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CP Nonconservation in Dynamically Broken Gauge Theories

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Abstract

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The recent proposal of Eichten, Lane, and Preskill for CP nonconservation in electroweak gauge theories with dynamical symmetry breaking is reviewed. Through the alignment of the vacuum with the explicit chiral symmetry breaking Hamiltonian, these theories provide a natural way to understand the dynamical origin of CP nonconservation. Special attention is paid to the problem of strong CP violation. Even though all vacuum angles are zero, this problem is not automatically avoided. In the absence of strong CP violation, the neutron electric dipole moment is expected to be $10^{-24} - 10^{-26}$ e-cm. A new class of models is proposed in which both strong CP violation and large $|\Delta S| = 2$ effects may be avoided. In these models, $|\Delta C| = 2$ processes such as $D^0 \bar{D}^0$ mixing may be large enough to observe.

1. Introduction

Since the discovery of CP violation in the neutral-K system [1], there have been many theoretical attempts to explain and understand it. Especially notable are the superweak model of Wolfenstein [2], the six-quark model of Kobayashi and Maskawa (KM) [3], and the Higgs-induced CP-violation models of Lee [4] and Weinberg [5]. Many of these proposals are attractive and, indeed, remain viable. However, none of them really explains the origin of CP violation. To some extent it is always put in by hand. For example, in the KM model, one arbitrarily assumes the fermion-Higgs couplings to be complex, while in the Lee and Weinberg models there is an arbitrary choice of Higgs self-couplings that leads to spontaneous CP-violation. Whatever the correct mechanism may be, one would very much like to understand the dynamical origin of the CP-violation phases.

Equally serious, and more immediate, is the problem of strong CP violation induced by QCD instanton effects [6]. The very attractive Peccei-Quinn proposal for eliminating strong CP violation inevitably leads to an axion [7], and the existence of this particle has, apparently, been ruled out experimentally [8]. Thus, one expects a neutron electric dipole moment $d_N \sim e m_u \sin \bar{\theta} / M_N^2 \sim 10^{-16} \sin \bar{\theta} \text{ e-cm}$, where $\bar{\theta} = \theta + \arg \det m_q$, θ is the QCD vacuum angle and m_q is the quark current-algebraic mass matrix [9]. Experimentally, $d_N \leq 10^{-24} \text{ e-cm}$ [10], so that a successful model must explain why $\bar{\theta} \leq 10^{-8}$ or, equivalently, why $\text{Im } m_u / \text{Re } m_u \leq 10^{-8}$. There is no natural way to understand such a small parameter in the currently popular models of CP violation. We view this as a serious deficiency of QCD as well as of the otherwise successful standard electroweak theory [11].

Dynamically broken electroweak gauge theories provide a natural explanation for the origin of weak CP violation and, possibly, for the absence of strong CP violation [12]. The proposal we describe here requires nothing beyond the new gauge interactions already deemed necessary for a realistic electroweak theory with dynamical symmetry breaking [13-15].

Spontaneous CP violation can occur when we carry out Dashen's procedure to identify the correct chiral-perturbative vacuum [17]. Whether this happens is determined in principal by only the gauge group and fermion representation content of the theory. If CP invariance is spontaneously broken, CP violation in the electroweak sector will be exactly as in the KM model, and the origin of the phases will be understood to be the alignment of the vacuum symmetry group within the full chiral flavor group. On the other hand, although there are no nonzero vacuum angles, strong CP violation is not automatically avoided [18]. We state a mathematical criterion for the absence of strong CP violation; if it is satisfied, CP-nonconserving phases in the quark mass matrix are naturally suppressed by a factor of order 10^{-9} . Together with additional CP-nonconserving phases in the new gauge interactions responsible for explicit chiral symmetry breaking [15,16], these produce a neutron electric dipole moment of order $10^{-24} - 10^{-26}$ e-cm. This moment is 4-6 orders of magnitude larger than in the standard KM model [19], and within the range of experiment in the next few years [20].

We also suggest a class of models in which strong CP violation may be absent. If this new proposal can be realized, all mixing angles and CP-violating phases appear in the "up" sector of the theory. This has the added benefit that the new gauge interactions do not mediate $|\Delta S| = 2$

processes and, in particular, off-diagonal terms in the $K^0-\bar{K}^0$ mass matrix are controlled by electroweak interactions, as usual. In this scenario, $D^0-\bar{D}^0$ mixing may be large enough to be detected with, say, the new Mark III detector to be installed at SPEAR.

Before describing the details of our proposal, a brief review of the general structure of dynamically broken electroweak gauge theories is in order. For a more complete discussion, see Refs. 13-16 and the review of Lane and Peskin [21].

2. The Structure of Dynamically Broken Electroweak Gauge Theories

These theories are based on gauge interactions only, without elementary scalar fields and without fermion bare mass terms. In addition to the familiar color [$G_C = SU(3)$] and electroweak [$G_W = SU(2) \otimes U(1)$] interactions, a new gauge interaction ["hypercolor," with gauge group G_H] is required [13,14]. We believe that ordinary color interactions become strong at a scale $\Lambda_C \sim \frac{1}{3}$ GeV, dynamically triggering the spontaneous breakdown of quark chiral symmetries. By analogy, it is assumed that G_H interactions are asymptotically free, that they grow strong at a scale $\Lambda_H \sim 1$ TeV, and that this triggers the spontaneous breakdown of "hyperfermion" chiral symmetries. It is this dynamical chiral symmetry breaking, in concert with the Higgs mechanism [22,23], which results in the breakdown of G_W to $U(1)_{em}$.

A theory based on $G_H \otimes G_C \otimes G_W$ interactions alone cannot be realistic. A "sideways" gauge interaction (with group G_S) is needed to break explicitly all chiral symmetries not gauged by G_W [15,16]. In plain language, quarks, leptons, and hyperfermions must acquire non-zero current algebra masses, and this task is accomplished by G_S interactions. At a scale $\Lambda_S \sim 100$ TeV,

G_S is dynamically broken down to a subgroup containing $G_H \otimes G_C$, and heavy G_S gauge bosons mediate transitions between quarks and leptons on the one hand and hyperfermions on the other. (See Fig. 1.)

