optical potential has been treated phenomenologically. Two sorts of parameterization are common:

(i) Take \( V_j \) (with perhaps an adjustable normalization factor) from a folding calculation, assume \( \delta V \) to be purely imaginary and parameterize it.

(ii) Parameterize \( V_{opt} \) directly.

By custom rooted in many years of study of light-ion direct reactions, the heavy-ion optical potential \( V_{opt} \) (or its imaginary part \( \text{Im} \delta V \) when a folded potential is used for the real part) is assumed to have a Woods-Saxon radial dependence:

\[
V_{opt} = -V_r f(r, R_r, a_r) - i V_i f(r, R_i, a_i) \tag{8}
\]

The parameters \( V_r, R, a \) are then determined in least-squares fits to elastic-scattering data, and \( f(r, R, a) \) is the usual Woods-Saxon radial function.

A large body of phenomenological lore has been acquired in this fashion. It has a peculiarly insubstantial flavor, quite unlike that of the phenomenology of nucleon-nucleus optical potentials. The parameter-values extracted from the data exhibit many ambiguities and few intelligible systematic trends. What seems to be fixed by the data in many cases is the value of the real part of the potential at some well-determined value of \( r \) in the nuclear surface, and the ratio \( \text{Im} V_{opt}/\text{Re} V_{opt} \) near the same value of \( r \). As we now discuss, even this apparently-solid nugget of information is something of an embarrassment.

Unconstrained Woods-Saxon fits to heavy-ion elastic scattering data do not yield unique sets of best-fit parameters. Numerous fits of comparable quality \( (\chi^2/\chi^2_{\text{min}} < 2) \) and widely-differing parameter-values are usually obtained. Furthermore, the data seldom contain more than four-parameters' worth of information. If, however, the real part of the potential is plotted as a function of \( r \) for each of the local-best-fit potentials, it is found that the various curves come surprisingly close to intersecting at a point. This phenomenon can be
illustrated in the case of the elastic scattering of 192 MeV. A series of five-parameter least-squares fits were carried out with the value of the diffuseness parameter \( a_r \) of the real part of the potential constrained to have fixed values ranging in steps of 0.05 fm from 0.45 fm to 0.90 fm; fits were obtained with values of \( \chi^2 \) per data-point ranging from 1.1 to 1.75. The resulting graphs of \( \text{Re } V_{\text{opt}} \) as a function of \( r \) are shown in Fig. 3. They can be seen to intersect at the 'point' \( r = 12.3 \) fm, \( \text{Vo}pt = 1.91 \) MeV, with uncertainties of a fraction of a percent in \( r \) and \( \text{Vo}pt \).

This result is in a sense correct; many fits to elastic-scattering data in a variety of heavy-ion systems yield such 'points' with uncertainties in \( r \) and \( \text{Vo}pt \) of 1% or less. It is also absolutely preposterous; the degree of radial localization implied is about an order of magnitude smaller than the wavelengths involved (typically around 0.5 fm). The sharp intersection and the associated suggestion that the data determine the value of the real part of the optical potential at a well-defined "sensitive radius" is in fact an artifact of the Woods-Saxon parameterization. That this is the case can be seen in a number of different ways.

1) The radial dependence of the potentials can be parameterized by powers of the Woods-Saxon function

\[
V_k[1+\exp(-R/k)]^{-n}, \quad n = 2, 3, 4, \ldots
\]

(\( k = r, 1 \)). Equally good fits are obtained, with equally well-determined sensitive radii. The values obtained for the sensitive radius, however, vary with \( n \) (the Woods-Saxon power) over a region about 1 fm wide.

2) Gaussian forms for the potentials also yield good fits to the data. Well-defined sensitive radii are again obtained, but with different values from those obtained with Woods-Saxons and powers thereof.
3) As a final test, a pointwise parameterization of the real part of the potential was studied,\(^{17}\) with a conventional Woods-Saxon form for the imaginary part. No explicit radial dependence is assumed for Re \(V_{opt}\); the free parameters are the actual values of Re \(V_{opt}\) at five or six fixed values of \(r\) distributed evenly across the region of the nuclear surface. The values of the potential at intermediate values of \(r\) are obtained from suitable interpolating polynomials and reasonable behavior at large and small \(r\) is ensured by a proper choice of extrapolating functions. (exponential decay as \(r \to \infty\), constant value as \(r \to 0\)) It is found that there is no point at which the real part of the potential is determined to better than 10%. There is however a well-defined band of \(r\)-values, about 1 fm in extent, throughout which the uncertainties in Re \(V_{opt}\) are least. This region in which the potential is best determined is in the tail of the overlap region of the nuclear densities and contains the "sensitive radii" obtained with special parameterizations of the potential.

Heavy-ion optical potentials, then, are treated phenomenologically on the basis of parametric forms that are without theoretical foundation. This basic flaw is not corrected by use of folded potentials for the real part of the potentials since the crucial absorptive part must still be parameterized in arbitrary fashion. The case of the sensitive radius clearly reveals how treacherous the road then is from data to physical statements about the potentials.

Distorted-wave Born approximation (DWBA) theories of peripheral direct reactions, based on the optical-model description of elastic scattering in each channel, have permitted a wealth of spectroscopic information to be extracted from analyses of light-ion induced inelastic-scattering and transfer data. A recent study\(^{18,19}\) of single-nucleon transfer over a wide range of bombarding energies indicates that the analogous approach to peripheral reactions induced by heavy ions is seriously defective. The system in question here is that used above to illustrate the phenomenon of the sensitive radius \(- 16^0_0 + 208^\text{Pb}\). The elastic scattering of \(16^0_0\) ions by \(208^\text{Pb}\) has been studied experimentally at
more than a dozen bombarding energies from 80 to 312 MeV (5 to 20 MeV/A).

Single-nucleon transfer reactions — \(^{(16,15)O}_{209}\) to \(^{209}\)Pb, \(^{(16,15)N}_{209}\) to \(^{209}\)Bi
and \(^{(16,17)O}_{207}\) to \(^{207}\)Pb — have also been studied at a number of energies across
the range specified above.

A detailed DWBA analysis, along the lines familiar from light-ion
physics, was carried out for these heavy-ion induced transfer reactions.

Various sorts of Woods-Saxon parameterizations of the elastic data are first
obtained — energy-dependent, energy-independent, individual fits at each energy
and so forth. Single-nucleon form factors are constructed so as to be consistent
with the single-particle level properties of \(^{207}\)Pb, \(^{209}\)Pb and \(^{209}\)Bi and
with what is known from electron scattering and light-ion-induced transfer on
\(^{208}\)Pb. The optical potentials are used to generate distorted waves that are
integrated over the single-nucleon form factors to obtain DWBA cross-sections
for the various heavy-ion induced transfer reactions on \(^{208}\)Pb.

It is found that while DWBA gives an excellent account of the relative
strengths of the transitions to different final states at each energy, it
predicts an energy-dependence for these transition-rates that is quite at variance
with experiment. These statements hold for all the Woods-Saxon parameter-
izations of the elastic data. It is found (Fig. 4) that

1) The observed angle-integrated cross-section to each distinct final state
increases rapidly through the Coulomb barrier, then levels off (at around 10 MeV/A) and in some cases begins to decrease again.

2) The angle-integrated DWBA cross sections continue to increase steadily through-
out the energy-range up to the highest energies studied (<20 MeV/A).

No other heavy-ion system has been studied in such detail over such
a wide range of bombarding energies. It is not, therefore, clear to what extent
the above discrepancy is a systematic feature of heavy-ion direct reactions
and their optical model-DWBA description. The suggestion is however unmistakable
that something is amiss, most probably with the form assumed for the absorptive
part of the optical potential. In the single-channel description of elastic scattering, the imaginary part of the potential has to take into account loss of flux due to two entirely different processes — complete fusion and direct reactions. Their contributions to the optical potential must be qualitatively different, yet the Woods-Saxon parameterization treats both on the same (probably inappropriate) footing.

We recall finally that the optical potential is an effective interaction and is therefore dependent on the precise choice of the model space in which it acts. Thus in going from a single-channel description of elastic scattering to a multi-channel theory designed to include, albeit in an averaged fashion, all channels directly and strongly coupled to the elastic channel, the optical potential and in particular its imaginary part may change radically. Indeed the direct contribution to the absorptive part of the potential may be small enough in the many-channel framework for a theoretically-guided heavy-ion optical potential to emerge.

6. Interpolation and Extrapolation in Angular Momentum

The partial-wave description of quasi-elastic heavy-ion reactions has two salient qualitative properties.

1) Many partial waves may contribute significantly.

2) The S-matrix elements are smooth complex functions of angular momentum. In this section we discuss how the simplifying aspects of the second property permit us to solve the computational problems posed by the first.

To illustrate the procedures involved, consider the proton-stripping reaction $^{208}_{\text{Pb}}(^{16}_{\text{O}},^{15}_{\text{N}})^{209}_{\text{Bi}}$ at $E_{\text{Lab}} = 312.6$ MeV, leaving $^{15}_{\text{N}}$ in its $1/2^-$ ground state and $^{209}_{\text{Bi}}$ in its $3/2^-$ excited state at an excitation energy of 3.12 MeV.

(One of the reactions discussed in Sec. 5) In the DWBA description of this reaction, only two channels are involved.
Incoming; \( J_T = J_P = 0 \). Channel-spin \( \delta^S = 0 \). Parity = +

Outgoing; \( J_T = 3/2^- \), \( J_P = 1/2^- \); \( \delta^S = 1 \) or 2. Parity = +

There are four basis states \( \beta \) in the outgoing channel, one in the incoming channel. For given \( J \), and the coupling scheme (1), the basis states are

\[ a: k_a = J \quad \delta_a = 0 \quad \pi = (-)^J \]
\[ b_1: k_B = J \quad \delta_B = 1 \quad \pi = (-)^J \]
\[ b_2: k_B = J - 2 \quad \delta_B = 2 \quad \pi = (-)^J \]
\[ b_3: k_B = J \quad \delta_B = 2 \quad \pi = (-)^J \]
\[ b_4: k_B = J + 2 \quad \delta_B = 2 \quad \pi = (-)^J \]

Other values of \( \pi_B \) are excluded by parity-conservation.

The DWBA \( S \)-matrix elements for transfer are proportional to the radial integrals

\[ I_{\alpha \beta}^J = \int R_{\alpha}(r)R_{\beta}^J(r,r')K_{\alpha \beta}(r')dr'dr' \]

where \( R_{\alpha}(r) \) is an optical-model radial wave function (distorted wave) and \( K_{\alpha \beta} \) is the single-nucleon transfer form factor (Eq. (4)). The radial integrals for a given pair of basis states \((\alpha, \beta)\) have the following properties;

1) The absolute magnitude \( |I_{\alpha \beta}^J| \) peaks at a "grazing" value of \( J \).
2) \( |I_{\alpha \beta}^J| \) falls off exponentially at large \( J \).
3) The behavior of the phase of \( I_{\alpha \beta}^J \) at large \( J \) is given to remarkable accuracy by the asymptotic expression

\[ \text{Arg } I_{\alpha \beta}^J \overset{\text{large } J}{\longrightarrow} \text{C}(\delta_{eJ}(k_{\alpha}) + \delta_{eJ}(k_{\beta})) \]

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where $\delta_1(t_{\alpha})$, $\delta_1(t_{\beta})$ are the elastic phase-shifts in channels $\alpha$, $\beta$ and $C(\delta_1(t_{\alpha}),t_{\beta},t_{\beta})$ is an $t$-independent constant.

The absolute magnitude of the matrix element $I_{a}^{J}$, in the notation of (10) above, is plotted as a function of $J$ in Fig. 5. It is seen that more than 100 partial waves (centered around $J = 150$) contribute significantly. However, the $J$-dependence of $|I_{a}^{J}|$, and of the corresponding phase shown in Fig. 6, is sufficiently simple that an accurate numerical representation can be obtained from explicit evaluations of the integrals at relatively few $J$-values.

The general procedure is as follows. The "grazing" $J$-value near which $|I_{a}^{J}|$ peaks can be predicted with adequate accuracy. Start at this value of $J$ and evaluate integrals at intervals $\Delta J$ in $J$ on both sides of the grazing $J$-value. Stop at low $J$ when successive integrals are encountered whose magnitudes are, say, less than $10^{-2}$ times that of the grazing integral; contributions from smaller $J$-values are omitted. Stop at the high-$J$ end when an integral is encountered whose absolute magnitude is less than a tenth of the size of the grazing integral. Use the values of the last two integrals computed to determine the parameters of a decaying Woods-Saxon function in $J$ to represent $|I_{a}^{J}|$ for larger $J$ and an average value of the constant $c$ in the representation (12) of the phase $\text{Arg } I_{a}^{J}$. Finally, fit a complex continued fraction to the computed integrals in the region $J_{\text{min}} \leq J \leq J_{\text{max}}$ and use this with the extrapolating functions discussed above to evaluate cross-sections.

The resulting representations of the function $I_{a}^{J}$ for the reaction $^{208}\text{Pb}(^{16}\text{O}, ^{15}\text{N})^{209}\text{Bi}(3/2^{-})$ at 312.6 MeV are shown in Figs. 5 and 6. The lower-$J$ cut-off is at $J_{\text{min}} = 110$ and the value beyond which extrapolation is used is $J_{\text{max}} = 182$. Various values of $\Delta J$ in the intermediate region are compared. No discernible difference is found in $|I_{a}^{J}|$ with $\Delta J \leq 8$, while significant differences in $\text{Arg } I_{a}^{J}$ for $\Delta J < 8$ are found only at $J$-values for which the magnitude of the integrals is negligible.
The corresponding cross-sections are shown in Fig. 7, with the remaining integrals $I^J_{ab} (\beta = \beta_2$ to $\beta_4$) treated in the same fashion as $I^J_{ab}$. The differential cross-section is reproduced with errors of less than 1% at all angles with $J_{\text{min}} = 110$, $J_{\text{max}} = 182$, $\Delta J = 8$. A reasonable approximation is still obtained with $\Delta J = 12$. It is found in the case under consideration that although 120 partial waves contribute significantly to the cross-section, radial integrals need be computed at only 10 values of $J$ for 1% accuracy. This represents a saving of a factor of ten in computational labor.

Similar interpolation and extrapolation methods have been developed for the S-matrix elements of other heavy-ion processes, including the S-matrix elements obtained in coupled-channel calculations. It is of crucial importance in all cases to identify the large-$J$ asymptotic behavior and to start using the appropriate extrapolating function at as small values of $J$ as possible. Otherwise, rational or polynomial representations must be used over ranges of $J$ in which the absolute value of the integrals varies through many orders of magnitude; this is not only wasteful of computational effort but, more significantly, is a serious source of numerical error.

The heavy-ion reactions in which the largest number of partial waves contribute tend to be those whose S-matrix elements are the smoothest functions of angular momentum. Thus interpolation and extrapolation in $J$ is most powerful where it is most needed. With the above numerical representations of the S-matrix elements as functions of $J$, 1% accuracy can be achieved in the cross-section with great savings in computational labor. There are no heavy-ion systems for which it should be necessary under normal circumstances to treat more than 20 or 30 $J$-values explicitly.
7. Treatment of Long-Range Coulomb Coupling

Three regions of configuration space should be distinguished in a discussion of heavy-ion direct reactions.

i) The interior region, $0 \leq r \leq R_N$, where $R_N$ is the outer limit of nuclear interactions. $R_N$ can range from 15 to 30 fm. for systems of physical interest.

ii) The intermediate region, $R_N < r < R_A$ extending from the outer limit of nuclear interactions to a distance $R_A$ beyond which all off-diagonal Coulomb couplings are negligible. $R_A$ may be as large as several hundred fm. for heavy-ion systems.

iii) Asymptopia, $r > R_A$, in which only the $1/r$ tails of the diagonal Coulomb interaction survive.

The standard procedure for the solution of the coupled equations (2) is to integrate $N$ times from $r = 0$ to $r = R_A$ (i.e. to asymptopia) with $N$ different sets of starting-values; the appropriate regular solution and the S-matrix elements are then determined by matching to the asymptotic form (5) at $r = R_A$.

This is extremely wasteful for heavy-ion calculations because

a) it involves repeated integration over the wide intermediate region wherein only Coulomb excitation is taking place but which may cover several hundred fm., and

b) it requires that the $N$ coupled equations be solved $N$ times.

Alternative procedures that avoid these pitfalls will now be discussed, the Coulomb-excitation problem in this section, the multiple-solution problem in the next.

Integration over the intermediate region, the region of Coulomb excitation, can be almost entirely eliminated by a procedure based on the Alder-Fauli factorization of the coupled-channel wave functions. Let us define $\mathbf{O}_s(r)$ (a vector whose $N$ components are labelled by channel basis states $a$) to be that outgoing solution obtained by starting in asymptopia (at $R_A$) with an outgoing wave $\mathbf{O}_s(k, r)$ in channel $s$, zero in all other channels, and integrating inwards.
The incoming solution \( \overrightarrow{\Pi}_s(r) \) is defined similarly. If these coupled-channel Coulomb functions can be computed efficiently in the intermediate region, then they can be used in place of the ordinary Coulomb function of Eq. (5) in a modified matching equation:

\[
R_s(r) = \frac{1}{2} \left[ \overrightarrow{\Pi}_{s,a_0}(r) - \sum \frac{k_{a_0}}{k_B} S_{0}\overrightarrow{\Pi}_{s,a_0}(r) \right]
\]

at \( r = R_N \).

If the coupled-channel Coulomb functions must be constructed by integration of the coupled equations across the intermediate region from \( R_A \) to \( R_N \), nothing has been gained. This, however, is not the case. Powerful alternative methods are at hand based on the Alder-Pauli factorization,

\[
\mathcal{O}_{s,a_0}(r) = \frac{1}{k_a} A_{s,a_0}(r) \tilde{O}_{0}(k_a r)
\]

with a similar expression for the incoming solution. The idea underlying this representation is that the coupled-channel Coulomb function in channel \( a \) is simply the asymptotic Coulomb function modulated by a slowly-varying amplitude \( a_{s,a_0}(r) \). Under the assumption that both \( a_{s,a_0} \) and the norm of the Coulomb function \( Q_a \) are very slowly-varying functions of \( r \), the coupled second-order equations satisfied by \( a_{s,a_0} \) (obtained by substitution of Eqs. (14) in Eqs. (2)) reduce to the much-simpler first-order form

\[
\frac{d a_{s,a_0}}{dr} = M(r) a_{s,a_0}(r)
\]

where the matrix is given by

\[
M_{\alpha\beta}(r) = -\frac{1}{2} \overrightarrow{U}_\alpha \overrightarrow{C}_{\beta\beta} \overrightarrow{U}_\alpha^* \overrightarrow{C}_{\alpha\beta}
\]

and \( \overrightarrow{U}_\alpha \) is the Coulomb part of the coupling matrix in Eqs. (2).
The approximations leading to Eqs. (15) have been tested in a variety of cases; they are found to be remarkably good and permit 1% accuracy or better in the cross-sections at nuclear matching radii R_N of 20 to 30 fm. (It is clear that all the approximations based on the factorization (14) tend to deteriorate as r decreases towards the region of classical turning points.) Replacement of Eqs. (2) by (15) is already a major simplification. Eqs. (15) can be integrated across the intermediate region with a step-size larger than that required with the original Eqs. (2) roughly in the ratio k/Δk, where k is an average wave-number for all channels and Δk is the largest difference in k-values between channels; k/Δk is typically a number of the order of 10 or greater.

A further enormous simplification is possible, one that entirely disposes of the need to integrate coupled equations across the region of Coulomb excitation. It turns out that for nearly all practical purposes, Eqs. (15) can be solved in first Born approximation:

$$\sum_{s,\alpha} b_{s,\alpha}(r) \sim \sqrt{K_0} \left[ \delta_{\alpha s} - \int_r^\infty M_{\alpha\beta}(r')dr' \right]$$  \hspace{1cm} (17)

It is of course obvious that this approximate solution must be valid for sufficiently large r. What is true but far from obvious is that the Born solution suffices (to accuracies of 1% or better in the cross-sections) all the way in to 20 or 30 fm. The integrals appearing in Eq.(17) are precisely those that give the DWIA matrix elements for first-order Coulomb excitation. These integrals obey recursion relations in angular momentum that permit the modulating amplitudes a_{s,\alpha}(R_N) to be computed for all J-values with little more effort than that involved in calculating them for one or two 'starting' J-values.

The upshot of all this is that the coupled-channel Coulomb functions \( \tilde{\Phi}_s(r) \) and \( \tilde{I}_0(r) \) can be computed at the edge of the interior region r = R_N with an effort comparable to that involved in a first-order DWIA calculation of inelastic scattering. The intermediate region has been completely eliminated and the problem reduced to that of solving the coupled equations (2) in the interior region (r ≤ R_N, R_N ~ 20 to 30 fm.).
8. Iterative Techniques for Coupled-Channel Problems

There are two classes of method for solving coupled radial equations that avoid the wasted effort of constructing $N$ regular solutions to find the one with the desired asymptotic behavior.

I. Basis-expansion methods $^{24,25}$

The radial functions $R_{n}(r)$ are expressed on $[0, R_{N}]$ as linear combinations of a suitably-chosen set of basis function $\phi^{(n)}(r)$

$$R_{n}(r) = \sum_{n=1}^{N} C_{n} \phi^{(n)}(r).$$

The coupled equations (2), with boundary conditions (13) at $R_{N}$, yield a set of $N$ inhomogeneous linear equations for the expansion coefficients $C$ and the $S$-matrix elements. The efficiency of methods of this sort depends on the ability to represent the radial functions to adequate accuracy with a reasonably small number $N$ of basis functions. Detailed studies have been confined to light-ion reactions. It is clear that heavy-ion wave functions will require more basis states than light-ion wave functions. If as seems likely $N \approx 60$ is a reasonable guess, then basis-expansion methods for heavy ions are likely to need more storage and to be significantly slower (by an order of magnitude) than the iterative methods described below, although still a great improvement over the brute-force approach.

II. Iterative Techniques

A number of iterative procedures have been used to solve coupled-channel equations for light-ion reactions. Only recently has a systematic effort been undertaken $^{26}$ to compare and assess their efficiency for heavy-ion reactions. All the methods under consideration are modifications, resummations or re-orderings of the Born-Neumann series for the set of coupled equations (2) in the interior region.
Suppose that \((k-1)\) steps of an iterative procedure have been completed, yielding a set of radial solutions \(R^{(k-1)}_a\) of Eqs. (2). These solutions are substituted in the right-hand sides of Eqs. (2) to yield a set of \(N\) uncoupled inhomogeneous equations

\[
L_a R^{(k)}_a = \sum_B U_{ab} R^{(k-1)}_B
\]

(19)

for the improved solutions \(R^{(k)}_a\). The \(S\)-matrix elements \(S^{(k)}_{\alpha\alpha_0}\) at the \(k\)'th step are obtained from the matching condition (13) by a procedure whose details are of no concern here. This is the straightforward Born-Neumann iteration. Various initial guesses \(R^{(0)}_a\) can be used to start the iteration. The precise choice of starting guess turns out to be relatively unimportant and we therefore start from the initial estimate

\[
R^{(0)}_a = \delta_{\alpha\alpha_0} f(r)
\]

(20)

that has the regular, optical-model solution \(f(r)\) of the homogeneous equation \(L_f = 0\) in channel \(\alpha_0\), zeroes in all other channels.

It is found that straightforward Born-Neumann iteration diverges for many cases of practical interest in heavy-ion physics. The following modifications, designed to improve convergence, have been studied.\(^{26}\)

1) Padé Acceleration

Given the sequence \(S^{(0)}_0, S^{(1)}_0, S^{(2)}_0, \ldots S^{(k)}_0\) of approximations to a given \(S\)-matrix element \(S^{j}_{\alpha\alpha_0}\), define a set of coefficients \(b_r\) by

\[
b_r = S^{(r)}_0 S^{(r-1)}_0 \quad (r > 1)
\]

(21)

\[
b_0 = S^{(0)}_0.
\]

By construction, the \(k\)'th approximation \(S^{(k)}_0\) is given by the sum of \(k+1\) terms of the power-series.
evaluated at $x = 1$,
\[ S^k = S^k(1). \] (23)
A Padé approximant to $S^k(x)$ is a rational function
\[ \frac{P_L(x)}{Q_M(x)} \] (LHN - k) (24)
whose coefficients are chosen such that the first $k$ terms of its Maclaurin expansion coincide with the polynomial $S^k(x)$. Sequences of rational functions such as
\[
\begin{align*}
[0/0] & [1/0] & [1/1] & [2/1] & \cdots & \cdots \\
k=0 & k=1 & k=2 & k=3 & \cdots & \cdots 
\end{align*}
\] (25)
evaluated at $x = 1$ converge more rapidly than the original sequence when it converges, and frequently converge when the original sequence diverges.

ii) Sequential Iteration

The straightforward iterative scheme described above is particularly ill-suited to the treatment of reactions in which a high-spin state can be excited only by a sequence of excitations of low multipolarity. For example, a $J = 20^+$ member of a rotational band is not reached from the ground state until the tenth iteration of the Born-Neumann procedure. To remedy this defect, Raynal proposed that the Born iteration be carried out sequentially. Suppose that the channels $\alpha$ are ordered in some fashion, with those channels most strongly coupled to the incoming channel $\alpha_0$ first, those strongly coupled to the first set next, and so on, with the elastic channel itself last. Then each improved solution $S^{(k)}_{\alpha}(x)$ is substituted in the inhomogeneous terms of all later equations. The equations to be solved are

\[
S^k(x) = \sum_{r=0}^{k} b_r x^r \] (22)
where $a = a_0 = 1$ refers to the incoming channel. Eqs. (26), with the starting guess of Eq. (20), constitute sequential iteration. The amount of computation involved per iteration is identical to that involved in straightforward Born-Neumann iteration.

iii) Sequential Iteration with Padé Acceleration

The convergence of the sequences $S^{(0)}$, $S^{(1)}$, ..., $S^{(k)}$ of approximate S-matrix elements obtained by sequential iteration can be improved with the aid of Padé approximants in the fashion described above for Born-Neumann iteration.

iv) Method of Moments

None of the above methods is guaranteed to converge for potentials of arbitrary strength. The restriction of the coupled equations to a relatively small interior region of $r$-space obviously improves their prospects, but no a priori mathematical conditions are known. The method of moments uses the sequence $R^{(0)}$, $R^{(1)}$, ..., $R^{(k)}$ of radial solutions generated by the Born-Neumann iterative process in such a way that convergence can be proved for all potentials, regardless of their strength, likely to be of interest in heavy-ion reaction theory.

The Born-Neumann radial functions are used to define a sequence of functions $\Pi^{(1)}(r)$ according to the relations

\begin{align*}
\Pi^{(1)}(0) &= R^{(0)} \\
\Pi^{(1)}(r) &= R^{(1)}(r) - R^{(1-1)}(r). \quad (k>1)
\end{align*}

These functions are used as basis-functions in an expanding sequence of finite vector spaces;
The coupled equations (2) are projected onto the finite vector space \( \mathcal{H}^{(k)} \) and the resulting linear equations solved for the expansion coefficients that specify the solutions \( \hat{g}_a^{(k)} \) within \( \mathcal{H}^{(k)} \). It can be shown\(^{26}\) that the corresponding S-matrix elements can be obtained from the Born-Neumann S-matrix elements \( S^{(k)} \) with the aid of the identity

\[
\hat{g}_a^{(k)} = \delta_{a0} S_{a0}^{0} + \sum_{i=1}^{k-1} A_i^{(k)} [S_{a0}^{(i+1)} - S_{a0}^{(i)}] \tag{29}
\]

where \( S_{a0}^{0} \) is the optical-model S-matrix element in channel \( a \) and the expansion-coefficients \( A_i \) are those that specify the radial solution

\[
\hat{g}_a^{(k)} = \sum_i A_i^{(k)} \hat{u}_i . \tag{30}
\]

Griffin and Koshcl\(^{27}\) have recently applied the method of moments to the solution of coupled equations for light-ion reactions. Further details will be found in their paper and in Ref. 26.

We have carried out\(^{26}\) extensive comparisons of the above methods in a wide variety of heavy-ion systems. To illustrate our results, consider the two-channel problem \(^{44}\)Ca(\(^{16}\)O,\(^{16}\)O')\(^{44}\)Ca(\(^2+\);1.156 MeV.) at a laboratory \(^{16}\)O energy of 60 MeV. There are four basis states in the two channels involved; for given \( J \) these are

\[
\begin{align*}
\alpha &= 1: & \beta &= 2: & \gamma &= 3: & \delta &= 4: \\
& a = J & & a = J-2 & & a = J & & a = J+2 \\
& \beta_a = 0 & & \beta_\beta = 2 & & \beta_\gamma = 2 & & \beta_\delta = 2 \\
& v = (-)^J & & v = (-)^J & & v = (-)^J & & v = (-)^J
\end{align*}
\tag{31}
\]
The values obtained, using the various iterative techniques, for the magnitude $|s_{13}^{J=30}|$ of the 1+3 S-matrix element in the near-grazing partial wave $J=30$, are shown in Table 1. The coupling potentials are derived by the standard procedure of deforming the optical potential; specifically, a value $\beta = .4$ (twice the physically-correct value) is used for the deformation in order to simulate strong coupling. It is seen that Born-Neumann and sequential iterations diverge, the Padé accelerated sequences and the method of moments converge and the Padé-accelerated sequential iteration converges most rapidly.

The results shown in Table 1 reflect what is found in a wide variety of reactions. Our main conclusions are summarized below.

1) Sequential-iteration with Padé acceleration is the most-rapidly convergent of all the iterative techniques. 1% accuracy in cross-sections can be achieved in 3 or 4 iterations. We have been unable to find a case where it diverges, even when the coupling potentials are arbitrarily multiplied by strength factors up to ten times the physical values in an effort to induce divergence.

2) The method of moments also converges fairly rapidly, roughly five or six iterations being necessary on the average for 1% accuracy in the cross sections. The method of moments involves significantly more computation and a great deal more computer storage at each iteration. In return, however, it yields converged radial solutions $R_n$ whereas the faster Padé-accelerated sequential method yields only S-matrix elements.

For large systems of coupled equations, the iterative methods discussed above converge sufficiently rapidly and involve sufficiently modest amounts of computation that they make the brute-force approach obsolete. The fastest available method is sequential iteration with Padé acceleration.

9. Summary and Conclusions

It is my guess that information of interest for nuclear structure will come predominantly from the lower-energy end of the range of energies discussed...
at this conference. At these lower energies, two theoretical approaches to heavy-ion reactions have been proposed that trace their origins to light-ion direct-reaction spectroscopy. These approaches can be described as statistical multi-step and giant-resonance coupled-channel theories. Neither has been implemented to anywhere near the point of answering the key physical questions. The technical advances in calculating direct-reaction matrix elements and solving coupled-channel problems described in Secs. 6 to 8, bring implementation of the averaged heavy-ion reaction theories well within the bounds of possibility.

Acknowledgments

I wish to thank my colleagues Steven Pieper and Mark Rhodes-Brown for permitting me to knit a fringe of my own opinions around a core of work done, in its entirety, in collaboration with them.
Table 1. Convergence of iterative calculations of the absolute magnitude of the S-matrix element $|S_{13}^3|_0$ (see the scheme (31) in the text) for the reaction $^{44}_{16}$Ca($^{16}_{0}, ^{16}_{0}$)$^{44}_{16}$Ca($^{2+}; 1.156$ MeV) at a laboratory energy of 60 MeV. Pure nuclear excitation is considered, with a deformed Woods-Saxon potential of first-order in B, with B = .4, and optical parameters (Eq. 8) $V_r = 110$ MeV, $V_t = 20$ MeV, $a_r = a_t = .5$ fm, $r_0 = 1.2$ fm (for all components of the potential).

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References and Footnotes

1) See, for example, D. K. Scott, *The Current Experimental Situation in Heavy-Ion Reactions*, Lectures at the NATO/NSF Advanced Studies Institute on Theoretical Methods in Medium-Energy and Heavy-Ion Physics, Madison, June 12-23, 1978. (In particular, Fig. 2-25) LBL Report 7727.


3) T. Udagawa and T. Tamura, Phys. Rev. Lett. 41, 1770 (1978);


9) K. S. Low, p. 267 of proceedings cited in Ref. 6.


17) S. C. Pieper and M. H. Macfarlane, to be published.


21) for "numerical representations" do not read "approximations."


27) P. J. Griffin and R. D. Koshel, to be published in Annals of Physics.

Figure Captions

Fig. 1  Energy-dependence and content of heavy-ion total reaction cross-section.

Fig. 2  Coupling scheme in the channel-spin representation.

Fig. 3  Real part in the nuclear surface of various Woods-Saxon potentials fitted to the observed elastic scattering angular distribution for $^{16}_0$ on $^{208}_Pb$ at 192 MeV.

Fig. 4  Comparison of angle-integrated cross-sections for the reactions $^{208}_Pb(^{16}_0,^{15}_N)^{209}_Bi$ to various single-particle states in $^{209}_Bi$ as obtained experimentally (heavy line) and from DWBA calculations (dotted line) (From Ref. 19).

Fig. 5  J-dependence of the absolute magnitude of the DWBA transfer matrix element $^{1}_O$ (scheme (10) in text) for the reaction $^{208}_Pb(^{16}_0,^{15}_N)^{209}_Bi (3/2^-; 3.12 MeV)$ at an $^{16}_0$ bombarding energy of 312.6 MeV. The parameters used in the DWBA calculation are given in Ref. 19.

Fig. 6  J-dependence of the phase of the transfer integral specified in the caption to Fig. 5.

Fig. 7  Comparison of differential cross-sections for the reaction specified in the caption to Fig. 5 obtained with various interpolation schemes described in the text.
Figure 1
Figure 4
Nuclear Collision at $E/A = T_F^*$

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1. Introduction

Nuclear beams with energies between 10 and 200 MeV per nucleon are now becoming available at several laboratories around the world. This development opens up the gate to a virtually virgin field of physics which might be called intermediate-energy nuclear physics. Nuclear collisions induced by such beams create a unique environment in which a number of novel phenomena are likely to occur. The exploration of this field will therefore undoubtedly prove a very rewarding venture.

In my contribution to this Symposium I wish to limit myself to nuclear collisions where the bombarding energy per nucleon is up to the order of the Fermi kinetic energy. The discussion will be

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qualitative rather than quantitative, with the emphasis on the basic physics of the situation. Although part of what I will present has the character of speculation, so far unsupported by experiment or elaborate calculations, I hope that it will provide some stimulation for theorists as well as experimentalists.

