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Backward Exotic Meson Production in $\pi^-n \rightarrow pX^-$ *

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Backward Exotic Meson Production in $\pi^- n \rightarrow p X^{--}$

by

David Mantell

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Chapter I

INTRODUCTION

As the number of known particles grows in an ever increasing fashion it becomes necessary to find classifications for them. Once this is done the significance of new particles is determined to a large extent by how well they fit into the classification. Unfortunately the scheme for such classification is not obvious, so trial and error must be used to find one that works for all existing particles. If new particles are found, ones that fit into a given scheme are evidence for that scheme. When new particles are found that do not fit, the scheme either must be expanded to include the additional members or abandoned altogether.

1.1 THE FORCES

Before going further it is necessary to say something about the fundamental forces of nature since all classifications of particles are related to how these particles interact with regards to these forces. The four forces that are recognized today are the gravitational, electromagnetic, strong and weak forces. It is expected that the understanding of all fundamental processes can be achieved through understanding the behavior of these forces.

The gravitational attraction between two objects like the earth and the moon is determined by the mass of those objects and their distance and some fundamental constant.

$$\text{force} = \text{constant} \cdot \text{mass1} \cdot \text{mass2} / \text{distance}^2$$

The earth and the moon are mutually trapped ('in a bound state') by the gravitational force between them. This means that their total energy is not great enough that they could ever be found apart. Even the earth and moon are composites of many atoms and molecules which are bound together by their gravitational attraction. On this scale, the gravitational force behaves so that composite systems which have properties very different from their constituents are formed. As the distances in atoms this force is essentially undetectable. However, the energy of a particle, which is related to its mass by Einstein's equation $E=MC^2$, has a fundamental relationship to the nature of the forces that are involved (e.g. Newtonian mechanics which describes motion in terms of forces is equivalent to Hamiltonian-Lagrangian dynamics which describes the motions in terms of energies). In fact the masses of different particles will be among the most useful clues in the effort to classify them.

The electromagnetic force is determined by the electric charges of the objects in a similar manner to the way the gravitational force is determined by the masses. There are also entities which are bound states due to this electromagnetic force: atoms! For atoms, the possible energies and angular momentum and other variables are all quantized (take on only discrete values) and the higher quantized energy states (excited states) are unstable and eventually decay (emitting a photon, which is a particle composed entirely of electromagnetic energy and angular momentum or spin) to the lowest possible quantized energy state. These excited states are also characterized by individual spin, but they have nearly the same total energy (or mass) and other similar electromagnetic properties.

The nucleus of an atom is an aggregate of many protons and neutrons. The proton and neutron have very similar properties which include their nearly equal masses. In a sense, the major difference is their charge (the proton has one unit of positive charge and the neutron has no charge). What is said is that these particles are in the same 'family'. A new variable called isospin is introduced. The isospin vector behaves just like the spin vector does (which will be discussed soon) except that it operates in some hypothetical isospin space. For spin and isospin the value of the projection along some direction is quantized, each value differing from the next by integer values. Thus the possible projection values for some vector S might be of length $n, n-1, \dots, 1-n, -n$ where n , the maximum value of the projection, is considered the actual length of S . For isospin, the projection represents the different charge states of a family according to the equation

$$t = Q + (S - B) / 2$$

(t is the projection and Q, S and B are the charge, strangeness and baryon number). The proton and neutron form a family with t of the neutron and the proton being $1/2$ and $-1/2$ respectively. They are the two possible projections of some isospin vector of length $1/2$. It will be seen that this type of grouping or classification of particles will become very important.

Inside the nucleus a strong force operates to bind the protons and neutrons called, logically enough, the strong force. The essential equivalence between the proton and the neutron is that they behave identically with respect to this force. The other force that operates only over nuclear distances is the weak force.

In processes for which both the strong and weak forces operate, the effects of the weak force are negligible. Only when there are no strong processes will a weak process be noticable. The defaulting of the strong force is quite common and is characteristic of 'strange' particles. Particles that could decay, meaning that in the change from an initial particle to some final particles energy, momentum and quantum numbers such as charge baryon number, strangeness etc. are conserved, do so by the strong force. When the decay involves a change of one unit of strangeness but conserves every thing else, weak decay is not forbidden even though strong decay is. These weak decays are characterized by being much slower than strong decays.

1.2 NOW FOR THE CLASSIFICATION

Spin can be used as a warm up for the game. Since the smallest possible spin is $1/2$, that can be used as the basic unit from which all other spins are constructed. When trying to measure the spin, one finds that measurements can only be made of the component of that spin along some direction. For spin $1/2$, the magnitude of the component is always $1/2$, but it can point in either direction along the axis. Using what is called a weight diagram, these states are represented by points along a line of the magnitude of the spin in some direction. For spin $1/2$, a weight diagram would look like this.

Larger spins can also be represented.

SPIN 1

SPIN 3/2

Again, each point represents a possible projection of the spin vector along some direction.

To add two spin one half vectors, one places one spin representation on top of every point of the other, making note of all the points on the first one. In other words, the projections simply add, so that for each projection representing the spin of the first particle, each of the projections of the second particle must be added.