It simplifies our subsequent discussion to assume that G_S commutes with $SU(2)_W$ (but not $U(1)_W$; see Ref. 15). Then all fermions--quarks, leptons, and hyperfermions--are contained in at most four irreducible representations (IR's) of G_S . More representations would imply extra chiral symmetries and an unwanted axion. Two of these, which we call $\mathcal{D}_{uL}^S$ and $\mathcal{D}_{dL}^S$, contain all fermions with $(T_3)_{\text{weak}} = +\frac{1}{2}$ and $-\frac{1}{2}$, respectively. These two representations are equivalent. The right-handed, weak isosinglet fermions are contained in (at most) two IR's, $\mathcal{D}_{uR}^S$ and $\mathcal{D}_{dR}^S$. Right-handed charge $+\frac{2}{3}$ quarks and hyperfermions to which they couple are contained in $\mathcal{D}_{uR}^S$; right-handed charge $-\frac{1}{3}$ quarks and their associated hyperfermions are contained in $\mathcal{D}_{dR}^S$; right-handed leptons belong to one (or both?) of these two IR's. To avoid degenerate up- and down-quark mass matrices, $\mathcal{D}_{uR}^S$ and $\mathcal{D}_{dR}^S$ must be inequivalent. This completes our review, and we turn now to the basic idea of our proposal.

3. Vacuum Alignment and Spontaneous CP Nonconservation

In the effective gauge theory which describes physics well below 100 TeV, each of the G_S representations transforms reducibly under $G_H \otimes G_C$. (From now on, we may ignore leptons, which are $G_H \otimes G_C$ singlets.) Neglecting the broken sideways interactions and the weakly coupled electroweak interactions, the $G_H \otimes G_C$ invariant effective Hamiltonian $\mathcal{H}_0$ of quarks and hyperfermions respects a global (chiral) flavor-symmetry group G_f . G_W is a subgroup of G_f . When hypercolor and color grow strong, G_f is dynamically broken to a subgroup S_f . Of the many Goldstone bosons

that result, three are absorbed by the $W^\pm$ and Z^0 bosons. The remaining ones acquire mass from the explicit chiral-symmetry-breaking perturbation $\mathcal{K}'$ generated by sideways and electroweak interactions.

The G_f -symmetric Hamiltonian $\mathcal{K}_0$ has many degenerate vacua, parameterized by the coset space G_f/S_f . If $|\Omega\rangle$ is the "standard" ground state of $\mathcal{K}_0$, with symmetry group S_f , then $|\Omega(W)\rangle = W(g)|\Omega\rangle$ has symmetry group gS_fg^{-1} , where $W(g)$ represents $g \in G_f/S_f$. As Dashen emphasized [17], $\mathcal{K}'$ lifts the vacuum degeneracy: The true chiral-perturbative vacuum,

$$|\Omega(U)\rangle = \lim_{\mathcal{K}' \rightarrow 0} |\text{ground state of } \mathcal{K}_0 + \mathcal{K}'\rangle, \text{ is determined by minimizing the vacuum energy}$$

$$E(W) = \langle \Omega(W) | \mathcal{K}' | \Omega(W) \rangle = \langle \Omega | W^{-1} \mathcal{K}' W | \Omega \rangle. \quad (1)$$

The minimum occurs at $W=U$. In the calculations described below, it is convenient to regard the vacuum $|\Omega\rangle$ (and S_f) as fixed. Then, from among all G_f -equivalent perturbations $\mathcal{K}'(W) = W^{-1}(g)\mathcal{K}'W(g)$, we determine that one which minimizes $E(W)$; and $|\Omega\rangle = \lim_{\mathcal{K}' \rightarrow 0} |\text{ground state of } \mathcal{K}_0 + U^{-1}\mathcal{K}'U\rangle$. This necessary procedure of matching the ground state to the chiral perturbation is known as "subgroup alignment" [13] or "vacuum alignment" [24].

Now we see how spontaneous CP violation can arise naturally. First, the vacuum angles are naturally zero. To see this, suppose that G_H and G_S , like G_C , are simple groups; extension to non-simple groups is straightforward. The fact that $G_H \otimes G_C \subset G_S$ implies equality of the vacuum angles: $\theta_H = \theta_C = \theta_S$. Equivalently, $\theta_S = \frac{1}{2}(\theta_H + \theta_C)$ and $\theta_- = \frac{1}{2}(\theta_C - \theta_H) \equiv 0$. The absence of bare masses implies that θ_S is unobservable, and may be freely rotated to zero. Thus, $\theta_H = \theta_C = 0$ and $|\Omega\rangle$ is (naively) P- and T-invariant. (Note: As we discuss below, this is not a solution to the strong CP problem [18].)

Second, we assume that the breaking of G_S does not introduce CP nonconservation, so that the effective interaction below 100 TeV is CP invariant. Thus, the time-reversal operations that leave $|\Omega\rangle$ and $\mathcal{K}'$ invariant are identical, and the vacuum energy is CP- and T-symmetric:

$$E(W) = E(W^*) \quad . \quad (2)$$

This equation tells us that E has T-invariant extrema corresponding to real unitary transformations $\bar{W}$. But, the minimum of E may be discretely degenerate, occurring at U and $U^* \neq U$. In that case, the true chiral perturbation $U^{-1}\mathcal{K}'U$ is not invariant under the T-operation appropriate to $|\Omega\rangle$, and T and CP are spontaneously broken. This possibility was suggested first by Dashen [17] and Lee [4].

We emphasize the naturalness of this scenario: The sideways group G_S and the pattern of its breakdown to $G_H \otimes G_C$ uniquely determine the separation of the fermion Hamiltonian into $\mathcal{K}_0$ and $\mathcal{K}'$. The structure of $\mathcal{K}_0$, i.e., the $G_H \otimes G_C$ representation content of fermions, uniquely determines G_f and, dynamically, the vacuum symmetry groups $g S_f g^{-1}$. And finally, $\mathcal{K}'$ determines whether E is minimized by a real or complex unitary transformation U .

Let us turn now to specific details of minimizing the vacuum energy E . While, unfortunately, no very satisfactory model of dynamically broken electroweak interactions has been found, we can say quite a lot about the structure of the flavor group G_f , the form of the perturbation $\mathcal{K}'$, and the possible outcomes and phenomenological consequences of vacuum alignment.

3.1. Global Flavor Groups, G_F and S_F

The flavor symmetry G_F of $\mathcal{H}_0$ is determined entirely by the $G_H \otimes G_C$ representation content of fermions. To ensure that G_W breaks down to a conserved electric charge and that the successful relation $\mu_W/\mu_Z \cos \theta_W = 1$ [13,14] is maintained, it is sufficient to assume the following [12,24]: With fermion fields denoted by ψ_{Lri}^0 and ψ_{Rri}^0 , left-handed (L) fields transform under G_W as isodoublets, while right-handed (R) fields transform as weak isosinglets. The index ρ identifies the particular irreducible representation $\mathcal{D}^\rho$ of $G_H \otimes G_C$ according to which ψ^ρ transforms. All $\mathcal{D}^\rho$ are assumed to be complex. The gauge group $G_H \otimes G_C$ acts on the (collective) index i . The index r labels the various flavors transforming as $\mathcal{D}^\rho$. There is an even number of such flavors, $r=1, \dots, 2n_\rho$. The first n_ρ may be thought of as "up" flavors and the second n_ρ as "down" flavors, so long as $G_H \otimes G_C$ commutes with G_W and vacuum alignment does not destroy electric charge conservation. Note that, under our assumptions, $G_H \otimes G_C$ interactions are vectorial.