2. The energy ladder

Let me start out by trying to put the energy region considered into the proper perspective in relation to other energy regions of nuclear physics. As the energy of the nuclear beams is steadily raised, a number of important thresholds are being passed. In the low end of the energy scale, where the characteristic energies are of the order of a few MeV, the fine details of the nuclear structure can be studied; this is the domain of nuclear spectroscopy. With increasing energy the individual quantum states lose their significance and the detailed structure of the intrinsic nuclear system largely dissolves.

When the energy has reached a few MeV per nucleon we are in the domain of strongly damped, or deep inelastic, collisions. In this regime, where the excitation per nucleon is still small in comparison with the intrinsic single-particle energies, the important degrees of freedom are a few macroscopic variables, particularly those associated with the nuclear surface. Since the macroscopic velocities are small in comparison with typical intrinsic speeds the communication of disturbances is rather fast and the corresponding macroscopic equations of motion are approximately local in time. The coupling of the macroscopic degrees of freedom to the structureless intrinsic reservoir
damps the macroscopic motion and is a source of dynamical fluctuations in these variables.

This situation will gradually change when the energy is raised further. When the macroscopic velocities are no longer small in comparison with the intrinsic speeds the intrinsic communication will require a non-negligible time and the retardation terms must be retained in the equations of motion. At the same time, inhomogeneities are likely to occur in the nuclear interior and additional degrees of freedom are activated.

An important threshold is probably reached when the excitation energy is of the same size as the nuclear binding energy. The nuclear system has then largely lost its cohesiveness and multifragmentation becomes increasingly dominant.

When the excitation energy per nucleon increases above the Fermi kinetic energy the quantal nature of the nucleons is expected to become less important and classical models gain increasing applicability. At the same time, the one-body mean field gives way to direct two-body collisions as the main governor of the nucleonic motion.

At still higher energies it becomes possible to produce pions by direct nucleon-nucleon collisions and we then enter into the mesonic regime. The basic microscopic degrees of freedom are now no longer conserved during the collision process.

The relativistic regime is reached when the kinetic energies are of the same order as the rest masses of the participating baryons. It is then possible to excite the intrinsic states of the baryon and the entire field merges more and more with elementary-particle physics.
One is presently contemplating nuclear beams with energies which are large in comparison with the rest masses. This regime of ultra-relativistic nuclear physics is still unexplored. There are speculations that sufficiently far into this domain the hadrons may "melt" and liberate their intrinsic constituents. They might then form an extended system of quark matter. The production of such an environment would clearly have a profound bearing on our fundamental understanding of nature.

3. The mean free path

The nucleons are fermions. This generally reduces the states accessible in a collision process. At densities and excitations typical of collisions with $E/A = T_F$ this effect may appreciably inhibit the occurrence of direct nucleon-nucleon collisions. This is well known from the lower end of the energy scale where the action of the exclusion principle largely eliminates two-body collisions, leaving the one-body mean field as the dominant governor of the nucleonic motion. Since the choice of approximations is very dependent on the nucleon mean free path it would be useful to obtain a semi-quantitative estimate of this quantity over a broad range of situations. We have made such an attempt, as will be described in the following.

Consider a uniform system of nuclear matter with a given temperature $T$. The distribution of nucleons in momentum space is then given by
\[ P(p) = \left(1 + e^{-(\epsilon - \epsilon_F)/T}\right)^{-1} \]  

(1)

as a consequence of the Fermi-Dirac statistics. Here \( \epsilon \) is the energy of the nucleon and \( \epsilon_F \) is the Fermi energy. We now imagine that a nucleon with momentum \( \vec{p}_0 = \vec{m}v_0 \) is moving through the system. Its notion will be degraded as a consequence of two-body collisions with the nucleons in the medium. If there were no Pauli blocking of the final states, the collision rate of the intruder nucleon would be given by

\[ \nu_0 = \frac{v_0}{\lambda_0} = \int \frac{d\vec{p}}{\hbar^2} f(\vec{p}) v_{\text{rel}} \sigma_{\text{NN}}. \]  

(2)

where \( \lambda_0 \) is the corresponding mean free path. Here \( \sigma_{\text{NN}} \) is the collision cross section for two nucleons in free space with the same kinematical conditions and \( v_{\text{rel}} \) is their relative speed.

However, the probability that a given final momentum \( \vec{p}' \) is available is reduced by the factor \( f(\vec{p}') = 1 - f(\vec{p}) \). Therefore, the actual collision rate is rather

\[ \nu = \frac{\nu_0}{\lambda} \]

\[ = \int \frac{d\vec{p}}{\hbar^2} f(\vec{p}) v_{\text{rel}} \int d\Omega \frac{d\sigma_{\text{NN}}}{d\Omega}(\vec{p}_0, \vec{p}') \frac{d\sigma_{\text{NN}}}{d\Omega}(\vec{p}', \vec{p}) \]  

(3)
where $d\sigma_d/d\Omega'$ is the differential collision cross section for the process $|p_o^+\rangle \rightarrow |p_o'^+\rangle$.

The occurrence of the blocking factors in the collision integral gives rise to a reduction in the actual collision rate and a corresponding increase of the mean free path between collisions. The exact size of this effect depends of course on the variation of $d\sigma_d/d\Omega'$ with angle and energy as well as on the distribution of nucleons in the medium. An approximate indication of the effect can be obtained by assuming that the free nucleon-nucleon collisions are isotropic and energy-independent. The mean free path is then increased by the factor

$$\eta = \frac{\int d\Omega' \nu_{rel}(p') \frac{d\sigma_d}{d\Omega'} \frac{1}{v_{rel}}} {\int d\Omega' \frac{d\sigma_d}{d\Omega'} \nu_{rel}}$$

We have calculated this factor as a function of the temperature $T$ of the medium and for different values of the incident energy $T_o$ of the intruder nucleon. The result is shown in Fig. 1. In general $\eta$ decreases when either $T$ or $T_o$ is increased. It is noteworthy that even at rather large values of these two parameters $\eta$ remains appreciably above unity. For example, for a particle in the Fermi surface ($T_o = T_F$) we find $\eta = 10$ even at a temperature of $T \approx 10$ MeV, and at $T_o = 2T_F$ (which would correspond to a physical approach energy of $T_{beam} = T_o - T_F - B \approx 30$ MeV) we still find an $\eta$-value of around 3. Since $\lambda_o$ would be around 2 fm we expect the actual mean free path $\lambda = \eta \lambda_o$ to remain at least of the order of the nuclear radius in the domain considered. The figure is useful for gaining a
quick impression of the blocking effect in a given situation. The calculated result suggests that the exclusion principle remains effective even at rather large temperatures and energies. The nucleons should therefore not be treated as classical particles.

4. Nuclear dynamics at moderate excitation

When the excitation energy per nucleon is small in comparison with the intrinsic kinetic energies the dynamics of the nuclear system can be discussed in terms of a few macroscopic degrees of freedom $\mathcal{L}$, such as those associated with the shape. The remaining degrees of freedom play a rather passive role and may be described statistically in terms of a temperature $T$. The temporal evolution of the macroscopic variables in governed by the Lagrange-Rayleigh equations of motion,

$$\frac{\partial}{\partial t} \frac{\partial \mathcal{L}}{\partial \dot{\mathcal{L}}} - \frac{\partial \mathcal{L}}{\partial \mathcal{L}} = \frac{\partial \mathcal{F}}{\partial \mathcal{L}}$$

(5)

where $\mathcal{L}$ is the macroscopic lagrangian and $\mathcal{F}$ is the Rayleigh dissipation function governing the damping of the macroscopic motion due to the coupling to the intrinsic system. We have reason to expect that this damping is relatively strong, possibly dominant.\footnote{This expectation can be easily argued for in the low-energy limit where the nucleon mean free path is long. However, it is worth noting that at higher energies where direct two-body collisions are frequent it is still to be expected that the nucleon dynamics is strongly damped.\footnote{Because the macroscopic variables are coupled to the intrinsic thermal system their dynamical evolution is not deterministic but}}
A dynamical equation of the above type can therefore only be expected to describe a mean trajectory in the macroscopic coordinate space. The statistical fluctuations inherent in the thermal reservoir will give rise to stochastic excursions away from the mean trajectory. It is therefore necessary to deal with the probability distribution $P(\xi; t)$. The dynamics can then be discussed within the framework of transport theory. In the simplest Fokker-Planck approximation the equation of motion takes on the form

$$\frac{\partial}{\partial t} P = -\nabla \cdot (V P) + \frac{\partial}{\partial \xi} \left( D \frac{\partial P}{\partial \xi} \right)$$  (6)

Here $V$ is the drift coefficient vector which governs the average motion of the centroid and $D$ is the diffusion coefficient tensor which determines the accumulated dispersions in the macroscopic variables. It is these transport coefficients that characterize the dynamical properties of the nuclear system.

Let us now specialize to the collision of two nuclei $A$ and $B$. When sufficiently close they may communicate by way of transferring nucleons. This mechanism appears to be an important, if not dominant, agency for the dissipation of the relative motion in the rather well-studied low-energy collisions. The transfer of an individual nucleon creates a particle-hole type excitation of the intrinsic nuclear system. It can be shown that the energy of this excitation amounts to approximately $\omega = f - U_p$ for a transfer from $B$ to $A$. Here $f = E_B - E_A$ is the difference between the Fermi levels in the two.
nuclei; it provides the static driving force for particle transfer. Furthermore, \( \vec{U} = \vec{U}_A - \vec{U}_B \) is the relative velocity of the two nuclear regions between which the transfer takes place. Finally, \( p = \frac{1}{2} m (\vec{v}_a + \vec{v}_b) \) where \( \vec{v}_a \) is the velocity of the nucleon relative to \( A \) and \( \vec{v}_b \) is its velocity relative to \( B \). Since it is the nucleons in the Fermi surface which are most readily transferred (due to the exclusion principle) we have \( p = P_F \) and the second term \( \vec{U} \cdot \vec{p} \) can be appreciable. (For example, if \( \frac{1}{2} mU^2 = 2 \) MeV we have \( UP_F = 16 \) MeV and if \( \frac{1}{2} mU^2 = 8 \) MeV we have \( UP_F = 32 \) MeV.)

It is therefore clear that the transfer of nucleons can be very effective in damping the relative nuclear motion.

It is possible to derive a transport theory for nuclear collisions on the basis of the nucleon-transfer mechanism. The general expressions for the transport coefficients entering into the Fokker-Planck equation (6) are given by

\[
\mathcal{V}_C = \alpha \int \frac{dp}{h} \left[ \frac{\hbar n}{2} \left( f^A(\vec{p}) - f^B(\vec{p}) \right) \right] \left( \vec{p} \right)
\]

\[
2 \epsilon_1 \epsilon_2 = \alpha \int \frac{dp}{h} \left[ \frac{\hbar n}{2} \left( f^A_{\epsilon_2} + f^B_{\epsilon_2} \right) \right] \left( \vec{p} \right)
\]

if the transfers occur over a plane contact surface of area \( \sigma \). The transfer rate is taken to be proportional to \( v_n \), the velocity component normal to the contact surface, as suggested by classical considerations. The dinuclear macroscopic variables \( \mathcal{C} \) include the proton and neutron numbers of the two nuclides, their relative separation

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and (radial and angular) momentum, their intrinsic angular momenta, and their intrinsic excitation. In the derivation of the above transport coefficients it is an essential assumption that the excitons created by the individual nucleon transfers are quickly dissipated into more complicated excitations so that the intrinsic nucleonic system remains in quasi-equilibrium throughout the collision; on grounds of the relatively long mean free path it is also assumed that the interior of the nucleus remains homogeneous. At relatively low energies, these assumptions are rather well justified. The theory then makes a number of general predictions about the correlations between the various observables.

But as the energy is raised those assumptions are expected to break down. The tacit assumption that the transferred nucleon is absorbed by its new host nucleus need no longer be generally valid; in the next section we shall discuss the possibility that transferred nucleons under favorable circumstances may be rejected promptly from the dinuclear complex. Furthermore, with the faster macroscopic motion the microscopic relaxation process can no longer be considered as instantaneous and the dynamical description in terms of time-local macroscopic equations of motion becomes dubious. By the same token the assumption of a quasi-equilibrated intrinsic system grows increasingly inadequate and one needs to include additional degrees of freedom describing inhomogeneities in the nuclear interior.
5. Promptly emitted particles

As mentioned earlier, it is usually a tacit assumption that a transferred nucleon is quickly assimilated in its new host. However, and this in particularly true at higher bombarding energies, the equilibration process may be aborted before completion, leading to the prompt emission of one or several particles. Such prompt ejectiles may teach us about the non-equilibrium properties of nuclei, and in particular, carry information on the early collision stage. It is therefore important to study in more detail the further fate of the transferred nucleons.

5.1 Fermi jets

The particle-hole excitation created by the transfer of a nucleon has a very special structure: it consists of a hole left behind in the donor nucleus A, say, and a particle moving through the recipient nucleus B. The velocity of the intruder nucleon follows from the kinematics of the situation,

\[ v_b = v_a + U \]  

(8)

Here \( v_a \) is the velocity of the nucleon as seen from A and \( v_b \) its velocity as seen from B; \( U \) is the velocity of A relative to B.

At moderately low excitation the intruder nucleon will proceed relatively uninhibited by two-body collisions, due to the action of the exclusion principle. It is then the bouncing around of the nucleon in the one-body nuclear container which provides the mechanism
for bringing the nucleon into equilibrium with the other nucleons. However, under special kinematical circumstances the nuclear potential, which has only a finite height, will not be able to reflect the nucleon which may thus escape. One may appreciate that such a process is indeed energetically possible by recalling that the kinetic energy of the intruder nucleon, as seen from the host nucleus, is boosted by an amount which is of the order of $U_p$. Therefore, when the relative radial nuclear velocity is sufficiently large, a transferred nucleon may be transmitted right across the recipient nucleus and emerge on the opposite side as an energetic ejectile. Since this phenomenon is a consequence of the kinematical coupling of the relative nuclear motion and the intrinsic nucleonic Fermi motion it has been called a "Fermi jet", although only one or a few nucleons can be ejected in a given collision. The Fermi jets form a subclass of Promptly Emitted Particles (so called PEPs) which refer to all light particles emitted at the early stage of a nuclear collision, whatever may be their production mechanism.

The Fermi-jet nucleons appear in a rather well-collimated angular region on the sides opposite to the interaction zone between the two nuclei. As time progresses, the dinuclear complex swings around and the relative motion is gradually degraded. The jets therefore appear in a narrow band in angle-energy space, a feature which should help in identifying the phenomenon experimentally. An extensive theoretical study of the Fermi jet mechanism has been carried out by Robel. An independent investigation has been made by Bondorf et al.
5.2 Two-body collisions

Although fairly long, the nucleon mean free path can not be considered as infinite. In fact, in typical situations it is known to be of the same order as the nuclear size. This fact has an impact on the Fermi jets since the transmission of the transferred nucleons is thus obscured. One may attempt to take account of this effect by assuming that the degradation of the nucleonic motion is dominantly due to two-body collisions. At the end of its free path the nucleon then collides with another one from the host nucleus, resulting in the creation of two quasi-free nucleons. Each of these may now propagate onward and possibly escape when reaching the surface. Such secondary ejectiles have been called two-body PEPs as opposed to the one-body Fermi-jet PEPs discussed above. The angular distribution of the two-body PEPs will be distinctly different from that of the one-body PEPs: they are in general aimed more sidewards than the predominantly forward-backward directed Fermi jets. This characteristics is the combined effect of the geometrical features of the distribution of the nucleons, in coordinate and momentum space.

As an illustration we show in Fig. 2 the calculated distribution of promptly emitted neutrons from the collision of 152 MeV $^{12}$C with $^{158}$Gd. One-body as well as two-body PEPs have been included; the total contribution of the two-body PEPs is around 30%. The bump around $\theta = 110^\circ$ appearing at $E_n = 11$ MeV results almost exclusively from the backward PEPs (i.e., those originating in the target and transmitted through the projectile, in the direction
opposite to the beam direction). The theoretical calculation is based on a statistical (Monte Carlo) method in which the fate of many individual nucleons are followed while the dinucleus evolves along an average dynamical trajectory (determined from an equation of the form $\langle \rangle$).

As the energy is increased, the two-body collisions will grow increasingly dominant and the Fermi-jet nucleons will give way to nucleons which are emitted after one or several collisions. The relative importance of the one- and two-body PKPs is illustrated in Fig. 3 for the C + Gd case.  

5.3 Hot spots?

When the nucleon mean free path $\lambda$ is of the order of the nuclear radius $R$ additional degrees of freedom are activated and the nuclear interior no longer remains homogeneous. Although the general expressions for the transfer of energy and momentum may still hold, it is now necessary to also consider the dynamics of the subsequent deposition of these quantities.

When $\lambda = R$ there is a large probability than an intruder nucleon will suffer its first collision in the front part of the recipient nucleus. If the flux of intruders is sufficiently high in comparison with the rate at which the local excitation resulting from the two-body collision is being dispersed the subsequent intruders will encounter an extra hot zone upon entering. Since the local mean free path is then diminished, they are therefore more likely to suffer an early collision. We are thus dealing with a self-amplifying process...
by which the deposition of the intruders’ energy in the front part of the host will produce an intransparent region: there will be a sudden phase transition where a relatively cold and transparent medium locally develops a hot and opaque zone. A hot spot has been formed.

Whether such a phenomenon will indeed occur is still an open question. After having indicated, the main line of argument in favor of it I would like to add a few words to the contrary: Although substantially reduced, the mean free path is still quite long, even at fairly high temperatures, if Fig. 1 can be trusted. Since one can hardly speak of a thermalized region with a size less than one mean free path the term "spot" appears somewhat misleading since it would occupy a sizable fraction of the nuclear volume. The size of possible temperature gradients occurring in the system is therefore severely limited and one ought to be very cautious about applying ordinary thermal-conduction theory.

6. Multifragmentation

When the beam energy per nucleon is of the same order as the Fermi kinetic energy the energy available for intrinsic excitation is comparable to the total nuclear binding energy. Hence, in principle, it would be energetically possible to totally dissemble the colliding nuclei into their nucleon constituents.

In such a situation the character of the nuclear system is profoundly altered. Rather than dealing with a dynamical situation dominated by two large nucleides, we are now faced with a competition between a large number of widely different fragmentations.
In the ordinary approach to nuclear dynamics the system considered is described in terms of a relatively small number of macroscopic degrees of freedom; in the exit channel these degrees of freedom describe a definite number of fragments, in various states of intrinsic excitation. Within such a framework we are now in the novel situation that the number of macroscopic variables to be dealt with is no longer an approximately conserved quantity. Quite to the contrary, macroscopic degrees of freedom can be created and destroyed in the course of the dynamical evolution as the system fluctuates back and forth between its different accessible fragmentations, ranging from a few hot or fast to many cold and slow fragments.

The theoretical methods presently in use for nuclear dynamics are inadequate for treating such a more complex phenomenon and there is a clear need for substantial formal development before a dynamical theory of such processes can be formulated.

Multifragmentation processes have already been observed experimentally. As an example Fig. 4 shows the result of 91 MeV/n C colliding with a Ag nucleus. A large number of fragments are ejected. Efforts are presently underway to pursue the study of this type of collision in a forthcoming experiment at CERN. Recently it has become possible to study multifragmentation processes at lower energies by the novel streamer chamber developed in Berkeley, as K. van Bibber has already discussed. Again, large parts of the initial nuclei emerge as relatively light fragments. Such pictures give a spectacular impression of the violent and complicated collision processes, the dynamics of which we must now try to understand.
7. Concluding remarks

Numerous interesting phenomena may occur in the intermediate energy region and my contribution is not intended to give a complete and balanced view of the field. I have discussed some of the basic features characterizing the new physical situation we are faced with and illustrated with a few specific phenomena which one might expect to occur. We are only just embarking on our venture into this new field and I foresee an exciting period when we try to identify the new phenomena and attempt to understand them. I expect that the journey will be exciting as well as rewarding, and feel confident that results will be established which will contribute not only to the field of nuclear physics but also to physics in general.

References

7. K. van Bibber, contribution to this symposium.
Fig. 1. The blocking factor $\eta = \lambda / \lambda_0$ defined by Eq. (4) as a function of the temperature $\tau$, for various values of the kinetic energy $T_0$ of the intruder nucleon; $T_F$ is the Fermi kinetic energy of the medium.
Fig. 2. The angular distribution of promptly emitted neutrons in the 152 MeV $^{12}$C + $^{158}$Gd collision, for selected cm energies.
Fig. 3. The energy dependence of the contribution from one and two-body PEFs in the C + Gd case for one selected impact parameter $b = 3.25$ fm.
Fig. 4. The result of a 91 MeV/n $^{12}$C nucleus colliding with a AgBr emulsion (S. Jacobsson, priv. comm.).
The completely relaxed reaction products resulting from the bombardment of $^{74}$Ge with $^{28}$Si ($E_{28Si} = 137$ MeV) and of $^{65}$Cu with $^{37}$Cl ($E_{37Cl} = 144.8$ MeV) have been measured by time of flight at the Strasbourg WP Tandem. Both reactions lead to $^{102}$Pd as a compound or composite nucleus and the relevant parameters are given in Table I. The grazing angular momenta have been deduced from elastic scattering measurements analyzed with the Optical Model computer code GENOA. The $L_{n}^*$ angular momentum characterizes the value at which the fission barrier $B_f$ coincides with the neutron binding energy ($B_n = 11$ MeV) corresponding to the liquid drop model. According to the parameters of Table I fusion-fission should be observable and since fission follows compound nucleus formation the mass distribution

<table>
<thead>
<tr>
<th>Reaction</th>
<th>$E^{lab}_{(MeV)}$</th>
<th>$E^*_{x}(MeV)$</th>
<th>Fissility</th>
<th>$E/V_c$</th>
<th>$L_{exp}^*(R)$</th>
<th>$L_{n}^{\text{LDM}} (R)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{28}$Si + $^{74}$Ge</td>
<td>137</td>
<td>92.4</td>
<td>1.64</td>
<td>68</td>
<td>0.41</td>
<td>60</td>
</tr>
<tr>
<td>$^{37}$Cl + $^{65}$Cu</td>
<td>144.8</td>
<td>81.3</td>
<td>1.40</td>
<td>67</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$^{*}$Visitor from Brookhaven National Laboratory, Upton, NY.
of relaxed fragments for both reactions should be identical. These mass
distributions are presented in Fig. 1. For a given system, they are similar
at various angles which implies a $1/\sin \theta$ dependence, for the cross sections
of a given fragment, in the laboratory system.

The total kinetic energy (TKE) of the fragments is plotted versus
their mass in Fig. 2 and shows that the kinematics of the residual products
are governed by the Coulomb repulsion between two spherical nuclei and
the total kinetic energies also agree with the Viola predictions 2) for
symmetric division. Such behavior strongly resembles those characterizing
fusion-fission phenomena.

The distributions for $^{37}$Cl + $^{65}$Cu exhibit a monotonically decreasing
behavior from $A \approx 50$ suggesting for the complete distribution a sharp
peak centered close to a mass which is one half of the compound nucleus,
in agreement with a fusion-fission distribution. The (FWHM)$_A$ is about
40 u in accordance with Nix predictions 3). The mass distributions for
the $^{28}$Si + $^{74}$Ge system are somewhat flatter and begin to decrease beyond
the target mass, they are also broader ($\approx 50$ u). This difference is in
conflict with the idea of compound nucleus formation in the two reactions
and its associated fission fragment distribution. If the two systems were
arranged to have close to the same grazing angular momentum $\ell^{\exp}$ and
therefore the same "band" of $\ell$ values should participate in fission in
both cases, however, the excitation energy reached in $^{102}$Pd is different
for the two reactions and may influence the fission mass distribution.
The different excitation also partially explains the fact that the cross
sections are smaller for the lower excitation in the $^{37}$Cl + $^{65}$Cu reaction
than for the $^{28}$Si + $^{74}$Ge reaction (11 MeV).
Our data seem to be in conflict with previous measurements by Barrette et al. for the $^{32}S + ^{50}Ti$ system. These authors have observed that when the excitation energy of the compound nucleus is increased the mass distribution changes from a double pronounced peak mass distribution (centered on projectile and target masses) to a broad flat distribution symmetric with respect to half the mass of the compound nucleus. The same trend in shape is also observed in the present work but evolves in the opposite way with an increase in excitation energy.

The present data may be understood in terms of a reaction mechanism faster than compound nucleus formation but slower than dissipative collisions like deep inelastic. If within a given time of contact between projectile and target nuclei a limited number of nucleons can be transferred and if $^{28}Si + ^{74}Ge$ and $^{37}Cl + ^{65}Cu$ are in contact for approximately equal times, the more symmetric system may have enough time to reach complete mass symmetry for the relaxed fragments whereas some mass asymmetry will persist for the other more asymmetric system.

The problem may also be examined in the framework of the liquid drop potential energy curves of two touching spheres versus mass asymmetry $\Lambda_p/\Lambda_p + \Lambda_t$, with two different points of injection (corresponding to $\Lambda_p/\Lambda_p + \Lambda_t = 0.27$ and 0.36 for $^{28}Si + ^{74}Ge$ and $^{37}Cl + ^{65}Cu$, respectively). The potential energy function may have a different slope at these two mass asymmetries which would lead to different mass drift and as a result to a different mass distribution for the residual fragments. Calculations to study this point are in progress.

The present results have to be compared to those relative to a strong change from asymmetric to symmetric in the mass distributions of relaxed fragments from systems with $91 < \Lambda_{Cl} < 121$, recently observed.
References
4) J. Barrette, F. Braun-Munzinger, C. K. Gelbke, H. E. Wegner, B. Zeidman,
5) H. Oeschler, F. Wagner, J. P. Coffin, P. Engelstein, and B. Heusch
   (to be published).
Fig. 1: Mass distributions plotted as a function of the mass of the relaxed fragments at various angles.
Fig. 2: Total kinetic energy of the relaxed fragments plotted as a function of their mass. The dotted lines correspond to the Coulomb repulsion of two spherical nuclei, calculated with $r_0 = 1.5$ fm and $d = 2.0$ fm, the solid line corresponds to Viola predictions.\(^2\).
FISSION PROPERTIES OF VERY HEAVY NUCLEI PRODUCED IN DEEP INELASTIC COLLISIONS

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Abstract:
Kinematically complete experiments have been performed on the three-body exit channels in the reactions 7.5 MeV/u $^{238}$U on $^{90}$Zr, $^{238}$U and $^{248}$Cm to investigate the fission properties of nuclei with a vanishing liquid drop fission barrier. Thus far, no evidence was found for true instantaneous three-body break-up of the collision system. Instead, a seemingly sequential fission pattern in the sense of unperturbed fission fragment kinetic energies (as found before for lighter systems) is observed even for the heaviest elements. However, a considerable broadening of the fragment mass distributions as well as angular distributions incompatible with the usual saddle point concept are also observed in this region. Possible explanations in terms of non-equilibrium processes or a revision of the equilibrium liquid drop picture near instability are discussed.


+ Visitor from Los Alamos Scientific Laboratory
I. Introduction

The availability of very heavy ion beams like Pb and U at energies well above the Coulomb barrier for all elements has raised expectations for the observation of a new type of fission process - "fast" fission under the influence of strong Coulomb and nuclear forces exerted by a heavy reaction partner. The first experiments in this direction using Pb and U beams on Ni and Zr targets did not find any evidence for such phenomena. Instead, all characteristics observed pointed to a sequential process which then proved itself as a powerful probe for the investigation of angular momentum transfer and angular momentum orientation in deep inelastic collisions. In continuation, we have consequently turned to the heaviest available target-projectile combinations, i.e. $^{238}$U+$^{238}$U and $^{238}$U+$^{248}$Cm. These systems are of particular interest because of their enhanced mass-diffusion and the strong external forces, combined with decreasing stability against fission. The behaviour of fission in this region has also important bearings on the possibility of super-heavy element production in heavy ion collisions.

In the present experiments, we have specifically concentrated on the three-body exit channel which allows the investigation of the fission properties of nuclei extending into the superheavy region. We find a strong increase of the width of the fission mass distribution in the region of the heaviest elements where the liquid drop fission barrier is expected to vanish. We find, in addition, that the standard liquid drop model together with the usual statistical distribution of K-states at the saddle point completely fails to describe the observed fragment angular correlations. It is not clear at present whether this failure is associated with more general deficiencies of the model for very heavy nuclei, or whether indeed non-equilibrium processes are observed for nuclei produced outside their fission saddle point.
II. Experimental

$^{238}$U and $^{248}$Cm targets of 100 to 200 μg/cm$^2$ were bombarded with a $^{238}$U beam of 7.5 MeV/u from the Unilac at GSI. Two and three-body exit channels were investigated in a kinematically complete way. One heavy fragment was analyzed in a 40x12 cm$^2$ position-sensitive dE-E gas ionization chamber centered at 50° lab, spanning CM angles between 45° and 110° (effectively 135° for the symmetrical system U+U). The atomic number Z was deduced from dE and time-of-flight rather than dE-E, yielding a better separation for slow particles. In the region of 60<Z<96 a Z-resolution of 4 to 5 units was achieved. The other binary reaction partner or its two fission fragments were detected in coincidence in a 1x1 m$^2$ position-sensitive parallel plate avalanche counter with an overall time-resolution of 0.5 ns (in connection with the bunched beam). Due to the high lab velocity of the fissioning nuclei as compared to the velocity of the fission fragments in the moving frame, the possible directions of the fragments are compressed into a narrow cone of less than 60° opening angle. Thus one angle setting of the detector was sufficient to cover the entire cone, resulting in 4π efficiency for the investigation of the various fission parameters. For a certain fraction of the events, only one fission fragment was detected due to losses on the support structure of the counter window and on an adjustable beam stop located in front of the detector, shielding it against small angle (<6°) Rutherford scattering.

In the off-line analysis, a two-step procedure on the basis of the ionization chamber data was employed, yielding the charge Z and the mass M of the non-fissioning deep inelastic collision fragment as well as the Q-value and the CM angle of the first binary reaction step. With the data from the other detector, the fission direction in the frame of the fissioning nucleus can then unambiguously be evaluated even in the case where only one fragment has been observed. With the detection of both, the full information including the total fission kinetic energy and fragment mass ratio is obtained. The background from four-body events stemming from double sequential fission is rejected very
effectively by testing the complanarity of the three observed particles in the CM system. The possibility of confusing a light transfer product of the first reaction step with a heavy fission fragment of the second step is strongly reduced for three-particle coincidences compared to one-particle inclusive measurements. We have verified in Monte-Carlo simulations of the system U+U, that essentially no contamination occurs down to Z=70 for the non-fissioning partner, corresponding to Z=114 for the fissioning nucleus.

III. Results

In a truly instantaneous break-up of the collision complex into three particles of comparable mass, one would expect strong Coulomb force influences of one particle on the total kinetic energies of the other two as well as specific directional correlations. Conversely, the non-existence of such effects in a sequential break-up process would allow to extract some lower limit for the time difference between the two reaction steps. We will therefore first discuss the fission fragment total kinetic energies and their azimuthal angular correlations.

The distribution of the vector difference $v_R = \mid \vec{v}_1 - \vec{v}_2 \mid$ of the fragment lab velocities, integrated over all other observables, is shown in Fig. 1 for two different bins for the charge $Z$ of the fissioning nuclei (determined as the $Z$-complement to the particles in the ionization chamber). The quantity $v_R$ essentially determines the total fragment kinetic energy $E_K = \frac{1}{2} \mu v_R^2$, $\mu$ being the reduced mass of the two fragments. All distributions clearly demonstrate the existence of intermediate fissioning nuclei as "resonances". Whereas unperturbed fragment energies for nuclei close to U are to be expected in view of large contributions from quasi-elastic reactions as well as sizable fission barriers at very large deformations in this region, the persistence of a seemingly sequential pattern up to nuclei with a vanishing liquid drop fission barrier is quite surprising. The only visible difference appears to occur in a significantly increased variance of the distributions. More quantitatively, the center of gravity, transformed into average
total kinetic energy \( \langle E_k \rangle \) and plotted versus the charge \( Z \) of the fissioning nuclei in Fig. 2, very well follows the known Viola-systematics \(^9\). It is also essentially independent from the fission direction and the energy dissipation in the first reaction step.

Taken together, these observations definitely rule out an instantaneous three-body break-up of the collision complex up to the heaviest nuclei. Simple Coulomb-trajectory calculations for the distortion of the fragment energies in the presence of the non-fissioning partner give a lower limit on the separation of the fissioning nucleus and the other partner at scission of 70 to 100 fm, corresponding to 2 to \( 4 \cdot 10^{-21} \) s ("scission-to-scission time"). For smaller separations a substantial distortion depending on the azimuthal fission angle should have been seen.

The fission fragment angular correlations are described in spherical coordinates in the rest-frame of the fissioning nuclei \(^2\). The normal to the reaction plane of the first step is chosen as the quantization axis \( (\theta=0^\circ) \). The reaction plane itself then corresponds to the equator \( (\theta=90^\circ) \); the beam axis defines the zero direction of the azimuth angle \( \phi \) for \( \theta=90^\circ \). The overall fragment azimuthal distributions, integrated over all polar angles, are shown in Fig. 3 for cuts in \( Z \) corresponding to the heaviest elements. Within our present accuracy, no significant dependences can be recognized.

For a really fast process, anisotropies like increased probabilities for fission parallel to the CM recoil axis of the first reaction step should have been observed. In qualitative accord with the findings discussed above, the isotropy found implies that the scission-to-scission time has to be comparable to or larger than the average rotational half-period of the fissioning system, assuming a symmetrical triangular angular momentum distribution. With an average angular momentum of \( 50 \hbar \) (compare Fig. 5 below), the rotational half-period of a mass 280 nucleus amounts to times between 7 and \( 25 \cdot 10^{-21} \) s, where the lower limit applies to rotation of a rigid sphere, the upper to that of two mass 140 spheres sticking together (demonstrating some necessary increase in the lower limit by the increasing deforma-
tion attained during the descent towards scission). Quantitatively, these times are significantly longer than the limit obtained from the lack of Coulomb distortions. They are also longer than the usual saddle-to-scission time estimates of about $4 \times 10^{-21}$ s and up depending on the type and strength of the friction force. It may be premature, however, to try to distinguish between different models on this basis. One should, in any case, expect very large fluctuations in the scission-to-scission time, considering the flat potential energy surface of a nearly spherical nucleus without a fission barrier (random walk with a superimposed path to scission).