Notice that the zero projection can be realized in two different ways, either adding the $-1/2$ projection of the second to the $1/2$ projection of the first or adding the $1/2$ projection of the second to the $-1/2$ projection of the first. These two ways represent mixtures of the zero projections of the spin zero and the spin one states. The spin 0 represents the situation in which the two spin vectors are pointing opposite $\longleftrightarrow = \bullet$ and the spin 1 represents the situation in which they point in the same direction $\longrightarrow \longrightarrow = \longrightarrow$. Larger spins also combine by adding all their projections. What holds above for spin holds identically for isospin!

The important step in the classification of particles is trying to extend this family structure to contain particles in larger groups (or multiplets) that have similar binding with respect to the strong force. To classify the hadrons, it is natural first to separate the mesons from the baryons. This is a natural line to divide at because baryons are half integer spin particles (fermions) and mesons are integer spin particles (bosons) which behave

quite differently. Further dividing by mass range gives the same groupings of particles, each of which consists of four or five families. By adding another dimension to the weight diagrams with strangeness on the new axis, the groups of particles can be arranged very nicely.

Just like the weight diagrams for spin that correspond to the mathematical structure of vectors, there is a mathematical structure ($su(3)$) that gives groupings that are like those above (see appendix).

As can be seen from the appendix, there are more $SU(3)$ multiplets than have been found experimentally. There is a very nice pattern to the multiplets for which particles do occur. The only meson multiplets that occur are the 1 and 8 which are constructed from $3 \times \bar{3}$; the only baryon multiplets that occur are the 8 and 10 which can be constructed from $3 \times 3 \times 3$. The hypothetical particles represented by the 3 and $\bar{3}$ are called quarks and antiquarks. By quark theory the mesons and baryons are bound states of different combinations of quarks and antiquarks. It would be reasonable to search for isolated quarks or, failing that, to search for indirect evidence of quarks. For example, protons and neutrons seem

to have internal structure. This internal structure can be imitated using a quark theory.

A fundamental question is: If there are quarks and they are bound together by twos and threes so tightly that it is very difficult ever to see one free, then why aren't particles that consist of four, five or six quarks and antiquarks seen? These hypothetical particles are called exotic. It is possible that such particles do exist, but for some reason they are hard to find. It is the purpose of this thesis to investigate this possibility!

Chapter II

BACKGROUND

2.1 INCLUSIVE REACTIONS

The particular reaction being studied here is $\pi^- n \rightarrow p X$, where the p , stands for forward proton, which indicates that in this reaction most of the original energy and momentum of the π^- is given to the proton, and where the 'X' means 'anything'. The experimental setup for this must consist of detectors to identify the π^- , a target that contains the n , and more detectors to identify the p (more about the experimental setup later). This is called a single particle inclusive reaction experiment where the energy and momentum of the initial particles and one given type of output particle are known or measured. The X may represent one or more particles that must take up all the energy, momentum and quantum numbers (i.e. charge, strangeness and isospin) that are unaccounted for by the single identified outgoing particle. An exclusive reaction experiment is one where all the final particles are identified $a \rightarrow b c d e f \dots$.

2.2 QUARK DIAGRAMS

Another way to represent this reaction is by a quark diagram, also called a duality diagram. The rules for such diagrams are that the quarks are represented lines with arrows that point to the right and antiquarks with ones that point to the left. The

initial particles start at the left and by rearranging their quark contents, annihilation of a quark and its antiquark or by pair creation of a quark and its antiquark all possible final states can be created . The rules are that the lines may not cross, that no quark line may be connected with an antiquark line from the same hadron, and that at junction of three particles ( ) each pair must share a line (Rosensweig 1977). One can think of the x-axis as time and of the quarks as points that trace out the lines in the course of the interaction (where antiquarks are thought of as quarks moving backwards in time).

2.3 PROPERTIES OF THE UNOBSERVED X

The X^{--} is all the left-over quarks and antiquarks and in the above diagram is shown as if there really were such a particle that is created by this reaction. If this picture is accurate, we can deduce many of its properties by measuring or knowing the properties of the other three particles in the reaction. The charge of X is $Q_\pi + Q_n - Q_p = -2$ (where Q_p means the charge on a p particle). The projection of its isospin vector is calculated the same way (remember $t_3 = Q - (B - S)/2 = -1$) $t_{3\pi} + t_{3n} - t_{3p} = -2$. Hence the magnitude of T must be at least 2. Since the sum of the lengths of the isospin vectors for the other three particles is equal to

$|T_\pi| + |T_n| + |T_p| = 2$, the maximum value that $|T_X|$ can have is also 2. Hence X^{--} (if it exists) is the member of a family with isospin 2 and it is the member with projection -2. To determine its mass:

$$E_{X^{--}} = \vec{p}_{X^{--}}^2 + m_{X^{--}}^2$$

and

$$\vec{p}_{X^{--}} = \vec{p}_{\pi^-} - \vec{p}_p$$

and

$$E_{X^{--}} = E_n + E_{\pi^-} - E_p = m_n^2 + (\vec{p}_{\pi^-}^2 + m_{\pi^-}^2) - (\vec{p}_p^2 + m_p^2)$$

yield

$$m_{X^{--}}^2 = m_n^2 + m_{\pi^-}^2 + m_p^2 + 2\vec{p}_{\pi^-} \cdot \vec{p}_p - 2\vec{p}_p^2$$

Notice that $m_{X^{--}}^2$ has been determined using only the masses and the momentums of the other three particles. Whether X is one particle or not, the right-hand side of this equation can be evaluated and is given the name 'missing mass squared'.