The flavor group of $\mathcal{H}_0$ is, then,

$$G_F = \prod_{\rho} \left[SU(2n_{\rho})_L \otimes SU(2n_{\rho})_R \otimes U_V^{\rho}(1) \right] \otimes U_A(1)'s \quad (3)$$

The axial-vector $U_A(1)$ symmetries are generated by those linear combinations of

$$j_{5\mu}^{\rho} = \sum_i \sum_{r=1}^{2n_{\rho}} \left[\bar{\psi}_{Rri}^{\rho} \gamma_{\mu} \psi_{Rri}^{\rho} - \bar{\psi}_{Lri}^{\rho} \gamma_{\mu} \psi_{Lri}^{\rho} \right] \quad (4)$$

which have no G_H - nor G_C -anomalous divergence. Suppose

$$\partial^\mu j_{5\mu}^\rho = 2n_\rho \left[T_\rho^H \frac{g_H^2}{8\pi^2} \text{Tr } F_H \cdot \tilde{F}_H + T_\rho^C \frac{g_C^2}{8\pi^2} \text{Tr } F_C \cdot \tilde{F}_C \right], \quad (5)$$

where T_ρ^H (T_ρ^C) is the trace of the square of the hypercolor (color) generators in the representation $\mathfrak{D}^\rho$. (Here we are assuming the G_H is simple; otherwise there is a term $T_\rho^H (g_H^2/8\pi^2) \text{Tr } F_H \cdot \tilde{F}_H$ for each simple non-abelian factor in G_H .) Then the $U_A(1)$'s are generated by

$$j_{5\mu}^{(\kappa)} = \sum_\rho \alpha_\rho^{(\kappa)} j_{5\mu}^{(\rho)}, \quad (6)$$

with $\alpha_\rho^{(\kappa)}$ chosen so that

$$\sum_\rho \alpha_\rho^{(\kappa)} T_\rho^H = \sum_\rho \alpha_\rho^{(\kappa)} T_\rho^C = 0. \quad (7)$$

In short, all G_f generators must be G_H - and G_C -anomaly free; currents with anomalies are not regarded as generating approximate symmetries of the theory.

The (standard) vacuum-invariance group is assumed to be

$$S_f = \prod_\rho \left[SU(2n_\rho)_V \otimes U_V^\rho(1) \right] \quad (8)$$

where $SU(2n_\rho)_V$ is the obvious diagonal subgroup of $SU(2n_\rho)_L \otimes SU(2n_\rho)_R$. This is in accord with recent work on spontaneous breaking of chiral symmetries of the form (3) [25]. Equivalently, the standard vacuum $|\Omega\rangle$ is defined to be the one in which fermion bilinear condensates are

$$\langle \Omega | \bar{\psi}_{Lri}^{\rho} \psi_{Rsj}^{\sigma} | \Omega \rangle = -\delta_{\rho\sigma} \delta_{rs} \delta_{ij} \Delta_{\rho} . \quad (9)$$

Here, Δ_{ρ} is real, positive and of order Λ_H^3 (Λ_C^3) for hyperfermions (quarks). It is understood that the operators $(\bar{\psi}_L^{\rho} \psi_R^{\sigma})$ have been renormalized and that Δ_{ρ} is finite.

Under G_f transformations,

$$\begin{aligned} \psi_{Lri}^{\rho} &\rightarrow \sum_{r'=1}^{2n_{\rho}} W_{\rho rr'}^L \psi_{Lr'i}^{\rho} , \\ \psi_{Rri}^{\rho} &\rightarrow \sum_{r'=1}^{2n_{\rho}} W_{\rho rr'}^R \psi_{Rr'i}^{\rho} , \end{aligned} \quad (10)$$

where W_{ρ}^L and W_{ρ}^R are unitary $2n_{\rho} \times 2n_{\rho}$ matrices. S_f transformations have $W_{\rho}^L = W_{\rho}^R$, for every ρ , while G_f/S_f is represented by the set of unitary matrices $\{W_{\rho} = W_{\rho}^R W_{\rho}^{L\dagger}\}$. The requirement that all G_f transformations be anomaly free means that the W_{ρ} are subject to the modularity constraints

$$\prod_{\rho} [\det W_{\rho}]^{T_{\rho}^H} = \prod_{\rho} [\det W_{\rho}]^{T_{\rho}^C} = 1 . \quad (11)$$

3.2. Chiral Perturbation, $\mathcal{K}'$

Both sideways and electroweak interactions contribute to the explicit G_f -breaking Hamiltonian $\mathcal{K}'$. Recall that this is an effective interaction which (together with $\mathcal{K}_0$) describes physics at energies of order 1 TeV and below. Thus, the G_S -contribution, $\mathcal{K}'_S$, is obtained from an operator

product expansion in which the very massive sideways gauge bosons are integrated out to obtain a $G_H \otimes G_C \otimes G_W$ -invariant effective interaction. The leading G_f -breaking terms in $\mathcal{K}'_S$ are four-fermion operators. Higher-dimension operators are suppressed by additional powers of Λ_S^{-1} ; e.g., six-fermion terms contribute to $E(W)$ a factor of order $(\Lambda_H/\Lambda_S)^2 \sim 10^{-4}$ times the four-hyperfermion terms. To the extent that we can ignore contributions to the four-fermion part of $\mathcal{K}'_S$ involving more than one internal massive G_S -boson (these contain additional powers of $g_S^2(\Lambda_S)/8\pi^2$, which may well be small at $\Lambda_S \sim 100$ TeV), $\mathcal{K}'_S$ is given by the current $\times$ current interaction

$$\mathcal{K}'_S = \frac{1}{8} g_S^2 (\mu_S^{-2})_{ab} j_\mu^a j_\mu^b. \quad (12)$$

Here, μ_S^2 is the massive sideways gauge boson mass matrix ($\mu_S^2 \sim g_S^2 \Lambda_S^2$), and j_μ^a is a broken sideways current:

$$j_\mu^a = j_{L\mu}^a + j_{R\mu}^a,$$

$$j_{L(R)\mu}^a = \bar{\psi}_{L(R)ri}^p \gamma_\mu t_{pri,\sigma sj}^{L(R)a} \psi_{L(R)sj}^\sigma \quad (13)$$

(sum over repeated indices).

