MASTER

THEORY OF ELECTRON
INDUCED REACTIONS

Progress Report
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Progress Report

In the original proposal (1978) we identified five areas involving high energy electrons and radiation interacting with nuclei which needed more precise calculations. In the current (second) year we have made substantial progress in two of these areas and added a third (electron production of pions) all of which is reported below. The state of the art for the others (Radiation Tail, Bremsstrahlung and Electron Scattering) is summarized in the new proposal.

a) Finite Size Effects in Virtual Photon Spectrum

We have attacked this problem from several directions to seek corroboration between different approaches, and in hope of covering a wide range of energies. There are two questions:

(a) To what extent is it possible to define a model-independent correction to the virtual photon spectrum to account for finite size of the target, and at what energy would this break down?

(b) How can the finite size correction be made simultaneously with the Coulomb (finite charge) correction and do these interact?

The answer to (a) is most simply sought in the plane wave Born Approximation where the calculation is fairly easily made.¹⁾ We have investigated a number of models which translate simply into momentum space, but avoid any unduly restrictive assumptions about the dynamics of the nuclear charge current distribution. We have, for example, liberalized the Helm model and worked out its configuration-space form so that it can be compared with other models.²⁾ The results show (see ref.(1) and appended text) that for the range of energies up to about 50 MeV the

correction is small (1 - 3%); from 50 - 200 MeV the correction is correctly given, over a wide range of possible charge-current distributions, by only one parameter--the transition radius. Fig. 1 shows drastically different current distributions for an M1 transition all of which share the same transition radius, and produce the same correction to the M1 virtual spectrum. This insensitivity to details of the transition is evidently due to a predominance of low momentum transfer components which selects only comparatively "low" moments of the corresponding form factors. Above 200 MeV this is evidently beginning to break down at the high momentum end of the spectrum, but it is equally true that the appearance of the nucleus at high energy-and-momentum transfer (quasi-elastic and deep inelastic region) also changes from that at the low energy nuclear excitation region for which our models were designed.

To answer question (b) we have looked at the problem in higher-order Born approximation and also in distorted wave approximations. Some earlier calculations^{3),4)} in second order Born approximation have shown that this is not a very useful calculation for the details of angular distributions at low energy transfer, but may work quite well for low momentum transfer. Such calculations made use of zero-mass and zero-energy-loss approximations which unfortunately are of no value in virtual photon calculations, and we have had to extend the calculations to take these into account, which has entailed some difficulties. The calculation involves the integral over two momentum variables and two form factors (one elastic and one inelastic). No suitable choice of form factor leads to a closed form result and thus far we have replaced the elastic form factor with a simple form (of the Yukawa type) and, integrating over all directions, are left with a double integral of the form

$$|H_{fi}|^2 = N \sum_{\ell, m} \frac{1}{2\ell+1} \int_0^\infty \int_0^\omega dq q' q^2 q'^2 \frac{\tau_\ell^{Mif}(q) \tau_\ell^{Mif}(q')}{(E^2 - q^2)(E^2 - q'^2)} G_\ell(q, q', \Delta, E) \quad (1)$$

where Δ is the momentum transfer and E the transition energy, τ_ℓ is the interaction form factor. Here G_ℓ splits into two parts which correspond to the result of the first Born approximation and to the first correction, respectively

$$G_\ell = G_\ell^{BA} + G_\ell^{(1)} \quad (2)$$

Thus far we have found a closed form for the correction term for M1 transitions and are presently calculating the interaction matrix element (Eq. (1)) by numerical integration, after dealing with the pole terms by subtraction.

The virtual photon spectrum is finally of the form

$$N(\omega) = \frac{N_c}{k_1^2} \frac{\omega}{\sigma_\gamma(\omega)} \int_{\Delta_{min}}^{\Delta_{max}} |H_{fi}|^2 \Delta d\Delta \quad (3)$$

which is a finite integral over reasonably well-behaved functions.

The alternative approach we have used is the distorted wave method described initially in this application in ref. (5). As in the plane wave case we have to introduce distributions for both elastic and inelastic processes, the former modifies the incoming and outgoing wave functions and the latter is in the interaction operator. In an approximation described earlier⁶⁾ an expansion can be made in terms of point Coulomb waves which is the same basis as used in our earlier distorted wave calculation (correct for 'point' nuclei). In this approximation the first correction term is relatively easily calculated--it is an integral which cuts off at the nuclear surface and can, in a number of cases, be expressed in a closed analytic form. Although we have some reservations about the approximation we are confident that it should have the right general behavior and our first results show an alarming interaction between the Coulomb and the finite-size corrections (see fig. 2).