The remaining aspects of the reactions investigated, on the other hand, do not favour an interpretation as fission of a truly equilibrated system either. Although upper limits for the scission-to-scission time cannot be given at this stage, both the fission fragment out-of-plane angular correlations and the behaviour of their mass distributions point, for the heaviest Clements, to a process which is not described by the standard liquid drop model.

As observed before for sequential fission of lighter elements, the out-of-plane angular distributions exhibit the strong concentration within the reaction plane expected for the decay of a system with considerable angular momentum aligned perpendicular to that plane. The half-width of these distributions as a function of the atomic number $Z$ of the fissioning system is shown in Fig. 4 for all three reactions. Due to the smaller mass diffusion in $^{238}\text{U}+^{90}\text{Zr}$, a full comparison can only be made up to $Z=100$. In this region, the behaviour is identical, i.e. the width decreases - within only 4 units of $Z$ - from about $110^\circ$ for nuclei close to U to a rather constant value of about $75^\circ$. As demonstrated before, this rapid drop basically reflects the rapidly increasing dissipation of kinetic energy and angular momentum (correlated with $Z$) rather than a direct influence of $Z$.

From these widths, average oriented spins $<I>$ of the fissioning nuclei can be deduced as in our previous work, relying on
empirical values for the variance $K_o$ of the Gaussian distribution of the spin projection $K$ on the symmetry axis of the fission saddle point configuration, and assuming the same amount of randomly oriented spin components ($M_o=13\hbar$) as established for the system $^{208}\text{Pb}+^{90}\text{Zr}$ $^2$. In all cases, a rise of $<I>$ from about $15\hbar$ for the quasi-elastic region to a plateau of $40-50\hbar$ is obtained, rather independent of the system (compare also ref. 2). This independence is due to an approximate cancellation between the target mass dependences of the orbital angular momentum brought in by the entrance channel, and that fraction of it transferred to the U-like nuclei in the sticking limit. Selecting only deeply inelastic events (TKE<340 MeV), the average spins $<I>$ deduced for $^{238}\text{U}+^{90}\text{Zr}$ are shown in the upper part of Fig. 5 as a function of the atomic number $Z$ of the fissioning system $^{12}$; the values of $K_o$ employed are given for comparison. The sticking-dependence $-A^{5/3}$ describes the data reasonably well.

For nuclei with $Z>100$, the observed widths of the out-of-plane distributions (Fig. 4) both for the $^{238}\text{U}$ and $^{248}\text{Cm}$ target remain surprisingly constant up to the heaviest elements, irrespective of the fact that the rotating liquid-drop-model for rotationally symmetric nuclei at these angular momenta (40-50 $\hbar$) leads to a complete loss of stability and a corresponding divergence of $K_o$ around $Z=107$ (Fig. 5, lower part). Empirical values for $K_o$ are not known in this region. Any attempt to deduce spin values for elements with $Z>100$ (as successfully done before for $Z<100$) on the basis of the theoretical values of $K_o$ obviously yields unphysical results. In the following, we therefore reverse the argumentation. Assuming, together with the previous value for $M_o$, the smooth sticking dependence of the average oriented spin $<I>$ established for $Z<100$ to remain valid up to $Z=110$, "experimental" values for $K_o$ can be deduced from the measured widths. As shown in Fig. 5, only a very slow increase up to about $25\hbar$ is found, contrary to the theoretical divergence.

The heavy elements under discussion are characterized by a more or less spherical saddle point shape with a near degeneracy of
levels with different $K$. Two alternative interpretations of our results are therefore possible. If we assume both (i) thermal equilibrium to be established at the saddle, and (ii) $K$ to be conserved throughout the descent from saddle to scission, then at least an indication of the divergence of $K_0$ should have been observed. Since it is not, either one of the assumptions has to be incorrect.

(i) $K$-non-conservation could solve the problem in the following way. During the descent towards scission, the near degeneracy of levels with different $K$ is removed. A strict conservation of $K$ would drive the system away from thermal equilibrium. However, due to the long scission times, relatively small perturbations may tend to reestablish equilibrium. Such reorientations will always favour small values of $K$ because of the higher density of such levels, resulting ultimately in effective parameters $K_0$ representative of nuclear shapes somewhere between spherical and scission. The influence of Coriolis forces will actually be increased for nearly spherical shapes because of the inverse relation between perturbation and moment of inertia.

(ii) A non-statistical, relatively "fast" process could alternatively solve the problem. It is conceivable that the fissioning system, formed already beyond its saddle point, receives the information about the final fission direction from the collision partner of the first reaction step. In an extremely simple picture, an initial direction collinear with the separation axis of the first step may be expected. If fluctuations are also considered, again effective parameters $K_0$ may arise which now do not resemble properties of the fissioning nucleus, but rather those of the deep inelastic collision (possibly related to $K_0$).
We finally turn to a discussion of the fission fragment mass distributions. The well-known asymmetric mass split of U-like nuclei is only observed in the vicinity of U, whereas a rapid transition towards symmetric fission occurs for heavier elements, mainly caused by the larger average energy dissipation. The RMS-widths of the mass distributions as a function of the atomic number of the fissioning nuclei are plotted in the upper part of Fig. 8. Rather system-independent, a strong increase is observed in the region of Z>100 up to the heaviest elements. The slight differences between the light and the heavy targets visible around Z=100 are possibly due to a specific selection of excitation energies for the latter, originating from losses by four-particle events. In the lower part of Fig. 8, the average trend of these data (solid line) is compared to various scattered results from heavy-ion fusion-fission reactions, compiled by Hanappe et al. Within the present accuracy, the overall behaviour appears to be rather universal.

The observed increase in the width of the mass distributions for heavy elements is much stronger than expected for a slight change in nuclear temperature, correlated with Z (see above). It is intriguing to speculate again about a relatively fast process, in which the shape of the observed mass distributions reflects an incomplete thermalization of the asymmetry mode, strongly excited in the first step of the reaction. The apparent independence of the entrance channel, however, rather suggests these results to reflect inherent properties of the liquid drop model, i.e. a close correlation between a decreasing stability against mass-asymmetry and the loss of stability in the fission degree of freedom for nuclei with a vanishing fission barrier. Further support for this interpretation stems from a possibly reduced increase in the widths observed for lighter elements with fission barriers reaching zero because of very high angular momenta. There does not seem to be a need to define a new reaction mechanism intermediate between fusion-fission and deep inelastic collisions.
IV. Conclusions

In our present experimental investigations of the heaviest collision systems available, no evidence has so far been obtained for instantaneous three-body break-up. The observed kinetic energy release of fission as well as the absence of any fragment azimuthal anisotropies confirm the existence of a dominantly two-step process up to the heaviest elements produced, with a lower limit of the scission-to-scission time of $\sim 10^{-20}$ s.

The strong fragment out-of-plane anisotropies for these elements are, however, not compatible with the current picture of a statistical equilibrium at the liquid drop saddle point, combined with strict K-conservation down to scission. In addition, the heaviest nuclei exhibit a considerable increase in the width of the fragment mass distributions. There is no way, at present, to decide whether these observations reflect deficiencies of the liquid drop model for nuclei at the limits of stability, or whether truly fast, non-statistical processes are being observed.
References:

2 SPECHT, H.J., MPI H-1978-W26, to be published in (Proc. Int. Conf. Nucl. Interactions, Canberra, 1978) and
4 HILDENBRAND, K-D., FREIESLEBEN, H., POHLHOFER, F., SCHNEIDER, W.F.W., BOCK, R., HARRACH, D.v., SPECHT, H.J.,
7 SANN, H., DAMJANTSCHITSCH, H., HEBBARD, D., JUNGE, J., PELTE, D., POHV, B., SCHWALM, D., TRAN THOAI, D.B.,
Nucl. Instr. Meth. 124 (1975) 509
9 VIOLA, V.E., Jr., Nucl. Data Sect. A 1 (1966) 391
12 CIVELEKOGLU, Y., thesis 1979, University of Heidelberg
14 DHAR, A.K., NILSSON, B.S., Phys. Lett. 77 B (1978) 50
15 HANAPPE et al., this conference
FLEROV, G.N., OGANESSIAN, Yu.Ts., Dubna preprint E7-6338 (1972)
Figure captions

Fig. 1 Distribution of the vector difference $|\vec{v}_1 - \vec{v}_2|$ of the fragment lab velocities for a Z-bin close to the entrance channel and for a high Z-bin in the systems $^{238}\text{U} + ^{90}\text{Zr}$, $^{238}\text{U}$, $^{248}\text{Cm}$.

Fig. 2 Average fission total kinetic energy release versus Z of the fissioning nucleus for the systems $^{238}\text{U} + ^{90}\text{Zr}$, $^{238}\text{U}$, $^{248}\text{Cm}$. The solid line is an empirical fit to older fission data (Viola 1966).

Fig. 3 Azimuthal angular distributions of fission fragments for a high Z-bin integrated over all off-plane angles $\theta$. Systematical errors may double the indicated statistical errors.

Fig. 4 FWHM of the out-of-plane fission fragment angular distributions for the systems $^{238}\text{U} + ^{90}\text{Zr}$, $^{238}\text{U}$ and $^{248}\text{Cm}$. The higher values close to the entrance channel are due to a strong contribution from quasi-elastic events.

Fig. 5 Comparison of the sticking-dependence with average oriented spins deduced for the deep-inelastic component of the reaction $^{238}\text{U} + ^{90}\text{Zr}$. The $K_0$-values used are also shown (top part). The bottom part contains $K_0$ values deduced from an assumed sticking behaviour of the spin for the systems $^{238}\text{U}$, $^{238}\text{U}$, $^{248}\text{Cm}$. The spin is normalized to $\approx 2.5$ h at $Z=92$. No points are evaluated up to $Z=96$, because sticking is not expected there to hold due to the quasi-elastic component. The liquid drop model prediction for $K_0$ is also shown.

Fig. 6 Width of the fission fragment mass distributions (defined as RMS of $A_i(A_i + A_j)$ for the systems $^{238}\text{U} + ^{58}\text{Ni}$, $^{90}\text{Zr}$, $^{238}\text{U}$ and $^{248}\text{Cm}$ (top). The bottom part shows a comparison of the trend of our data with fusion-fission data from refs. 14).
Fig. 1
7.5 MeV/U $^{238}\text{U}$ on

Charge of fissioning nucleus $Z$

Total kinetic energy $<E_K>$ [MeV]

- $^{90}\text{Zr}$
- $^{238}\text{U}$
- $^{248}\text{Cm}$

$0.107 \frac{Z^2}{A\sqrt{3}} + 22.2$

Fig. 2
Normalized CountRate

Azimuth-angle $\phi$

- $^{235}\text{U} + ^{238}\text{U}$
- $Z = 103 - 111$
- $^{238}\text{U} + ^{248}\text{Cm}$
- $Z = 104 - 112$

Figure J
Fig. 4

FWHM of Out-of-plane fission distribution

Charge of fissioning nucleus $Z$

- $^{90}\text{Zr}$
- $^{238}\text{U}$
- $^{248}\text{Cm}$

7.5 MeV/U $^{238}\text{U}$
Fig. 5

- Charg of fissioning nucleus Z
- \( \langle I \rangle, K_0(f) \) vs. Charge of fissioning nucleus Z
- \( U+Zr \)
- TKE < 340 MeV
- \( K_0 \) (theory)
- \( I \) (sticking U+U)
- \( K_0(f) \) (sticking U+U)
ANGULAR MOMENTUM DISSIPATION AND SPIN ORIENTATION OF LIGHT AND HEAVY FRAGMENTS IN DEEP-INELASTIC REACTIONS INDUCED BY $^{16}$O ON Ti AND Ni.

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I. INTRODUCTION

The question of the dynamical behaviour of a system of two interacting heavy ions is very complex and calls for various approaches. One aspect concerns the mechanism of angular momentum transfer into excitation spins of the reaction partners which has been investigated by various methods. We report in this paper our results of a study of the intensities and angular correlations of yrast transitions in the heavy fragments as well as the deexcitation gammas of excited ejectiles measured in coincidence with the light fragments from the 96 MeV $^{16}$O + $^{48}$Ti reaction. Combining these results with our earlier measurements\(^1\) of the circular polarization of gamma radiation from the reaction $^{16}$O + $^{58}$Ni we try to find some systematic features of the reaction dynamics in mass asymmetric systems at modest bombarding energies (see tab. 1).

II. POPULATION OF YRAST STATES

In order to study the yrast states we have used a 120-cm\(^3\) Ge(Li) detector positioned 9 cm from the target at 45\(^\circ\), 90\(^\circ\), and 180\(^\circ\) with respect to the mean recoil direction in the in-plane geometry, and normal to the scattering plane in the out-of-plane geometry. Light fragments were detected at 35\(^\circ\) with a resolution of $\Delta Z/\Delta Z = 25$ using a $\Delta E$-E telescope which consisted of an axial-field ionization chamber and a 9 cm\(^2\) surface barrier Si-detector. As an example, Fig. 1a gives $\gamma$-ray spectra recorded in coincidence with $Z = 6$ light fragments which represent the strongest $Z$ channel. We observe the yrast sequence in $^{50}$Cr up to spin 12 h, the highest yrast state known so far\(^2\), and in $^{51}$Cr up to spin 17/2. The yields of Cr, V, Ti and Sc nuclei as derived from the intensities of the ground state transitions are consistent with evaporation calculations (GROGI2K) assuming an excited $^{52}$Cr fragment. Summing all ground state transitions we obtain a total $\gamma$-ray strength of $\approx 85\%$.
limiting the direct particle decay into ground states to 20%. For \(|Q| < 40\) MeV an upper limit of 5% can be set for a break-up of \(^{16}\text{O}\) ejectiles which would lead to excited Ti fragments. A strong alignment of the heavy fragments can be seen in the large in-plane yields of stretched quadrupole and the large out-of-plane yields of stretched dipole transitions (see fig. 1b). This renders rather difficult the type of analysis reported recently\(^3\) where a small measured anisotropy of \(\gamma\) transitions summed over different multipolarities has been used to deduce an equally small alignment of the decaying nuclei.

III. ANGULAR MOMENTUM DISSIPATION

The yields per \(Z = 6\) fragment of \(^{50}\text{Cr}\) yrast transitions, corrected for the measured angular correlation, are shown in Fig. 3 for several \(Q\)-windows. We use the observed population patterns to determine the initial fragment spin. In order to account for the effects of neutron evaporation and the pre-yrast \(\gamma\)-decay we have performed statistical model calculations (GROGI2K) with standard parameters. As already remarked the observed yields of the Cr, V, Ti and Sc isotopes can only be described by taking \(^{52}\text{Cr}\) as strongly prevalent primary fragment with spins \(J_i\) as given in Fig. 4c, which were obtained by fitting the absolute yields of the \(^{50}\text{Cr}\) \(\gamma\)-transitions. The deduced values of \(J_i\) approximately span the range between the value of 8 \(\hbar\) obtained for an \(\alpha\) particle transferred with the projectile velocity in a grazing collision\(^4\) and the value of 14 \(\hbar\) obtained for sticking rigid spheres with the total angular momentum \(H_{\text{crit}} = 40\) \(\hbar\).

IV. NUCLEAR ALIGNMENT OF HEAVY FRAGMENTS

The in-plane/out-of-plane intensity ratios \(I(90^\circ)/I(0^\circ)\) averaged over the observed stretched E2 transitions following \(2n\) evaporation in the \(^{12}\text{C}\) and \(^{16}\text{O}\) exit channels are displayed as a function of \(Q\) in Fig. 4a. For comparison we also show the anisotropy of the energy integrated \(\gamma\)-yield recorded with the large NaI detector which obviously is less sensitive to the fragment spin alignment. Taking our detector geometry and assuming a Gaussian substate population of the initial state

\[
P(m) \propto \exp \left[-(J_i - m)^2 / 2\sigma^2\right]
\]
we have used our measured anisotropies to deduce the initial
alignment $P_{zz}$, defined in the classical limit as

$$P_{zz} = \frac{3}{2} \left< \frac{m^2}{J^2} \right> - \frac{1}{2}$$

The results are shown in Fig. 4b.

We can parametrize this result by following a recently published
procedure and decomposing the transferred angular momentum $I$
into a fully aligned component $I_{aligned}$ on which a random spin
$I_{non-aligned}$ is added which leads to a dandelion like construction
for $I$. The result is given in Fig. 5. If one associates increasing
inelasticity with increasing reaction times this picture is
suggesting a time sequence in the $I$-transfer process which should,
however, not be taken too serious: the buildup of $I_{aligned}$ up to
about the optimum $Q$-value is followed by an equally rapid increase
of $I_{non-aligned}$ at more negative $Q$.

V. NUCLEAR ALIGNMENT OF LIGHT FRAGMENTS

The $\gamma$-radiation coming from the light fragments was measured
with a $27 \text{ cm} \times 33 \text{ cm}$ NaI crystal. The crystal was positioned at
$70^\circ$ and $-142.5^\circ$ with respect to the particle detector in the in-
plane geometry and normal to the reaction plane in the out-of-plane
geometry.

In the NaI spectra coincident with the light fragments between $Z = 5$ and 10 the photo-peaks corresponding to the first excited
states of the most abundant isotopes clearly stand out against
the high-energy continuum from the heavy fragments (Fig. 2). The $\gamma$-yields per $Z$ fragment summed over all observed transitions and corrected for the angular correlation give the fragments
excitation probability (tab. II) which are approximately constant
over a large range of $Q$-values. This is in marked contrast to the
partition of the energy loss proportional to the mass number $A$
which is observed for heavier systems in accordance with diffusion
models. The generally low excitation probability for $Z = 6$ and 8
is most likely caused by the sparsity in low lying levels, in
contrast to the situation in Ne.

The measured anisotropies of the $^{12}\text{C}(2^+)$ and $^{16}\text{O}(3^-)$ de-
excitation $\gamma$-rays are displayed in Fig. 6. In the quasi-elastic
region, the $^{12}\text{C}(2^+)$ decay shows a strong in-plane anisotropy. A
small in-plane anisotropy is still indicated in the deep-inelastic
region where negative-angle scattering dominates. In a classical picture of frictional collisions—which may be invalid for the light ejectiles—symmetry about their scattering plane is expected. At \( Q < -15 \text{ MeV} \) our data for the \(^{16}\text{O}(3^-)\) de-excitation and the heavy fragment yrast decays (section II) are in agreement with this picture.

VI. CONCLUSION

If we combine the results presented here with the data of our \(^{16}\text{O} + ^{58}\text{Ni}\) experiment\(^1\) which has very similar dynamical parameters (see tab. I) we get the following qualitative picture:

The \( \gamma \)-decays of the residual nuclei in both reaction channels show an in-plane/out-of-plane anisotropy which, however, is for \( Z=6 \) considerably larger and more \( Q \)-dependent than for \( Z=8 \) (fig. 4). This is confirmed by the \(^{16}\text{O} + ^{58}\text{Ni}\) results (fig. 7) which showed\(^1\) already that the friction model provides a very good description for the heavy fragments in deep-inelastic collisions. The \(^{16}\text{O}\) ejectiles show also an alignment (fig. 6) which is roughly constant with \( Q \). This behaviour is in agreement with the friction model if one takes into account that a rather low excitation probability is expected since the ejectile possesses only a small number of bound levels (as opposed to Ne, see tab. II). The \(^{12}\text{C}\) ejectiles show a considerable alignment which seems rather \( Q \)-dependent (fig. 6) and reflects the corresponding behaviour of the \( Z=6 \) residuals (fig. 4). The strong in-plane correlation for the ejectiles which disappears in the region \( Q<Q_{\text{opt}} \) and which is known\(^1\) not to exist in the heavy fragments suggests that the \( \alpha \)-transfer mechanism leads to a strong coherence in the \( m \)-population of the ejectile which gradually vanishes with increasing inelasticity. Clearly, a deeper understanding of this behaviour requires the measurement of the circular polarization of the ejectile gammas which is presently in progress.

This work was supported in part by BMFT
References


2) C. Signorini et al., Proc. Int'l Conf. on Medium Light nuclei, Florence 1977, p. 198


4) D. M. Brink, Phys. Lett. 40B (1972), 37

Table I: Dynamical parameters for the 96-MeV $^{16}\text{O} + ^{48}\text{Ti}$ and $^{16}\text{O} + ^{58}\text{Ni}$ systems

<table>
<thead>
<tr>
<th>System</th>
<th>$E_{cm}/B$</th>
<th>$v_{surf}/c$</th>
<th>$\theta_{graz}$</th>
<th>$\theta_{crit}$</th>
<th>$\Delta E_{sticking}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{16}\text{O} + ^{48}\text{Ti}$</td>
<td>2.7</td>
<td>9%</td>
<td>26°</td>
<td>48°</td>
<td>13 h</td>
</tr>
<tr>
<td>$^{16}\text{O} + ^{58}\text{Ti}$</td>
<td>2.3</td>
<td>8.5%</td>
<td>33°</td>
<td>52°</td>
<td>15 h</td>
</tr>
</tbody>
</table>

Table II: Excitation probabilities of light fragments at the most probable reaction Q-values for 96-MeV $^{16}\text{O} + ^{48}\text{Ti}$

<table>
<thead>
<tr>
<th>Z</th>
<th>Excited States</th>
<th>Yield per Z Fragment</th>
<th>Q [MeV] most probable</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>$^{12}\text{C}(2^{+})$, $^{13}\text{C}(3^{-})$</td>
<td>21%</td>
<td>-33.0</td>
</tr>
<tr>
<td>7</td>
<td>$^{15}\text{N}(5^{+}, 1^{+}, 3^{-})$, $^{14}\text{O}(0^{+})$</td>
<td>22%</td>
<td>-31.5</td>
</tr>
<tr>
<td>8</td>
<td>$^{16}\text{O}(3^{-}, 1^{-})$, $^{18}\text{O}(2^{+})$</td>
<td>22%</td>
<td>-30.0</td>
</tr>
<tr>
<td>10</td>
<td>$^{18,20,22}\text{Ne}(2^{+})$, $^{21}\text{Ne}(5^{+})$</td>
<td>90%</td>
<td>-31.0</td>
</tr>
</tbody>
</table>
Fig. 1: a) Gamma-ray spectra recorded with a 120 cm$^3$ Ge(Li) Diode in coincidence with $Z = 6$ ejectiles (Q-integrated)

b) Gamma-ray spectrum of $Z = 6$ channel, recorded with large NaI spectrometer (see fig. 2). The plot shows the difference of in-plane (N(90°)) to out-of-plane (N(0°)) intensities. Summation over all $E_Y$ gives isotropy.

Fig. 2: Gamma-ray spectra recorded with a 27 cm x 33 cm NaI crystal in coincidence with

a) $Z = 6$, b) $Z = 7$, c) $Z = 8$ ejectiles from $^{16}O + ^{48}Ti$

at $E_{lab} = 97$ MeV and $\theta_{lab} = 35^\circ$ (Q-integrated)

Fig. 3: Yields per $Z = 6$ fragment of yrast transitions in $^{50}Cr$. Statistical model fits for $J^{(52)Cr}$ are shown for most probable deep inelastic Q-value.

Fig. 4: a) Anisotropy of stretched E2 yrast transitions recorded with Ge(Li) detector (circles) and of energy integrated NaI Spectrum (squares), both in coincidence with ejectiles.

b) Spin alignment of primary $^{52}Cr$ fragment coincident with $Z = 6$. Upper and lower limits of $P_{zz}$ (dashed lines) are defined in the text.

c) Spin of primary $^{52}Cr$ fragment coincident with $Z = 6$.

Fig. 5: Decomposition of a spin I into fully aligned and completely unoriented components.

The data points indicate the $P_{zz}$ and $J_1$ values of fig. 4.

Fig. 6: Intensities of $\gamma$-rays from ejectile deexcitation.

a) in coincidence with $Z=6$ at two in-plane angles (relative to the ejectile angle of 35°) and 90° out-of-plane

b) the same for $Z=8$

The given excitation probabilities are normalized to gamma emission into $4\pi$ solid angle.

Fig. 7: a) Particle spectra in coincidence with $\gamma$-rays in the reaction $^{16}O + ^{58}Ni$.

b) Circular polarisation of $\gamma$-rays. (positive $P_\gamma$ corresponds to negative scattering angles)

c) Average anisotropy of stretched E2 transitions in $^{58}Ni$, $^{60}Ni$ and $^{61}Cu$ ($Z=6$) and $^{58}Ni$, $^{56}Fe$ and $^{56,57}Co$ ($Z=8$), measured with Ge(Li) detector.

764
Fig. 3

Fig. 4

Fig. 5
Fig. 6

Fig. 7
Study of the $^{20}\text{Ne} + ^{197}\text{Au}$ Reaction at Incident Energies between 150 and 400 MeV

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Abstract

The $^{20}\text{Ne} + ^{197}\text{Au}$ reaction has been studied at incident energies of 7.5, 11.0, 14.5 and 20 MeV/N. Total isotopic yields, energy and angular distributions have been measured. From systematics in the energy spectra and the comparison of light-particle to heavy fragment yields evidence for a rapid onset of nonequilibrium processes above 10 MeV/N is obtained. It is found that for the explanation of final fragment yields and energy distributions more than one-step processes have to be taken into account.

Introduction

The evolution with energy of heavy ion reactions in the energy region above 10 MeV/N has recently become subject to extensive investigations. One of the most important questions is how the reaction mechanism evolves from statistical equilibrium observed at incident energies of 5 MeV/N to completely nonequilibrium processes seen at very high bombarding energies (>200 MeV/N). In this paper we present a study of the $^{20}\text{Ne} + ^{197}\text{Au}$ reaction at bombarding energies of 7.5, 11.0, 14.5 and 20 MeV/N. From our data we conclude that the transition region is between 10 and 20 MeV/N. In addition we find that more than one-step processes have to be taken into account.
Experimental procedure

The experiments were performed with the $^{20}\text{Ne}$ beam of the VICKEI accelerator at the Hahn-Meitner-Institut, Berlin. Using standard AE-E techniques with solid-state detector telescopes we measured angular distributions of charged particle spectra between charge 2 and 15 emerging from a $^{197}\text{Au}$ target at four bombarding energies, 150, 220, 290 and 400 MeV. Generally, mass separation was achieved up to mass 20. Except for the 150 MeV data angular distributions were taken over the whole contributing angular range.

Results

The gross structure of the change of particle spectra between low and high bombarding energies can best be demonstrated in fig. 1. Shown are raw data spectra at 150 MeV (a) and 400 MeV (b) taken with a very thin (15 μ) AE detector. At the low bombarding energy the cross section for ejectiles with $Z > Z_{\text{proj}}$ is practically zero (except for fission fragments). The cross section for elements with $Z < Z_{\text{proj}}$ drops rapidly with increasing $\Delta Z = Z_{\text{proj}} - Z_{\text{final}}$. The energy spectra at all angles are peaked well above the Coulomb barrier (solid line) of the two final fragments. At 400 MeV bombarding energy two components are seen. In the high-energy component again we find hardly any cross section for elements with $Z > Z_{\text{proj}}$. The cross section for $Z < Z_{\text{proj}}$ drops smoothly with increasing $Z$. The low energy component is characterized by a broad $Z$ distribution with energies extending below the Coulomb barrier (solid line) of the final partners. As an example for the general trend a Wilczynski diagram for $^{13}\text{C}$ is shown in fig. 2. The high-energy part concentrates at forward angles and contains by far the dominating cross section. The same is true for the two intermediate bombarding energies, where practically no (220 MeV) or very small contribu-
tions were observed in the completely damped components. Fig. 3 summarizes this part of the gross structure showing the total angle- and energy-integrated cross section for $Z < Z_{\text{proj}}$. For the different bombarding energies normalized to $Z_{\text{proj}} - Z = 1$. As mentioned above, the Z-distribution is rather steep at low incident energy, jumps strongly from 150 to 220 MeV and flattens then gradually towards the 400 MeV limit.

The energy spectra for the different isotopes look very much alike at 150 MeV (fig. 4). They are nearly bell shaped with a small tail to the low energy side. At 220 MeV the spectra have similar shapes (fig. 5) with the exception that the width of the $^{16}$O spectrum is definitely smaller than for the neighbouring isotopes ($\sim$20-30 %). This could also be interpreted as a component with a smaller width sitting on top of a broader distribution. The same effect is also observed at higher bombarding energies. Fig. 6 shows the angle-integrated center-of-mass energy spectra of the oxygen and nitrogen isotopes. Again the $^{16}$O spectrum has the smallest width. The slope to the low energy side varies for the different isotopes.

The very significant change of the light-particle spectra is demonstrated in fig. 7 where center-of-mass a-spectra are shown for three bombarding energies (220, 290 and 400 MeV). Clearly two components are visible, one being centered at 18 MeV - the Coulomb barrier of an Au-like emitter - the other one increasing with beam energy and being centered at energies below the ones corresponding to the beam velocity. This gap gets wider with increasing bombarding energy (fig. 8a). The total cross section for the fast component rises strongly from 220 to 290 MeV and increases only by $\sim$25 % from 290 to 400 MeV whereas the slow component exponentially depends on the bombarding energy (fig. 8b).
Discussion and comparison with other experiments

To explain the major part of the cross section of the high energy component we assume as a first simple approach that at 20 MeV/N the projectile is excited in a sudden process and then sequentially decays into the observed fragments. The particle yields should then be given \(^3\) by

\[
\sigma(N,Z) = \sum_i \exp(\frac{Q_i}{T})
\]

where the \(Q_i\) are the threshold \(Q\)-values for the various projectile fragmentation channels and \(T\) is a parameter which can be interpreted as the effective temperature of the excited projectile. This model has been successfully applied to data at 20 MeV/N and 2 GeV/N with \(^{16}\)O induced reactions on various target nuclei.\(^1\) The temperature found there was around 7 MeV for both incident energies. Fig. 8 shows the ratio \(\sigma_{\text{exp}} / \sum_i \exp(\frac{Q_i}{T})\) for the different fragment elements and isotopes, where \(\sigma_{\text{exp}}\) is the total reaction cross section of the high energy component of the particle spectra and \(T\) has been taken as 8 MeV. Within a factor of two the relative particle yields are reproduced. The agreement is comparable to the one obtained by the Berkeley group.\(^1\) A systematic deviation from a constant ratio is observed which is similar in both experiments.

If we believe that the main part of the fragment cross section is due to projectile fragmentation the sum of the absolute cross sections of fragments associated with one or more \(\alpha\)-particles should be comparable to the measured yield of fast \(\alpha\)-particles. At 400 MeV the total \(\alpha\)-yield of the high energy \(\alpha\)-component is 970±150 mb compared to 1050±100 mb deduced from the fragment cross section. For the \(^3\)He the measured cross section is 50±25 mb. The \(^3\)He yield predicted from the fragment cross section associated with \(^3\)He emission of an excited \(^{20}\)Ne is 160±40 mb. This discrepancy will be discussed later.
The comparison of the total fast-\(\alpha\)-yield and the one predicted by the fragment cross sections can also be done for the two other energies where fast \(\alpha\)'s have been observed. It is found that at 220 MeV only 20\% at 290 MeV 70\% of the fast-\(\alpha\)-yield could be explained. These observations are in accordance with the ones presented in a recent paper by Wilczynski and co-workers.\(^2\) For the case of \(^{12}\)C induced reactions on \(^{16}\)O\(_{\text{d}}\) these authors have shown that at around 10 MeV/N projectile break-up into the lowest-energy break-up channel starts giving rise to fast-\(\alpha\)-particle emission, and that the heavy partner which is below the critical angular momentum undergoes fusion with the target nucleus. For our case this implies that with increasing bombarding energy the heavy break-up partner undergoes no more fusion with the target nucleus because its angular momentum exceeds the critical one. Nevertheless, the particle can interact with the target nucleus and will exchange nucleons. The strongest channels are neutron pick-up and proton stripping as is known from many heavy ion reactions. This would lead to an enhancement of the neutron-rich final products as compared to the neutron-deficient products. This is indeed seen in both the Berkeley and our experiment. At 2 GeV/N there is no time for a subsequent particle exchange and the enhancement is not found. The exchange effect would lead to broader energy spectra for isotopes adjacent to a strong break-up channel (fig. 6).

The too large cross section for \(^3\)He predicted from the fragment yields relative to the one experimentally observed is mainly due to the contribution of \(^{17}\)O and \(^{13}\)C. These isotopes neighbour the very abundant \(^{16}\)O and \(^{12}\)C channels. The very likely neutron pick-up of \(^{12}\)C and \(^{16}\)O fragments contributes to the \(^{13}\)C and \(^{17}\)O cross section. A re-evaluated primary distribution that takes such a process into account would decrease the estimated \(^3\)He cross section.
Summary

We would like to summarize our results by proposing the following model:

At low energy there is full equilibration of the total available kinetic energy between target and projectile. As the bombarding energy rises the projectile gradually acquires a larger fraction of the available energy. This results in the onset of projectile break-up into the lowest Q-value favoured break-up channels. At 20 MeV/N the projectile then is so highly excited that all break-up channels are open. The observed fragment distribution is not the primary distribution. From the systematics in the data we suggest that single-nucleon exchange following fragmentation should be taken into account at 20 MeV/N.

References

3) V.K. Lukyanov, A.I. Titov, Physics Letters 57B (1975) 10
Fig. 1: ΔE-E experimental spectra at 150 (a) and 400 (b) MeV bombarding energy.

Fig. 2: Wilczynski plot of $^{13}$C at 400 MeV. The cross section $d\sigma/d\theta$, represented as a linear function of the square size, is shown versus center-of-mass angles and energies.
Fig. 3:  
Z distribution for final products with Z < Z_{projectile} normalized to AZ = 1 for the different bombarding energies.

Fig. 4:  
Energy spectra of nitrogen and oxygen isotopes at 150 MeV bombarding energy. The spectra of all isotopes have approximately the same widths.
Fig. 5:
Energy spectra of nitrogen and oxygen isotopes at 220 MeV. The $^{16}$O spectrum has a smaller width than the other isotopes.