The Hermitian G_S -generator matrices t^{La} and t^{Ra} represent $\mathcal{D}_{u_L}^S \oplus \mathcal{D}_{d_L}^S$ and $\mathcal{D}_{u_R}^S \oplus \mathcal{D}_{d_R}^S$, respectively. They are block-diagonal into "up" and "down" blocks, with $(t_{\rho,\sigma}^{La})_{up} = (t_{\rho,\sigma}^{La})_{down}$, but $(t_{\rho,\sigma}^{Ra})_{up} \neq (t_{\rho,\sigma}^{Ra})_{down}$ (at least when ρ refers to quarks and σ to hyperfermions). Because of the structure of

G_f and S_f , only the $j_L j_R$ part of $\mathcal{H}'_S$ contributes nontrivially to $E(W = W_R W_L^\dagger)$:

$$\mathcal{H}'_S = \Lambda_{ab}^* \bar{\psi}_{Lr}^\rho \gamma_\mu t_{\rho r, \sigma s}^{La} \psi_{Ls}^\sigma \bar{\psi}_{Rs'}^{\sigma'} \gamma^\mu t_{\sigma' s', \rho' r'}^{Rb} \psi_{Rr'}^{\rho'} + \text{irrelevant terms} + \text{higher-dimension operators} \quad (14)$$

Here, $\Lambda_{ab} = \Lambda_{ba} = \Lambda_{ab}^* = \frac{1}{2} g_S^2 [(\mu_S^{-2})_{ab} + (\mu_S^{-2})_{ba}]$; repeated indices are summed over; and $G_H \otimes G_C$ indices are suppressed. Time-reversal invariance of $\mathcal{H}'_S$ requires

$$\Lambda_{ab} t_{\rho r, \sigma s}^{La} t_{\sigma' s', \rho' r'}^{Rb} = \Lambda_{ab} t_{\rho r, \sigma s}^{La*} t_{\sigma' s', \rho' r'}^{Rb*} \quad (15)$$

The electroweak perturbation $\mathcal{H}'_W$ is interesting in its own right; indeed, until acceptable candidates for G_S and G_H emerge, it is the only part of $\mathcal{H}'$ we can calculate with confidence. With G_f and S_f as specified, nontrivial electroweak contributions to $E(W)$ do not occur until fourth order in the $SU(2) \otimes U(1)$ couplings g_2 and g_1 [15,24]. Their leading contribution to E will be $O(\alpha^2 \Lambda_H^4 \ln 1/\alpha)$. Taking advantage of the fact that the scale at which G_W breaks down is precisely Λ_H , this contribution may be calculated from [26]

$$\begin{aligned} \mathcal{H}'_W = & -i \frac{g_1^2 g_2^2}{4} \sum_{\alpha=1}^3 \int d^4x d^4y d^4z \Delta^{\mu\nu}(x-y) \Delta^{\lambda\rho}(z) \\ & \times T(j_{L\mu}^\alpha(x) j_{L\nu}^\alpha(y) j_{R\lambda}^3(z) j_{R\rho}^3(0)) \quad (16) \end{aligned}$$

In Eq.(16), $i \Delta^{\mu\nu}$ is a massless spin-one propagator, $j_{L\mu}^\alpha$ is a $SU(2)_W$ current, and $j_{R\mu}^3$ is the right-handed T_3 part of the $U(1)_W$ current. The $O(\alpha^2 \ln 1/\alpha)$ contributions to $E(W)$ are determined from the infrared divergent part of $\langle \Omega | W^{-1} \mathcal{H}'_W | \Omega \rangle$; the infrared cutoff is provided ultimately by the Z^0 -mass. We remark that the infrared-finite terms of $O(\alpha^2)$ involve incalculable strong hypercolor interactions.

3.3. Minimizing the Vacuum Energy

To write an explicit expression for $E(W)$, we define first the four-fermion condensates $\Delta_{\rho\sigma}$ and $\Delta'_{\rho\rho}$, by

$$\begin{aligned} \langle \Omega | \bar{\psi}_{Lr}^\rho \gamma_\mu \psi_{Ls}^\sigma \bar{\psi}_{Rr'}^{\sigma'} \gamma_\mu \psi_{Rr'}^{\rho'} | \Omega \rangle = & -\delta_{\rho\rho'} \delta_{rr'} \delta_{\ell\ell'} \delta_{\sigma\sigma'} \delta_{ss'} \delta_{jk} \Delta_{\rho\sigma} \\ & -\delta_{\rho\sigma} \delta_{rs} \delta_{ij} \delta_{\rho'\sigma'} \delta_{r's'} \delta_{k\ell} \Delta'_{\rho\rho'} \end{aligned} \quad (17)$$

As in Eq.(9), it is understood that the four-fermion operators have been $G_H \otimes G_C$ renormalized. $\Delta_{\rho\sigma}$ and $\Delta'_{\rho\rho}$, can be related to integrals of spectral functions for sideways currents; they arise from graphs of the type shown in Figs. 2a and 2b. The $\Delta'_{\rho\rho}$ term in Eq.(17) is a G_F -invariant, contributing a trivial additive constant to $E(W)$. The condensate $\Delta_{\rho\sigma}$ is of order $\Delta_\rho \Delta_\sigma$, i.e., of order $\Lambda_\rho^3 \Lambda_\sigma^3$. The $O(\alpha^2 \ln 1/\alpha)$ part of the electroweak contribution to $E(W)$ comes from the graph in Fig. 3 [26].

Then, apart from trivial constants, the contribution of higher dimension operators, and $O(\alpha^2)$ electroweak terms, the vacuum energy is given by

$$\begin{aligned}
E(W) = & -\Lambda_{ab} \sum_{\rho, \sigma} \text{Tr} (t_{\rho, \sigma}^{La} W_{\sigma}^{\dagger} t_{\sigma, \rho}^{Rb} W_{\rho}) \Delta_{\rho\sigma} \\
& - \sum_{\rho, \sigma} \sum_{\alpha=1}^3 \text{Tr} \left(\frac{\tau_{\rho}^{\alpha}}{2} W_{\rho}^{\dagger} \frac{\tau_{\rho}^3}{2} W_{\rho} \right) \text{Tr} \left(\frac{\tau_{\sigma}^{\alpha}}{2} W_{\sigma}^{\dagger} \frac{\tau_{\sigma}^3}{2} W_{\sigma} \right) I_{\rho\sigma} .
\end{aligned} \tag{18}$$

Here τ_{ρ}^{α} is an $SU(2)_W$ generator, normalized to $\text{Tr}(\tau_{\rho}^{\alpha} \tau_{\rho}^{\beta}) = 2n_{\rho} \delta_{\alpha\beta}$, and $I_{\rho\sigma} = I_{\sigma\rho}$ is a real integral over a product of spectral functions--one for the ψ^{ρ} part of the weak current, the other for the ψ^{σ} part. $I_{\rho\sigma}$ is of order $\alpha^2 \Lambda_{\rho}^2 \Lambda_{\sigma}^2 n_{\rho} n_{\sigma} / \alpha$. Using Eq.(15), one readily verifies that $E(W) = E(W^*)$.