2.4 CROSSECTIONS

One complication with studying this reaction is that there are no pure neutron targets. A target can be made of deuterium, which can be regarded as a half proton and half neutron target, and a pure proton target can be made of hydrogen (in both cases the interactions with the electrons are rare and can be ignored). In some way the effects of the protons must be subtracted from the deuterium reaction to give the pure neutron interaction. To do this, some method must be used to talk about rates of reactions. A convenient way of doing this is to introduce the idea of cross-section. One intuitively expects the number of reactions $a+b \rightarrow c+d$ to be proportional to the number of incoming 'a' particles and to the number of possible target 'b' particles that are in the path

of the a's . These numbers are the flux of the a's in the beam and the number of b's per unit volume times the length of the target.

$$N = \sigma (\#/\text{Volume}) \cdot \text{Length} \cdot \text{Flux}$$

This proportionality constant σ is called the cross section and has the units of area. The area that σ represents is the effective area that one target particle takes up as seen by the beam particles. In the proton and deuterium case, one wants to subtract the area of a proton from the area of a deuterium to give the neutron area (this should give approximately the correct value).

Sometimes it is important to study the angular distribution of the products of a reaction. If one measures only a given type of particle as detected within a small solid angle as a function of the angle, the proportionality constant becomes the differential cross section $\frac{d\sigma}{d\Omega}$. An even more specific experiment might measure a given particle as a function of angle and also for a given energy range giving a double differential cross section $\frac{d\sigma}{d\Omega dp}$. Since $d\vec{p}/E$ is the same in any reference frame (it is said to be lorentz invariant), a cross section called the invariant cross section which is also lorentz invariant $E \frac{d^2\sigma}{d^2p}$ is generally used and is related to the above double differential cross section by kinematical factors.

$$E \frac{d\sigma}{d^2p} = E \frac{d\sigma}{p^2 dp d\Omega} = \frac{E}{p^2} \left(\frac{d\sigma}{dp d\Omega} \right)$$

For this experiment a different form of the invariant cross section is used:

$$E \frac{d\sigma}{d^2p} = E \frac{d\sigma}{p^2 dp d\Omega} \approx \frac{1}{p^2} \left(\frac{d\sigma}{dp/p d\Omega} \right) \quad \text{at high energies}$$

Chapter III

EXOTIC MESONS

3.1 SOME TOOLS OF THE TRADE

It is convenient when considering a reaction $ab \rightarrow cd$ to use the Mandelstam variables defined as follows:

$$s = -(\vec{P}_a - \vec{P}_b)^2 + (E_a - E_b)^2$$

$$t = -(\vec{P}_a - \vec{P}_c)^2 + (E_a - E_c)^2$$

$$u = -(\vec{P}_a - \vec{P}_d)^2 + (E_a - E_d)^2$$

where the E's are total energies of the particles and the P's are their momentums. The convenience of these variables is that they are Lorentz invariant. The missing mass squared, as defined before is described very easily using s, t and u:

$$mm^2 = s + t + u - (m_a^2 + m_b^2 + m_c^2)$$

If the incoming particles combine to form one particle that then decays into the outgoing particles, this is called an s channel resonance since the s variable gives the energy or mass of this intermediate state. Another possible process is that the incoming and outgoing particles exchange a particle-like entity called a Regge pole. This is called a t or u channel exchange (the distinction will be clarified later).

It is also convenient in such reactions to define two additional variables in the laboratory frame of reference for the emerging particles called rapidity and Feynman X and defined as follows:

$$y = \frac{1}{2} \ln \left(\frac{E + P_{||}}{E - P_{||}} \right)$$

$$X = \frac{P_{||}}{P_{||}^{max}}$$

where $p_{||}$ is the projection of the particle's momentum along the direction of the initial particle and E is its energy. The rapidity is sometimes approximated at high energies by the pseudorapidity

$$y \approx \eta = \ln \frac{1}{\sin \theta}$$

so θ is the scattering angle in the lab frame. Hence high energy particles tend to come out of an interaction with small angles and lower energy particles tend to emerge at much larger angles. Plots of the rapidity distribution may be seen in figure 1. The particles that come off at small angles are often called forward particles and since their energy and momentum are closer to that of the incoming particle, they are put at the top of the reaction diagrams. The particles with larger angles are called backward particles and are considered to be produced at the target.

3.2 DUALITY AND CROSSING

Regge recurrences are sets of particles with identical quantum numbers except for their spins. When plotted against their masses squared they form straight lines of nearly identical slope (figure 2). These are to be added to the particles discussed in the first chapter, each of which is the lowest spin member of a set.