Fig. 6:
Angle integrated center-of-mass spectra of N and O isotopes at 400 MeV. The $^{16}$O has the smallest width. The tail extending towards the low energies is different for each isotope.
Fig. 7: c.m. α-spectra at 220, 290 and 400 MeV for different angles (see text)

Fig. 8a: Dependence of the most probable α-energy of the fast α-component on the bombarding energy

Fig. 8b: Dependence of the total cross section of the fast and slow α-component on the bombarding energy

Fig. 9: Plot of the ratios \( \frac{\sigma_{\exp}}{\sigma_{\text{exp}(Q/T)}} \) versus mass number at 400 MeV. Symbols represent different elements.
COUPLED CHANNEL EFFECTS IN PERIPHERAL REACTIONS
IN THE CLOSED FORMALISM*†

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ABSTRACT

We generalize the closed formalism for heavy-ion elastic, inelastic and transfer reactions to include coupled-channels effects. We use the radial Gell-Mann-Goldberger equation to obtain perturbative corrections to the normal amplitudes. These corrections are, in principle, completely determined by the normal elastic amplitude provided the coupling strengths to the different channels that give rise to these corrections are predetermined experimentally. All the corrections are evaluated in closed form. This aspect of the theory, i.e., the dynamical relation between the different amplitudes, is particularly emphasized in connection with the application of the formalism to discuss heavy-ion elastic, inelastic and transfer data at backward angles.

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I. INTRODUCTION

In a series of papers one of us developed a closed formalism for heavy-ion elastic, inelastic and transfer reactions. A basic input in this theory is the strong absorption profile assumed for the nuclear reflection coefficient. Since the inelastic and transfer amplitudes were assumed to be given by DWBA, no feedback into the elastic channel was considered. Moreover, the possible influence of strongly coupled channels on the transfer amplitude was omitted. It is now, however, clear that such coupled-channel effects are often quite important in heavy-ion reactions. Full coupled-channels calculations as well as more approximate coupled-channels Born approximation calculations have been performed by several groups. In cases where the number of channels involved is large, these calculations become quite involved and costly, and the physical interpretation of the computer output becomes more and more difficult. One of the aims of the closed formalism approach is to display the analytic form of the different amplitudes so that a simple interpretation of the general structure of the cross sections can be given. In this paper we report on our recent work in which we extend the closed formalism to cases where coupled-channels effects are important.

II. THE BASIC PROCESSES

Two basic processes that can modify the elastic amplitude are depicted schematically in Fig. 1. From the coupled-channels equations for these transitions we derive expressions for the modified elastic nuclear $S$-matrix
which we write as

\[ S^{(N)}_k = S^{(N)}_k + S^*_k, \quad (1) \]

where \( S^{(N)}_k \) is the usual partial-wave elastic nuclear S-matrix with strong-absorption profile and \( S^*_k \) is the correction associated with one of the processes depicted in Figs. 1a and 1b. Since the elastic S-matrix enters into the DWBA amplitudes for inelastic scattering and transfer, an improved distorted-waves theory is obtained by using Eq. (1) for the elastic partial waves in the initial and final channels. If the coupling strengths between the physical channels are known, the modified amplitudes are completely determined by \( S^{(N)}_k \) and the form factors of the transitions involved. After having calculated \( S^{(N)}_k \) in explicit form, the partial-wave summations in the amplitudes can be carried out analytically by using the methods developed in Refs. 1-3.

III. ELASTIC SCATTERING

Starting from the radial coupled-channel equations that describe the coupling of the elastic channel to inelastic or transfer channels, we determine, as an intermediate step, an effective potential in the elastic channel by formally eliminating the nonelastic channels. From this potential we obtain the modified partial-wave S-matrix by using the "radial Gell-Mann-Goldberger relation". Closed-form expressions for \( s_k^{(N)} \) are obtained by means of the following approximations:

1) The Green's function in the intermediary channel is replaced by its on-shell form,

\[ \frac{1}{G^{(N)}_n(k, r, r')} = \frac{1}{S^{(N)}_n(k_n)} f_n(k_n, r_n) f_n^{\dagger}(k_n, r_n), \quad (2) \]
where $f_{\lambda}(k_n, r)$ is the regular solution of the radial optical model
Schroedinger equation describing channel $n$. This approximation has been
shown to be quite adequate for energies not too far from the Coulomb barrier.$^4$ A criticism has been raised$^5$ that with (2) the coupled-channels effects would be overestimated for low partial waves since $S_{\lambda}^{(N)}(k_n)$ is small for small $\ell_n$. In our calculation, however, the $S_{\lambda}^{(N)}(k_n)$ appearing in the denominator in (2) is cancelled by another $S_{\lambda}^{(N)}$ which appears in the second term of the Gell-Mann-Goldberger relation.

2) In considering the coupling of the elastic channel to a transfer channel we encounter the problem of channel non-orthogonality. The effective potential in the elastic channel which simulates this coupling then has the formal structure

$$V_{\text{eff}} = V_{\text{oT}} G_T V_{\text{TO}} - V_{\text{oT}} S_{\text{OT}}, \quad (3)$$

where $V_{\text{oT}}$ is the form factor, $G_T$ is the Green's function in the transfer channel, and $S_{\text{OT}}$ is the usual non-orthogonality integral. In our present formulation we have dropped the non-orthogonality term for simplicity, knowing, however, how to include it if necessary. We emphasize that while in the one-step transfer DWBA theory one does not encounter the non-orthogonality problem, it is relevant for the feedback of the transfer channel into the elastic channel, a two-step process. Now we summarize some of our results.

A. Dynamic Polarization

The effects of the coupling of the elastic channel to a collective inelastic channel (dynamic polarization) have been discussed recently in
terms of an effective optical potential. In our approach there is no need to determine this potential explicitly since the amplitude can be calculated directly. Fig. 1a represents the dynamic polarization effect. Considering the specific case of Coulomb and nuclear excitation of 2^+ state, we obtain the following expression for the partial wave amplitude \( \tilde{S}_{DP}^{(N)} \) (using the notation of Refs. 1-3)

\[
\tilde{S}_{DP}^{(N)}(\lambda) = g^{(N)}(\lambda) \left\{ 1 - \frac{k^2}{2} \sum_{\kappa=2}^{2} \left\{ a_{\kappa}(\lambda) \right\}^2 \times \right. \\
\left. \left\langle \left( \delta_{2}^{(C)} \right)^2 \left[ c_{2-\kappa}(\lambda) \right]^2 - 12 \delta_{2}^{(C)} \delta_{2}^{(N)} c_{2-\kappa}(\lambda) \left[ \frac{D_0(\lambda)}{\eta_0(\lambda)} + \eta_0^{(N)}(\lambda) \right] \right. \right. \\
\left. \left. - \left[ \delta_{2}^{(N)} \right]^2 \left[ \frac{D_0(\lambda)}{\eta_0(\lambda)} + \eta_0^{(N)}(\lambda) \right]^2 \right\rangle \right\} \tag{4}
\]

where \( \lambda \equiv k + \frac{1}{2} \),

\[
g^{(N)}(\lambda) = \eta_0(\lambda) e^{i2\delta_{N}(\lambda)}
\]

\[
\delta_{0}(\lambda) = 2 \frac{d \delta_{0}(\lambda)}{d \lambda}, \quad D(\lambda) = \frac{d}{d \lambda} \eta_0(\lambda)
\]

\( \delta_{2}^{(C)} \) and \( \delta_{2}^{(N)} \) are the Coulomb and nuclear quadrupole deformation lengths, respectively,

\[
c_{2-\kappa}(\lambda) = \frac{3}{5} \left( \frac{kR}{n} \right) l_{2-\kappa}(\alpha, \xi) \quad \text{(see Ref. 2),}
\]

and \( a_{\kappa}(\lambda) = \frac{5}{2} \left( \frac{\sqrt{5}}{\pi} \right) \int_{0}^{\infty} \int \lambda \lambda+\kappa \lambda+2 \lambda \lambda \xi(\lambda, \xi) \)

After separating in expression (4) the slowly \( \lambda \)-varying and rapidly \( \lambda \)-varying terms, we can evaluate the elastic scattering amplitude in closed form by performing the \( \lambda \)-sua (\( \lambda \)-integral) using the methods of Refs. 1-3. (Details are given in Ref. 6.)
For strong Coulomb polarization, the Coulomb excitation contributions to $S_{pq}^{(N)}(\lambda)$ have to be calculated to higher orders, leading to an exponentiation of the pure-Coulomb terms in Eq. (4). The result is identical to the modified $S$-matrix derived previously from the dynamic polarization potential of Ref. 4 in the "Coulomb distorted eikonal approximation".

B. Coupling to a Transfer Channel

For simplicity we consider the effect of the transfer of a zero-spin particle and restrict ourselves to zero angular momentum transfer $L=0$. Then we obtain for the modified $S$-matrix corresponding to the process of Fig. 1b, shown more specifically in Fig. 2,

$$S_{pq}^{(N)}(\lambda) = \frac{g^{(N)}}{2} \left[ 1 - \frac{1}{2} \frac{\mu}{\mu_0} \frac{a_2(k_1 k_2)}{k_1 k_2} \right] \times$$

$$\frac{\langle \epsilon_1 \rangle}{\hat{\Gamma}_{oo}(\alpha, \xi)} \frac{\langle \epsilon_2 \rangle}{\hat{\Gamma}_{oo}(\alpha, \xi)}$$

(5)

where $\mu/\mu_0$ is the reduced mass in the elastic (transfer) channel, $k_0$ is the asymptotic wave number in the elastic channel and $k_T^*$ is related to the asymptotic wave number in the transfer channel, $k_T$, by $k_T^* = \frac{m_1}{m_f} k_T$, where $m_1$ and $m_f$ are the mass of the target (residual) nucleus, respectively. $a_2(k_1, k_2)$ is the transfer equivalent of $a_2(\epsilon_1, \epsilon_2)$ of Eq. (4) and contains all spectroscopic information. $k_1$ and $k_2$ are the imaginary wave numbers of the transferred particle in the initial and final bound states, respectively. The functions $\hat{\Gamma}_{oo}(\alpha, \xi)$ and $\hat{\Gamma}_{oo}(\alpha, \xi)$ are standard Coulomb-transfer integrals depending on the mean Rutherford deflection functions $\alpha = \theta_{\alpha}(\lambda)$ and the adiabaticity parameter $\xi$.  

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The $\xi$-sum in the elastic scattering amplitude can be performed analytically and is given in Ref. 6. It is interesting to note that in cases where a sequential transfer leads into the elastic channel ("double elastic transfer", in Fig. 3), the intermediate state is the same as the one appearing in Fig. 2. Then the partial wave amplitude is given by that of Fig. 2 except for the phase factor $(-1)^\xi$, i.e.,

$$(\text{Figure 3}) = (-1)^\xi (\text{Figure 2}).$$  \hspace{1cm} (6)

Since the amplitudes corresponding to Fig. 2 and Fig. 3 add up coherently, the total partial-wave nuclear S-matrix is then given by

$$\hat{S}^{(N)}_{\text{DT},k} = \hat{S}^{(N)}_k \left[ 1 + [1 + (-1)^\xi] \left\{ -\frac{1}{2} \frac{\epsilon_{\alpha_T}^0 \epsilon_{\alpha_T}^0}{\epsilon_{\alpha_T}^0} \frac{s_{\alpha_T}(\kappa_1 \kappa_2)}{\kappa_1} \right\} \right]$$

$$\times \left\{ -\frac{s_{\alpha_T}(\kappa_2)}{\kappa_2} \right\} \left\{ -\frac{s_{\alpha_T}(\kappa_1)}{\kappa_1} \right\}$$

Equation (7) is an example of an even-odd staggering effect which may be relevant in certain backward-angle elastic scattering phenomena. In situations where a single elastic transfer is important (Fig. 4), the modified elastic partial-wave S-matrix has the form:

$$\hat{S}^{(N)}_{\text{ET},k} = \hat{S}^{(N)}_k \left[ 1 - (-1)^\xi \right] \left\{ -\frac{1}{2} \frac{\epsilon_{\alpha_T}^0 \epsilon_{\alpha_T}^0}{\epsilon_{\alpha_T}^0} \frac{s_{\alpha_T}(\kappa_1 \kappa_2)}{\kappa_1} \right\} \left\{ -\frac{s_{\alpha_T}(\kappa_1)}{\kappa_1} \right\} I_{\alpha,0}.$$

Again the phase factor $(-1)^\xi$ indicates the exchange character of the transfer contribution. A significant feature of our calculation is that the overall phase of the elastic transfer amplitude is fixed. This is in contrast to DWBA calculations of the elastic transfer contributions which has to be added to $\hat{S}^{(N)}_k$ with an undetermined relative phase.
IV. THE INELASTIC AND THE TRANSFER AMPLITUDES

It is clear that corrections to the distorted wave Born approximation for the inelastic and the transfer amplitudes can now be made in a straightforward way by substituting for the partial-wave elastic S-matrix elements the modified expressions \( S^N \) obtained in the previous section (see Figs. 1c, 1d, 1e). For each case the partial-wave summations in these improved DWBA amplitudes can be carried out analytically following the procedures developed in Refs. 1-3 (see details in Ref. 6).

V. CONCLUSION

Although our method is based on a perturbative treatment of the channel coupling, the analytic form of our results provides a clear understanding of the physics involved and facilitates the interpretation of more exact coupled-channel calculations. Numerical comparison between our results and full coupled-channel calculations is in progress. Our formulae can be extended to the backward-angle region following the methods developed in Ref. 9. Applications of our theory to large-angle heavy-ion scattering data with the aim of identifying the dynamical mechanisms that cause backward-angle enhancement of the elastic, inelastic and transfer cross sections, are in progress.

A detailed account of our theory will be given in a forthcoming publication.

REFERENCES

3. W. E. Frahn, Closed-form description of heavy-ion transfer reactions based on distorted-waves theory, to be published.


6. W. E. Frahn and M. S. Hussein, Coupled-channels extension of the closed formalism for heavy-ion collisions, to be published.


FIGURE CAPTIONS

Figure 1  The basic processes considered. The initial target (final residual) nucleus is represented by $A_i(A_f)$ with the corresponding ground and an excited state indicated by the lines. Figures 1a and 1b correspond to elastic scattering. Figures 1c and 1d correspond to inelastic scattering. Figure 1e refers to a transfer process.

Figure 2  The correction to the elastic scattering amplitude due to the coupling to a transfer channel (Figure 1b).

Figure 3  Correction to the elastic scattering amplitude due to double elastic transfer.

Figure 4  Single-elastic transfer process.
FIG. 1
FIG. 2

FIG. 3

FIG. 4
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ABSTRACT

The coupled channel radial equations for multiple Coulomb excitations are used to derive the Coulomb polarization potential in the elastic channel. The potential is seen to have the general form

\[ V_r(r) = \frac{a_r}{r^3} + \frac{b_r}{r^4} + \frac{c_r}{r^5} \]

where the coefficients \( a_r \), \( b_r \), and \( c_r \) are oscillatory functions of the coupling constant. We have explicitly calculated \( V_r(r) \) up to fourth order in the coupling and used the resulting expression to obtain the elastic scattering cross section in the WKB approximation. We were able to account reasonably well for the data on \(^{20}\)Ne + \(^{148}\)Sm, \(^{150}\)Sm and \(^{152}\)Sm at \( E_{\text{Lab}} = 70 \) MeV. Including higher-order effects in \( V_r(r) \) is straightforward.

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Recently a simple expression for the Coulomb polarization potential for heavy-ion scattering at sub-barrier energies was derived.\textsuperscript{1,2} Although this potential was obtained assuming the validity of the usual semiclassical conditions and incorporating the loss of energy due to the excitation of the $2^+$ state involved through a simple energy loss factor, the overall agreement of the resulting semiclassical elastic cross section with the full two coupled-channels results was impressive. The potential of Refs. 1 and 2 is, however, necessarily limited to the weak coupling situation. This fact was demonstrated through fitting the elastic scattering data of $^{20}\text{Ne} + ^{148}\text{Sm}$, $^{150}\text{Sm} + ^{152}\text{Sm}$ at $E_{\text{lab}} = 70$ MeV\textsuperscript{3} where it was found that the simple WKB elastic cross section based upon the second-order potential of Ref. 1 could not account well for the back-angle data points. A theoretical study based on the semiclassical theory of Coulomb excitation has recently been made.\textsuperscript{4} In that study the limitation of the second-order potential was discussed in detail and a more exact potential which incorporates multiple Coulomb excitation effects was derived. In this paper we show that the simple $r$-dependence of the potential in Ref. 1 is maintained even when including higher order, multiple Coulomb excitation effects.\textsuperscript{5} We exhibit this property in detail for the absorptive part of the Coulomb polarization potential by incorporating the reorientation effect to second-order. With this improved potential we account rather well for the data of Ref. 3.\textsuperscript{5}

Our starting point is the coupled radial equations describing multiple Coulomb excitation between even-even nuclei at sub-barrier energies as given in Ref. 6. Eliminating all other channels in favor of the elastic one we end up with an effective equation for elastic channel radial wave function which has besides the monopole-monopole Coulomb potential, an extra potential coming
coupling to the eliminated channels. We follow the same procedure as in Ref. 1, namely we replace the Coulomb Green's function by its on-energy-shell form and set the energy loss in the different excitation processes to be zero. We correct for this approximation later by inserting the appropriate semiclassical energy loss factors that were discussed and tabulated in Ref. 7. By using the semiclassical recursion formulae for the Coulomb wave functions\(^8\) we were able to derive the following simple form for the trivially equivalent local Coulomb polarization potential (TELP)

\[ V_L(r) = \frac{a_k}{r^3} + \frac{b_k}{r^4} + \frac{c_k}{r^5}. \] (1)

The coefficients \(a_k\), \(b_k\) and \(c_k\) are functions of the center-of-mass energy \(E\), the Sommerfeld parameter \(n\), the quadrupole matrix elements \(\langle l_m | M(E^2) | l_m \rangle\), and the orbital angular momentum \(l\). Although the coefficients above are infinite series in powers of \(\langle l_m | M(E^2) | l_m \rangle\), the \(r\)-dependence of the potential in the zero energy loss limit considered above is the same as that of the second order potential of Ref. 1. In Ref. 4, the coefficient \(a_{l=0}\) was evaluated numerically for the case of coupling to pure rotational states using a different method based on the semiclassical theory of Coulomb excitation. The results of Ref. 4, summarized in Fig. 1, show that the imaginary part of \(a_{l=0}\) has an oscillatory behavior as a function of the quadrupole coupling strength \(q_{O^+2}\) defined through

\[ q_{O^+2} = \sqrt[3]{\frac{k^2}{5 \pi n}} \frac{\langle 0 | M(E^2) | 2 \rangle}{Z \pi^2}. \] (2)

The real part of \(a_{l=0}\) is seen to be negative and non-oscillatory as a function of \(q_{O^+2}\). Based on these observations we are led to conclude that these properties hold for the potential appearing in Eq. (1) also.
Moreover, the above behavior of \( a_{\ell=0} \) would also hold for \( \ell \neq 0 \) and it is certainly shared by the other coefficients \( b_{\ell} \) and \( c_{\ell} \). These last two coefficients identically vanish for head-on collisions \( \ell = 0 \). It is clear that the lowest order (third) of the real part of the potential (4) is due entirely to the reorientation of the \( 2^p \) state. As will be clear later, the real part of the potential \( V_{\ell}(r) \) is quite insignificant as compared to the dominant monopole-monopole Coulomb potential. We turn now to the higher-order corrections to the imaginary part of \( V_{\ell}(r) \). The imaginary part of \( V_{\ell}(r) \) has been calculated up to and including fourth order in the quadrupole coupling coming from the reorientation of the \( 2^p \) state in both projectile and target.\(^5\)

The formula for the sub-barrier elastic cross section is then easily obtained following the same WKB prescription as in Ref. 1:

\[
\frac{\sigma(\theta)}{\sigma_R(\theta)} = \exp\{-K(\theta) f(\theta)\}.
\]

where

\[
K(\theta) = \frac{16}{45} [q_0 q_2(p) [1 - \frac{4}{45} q_2 q_2(p) f_1(\theta)] g_p(\xi) + q_0 q_2(T) [1 - \frac{4}{45} q_2 q_2(T) f_1(\theta)] g_T(\xi)] ;
\]

\[
f_1(\theta) \equiv [(1 - \sqrt{\frac{\ell-1}{2}}) \tan \frac{\theta}{2} \tan \frac{\theta}{2} + \frac{1}{3} \sin^{4} \frac{\theta}{2}]
\]

and \( f(\theta) \) is the universal angular function given in Ref. 1 and 2. The symbol \( \sigma_R(\theta) \) above is the Rutherford cross section. \( T \) and \( p \) refer to the target and projectile respectively. The factors \( g_p(\xi) \) and \( g_T(\xi) \) are the semiclassical energy-loss factors for target and projectile respectively. These factors are tabulated\(^7\) for different values of the adiabaticity parameter, \( \xi = \frac{nAE}{E} \), where \( E \) is the energy loss in a given inelastic
transition. We have introduced the parameter \( q_{2+2} \) which is related to the reorientation reduced matrix element \( <2||M(E2)||2> \) through:

\[
q_{2+2}(T(p)) = \frac{\sqrt{2}}{\sqrt{5}} \frac{k^2}{\pi} \frac{<2||M(E2)||2>T(p)}{\langle E^2 \rangle \langle T(p) \rangle} \tag{4}
\]

The definition of \( q_{2+2} \) above is convenient as it becomes equal to \( q_{0+2} \) in the case of pure rotational band. In the limit of a pure harmonic vibrational \( 2^+ \) state \( <2||M(E2)||2> = 0 \) and thus \( q_{2+2} = 0 \), we recover the result of Ref. 1.

In Fig. 2 we compare the value of \( \frac{q(\theta)}{q_R(\theta)} \) calculated from Eq. (3) with the experimental data of Ref. 3. We have assumed \( q_{2+2} = q_{0+2} \) throughout and calculated \( q_{0+2} \) and the energy loss factors from the \( B(E2) \) 's and the \( E_{2+} \) 's used in Ref. 3. The agreement is quite satisfactory. It should be clear, however, that insofar as \( ^{152} \)Sm is concerned, the inclusion of the excitation of the \( 4^+ \) state at back angle is called for as it has an excitation energy of 0.366 MeV and therefore its effect on \( \sigma_{\text{eff}} \) to leading order in the coupling is comparable to that of the reorientation of the \( 2^+ \) state.

In Ref. 9 a calculation of \( \text{Im}V(r) \) for a pure rotational band up to and including the eighth order term (altogether 59 terms) was performed. The exponent \( K(n) \) in the \( \xi=0 \) limit was found to be

\[
K(n) = \frac{16}{45} q^2 \left( 1 - 0.1 q^2 + 0.02 q^4 - 0.008 q^6 \right) \tag{5}
\]

Clearly in cases where \( q \) is not so small the higher order terms appearing in \( K(n) \) must be included. Furthermore, in assuming \( q_{2+2} = q_{0+2} \) we are certainly overestimating the reorientation effect in the \( ^{148} \)Sm and \( ^{150} \)Sm.
nuclei. It seems that for these nuclei a more important refinement of the potential would come from a better treatment of the energy loss. A semiclassical calculation of the TELP for a pure harmonic quadrupole vibrator valid for any value of the adiabaticity parameter, \( \xi \), was performed in Ref. 10.

So far we have not discussed the role of the real part of the Coulomb polarization potential. To see how important this real potential is, we estimate its ratio to the monopole-monopole Coulomb potential at the distance of closest approach for the system \(^{20}\text{Ne} + ^{152}\text{Sm}\) at \( E_{\text{lab}} = 70 \text{ MeV} \).

From Fig. 1 we obtain for \( \xi = 0 \)

\[
\frac{\text{Re} V_{2\pi 0}(r)}{2 \frac{Ze^2}{2a}} = 0.56 \frac{n}{n} = -1.07 \times 10^{-2} \tag{6}
\]

Therefore, the real part of the Coulomb polarization potential, as calculated within our approximations, acts as a very small reduction factor in the Coulomb repulsion between the heavy ions.

We have demonstrated that the heavy-ion Coulomb polarization potential as calculated under the usual semiclassical approximations, has the very simple \( r \)-dependence of Eq. (1) even in the case of strong multiple Coulomb excitation coupling. By incorporating the contribution from the reorientation of the \( 2^+ \) state into the second-order potential of Ref. 1, we have been able to account rather well for the recent sub-barrier elastic scattering data of Ref. 3.
REFERENCES

1. A. J. Baltz, S. K. Kauffmann, N. K. Glendenning and K. Pruess, 

to Nucl. Phys. A.

A. Menchaca-Rocha, D. K. Scott, T.J. M. Symons, K. Van Bibber, Y. Viyogi, 

422.

5. M. S. Hussein; to be published.

6. K. Alder and Aa. Winther, Electromagnetic Excitations (North-Holland), 
1975.

28 (1956) 432.

8. Milton Abramowitz and Irene A. Stegun, Handbook of Mathematical Functions 
(Dover, New York, 1970).


1801.

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FIGURE CAPTIONS

Figure 1  The semiclassical potential \( v_q(0) \) of Ref. 4. What is plotted are the real and imaginary parts of \( a_q(0) \):

\[
a_q(0) = \frac{3}{q} \left[ 2 + i \omega(q) \right]
\]

where \( a \) is half the distance of closest approach for \( L=0 \). Also indicated is the corresponding \( \omega(q) \) of Ref. 1.

Figure 2  Angular distributions for elastic scattering of \(^{20}\)Ne on Samarium nuclei. The solid curves are calculated from Eq. (3) in the text. The data are from Ref. 3.
Figure 1
Multi-Phonon and Giant Resonance Excitation in
Inclusive Heavy Ion Inelastic Scattering

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I. INTRODUCTION

In this investigation, we will study the potential usefulness of high energy heavy ion reactions in probing collective surface modes of nuclei. The hope is that with heavy ions as projectiles one may strongly and selectively excite a variety of collective degrees of freedom. Especially the finding of collective modes with high multipolarity is important for the understanding of nuclear collective motion\(^1\) and may shed light on the role of such states as doorways\(^2\) to more complicated reaction processes such as deep inelastic scattering and fusion reactions. Furthermore, the position and widths of these giant resonances can be compared to predictions of RPA theories and, thereby, provide a test of the effective force of nucleons in nuclei\(^3\).

Inelastic collective excitations are usually treated in the distorted wave Born approximation (DWBA)\(^4\). Yet the DWBA calculation fails to account for the inclusive inelastic spectrum observed in heavy ion and a-particle induced inelastic scattering\(^5\). One possible reason for the observed discrepancy may be the neglect in DWBA of multi-phonon excitations. A recent

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coupled channels calculations of the $^{208}$Pb(α,α') reaction, in classical approximation, confirms specifically that multi-phonon excitations strongly contribute to the "background" under the giant resonance peaks. It is, however, unclear how reliable classical trajectory calculations are for α-induced reactions especially at the relatively low energy of $E=65$ MeV. On the other hand, the fully quantum mechanical coupled-channels approach will soon become less and less tractable for heavy ion reactions at high energy, e.g., ($^{16}$O,$^{16}$O') etc.

Based on these considerations, we wish to design a scheme in which one can reliably calculate the excitation probability of the low-lying collective states as well as the multi-phonon excitation. This method is based on a Taylor series expansion of the collective Hamiltonian in terms of the deformation length $δ$. In the degenerate harmonic oscillator model, this deformation length is approximately $δ(AN) \approx (AN)/2.5$ fm for a $(AN)\hbar \omega$ excitation. Giant resonances usually consist of $1\hbar \omega$ or $2\hbar \omega$ excitations. Therefore, the deformation length is probably less than 1 and the Taylor series expansion of the collective Hamiltonian should converge. Physically, one can also argue that surface vibrations should be of the order of the surface diffuseness, and therefore the Taylor expansion is meaningful (the expansion parameter is then roughly equal to the diffuseness $\alpha_0$, 0.6 fm).

A powerful method, related to the Taylor series expansion of the nuclear Hamiltonian, is the Austern-Blair (A-B) model of the inelastic excitation. In the A-B model, the transition S-matrix element is related to the elastic S-matrix by simple algebraic operations. The A-B model as applied to single excitations has been tested to be very reliable in many occasions. We will, in the following, use the A-B model to describe single
and multiple phonon excitation induced by α-particles and heavy ions impinging on $^{208}\text{Pb}$.

II. REACTION DYNAMICS

A. Austern-Blair Model for Inelastic Scattering

In order to treat inelastic scattering to collective phonon excitations, the transition potential $V_T$ is written in a Taylor series

$$V_T = R \frac{dU}{dR_0} \Sigma a_{\lambda \mu} Y_{\lambda \mu}^{*}(\hat{r}) + \frac{1}{2} \frac{d^2U}{dR^2} \Sigma a_{\lambda \mu} Y_{\lambda \mu}^{*}(\hat{r}) \Sigma a_{\lambda' \mu'} Y_{\lambda' \mu'}^{*}(\hat{r}) + \ldots, \quad (1)$$

where $U(r-R_o-R_p)$ is the ion-ion potential, $a_{\lambda \mu}$ is the phonon amplitude of multipolarity $\lambda$, and $R_o$ and $R_p$ are the radius of target and projectile, respectively. For simplicity, and since we are only interested in target excitations, deformation of the projectile is neglected in eq. (1). With the above expansion, the radial integral for single step excitation in the A-B approximation is given by

$$\frac{\mu}{\hbar^2} \int r^{(1)} = \frac{4\pi n^f n^i}{2A_f 2A_i}, \quad (2a)$$

while the "direct" plus "two step" second order radial integral is expressed as

$$\frac{\mu}{\hbar^2} \int r^{(2)} = 4K_f K_i \frac{2n^f}{2A_f} \frac{2n^i}{2A_i}, \quad (2b)$$

In the above equations, $\mu$ is the reduced mass, $(n^f, n^i)$ and $(A_f, A_i)$ are the elastic S-matrices and the grazing angular momenta for the final and initial channel, respectively. Following Hahne $^3$, the geometric mean of the derivative of the S-matrices in entrance and exit channel is used to take effects of the reaction Q-value into account. This prescription has been extensively tested and found to be remarkable accurate for describing
collective excitations even at fairly high excitation energies. The approximations for second order transitions have not yet been studied numerically, except for the effect on elastic scattering of virtual transitions to collective states. A detailed numerical investigation of this approximation is under way.

For monopole excitations, special treatment of the radial integral is necessary. Following Satchler, the perturbing potential $U_T$ can be expressed in terms of the ion-ion potential $U$ as

$$U_T(r) = -a_0 [3U(r) + r \frac{dU}{dr}]$$

(3)

where $a_0$ is the monopole deformation parameter. The term proportional to $U$ arises from particle number conservation. The corresponding radial integral in the A-B formalism then reads

$$\frac{\mu}{\pi n} I^{f}_{t} \frac{d}{dt} \frac{3n}{2} \left( \frac{3U}{3U} - \frac{3U}{3U} \right)$$

Assuming that the scattering phase shifts $\delta(\lambda+1/2)$ depend linearly on $U$, eq. (4a) can be rewritten as

$$\frac{\mu}{\pi n} I^{f}_{t} \frac{d}{dt} \frac{3n}{2} \left( \frac{3U}{3U} - \frac{3U}{3U} \right)$$

(4b)

The terms $\sim n$ in eq. (4b) are not surface peaked and, due to strong absorption, phase averaging and large grazing angular momenta, are relatively less important for heavy ion scattering.

B. Nuclear S-Matrix Elements

The elastic nuclear S-matrix elements which are necessary to calculate the inelastic cross sections in the A-B model, are generated from a simple strong absorption model. At high energies ($E/A \approx 10$ MeV/nucl) where
internal waves reflected from the inner barrier are almost completely absorbed, this should be a reasonable approximation. We, therefore, use the Ericson parameterization

\[ \eta(\lambda) = \frac{1}{1 + \exp(-i\gamma + \frac{\lambda - \Delta}{\Delta})} \]  (5)

The grazing angular momentum \( \lambda \) is calculated using the proximity potential for the ion-ion interaction. The width parameter \( \Delta \) is energy dependent via \( 2a \). Here, \( a \) is the diffuseness of the ion-ion potential, i.e., \( a \approx 0.6 \) fm. Since the inclusive cross sections turn out to be fairly independent of the nuclear phase \( \gamma \), the latter is fixed at \( \gamma = \pi/2 \). No Coulomb excitation is included at present. This approximation should be reasonable for light projectiles and high energies.

C. Nuclear Structure

In order to calculate inclusive inelastic scattering of \( \alpha \)-particles and heavy ions from \( ^{208}\text{Pb} \), we assume that the spectrum of \( ^{208}\text{Pb} \) can be described in terms of low lying surface vibrations and giant resonances. The excitation energy and the fraction of the sum rule exhausted by these phonons are taken from a recent theoretical investigation. Since, in our scheme, excitation probabilities are calculated up to second order in the deformation lengths, only one and two phonon states can be populated. The separation of a giant resonance from its background, therefore, depends on the relative cross sections for direct giant resonance excitation vs. two-phonon excitation. In order to roughly account for neglecting the phonon-phonon interaction, the two-phonon states are assumed to have a width
$E_2 = 0.5E^*$, $E^*$ being the excitation energy in $^{208}\text{Pb}$. This recipe roughly agrees with recent calculations\textsuperscript{10).} The width of the giant resonances is taken as in Ref. 6).

III. RESULTS AND DISCUSSION

The inclusive $^{208}\text{Pb}(a,a')$ cross section is shown in Fig. 1 for $E_a = 65$ and 130 MeV, respectively. Both calculations have been obtained using a width $\Delta = 1.3$. Both spectra exhibit pronounced peaks corresponding to the various giant resonances superimposed on a relatively smooth background due to double phonon excitation. At 65 MeV, second order transitions contribute to the total inelastic cross section at a fraction of $\%30\%$, i.e., about half as much as obtained in Ref. 6). The calculation of Ref. 6) (solid line in Fig. 1) deviates most strongly from our results at high excitation energies ($E^* > 20$ MeV). Since excitation of 3 or more phonons is expected to be small (see below) we attribute this discrepancy to the different treatment of angular momentum matching in the classical trajectory calculation. This also results in a much larger average energy loss of $\%20\text{ MeV}$ obtained in Ref. 6) as compared to $\%10.5\text{ MeV}$ in our calculation. These results remain essentially unchanged if $\Delta$ is increased to $\Delta = 1.85$, as expected in semiclassical models for $E_a = 65\text{ MeV}$.

It is interesting to point out that at $E_a = 130\text{ MeV}$, the peak to background ratio for the giant resonances is larger than at $E_a = 65\text{ MeV}$, as is especially obvious for the E4 resonance. At 130 MeV, multi-phonon excitations comprise about $\%40\%$ of the total inelastic cross section.