Contributions to dimension-four operators in $\mathcal{H}'_S$ from higher-order sideways exchanges (involving additional powers of $g_S^2(\Lambda_S)/8\pi^2$) do not change the structure of $E(W)$ in any very essential way. Dimension-six and higher operators contribute additional factors of W_{ρ} , but they are suppressed by powers of Λ_S^{-2} . The largest terms in $E(W)$ involve integrations over hyperfermions only. In the sideways term, these are nominally $\sim (1 \text{ TeV})^6 / (100 \text{ TeV})^2 \cong (100 \text{ GeV})^4$ times group-theoretic factors; we expect, but have not proved, that $\Delta_{\rho\sigma}$ is positive. In simple model calculations of $I_{\rho\sigma}$, we find that it is positive and that the electroweak term is comparable or somewhat less in magnitude than the sideways term. Finally, these same calculations indicate that the neglected $O(\alpha^2)$ terms are 5-10 times smaller than the $O(\alpha^2 n_{\rho} n_{\sigma} / \alpha)$ ones.

The vacuum energy is to be minimized subject to the constraints that W_{ρ} is unitary and satisfies the modularity conditions (11). Actual minimization requires a specific model, of course. However, an important condition on all extrema of E can be obtained from a

simple generalization of Nuyts' method [27] of introducing Lagrange multipliers and extremizing the quantity

$$\begin{aligned} \mathcal{F}(W; B, \nu_H, \nu_C) = & E(W) + \sum_{\rho} \text{Tr}[B_{\rho} (W_{\rho}^{\dagger} W_{\rho} - 1_{\rho})] \\ & + i\nu_H \left[\prod_{\rho} (\det W_{\rho})^{T_{\rho}^H} - 1 \right] + i\nu_C \left[\prod_{\rho} (\det W_{\rho})^{T_{\rho}^C} - 1 \right]. \end{aligned} \quad (19)$$

The matrices B_{ρ} and the real parameters ν_H and ν_C are the Lagrange multipliers. We find that E is stationary at W provided that

$$(M_{\rho} - M_{\rho}^{\dagger}) \Delta_{\rho} = i(\nu_H T_{\rho}^H + \nu_C T_{\rho}^C) 1_{\rho} \quad (20)$$

where the "mass" M_{ρ} is defined by

$$M_{\rho rr'} \Delta_{\rho} = -W_{\rho rs}^{L\dagger} \frac{\partial E}{\partial W_{\rho s' s}} W_{\rho s' r'}^R \bigg|_{W \text{ extremizes } E} \quad (21)$$

In particular, at the minimum $W=U$ of the vacuum energy, M_{ρ} is given by

$$\begin{aligned} M_{\rho} \Delta_{\rho} = & \Lambda_{ab} \sum_{\sigma} U_{\rho}^{L\dagger} t_{\rho, \sigma}^{La} U_{\sigma}^{\dagger} t_{\sigma, \rho}^{Rb} U_{\rho}^R \Delta_{\rho \sigma} \\ & + 2 \sum_{\sigma} \sum_{\alpha=1}^3 \left(U_{\rho}^{L\dagger} \frac{\tau_{\rho}^{\alpha}}{2} U_{\rho}^{\dagger} \frac{\tau_{\rho}^3}{2} U_{\rho}^R \right) \text{Tr} \left(\frac{\tau_{\sigma}^{\alpha}}{2} U_{\sigma}^{\dagger} \frac{\tau_{\sigma}^3}{2} U_{\sigma} \right) I_{\rho \sigma} \end{aligned} \quad (22)$$

The separate determination of U_ρ^R and U_ρ^L will be explained soon. First, let us improve Eq.(20): It is easy to show from Eqs.(20) and (22) that

$$2i \sum_\rho n_\rho (T_\rho^H v_H + T_\rho^C v_C) \equiv \sum_\rho \text{Tr} (M_\rho - M_\rho^\dagger) \Delta_\rho = 0.$$

Also, since G_H and G_C are embedded in G_S , $2 \sum_\rho n_\rho T_\rho^H = 2 \sum_\rho n_\rho T_\rho^C$. Thus, $v_H = -v_C$, a reflection of the fact that θ_S is unobservable. Eq.(20) becomes

$$(M_\rho - M_\rho^\dagger) \Delta_\rho = i v_C (T_\rho^C - T_\rho^H) 1_\rho \quad (23)$$

Most of our remaining discussion follows from Eqs.(22) and (23).

First, we can illuminate the meaning of the mass M_ρ and the Lagrange multiplier v_C by considering the problem of vacuum alignment in QCD. We assume n flavors of quark q_r with $G_f = SU(n)_L \otimes SU(n)_R \otimes U_V(1)$, vacuum symmetry $S_f = SU(n)_V \otimes U_V(1)$ corresponding to the condensates $\langle \Omega | \bar{q}_{Lr} q_{Rs} | \Omega \rangle = -\delta_{rs} \Delta_q$ ($\Delta_q = \Delta_q^* > 0$), and chiral perturbation

$$\mathcal{K}'_q = \bar{q}_L m q_R + \bar{q}_R m^\dagger q_L,$$

with m an $n \times n$ "naive" quark mass matrix. Any vacuum angle (θ_C) dependence is contained in m_q . We are to minimize

$$E_q(W) = \langle \Omega | W^{-1} \mathcal{K}'_q W | \Omega \rangle = -\text{Tr} (mW + W^\dagger m^\dagger) \Delta_q$$

subject to the constraints that $W = W_R W_L^\dagger$ is unitary and unimodular (i.e., there is no approximate $U_A(1)$ symmetry which is spontaneously

broken). At the extrema of E , we find

$$(M - M^\dagger) \Delta_q = i\nu 1 \quad ,$$

where ν is the Lagrange multiplier for the constraint $\det W = 1$,

and (at the minimum of E_q , in particular)

$$M \Delta_q \equiv - W_L^\dagger \frac{\partial E}{\partial \tilde{W}} q W_R \Big|_{W=U} = U_L^\dagger m U_R \Delta_q .$$

Minimizing E determines only $U = U_R U_L^\dagger$. However, M is the true quark mass matrix, to be used in calculations of chiral symmetry breaking, and it ought to be diagonal. Since $M - M^\dagger$ is proportional to 1, M and $M^\dagger$ can be simultaneously diagonalized by a unitary S_F -transformation matrix V . This removes the ambiguity in U_R and U_L , and we assume henceforth that M and $M^\dagger$ are diagonal.