The idea of crossing links the Regge recurrences that are created in the s channel and the Regge poles of the same quantum numbers that are exchanged in the t and u channels of other reac-

tions. Crossing says that if and only if resonances (and other particles of the recurrence) can be found in the s channel of a reaction then exchange of the Regge pole of the same quantum numbers can be found in the t channel (u channel) of the 'crossed' reaction $a\bar{c} \rightarrow \bar{b}d$ ($a\bar{d} \rightarrow \bar{b}c$) (where the overstrike means the antiparticle).

In view of this, the exchange of a Regge pole is in some sense the exchange of some type of average of the members of a recurrence.

The concept of duality also links the seemingly unrelated processes of resonance formation and pole exchange. Duality states that they are actually different descriptions of the same phenomenon and that either one is sufficient to completely describe a scattering process (Rosensweig 1977). The resonance picture is used at lower energies because the crosssections can be described in terms of relatively few such resonances, making the mathematical description very simple, whereas at higher energies there are so many overlapping resonances that the behavior of the crosssection becomes fairly smooth, thus making the description in these terms very difficult. At these higher energies the Regge picture becomes very simple, the crosssections being describable by relatively few Regge pole exchange terms.

The combination of duality and crossing has been successful in describing meson meson and meson baryon processes. But to apply them to baryon antibaryon processes requires the existence of

exotic two quark two antiquark meson as dual to ordinary mesons. If meson resonances are found in $B\bar{B}$ reactions, then they must be dual to exotic Regge trajectories which by crossing must be realized as s channel resonances (this statement shows a bias towards the resonance picture, the equivalent statement for one biased toward Regge exchange might be; if meson trajectories are exchanged in BB reactions they must be dual to exotic s channel mesons resonances which by crossing must be realized as an exotic trajectory. Duality insures that neither is more correct).

The quark graphs illustrate how normal meson trajectory exchange and four quark meson resonance production and vice versa are dual.

3.3 WHAT TO LOOK FOR

Unfortunately the nucleon anti-nucleon system has no exotic channels. The productions and exchanges have non-exotic quantum numbers, making positive identification of four quark mesons very difficult. Hence until high intensity beams of other antibaryons can be produced, absolute confirmation of exotics and duality for those systems will have to wait (Lipkin 1973). Of the experiments that can be done in meson baryon channels, the most promising seems to be backward production of exotics from non exotic baryon exchange (for example $\pi^- n \rightarrow p \bar{X}$) which should occur at the same order of magnitude as nonexotic production for the same baryon exchanges (Jacob 1970).

Different types of exotic mesons may be found that are not required by duality arguments. These would not have any special relationship to BB systems and would be more likely to decay into mesons. It is expected that these would be short lived (very broad) and most likely difficult to find.

The mesons required by duality arguments, on the other hand, should be fairly easy to find. At high energies they should also be broad, but should have crosssections on the order of $7.0 \pm 3.1 \mu\text{B}$ (Pennington 1978). At the baryon antibaryon threshold ($\sim 2 \text{ gev}$) the lifetime of these states should increase dramatically since the phase space for $B\bar{B}$ decay modes becomes very small and, below the threshold they are banned altogether, forcing a meson decay (Jacob 1970). In either case there is plenty of reason to look closely for these new types of particles.

Chapter IV

THE EXPERIMENT

4.1 HOW IT WORKED (*LEVINSON 1978)

This experiment was conducted at Fermi National Accelerator Laboratory using the M6 Beam Line and Single Arm Spectrometer Facility. It is called a spectrometer in analogy to an optical spectrometer that disperses a light beam by its wavelength (momentum) in order to measure special characteristics of the light source as a function of momentum. When looking at a beam of light that has passed through a prism a spectrum is seen. In a sense, the prism and the eye act as a spectrometer, the prism dispersing the light for the eye that is able to distinguish between the momenta for the light of different wavelengths because of their physical separation. The Single Arm Spectrometer works on the same principle looking at electrically charged particles that emerge from interactions of beam particles with a stationary target. These particles are dispersed according to their momentum by magnetic fields and are measured with various types of detectors and electronics. The magnetic fields disperse, collect and bend the paths of the particles the same way that optical devices such as prisms and lenses do for light beams. Additionally, though, the spectrometer has the job of distinguishing between different types of particles, a complication that the optical version does not have to deal with. The M6 Beam line also measures the momenta and

types of the beam particles before recombining them at the desired angle and allowing them to collide into the target; so in a sense it, too, acts as a particle spectrometer.