Figure 2 compares inclusive cross sections for $^{208}\text{Pb}(^{160},^{160}')$ at $E_{\text{Lab}} = 240$ and 300 MeV with the $(a,a')$ reaction at 65 MeV. For all calculations, $\Delta = k-a$ has been used. It is seen from this figure that while
the excitation probabilities for giant resonances increase when going from a to $^{16}$O as a projectile so does the multi-phonon background (to about 40% of the total inelastic cross section) so that only slight advantages are expected by using heavier ions instead of a-particles as projectiles. It should also be noted that in Fig. 2, the EO giant resonance strength has been increased to 200% of the energy weighted sum rule in order to make the peak more clearly visible. The dotted line in Fig. 2 shows the result for the EO peak assuming a strength corresponding to 100% of the sum rule. The EO peak will be unobservable in the inclusive spectrum if the strength exhausts less than 50% of the sum rule.

A. Estimate of the Multi-Phonon Excitation Probability

At high incident energy, heavy ions behave like classical particles, $\Delta \gg 1$. The angle integrated cross section for single excitation of a phonon with multipolarity $\lambda$ can then be expressed as

$$
\sigma_{\lambda} \propto \frac{2}{64\Delta^2} \sum_{\nu} \left[ a^\lambda_{\nu} \right]^2 \frac{1}{2\lambda} \left( \frac{\lambda + \lambda^*_F}{\Delta} \right) \cosh \left( \frac{\lambda - \lambda^*_F}{\Delta} \right) \cosh \left( \frac{\lambda^*_F}{\Delta} \right)
$$

where the second equality in eq. (6) is obtained by replacing the sum over $\nu$ with an integral. Equation (6) gives a very good estimate of the integrated cross section (error is less than 10%) for the ($^{16}$O,$^{18}$O') reaction. Since $\Delta = (A_1 + A_2)/2 \geq kR$ and $\Delta \gg kR$, we find that the excitation probability of the surface mode is proportional to $R/a$, the ratio of the radius to the surface diffuseness.
Similarly, the multi-phonon excitation probability is given by

\[ \sigma^{(2)}_{1,2}\lambda = \delta^2 \frac{(2t_1^*+1)(2t_2^*+1)}{6\pi n^2} \left(2\lambda+1\right) \delta_{1,2} \delta_{\lambda,0} \frac{1}{a} \]  

and

\[ \frac{\sigma^{(2)}_{1,2}\lambda}{\sigma^{(1)}_{1,2}\lambda} \approx \frac{(2t_1^*+1)(2t_2^*+1)}{6\pi n^2} \left(2\lambda+1\right) \delta_{1,2} \delta_{\lambda,0} \frac{1}{a} \]  

In the same approximation,

\[ \frac{\sigma^{(2)}_{1,2}\lambda}{\sigma^{(1)}_{1,2}\lambda} \approx \frac{(2t_1^*+1)(2t_2^*+1)(2t_3^*+1)(2t_4^*+1)}{6\pi n^2} \left(2\lambda+1\right) \delta_{1,2} \delta_{\lambda,0} \frac{1}{a} \]  

Equation (8) agrees with the numerical result to within 10%.

We can see now that the two-phonon excitation probability is smaller than the one phonon excitation, e.g.

\[ \frac{\sigma^{(2)}_{1,2}\lambda}{\sigma^{(1)}_{1,2}\lambda} = \frac{1}{7} \delta_{\lambda,1}^2 \delta_{\lambda,2}^2 \delta_{\lambda,3}^2 \delta_{\lambda,4}^2 \frac{1}{a} \]  

Expanding the scattering amplitudes in a power series in the number of phonons is surely a convergent procedure provided that \( \delta^2 < 1 \). Since the number of possible two phonon states is much larger than the single phonon states, we obtain therefore that the two-phonon background amounts to 40% and 30% of the total inelastic cross section in \((^{16}O,^{16}O')\) and \((a,a')\) reactions, respectively. However, the 3rd order, as calculated using eq. (9), contributes less than 5% to the total inelastic cross section.

IV. CONCLUSION

The present investigation has shown that despite an increasing background due to multi-phonon excitation, giant resonance peaks should show up strongly in inelastic heavy ion scattering. This seems to be borne out by
recent experiments\textsuperscript{11,12}). The present model provides a convenient scheme for calculating inelastic cross sections for single and double excitations of phonon and giant resonance states especially at very high energy, where the large number of angular momenta involved make fully quantum mechanical coupled channels calculations intractable. Extensions of the present calculations to include Coulomb excitation and to calculate spectra at fixed angles are under way.
REFERENCES


Fig. 1: Inclusive inelastic cross sections as a function of excitation energy for the reaction $^{208}$Pb$(a,a')$ at $E_a=65$ and 130 MeV. The spins and parities of the phonon states and giant resonances used in the calculations are indicated. The solid line at $E_a=65$ MeV is the prediction of Ref. 6. The dotted line represents the single and double excitation probability for O$^-$ states.

Fig. 2: Inclusive inelastic cross sections as a function of excitation energy for the reaction $^{208}$Pb$(^{16}$O,$^{16}$O$'$) at $E_{16}=240$ MeV (solid line) and 500 MeV (dashed line). For comparison, the $^{208}$Pb$(a,a')$ inclusive cross section at $E_a=65$ MeV is shown as dashed-dotted line. For further details see caption in Fig. 1 and text.
We consider reactions in which a target is bombarded by a relatively light projectile nucleus like $^{16}$O, and a still lighter nucleus, presumed to be a fragment of the projectile, is observed to emerge. As specific examples we consider recent data by Gelbke et al. and by McGrath et al., in which $^{16}$O bombards $^{208}$Pb or $^{58}$Ni at energies of 3-15 MeV/nucleon above the Coulomb barrier, and spectra are measured, at a variety of angles, for outgoing fragments between $^6$Li and $^{15}$N.

Spectra of $^{15}$N and $^{12}$C taken at the grazing angle invariably exhibit a prominent peak centered at essentially the incident beam velocity; this strongly suggests a peripheral direct reaction, and it is this "quasi-elastic peak" in which we are primarily interested. At angles away from the grazing angle, the $^{15}$N and $^{12}$C spectra often exhibit a separate, additional peak at a lower outgoing energy, which seems appropriately identifiable as a "deep-inelastic" component of the spectra. Finally, the single peak which occurs, at any angle, in the spectra of lighter fragments, is shifted so far down in energy toward the final-channel Coulomb barrier (even if it occurs at the beam velocity) that no clear distinction can be made between these two types of components, and consequently we make no attempt to discuss them in terms of a direct-reaction mechanism.

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Our purpose is to investigate the extent to which the quasi-elastic peak in the grazing-angle spectra of $^{15}$N and $^{12}$C can be understood in terms of a direct-reaction model. In particular, an intriguing aspect of the data of Gelbke et al., particularly emphasized by Scott is the fact that the widths of these spectra (especially if expressed as momentum widths rather than energy widths) exhibit an increase by nearly a factor of two as a function of bombarding energy, the increase occurring over a narrow energy range near 15 MeV/nucleon above the entrance-channel Coulomb barrier.

In brief, our model attributes this spectral width to "Doppler broadening" or "Fermi motion" within the projectile before it fragments. Taking the ($^{16}$O, $^{15}$N) reaction as an example, the $^{15}$N and the proton are moving randomly relative to each other inside the $^{16}$O before the collision. Recalling that the proton is moving relative to the target rapidly enough that it is not likely to be captured into a bound state of definite energy, the $^{15}$N can be moving either forward or backward relative to the $^{16}$O CM at the time the proton is removed, and it is this randomness which is primarily responsible for the range of energies in the quasi-elastic peak of the $^{15}$N spectrum. We conjecture that the two most-likely direct reactions leading to this spectrum are projectile fragmentation (breakup) and transfer. Our above discussion applies only to fragmentation; as we show below, transfer narrows the spectrum further. Consequently, we conjecture that the increase observed in the widths of the spectra at 15 MeV/nucleon above the barrier may indicate a change in mechanism from transfer to fragmentation.

We attempt to obtain a qualitative understanding from the simplest possible direct model for these two processes, the plane-wave Born approximation -- or rather, an improved version of this approximation in
which the local (Coulomb-corrected) momenta at the point of transfer are
used in place of the asymptotic momenta. Taking $^{16}\text{O} \rightarrow ^{15}\text{N} + p$ as our
example, we write its incident wave function in the form

$$
\psi_{\text{incident}} = e^{i\mathbf{k} \cdot \mathbf{r}} e^{i\mathbf{p} \cdot \mathbf{r}} \phi_p(\mathbf{r}_p - \mathbf{r}_N) \tag{1}
$$

Assuming, for simplicity, an infinitely heavy target which remains in its
ground state, its wave function is inert and so need not be included in the
matrix element. The final state has the form

$$
\psi_{\text{final}} = e^{i\mathbf{K} \cdot \mathbf{r}_N} \phi_{\text{PT}}(\mathbf{r}_N - \mathbf{r}_p) \tag{2}
$$

where $\phi_{\text{PT}}$ will be a plane wave for the final proton state for fragmentation,
and a function localized at the target for a transfer reaction. Leaving
$\phi_{\text{PT}}$ unspecified for the moment, the PWBA matrix element becomes (neglecting
normalization) in the "prior" representation

$$
T = \int d^3r_p \int d^3r_N e^{i(\mathbf{k}_N \cdot \mathbf{r}_N) + \mathbf{k}_p \cdot \mathbf{r}_p} e^{i\mathbf{p} \cdot \mathbf{r}} \phi_{\text{PT}}(\mathbf{r}_N - \mathbf{r}_p) \cdot \mathbf{V}(\mathbf{r}) \cdot \phi_{\text{PT}}(\mathbf{r}_p)
$$

$$
= \hat{\phi}_{\text{PT}}(\mathbf{k}_N - \mathbf{K}_p) \int d^3k \hat{\phi}_{\text{PT}}^*(\mathbf{k}) \mathbf{V}_{\text{PT}}(\mathbf{|K}_0 - \mathbf{K}_p |) \tag{3}
$$

where $\mathbf{K}_0 \equiv \mathbf{K}_N + \mathbf{K}_p$ is the incident $^{16}\text{O}$ momentum, and $\hat{\phi}(\mathbf{k})$ is the Fourier
transform of $\phi(\mathbf{r})$.

The projectile-fragmentation limit is particularly simple. Then
$\phi_{\text{PT}}^*(\mathbf{k}) = \exp(-i\mathbf{k}_p \cdot \mathbf{r}_p)$, so
$\hat{\phi}_{\text{PT}}^*(k) = \delta(\mathbf{k} - \mathbf{k}_p)$, and

$$
T^{\text{PF}}(\mathbf{k}_N, \mathbf{k}_p) = \hat{\phi}_{\text{PT}}(\mathbf{k}_N - \mathbf{K}_p) \mathbf{V}_{\text{PT}}(\mathbf{|K}_0 - \mathbf{K}_p |) \tag{4}
$$

In the $^{15}\text{N}$ spectra of interest, the accompanying proton is not detected,
so $|T|^2$ must be integrated over $d^3k_p$. If this integration includes a

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sufficiently wide range of proton momenta, \( \int d^3k' |v_{pt}|^2 \) will no longer depend on \( k_N^r \), so the \(^{15}\text{N}\) spectrum is given simply by

\[
T^{DF} (k_N^r) = |\langle \psi^{*}_{pN}(k_N^r-R_0) \psi^{*}_{pT}(k_N^r) \rangle|^2.
\]  

(5)

Shifted to the projectile's frame of reference, where \( k_N = 0 \), this is just \( |\langle \tilde{\psi}_{\text{FN}}(k_N^r) \rangle| \), the "Fermi distribution" or momentum spectrum of the \(^{15}\text{N}\) in the original \(^{16}\text{O}\).

In the opposite limit that the unobserved proton is transferred to the target (i.e., to an unstable state, but one which lasts until the "spectator" \(^{15}\text{N}\) has departed), \( \phi_{pt}^p \) must describe this state. In particular, we take it to be an angular momentum eigenstate, \( \phi_{pt}^{l} \), with \( l \) in the exit-channel angular momentum window determined by the local momentum \( k_p \) of the proton in the initial channel, centered at \( \frac{k}{p} R_0 \). For notational simplicity we define

\[
\langle \phi_{pt}^{l} | \phi_{pt}^{*} \rangle = \int \phi_{pt}^{l} (\vec{k} - \vec{k}) \phi_{pt}^{*} (\vec{k}) d^5k,
\]  

(6)

in terms of which Eq. (3) gives the matrix element for transfer as

\[
T^{Tr} = \langle \phi_{pN}(k_N^r-R_0) | \phi_{pt}^{l} (k_N^r) \rangle^* \).
\]  

(7)

In the case of projectile fragmentation, it was the integration over the unobserved \( k_p' \) which eliminated the dependence of the second factor on \( k_N^r \). Since \( k_p' \) does not appear now, this elimination no longer occurs. Hence \( k_N^r \) is limited by both factors in the case of transfer, and so has a narrower spectrum.

In order to compare these expressions with the experimental spectra mentioned, we note that fortunately the same spectra were previously measured,\(^4\) also with an \(^{16}\text{O}\) beam, but at the much higher energy of 2.1
GeV/nucleon, where the reaction is believed to be definitely fragmentation. The observed spectra were fit with a Gaussian approximation to the momentum-space $^1_6O$ wave function,

$$|\phi_{pN}(k)|^2 = C e^{-k^2/2a^2}.$$  

The fitted values of $a$ were $95 \pm 3$ MeV/c for $^{15}N$ and $120 \pm 4$ MeV/c for $^{12}C$, and these are the values we have employed in comparing our results with the lower-energy spectra.

As for the "transfer" reaction, the fact that the $^{15}N$ has a continuous spectrum clearly indicates that at these high energies the proton is not captured by the target into discrete bound states. What we shall understand by "transfer into the continuum" is that the proton is captured into a wave packet localized at the surface of the target. It is not an energy eigenstate and so not stationary, but we suppose the reaction to differ from fragmentation in that the captured state lasts at least until the spectator $^{15}N$ has moved out of range. As a specific model, we choose a surface-peaked Gaussian shape,

$$\phi_{pT}(r) \sim e^{-((r-R)/b)^2} r_n(r),$$  

with $b = 2$ Fm and $R = 7.8$ Fm in the case of a $^{208}Pb$ target and 5.2 Fm for a $^{58}Ni$ target—in both cases just a bit inside the Coulomb barrier. Actually, rather than using a single $l$-value we have employed a small range of $p$-target $l$-values; the details are given in Ref. 5.

Using these wave functions, we obtain the results shown in Fig. 1 for the data of Gelbke et al. This direct-reaction model is seen to produce quasi-elastic spectra which are indeed centered at the beam velocity and have approximately the right widths. What is not reproduced is the low-energy tail which most spectra have; since this corresponds to internal
excitation (presumably of the target, which has the higher level density), it is reasonable that it should arise from multi-step processes not included in a direct-reaction model. The widths we obtain for fragmentation and transfer spectra generally differ by not more than 30%. It is by no means clear to us that a PWBA can be trusted to this accuracy, but if it can, Fig. (2) suggests that the $^{15}$N spectra do result from transfer for $^{16}$O incident energies of 140, 218 and 250 MeV, but from fragmentation for 315 MeV. The $^{12}$C spectra seem better fit by fragmentation throughout this energy range. However, there is some reason to believe that the $\alpha$ to be used for $^{12}$C should be some 20% higher than the 120 MeV/c employed in these figures; doing so would lead to the conclusion that $^{12}$C also results from transfer at the lower energies and from fragmentation at 315 MeV (15 MeV/nucleon about the barrier).

Our fits to the Stony Brook grazing-angle data for $^{58}$Ni ($^{16}$O, $^{12}$C) at $^{16}$O lab energies of 80, 100 and 115 MeV are shown in Fig. 3. We have again used $\alpha = 120$ MeV/c, and from these results would conclude in all three cases (3.4.4 and 5.3 MeV/nucleon above the barrier) that fragmentation is more likely than transfer. In either interpretation, the direct nature of the main peak seems unambiguous, with the low-energy tail presumably arising from a more complex multi-step or "statistical" mechanism.

REFERENCES


Figure 1
Figure 2

Figure 3
A COMPARISON OF FRAGMENTATION CALCULATIONS WITHIN
MICROSCOPIC AND MACROSCOPIC FRAMEWORKS*

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ABSTRACT
The calculation of the isotopic production cross sections from high energy heavy ion reactions has proceeded along two divergent paths. From the regime of relativistic heavy ion reactions comes the abrasion-ablation model which is dependent on the quick nature of these collisions and the separation of the reactant nuclei into participants and spectators. From the intermediate energy regime comes the intranuclear cascade model where the reaction is treated as a sum of nucleon-nucleon scattering events inside a nuclear potential well. Both frameworks are able to treat projectile as well as target fragmentation. In this presentation the results of both model calculations will be compared to recent experimental data for the fragmentation of 213 MeV/A $^{40}$Ar projectiles by $^{12}$C nuclei and for the fragmentation of $^{209}$Bi target nuclei by a 400 MeV/A $^{20}$Ne beam. These comparisons show the importance of the statistical decay process in determining the distribution of final products as well as the deficiencies of the two models.
The recently reported production of neutron-rich light isotopes in relativistic heavy ion reactions\textsuperscript{1} has brought new interest into the calculation of the fragment production cross sections. These calculations arose in the regimes of high energy proton induced reactions and cosmic ray physics and were carried over into the new field of relativistic heavy ion reactions.\textsuperscript{2} Just as their backgrounds are different, the calculational approaches to fragmentation reactions are also very different.

In this paper we will present the results of a microscopic intranuclear cascade-evaporation calculation and a macroscopic abrasion-ablation calculation for projectile fragments from the reaction of 8.52 GeV \textsuperscript{40}Ar with \textsuperscript{12}C and for target fragments from the reaction of 8.0 GeV \textsuperscript{20}Ne with \textsuperscript{209}Bi. The comparison of the results of these model calculations with each other and with experimental data will be especially useful because the forms of both models that we have used are parameter free.\textsuperscript{3} The comparisons are also interesting because they should shed light on those factors that play important roles in the production of these fragments.

The collision of the relativistic heavy ion (RHI) projectile with the target nucleus is treated as a two-step process in the Monte Carlo cascade calculation. A fast step occurs with cascading collisions of nucleons from one reaction partner inside the nucleus of the other partner, which is followed by a slow statistical evaporation of the primary fragments after
the fast cascading nucleons have escaped or have been captured by the primary fragments. The calculation is made using an extension of the intranucleon cascade code, VEGAS, for proton induced reactions which has been modified to treat two colliding nuclei. The two nuclei have diffuse nuclear density distributions and Fermi motion of the nucleons is also included. The neutron or proton nature of the collision partners is selected at random in proportion to their number in the nucleus. The impact parameter is also selected at random, and the final cross sections were integrated over impact parameter. The primary fragments are subsequently individually deexcited using a version of the Dostrovsky, Fraenkel and Friedlander statistical model Monte Carlo calculations. The excitation energy of each fragment was obtained from the fast cascade code.

In the abrasion-ablation view of the collision of the RHI with a target nucleus the two nuclei are taken to be uniform hard spheres which move on straight line trajectories. Those nucleons that lie in the region of overlap of the two nuclei are sheared off in the abrasion (or fast) stage of the collision. The spectator fragments of the target (and projectile) which consist of the nucleons that were outside the region of overlap are assigned an excitation energy that is proportional to their excess surface area. This is the minimum excitation energy that such fragments would be expected to have, and increases due to frictional forces are likely to be present. The variance
of the neutron to proton ratio can be calculated in the statistical limit\textsuperscript{7} or through ground state fluctuations of normal nuclear matter.\textsuperscript{8,9}

The results of these calculations can be seen in Fig. 1 for the case of fragmentation of a 213 MeV/A $^{40}$Ar with $^{12}$C. Both the cross sections from the fast stage of the reaction (primary) and the cross sections from the statistical stage (final) are shown for three different calculations. In Fig. 1A and 1C the abrasion-ablation model was used along with an uncorrelated\textsuperscript{7} and a highly correlated\textsuperscript{8} neutron-proton distribution, respectively. These products are allowed to deexcite and produce the distributions of products seen in Figs. 1B and 1D. The distribution of nuclei produced by the fast cascade are shown in Fig. 1E and after deexcitation in Fig. 1F. Comparison of the primary fragment cross section distributions from the three models show that these distributions are very dissimilar. The difference in the correlations among the removed nucleons can be seen in the width of the Z distributions at constant mass number. The two uncorrelated distributions are approximately twice as wide as the correlated one. However, the final residue cross sections are quite similar. This comes about because of the relatively large amounts of excitation energy deposited in the primary fragments by the collision process. For comparison the measured data of reference 1 is shown in Fig. 2. In general, the data is well reproduced by
the correlated abrasion-ablation and the cascade-evaporation models, the uncorrelated abrasion-ablation final distribution is too broad. For these light nuclei the final isotopic distributions bear the characteristics of the valley of beta stability and pre-equilibrium features of the distributions are washed out by the statistical process.

The influence of the statistical process may be lessened by considering the fragmentation of a high mass nucleus because the valley of beta-stability is broader and statistical evaporation of charged particles is inhibited. In Fig. 3 we present the results for the final product cross sections for gold and thallium isotopes from the reaction of 400 MeV/A $^{20}$Ne with $^{209}$Bi. Only the correlated abrasion-ablation results (solid curve) and the uncorrelated cascade-evaporation results (histograms) are shown. Large differences are immediately apparent between the two calculations. The uncorrelated final product distribution is approximately three times as wide as the correlated one, and although flatter contains slightly more cross section. For comparison the results of reference 10 for the production of these isotopes are shown by the solid points. The cross sections were measured radiochemically and corrected for beta-decay feeding; unfortunately, the most neutron deficient nuclei had half-lives too short for measurement. The abrasion-ablation calculation predicts nuclei which are too neutron excessive, this may be due to an under-estimation of the excitation energy of the fragments. The
width of the distribution is not obtainable from the measurements and must wait for future experiments.

In conclusion we can say that the measured isotope production cross sections for the fragmentation of $^{40}\text{Ar}$ on a $^{12}\text{C}$ target are in general agreement with the absolute predictions of the uncorrelated cascade-evaporation model and the correlated abrasion-ablation model. Because of the high excitation energies deposited in the primary fragments by the fast interactions, and the nature of statistical processes, the pre-equilibrium features of the distributions are suppressed in the fragmentation of light nuclei. These features are visible with fragmentation of heavy nuclei, but measured results for such processes are incomplete.
References

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†The calculations presented here are taken in part from reference 3, and we would like to acknowledge the contributions of the other authors of reference 3 for their assistance in these calculations. We would also like to thank Dr. K. Aleklett for the use of his data prior to publication.

4. Y. Yariv and Z. Fraenkel, to be published.
7. See for example, L.F. Oliveira, R. Donangelo and J.O. Rasmussen, Phys. Rev. C19, 826 (1979), and references therein.


Figure Captions

1. The primary fragment isotopic production cross sections for $^{40}\text{Ar}$ fragmentation are shown in (A), (C) and (E) for the three models discussed in the text. The final products, after statistical deexcitation, are shown in (B), (D) and (F).

2. The data from reference 1 is shown as a contour diagram for comparison to the calculations represented in Fig. 1.

3. The calculated values of the production cross sections for gold and thallium isotopes are shown for the cascade-evaporation (histograms) and for the abrasion-ablation (solid curve) calculations. The data from reference 10 is shown as the solid points.
Figure 1

Contours in $d^2\sigma / dZ \, dA$
(mb / Z unit / A unit)
Figure 2

Contours in $d^2\sigma/dZ\,dA$
(mb / Z unit / A unit)
Figure 3

\[ \frac{d^2\sigma}{d\Omega dA} \] (mb/\(Z\) unit/Aunit)

$^{20}_{\text{Ne}} + ^{209}_{\text{Bi}} \rightarrow ^{79}_{\text{Au}}$  
($\Delta Z = 4$)

$^{20}_{\text{Ne}} + ^{209}_{\text{Bi}} \rightarrow ^{81}_{\text{Tl}}$  
($\Delta Z = 2$)

Mass number, A

XBL 797-2125
Heavy-Ion Reactions Leading to Continuum Spectra
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In the past years, we have been studying various aspects of heavy-ion physics, using -10 MeV/amu light heavy ion beams. In this report, I discuss a project dealing with reactions leading to continuum energy spectra. The emphasis in this project is on understanding the reaction mechanism quantitatively in terms of quantum mechanical treatments in a microscopic picture. Theoretical analyses are made using the DWBA theory together with the shell model. The study is being carried out in collaboration with Udagawa and Tamura at University of Texas at Austin.

Because an attempt such as this involves new and often daring approaches, we try to perform different experimental measurements to which a common theoretical model is applied. I present here examples of three experimental measurements which illustrate our program: a study of alpha-transfer reactions to highly excited continuum regions, a study of projectile breakup processes, and a study of spin-polarization measurements. While these investigations are aimed at bettering our understanding of the reaction mechanism, we are also seeking heavy-ion reactions which will provide us with direct access to spectroscopic information. An example of this, which I describe, is a transfer reaction leading to nuclear molecular resonances.

In this project, we concentrate our attention on reactions with
direct reaction mechanism. Therefore, we first select the reactions which demonstrate proper characteristics of the direct process; that is, we make our approach from direct channels among numerous outgoing channels produced in heavy-ion reactions. Various outgoing channels from an incident channel $^{232}$Th $+ ^{20}$Ne at 261 MeV incident energy provide an example. In Fig. 1, some of the energy spectra recorded simultaneously with a solid-state counter telescope are displayed. Although one can extract interesting features from the investigation, itself, of these spectra, I will only offer here a few remarks relevant to the present study.

First, it should be noted that these reactions all show continuum energy spectra except in a few channels, such as the elastic-inelastic channel of $^{20}$Ne. (In this particular experiment, the energy resolution was quite large; thus low lying states do not appear to be separated, though intrinsically they are.) In Fig. 2 these reactions are summarized in terms of the energy integrated cross sections versus the reaction Q-values, in particular $Q_{yy}$'s (the Q-values for both outgoing nuclei in their ground states).

It should be remembered that the cross section for any reaction can be expressed by two factors: a reaction mechanism, and a level density with proper strengths reflecting structures involved in the projectile and the residual nuclei. If the reaction considered is very complicated, such as a compound process, the reaction mechanism will vary smoothly among neighboring isotopes and the cross section will be determined mainly by the level density. The level density, in general, has an exponential dependence on the excitation energy, thus on $Q_{yy}$. We may thus expect the linear variation of cross sections for isotopes of a given Z if processes are complicated (note that the cross section scale in Fig. 2
is in logarithmic). The lithium isotopes, for example, show such a feature, indicating the complicated reaction mechanisms involved. On the other hand, if deviations are observed, the reaction mechanisms must involve individual structure effects, and thus are not likely to be compound-type processes.

A consideration of the characteristic feature of the heavy-ion reaction dynamics can provide insights as to the observation of continuum energy spectra, as seen in the spectra in Fig. 1. A most important characteristic in the heavy-ion reaction is the well known "Q-value and transferred angular momentum effect," which is essentially a consequence of the inherent short wave length and strong absorption in nucleus-nucleus collisions. This effect produces a spin selectivity, such that for a given Q-value transition strengths are strongly concentrated in a narrow interval of residual spin values. The cross section is determined by the availability of residual states with the spins so selected; instead of the spectroscopic factor in the ordinary transition to a single state, it is the level density weighted by spectroscopic strengths which we call "spectroscopic density $\rho_s(E_x, I)$," where $E_x$ and $I$ are the excitation energy and spin, respectively.

To illustrate this effect, four types of reactions are considered. In Fig. 3, actual energy spectra of these reactions are displayed on the left side, and the relationships between the spin selectivities and the residual level distributions are shown on the right. Here the spectroscopic strengths associated with each state are not taken into account. The expected enhancements caused by the spin selectivities are displayed by the solid lines as function of $E_x$ and $I$. Note that they may be called
"entry lines," as is often done in the high-spin studies. The states in
the residual nuclei are indicated by dots for the known levels and by
shaded area for many levels expected. In addition, the yrast lines are
indicated by the dashed lines.

In the first reaction, \(^{12}\text{C}(^{14}\text{N}, ^{12}\text{C})^{14}\text{N}\), the spin selectivity lies
just below the yrast line. Thus, one should expect enhanced transitions
to high-spin states in a wide range of \(E_x\), together with underlying
lower-spin states. The spectrum, indeed, shows this, with many peaks
_corresponding to high-spin states above the continuum. In contrast, the
simultaneous measurement of the second reaction, \(^{12}\text{C}(^{14}\text{N}, ^{12}\text{B})^{14}\text{O}\), shows
extremely small yield in the entire energy range, indicating a typical
mismatched case, as seen in the right figure. The third reaction, \(^{12}\text{C}(^{10}\text{B}, ^{7}\text{Li})^{15}\text{O}\), provides an interesting case. The spin selectivity
shows a mismatched condition in the low \(E_x\) region, resulting in almost
no transitions, as seen in the actual spectrum. It then crosses the
yrast line, and indeed strong transitions to the \(11^+/2\) and \(13^+/2\) states
are then observed. In the higher \(E_x\) region, the transitions favored by
the selectivity go under the yrast line and populate many overlapping
states with spins lower than the yrast values, thus yielding continuum.

When we use heavy targets, the situation is again different. The
last example, \(^{58}\text{Ni}(^{14}\text{N}, ^{10}\text{B})^{62}\text{Zn}\), shown in Fig. 3 illustrates such a
dynamic condition. The selected transitions always lie below the yrast
line (here the level distribution is simply taken under the estimated
yrast line of the rigid rotator), hence the process populates many
overlapping states in the entire energy region. According to this
argument, the observation of continuum spectra in heavy-ion reactions is
understood to be a consequence of the interplay between the inherent
characteristics of the reaction dynamics and the spectroscopic density

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reflecting the structure of the residual nucleus.

It is worth noting that even in reactions using heavier projectiles at higher bombarding energies, this relation generally remains, and one should expect dominance of continuum spectra. Considering the next generation of heavy-ion machines, it is important to understand thoroughly the continuum spectra. The spin selectivity provides a powerful way to study individual high-spin states. For instance, the reaction $^{12}_c(^{10}_B, ^7_{Li})^{15}_{O}$, mentioned above, demonstrates a very clear filtering of low spin states at such high excitation region, i.e. $E_x \approx 12-15$ MeV, thus enabling us to identify the $11^+ / 2$ and $13^+ / 2$ states. In fact, we have carried out a number of spectroscopic investigations of light nuclei. Although it is not the subject of this paper, I would like to stress that I believe this is one of the most interesting and fruitful uses of the heavy-ion reaction, and I expect increased attention to be paid to it in the future.

1. Alpha Transfer Reactions with Large Energy Losses

We selected three alpha transfer reactions, $(^{20}_{Ne}, ^{16}_{O})$, $(^{14}_{N}, ^{10}_{B})$, and $(^{12}_{C}, ^9_{Be})$ on $^{40}_{Ca}$. These projectiles have relatively large alpha spectroscopic factors. The bombarding energies were chosen at 262, 153, and 149 MeV for $^{20}_{Ne}$, $^{14}_{N}$, and $^{12}_{C}$, respectively, which all correspond to $-7$ MeV/amu above the Coulomb barriers. Energy spectra at relatively forward angles are displayed in Fig. 4. They all show continuum spectra as expected. Transition strengths are distributed in wide energy ranges, and maximum cross sections are at energies quite low compared to the incident energies; that is, they display large energy losses. The energy-integrated angular distributions are shown in Fig. 5.

To these experimental data, we apply theoretical analyses based on
the exact finite-range (EFR) DWBA theory. Because these analyses have been detailed elsewhere,\textsuperscript{4,5} only a brief outline is given here. We consider the process in terms of one-step direct transfer to many overlapping alpha-like states in \textsuperscript{44}Ti. The cross section is then written as

\[
\frac{d^2\sigma}{dE\,dQ} = \rho_S(E_x, I) \cdot \frac{d\sigma}{d\Omega} \text{DWBA}
\]

This DWBA calculation exhibits many complications.

First, the calculation of the form factors is not trivial, since it deals with highly excited unbound residual states. After careful study, it was found that because of the strong localization associated with the heavy-ion reaction, one can effectively use a real function peaked at the surface region. The actual evaluation of spectra poses tremendous complications, since there are a huge number of overlap integrals involved with a number of variables, such as the incident, outgoing and transferred angular momenta, and the Q-value; typically, a calculation of one spectrum requires several million overlap integrals. In order to make the calculation feasible, an analytic formula of the DWBA cross section, which contains several parameters, is derived. To determine these parameters, samples of several thousand overlap integrals are calculated with the EFR-DWBA, and a $\chi^2$-fitting procedure is used.

The spectroscopic density $\rho_S(E_x, I)$ also presents a problem, since it deals with highly excited and high-spin cluster strength. Several methods were investigated, and it was found that a use of the SU$_3$ shell model reproduces the experimental results. We consistently use the same $\rho_S(E_x, I)$ for all these three reactions.
The calculated results are displayed in Fig. 4 by the solid curves. The theoretical results fit the shapes of the spectra very well for the \( ^{14}\text{N},^{10}\text{B} \) and \( ^{13}\text{C},^{9}\text{Be} \) reactions. For the \( ^{20}\text{Ne} \) induced reaction, however, the theoretical results agree with the experimental spectra only at backward angles, and large discrepancies are seen at forward angle spectra, as shown by the dashed curves and shaded areas. This disagreement is the subject of the next sections. The theoretical spectra were mutually normalized in each reaction; thus, the shape of the angular distribution is properly calculated. The results are displayed in Fig. 5, and again the calculation properly reproduced the experimental results except for the forward angle region of the \( ^{20}\text{Ne},^{16}\text{O} \) reaction.

It may seem surprising that a simple direct transfer mechanism reproduces all these seemingly complicated processes with large energy losses. It should be emphasized that the present analysis is based on a fully quantum mechanical treatment and a microscopic picture, without the introduction of any phenomenological assumptions.

II. Projectile Breakup Process in the \( ^{20}\text{Ne},^{16}\text{O} \) Reaction

As mentioned in Section I, the transfer calculation could not explain the spectra at forward angles of the \( ^{20}\text{N},^{16}\text{O} \) reaction. It was thus speculated that the unexplained extra part must correspond to a different process. Accordingly, we considered a direct breakup of the projectile \( ^{20}\text{Ne} \).

In order to examine such a scheme, we performed additional experiments: measurements of the same \( ^{40}\text{Ca},^{20}\text{Ne},^{16}\text{O} \) \( ^{44}\text{Ti} \) reaction at 200 MeV and 149 MeV, and the \( ^{20}\text{Ne},^{16}\text{O} \) and \( ^{14}\text{N},^{10}\text{B} \) reactions on \( ^{58}\text{Ni} \) and \( ^{94}\text{Mo} \) targets.
The result indicates consistent features of the projectile breakup: 1) this extra component becomes larger at higher bombarding energies, and 2) it also appears in the \(^{20}\text{Ne},^{16}\text{O}\) reaction on \(^{58}\text{Ni}\) and \(^{94}\text{Mo}\), while the \(^{14}\text{N},^{16}\text{O}\) reaction on them were explained simply by the transfer processes.