Now, if $\nu \neq 0$, we know that the true chiral perturbation, $U^{-1} \mathcal{H}'_q U = \bar{q}_L M q_R + \text{h.c.}$, has a P- and T- violating part, $(i\nu/\Delta_q) \bar{q} \gamma_5 q$, and that ν/Δ_q vanishes as the lightest quark mass [9]. However, it is easy to see that $\nu = 0$ if m can be made diagonal, with real positive eigenvalues, by unitary U_L and U_R with $\det U = 1$: Suppose we could remove the constraint $\det W = 1$. Then the absolute minimum of E_q would occur for U'_L and U'_R such that M is real, diagonal, and positive. If $\det U'_R U'^{\dagger}_L = 1$, then $\nu = 0$. With the θ_C -dependence absorbed in m_q , this condition for ν to be zero corresponds to $\bar{\theta} = \arg(\det m) = 0$.

Now return to the main problem. As in QCD, all ambiguity in U^L_ρ and U^R_ρ is removed by demanding that M_ρ and $M^\dagger_\rho$ are diagonal. This is a sensible prescription, especially for quarks ($\rho = q$). Indeed, we now argue that M_q is essentially the same as the quark current-algebra mass matrix m_q and, in particular,

$$\begin{aligned}
\text{Im } m_q &= \text{Im } M_q + \text{relative order } 10^{-9} \\
&= (\nu_C T_q^C / 2\Delta_q) 1_q + \text{relative order } 10^{-9}.
\end{aligned} \tag{24}$$

By definition, $m_{qrr'}$ is the coefficient of $\bar{q}_{Lr} q_{Rr'}$ in an effective low-energy interaction obtained from $\mathcal{K}'$ by integrating out all degrees of freedom (hyperfermions, hypergluons, and hard G_C -gluons) above ~ 100 GeV.

First, notice that electric charge is conserved after vacuum alignment if and only if all $U_\rho^{L,R}$ are block-diagonal:

$$U_\rho^{L,R} = \begin{pmatrix} U_{\rho, \text{up}}^{L,R} & 0 \\ 0 & U_{\rho, \text{down}}^{L,R} \end{pmatrix} \tag{25}$$

Our assumptions on the structure of G_S -representations and of G_f and S_f were designed, in part, to ensure that charge is conserved, and we assume henceforth that it is. From Eq.(22), it is clear that the electroweak contribution to M_ρ involves only $SU(2)_W$ -index $\alpha=3$, and that this contribution is real for every ρ .

The quark mass matrix m_q is generated mainly by the graph shown in Fig.1b; electroweak contributions can be ignored. In the same approximation that Eq.(12) is valid, the sideways contribution to m_q is given by

$$\begin{aligned}
(m_q)_S &= 2\Lambda_{ab} \sum_{\sigma \neq q} (U_q^{L\dagger} t_{q,\sigma}^{La} U_\sigma^\dagger t_{\sigma,q}^{Rb} U_q^R) \Delta_\sigma \\
&\quad + \text{terms of relative order } 10^{-9}.
\end{aligned} \tag{26}$$

Here, the sum runs over hyperfermions (ψ^σ) only, and the neglected terms are of order $\Delta_q/\Delta_\sigma \sim 10^{-9}$. (Higher-order sideways exchanges,

whether suppressed by powers of $g_S^2(\Lambda_S)/8\pi^2$ or Λ_S^{-2} , alter $(M_q)_S$ and $(m_q)_S$ in the same way.) If hyperfermions ψ^σ have no ordinary (G_C) color interactions, it is easy to show that $\Delta_{q\sigma} = 2\Delta_q \Delta_\sigma$, in which case $(m_q)_S = (M_q)_S$, up to terms of relative order 10^{-9} due to graphs in Figs. (2a) and (4a) involving quarks alone. Allowing for the possibility that hyperfermions also carry ordinary color, and taking into account the usual renormalization of Δ_q , Δ_σ , and G_S -vertices, it can be shown that [26]

$$\Delta_{q\sigma} = 2\Delta_q \Delta_\sigma \left[1 + C_\sigma \cdot O\left(\frac{\alpha_C(\Lambda_H)}{2\pi} \frac{\Lambda_C^2}{\Lambda_H^2}\right) \right]. \quad (27)$$

Here, C_σ is a group-theoretical factor and $(\alpha_C(\Lambda_H)/2\pi) \Lambda_C^2 / \Lambda_H^2 \sim 10^{-9}$.

This establishes Eq.(24). In particular, even if $v_C = 0$ so that $M_q = M_q^*$, m_q is not necessarily real; m_q may have an imaginary part of order 10^{-9} times its real part. (Note, however, that if only one hyperfermion species contributes to M_q and m_q , both are real or complex together.)

Eq.(24) is of central importance to the discussion of CP violation which comes next. Although the effective low-energy quark Hamiltonian may involve additional CP-violating terms, they are at most $O(10^{-9})$ times the quark mass term. Thus, the possibility of strong CP violation is controlled entirely by the magnitude of v_C .

4. The Character of CP Nonconservation

There are three possible outcomes to the vacuum alignment problem:

(i) $U = U^*$. In this case, CP invariance is not spontaneously broken. $M_\rho M_\rho^\dagger$ is a real symmetric matrix, for each ρ , and so U_ρ^L and U_ρ^R are real orthogonal matrices also. $v_C = 0$, and there is no CP violation of any kind. We know one class of theories in which this happens, namely vectorial sideways interactions. These have

$t_{\rho,\sigma}^{La} = t_{\rho,\sigma}^{Ra}$ in both the up and down sectors, hence $E(W) = E(W^\dagger)$, and it can be shown that the trivial $U_\rho = 1_\rho$ minimizes E . It is both interesting and fortunate that vectorial theories are already ruled out on other grounds: Because they have an overall $SU(2)_R$ as well as $SU(2)_L$ symmetry, up-and down-quark masses are equal in pairs, and all Cabibbo-KM angles vanish [15,28].