Particles $\pi^\pm$, $K^\pm$, $p^\pm$ are created at the beginning of the beam line when protons extracted from the main ring of the accelerator interact with a beryllium target and at the beginning of the spectrometer when those particles interact with either a liquid hydrogen or deuterium filled target, or an empty target (Empty targets are used to measure the amount of stray events in the spectrometer that are created in interactions with the containers for the hydrogen or deuterium or other parts of the apparatus). The momentum range of the particles that enter the beam line is selected by first spreading the particles by their momentum using a dipole magnet which acts like a prism and passing them through a steel slit. The sign of the electric charge of the particles is selected when doing this since opposite sign particles are bent in opposite directions by a magnetic field. A finer determination of the momentum is later done by further spreading the momentum and passing the spectrum through a hodoscope, which is an array of detectors that scintillate when charged particles pass through them. The light is collected and turned into an electronic pulse. The detectors in the hodoscope are arranged in a row so that they sample different parts of the momentum spectrum. Hence a range of the possible momenta for the particle is determined by the physical extent of each detector. Similar arrays are used throughout the beam line to get finer measurements of angles and positions coming at and emerging from the target. In this way, a resolution of about .1 percent was obtainable for beam momentum at 100 gev.

Hodoscopes were also used around the target and in the spectrometer to count the charged particles emerging from the interaction and to give a crude estimate of the angle at which they emerge.

Also used to reconstruct the tracks of the charged particles that enter the spectrometer are Multi Wire Proportional Chambers (MWPC). These work on the principle that high energy charged particles will ionize a gas along their path. In the MWPC's there are many parallel wires with a two millimeter separation between them surrounded by a gas. A voltage is applied between neighboring wires so that the freed electrons create a cascade of electrons as they also ionize the gas enroute to one of the wires. When this cascade reaches the wire nearest to where the electron passed, a pulse is created in the wire which is read out electronically giving the approximate position at which the charged particle passed through the MWPC.

The particle types are distinguished by using Cerenkov detectors. There are many variations of Cerenkov detectors, but they all work on the principle that when a charged particle travels through a medium, at faster than the speed of light in that medium radiation is given off in a conical shock wave that follows the particle. The angle of the cone is given by $\Theta = \cos^{-1}(c/nv)$ where c is the speed of light in a vacuum and n is the refractive index of the medium (c/n is the speed of light in the medium) and v is the velocity of the particle. Many of these detectors are long tubes filled with gas and contain mirrors to reflect the Cerenkov radiation into photomultiplier tubes. The pressure of the gas in the tubes can be adjusted very accurately enabling the index of re-

fraction to be set at values very near to one. Hence different velocities near the speed of light for which the detector is sensitive can be chosen. The velocity and momentum give enough information to determine a particle's mass. If the sensitive range of the Cerenkov detectors is set cleverly, the velocity range will give a mass range for each particle that enables one to distinguish between π , K and p's. These ranges are selected by Cerenkovs that either detect all particles above the velocity of light for the pressure of the gas (called threshold Cerenkov detectors) or by just detecting only some angular range of light cones (called a differential Cerenkov).

For example, in the SAS three threshold Cerenkov detectors are able to distinguish between π 's, k's and p's. The first one is very long and therefore very efficient. It is set to detect π 's but not the slower (heavier) k's and p's. The next two, which are less efficient, are set for both π 's and k's but still give no electronic pulse for p's. Using just these three detectors, π 's are identified by a pulse from the first Cerenkov regardless of the other two, k's are identified by nothing from the first but a pulse from one of the other two detectors, and p's are identified by no pulses at all. To aid in the identification of p's, a differential Cerenkov is used that is set to detect p's but reject the faster moving π 's and k's.

4.2 NORMALIZATION (*CUTTS 1977)

Since the magnets used in the beam line and the spectrometer have such regular magnetic fields, the 'optics' of the apparatus can be predicted as long as the particles are within some optimum

solid angle and momentum range called the uniform acceptance region. The spectrometer can be mapped for efficiency by transporting particles through the beam line directly into any part of the spectrometer. It is less important to make such a map of the beam line because ambiguous particle, momentum or angle decoding do not create biases since such events are ignored and also because the missing mass is much more sensitive to the decoding in the spectrometer. How events are selected in the beam does not effect the crossection since the number of scatters is normalized by the accepted particle flux. Once this was done careful determinations of the crossections for a common reaction such as $\pi^- p \rightarrow \pi^- X$ can be made.

The way the mapping is done, in order to include as many events as possible in the crossection determination for rare reactions, is to use these carefully determined crossections from $\pi^- p \rightarrow \pi^- X$ and the actual data to probe even the nonuniform regions of the spectrometer. If this is done with data in a Feynman X region ($X=.7$ or $.8$) where the variation of the double differential crossection is slow across the X acceptance of the SAS (single arm spectrometer), it will give a reliable test of the efficiency of the SAS that is independent of the data used for the normalization. For this normalization data, the aperture of the SAS can be determined in the uniform acceptance region by

$$\left(\frac{\Delta p}{p} \Delta \Omega\right)_{SAS} = \text{Scatters} / \text{Flux} \left(\frac{\text{Nuclei}}{\text{Volume}} \text{Length} \right) \frac{d\sigma}{dp d\Omega}$$

$$\cong \text{Scatters} / \text{Flux} \left(\frac{\text{Nuclei}}{\text{Volume}} \text{Length} \right) P_{SAS}^2 \left(E \frac{d\sigma}{d^3p} \right)$$