The problem with calculating distorted wave matrix elements without approximation lies with the long ranged part of the retarded Green function. It is possible to see that, as a radial integral, this does not lead to satisfactory convergence even if carried over a range several thousand times the nuclear radius. The integral moreover has evaded several ingenious convergence techniques we have dreamed up for its benefit. Currently we are having some success with a method which calculates directly the difference between the radial integral sought, and the point radial integral. Another promising method, which is closely related, calculates the difference with a function derived from the asymptotic expansion. The latter has been tried only on certain "model" integrals which substitute Bessel functions for the Coulomb functions so that we have an exact indefinite integral for comparison.

b) Pair Production

During the past year we have expended considerable effort on calculating the Coulomb distortion effects in pair production. Although the analysis and the debugging of the computer code was largely complete a year ago, we have had to calculate considerably more terms in the partial wave series than we originally contemplated. The analysis and our successful evaluation of the pair production radial integrals is described in detail in Kosik's dissertation⁷⁾ which was completed in August 1980. A manuscript on pair production is being prepared and will be sent to D.O.E. upon completion. Here we will just discuss our results to date.

In fig. (3), we show the distortion factor D which is the ratio of the total pair production cross section with Coulomb distortion to the plane wave (Bethe-Heitler) result as a function of photon energy ω for

$Z = 92$. The result obtained using the Sommerfeld-Mave approximation is labelled DBM, while the other curve is calculated exactly by Øverbø et.al.⁸⁾ from threshold to approximately 5 MeV, and the dashed curve is the so-called "bridging" formula of Øverbø.⁹⁾ The point shown in fig. (3) is our calculated value of the Coulomb distortion. The values of our calculated cross section and distortion factor are 15.87 barns and 0.887 respectively, while the Øverbø interpolation gives 15.98 and .894 respectively. Another interpolation formula was created by Maximon and Gimm¹⁰⁾ to demonstrate that with the available input one could not guarantee that the interpolation was good to 1% as claimed by Øverbø. Their formula gives $\sigma = 15.86$ which agrees very well with our result, but only differs by $\approx 0.8\%$ from Øverbø's result for this case. At higher photon energies the two interpolation formulae differ by larger amounts, so additional calculated points are needed. Those differences may seem small, but recall that in the energy region of interest for photonuclear studies the pair production cross section is often 90-95% of the total, and thus an error of 1% in the pair cross section leads to an error of more than 10% in the extracted photonuclear cross sections.

c) Electroproduction of Pions

Using a model based on the pseudo-vector Born terms, the Δ (1230) in the s and u channels, and the ω meson, we have obtained pion electroproduction amplitudes from the nucleon in the non-relativistic limit¹¹⁾. We have imposed gauge invariance after taking the non-relativistic limit, and our operator is valid in any frame of reference to order p^2/m^2 where p is the three momentum of any particle involved in the process and m is the rest mass of the nucleon. Thus, we must restrict the use of our

amplitudes to four momentum transfer squared $|k^2|$ less than approximately 0.3 GeV^2 . We find good agreement with the available experimental data and give sets of parameters for various production processes. In the real photon limit where the four momentum transfer squared goes to zero, our amplitudes reduce to those of Blomquist and Laget¹²⁾ which have been quite successful in analyzing pion photoproduction¹³⁾. We are currently applying our pion electroproduction operator to the deuteron for values of $|k^2| > 0$.

Fig. 1

Fig. 2

Fig. 3 The ratio of the Coulomb distortion cross section to the plane wave result for $Z = 92$ as a function of photon energy.

References

1. P. Durgapal and D. Onley, Bull. Am. Phys. Soc. 25 (1980) 746. (Abstract only) text appended to this report.
2. P. Durgapal and D. Onley, "Charge and Current Distributions in the Helm Model," to be published. (Submitted as a report to DOE)
3. L. S. Cutler, Phys. Rev. 157 (1967) 885.
4. J. C. Bergstrom, Phys. Rev. C11 (1975) 1514.
5. W. Gargaro and D. Onley, Phys. Rev. C4 (1971) 1032.
6. P. Durgapal and D. Onley, "Coulomb Wave Expansion in Electron Scattering," to be published. (Submitted as a report to DOE)
7. D. W. Kosik, Ph.D. Dissertation, Ohio University, August 1980. (To be submitted to DOE as a report)
8. H. Davies, H. Bethe and L. Maximon, Phys. Rev. 93 (1954) 788.
9. I. Øverbø, Phys. Lett. 718 (1977) 412
10. L. Maximon and H. A. Gimm, Report No. NBSIR78-1456, U.S. Govt. Printing Office.
11. S. Mehrotra and L. E. Wright, "Pion Electroproduction in the Non-Relativistic Limit," accepted by Nuclear Physics. (Submitted as a report to DOE)
12. I. Blomquist and J. M. Laget, Nucl. Phys. A280 (1977) 405.
13. G. N. Epstein, M. K. Singham and F. Tabakin, Phys. Rev. C17 (1978) 702. J. S. O'Connell, W. M. McDonald and E. T. Dressler, Phys. Rev. C19 (1979) 455.