A theoretical analysis was carried out assuming a direct inelastic process: that is, the \(^{40}\text{Ca} + ^{20}\text{Ne} \rightarrow ^{40}\text{Ca} + ^{20}\text{Ne} + ^{16}\text{O} + \alpha\) process. The calculation is made in terms of the ordinary DWBA formalism in which the interactions are initially taken between \(^{16}\text{O}\) and \(^{40}\text{Ca}\), and \(\alpha\) and \(^{40}\text{Ca}\).

Those interactions are transformed to the interaction between the center of mass of the \(^{16}\text{O} + \alpha\) system and \(^{40}\text{Ca}\), and the interaction between \(^{16}\text{O}\) and \(\alpha\). The former interaction causes the inelastic excitation of \(^{20}\text{Ne}\) to the \(^{16}\text{O} + \alpha\) system, and the relative scattering state of \(^{16}\text{O}\) and \(\alpha\) is constructed using the latter interaction. The calculation indicates interesting features. For instance, strong cross section is attributed to the \(l = 0\) component of the \(^{16}\text{O} + \alpha\) system, and contributions from higher \(l\) components become progressively smaller. In terms of the excitation energy in the \(^{20}\text{Ne}\) system, strong transitions are concentrated at \(-4\) to \(-5\) MeV above the alpha particle threshold.

The results are shown in Fig. 6 for the \(^{40}\text{Ca}(^{20}\text{Ne},^{16}\text{O})\) at 262 and 149 MeV. The dotted curves show the theoretical results of the transfer process, and the dashed curves correspond to the breakup process. The final spectra, constructed by summing these two processes, are shown by the solid curves. As in the transfer calculations, the cross sections are mutually normalized to obtain the proper shape of the angular distributions. The result clearly indicates that the model reproduces these inclusive spectra. Therefore, the reaction \(^{20}\text{Ne},^{16}\text{O}\) can be understood.
in terms of the transfer and breakup processes.

At this stage, questions arise concerning the reactions induced by $^{14}\text{N}$ and $^{13}\text{C}$, where the transfer scheme without the breakup process apparently explains the whole spectra. There are two crucial factors which make a difference when the $^{14}\text{N}$ and $^{13}\text{C}$ reactions are compared to the $^{20}\text{Ne}$ reaction. It is well known that $^{20}\text{Ne}$ is one of the most alpha-like nuclei; in fact, the alpha breakup threshold is the lowest particle channel in $^{20}\text{Ne}$. In $^{14}\text{N}$ and $^{13}\text{C}$ nuclei, the other channels open at lower energies. Therefore, even if the inelastic excitation has taken place equally, the breakup must favor other channels. Another factor is the difference between the internal structures of these projectiles. The alpha cluster in $^{20}\text{Ne}$ is in the $I=0$ state around the core, while in $^{14}\text{N}$ and $^{13}\text{C}$, it is dominantly in the $I=4$ and $2$ states, respectively. As noted, the breakup strength in $^{20}\text{Ne}$ lies mainly in $I=0$ excited states at relatively low excitation region. For $^{14}\text{N}$ and $^{13}\text{C}$, excitation to $I=0$ scattering states at low energy should be depressed because of these ground state configurations.

In conclusion, we believe that the model has explained all inclusive data. Further tests must be made, especially studies of correlation experiments, which will be more conclusive. In fact, we have carried out such an investigation of the $\alpha$ and $^{16}\text{O}$ correlation experiment, and a preliminary finding shows that the theoretical prediction is indeed in agreement with the data.

III. Spin Polarization Measurements

Sugimoto et al. have applied the well established technique of spin polarization detection of $^{12}\text{B}$ to heavy-ion reaction studies. A
determination of the spin polarization \( P \) of an ejectile can be crucial in the understanding of the reaction mechanism. For instance, a classical model with the concept of frictional force will predict the spin polarization. The prediction is schematically shown in Fig. 7a. Since the frictional force on the projectile-ejectile system always acts in the opposite direction of \( k_1 \), the direction of spin of the ejectile is either counterclockwise (1) or clockwise (2), depending on whether its scattering path goes through the near-side or far-side of the target nucleus. Therefore, the spin polarization is predicted to be negative for the quasielastic process and positive for the deep inelastic process, since the polarization axis is defined to be \( \mathbf{k}_1 \times \mathbf{k}_f \).

In experimental measurements in the reaction \(^{100}\text{Mo}(^{14}\text{N},^{12}\text{B})\), Sugimoto et al. found the spin polarization to be opposite of that predicted by this classical model. \( P \) starts large positive at the highest energy end of the \(^{12}\text{B}\) spectrum, then decreases rapidly towards the lower energy region. Ishihara et al.\(^9\) accordingly, made a theoretical interpretation using a semiclassical treatment. The process is assumed to be the direct two-proton transfer, and qualitative agreement with the observed data was obtained.

A crude but physical explanation of the transfer model can be made schematically, as illustrated in Fig. 7b. For simplicity, we consider the process only on the reaction plane. One can then see that there are four different modes of the transfer, each dependent on various combinations of \( m \) component of the transferred cluster (the \( m \)-axis is taken to be \( \mathbf{k}_1 \times \mathbf{k}_f \)). For instance in (1), the cluster is in the negative \( m \) state in the projectile and transfers to the positive \( m \) state in the residual nucleus. Since in the projectile the core and the transferred cluster...
are coupled to zero spin, the $m$ of the core must be opposite to that of the cluster. The ejectile is, in fact, the core; hence, the spin polarization of the ejectile in the mode (1) is positive. The mode (2) corresponds to the residual cluster in the positive $m$ state. The modes (3) and (4) correspond to the combinations of the opposite $m$ state to those in (1) and (2). The paths in the modes (2) and (4) may appear strange because they show discontinuity, but one must remember the addition of the momentum caused by the relative motion of the colliding nuclei.

These four modes compete in the process, and if we examine the actual case, the first two modes, especially (2), dominate at the highest energy end of the spectrum. The rapid decrease of $P$ towards lower energy is also understood as the interplay of these modes. A more quantitative theoretical analysis, which is the same DWBA theory described above in (1), was carried out by Udagawa and Tamura, and the observed experimental results were reproduced.

We have extended this study as one of our projects to investigate the reactions leading to continuum spectra. As the first step, we considered it important to establish the characteristic of the spin polarization which we observed in the $^{14}$N, $^{12}$B reaction as a signature of the direct transfer process. Therefore, the reaction $^{232}$Th($^{13}$C, $^{12}$B) at 149 MeV was first investigated, because it can safely be assumed to be a proton transfer process. The result is shown in Fig. 8, where the energy spectrum is displayed on the top and the polarization on the bottom. The energy spectrum shows a typical quasielastic spectrum enhancing the high energy end. The polarization does show the same characteristic feature we observed in the $^{14}$N, $^{12}$A reaction discussed above. The theoretical analysis is shown by the solid curves in Fig. 8,
and the result confirms the argument described above.

The measurement is then extended to a more exotic reaction, $^{197}\text{Au}(^{19}\text{F},^{12}\text{B})$ at 189 MeV, and the result is shown in Fig. 9. To our surprise, the polarization shows again the same feature, although it involves a transfer of seven nucleons with large energy loss, as seen in the energy spectrum. The theoretical analysis indicated by the solid curves reproduces both the energy spectrum and the polarization in the high energy region. One must, therefore, conclude that despite its apparent complexity, the process is dominated by the simple transfer mechanism. The theoretical prediction, however, deviates from the experimental results in the low energy region (shown by the dashed curve), thus indicating the involvement of more complicated processes. If we attempt a pure phenomenological treatment such that this extra part of the spectrum causes no polarization, the polarization is corrected, as shown by the dash-dot curve which reproduces the observed result. Since such a treatment is purely phenomenological, the results are not conclusive, though they remain suggestive.

From these polarization studies, we have again seen the direct transfer process play a dominant role in these heavy-ion reactions, and the present theoretical analyses quantitatively explain the experimental results. I might add that our present and future program is to continue the same collaborative investigations of theory and experiment, extending various types of reactions and higher bombarding energies to find the validity and limitation of the simple direct mechanism.

IV. Reaction $^{12}\text{C}(^{16}\text{O},\pi)^{24}\text{Mg}$ Leading to the Molecular Resonances$^{11}$

An experimental study which indicates the possibility of many
fruitful uses of heavy-ion reactions in spectroscopic studies of exotic structure is presented here. The reaction $^{12}_C(^{16}O,\alpha)^{24}Mg$ was investigated using an Enge split-pole magnetic spectrograph. The results are summarized in Fig. 10. Total energy spectra observed at three angles are shown with the logarithmic vertical scale. The yields for these spectra are relatively normalized. Laboratory energies are given on the horizontal axis, while excitation energies in the $^{24}Mg$ system are displayed above the spectra.

Several intriguing features can be ascertained from these spectra. The spectra at 7° and 15° show irregular structures above smooth continuum in the low energy region, in addition to weak but definite peaks corresponding to the bound states of $^{24}Mg$. In order to see these structures more clearly, we subtracted smooth spectra, indicated by the solid curves. (We operationally call them background spectra, although we recognize that to categorize this way, without understanding the nature of the continuum, can be quite dangerous.) The resulting spectra are displayed below the total energy spectra. These broad peaks show a striking correlation with the well known molecular resonance states in the $^{12}C + ^{12}C$ system, as labeled by the spin-parities. We also measured the corresponding spectra from the reaction $^{13}C(^{16}O,\alpha)^{25}Mg$ and found only smooth spectra without such structures. Since resonance-like structures have not been observed in the $^{12}C + ^{13}C$ system, this contrast gives additional evidence for the fact that the present reaction populates the molecular resonance states.

In the absence of quantitative theoretical understanding of the reaction mechanism, we can only state that the data is highly suggestive of a direct transfer. It should be remembered that experimental measurements
of the molecular states have so far been restricted to observations of excitation functions for various outgoing channels induced by the $^{12}\text{C} + ^{12}\text{C}$ incident channel. Therefore, the present reaction provides an entirely new and more versatile way to study such exotic structures. It is also interesting to investigate the transitions to low lying states of $^{24}\text{Mg}$. If the reaction is the $^{12}\text{C}$ transfer, these transitions would indicate direct overlaps of the $^{12}\text{C} + ^{12}\text{C}$ configuration to the normal $^{24}\text{Mg}$ low lying states. This reaction may even become useful for astrophysical investigations, once the reaction mechanism is quantitatively understood. All the possibilities implicit in our findings suggest that our study may be analogous with the event which resulted in the recognition of the use of the (d,p) reaction for spectroscopy instead of the n-capture reaction.

I wish to thank all my colleagues. Theoretical work has been carried out entirely by T. Udagawa and T. Tamura. M. Ishihara initiated the spin-polarization experiment, as well as provided much of the initial motivation in the study of the continuum spectra.

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References

1. Summary of the work at Texas A&M was given in K. Nagatani,
2. K. G. Nair, C. R. Wotsley, R. Hanus, M. Hamm, and K. Nagatani,
3. K. Nagatani, D. H. Youngblood, R. A. Kenefick, and J. D. Bronson,
4. H. Fröhlich, T. Shimoda, M. Ishihara, K. Nagatani, T. Udagawa,
   (Hakone), p. 3 (1977).
6. T. Udagawa, T. Tamura, T. Shimoda, H. Fröhlich, M. Ishihara, and
7. M. Ishihara, T. Shimoda, H. Fröhlich, H. Kamitsubo, K. Nagatani,
8. K. Sugimoto, M. Tanaka, A. Mizobuchi, Y. Nojiri, T. Minamisomo,
   M. Ishihara, K. Tanaka, and H. Kamitsubo, Phys. Rev. Lett. 39,
11. K. Nagatani, T. Shimoda, D. Tanner, R. Tribble, and T. Yamaya,
    to be published.
12. T. M. Cormier, J. Applegate, G. M. Berkowitz, P. Brun-Munzingen,
    18, 940 (1977) and references therein.
Figure Captions

Fig. 1 Energy spectra from the bombardment of 261 MeV \(^{20}\)Ne on \(^{232}\)Th taken simultaneously by a solid-state counter telescope at 35° are shown.

Fig. 2 Energy integrated cross sections for the reactions shown in Fig. 1 are displayed; cross sections are plotted in the logarithmic vertical scale versus \(Q_{yy}\). The lines indicate \(e^{-Q_{yy}/T}\) with \(T = 3.0\) MeV.

Fig. 3 On the left, the energy spectra of the reactions indicated in the corresponding right figure are shown. Figures on the right illustrate the level distributions (dots for the known states and shaded areas for the expected states), the yrast lines (dashed curves), and the spin selectivities (solid lines) in terms of the excitation energies \(E_x\) and spins \(I\) in the residual nucleus.

Fig. 4 Energy spectra of the reactions indicated are displayed at labeled angles. The solid curves are the results of the transfer calculations. The theoretical spectra are normalized to the experimental cross section at one angle for each reaction. The dashed curves and shaded areas indicate the extra component unexplained by the transfer scheme.

Fig. 5 Energy integrated cross sections (dots) are compared to the theoretical results (solid curves).

Fig. 6 Energy spectra are shown with the theoretical results; the dotted curves display the transfer spectra, the dashed curves show the projectile breakup spectra, and the total spectra are shown by the solid curves.
Fig. 7 Schematic illustration of the spin polarization is shown. Figure (a) shows the classical model with the frictional force. The projectile (P) and the target (T) interact causing the frictional force (dashed line) to rotate the ejectile in the direction indicated by the arrow, resulting in the positive P for the near-side path (1) and the negative P for the far-side path (2). Figure (2) shows the transfer scheme (only for the near-side path, i.e. quasielastic process). The classical paths of the transferred cluster are indicated. The core, which is the ejectile $^{12}$B, is labeled by C with the polarization as shown. Restricting the transfer only on the reaction plane, these four different modes compete as discussed in the text.

Fig. 8 Energy spectrum and spin polarization of the reaction $^{13}$C, $^{12}$B on $^{232}$Th at 149 MeV incident energy measured at 25° are shown in (a) and (b), respectively. The solid curves show the theoretical spectra.

Fig. 9 Energy spectrum and the spin polarization from the reaction $^{197}$Au, $^{19}$F, $^{12}$B at 189 MeV measured at 25° are shown in (a) and (b), respectively. The histogram shows an energy spectrum using a solid-state counter in a separate experiment. The solid curves show the calculated results; the dashed curve indicates the difference between the experimental and theoretical energy spectra. The dash-dot curve shows the corrected polarization, assuming zero polarization for the component (dashed curve) unexplained by the present theory.
Energy spectra of the reaction $^{12}$C($^{16}$O,$\alpha$)$^{24}$Mg taken by an Enge split-pole magnetic spectrograph are shown; the total spectra are displayed in the logarithmic vertical scale, while the background (smooth curves just below the total spectra) subtracted spectra are displayed in the linear scale on the bottom. The peaks are labeled by excitation energies in $^{24}$Mg, the spin-parities for the molecular resonance are from ref. 12.
Figure 1

$^6\text{Li}$, $^7\text{Li}$, $^6\text{Li}$, $^9\text{Li}$

$^7\text{Be}$, $^9\text{Be}$, $^{10}\text{Be}$

$^{20}\text{Ne} + ^{232}\text{Th}$

261 MeV

$\theta_{\text{lab}} = 35^\circ$

$d^2\sigma/d\Omega \, dE_{\text{lab}}$ (mb/sr MeV)
Figure 3
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\[ \frac{d^2\sigma}{dE_{\text{lab}}} \] (mb/MeV-sr)

\begin{align*}
\text{40Ca}^{20}\text{Ne},^{16}\text{O} & \quad \text{44Ti} \\
262 \text{ MeV} & \\
\text{120} & \quad \text{100} & \quad \text{80} & \quad \text{60} & \quad \text{40} & \quad \text{20} & \quad \text{0} \\
5^\circ & \quad & \quad \cdot & \quad \cdot & \quad \cdot & \quad \cdot & \quad \cdot & \quad \cdot & \quad \cdot \\
6^\circ & \quad & \quad \cdot & \quad \cdot & \quad \cdot & \quad \cdot & \quad \cdot & \quad \cdot & \quad \cdot \\
7^\circ & \quad & \quad \cdot & \quad \cdot & \quad \cdot & \quad \cdot & \quad \cdot & \quad \cdot & \quad \cdot \\
8^\circ & \quad & \quad \cdot & \quad \cdot & \quad \cdot & \quad \cdot & \quad \cdot & \quad \cdot & \quad \cdot \\
9^\circ & \quad & \quad \cdot & \quad \cdot & \quad \cdot & \quad \cdot & \quad \cdot & \quad \cdot & \quad \cdot \\
10^\circ & \quad & \quad \cdot & \quad \cdot & \quad \cdot & \quad \cdot & \quad \cdot & \quad \cdot & \quad \cdot \\
11^\circ & \quad & \quad \cdot & \quad \cdot & \quad \cdot & \quad \cdot & \quad \cdot & \quad \cdot & \quad \cdot \\
12^\circ & \quad & \quad \cdot & \quad \cdot & \quad \cdot & \quad \cdot & \quad \cdot & \quad \cdot & \quad \cdot \\
13^\circ & \quad & \quad \cdot & \quad \cdot & \quad \cdot & \quad \cdot & \quad \cdot & \quad \cdot & \quad \cdot \\
14^\circ & \quad & \quad \cdot & \quad \cdot & \quad \cdot & \quad \cdot & \quad \cdot & \quad \cdot & \quad \cdot \\
15^\circ & \quad & \quad \cdot & \quad \cdot & \quad \cdot & \quad \cdot & \quad \cdot & \quad \cdot & \quad \cdot \\
16^\circ & \quad & \quad \cdot & \quad \cdot & \quad \cdot & \quad \cdot & \quad \cdot & \quad \cdot & \quad \cdot \\

\text{E}_x \text{ (MeV)} & \quad \text{E}_{\text{lab}} \text{ (MeV)}
\end{align*}

\[ \text{40Ca}^{14}\text{N},^{10}\text{BF}_{4}^{44}\text{Ti} \]

153 \text{ MeV}

\[ \text{40Ca}^{35.9}\text{C},^{16}\text{Be}^{44}\text{Ti} \]

149 \text{ MeV}

Figure 4
$^{40}\text{Ca} (^{20}\text{Ne}, ^{16}\text{O}) ^{44}\text{Ti}$

262 MeV

$^{40}\text{Ca} (^{14}\text{N}, ^{10}\text{B}) ^{44}\text{Ti}$

153 MeV

$^{40}\text{Ca} (^{13}\text{C}, ^{9}\text{Be}) ^{44}\text{Ti}$

149 MeV

$\frac{d\sigma}{d\Omega}\Bigg|_{\text{lab}}$
Figure 6

\[ ^{40}\text{Ca}^{20}\text{Ne}_{6}^{16}\text{O} \rightarrow ^{44}\text{Ti} \]

262 MeV

149 MeV

\( \frac{d^2\sigma}{dE\,d\Omega} \) (mb / MeV \( \cdot \) sr)

\( E_{\text{lab}} \) (MeV)

\( E_{\text{x}} \) (MeV)
Figure 7
$^{232}\text{Th}(^{13}\text{C},^{12}\text{B})^{233}\text{Pa}$

(a)

(b)

Figure 8
Figure 9
ON THE DISTINCTION BETWEEN DIRECT AND STATISTICAL CONTRIBUTIONS TO PROJECTILE-FRAGMENT SPECTRA

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ABSTRACT

Collisions of $^{16}_6$O with heavy targets have recently been investigated at incident energies in the range 143 to 315 MeV, from which spectra $\sigma(E')$ of outgoing fragments have been obtained (1). At angles larger than the grazing angle $\sigma(E')$ is found to have two peaks which clearly correspond to "quasi elastic" and "deeply inelastic" events, but at the grazing angle only a simple peak occurs, making it difficult to distinguish these two reaction mechanisms from one another in $\sigma(E')$ itself. We have found empirically, however, that the quantity (2) $I(E') = \frac{\sigma(E')/\sigma_{\text{phase space}(E')}}{\text{phase space}(E')}$ appears to be more sensitive to this distinction. Even in the case of "overlapping" quasi-elastic and deep-inelastic components, $I(E')$ has a structure consisting of two linear components, which we suggest may be associated with these two mechanisms. $I(E')$ seems to sharpen the distinction between the two processes, and thus might provide a useful form for displaying the data.

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The analysis of data by means of information theory, standard in atomic collisions (2), has recently been introduced in nuclear reactions (3) and (4). Fundamentally the method rests upon introducing an a priori probability as a statistical assumption and adding additional information as available, subject to the constraint of minimal gain of information with respect to the a priori distribution.

Information theoretical analyses of data have pointed to the conclusion that while the final phase space is not uniformly filled, the constraints leading to the non-statistical final state distribution are simple and can often be determined directly from the data.

This constraint is one of the two basic ingredient of the theory allowing one to use physical intuition to characterize the reaction in terms of a few selected features. The other ingredient is the prior distribution which would describe the reaction if the distribution of final states were purely statistical, that is, if dynamics played no role in selecting the final quantum states.

In the reaction analyzed in (2), for example, \(^{232}_{\text{Th}}(^{16}_N, X) X\) being \(^{16}_N, ^{15}_N, ^{14}_C, ^{13}_C, ^{12}_C\) or \(^{11}_B\), the distribution of final states was obtained by assuming maximal entropy subject to a single kinematical constraint on the excitation energy with a prior distribution obtained from the Fermi gas model. Minimizing the gain of information \((I = -\ln \frac{\sigma}{\sigma_0})\) subject to this constraint yields,

\[
- \ln \frac{\sigma}{\sigma_0} = \lambda U + \lambda_0
\]

where \(U = E' - E_{\text{rot}} - E_{\text{pair}}\), \(E'\) being the excitation energy, \(E_{\text{rot}}\) and \(E_{\text{pair}}\) the rotational and pairing energies, respectively. \(\lambda_0\) insures the
The success of this description of transfer reactions seems to indicate that they can be characterized mainly by the amount of energy transferred during the process and that the spectra are well predicted by the density of states of the residual nucleus. This is what makes it meaningful, we believe, to apply a statistical method in order to describe the reaction.

Another kind of nuclear reaction involving heavy ions which can be described by statistical theories is the deeply inelastic collision (5). Naturally, we expect that the average energy transferred in a reaction of this type should be larger than in the case of direct reactions, for example, transfer or breakup. If deeply inelastic collisions can also be characterized by a single constraint on the average excitation energy, than an information theoretical analysis of spectra could provide a useful means of distinguishing between the contributions from direct reaction mechanisms and the so-called "statistical" processes. As pointed out in the abstract, this distinction could be difficult to make at angles near the grazing angle.

In order to investigate the plausibility of this idea, we have analyzed some of the spectra measured by Steadman et al. (6). We constructed the quantity

$$I(E') = - \ln \frac{\sigma(E')}{\sigma_o(E')}$$

where $\sigma(E')$ is the experimental cross section, $\sigma_o$ the "prior" distribution (Fermi gas model) (7)

$$\sigma_o = \frac{1}{4\pi} \exp \sqrt{\frac{U}{\hbar^2}}$$
with \( a = \frac{A}{5} \) where \( A \) is the mass number of the residual nucleus, and \( \theta \) is as defined previously.

We plotted this quantity \( I(E') \) as a function of the excitation energy \( E' \) for three different reactions as shown in Fig. 1, 2 and 3. We found that \( I(E') \) has a structure consisting of two linear components which we suggest may be associated with these two mechanisms. The component with the larger slope \( \lambda \) and corresponding smaller excitation can be associated with the quasi-elastic reactions while the component with smaller slope and larger average excitation energy corresponds to deeply inelastic reactions. The change in slope clearly displays the change in mechanism.

The density of states used varies many orders of magnitude over a typical range of excitation energy. This makes it difficult to detect variations in the ratio of the experimental cross section to the phase space, which makes it hard to draw definite conclusions concerning the usefulness of the surprisal analysis, as pointed out by W. Friedman et al. (8). Conclusive evidence requires cross sections which fall by many orders of magnitude.
REFERENCES

(6) S. G. Steadman et al., to be published.
(8) W. Friedman, M. S. Hussein and M. C. Nemes, unpublished.
FIGURE CAPTIONS

Fig. 1a Cross section as a function of excitation energy for the reaction $^{232}$Th($^{16}$O, $^{15}$N) at $E_{lab} = 105$ MeV.

1b Plot of the surprisal as a function of excitation energy. The two straight lines are fits to the data in the direct and "deep inelastic" regions. We have plotted $\log \sigma_0$ (normalized) as well to show to what extent the surprisal deviates from phase space alone.

Fig. 2a Cross section as a function of excitation energy for the reaction $^{197}$Au($^{16}$O, $^{15}$N) with $E_{lab} = 218$ MeV (data from Ref. (6)).

2b Surprisal versus excitations for the reaction in Fig. 2a. The straight lines are fits to the data in the direct and "deep inelastic" regions.

Fig. 3a Cross section as a function of excitation energy for the reaction $^{197}$Au($^{16}$O, $^{15}$N) with $E_{lab} = 250$ MeV (data from Ref. (6)).

3b Surprisal versus excitation energy for the reaction in Fig. 3a. The two straight lines are fits to the data in the direct and "deep inelastic" regions.
Figure 1a

$^{232}\text{Th} (^{16}\text{O}, ^{15}\text{N})$

$E_{\text{LAB}} = 105 \text{ MeV}$
$^{197}\text{Au}(_{16}\text{O}, _{15}\text{N})$  
$E_{\text{LAB}} = 218 \text{ MeV}$
$^{197}\text{Au}(^{16}\text{O},^{15}\text{N})$

$E_{\text{LAB}} = 250 \text{ MeV}$

Figure 3a
Figure 3b

$^{197}\text{Au}(^{16}\text{O},^{15}\text{N})$

$E_{\text{LAB}} = 250$ MeV
Neutron Emission in Reactions Between $^{16}$O and $^{93}$Nb at 12.75 MeV/Nucleon*

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Measurements of neutrons coincident with various reaction products have been made for the reaction $^{16}$O + $^{93}$Nb at 204 MeV. Eight neutron detectors consisting of NE213 liquid scintillator in direct contact with phototubes were arranged at various angles and at distances of 70 cm around a thin-walled spherical aluminum chamber. Seven of the detectors were located in the reaction plane. The coincident reaction products were detected by means of two $\Delta E$-$E$ telescopes located inside the chamber at 22.5° with respect to the beam. In one telescope the $\Delta E$ element consisted of a silicon surface barrier detector, while in the other, a miniature $\Delta E$ gas ionization chamber was used. Each neutron detector was operated in conjunction with a thin NE102 plastic scintillator paddle located in front of it to identify energetic protons.

A CAMAC-based data acquisition system was used to record energies of charged reaction products, flight times of neutrons and the energy outputs of the phototubes. Pulse-shape discrimination was used to distinguish between fast neutrons and $\gamma$-events. Calibration measurements were made by means of a $^{252}$Cf source placed at the target position.

Energy spectra of neutrons in coincidence with projectile-like deeply inelastic products and with evaporation residue products were generated from the raw data. These spectra appear to have two components, a low energy component that can be described by a Boltzmann distribution with a 1.5-2 MeV temperature and a higher energy component consistent with a temperature of about 8 MeV. Average neutron multiplicities associated with projectile-like products were found to range from about 2 to 9 as the product energy decreased from 170 MeV to 60 MeV. The possible origin of the high-energy neutrons will be discussed in the context of various models.


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ANGULAR MOMENTUM TRANSFER IN INCOMPLETE FUSION REACTIONS

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Spectra of charged particles from 153-MeV $^{160}$O on $^{154}$Sm were measured from 10° to 160° in coincidence with characteristic lines from Er, Dy and Gd nuclei and with associated γ-ray multiplicity information. The energetic $^4$He and $^8$Be particles are strongly forward peaked. The transferred angular momentum deduced from $\langle H_Y \rangle$ for the Gd, Dy and Er products increases linearly with captured mass assuming that the captured species are $^4$He, $^8$Be, and $^{12}$C. These observations are interpreted in terms of projectile fragmentation and capture of part of the projectile by the target. Assuming that the incident angular momentum is divided between the fragments according to their masses, we deduce $\langle \zeta \rangle = 52, 60$ and 74 for $^{12}$C, $^8$Be and $^4$He capture, respectively. These are consistent with a model based on the concept of successive critical angular momenta for incomplete fusion.

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The Decay of a Hot Zone in Heavy
Ion Reactions

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We study the emission of nucleons from a localized hot zone possibly
formed in the early stage of heavy ion reaction at intermediate energies
(10 MeV/A to 100 MeV/A). We assume that the hot, possibly compressed, zone
is initially surrounded by cold nuclear medium of normal density. Furthe-
more we assume that the time scale for the motion of the zone inside the
cold medium is slow as compared to average motion of the emitted nucleons.
The compressed zone is characterized by two parameters, namely, the ratio
between the size of the zone \( R_s \) to the size of the system \( R \), \( \xi \)

\[ \xi = \frac{R_s}{R} \]

and the ratio \( \eta \) between the density of the zone \( \rho_s \) to the density of nuclear
matter \( \rho_0 \)

\[ \eta = \frac{\rho_s}{\rho_0} \]

The temperature of the zone (assumed to be locally equilibrated) is deter-
mined from Fermi gas considerations. Therefore, according to our model,
the nucleons in the hot spot have a Fermi distribution such that

\[ F(E_s) = \left[ \exp\left(\frac{E_s - u_s}{T_s}\right) + 1 \right]^{-1} \]

The nucleons are supposed to emerge from the hot spot and to be refracted
at the nuclear wall. We estimate the pre-emission time from the hot spot
\( \Delta t \) to be

\[ \Delta t = \frac{R - R_s}{\langle v \rangle} \]
Here \( \langle v \rangle \) is the average velocity of nucleons in the zone.

The differential multiplicity, \( M_s \), of the nucleons emerging from the hot spot during a \( \Delta t \) is given by

\[
M_s = \frac{d^2N}{dp_d\psi d\varphi_d} = \frac{4\pi R^2 \psi}{h \sin \theta} \frac{\Delta t\epsilon(E_s)}{A}
\]

The differential multiplicity of the nucleons emerging from the nucleus is given by

\[
M_n = M_s c(p_s - \frac{\sqrt{2mV_o}}{\cos \varphi})
\]

The step function \( c \) is introduced so as to allow only those nucleons which have sufficient kinetic energy in the direction normal to the surface of the nucleus to escape from a well of depth \( V_o \). The angle \( \alpha \) is defined in Fig. 1.

We now express the differential multiplicity \( \frac{d^2N}{dE_d} \) with respect to the laboratory frame (using conservation of mechanical energy) and obtain

\[
\frac{d^2N}{dE_d} = \frac{4\pi R^2 (E+V_o)^{2m_n}}{h^3V_o \sin \theta} \left( E + V_o \right) \left( E + V_o \right).
\]

Here \( \frac{30a}{\partial} \) is the partial derivative at constant laboratory momentum. In Figs. 2 and 3 we show the energy spectra \( \frac{dN}{dE} \) for the reaction \(^{197}\text{Au}(^{16}\text{O}, \text{nucleon})X\) at 315 MeV. In Fig. 2 the effect of compression for fixed size of the hot spot is shown. In Fig. 3 we show the effect on spectra of the size of the hot spot for a fixed compression. The parameter \( \xi \) determines the position of the peak of the spectrum while the parameter \( \eta \) determines the slope of the spectrum. In Fig. 4 the preliminary experimental data of Symons et al. have been shown and compared with our model with \( \xi = 0.5 \) and \( \eta = 1 \). As seen these parameters reproduce the data very well.
For energies of about 15 MeV/A it is expected that the data should be consistent with a large hot spot and small compressibility.

It is clear from our discussion that experiments at higher energies may shed light on the energy dependence of the size and compressibility of localized nuclear excitation.

References
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Figure Captions

Figure 1. The assumed geometry for the hot spot emission is displayed. The angle $\theta_s$ is the angle between the beam axis and the momentum of the emitted particle inside the nucleus. The angle $\theta$ is the angle between the beam axis and the momentum of the emitted particle outside the nucleus.

Figure 2. Energy spectra of nucleons as a function of kinetic energy of the nucleons for the reaction $^{16}O + ^{197}Au \rightarrow$ nucleons + X at 315 MeV incoming energy. Four theoretical curves are shown. The values of the free parameter were: solid line ($\eta = 1$, $\xi = 0.5$), dotted line ($\eta = 1.5$, $\xi = 0.5$) dashed line ($\eta = 2$, $\xi = 0.5$) and finally, dashed-dotted line ($\eta = 2.5$, $\xi = 0.5$). All curves are arbitrarily normalized to 100 at their maxima.

Figure 3. Energy spectra of nucleons as a function of kinetic energy of the nucleons for the reactions $^{16}O + ^{197}Au \rightarrow$ nucleons + X at 315 MeV incoming energy. Three theoretical curves are shown. The values of the free parameters were: solid line ($\eta = 2$, $\xi = 0.4$), dashed line ($\eta = 2$, $\xi = 0.5$) and finally dashed-dotted line ($\eta = 2$, $\xi = 0.6$). All curves are arbitrarily normalized to 100 at their maxima.

Figure 4. Energy spectra of nucleons as a function of kinetic energy of the protons for the reaction $^{16}O + ^{197}Au \rightarrow$ protons + X at 315 MeV incoming energy. One theoretical curve (solid line $\eta = 1$, $\xi = 0.5$) is drawn. The points are preliminary experimental data from D. Scott. The theoretical curve is arbitrarily normalized to the maximum value of the experimental data points.
Figure 3

Figure 4

ISO

Figure 3

Figure 4
A Study of Heavy-Ion Collisions in terms of Nuclear Hydrodynamics*

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I. Introduction

Among the most important motivations for recent interest in medium- and
high-energy heavy-ion reactions are the probing of the equation of state of nuclear
matter, the possible study of nuclear systems at high densities and temperatures
and their manifestations in exotic phenomena such as pion condensation and shape
isomers.† For many such investigations, nuclear hydrodynamics appears to pro-
vide an appropriate framework.