In the nonuniform acceptance region, the aperture must be multiplied by the efficiency of the SAS to give the acceptance (which, in any case, is given by the right hand side of the above equation). The efficiency is now determined by comparing the acceptance to those in the uniform area where the efficiency is one. In fact, the acceptance can be determined for any solid angle ($d\Omega$) or relative momentum bite ($\Delta p/p$) of the SAS and the SAS can be cut up into 'pieces' which correspond to different momentum bites, thus forming a mapping of the SAS in momentum space. In other words, the deviations of the event rate, with respect to the momentum of the spectrometer, are due either to statistical fluctuations in the data or to inefficiencies of the SAS since the momentum of particles in the SAS are equal to the beam momentum times the X measured in the SAS for which the double differential cross-section is flat by choice. The normalization is being done in terms of missing mass squared, which depends only on the X setting of the spectrometer, rather than $\Delta p/p$, which depends only on the magnetic optics of the SAS. To actually make this normalization correspond to various X settings of the data to be studied, those X settings are used while doing the normalization rather than the real X setting of the normalization data. Since this procedure that maps the SAS is independent of the actual setting of the SAS, but only to the momentum deviations $\Delta p/p$ and angles, values assumed for the spectrometer momentum settings do not effect the quantities 'efficiency times ($\frac{\Delta p}{p} d\Omega$)' but does allow one to determine such quantities (acceptances) as they will correspond to different missing mass squared values when data is taken at the various X settings. This allows one to bin events by missing mass and

to normalize by the efficiency after all events have been collected (see figure 3 of normalization data assuming $X=.94$). This also gives a separate normalization for each X setting, each of which is essentially the same as normalizing by $\Delta p/p$ because $\Delta(MM^2) = 2m_p(\Delta p_s)$ (all other terms being less by factors of p_{beam}), meaning that a distribution such as the normalization data that is flat in : missing mass squared will also be flat in $\Delta p/p$. So it does not matter what X setting is assumed since the SAS efficiency depends only on $\Delta p/p$ and is blind to the actual momentum setting.

Chapter V

THE ANALYSIS

The analysis of the data using the normalization technique is very straightforward. But since nonuniform areas of the spectrometer are used, some checks and careful cuts on the data must be made to avoid such things as scattering off parts of the apparatus or small effects in insensitive parts of the spectrometer from being counted too heavily. The second effect is handled by limiting the amount that efficiency will be tolerated to and by ignoring areas for which the efficiency goes below that. Also, cuts on the vertical and horizontal angles of particles entering the spectrometer clean up the data to a very large degree (figures 4 and 5). A check on this is if all the particles are within the radii of the Cerenkov detectors. This is especially important since no pulses in the Cerenkovs is used as the proton identification.

Also, cuts can be made to insure that the accepted events fit the description of backward production of a charge two particle. A simple requirement that at least two charged particles emerge from the decay of the produced particle might be reasonable. Also, the average rapidity of these emerging tracks could be required to be near the lower end of the rapidity distribution. At $s=190\text{Gev}^2$, the middle of the rapidity distribution should be around 2.5. Unfortunately the statistics, which are poor to begin with, can be dealt crippling blows by such requirements. Hence it

may be that the overall statistics of the experiment are not good enough to resolve expected fluctuations in the data from actual resonant cross section enhancements ($\sim 7\text{ dB}$ as quoted before from Pennington 1978).

Also supporting this view is the lack of nonexotic bumps in the isospin one cross section. If Jacob and Weyers are correct when they predict that the exotic cross sections should be comparable to the nonexotic ones, then the absence of both indicates that this data is unable to confirm, deny or set meaningful upper limits on the cross sections of such resonances in the exotic channel. The missing resonances in the isospin one channel cannot be explained by the normal uncertainties in the momentum decoding in the SAS (which would be the major source of error) since they were less than .1. The missing mass is approximately equal to $2m_p p_B (1 - p_B/p_0)$. Differentiating this gives $\Delta m^2 = 2m_p p_0 (\Delta p) = .2$ so $\Delta m = .2/\sqrt{2} \text{ (mm)}$ which ranges from its maximum at about .1 in the lower mass regions and down to about .02 near 5 GeV.

The data is fit assuming the interaction involved is Δ^{++} exchange and significance tests are used to evaluate this assumption. The triple Regge model relates an inclusive scattering cross section and its possible exchanges to Pomeron exchange at $t=0$.

From Ferbel Relationship between an inclusive cross section and the relevant triple-Regge graph.

At high missing mass the triple Regge model predicts

$$E \frac{d\sigma}{d^3p} \sim \left[\frac{s}{(MM)^2} \right]^{\lambda(\alpha_{\Delta}(t) - \frac{1}{2} \alpha_{\text{Pomeron}}(0))} \int (\alpha_{\Delta}(t) - 1)$$

($\alpha(t)$ is the equation for the line through the resonances) (Ferbel 1976)

$$\alpha_P(0) = 1 \quad (\text{pomeron})$$

$$\alpha_\Delta(t) = .2 + .92t \quad (t = -.1) \Rightarrow \approx .08$$

$$E \frac{d\sigma}{d^3p} \propto \left[\frac{s}{(MM)^2} \right]^{2[.08-.5]} s^{[1-1]}$$

$$\propto \left[\frac{(MM)^2}{s} \right]^{-2(.08-.5)}$$

$$\propto (MM^2)^{.84} \quad (\text{at fixed } s)$$

The data is therefore least squares fitted (weighted by the errors) to the form $A+B(MM)^2$ for the parameters A,B. Here the assumption that over the small range the difference between M and $M^{.84}$ is small.