APPENDIX

Abstract Submitted
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Session
Electron
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Finite Size Effects in Virtual Photon Spectra.
P. DURGAPAL AND D. ONLEY, Ohio Univ.*--The generalized
Helm model has been used to evaluate nuclear finite size
effects on virtual photon spectra¹. But the idea of a
spectrum of virtual photons is not useful if its shape
is strongly model dependent. To determine to what ex-
tent these results are model specific we have adapted
Helm-like form factors for both longitudinal and trans-
verse contributions, but with arbitrary parameters.
Looking simultaneously at the nuclear charge-current
distributions and the virtual photon spectra while vary-
ing these parameters we are able to generate a variety
of transition distributions and examine the effect on
the virtual photon spectrum.

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¹A. C. Shotter, J. Phys. G: Nucl. Phys., Vol 5,
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Text follows:

FINITE SIZE EFFECTS IN VPS

In the original VPS idea, the effect of a charged particle near a scattering centre is considered to be equivalent to the incidence of a burst of radiation with the spectrum $N^{(\lambda L)}(E_e, \omega)$, where $\omega = E_i - E_f$ is the transition energy and λL denotes the multipole components (E2, M1, etc.).

If $\sigma_Y^{\lambda L}(\omega)$ is the photo-absorption cross section for photons of multipolarity λL and energy ω , then the integrated cross section for the production of final nuclear states f due to bombardment by electrons of energy E_e is analogous to an expression for the yield in photo nuclear work.

$$\sigma_f(E_e) = \int_{E_{\text{thres}}}^{E_e - m_e} \sum_{\lambda L} \sigma_Y^{\lambda L}(\omega) N^{\lambda L}(E_e, \omega) \frac{d\omega}{\omega}$$

In order to find this we need to calculate the virtual photon spectrum $N^{\lambda L}(E_e, \omega)$.

Earlier calculations have used two approximations to calculate this spectrum:

1. Plane wave Born Approximation
2. No penetration of the target (long wavelength lim. or pt. nucleus approximation).

This result is a relatively simple set of expressions somewhat resembling the Bethe Heitler formula for bremsstrahlung.

Replacement of plane waves by distorted waves has already been done but long wavelength approximation still has to be used. There are a couple of publications in which form factors have been used in place of the long wavelength approximation.

We are studying finite size effects of the nucleus systematically using PWBA, and have made an effort to answer the following questions.

1. For what energies is the long wavelength approximation acceptable?
2. For what energies does the effect of finite size simply reduce to introduction of an effective transition radius--and what effect does this have?
3. For what energies will more details of the nuclear transition densities be necessary?

We have used Helm model form factors for our calculations, since they have simple forms with adjustable parameters. We have also calculated analytic expressions for the charge and current distributions corresponding to these form factors so that we can see what they look like.

We first investigate magnetic transitions for which only one form factor is necessary, and which we have chosen to be

$$F_{TM}^{\ell}(q) = \frac{cq}{\sqrt{2\ell+1}} \exp\left(-\frac{g^2 q^2}{2}\right) \left\{ \sqrt{\ell} \gamma_{\ell+} j_{\ell+1}(qR) + \sqrt{\ell+1} \gamma_{\ell-} j_{\ell-1}(qR) \right\}$$

where g , R and γ_s are adjustable parameters. The precise values of the parameters chosen are unimportant, except of course that they should be realistic and R is proportional to $A^{1/3}$.

The expression for the virtual photon spectrum is an integral over the square of the momentum transfer between kinematic limits $q_{\min}$ and $q_{\max}$.

$$N_{(E_e, \omega)}^{ML} = \frac{\alpha}{4\pi} \frac{\omega^2}{k_1^2} \int_{q_{\min}^2}^{q_{\max}^2} d(q^2) \frac{[(q_{\max}^2 - q^2)(q^2 - q_{\min}^2) + 2q^2(q^2 - E^2)]}{q^2(q^2 - E^2)^2} \frac{|F_{TM}^{\ell}(q)|^2}{|F_{TM}^{\ell}(\omega)|^2}$$