However, for the validity of nuclear hydrodynamics it is crucial that the in-
teraction time be long compared with the time necessary to bring the system into
local equilibrium. Head-on collisions involving large nuclei present the most
favorable cases where nuclear hydrodynamics may provide a valid description for
the reaction process. Accordingly, a critical test of nuclear hydrodynamics will
be the confrontation of theoretical predictions and experiments for head-on and

* This work was done in collaboration with Dr. C. Y. Wong of Oak Ridge National
Laboratory. Further details of this investigation will be reported in a forthcoming
article.

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The recent observation by Gutbrod and co-workers at Berkeley that many features of the heavy-ion collisions at intermediate energies may have hydrodynamical origins gives additional impetus to the treatment of heavy-ion reactions in terms of nuclear hydrodynamics. In their experiments, attention was focused on central collisions. Also a concept of centrality has been introduced by Wong to facilitate comparisons of theoretical and experimental results. In the present discussion, I shall report some of the results of hydrodynamical calculations for three-dimensional head-on collisions. The system of $^{20}_{\text{Ne}} + ^{197}_{\text{Au}}$ is chosen here with the hope that significant comparisons with recent experiments may be made in the near future. Further details of this investigation and the study of non-head-on collisions will be reported elsewhere.

II. Basic Equations and Parametrization

The connection between nuclear hydrodynamics and a more fundamental quantum many-body has been extensively studied by various authors. The basic dynamical variables are the density field $n$, the current density field $\mu$, and the thermal energy density field $\mu E_T$. The basic equations are the continuity equation:

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mu) = 0, \quad (1)$$

the Navier-Stokes equations:

$$\frac{\partial (n\mu)}{\partial t} + \sum_{j=1}^{3} \nabla_j [n\mu J_j] + \frac{1}{m} (p_{ij} + p_{ij}^+) = \frac{e}{m} \nabla_i \int d^3 \tilde{r} n(\tilde{r}, t) \nabla_j (\tilde{r}, \tilde{r}^+) \quad (2)$$

and the thermal energy density equation:
The parametrization of the equation of state and the long-range part $V_L$ of the nucleon-nucleon interaction has been given in detail in references 5,7 and 11.

The total pressure $p$, thermal pressure $p_T$ and the temperature field $T$ can be determined once the equation of state is given.

III. Results and Discussion

Eqs. (1) - (3) are solved to simulate the collisions of heavy-ions. The numerical techniques employed are the flux-corrected-transport method and the time-step-splitting method of Boris and Book 12 which allow a proper incorporation of the realistic features of the bulk properties of nuclei. 5,7

To study the effects of dissipation on the dynamics, calculations using two sets of transport coefficients are compared. They are

$$\eta, \zeta = 10^{-4} \text{ MeV/(fm}^2\text{-c)}$$  \hspace{1cm} (5a)$$

and $$\kappa = 10^{-4} \text{ c/fm}^2$$  \hspace{1cm} (5b)$$
which will be referred to as the "small viscosity" or non-viscous case in the text and

\[ \eta = 0.75 \text{ MeV/(fm}^2\text{-c)}, \quad (6a) \]
\[ \zeta = 18.76 \text{ MeV/(fm}^2\text{-c)} \quad (6b) \]
and \[ \kappa = 0.014 \text{ c/fm}^2 \quad (6c) \]

which are estimated from the Landau liquid theory and the width of the giant monopole resonance of $^{208}_{\text{Pb}}$, and will be referred to as the "large viscosity" or viscous case.

Figure 1 depicts the time evolution of the density field in the center-of-mass system for the collision. The plots shown are the cuts of the density contours in the reaction plane, graded in levels of 0.025 fm$^{-3}$. To standardize our notations in subsequent discussions, we take 0.15 fm$^{-3}$ to be the "normal value" $\rho_o$ of nuclear density which is designated by the contour label "6" in the figure.

The calculation shown in figure 1 is done with the small values of transport coefficients given in eqs. (5a) and (5b). The prominent features are the formation of a compressed region with a maximum density of 2.8 $\rho_o$ and sharp "edges" in the profile and the ejection of nuclear matter in a "side-wing" along a direction of about 120° in the center-of-mass system. After about 70 fm/c, the central density of the composite system drops back to the normal value and continues to decrease while the volume expands outwards irretrievably. We adopt the operational criteria that for such a case the calculation is to be terminated once the maximum density drops to below 50% of the normal value. The final result of this
reaction is interpreted as a complete dissociation into single nucleons. The knowledge of the density and velocity fields at the end of the calculation allows us to compute the angular and energy distributions of the outgoing particles.

Figures 2 and 3 are the angular distributions in the center-of-mass and laboratory systems, respectively. The underlying components of this distribution can be further analyzed by the energy distributions. The forward peak comes from slow particles of below 5 MeV in the laboratory system and can be easily understood as being due to the stopping of the nucleons by the heavy target. The backward peak (figure 4) comes from a group of particles of below 12 MeV and a smaller group of about 34 MeV. In the angular range of between 70° and 110°, the energy distribution (figure 5) has peaks at about 22 MeV and 38 MeV.

To study the effect of the viscosity and thermal conductivity on the dynamics, the calculation is repeated with the large values of transport coefficients given in eqs. (6a), (6b) and (6c). The time evolution of the density field is given in figure 6. Here, despite some apparent similarities with the non-viscous case, there are several distinct differences. Firstly, in the non-viscous case, the thermal energy remains negligible throughout the entire reaction (about 1 MeV at the most). In contrast, the larger viscosity and thermal conductivity lead to the generation of considerable thermal energy during the compression stage, with a maximum of about 2500 MeV. The second difference lies in the density field. With large viscosity, the density profile is smoother and the maximum density reached (1.7 n_0) is smaller. This effect can be understood as a consequence of the Navier-Stokes equations. Thirdly, in the presence of dissipation, the time scales involved in the
Figures 7 and 8 are the angular distributions for the viscous case in the center
-of-mass and laboratory systems, respectively. They are distinctly different from
the corresponding distributions (figure 2 and 3) in the non-viscous case in shape and
the positions of the angular peaks. Both the forward and backward peaks come from
slow particles of below 8 MeV. For the angular range between 60° and 90°, the
energy distribution (figure 9) shows a wide spectrum of particles from 4 MeV to
about 120 MeV, with a peak at about 26 MeV.

IV. Summary and Conclusion
The features of nuclear hydrodynamics illustrated in the above case were also
found in a systematic study spanning the energy region from 50 MeV to 400 MeV
(laboratory) per nucleon for the 20Ne projectile. The predicted angular distri-
butions of the reaction products have prominent features. Apart from sharp peaks
in the forward and backward directions which come from slow particles, there are
distinct peaks predicted between about 70° and 130° which come from more ener-
ggetic particles. The positions of the peaks in this angular range tend to move to-
wards larger angles with increasing bombarding energies. The presence of dis-
sipation displays significant effects on the dynamics, in particular, in the maximum
density reached in the reactions and the angular and energy distributions of the final
products. The viscosity dependence of the density compression can be summarized
in the curves in figure 10. For the angular distribution, the presence of viscosity
can lead to a difference of as much as 10° to 20° in the positions of the angular
peaks. The energy distributions tend to be broadened by the presence of dissipa-
tion. Accordingly, detailed comparisons with experiments will provide a useful means to obtain the nuclear transport coefficients.

Acknowledgments

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References


3) H.H. Gutbrod, private communication.

4) C.Y. Wong, "Selection of Truly Central Events in Intermediate Energy Heavy-Ion Collisions", ORNL preprint and to be published.


7) H.H.K. Tang, Ph.D. Thesis, Yale University (1979) and to be published.

9) C.Y. Wong, J.A. Maruhn and T.A. Welton, Nucl. Phys. A233, 469 (1975),
    Boris and K. Hain, J. Comp. Phys. 18, 248 (1975), J.P. Boris and D.L. Book, 

Captions

Figures 2, 3, 7 and 8:

The angular distributions are given in the same, but arbitrary unit. A given 
angular distribution can be converted into the absolute unit (nucleon number /sr) 
by multiplying by a factor of 180/π.

Figures 4, 5 and 9:

The energy distributions are given in the same, but arbitrary, unit. A given 
energy distribution can be converted into the absolute unit (nucleon number/(MeV-sr)) 
by multiplying by a factor of 180/π.
\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure1}
\caption{\textit{20Ne + 197Au} \hspace{1cm} \textit{E=400 MeV \hspace{0.1cm} b=ofm\text{(non-viscous)}}}
\end{figure}
$^{20}\text{Ne} + ^{197}\text{Au}$

$E=400\text{ MeV} \ b=\text{ofm (viscous)}$

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</table>
Figure 7

$^{20}\text{Ne} + ^{197}\text{Au}$

$E = 400 \text{ MeV}$, $b = 0.25$ (viscous)

$4.29 \times 10^2$

4.48

$\frac{dN}{d\Omega}$

Figure 8

Figure 9

$60^\circ < \theta < 90^\circ$
Figure 10 INCIDENT ENERGY/PROJECTILE NUCLEON (MeV)
DOMINANCE OF STRONG ABSORPTION IN $^9\text{Be} + ^{28}\text{Si}$ ELASTIC SCATTERING

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For several years we have been involved in a systematic study of elastic scattering of light heavy ions from $^{28}\text{Si}$. Our goal is to gain a better understanding of this relatively simple heavy ion interaction process. For example, we wish to know about the energy dependence of the optical potentials, whether we can determine the real or imaginary well depths, the "projectile dependence" of the interaction, and whether heavy ions show evidence for nuclear "rainbow" scattering (as is found for light ion projectiles).

From our study of $^{16}\text{O} + ^{28}\text{Si}$ elastic scattering we found that, in contrast to light ion behavior, the high energy data show no evidence for being dominated by rainbow scattering. Moreover, a shallow 6-parameter Woods-Saxon (WS) potential was found (816) which is capable of fitting data from $E_L = 33$ to 215 MeV without explicit energy dependence. A similar study of the $^{12}\text{C} + ^{28}\text{Si}$ system gave essentially the same results.

In contrast to these results, however, we found that the $^6\text{Li} + ^{28}\text{Si}$ system behaved in a qualitatively different fashion. In particular, the $^6\text{Li}$ data at $E_L = 135.1$ MeV do exhibit the characteristic exponential fall-off of nuclear rainbow scattering. As is true for $\alpha$-particle elastic
scattering, we found that a reasonable fit to the high energy data required a real well depth $V_{0} \geq 100$ MeV. Furthermore, it was found to be impossible to fit $^6\text{Li} + ^{28}\text{Si}$ data over a large energy range with an energy independent WS potential. On the basis of the qualitative differences described above, we term the $^6\text{Li} + ^{28}\text{Si}$ system to exhibit "light ion" behavior while the $^{12}\text{C} + ^{28}\text{Si}$ and $^{16}\text{O} + ^{28}\text{Si}$ systems show "heavy ion" behavior.

Because the character of the scattering changes markedly from $^6\text{Li}$ to $^{12}\text{C}$ projectiles, we have undertaken a study of the $^9\text{Be} + ^{28}\text{Si}$ system to map out the transition region. Data were measured at 121.0 and 201.6 MeV using $^9\text{Be}(3^+)$ and $^9\text{Be}(4^+)$ beams from the LBL 88-inch cyclotron. In order to carry out global optical model searches, low energy $^9\text{Be} + ^{28}\text{Si}$ data, measured by other groups, were also utilized.

Compared with our results for the other projectiles, we find that demanding an energy independent fit does not constrain the depth of the real potential in the case of $^9\text{Be}$ scattering. This is demonstrated by the wide variety of potentials listed in Table I, fits from some of which are illustrated in Fig. 1. For example, we note that potential sets G92 and G95 have essentially the same imaginary well but very different central well depths for the real potential. Clearly the fits in Fig. 1 are not very sensitive to the depth of the real well. There is some problem with most of the potentials listed in Table I in reproducing the trend of the back angle data ($\theta_{\text{c.m.}} = 25^\circ$) at 201 MeV. This deficiency is improved (see Fig. 1) by using a potential such as G38.

One question we had hoped to answer in this study concerned the existence of rainbow scattering. We know that high energy light ion elastic scattering angular distributions, up to $^6\text{Li}$, do show behavior which is
dominated by nuclear rainbow scattering, while $^{12}\text{C}$ and $^{16}\text{O}$ projectiles do not.\textsuperscript{1,2} The potentials in Table I have predicted rainbow angles ranging from $-81^\circ$ (G90) to $-3^\circ$ (G38). Thus, the data would be expected to show characteristic rainbow behavior (at least if $V > 75$ MeV) unless the absorption is too strong. It turns out that the absorptive strength required to fit the data is enough to remove the observable effects of the rainbow scattering in the calculated angular distributions. This is confirmed by model calculations which show that reducing the strength of the imaginary potential by a factor of 2 for set G92 gives rise to a factor of 10 increase in the back angle cross sections at 201 MeV, as well as giving rise to the smooth angular distribution which is typical\textsuperscript{3} of nuclear rainbow scattering. Although the calculation with potential G38 does show some flattening out at back angles, this is not related to rainbow effects. For this potential a reduction of the imaginary strength by a factor of 100 (to $W = 5$ MeV) makes essentially no change in the magnitude of the predicted cross sections, although the phase of the oscillations shifts markedly.

In order to understand why the various projectiles behave differently, we have examined the potentials which fit the data. Figure 2 shows the radial form of the potentials for $^6\text{Li}$, $^9\text{Be}$, and $^{16}\text{O}$ elastic scattering from $^{28}\text{Si}$. The "sensitive regions" shown in the figure were obtained from "notch perturbation" calculations.\textsuperscript{5} (Potential parameters for ions other than $^9\text{Be}$ are given in Refs. 1 and 2.) Although we have used potentials having no explicit energy dependence, the different real and imaginary well geometries nonetheless give rise to an implicit energy dependence of the potential. This is demonstrated in Fig. 3, where we plot the ratio of $V/W$ (evaluated at the radius of maximum sensitivity obtained in the notch
perturbation calculations) as a function of energy.

For the $^{16}\text{O} + ^{28}\text{Si}$ potential E18, we see that the ratio slopes downward as the energy increases, signifying a gradual increase in the strength of the imaginary compared with the real potential. Potential A23 of Ref. 1 is a deep real potential which was adjusted (by increasing the imaginary strength) to fit the high energy data. Clearly it will work less well at low energies. For comparison, the behavior of Satchler's "A-type" potential $^6$ is shown in Fig. 3. This potential has essentially the same real well as potential A23 but has an imaginary diffuseness which increases with energy. Looking at Fig. 3, it is not surprising that this potential does an equivalently good job at fitting the data.

In the case of $^6\text{Li} + ^{28}\text{Si}$, just the opposite trend is observed, with the $V/W$ ratio increasing as the energy increases. For $^6\text{Li}$ it has not been possible to find an energy independent potential which fits the high and low energy data sets simultaneously; the curve marked "fits" is obtained from different potentials at each energy. Part of the problem is related to the very rapid change in the $V/W$ ratio between $E_{\text{c.m.}} = 10$ and 40 MeV. An energy independent WS potential has trouble following such a rapid variation; the curve for potential R22 is representative of about how well one can do with a WS shape. Potential Z8 is a shallow, surface transparent potential similar to the $^{16}\text{O} + ^{28}\text{Si}$ potential E18 and has entirely the wrong trend for the data.

Looking now at the $^9\text{Be} + ^{28}\text{Si}$ potentials, we see that the overall trend is quite flat, with the imaginary potential always stronger than the real. If we stretch our faith in the representation shown in Fig. 3, we can infer that for this particular system a 4-parameter WS potential (that is,
\( r_R = r_1, a_R = a_1 \) might be adequate over the whole energy range. This is indeed correct, since calculations with potential G05 in Table I yield fits equal in quality to those from the 6-parameter potentials.

If we look at these trends for the various projectiles, we see that the \(^{16}\text{O}\) potential behaves in a rather intuitive way, that is, the imaginary potential gets more important as the energy increases. On the other hand, \(^{6}\text{Li}\) behaves in the opposite fashion and \(^{9}\text{Be}\) shows domination by the imaginary potential at all energies. Particularly in the case of \(^{6}\text{Li}\), it is tempting to ascribe the low energy behavior to breakup in the Coulomb field of the target. This mechanism has been demonstrated\(^7\) for \(^{6}\text{Li}\) at near-barrier energies, although even at the barrier the nuclear potential plays a role.\(^8\) In addition, there is some evidence\(^9\) that the breakup changes to a direct (presumably nuclear) process at high energies. We note again that Fig. 3 shows almost a discontinuity in the V/W ratio at low energies which makes these data difficult to fit along with the higher energy data even when using an explicitly energy dependent potential. However, it is possible to get fairly reasonable fits to the higher energy data sets (\(E_L = 46 - 135.1\) MeV) with an energy independent potential.

In the \(^{9}\text{Be}\) case, Fig. 3 does not indicate any anomaly in the low energy data. However there are some differences in \(^{9}\text{Be}\) compared with \(^{6}\text{Li}\) which might explain this fact. Although \(^{6}\text{Li}\) breakup at low energies goes mainly through a single state (the \(3^+\) at 2.18 MeV), which is excited primarily via Coulomb excitation,\(^7\) the \(^{9}\text{Be}\) breakup\(^10\) goes through a number of low-lying states. One of these states is reached by an E2 transition with a \(B(E2)\) similar to that for the \(^{6}\text{Li}\) excited state, but the others are not reached by E2 transitions and might be predominantly excited by nuclear inelastic
scattering. Depending on which states dominate the breakup process, it is possible that the V/W ratio does not change much near the barrier.

In summary, we find that the elastic scattering of $^9$Be from $^{28}$Si is dominated at all energies by relatively strong absorption. This removes much of the sensitivity to the real potential and even elastic scattering data spanning a range of energies from 13 to 201 MeV do not allow a unique determination of the potential parameters. There is at least circumstantial evidence that $^6$Li scattering at low energies (and by implication also $^9$Be scattering) may be strongly influenced by breakup processes, although it is not entirely clear that the breakup mechanism is the same in both cases.

References

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Table I. $^9\text{Be} + ^{28}\text{Si}$ Optical Potentials

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<th>$\delta_0$</th>
<th>$W$</th>
<th>$r_I$</th>
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<td>(MeV)</td>
<td>(fm)</td>
<td>(fm)</td>
<td>(deg)</td>
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<tr>
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Fig. 1. $^9\text{Be} + ^{28}\text{Si}$ elastic scattering angular distributions at $E_L = 13$, 121 and 201.6 MeV. Optical potential parameters are listed in Table I.
Fig. 2. Radial shapes of potentials found for $^{16}\text{O} + ^{28}\text{Si}$ (Ref. 1), $^6\text{Li} + ^{28}\text{Si}$ (Ref. 2), and $^9\text{Be} + ^{28}\text{Si}$ elastic scattering.

Fig. 3. Ratio of real to imaginary potential (evaluated at the radius of maximum sensitivity) for various heavy ion elastic scattering systems.

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Conference Summary

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We have now almost come to the end of a very long, stimulating and enjoyable conference—stimulating for us all, enjoyable for you, and I can assure you at this point I would treat you to a long, rich and stimulating summary, except that I have a bus to catch at a quarter to one. I feel that this has been a successful conference, and is the first one that I have attended for quite some time that the talks have continued to interest me right up until the very last day. This was partly due to the lucidity of the presentations, partly due to the ordering of the talks by the clever organizers of the conference. It may also have something to do with the fact that I had to make the summary. As Oscar Wilde so aptly put it, "When a man is under sentence to be hanged, it concentrates the mind wonderfully."

Let me give some general impressions that I shall take away with me from this conference. The first concerns the general behaviour of heavy ion reactions in the region from 10 to a few hundred MeV/nucleon. They look like Fig. 1—you see a lot of junk. The illustrated analogy is profound because what you see here is a large massive projectile, skimming into the stratosphere and breaking up under very high temperature and high pressure conditions. There follows that terrible moment...
of silence which NASA and NORAD had to endure before the debris from this event landed on a remote detector in the Australian Outback. Although this is a purely classical problem, not all the computer power of NASA or NORAD could predict the outcome, and in that sense I feel that what we have heard at this conference about projectiles in the nuclear stratosphere represents a considerable successful achievement by our nuclear scientists. There is another important aspect to this diagram at the top, namely "Time," because if there is one central theme that runs through understanding the sort of phenomena that we have heard about for these five
days, it is the concept of time. Another illuminating detail in the
top corner, summarizes the relationship between the lecturers and the
audience, in particular the relationship between our distinguished chair-
man (Lee Grodzins) and Gomez del Campo in discussing the Hauser-Feshbach
calculations. As I said, this diagram is profound, because basically
we are dealing with the "Humpty-Dumpty problem," not all the king's
horses and all the king's men can ever put Humpty Dumpty together again.
Unless, as Sid Kahana noted a few months ago (quoting from the King of
England), it just goes to show that "we need more horses and more men."
That is also rather appropriate since part of the reason why we are
here this week is to discuss the need for more accelerators and more re-
searchers to find out how to put the nucleus together.

Now this was essentially the first major conference to discuss this
intermediate energy domain, following an earlier workshop at Saclay last
year. I think the first serious mention of this area took place at a
Bevalac Summer Study in 1976, so it is actually a relatively recent
growth of our field. At that time my own view of the subject of high
energy, heavy ion collisions was very much of a Scientific Wonderland.
I think we are seeing now a rapid maturing of the field, with some firm-
ing up of the data and of the concepts in these intervening three years.
It is fair to say that one has left behind the wonderland, but I would
still like to take my inspiration from the very clever works of Lewis
Carroll and say that although we have gone through the looking glass,
as the walrus said,
"The time has come to talk of many things,
of shoes and ships and sealing wax,
of cabbages and kings—
and why the sea is boiling hot,
and whether pigs have wings."

The thoughts here are highly relevant to our conference. The shoes and the ships are the accelerators that we have discussed. Sealing wax is the instrumentation we heard about. The cabbages and the kings—well, the kings are the lecturers, and the cabbages are the rest of us who sat in the audience. Unhappily I was able to be a little bit less of a cabbage than most of you, and I do not think anyone can accuse our chairman of being a cabbage either. To the last two lines I would particularly draw your attention. They are extremely profound. "And why the sea is boiling hot"—you see here one accepts an absurd concept without any worry whatsoever. If you listen to Dr. Wong, Dr. Bertsch and Dr. Tang talking to you, they put over the most incredible concepts. They show the nucleus with volcanos in the middle of it, erupting puffs of Fermi smoke. I carefully looked around the audience as Dr. Wong was describing it, and there were no signs of alarm whatsoever. So, we don't worry about discussing why the sea is boiling hot, but try raising the question of high density in these heavy ion collisions, and you are into the area of whether pigs have wings. Nevertheless, it is very interesting to have these ideas, because they sum up a sense of the spirit of heavy ion science and the excitement that people have about the field. Once
again one can walk and talk and discuss physics; even if we do not un-
derstand what we are talking about, we can still talk.

Another impression that I shall carry away from this conference is
the remarkable youth of the speakers. Even those who are not so young
certainly give the impression of having drunk at the fountain of eternal
youth. Now this observation should not be taken as an indication of
Chargaff's view of another field of science when he said, "That in our
days such pygmies cast such giant shadows, only shows how late in the
day it has become." What he overlooked, of course, is that pygmies also
cast giant shadows at dawn. Or as it can expressed in an unfortunate
metaphor by the summarizer of another modern nuclear physics conference,
"These days we have the unique privilege of sitting side by side with
the giants upon whose shoulders we stand." But we are at a dawn in a
sense; there is a new excitement in the air among young people. But the
dawn in this field breaks not in the East, nor indeed even in the West,
but with a broad blaze across the middle of the country—from Chalk River
to Argonne, MSU and Oak Ridge. The dawn is also breaking in the real
East with the GANIL project, and perhaps projects at GSI and in Japan.

Summarizing conferences does have one advantage, because I was
given a very elegant suite of four rooms in order to conduct my delibera-
tions properly. Since I found that the conference fell naturally into
four parts, the situation was very convenient for me. I could keep
track of the vast mass of material by assigning one area to each room.
The four rooms were the kitchen, the bathroom, the living room and the
bedroom. Now the kitchen was very easily designated for its proper work. I put all the accelerator and instrumentation work in there, where all the good ingredients start to get cooked. The area of macroscopic studies, by sheer size alone, had to be assigned to the living room. The exotic I put in the bedroom where I believe it rightly belongs. That, unfortunately left me with only one room for the microscopic, which should not be taken as any reflection at all on what I feel about this subject. A number of important things are done in the bathroom and also a lot of the groundwork takes place before one is fit to enter the other rooms. It is also I believe a rather appropriate scheme of things in the wider sense because so much of what these high energy heavy ion collisions are about, is to find out how the nucleus first of all excites itself, by one-body dissipation, two-body dissipation or even many-body dissipation, and subsequently how it relieves itself.

Since most good things begin in the kitchen followed by some groundwork in the bathroom, and then relaxation in the living room and finally the bedroom, I shall follow this sequence in trying to map out the four main areas of the conference as I saw them.

Let me just first of all remind you of a famous picture (Fig. 2) which has been seen so many times and which represents a sort of map or chart to guide us through the rich and diverse phenomena. This chart has its origins in Scandinavia and is probably a suitable corollary to the Randrup model of energy dissipation in two Viking longships. The map is constructed, like its predecessor the Vinland map, from a great deal of
imagination, some definitely wrong information and some startling tales of intrepid explorers who have already ventured out into these strange seas. I have added something to this map through the names of all the speakers and the room I saw them in for most of the time. These are coded. This is the only time that I shall mention all the speakers' names. Of course I did select material from every speaker, but I had to cut most of it out because of this arbitrary reduction in time by our chairman. So even though I will not now mention all these names
again because I have little time, believe me that all your thoughts are with me and shall be carried away from here preciously. The purple is the exotic, and you see MacDonald plunging into the negative energy sea. The rest of them are arranged around, sometimes exotic, sometimes macroscopic in green, the microscopic along the top, and the exotic in the purple, and the black is reserved for instrumentation. This diagram is important because it shows a number of important boundaries which we have heard about many times. An important part of the research with the machines from 10 to a few hundred MeV per nucleon will be devoted to map out these boundaries, to find out if they exist, how sharply they are delineated, and therein lies a great deal of interesting physics. In general perhaps people have avoided the boundaries in their excursions and talks about this chart.

So let me first then go to the accelerators. It is appropriate that they should come first because we are all dominated by the accelerator builders. Even the simple proton has been referred to as a hydrogen ion in some recent APS bulletins! Harar in describing the GANIL project referred to it as "monolithic." I am not quite sure what he meant by the use of this word, but I think it is quite an appropriate one in terms of our chairman's discussion of heavy ion physics in terms of tombstones as the milestones on Randrup's postage-stamp representation, because in a sense accelerators are like our great churches and great cathedrals. They are monuments to our twentieth century culture, and to the religion that all of us gathered here today have chosen to follow. I certainly
cannot fail to be impressed by listening to the lighthearted way in
which Blosser, Bollinger and Heighway describe these remarkable techni-
cal developments. When one considers what the staggering, even frighten-
ing, problems must have been in the years of development, and then listens
to Blosser saying that "one just turns the magnet on taking a casual look
to see if the helium gauge has got its needle up somewhere," I find it
truly impressive. We can learn a lot from the accelerator builders. I
found a very nice little description from a talk given by Hine back in
the fifties when the construction of the CERN machine was under discussion.
He said, "Here we enter a field where the unknowns and the unpredictables
in fact are determining. I have mentioned before that the planning of
experiments in the minds of physicists was in terms of the search for new
particles rather than the detailed quantitative comparison of measurements
with theory. We as machine designers had to fight to get agreement to
spend money on two experimental areas or on provision for all conceivable
uses of the machine when the only physics directive we had was to find
out if antinucleons exist. This is the situation which is again with us
because of the possibility of a 300-GeV machine. Should it be 100 or 300
GeV? Or should it be some other kind of machine entirely? The conclusion
I draw from our experience with the overall design of the CERN PS is that
detailed predictions are a waste of time and should not be considered
very seriously in finally deciding on a machine. They are unfortunately
a necessary part of the process of raising money, but that is another
story. In fact the important thing in designing a new accelerator is to
get one, or preferably more, of its parameters one order of magnitude
better than in any current machine, and then to trust to the generosity of nature to provide new fields of research and exciting results. In the past she has never let us down which is more than can be said of most theoreticians." That is basically what the machines that we have heard about at this conference have done. They have made one or more of the important parameters an order of magnitude better than it ever was before. In some cases, e.g., MSU and GANIL, the energy is an order of magnitude higher. In the case of the Argonne and Chalk River machines, there are levels of precision in the machine that are an order of magnitude better.

These four machines are shown on the energy mass plot in Fig. 3. This familiar diagram is actually a little bit different from those you have seen before in covering two more decades (two more levels of purgatory in the tombstone metaphor). It actually goes up to TeV per nucleon. This is just to give a broad panoramic view, a little bit the same way as Randrup did in his representation of physics as a function of energy. One starts off from low energy which is dominated by mean field phenomena, up through the Fermi energy where transitions from quantal to classical type of behavior are expected. Maybe at higher energies in central collisions one can look for exotic effects, as other thresholds of pions, strangeness, and the antinucleon are crossed. Finally there may be encountered astrophysical problems, the cosmology of the early universe, the possible quark matter and limiting hadronic temperatures, and possible investigations at Tev per nucleon energies of Centauro events which are
conjectured by some high energy theorists in cosmic ray observations to be blobs of quark matter flying in at us from the primordial events in the early universe. Therefore I label this region as heaven in accord with our chairman's reference to tombstones. At the worst they are leading us towards heaven and to Ray Mix's nirvana.

One thing we missed hearing about in detail at this conference, but which is probably very important in accelerator technology are the very remarkable developments in EBIS ion sources that have taken place.
in Orsay and SACLAY over the last year or so. It is indicative of the interdisciplinary nature of heavy ion research that the only mention of these sources was by the atomic physicist, who visited us briefly. It is now apparently considered quite feasible to get 50% duty cycles for fully stripped argon or calcium beams and even krypton $^{34+}$ beams. Such sources, installed on some of the cyclotrons for example that we heard about at this conference, would give comparable or possibly even better performance than the injectors that are being used. This was a totally unexpected development taking place over the past year, but one which could have a lasting impact on the cost of future accelerator facilities in the few hundred MeV/nucleon region. The present day cost of coupled accelerator facilities like MSU is certainly substantial. Fig. 4 represents a cost vs energy graph based on a recent study of an intersecting ring and high energy synchrotron (the VENUS project at Berkeley). We see that a linear extrapolation gives a good approximation to the cost of the MSU facility—or to the ORNL 1 GeV/nucleon proposal! (Note that the difference between the linear extrapolation and one which tends more reasonably to zero cost as the energy tends to zero is roughly the difference between NSF and DOE figures at MSU). Clearly any developments in ion source technology which reduce the burden of these path-breaking heavy ion facilities in the nuclear community must be taken seriously.

There is an extension to this philosophy of machine building that can be extended to the design of experimental equipment which is also
becoming very extensive. More and more experiments are making use of beams and detectors built not for one experiment, or to solve any particular problem in physics, but because they make measurements possible in a certain field of phenomena. It is then up to the ingenuity of the physicists to extract physics with the means at hand. It is rather surprising to see how much of the development in heavy ion physics instrumentation has taken place in Germany and on the continent rather than in the United States. Therefore I think it was very
refreshing to hear some of the novel concepts for detectors, not only from Germany at this conference, but also from people in the United States. I must say I was very impressed with the imaginative ideas that are being put forward by Gruhn, Van Bibber, Glässel, Sarantites, Coffin, Nolen and Harwood in their descriptions of diverse large pieces of equipment that are expensive, but which are essential to deal with high multiplicities. One can see that there are a lot of new ideas in the air—the use of the streamer chamber, for example, was in low energy physics a pioneering and imaginative idea of Karl Van Bibber. The use of charged coupled devices for reading these out or for reading out the Bragg curve spectrometer that Chuck Gruhn talked about are all very novel and exciting ideas, ideas which are being transplanted from high energy physics. There is certainly one immediate important outcome of our discussions of instrumentation for our chairman (I'm sorry I'm focusing on you so much) who has already received a design from Chuck Gruhn for a Bragg curve spectrometer that he is going to have undergraduate physics lab students build. You see, there are instructions here in Fig. 5 like connect chain to grid. (Now we know where Grodzins gets his equipment money.)

We now proceed rapidly on to the bathroom with some brief words about the microscopic aspects of this conference. For some time people have come to think that heavy ion reactions have made little contribution to spectroscopy. I must say since I have not myself been working in that field for some years, I was extremely impressed by listening
to Walter Henning and Malcolm Macfarlane describe the excellent state of microscopic direct reaction theory. There have been long standing problems around in the literature, problems of shifts of peaks in angular distributions that were extensively studied here by the group at Brookhaven, and also problems with forward rising cross sections. As I understand it, all these effects can now be elegantly understood in the framework of the model that Seglie and Ascuitto have put forward involving the molecular orbital approach or the two-center shell model.
approach in calculating the wave functions. It may be that we are wit-
nessing the closing of a chapter of nuclear physics, but equally it is
not clear that the explanations that have been put forward for these
anomalies will have anything to do with the anomalies that are appear-
ing in simple direct reactions at much higher energies, because the
concepts of two-center shell model and so on should become less and
less important as one goes up to very high energies, i.e. as the speed
of the collision goes beyond the intrinsic nucleon time of motion of a
particle around the nucleus. I was also very impressed by the results
that Henning showed (see Fig. 6) on the evolution of simple transfer
reactions of one and two nucleon transfer as a function of the excitation energy. The evolution of these is very similar to the evolution that Wilczynski first demonstrated for deep inelastic reactions. We are here seeing an interesting sociological phenomenon, in that nuclear chemists led us into the study of deep inelastic scattering and of course, its predecessor, fission. Now one can see that the nuclear physicists have leapt in and begun to take over the description of these interesting phenomena in terms of a more microscopic model. And the corollary, which we have also seen at this meeting, is that subjects dear to the nuclear physicists for a long time, like simple transfer and multinucleon transfer reactions, have now suddenly been swooped on by the nuclear chemists. We had to be enthused, listening to Sarantites talking about alpha transfers and two-alpha transfer, and could not point out that some of us have actually been studying transfer reactions to discrete states for quite a long time. But this give and take between the two fields has been very good and very fruitful on the whole.