5.1 THE TESTS

The first test of this fit is a chi squared test. Chi squared basically counts the number of standard deviations that all the points lie from the fitted curve. When forty five bins are used in the fit this test gives approximately one per degree of freedom (nothing that anyone would get excited about). A problem with chi squared is that it does not test correlations between neighboring points so that a bump, which is a number of points that are expected to have some correlation, is no different from the points alternating about the fit with with the same deviations. Also this test is not invariant to the bin size. For example with only six bins the chi squared jumps to about twenty per degree of freedom.

A method commonly used to test bumps is to combine all the data in the bump into one bin, fit to the rest of the data and count the number of standard deviations the one bin lies from the fitted curve. For the largest bumps in this data this method gives two standard deviations while four or five is generally con-

sidered sufficient evidence for a resonance. An objection to this method is that if one can really measure to the accuracy of the original binning then information is lost by combining the data into one bin.

Another test which attempt to circumvent this problem is the Durbin-Watson statistic. It is easily calculated as $c = \sum x_i x_{i+1} / \sum x_i^2$ where x_i is the deviation of the i^{th} value from the fitted curve. It does not use the errors but it is very sensitive to fluctuations in the data. The statistic is meant to test if points are correlated beyond the variables used to fit them or if the fit can adequately describe them. When the number of bins, N , is large each standard deviation away from the conclusion that the fit is adequate is $1/\sqrt{N}$. So when $C \cdot \sqrt{N}$ begins exceeding 4 (standard deviations) just like for the one bin test one may conclude that the structure is significant. The problem encountered with this test is that when the number of bins becomes small, since C can get no larger than $N-1/N$ (which is the case when all the points are the same distance from the line), the maximum value of $c \cdot \sqrt{N}$ is already less than four standard deviations. This means that the distribution of C values does not adequately approximate a gaussian and the value of C loses its meaning when it approaches one (see figures 6 and 7).

Appendix A

SU(3)

The basic units for the mathematical group SU(3) are called the 3 and $\bar{3}$ representations (three is the number of members in these representations). Two variables are used to describe each member, enabling one to plot them on a two dimensional weight diagram.

The scales on the axes are arbitrary; this particular scale is used because it is convenient for describing elementary particles. All the other representations can be generated by taking 'product' of threes and three bars. As with spin this multiplication is performed by adding the values on one weight diagram to the values on the other. The 3x3 gives

$$\begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \times \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} = \begin{array}{c} \cdot \cdot \cdot \\ \cdot \cdot \cdot \\ \cdot \end{array} = \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} + \begin{array}{c} \cdot \cdot \cdot \\ \cdot \cdot \cdot \\ \cdot \end{array}$$

3 3 $\bar{3}$ 6

a $\bar{3}$ and a 6. The 3x6 gives

$$\begin{array}{c} \cdot \cdot \\ \cdot \end{array} \times \begin{array}{c} \cdot \cdot \cdot \\ \cdot \cdot \cdot \\ \cdot \end{array} = \begin{array}{c} \cdot \cdot \cdot \cdot \cdot \\ \cdot \cdot \cdot \cdot \cdot \\ \cdot \cdot \cdot \cdot \cdot \\ \cdot \cdot \cdot \cdot \cdot \end{array} = \begin{array}{c} \cdot \cdot \cdot \cdot \cdot \\ \cdot \cdot \cdot \cdot \cdot \\ \cdot \cdot \cdot \cdot \cdot \end{array} + \begin{array}{c} \cdot \cdot \cdot \cdot \cdot \\ \cdot \cdot \cdot \cdot \cdot \\ \cdot \cdot \cdot \cdot \cdot \end{array}$$

3 6 10 8

Some more examples are

$$\begin{array}{c} \cdot \cdot \\ \cdot \end{array} \times \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} = \begin{array}{c} \cdot \cdot \cdot \\ \cdot \cdot \cdot \\ \cdot \end{array} = \begin{array}{c} \cdot \cdot \cdot \\ \cdot \cdot \cdot \\ \cdot \end{array} + \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array}$$

3 $\bar{3}$ 8 + 1

Figures

Figure 1: Rapidity distribution for different values of center of mass energy

Figure 2: Regge Recurrences plotted spin vs. mass squared

Figure 3; The normalization data ($\pi p \rightarrow \pi p$ at $X=.7$)

invariant crossection vs. mass squared processed assuming
X=.94

Figure 4: $\delta\Delta$ vs. Δ before any angle cut in SAS (beam + SAS z deviation from nominal magnetron)

Figure 5; " " " after the angle cuts

Figure 6; Histogram of Durbin-Watson statistic for 10 bins of random gaussian values. Notice that the standard deviation is .272 rather than $1/\sqrt{10}=.316$ predicted for C to be a gaussian distribution. Also note that no values exceed one.

