STATISTICAL APPROACH TO LHCD MODELING USING THE WAVE KINETIC EQUATION

by

K. KUPFER,* D. MOREAU,** and X. LITAUDON**

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K. KUPFER*

General Atomics, San Diego, CA 92186-9784

D. MOREAU and X. LITAUDON

Association EURATOM-CEA sur la Fusion Contrôlée
CE Cadarache, Saint Paul lez Durance, France

ABSTRACT

Recent work has shown that for parameter regimes typical of many present day current drive experiments, the orbits of the launched LH rays are chaotic (in the Hamiltonian sense), so that wave energy diffuses through the stochastic layer and fills the spectral gap.¹ We have analyzed this problem using a statistical approach, by solving the wave kinetic equation for the coarse-grained spectral energy density. An interesting result is that the LH absorption profile is essentially independent of both the total injected power and the level of wave stochastic diffusion.

THE WAVE KINETIC EQUATION

We consider the propagation of short wavelength lower-hybrid waves in a tokamak. The local spectral energy density of the rf field is denoted $U(\mathbf{x}, \mathbf{k}, t)$, where $\mathbf{x}$ is the position vector, $\mathbf{k}$ is the wavevector, and $U d\mathbf{x} d\mathbf{k}$ is the energy in the six-dimensional volume element $d\mathbf{x} d\mathbf{k}$. For simplicity we assume there is no mode conversion to the fast wave, so that every point in the $(\mathbf{x}, \mathbf{k})$ phase space is associated with a unique polarization vector. Also, the background plasma is assumed to be stationary. In this case, U obeys the wave kinetic equation (WKE)

$$\left(\frac{\partial}{\partial t} + \dot{\mathbf{x}} \frac{\partial}{\partial \mathbf{x}} + \dot{\mathbf{k}} \frac{\partial}{\partial \mathbf{k}} \right) U + 2\gamma(\mathbf{x}, \mathbf{k})U = S(\mathbf{x}, \mathbf{k}, t) \quad , \quad (1)$$

where γ is the damping rate (electron Landau damping) and S is the rf source. The quantities $\dot{\mathbf{x}}$ and $\dot{\mathbf{k}}$ are time derivatives of $\mathbf{x}$ and $\mathbf{k}$ along the ray trajectories induced by the local dispersion relation $D(\mathbf{x}, \mathbf{k}, \omega) = 0$, where ω is the frequency of the waves. Upon inverting the dispersion relation to obtain $\omega = \Omega(\mathbf{x}, \mathbf{k})$, the ray trajectories can be written in Hamiltonian form,

$$\dot{\mathbf{x}} = \frac{\partial}{\partial \mathbf{k}} \Omega \quad , \quad \dot{\mathbf{k}} = - \frac{\partial}{\partial \mathbf{x}} \Omega \quad . \quad (2)$$

A detailed derivation and discussion of the WKE has been given by Ref. 2.

Once the source function is specified, Eq. (1) can be solved by following increments of injected power along ray trajectories in phase space. Appropriate canonical coordinates in tokamak geometry are $(\mathbf{x}, \mathbf{k}) \rightarrow (r, \theta, \phi, k_r, m, n)$, where m and n are the poloidal and toroidal mode numbers. In the cylindrical approximation both θ and ϕ are ignorable coordinates, so that m and n are conserved; in which case the ray trajectories are integrable and the solution of (1) can be written explicitly. We consider the point

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source $S = (2\pi)^{-2} \delta(r - r_o) \delta(k_r - k_{ro})$, where r_o is just inside the cutoff near the plasma edge and k_{ro} satisfies the dispersion relation at $r = r_o$. (Here m and n are considered parameters.) In this case, the solution of (1) is

$$U = (2\pi)^{-2} \delta[\Omega(r, k_r) - \omega] \frac{\cosh[\nu y(r)]}{\sinh \nu} ; \quad \text{for } r_1 \leq r \leq r_2, \quad (3)$$

where

$$\nu = \int_{r_1}^{r_2} dr 2\gamma(r)/v_r(r) \quad \text{and} \quad y(r) = \nu^{-1} \int_{r_1}^r dr' 2\gamma(r')/v_r(r'). \quad (4)$$

Here $v_r = |\partial\Omega/\partial k_r|$ and the dispersion relation is used to solve for k_r as a function of r . The turning points where $v_r = 0$ are denoted by r_1 and r_2 , so that the ray is confined to the region $r_1 \leq r \leq r_2$, where r_1 is the caustic and r_2 is the cutoff. From (3) we can calculate the absorbed power density, $P(x) = \int dk 2\gamma(x, k) U(x, k)$, and the energy density in the rf parallel electric field (which determines the electron quasilinear diffusion coefficient), $\epsilon_o |E_\parallel|^2/2 = \int dk \alpha_\parallel(x, k) U(x, k)$. Here $\alpha_\parallel = 2 \partial D / \partial \epsilon_\parallel (\omega \partial D / \partial \omega)^{-1}$ follows from the cold plasma dispersion relation, where $\epsilon_\parallel$ is the parallel component of the dielectric tensor.