We have evaluated the VPS for M1 transitions. The upper part of Fig. 1 shows that at low energies such as 10 MeV finite size does not produce any observable effect. The virtual photon spectra for mass numbers $A = 1$, $A = 240$ and that calculated in the long wavelength limit all coincide. The lower part of Fig. 1 shows the same at a higher energy, 100 MeV. Here finite

size causes a depression in the virtual photon spectrum. At these energies, we have discovered that irrespective of the details of the form factor if we keep the transition radius R_m defined by a Maclaurin expansion in powers of q^2 of the form factor

$$\frac{|F_{TM}^{\ell}(q)|}{|F_{TM}^{\ell}(\omega)|} = \frac{q^{\ell}}{\omega^{\ell}} \left[1 - \frac{q^2}{2(2\ell+3)} \frac{(\ell+3)}{(\ell+1)} R_m^2 + \dots \right]$$

constant, we get the same virtual photon spectrum. To test this we have tried a very wide variety of shapes for the current distributions. The component of the total current distribution that contributes to magnetic transitions is written as a product of a radial part $J_{\ell,\ell}(r)$ and the appropriate vector spherical harmonic

$$\vec{J}_{M\ell}(\vec{r}) = J_{\ell,\ell}(r) \vec{Y}_{\ell\ell}^m(\hat{r})$$

The radial part $J_{\ell,\ell}(r)$ can be obtained from the form factor and the transition radius can then be obtained as a moment of this radial part.

$$R_m^2 = \frac{(\ell+1)}{\ell+3} \frac{\int d^3r r^{\ell+2} J_{\ell,\ell}^{if}(r)}{\int d^3r r^{\ell} J_{\ell,\ell}^{if}(r)}$$

The upper part of Fig 2 shows the radial part for three different distributions, the Helm Model, a degenerate Helm model with zero skin thickness and a modified exponential shape $[e^{-ar} P(\epsilon r)]$. Note that when we take the skin thickness to be zero in the Helm model we get a spike-like distribution. All these three distributions shown have the same transition radius. The lower part of Fig. 2 shows the virtual photon spectra corresponding to these distributions and interestingly enough they coincide. The main advantage of this result is that we can use any of these distributions to evaluate the virtual photon spectrum. If we use the form factor corresponding to the modified

exponential distribution, we can find an analytic expression for the virtual photon spectrum. At higher energies the VPS for the three distributions start deviating as we approach the end point and at still higher energies the Helm model form factors which were designed for nuclear transitions are not appropriate as we start getting zeros in the cross section which appear as spikes in the effective virtual photon spectrum (lower part of Fig. 2). Even at these energies the modified exponential distribution gives reasonable looking virtual photon spectrum. But because of the deviations occurring near the end point, at these energies, we need more information about the transition densities than just the transition radius.

We can make similar observations in the case of electric transitions also, but now, things are more complicated because we have both a charge and a current distribution contributing to the virtual photon spectrum in the form of its Coulomb and transverse components. The Coulomb and the transverse form factors are not necessarily related except by Siegert theorem at low momentum transfer

$$F_{TE}^{\ell}(q) \simeq \frac{\omega}{q} \sqrt{\frac{\ell+1}{\ell}} F_C^{\ell}(q) \quad , \quad q \simeq \omega.$$

Initially, we simplify things by assuming a modified form of Helm model for which this relation is satisfied for all values of momentum transfer. In collective dynamical terms this is equivalent to assuming an irrotational, incompressible fluid flow; also known as the Tassie term. The charge density and the components of the total current that contribute to electric transitions are

$$\rho(r) = \rho_{\ell}(r) Y_{\ell m}(\hat{r})$$

$$\vec{J}_E(\vec{r}) = J_{\ell\ell-1}(r) \vec{Y}_{\ell\ell-1}^m(\hat{r}) + J_{\ell,\ell+1}(r) \vec{Y}_{\ell\ell+1}^m(\hat{r})$$

Under the assumption of incompressible fluid flow, the $J_{\ell, \ell+1}$ component of the current is zero. Fig. 3 shows a plot of the transition charge and current distribution for dipole and quadrupole transitions. [In case of degenerate Helm model i.e. if we take skin thickness to be zero, the charge distribution becomes a delta function $\delta(r-R)$ for all transitions, the current distribution is constant for $\ell = 1$ and varies as r for $\ell = 2$ up to radius R .]

Fig. 4 shows the virtual photon spectra for E1 and E2 transitions corresponding to these distributions. The effect is larger for E2 than E1. In this case again we can show that keeping transition radius constant we get same results with different form factors.

Now if we relax the Tassie term i.e. equivalent to introducing compressibility in the fluid or magnetic current in the model, the parameters in the transverse form factor are then unrelated to those in the Coulomb form factor except that continuity must be observed. We then have a non-zero $J_{\ell \ell+1}$ component, the effect of which is shown in Fig. 5 and depends on how drastic the change in R and g is.

The introduction of compressibility in the distribution of matter is equivalent to introducing magnetization due to anomalous magnetic distribution or both. As far as electron scattering is concerned they both give rise to the same effect as can be seen from a comparison of current densities in the two cases, Fig. 6.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6