As you just heard from Ken Nagatani's talk, there are now quite important steps also underway to understand the evolution from quasi-elastic processes towards deep inelastic processes in terms of transfer and coupled channels multi-step direct reaction theory. This also I think is a very important and fruitful development, and it looks as if one will very soon be able to trace the evolution towards these large damped processes from the simple transfer reactions, fragmentation reactions, that we heard so much about. There lies a golden opportunity
to link the microscopic and the macroscopic, and the marriage is already taking place. Fig. 7 shows very nicely how many of the features of deeply inelastic scattering in heavy systems like Xe + Bi are also present in $^{20}$Ne + $^{40}$Ca. But here as shown by the solid curves, the evolution to large energy losses can be explained by microscopic theories of fragmentation (dashed) and transfer (dotted). Let me just summarize what seems to be going on in this field from the very elegant and inimitable prose of Malcolm Macfarlane. (I couldn't
possibly improve on it, so I just took it straight from his talk.) He said that "the smooth dependence of S matrix elements on angular momentum and the semiclassical nature of Coulomb excitation should be fully exploited in the numerics of DWBA coupled channel calculations. This defangs the huge L and R ranges endemic to the heavy ion reactions. It should be possible to see how much of deep-inelastic cross sections arise from excitations of giant resonances. The CC giant resonance theory can be worked out for not too heavy systems up to energies per nucleon of 20 or 30 MeV. The multistep direct reaction theories can probably be extended to much higher energies."

Now the question of course of the role of giant resonances in heavy ion reactions raises its head. Sandorfi gave us a beautiful talk with viewgraphs which were the only ones that in any way approach the quality of my own. So I have certainly included one of his. He showed us calculations on the cross sections for exciting various giant resonances. In Fig. 8 you see L = 2 and L = 4 cross sections, as a function of the energy per nucleon, comparing an 16O projectile with the alpha particle. Both actually seem to follow the same trend, with the cross section increasing by 2 orders of magnitude over the range from 10 to 40 MeV per nucleon. In spite of the similarity, I think it is still worthwhile to pursue the excitation of such resonances in heavy ion reactions because, as he showed, one of the great bugbears in analyzing the spectra lies in the subtraction of the background processes (even in the electron scattering case). If one has a large range of projectiles, from electrons all the way to very
massive projectiles, exciting the same resonances, one will get a better constraint on extracting the intrinsic strength (which frequently varies in experiments by 100%) of the giant resonances relative to that background. But there is something else that is very interesting about these resonance in the energy range we are talking about at this conference;
Sandorfi sees the cross sections for these resonances increasing by orders of magnitude. We suspect that over the same energy range, the cross section for deep inelastic processes might be decreasing by orders of magnitude. Therefore, one of the challenges about this energy region is the subject of our theoretical models and ideas to the limits, and the opportunity to make some very sensitive inroads in the understanding of the theory of all these processes.

The situation at the moment in understanding these evolutionary transfer and fragmentation processes, and the role of giant resonances, is really confined to extreme situations at high and low energy. A neat illustration, shown by many people at this conference, appears in Fig. 9. (I've put some summary things on it.) This diagram is a guide
to focus attention on the possible behaviour as the function of energy in heavy ion collisions. What we are seeing is oxygen on gold going to fragments, to the carbon fragments as plotted. The evolution of the width of the spectrum as a function of the energy from the lower energies up to GeV per nucleon energies is drawn. Now we heard a lot of talks by people from the Texas groups and from McVoy, Nemes and Sarantites about various simple models which help us to understand the onset of fragmentation, represented by the constant width of the Fermi momentum. Note that there are points (mainly from Karl Van Bibber's work in conjunction with Lee Schroeder's group in Berkeley) which haven't been shown at this conference, in the region of 50 to 100 MeV per nucleon. So in this region one has nuclear break-up like the relativistic regime. In the low energy regions one may have some equilibration and deep inelastic going on, or one may have simple transfer processes. That is the way that McVoy and the Texas groups are looking at the processes at the moment. Whatever is the ultimate explanation, the study of break-up of nuclear matter by any means must contain much interesting physics about the properties of nuclear matter.

There may also be something interesting to study about giant resonances across the whole space of this diagram. We heard some information about their role at low energies, and we heard briefly from Morrissey on their possible role at much higher relativistic energies. Now I want to show his picture, since I want to emphasize that there are precise calculations being done in high energy heavy ion collisions, of a
quality as high as in the more familiar low energy collisions. Also these calculations are on data of high quality and high precision. The data that James Symons showed on argon and calcium-induced reactions taken at the BEVALAC are high quality data with separation of isotopes. You will recall the calculations of Morrissey which describe such data by calculating the distribution of isotopes first of all in the case where the products are produced as a result of the uncorrelated cascade and in another model where correlations are built into the ground state of the nucleus. These correlations change the isotope distributions. In other words, the nucleus comes by at high speed, and the abrasion process takes a fast exposure snapshot or sampling of the motion of the neutrons and protons inside the nucleus. If they were completely correlated, then every scoop taken out of the nucleus would look the same, leading to a very narrow correlation, or a very narrow distribution in isospin. If the neutrons' and protons' motion is completely uncorrelated, the distribution will be much broader. (Fig. 10(a)) But these distributions are perturbed by secondary decays from the very highly and unknown excitation of the primary abraded nuclei. The resultant distributions of cross section contours, shown in Fig. 10(b) are very similar.

Here one faces precisely the same problem that Gomez del Campo described in his talk of trying to extract from the observed secondary decay products the information about the primary distribution (or maybe it was the opposite sequence). There he dealt with excitations of few
MeV, whereas Morrissey deals with excitations of hundreds of MeV; but the calculations are being done in both regimes, and the data exist in both cases. In the very low and very high energy cases, there may also be interesting parallels in the physics, as illustrated in Fig. 11. The high energy abrasion leads to dispersion of mass and charge determined by zero point quantal motion. The low energy reactions produce nuclei by diffusion and statistical motion. Note, however, that there are now suggestions that an alternative picture of zero point motion of the giant dipole in the composite system may play some role. In that case, one has the peculiar problem of observing the zero point motion more from the complex to the individual nuclei at high energies.
Sociologically the two regimes appear to be poles apart. You would have noticed there was very little criticism of Morrissey's talk, compared to the heated argument with Gomez del Campo, and this you see is a phenomenon very like the heavy ion collision. As the energy gets higher, the spectators and the participants separate out a lot more. But as the energy comes down, all these things get mixed together, and you get this hot interaction between the participants and the spectators.

That leads me on, I think sort of naturally, to the next subject that I want to discuss, namely the whole question of sources of particles.
in heavy ion collisions. Nagamiya showed us these very pretty pictures of the abrasion-ablation model. I noticed at conferences a year or so ago whenever these pictures were shown there were hoots of derision from the audience, but that has all died down now. One doesn't laugh any more. Let me try to give you a graphic illustration of what the sources in the high energy heavy ion collision are like. These again I would say are high quality transparencies—(I beg your pardon)—high quality calculations, actually by Smith and Danos in this case, of the collision of a neon on uranium at 250 MeV per nucleon, the top of the energy region
that we are discussing here. There is the distribution in coordinate space and the distribution in momentum space. You can see here a visual representation from the output of the microscopic cascade calculation of the type of sources that Nagamiya was talking about. The sources are most clearly seen in the momentum representation, by following the momentum distributions of the projectile and the target from the initial time (the frames represent snapshots as a function of time). As the collision proceeds, the intermediate rapidity space starts building up at the expense of depleting the other two. Then you see an intermediate glob that might be (in the naive mind) associated with the fireball description. The important point here is that the sources are separated out in the high energy collision so that one has projectile, target and intermediate distributions or sources for the emission of particles. A more mathematical description, closer to the data, of the remarkable experiments of Nagamiya's group is given by the rapidity diagram plot in Fig 13 for the production of protons in the collision 800 MeV argon on lead. The top half is just the unbiased production of the protons. The bottom half shows the selection of the protons when there is a high multiplicity cut imposed. At the top you can see a hint of the two projectile and target contours. Then there is this huge yield (well, not so huge because the cross section is falling off here rapidly), represented by the distribution of the contours at the intermediate rapidities associated with the fireball. As you impose the high multiplicity selection on the data (which is believed to confine the reaction closer and closer to more central collisions), one sees that the projectile and
target sources disappear, and one is left with the intermediate rapidity distribution, centered at the fireball rapidity. So the experiments are underway, and the methods are at hand for isolating the sources in the high energy heavy ion case.

The problems and the physics are very similar to what we heard from Guerreau at the much lower energies below 10 MeV per nucleon in deep inelastic collisions. The type of reaction mechanism here is quite different, but again one is faced with the problem of trying to extract from
the data what are the sources producing the emitted light particles. (Fig. 14).

\[ ^{40}\text{Ar}(280 \text{ MeV}) + ^{58}\text{Ni} \]
\[ \text{a in coincidence with } \theta = 23 \text{ at } 30^\circ \]
\[ \text{a in plane invariant cross sections} \]
\[ \frac{d^5\sigma}{dp_1 dE_1 d\Omega_2 d\Omega_3} \]

Figure 14

You can see that in this collision of argon and nickel that there are probably at least two sources—one associated with the projectile-like fragment and one associated with the target-like fragment (and maybe there is another source associated with something like the composite compound nucleus). Guerreau described very nicely how one can extract from such
data the information we have amassed about deep inelastic collisions, that the particles are emitted from the equilibrated projectile, and target fragments and various degrees of freedom have been relaxed, the temperatures have become the same, that the energy is divided according to the mass and so on. So now we have two extremes—we have this one, and I think that from Guerreau's talk one might believe that the physics is broadly well understood of various relaxations and emissions of particles from these sources in low energy collisions and to some extent they are also understood in the high energy case of Nagamiya's data. In both regimes the experiments are powerful. The challenge now obviously is to try and understand how one picture evolves into the other. Can one sort that out? Does it happen suddenly, or does it happen smoothly? Or is it just a horrible grey mess across the whole energy region?

We began to see some information on that from Lynen's talk on slightly higher energy per nucleon collisions, but still deep inelastic. I remind you of these distributions, which are plotted in Fig. 15 as the charge distribution as the function $Z$ for several different energies of krypton on erbium. At the low energies of 5 MeV and 8 MeV per nucleon, the distributions are roughly symmetrical and peaked around the $Z$ of the projectile. With increasing energy of 12.2 MeV per nucleon, the distribution broadens out to give a larger yield of the light fragments. Now Lynen, again in some remarkable experiments, interpreted this result, the increased production of these light fragments, as essentially
related to fission of the projectile. Now I can tell you that at much higher energies, at least with an argon projectile, one certainly has projectile fragmentation setting in in much the same way it was studied for the oxygen case (Fig. 9). The charge distributions are shown in Fig. 16 from the work of Symons and Westfall et al. We see how the distribution becomes almost flat as the abrasion process sets in. For 213 MeV/nucleon for argon on thorium, compared to the gaussian distributions (only one-half shown) in the low energy data at 6.5 and 10
MeV/nucleon (the original Volkov data of deep inelastic scattering). I think here is an interesting problem that is now accessible with the argon-induced reactions, to trace the evolution of the deep inelastic process, into (if Lynen's interpretation is right) the fragment or fission of the projectile-like part, into the eventual complete explosion in Bondorf's description of the projectile into a large number of pieces.
As I said, a key question, which I repeat once again, is understanding the sources of the emission of light particles in these different reactions as a function of the energy. Here is a little summary that I made from a number of talks that were presented at this conference dealing with the problem of light particle emission. As a function of the reaction energy above the barrier the table shows the percentage of pre-equilibrium or high energy-like particles being produced. For example, the reaction, krypton on erbium, studied by Huizenga et al. at
2.5 MeV/nucleon above the barrier, essentially no pre-equilibrium particles were observed at the accuracy of the experiment. As one goes up in energy we heard results from Plasil for oxygen on niobium at 8 MeV/nucleon. Here one observes some percent--there is not a quantitative number, but there is certainly some pre-equilibrium neutron emission. Similarly, Sarantites reported an experiment, and Gelbke talked about reactions of oxygen on gold at energies of 15 MeV/nucleon above the barrier where there was something like 20% production of fast light particles. Nagamiya mentioned experiments at 800 MeV/nucleon for argon on mass 40 where there was 50% of what he called clean knock out or simple nucleon-nucleon scattering. (By the way, be very careful not to confuse clean knock out (CKO) with (KNO), the high energy scaling concept which is also often discussed for these reactions, and among the less literary-minded people could also be mistaken for klean-nockout. I was actually rather confused by this for a while when I was listening to Nagamiya, but I did manage to sort it out.) What we observe here is an apparent sudden increase in pre-equilibrium emission, or knock out, of high energy light particles. My contention is that this does not proceed smoothly from low energies up to high energies, but probably sets in suddenly with the energy per nucleon. There are several different processes lumped together here, but I am taking a very broad view. Now once again I have to take my hat off to the BEVALAC people here, and to Nagamiya's groups in particular. These guys have gone in there and by god, they've done the experiments, and they come out with a firm number. Their experiment is the only one on
this diagram that gives you a number of any accuracy; he tells you it's 50% from a rather convincing analysis.

Now, these fast light particles could be produced by all sorts of mechanisms floating around, and Randrup mentioned them—peps, Fermi jets, pre-equilibrium and so on. We have actually studied some of these things at Berkeley. (This is where I get into the phase of presenting our own data, rather than proceeding with the summary of the conference—it is a well-known technique, I believe.) One of the interesting concepts, which may be nonsense, but interesting to think about, is the production of fast light particles by hot spots. Randrup described some modern calculations on this topic. Actually this subject has been discussed in the literature since 1932. Fig. 17 shows some old calculations from about 1950 about the effects of a hot spot. Plotted here is the relation of time to temperature; the left curve is essentially the lifetime for single particle emission. If you excite a state, you can calculate how long its going to live and emit a particle. Of course as the temperature goes up, the lifetime of the state goes down. Then you can plot another time for relaxing the energy over the system (right-hand curve) which increases as the temperature increases (this behaviour is just a reflection of the shortening of the mean free path as more and more particles are raised out of the Fermi sea and more phase space is available for collisions to take place). Now these curves cross over at some point. Once they cross, the time for emission of a single particle is faster than the time it takes to spread the energy out over the
whole system, just the condition for forming a hot spot. This was discussed by Tomonaga in 1936 or so. He showed how one could from such considerations derive all sorts of properties of the nucleus like the thermoconductivity, the specific heat and so on based on the Fermi gas. Let's not go into that, but they are concepts that are very much with us today and at this conference, and we have precious little more information about them now than we had in 1932. However, I think that this again reflects the interesting physics as a function of the energy.
You also heard in various talks by Bertsch, Randrup and Wong about concepts of relaxation time and equilibration at very high energies, of GeV/nucleon. I have inserted some typical relaxation times that they mentioned, and you see these are numbers like $10^{-23}$ second, whereas the relaxation times in deep inelastic scattering are more typically $10^{-21} - 10^{-22}$ second. As John Huizenga pointed out to me, these relaxation times at low and high energy appear to vary in the opposite way with energy; that looks like an interesting problem! Eventually we must understand the transport properties of nuclear matter across the whole board.

As a first attack on the problem, Conrad Gelbke showed some results from the work of Symons et al. on the study of pre-equilibrium protons from $^{16}$O on Au at 20 MeV/nucleon. The energy spectrum in Fig. 18 shows very high energy protons going out to 80 MeV, with an analysis based on a pre-equilibrium model with 16 or 20 excitons initially. Yesterday, Sperber took precisely these data and compared them with a hot spot model. Unfortunately, he left before I got the transparency from him, but the results of his theory go right through the data! There are other calculations that have been done with the fireball model which also do a tolerable job. Now one has the task of understanding the relation between these models. The exclusive data are likely to be more restrictive as Gelbke showed for much more sophisticated experiments, involving correlations and multiplicities. But I am convinced that it is still important to measure the inclusive data because very often they trigger the important germ of a new idea. In
fact it is probably a radical observation to make for the whole of this field, calling for huge walk-in scattering chambers, multiplicity arrays, detectors measuring multiplicities of hundreds costing the same number of millions of dollars. One has to have some simple focussing idea, and that is where the theoreticians come in—even if they're absolutely wild, blowing puffs of Fermi smoke out of the nuclear volcanos. The idea gives a focus to the experimenter, and an experimenter likes that sort of challenge. He goes and looks for the
puffs of Fermi smoke or the hot spot emission. Probably these things are not there at all, but other phenomena will turn things up.

Let me just show one other picture (Fig. 19), bridging low and high energy, that Nagamiya showed. This concerned the back angle production of particles in various collisions of heavy ions. You look at the 180°-production of protons and pions, and you put them in this representation, due to Frankel. I won't go into it. Nagamiya discussed

\[ \frac{d\sigma}{d\Omega} \propto e^{-K_{mn}/K_0} \]

Figure 19
the $K_o$ parameter which is essentially a measure of the momentum dis-
tribution of the struck nucleons inside the target, and we can extract
this number as a function of the collision energy. Now you see the
number increases and then saturates at some hundred MeV/c which is
supposed to be reflection of the Fermi momentum, or tail of the Fermi
momentum distribution. But I think it's interesting again to discover
what happens to that curve as you come all the way down almost to the
zero energy point, and I just put on some points here that I got from
the literature (Maryland and Virginia) for low energy proton-induced
reactions producing protons at a backward angle and alpha-induced re-
actions producing protons at a backward angle. The points lie right
down at the bottom but connect smoothly to the higher energies. Weiner
and his collaborators at Marburg have analyzed the Maryland data in
terms of a hot spot model, with the solution of the equation of trans-
port through the whole nuclear medium (which Sperber has not done in
his calculations so far). You have two models then, and you have to
understand how to connect the hot-spot description onto Fermi distri-
bution picture and the saturation at energies of 3 GeV or so. Again,
I think that's an interesting problem to be solved by studying energies
up to a few hundred MeV per nucleon.

Now I am coming to the end of the macroscopic section and am about
to go into the exotic section. But there is one important area of macro-
scopic phenomena that I must discuss concerning the fusion studies de-
scribed by Huizenga. I like the approach that he takes toward fusion,
in spite of the criticisms that were leveled at it. If you remember,
He essentially calculates fusion by calculating the trajectories of particles, finding the solution of the equations of motion in a model involving conservative and dissipative forces. He can make a uniform description using a proximity model for the potential and study the fusion cross sections right across the periodic table in a non-arbitrary and unselected way of dealing with the parameters. He showed this diagram (Fig. 20), which is the fusion cross sections for $^{12}_C$ and $^{16}_O$.

![Figure 20](image)

induced fusion on a variety of targets and pointed out that he had remarkably good agreement between the experiments and the data across that
whole region. I think one has to agree that is the case, and therefore, we can follow the fusion process in an unambiguous and parameter-free way. But in all fairness it has to be pointed out that there are data around at the present time, which do not fit into this elegant mould. Stokstad gave me these figures for oxygen on boron, and nitrogen on carbon-induced fusion as a function of energy. There is a huge difference between these. For comparison I put on here a rough transposition of Huizenga's prediction for this particular reaction. (Please do not take it literally; I had to read it off of one of his publications, but clearly it fails to describe these data.) The large difference between two reactions leading to the same compound nucleus is also puzzling. In Huizenga's model this difference would have to be accounted for by much greater dissipation in one reaction than in the other, but there is no understanding of what such difference in dissipation would be in that model at the present time; but it is obviously an interesting question. There is one other model of course that claims to be microscopic in its basis, which is also frequently applied to fusion reactions, and that is the Time Dependent Hartree-Fock. I do not understand why this word has barely been mentioned at this conference; maybe it has become a four-letter word, TDHF, and in fact it would probably fit in with Bob Stokstad's description of whatever movie it was that he saw with his friends one evening of these strange contorting shapes in TDHF calculations. However, there is a very interesting description by this model of the fusion cross sections as a function of energy for oxygen on calcium, shown with the experimental data (Argonne and others) in
Fig. 21. Here also is a transposition of Huizenga's calculation with his non-conservative forces model, and I think one must admit that it does a pretty reasonable job. However, the calculation with this TDHF model, which is a parameterless model in a sense (except that you've got 3 different possibilities up here) is also on the right track. And that surely is a very exciting possibility, to understand the relationship between the approaches.
Now we pass on to the exotic. First, I must ask Mike Nitschke to forgive me for not discussing the superheavy question extensively, since he's a friend and colleague of mine at Berkeley. Actually, I was very glad to see that he gave me some credit for making a contribution to this field of superheavy elements by my quotation from Shakespeare: “To fuse or not to fuse?” I think that I can now give him some more advice by completing that quotation, which summarizes the situation that Mike Nitschke, and the other intrepid people who study this field, are up against. "To (fuse) or not to (fuse), that is the question. Whether 'tis nobler in the mind to suffer the slings and arrows of outrageous fortune, or to take arms against the sea of troubles and, by opposing, end them." These guys are up against the slings and arrows of outrageous fortune, and you will find the figures and the numbers to prove it in Nitschke's talks. But like Hamlet, they will have to soliloquize and debate with themselves whether further actions or words is the best procedure at this time. Probably, one can never give up.

One can ask the question: How many new isotopes are worth one new element? This question came up at several times, and I think the answer to it essentially depends on whether you are a nuclear chemist or a nuclear physicist. A nuclear chemist regards one new element as extremely important. A nuclear physicist regards one new isotope as a worthwhile trophy. In any case the experiments that James Symons described on producing new isotopes were conducted at precisely the same time as Ghiorso and Nitschke were up at the SuperHilac with a calcium 48 beam, trying to produce superheavies. They got none and Symons got 18.
Probably 18 is worth one element. It is certainly the singly major advance in production of exotic nuclei since Volkov's pioneering experiments in deep inelastic scattering in 1970. This subject of exotic nuclei is a strange one, somewhere in the grey zone between the fundamental and technical virtuosity. It is the prerogative of a few individuals, many of them fired in their enthusiasm by Joe Cerny who goes around the world stopping off at odd laboratories, telling people to pursue the most ridiculous reactions, and then leaving them for years! I strongly suspect that he selects reactions that are quite impossible and then goes back to Berkeley and carries on with the feasible work.

Clearly the high energy abrasion process is now with us to stay as a means of advancing to the limit of stability in the future. There is the possibility of doing the type of experiment that took two days with calcium 48 in only half an hour at MSU. And clearly if so much could be done in half an hour, maybe the whole map of stable nuclei can be completed in two days! As Symons pointed out, as the limits of stability are approached, one may find strange and interesting nuclei. Wheeler and Wong in particular have pointed out for some time, that nuclei may assume new equilibrium forms once they carry extremely large neutron excess. The density may change, or they may assume polycentered forms, instead of the normal single-centered nuclei that we are familiar with for most of the periodic table. Exotic forms of nuclear matter, like pion condensates may also lurk in neutron-rich nuclei, manifesting itself by delayed neutron emission. One thing is clear. If significant differences between theoretical mass predictions are to be established,
the best course available is to check their predictions on stable and unstable nuclei close to the end of the chart. These experiments are now within our grasp.

Let us proceed to other exotic matters. A good framework for such research was given by Walt Benenson in the Unicorn analogy. Walter Benenson is a little bit like an old violin—he gets better every time you hear him. I've heard his talks several times, but this was definitely the best. In elaboration of his unicorn I would like to show this picture of what is involved in searching for the exotic. This picture, in case you don't recognize it, is a detail from the famous Unicorn tapestry in the Cloisters Museum in New York (Fig. 22). It shows the hunt of the Unicorn, which is surrounded by these fierce guys with sharp spears and
daggers, raised to make a kill. Now these I identify as the complicated pieces of instrumentation which we heard about earlier in this conference. The spears are expensively sharpened and ready to do the job! The area of the hunt is narrowed down. There are other interesting characters around: someone is blowing the trumpet very loudly proclaiming that the exotic has already been discovered. Sadly, we have missed them at this conference. In the center is the conference summarizer, reflecting profoundly and seriously from a considerable distance away. There are other people involved in calm discussions.

In the search for the exotic, it is usually thought that very high energies are a necessary prerequisite. However, lower energies could also be important. Here is this famous plot of the ground state energy per nucleon related to the nuclear density (Fig. 23). In the meanwhile

![Figure 23](image-url)
Greiner's group has moved the density isomer from 4 or 5 times normal nuclear density down to only 2.5 times normal nuclear density. At much higher density, as Randrup mentioned, there are other possible phases of quark matter. Now I tried to compile from this conference and from elsewhere some numbers on densities that are reported as the function of the incident energy in heavy ion collisions. Along the right hand axis is the incident energy in GeV per nucleon for some system like neon on gold. The line is the result of some calculations of the achievable density. Here for example are the results that you heard Dr. Tang speak about yesterday, with densities of something like 2 or 3 times normal, quite reasonable at low energies of 100 or 200 MeV per nucleon, the region where they believe nuclear hydrodynamic models to be valid. Also shown is a result from a cascade calculation of Cugnon et al. which gives at higher energies of 1 or 2 GeV a density of about 3.5. There are some calculations of Iwe, derived in a model, which I have tried to understand several times, and I read the paper again last night while most of you were enjoying yourselves at the banquet. I do not quite understand the mechanism, but in any case, it gives the results of 6 times normal at some 4 or 5 GeV. Finally there is a point at very high energy from Goldhaber, who deals with a model for the collision of relativistic slabs and predicts that you will get 10 times normal density at an energy of 10 GeV per nucleon. I show this picture because I think that it is not always appreciated that the region of a few hundred MeV per nucleon should give rise for some small fraction of the time to densities substantially
higher than normal (according to calculation). Even Ralph DeVries with his elastic scattering calculations predicted densities of 2ρ₀, but perhaps that is not so profound. So much for the theoretical situation.

Now what have we got experimentally, and what evidence has one seen at this conference?

Here are the calculations (Fig. 24) of Wong and Tang again, which they reported at this conference, on 400 MeV per nucleon neon on gold in head-on collision using a hydrodynamic model. [God, a message has fallen from on high! It says, "People who think they know everything are particularly aggravating to those of us who do." I thought it was a message to tell me to stop—and indeed it may be just that; in any event it could be a text for all summarizers.] The question is, "Is this valid?"

![Figure 24](image)
(The hydrodynamical description, that is to say) or must I put a bar across it and censure it? According to Randrup, yes. Hydrodynamics may never be valid. In any case they have done the calculations, and I am in no position to judge the answer to that question. But what I can tell you, and what Nagamiya did not have time to show in his talk, is that there is some evidence for side splashing events in collisions at only a few hundred MeV per nucleon from the Poskanzer-Gutbrod group at the BEVALAC. These are the energy spectra of protons emitted in the collision with a selection for high multiplicity. You see in Fig. 25, the statistical-type thermal spectra for many angles, but at forward angles, the data plunges below the data at wider angles. So transform this into an angular distribution, and you have a peak. A few years ago.
something like that would have been hailed as a major triumph and unquestionably evidence for compression and shock wave effects. Now one is a little more cautious, but there is hope. Let me show briefly another aspect of possible exotic phenomena at energies relevant to this meeting, the result on the pion, π⁺, π⁻ production in Benenson's talk. He accidentally left the punch line out of his story from Thurber when the wife was carted off to the lunatic asylum, which I understand is called the "booby hatch" in this great country. It says "don't count your boobies before they're hatched." That's probably good advice for anyone looking for exotic phenomena. But the dramatic thing about his results in Fig. 26 is the appearance of something unusual; no longer are we faced with smooth, uninteresting, continuous, dull-looking data from the BEVALAC. There are peaks, there are things jumping around, maybe even unicorns. Another interesting sidelight of these data is the stimulation they gave MacDonald to seize seriously upon the fireball model as a means of describing one atom plowing through the other.

Nagamiya showed you other new results on blast wave phenomena, producing an explosion of the fireball with an outward radial flow in addition to the random thermal motion. Such a phenomenon is rather universal throughout the whole of physics. At the other end of the scale, Fig. 27 is a clear example of an explosive event in the crab nebula. To indicate this unity, I put the BEVALAC inside it which many people have told me is possibly the best place to put it!

The BEVALAC has certainly pointed the way to the search for the exotic. Many of the techniques, the insights and the types of experiments
$\text{Ne} + \text{NaF}\ 0^\circ$

- $125 \text{ MeV/nucleon}$
- $150$
- $200$
- $250$
- $400$

$R = \frac{\pi^-}{\pi^+}$

Figure 26
will be repeated in the energy regime of this conference—experiments on compression, on interferometry, localisation and the like. It is likely that the exotic must be sought in the events of the type shown for 1.8 GeV/nucleon argon collisions in Fig. 28. These high multiplicity events are already appearing at 28 MeV/nucleon, as illustrated in Fig. 28(b) from Van Bibber's talk. These reactions are already very different from the dominant two-body processes of low energy heavy-ion collisions.
Bertsch and Wong showed us how these central collision events might be isolated experimentally. I found their treatments very refreshing, because they illustrated how sometimes even the most abstract thinkers are seriously devoting themselves to the real world. Bertsch suggests a measurement of the longitudinal component of the kinetic energy of the reaction products. A study of this quantity as a function of impact parameter could delineate between hydrodynamic, mean field and fireball descriptions. A second kind of measurement concerns the coefficients of the spherical harmonic expansion of the angular distribution of reaction products. Besides giving information on the impact parameter, these coefficients could quantify the stiffness of the equation of state of nuclear matter. On the experimental side, Gelbke showed us techniques at low energies to divide reactions into central and peripheral categories by looking at the coincident fusion fragments. These methods might be profitably extended into the higher energy regions pertinent to this meeting, to supplement the high multiplicity selection.

Finally, on this exotic question, it is appropriate to end with the microscopic, by showing the work of DeVries and Peng presented here. The implications are that two heavy nuclei may exhibit greater transparency than a nucleon on the nucleus. This calculation is based on a microscopic Glauber theory with input from the nucleon-nucleon cross sections. As usual, DeVries makes a very good point. In fairness to scholarship, however, this transparency has been recognized by
high energy aficionados for sometime. They allowed for it by intro-
ducing a surface transparency parameter $\delta$ into the usual expression for
the reaction cross section. The center of the nucleus is presumably
still rather black. Nor should people jump to the conclusion from
Fig. 29, that all heavy ion collisions are a superposition of nucleon-
nucleon scatterings. The point is that precise measurements and
calculations can be made which may indicate that if something unusual
happens, we may actually see it, possibly even from a measurement of
elastic scattering. However, Ralph DeVries knows very well that al-
though nuclei may be transparent to some extent, occasionally they are
not, and in any case exotic and exciting things do not have to happen
frequently in order to be worthwhile. Here is one picture of an event,
where clearly the nucleus is not transparent—the collision of a 70-GeV proton from Serpukhov on a nucleus in an emulsion where the whole nucleus is completely blown apart (Fig. 30). And even if that only happens once,

Complete destruction of Ag, Br, Pb nuclei by 70 GeV/c protons and 17 GeV/c alphas (150 events analyzed)

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Figure 30

it did happen. One has to understand how the energy was deposited in that system, or was it simply a "statistical fluctuation" in the cascade calculation?

Peter Haustein showed us similar things at very high energies for proton-induced reactions going up to 28 GeV. Coming up from low energies, initially the fragments that are produced initially go forward. At higher energy, they go even more forward, which is what every reasonable physicist would expect. But there comes a point up around 28
GeV where it seems that no more energy is deposited in the nucleus. Also these fragments do not go forward anymore but are produced side-ways. He gave an interpretation in terms of possible hydrodynamics and splashing effects. Interestingly enough, the onset of this effect coincides with the flattening of the mass yield curve for Cu spallation as a function of energy (Fig. 31). These effects appear to be associated

Figure 31
with a total kinetic energy of around 5 GeV and coincide with the
saturation of the momentum spectrum state parameter $K_0$ we discussed
in Fig. 19. Someday this limiting behaviour will surely have a uni-
ified interpretation. At present one can only speculate whether it
reflects some subtle hydrodynamic effect, or in the case of the $K_0$
parameter a scattering from quark clusters in the nucleus—as Baldin
has suggested—or some more trivial effect.

I must come rapidly to a conclusion and omit the other very fas-
cinating and interesting things that have been described at this con-
ference. I think that we have heard about a wide variety of phenomena,
which have been discussed with interest and enthusiasm. I think this
conference has been largely free of the type of intellectual bias which says
that high energy heavy ions are presently dull and uninteresting and
must prove themselves fast. Presumably such statements mean that they
prove themselves by the production of some fantastic, spectacular event—
like a shock wave or a density isomer. In fact there is nothing to
prove. The studies, the ideas and the calculations, that we have heard
discussed at this conference, are surely an integral part of our physics
and must someday appear in our textbooks. Otherwise we will also have
failed future generations. I am reminded of the religious sect some-
where out on the east coast of this country who decided that the 19th
century was the ideal place to stop. There was a time also when physi-
cists believed that the 19th century was also an ideal stopping place!
I think it is very difficult for us to be able to stand here today and
say, if I may continue with my Shakespearean applications to nuclear physics, "Enough, no more, it is not so sweet now as it was before."

Personally, I am convinced that there is sufficient richness and difficulty in the area of physics addressed by this meeting to warrant more than one machine of 200 MeV per nucleon. As Randrup has said somewhere, "It is neither a classical nor a quantum-mechanical regime, neither adiabatic nor sudden, and it will carry our traditional models to the limits of their validity. It is a region which will compare TDHF, hydrodynamic models, and find how all these things are merging together. Therefore, I can only wish BNL well in their plans to proceed with a machine in this energy regime. However, in the present climate of opinion, and with a NUSAC meeting coming up next week, I am reminded very much of Somerset Maugham, when he was seriously ill in the hospital and visited by a friend, who on leaving, possibly for the last time, asked if there was anything that he could bring him. And he said weakly, "No, it's too late for fruit, too soon for flowers." In any case, whatever the outcome, you have staged a splendid conference. We must all thank you for bringing together the shoes, the ships, the cabbages, the sealing wax, the kings—and the pigs. I am sure that I can speak for everybody here, when I say that I leave this conference with the feeling that BNL really cares about its visitors whether at conferences or whether they are coming to do physics. I cannot imagine a better environment in which to do physics. The entertainment, the food and the coffee were always right, and they were always there.
High-level physicists and secretarial staff were eager, in an understanding and courteous way, to help with even the smallest detail. We even heard about on-shell clans, and off-shell clans, and a new concept has been introduced of clans-on-the-half-shell. I think for all this we are really very much indebted to Jean Barrette, Peter Bond, Chellis Chapman, Sid Kahana—and I must confess I regret that he was unable to cast his mercurial insight on the topics of this conference and also since this was the first time I had a chance to summarize a conference at which he spoke, whereas he has summarized several at which I spoke. And, finally, Arthur Schwarzschild. Equally from my point of view, I am just as much indebted to Lore Barbier and Jackie Hooney, who have only been names on the telephone to me for many years. Certainly, I cannot imagine a more congenial and considerate environment. I wish you well.