There are two distinct regimes: (1) the multipass regime (for $\nu \lesssim 1$), when the absorption is peaked at the caustic; and (2) the singlepass regime (for $\nu \gtrsim 5$), when nearly all the power is absorbed before the ray reaches the caustic. Figure 1 shows the

FIG. 1. Radial profiles of the absorbed power (top) and parallel electric field energy (bottom) in the multipass regime ($\nu < 1$) for a single field harmonic ($m = 100, n = 450$) in cylindrical geometry. The solid curve is the WKE solution from Eq. (3) and the dashed curve is the numerically determined fullwave solution. Parameters are typical for LHCD on Tore Supra.

radial profiles of P and $|E_\parallel|^2$ for a single field harmonic (m, n) in the multipass regime. The solid curve is the solution of the WKE, as given above, and the dashed curve is the corresponding full-wave solution, determined numerically by a cylindrical fullwave code.³ The figure shows three important results: (1) that the full-wave solution in the multipass regime develops a node structure due to interference between the ingoing going and outgoing waves; (2) the WKE solution averages accurately over this fine scale interference pattern; and (3) the singular behavior of the WKE solution in the vicinity of the turning points (caustic and cutoff) occurs over a very short scale-length and is a negligible effect. We have also compared fullwave and WKE solutions in the singlepass regime, in which case the two methods converge, as expected.

THE WAVE DIFFUSION MODEL

In toroidal geometry the inherent poloidal asymmetry typically causes the formation of a thick stochastic layer in ray phase space.¹ This allows m to upshift (and downshift) as the rays fill the available stochastic phase space, until all the injected power is absorbed. When the spectral gap is large and the electron Landau damping is weak, the rays make many passes through the plasma before being absorbed. In this case, the chaotic dynamics allows the WKE to be approximated as a diffusion equation for the appropriately coarse-grained phase space energy density. The quasilinear treatment of (1) leads to the result that $\langle U \rangle \approx (2\pi)^{-2} \hat{U}(m) \delta(\Omega_o - \omega)$, where $\Omega_o(r, k_r, m) = \omega$ is the unperturbed (cylindrical) approximation to the dispersion relation, and $\hat{U}$ obeys the diffusion equation,

$$\tau(m) \frac{\partial}{\partial t} \hat{U} - \frac{\partial}{\partial m} D \frac{\partial}{\partial m} \hat{U} + \nu(m) \hat{U} = \delta(m) P_{\text{in}} \quad (5)$$

Here $\tau(m) = \int_{r_1}^{r_2} dr |\partial \Omega_o / \partial k_r|^{-1}$ is the radial transit time, D is the diffusion in m per radial transit, and $\nu(m)$ is the damping decrement per transit, as defined in (4) using the unperturbed orbits. Also, P_{in} is the total injected power and we have idealized the LH source as being a delta function spectrum in n and m , with n treated as a parameter.

Equation (5) is solved on the domain $m_a \leq m \leq m_b$ with a reflecting boundary condition ($\partial \hat{U} / \partial m = 0$) at m_a and an absorbing boundary condition at m_b . The lower boundary, as discussed in Ref. 1, corresponds to a propagation limit, since the dispersion relation cannot be solved for real k_r if $m < m_a$, where m_a can be calculated in the cylindrical approximation. The upper boundary m_b is somewhat arbitrary because the electron Landau damping becomes very large at large m , so that $\hat{U}$ decays rapidly. It is sufficient to chose m_b large enough that the power flow across the boundary is negligible. Thus in steady-state (5) yields the conservation relation $P_{\text{in}} = \int dm \nu(m) \hat{U}(m)$.

The effect of the chaotic dynamics on the evolution of the spectral energy density enters (5) through the diffusion coefficient D . Although D can be a function of m , we have found that taking D to be a constant is an adequate approximation. Typical values for D obtained from toroidal ray-tracing codes are in the approximate range 10^3 to 10^4 (assuming a circular plasma cross-section and parameters typical of Tore Supra and JT-60). Since the characteristic timescale for the divergence of neighboring trajectories in the chaotic phase space must be much shorter than the time scale for absorption, one finds that a rough criterion for the validity of the diffusion model is that $\sqrt{D}$ must be smaller than the typical m upshift required for strong electron Landau damping.

Because the damping decrement in (5) depends implicitly on the electron distribution function, one must solve (5) together with the electron Fokker-Planck equation, where the latter includes rf driven quasilinear diffusion. We refer to this coupled nonlinear system as the wave diffusion/Fokker-Planck system, or simply the WD/FP. The situation is entirely analogous to the usual coupled ray-tracing/Fokker-Planck approach (RT/FP), except that in the RT/FP one seeks an exact solution of (1), which requires the use of a toroidal ray-tracing code to calculate the orbits of a large ensemble of rays for many radial transits. We have implemented numerical solutions of the WD/FP using a simple one-dimensional Fokker-Planck model at each radial grid point. A typical steady-state solution is shown in Fig. 2 for JT-60 like parameters with

$n_{eo} = 1.5 \times 10^{19} \text{ m}^{-3}$, $T_{eo} = 5 \text{ keV}$, and a launched n_e of 1.5. Good agreement is found between the WD/FP and RT/FP, as long as the rays make many passes before being absorbed, in which case the effect of the chaotic dynamics is well described by (5).

FIG. 2. Typical solution of the WD/FP. The rf energy density in the shaded region is large enough to produce quasilinear flattening of the electron tail. [Here the parallel velocity is normalized to the local electron thermal velocity $\{T_e(r)/m_e\}^{1/2}$.] The profile of the absorbed rf power density is superimposed.

Numerical solutions of the WD/FP over a large range in the parameters D and P_{in} indicate the following simple scaling. Defining $W(m) = \nu \hat{U}/P_{in}$ as the normalized absorption per unit m , one finds that in steady-state W is essentially independent of $\hat{U}$ due to strong quasilinear flattening of the electron tail, i.e., $\gamma \propto \partial f_e / \partial v_{||} \propto 1/\hat{U}$. It follows from (5) that $\hat{U} \propto P_{in}/D$, so that quasilinear flattening occurs as long as P_{in}/D is larger than some critical value. For typical LHCD parameters P_{in}/D is 100 to 1000 times the critical value, so that the radial profiles of the absorbed power and the local current drive efficiency are independent of substantial variations in either D , or P_{in} .

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