(ii) $U \neq U^*$, $v_C \neq 0$. In this case, strong CP nonconservation occurs. As noted earlier, the CP-nonconserving term $i(v_C T_q^C / 2\Delta_q) \bar{q} \gamma_5 q$ in the effective Hamiltonian leads to a neutron electric dipole moment $d_N \sim 10^{-16} (v_C T_q^C / m_u \Delta_q) \text{ e-cm}$ [9], where $m_u \sim 5 \text{ MeV}$ is the up quark mass. But, it is clear from Eqs.(22) and (23) that, if $v_C \neq 0$, its natural scale is $m_u \Delta_q$, and d_N exceeds the experimental bound by 10^8 [10]. As far as we know, this case is a logical possibility: Even though all bare masses (and vacuum angles) are zero and all symmetry breaking dynamical, there may be strong CP violation.

(iii) $U \neq U^*$, $v_C = 0$. In this case, CP invariance is spontaneously broken, but there is no large contribution to d_N . If m_q is not real, it contributes at most $10^{-24} - 10^{-25} \text{ e-cm}$ to d_N . Other G_s -induced contributions to d_N include:

(1) A CP-violating piece in the operator $\bar{q}_L \sigma_{\mu\nu} F^{\mu\nu} q_R$, where $F_{\mu\nu}$ is the electromagnetic field. This arises from the graphs shown in Fig.4. Even if M_q is real, there is no reason for these to be real unless only one species of hyperfermion appears in the intermediate state. The contribution of Fig.5a is naturally

$$(d_n)_{4a} \sim e g_S^2 \frac{\Lambda_H^3}{\mu_S^4} R_1 \sim e \frac{m_{u,d}}{\mu_S^2} R_1 \sim 10^{-26} R_1 \text{ e-cm}. \quad (28)$$

where R_1 is a mixing-angle factor and we have used $\mu_S \sim 100$ TeV as is appropriate for generating the small u and d masses. Naively, the graph in Fig. 4b contributes $\sim e g_S^2 \Lambda_H^2 / \mu_S^2 \sim 10^{-21}$ e-cm to d_N . This estimate is wrong. Because of the dynamical nature of the hyperfermion mass, dropping off like Λ_H^3 / P^2 above $\Lambda_H \sim 1$ TeV[29], this graph's contribution is suppressed to the same level as $(d_N)_{4a}$, namely 10^{-26} e-cm times mixing-angle factors.

(2) The Hamiltonian $\mathcal{H}'_S$ (Eq.(12)) contains four-quark terms involving products of both flavor-conserving and nonconserving G_S -currents. The coefficient of terms such as $\bar{u}_L \gamma_\mu c_L \bar{c}_R \gamma^\mu u_R$ and $\bar{d}_L \gamma_\mu s_L \bar{s}_R \gamma^\mu d_R$ will, in general, be complex with phases of order one and magnitude $\lesssim 10^{-10} (\text{GeV})^{-2}$. Allowing for an additional suppression of $10^{-1} - 10^{-2}$ due to the difficulty of finding a strange or charmed quark pair inside a neutron, these terms are expected to give

$$\begin{aligned} (d_N)_{4\text{-quark}} &\sim e M_N (10^{-11} - 10^{-12} \text{ GeV}^{-2}) R_2 \\ &\sim (10^{-25} - 10^{-26}) R_2 \end{aligned} \quad (29)$$

R_2 is another unknown mixing-angle factor.

(3) The only other source of a neutron electric dipole moment is the electroweak interactions. The Cabibbo-Kobayashi-Maskawa matrix in the charged weak current for quarks is $U_{q,\text{up}}^{L\dagger} U_{q,\text{down}}^L$. If U_q is complex, then we expect that both U_q^L and U_q^R will be also. And if $U_{q,\text{up}}^{L\dagger} U_{q,\text{down}}^L$ contains phases which cannot be eliminated by an anomaly-free redefinition of quark-field phases, charged weak interactions violate CP conservation à la the KM model[3]. However, this contributes only $\sim 10^{-30}$ e-cm to d_N [19].

To sum up: A variety of sideways-induced effects produce a neutron moment $d_N \sim 10^{-24} - 10^{-26}$ e-cm, which is $10^6 - 10^4$ times larger than expected in the standard KM model. While one can think of situations in which some of these effects vanish ($\text{Im } m_q = 0$, e.g.), it is difficult to imagine that they all do so long as U is complex.

Finally, let us turn to CP violation in the neutral-K system. Electroweak contributions to the CP-violating parameters ϵ and ϵ' have been discussed extensively elsewhere [30], so we will concentrate on those due to broken sideways interactions. The nominal strength of four-quark terms in $\mathcal{H}'_S$ is $(100\text{TeV})^{-2} = 10^{-5} G_F$ times (possibly) complex mixing-angle factors. Thus, there is negligible contribution to ϵ' from any $|\Delta S| = 1$ terms appearing in $\mathcal{H}'_S$. If $|\Delta S| = 2$ terms occur, especially $\bar{s}_L \gamma_\mu d_L \bar{s}_R \gamma^\mu d_R$, a current-algebra calculation of their contribution to Δm_K is in agreement with experiment if the real part of the mixing-angle factor is $\leq 10^{-1} - 10^{-2}$ [15]. This is a reasonable expectation for the coefficient of $|\Delta S| = 2$ terms. But, equally reasonable the imaginary part should be comparable to the real part; unless something special happens, all CP-violating phases are expected to be order one. This produces $\epsilon \sim 1$, three orders of magnitude too large!

This is one of the most serious problems facing the general program of constructing dynamically broken electroweak gauge theories. It emphasizes once again the need for a model realistic enough to have a non-trivial, CP-nonconserving solution to vacuum alignment. Only then can one critically assess the magnitude of the $|\Delta S| = 2$ problem. A possible resolution of the $|\Delta S| = 2$ problem will be outlined below.

We have stressed that the structure of G_S and the pattern of its breakdown determines, in principle, the CP-transformation properties of the low-energy quark interaction. Clearly, only the type (iii) models, those for which $U \neq U^*$ and $v_C = 0$, have any hope of describing

the observed CP violation. We should ask, then: When is $v_C = 0$? We would like to have a physical criterion for which sort of models will be type (iii), and which will not. So far, our only definite criterion is a mathematical one, already apparent in our discussion of QCD: Suppose we remove the constraint proscribing chiral transformations with G_H - or G_C -anomalies, thereby generalizing $E(W)$ to include dependence on $\theta_- = \frac{1}{2}(\theta_C - \theta_H)$. Then it is easy to show that $v_C = 0$ if we find that (a) with respect to W -variations, E is minimized at $W=U$, $\theta_- = 0$; and (b) with respect to θ_- -variations, E is stationary at $\theta_- = 0$, $W=U$.