Figure 7; Now for twenty bins, the curve is a much better approximation to a gaussian

Figure 8: vertical angle (ϕ) vs horizontal angle (θ) in SAS

Figure 9-12: results with an angle cut in phi and theta of -1.1 to 1.1 and -1.8 to 2.8 respectively and an efficiency cutoff of 50%. (the I=2 is the $\pi n \rightarrow pX$)

Figure 13-14: results with additional cuts requiring average rapidity of at least two charged tracks (multiplicity) to be less than 2.5

Figure 15: log plot of $I=2$ with bin size $.5 \text{ geV}^2$

Figure 1

Figure 2

Figure 1.6. Regge recurrence patterns for 1, 8, and 10 SU(3) multiplets of baryons: from V. Barger and Cline, *Phys. Rev.* 155, 1792 (1966).

TOT= 2628. IN= 2582. UNDX= 30.00 OVR= 18.00 MIN= -7.558 MX= 28.71 AVE

Figure 7

SCATTER PLOT

RUN 11000
X=RELU1A Y=RELU1A
Z=0

IN=100. PI=.3 X=.7 IGFCIP=1. H=0240

LE=IN,0) FOR Y ME: 0 10 20 30 40 50 60 70 80 90 100 FOR Y LUT: 0 10 20 30 40 50 60 70 80 90 100

DATA	11	DATA
DS X	21	DS Y
AXIS	0000	AXIS
7.5000	+	1
7.2000	+	1
6.9000	++	2
6.6000	2+	3
6.3000	2++	4
6.0000	++	2
5.7000	2	2
5.4000	3+	4
5.1000	++	3
4.8000	42	3
4.5000	+	0
4.2000	256	11
3.9000	+AA	27
3.6000	LA	42
3.3000	2Ch4	55
3.0000	3004	82
2.7000	+LL	91
2.4000	2DD4	96
2.1000	5ED4	103
1.8000	2LE	108
1.5000	+LL+	107
1.2000	6DF3	106
0.9000	6LP	129
0.6000	3LE3	115
0.3000	4ED2	111
-0.0000	5L02	110
-0.3000	9ED3	122
-0.6000	+F02	113
-0.9000	7FC0	121
-1.2000	+FF4	133
-1.5000	5EE6	119
-1.8000	6FF5	126
-2.1000	3FD+	110
-2.4000	4DD+	100
-2.7000	5FDB	120
-3.0000	9DC3	91
-3.3000	+BA3	46
-3.6000	+AA	71
-3.9000	7++	9
-4.2000	42+	7
-4.5000	7+	8
-4.8000	+	1
-5.1000	++	2
-5.4000	2	2
-5.7000	+	0
-6.0000	+	2
-6.3000	++	2
-6.6000	2	2
-6.9000	+	1
-7.2000	++	2
-7.5000	+	0

LEFT
BIN
7766665554443333222111000000011122233344455566667

X=SDelta Y=DEL.TA

[illegible]

Figure 6

TEST OF DURBIN-WATSON STATISTIC
7000. -

05/03/78 19:30:41

C X1*X2/ XXX 5833. -

"TEN " DEF'N:
-1.000 1.000
0.5000E-01 40 4666. -

TOT 100000
IN 100000
UNDR 0.0 3500. -
OUR 0.0
MIN -0.85249
MAX 0.87535
AVE -0.66073E-03
SIG 0.27199 2333. -
NOBS 100000

Figure 7

TEST OF DURBIN-WATSON STATISTIC
5000. -

05/03/78 19:30:41

C X1*X2/ X*X 4166. -

"TWEEN" DEF'N:

-1.000 1.000

0.5000E-01 40 3333. -

TOT 50000

IN 50000

UNDR 0.0

OUR 0.0 2500. -

MIN -0.75742

MAX 0.75876

AVE -0.60114E-03

SIG 0.20656 1666. -

NOBS 50000

>

SCATTER PLOT

```

RUN      11804
A=1HF  IA      Y=PIII
S      i

```

A(0) - PLOT OF LOG RATIO: 1 1 1 FOR Y LU: 65 5137 62 FOR Y LU:

[illegible][illegible]

11111 54444433333222221111100000000011111222223333344444

Figure 9

H2

05/27/78

11,33,32

INV. CRØSECTION: UB/GEV**2

Figure 10

D2

05/27/78 11,33,32

INV. CRØSSECTION: UB/GEV**2

Figure 11

I=1

05/27/78

11,33,32

INV. CROSSSECTION: UB/GEV**2

Figure 12

I=2 05/27/78 11,33,32

INV. CRØSECTION: UB/GEV**2

Figure 13

H2

05/28/78

13,42,52

INV. CROSSSECTION: UB/GEV**2

Figure 14

D2

05/28/78

13,42,52

INV. CRØSSECTION: UB/GEV**2

Figure 15

I=2

05/25/78

16,14,45

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