One class of models in which it may be possible to find $U \neq U^*$, but $v_C = 0$, is suggested by the fact that the vacuum energy for a vectorial G_S -theory is minimized by $U=1$. Consider a model which is vectorial in the down sector, but not in the up sector:
 $(t_{\rho,\sigma}^{La})_{up} = (t_{\rho,\sigma}^{La})_{down} = (t_{\rho,\sigma}^{Ra})_{down} \neq (t_{\rho,\sigma}^{Ra})_{up}$. So long as electric charge is conserved, $U_{\rho, down} = 1_{\rho, down}$ extremizes the vacuum energy and, in fact, minimizes the down-sector contribution to E . From Eqs.(22) and (23), this extremum has $v_C = 0$. Of course, $U_{\rho, up}$ is tied to $U_{\rho, down}$ through the modularity constraints. But if these permit E to be minimized with a trivial $U_{\rho, down}$ and complex $U_{\rho, up}$, the model has type (iii) CP-violation.

There is an important bonus in this scenario: it solves the $|\Delta S| = 2$ problem, as well as the strong CP problem. With vectorial sideways interactions in the down sector, it seems likely to us that $M_{\rho, down}$ will be diagonal (as well as real and positive) with $U_{\rho, down}^L = U_{\rho, down}^R = 1_{\rho, down}$. To see this, examine the form Eq.(22) takes when, in the down sector, we put $t^{La} = t^{Ra} \equiv t^a$ and $U^L = U^R = 1$:

$$(M_{\rho r r'})_{\text{down}} \Delta_{\rho} = \Lambda_{ab} \sum_{\sigma, s} t_{\rho r, \sigma s}^a t_{\sigma s, \rho r'}^b \Delta_{\rho \sigma} + \frac{1}{4} \delta_{rr'} \sum_{\sigma} I_{\rho \sigma}. \quad (30)$$

Gauge group generators $t_{\rho r, \sigma s}^a$ are generally such that the first term on the right side of Eq.(30) also is proportional to $\delta_{rr'}$. Thus, there are no mixing angles at all in the down sector, as well as no reason to expect $|\Delta S| = 2$ sideways interactions.

In this scenario, all mixing angles reside in the up sector. And, in general, we would expect $|\Delta C| = 2$ terms such as $\bar{c}_L \gamma_{\mu} u_L \bar{c}_R \gamma^{\mu} u_R$. A crude estimate of their contribution to ΔM_{D^0} gives

$$\Delta M_{D^0} \approx (100 \text{ TeV})^{-2} f_D^2 M_{D^0} R. \quad (31)$$

R is another (model-dependent) mixing-angle factor, and f_D is the D -meson decay constant, expected to be approximately equal to $f_{\pi} = 100 \text{ MeV}$. This gives $\Delta M_{D^0} \approx 10^{-12} R \text{ GeV}$. With R expected to be $\sim 10^{-1}$ to 10^{-2} , this is surprisingly close to the canonical estimate of the D^0 -width: $\Gamma_{D^0} = 5G_F^2 M_D^5 / 192\pi^3 \approx 2 \times 10^{-12} \text{ GeV}$. In view of this, we strongly urge a renewed search, with high statistics, for $D^0 - \bar{D}^0$ mixing. Such a search should be carried out by the MARK III detector at SPEAR, utilizing the 1^3D_1 charmonium state ψ'' (3772).

4. Summary

Dynamically broken gauge theories of the electroweak interactions provide the only clear insight to the origin of CP-violation as we observe it. Though the dynamically determined Hamiltonian $\mathcal{H}_0 + \mathcal{H}'$ and the ground state $|\Omega\rangle$ may appear to be CP invariant, this can be an illusion. Spontaneous CP nonconservation can occur as a direct consequence of the need for vacuum alignment in any theory with both spontaneous and explicit symmetry breaking.

Even though all vacuum angles naturally can be zero, there still may be a strong CP problem, manifested by a large imaginary part of the quark mass matrix and a neutron electric dipole moment d_N which is eight orders of magnitude larger than allowed. With the Peccei-Quinn mechanism for avoiding strong CP violation apparently closed to us, the only possibility appears to be that the energy of the true ground state is stationary with respect to variations in the vacuum angles. If this happens, the overall phase of the quark mass matrix is at most $O(10^{-9})$. We emphasize that this strong suppression is very specific to models with dynamical symmetry breaking, characterized by fermion condensates instead of scalar field vacuum expectation values.

In the desirable type (iii) models, CP violation in the electroweak interactions is precisely as proposed by Kobayashi and Maskawa. Now, however, the origin of the phases is understood and they are calculable, at least in principle. But there is an even more immediate departure from the standard KM model: Additional sources of CP violation, in the quark mass matrix and other low-energy effects of the sideways interaction, seem to lead inevitably to $d_N \sim 10^{-25 \pm 1}$ e-cm. Thus, if the electric dipole moment of the neutron is not found at this level, our general approach to CP nonconservation will almost certainly be ruled out.

Finally, in this paper we have proposed a particular class of models in which there are good reasons to believe that the strong CP problem is solved automatically. These "vectorial down-sector" models simultaneously solve the problem of unacceptably large $|\Delta S| = 2$ effects mediated by the sideways interactions. With all nontrivial angles and phases in the up sector, it is expected that these models

will have $|\Delta C| = 2$ processes such as $D^0 - \bar{D}^0$ mixing. Our estimate of ΔM_D suggests that $D^0 - \bar{D}^0$ mixing may soon be observable in e^+e^- annihilation experiments.

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Figure Captions

1. (a) The basic transition from a hyperfermion to an ordinary fermion, mediated by a heavy sideways gauge boson.
 (b) Typical graph producing a mass for a quark or lepton. The shaded blob represents the dynamical mass of the hyperfermion. It is understood that all possible gluon and hypergluon corrections to the fermion lines are included.
2. Graphical structure of (a) $\Delta_{\rho\sigma}$ and (b) $\Delta'_{\rho\sigma}$. The heavy dots are G_S -vertices. It is understood that all possible gluon and hypergluon corrections to the fermion loops are included.
3. The $O(\alpha^2 \ln 1/\alpha)$ contribution of electroweak interactions to the vacuum energy. The cross-hatched blobs represent the polarization tensor $i \int d^4x e^{-ik \cdot x} \langle \Omega | W^{-1} T(j_{L\mu}^\alpha(x) j_{R\lambda}^3(0)) | \Omega \rangle$; the wavy lines are (massless) G_W -boson propagators.
4. Contributions to the CP-violating part of $\bar{q}_L \sigma_{\mu\nu} F^{\mu\nu} q_R$. The notation is as in Fig.1. These graphs assume that the relevant G_S -gauge bosons and hyperfermions are electrically charged.

Figures 1(a) and 1(b)

Figure 2

Figure 3

(a)

(b)

Figure 4