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ER-548
Research Project 236-2

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Prepared by

UNIVERSITY OF CALIFORNIA, LOS ANGELES
Chemical, Nuclear, Thermal Engineering Department
School of Engineering and Applied Science
Los Angeles, California 90024

Principal Investigators
William E. Kastenberg
David Okrent

Prepared for

Electric Power Research Institute
3412 Hillview Avenue
Palo Alto, California 94304

EPRI Project Manager
Noel A. Amherd
Fossil Fuel and Advanced Systems Division

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Principal Investigators
William E. Kastenberg
David Okrent

Authors

V. Badham	T. H. K. Frederking	G. C. Pomraning
C. K. Chan	W. E. Kastenberg	A. Z. Ullman
W. J. Ferrell	T. E. McKone	Y. D. Ting
J. Grzesik	D. Okrent	J. Yakura

Chemical, Nuclear, Thermal Engineering Department
School of Engineering and Applied Science
University of California, Los Angeles
Los Angeles, California 90024

ABSTRACT

The objective of this study was to make a preliminary examination of some potential safety questions for conceptual fusion-fission hybrid reactors. The study and subsequent analysis was largely based upon reference to one design, a conceptual mirror fusion-fission reactor, operating on the deuterium-tritium plasma fusion fuel cycle and the uranium-plutonium fission fuel cycle. The blanket is a fast-spectrum, uranium carbide, helium cooled, subcritical reactor, optimized for the production of fissile fuel. An attempt was made to generalize the results wherever possible.

The major potential hazards were found to be:

- (a) Fission products,
- (b) Actinide elements,
- (c) Induced radioactivity, and
- (d) Tritium.

Event trees were constructed for the following accidents:

- (a) Blanket loss-of-flow,
- (b) Blanket loss-of-coolant,
- (c) Steam ingress, and
- (d) Plasma density-temperature transient.

Important phenomena studied were loss-of-coolant, steam ingress, and blanket criticality. As a result of these studies it appears that highly reliable and even redundant decay-heat removal must be provided. Loss of the ability to remove decay heat results in the melting of fuel, with ultimate release of fission products and actinides to the containment. In addition, the studies indicate that blankets can be designed which will remain subcritical under extensive changes in both composition and geometry.

Magnet safety and the effects of magnetic fields on thermal parameters were also considered. Improvements in magnet design must focus on:

- (a) The superconductor,
- (b) Its normal-metal stabilizer
- (c) Normal-metal barriers, and
- (d) Coolant conditions.

EPRI PERSPECTIVE

PROJECT DESCRIPTION

Controlled thermonuclear fusion programs around the world are now formulating plans for the development of fusion energy for practical applications. Whereas scientific breakeven still awaits the results from experimental devices now under construction in several countries, many detailed conceptual fusion reactor designs have been produced during the last few years. These designs have spanned various concepts, including tokamak, mirror, and laser-fusion configurations, and various time scales, from experimental power reactors--to be built possibly in the 1980s--to commercial-scale electric power producers.

Early in 1974, EPRI initiated activities to investigate the safety aspects of magnetic and inertial confinement fusion power systems that burn deuterium-tritium (D-T). The objective of these activities was to critically evaluate the conceptual fusion designs from a point of view that is a major element of power system availability, licensing, and regulation. Subsequently, in 1976, fusion-fission systems were added to the scope of work. The results of these activities are presented in three companion reports, EPRI ER-546, 547, and 548. Enlarged and more detailed future studies are expected to build on these results to help resolve potential safety problems.

PROJECT OBJECTIVES

Three major questions have been used in these analyses of fusion energy systems: (1) Are there potentially hazardous materials that can affect the health and safety of the public? (2) Are there mechanisms that can release or volatilize these materials? (3) Are there mechanisms that can threaten containment integrity? The current safety studies are preliminary and admittedly limited owing to the time, scope, level of effort, and stage of development of fusion power. Since fusion system designs to date are conceptual, it is appropriate to investigate generic types of potential safety problems using mechanistic techniques rather than probabilistic ones. Probabilistic analyses require detailed information that is not yet available.

CONCLUSIONS AND RECOMMENDATIONS

These studies have identified several hazards and accident pathways that could lead to potential safety issues. For example, the principal radioactive hazards are tritium and induced structural radioactivity. Fusion-fission systems in addition contain actinides and fission products. While hazardous materials likely will be present in fusion reactors, a measure of the seriousness of the hazard and the ability to make use of other materials and design options must be investigated. Hence, some appropriate safety-oriented activities suggested by these early results would include the following: incorporation of safety considerations in the next generation of fusion reactor design, definition of safety-related R&D required to remove uncertainties and quantify phenomena, establishment of the requirements for safety criteria and testing specifications, and development of a procedure, methodology, and plan for future safety-oriented endeavors.

Fusion has the potential for attractive environmental and safety characteristics, but early and sustained effort must be applied to ensure the achievement of this potential. There are many possible options and choices in fusion, including confinement concepts, design features, materials, fuel cycles, and applications, and every selection from among these choices determines a specific safety characterization. Identification of the most appropriate fusion system for future deployment should be based on analyses that include safety requirements and criteria.

Noel Amherd, Project Manager
Fossil Fuel and Advanced Systems Division

PREFACE

This report summarizes work completed during the period June 1, 1976 through August 30, 1977 on "Safety Studies of Fusion-Fission Power Reactors" supported by the Electric Power Research Institute under Contract RP236-2. Five students, one post-doctoral fellow, and six faculty members contributed to the project.

Although an attempt was made to keep the study as generic as possible, most of the calculations presented are for one particular design, the Lawrence Livermore Laboratory, Fusion-Fission Hybrid Reactor. Any conclusions, recommendations, and suggestions should be taken in this context. While this design and the calculations presented herein are preliminary, it is hoped that the results can be factored into future designs so that attempts can be made to avoid areas of reactor hazard where possible, or to provide adequate engineered safeguards, where appropriate.

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Section 1

INTRODUCTION AND SUMMARY

Since the inception of nuclear energy, in the form of fission reactors, there has been a concern for public health and safety. With the emergence of conceptual designs for power-producing fusion reactors, there have also been preliminary investigations of their potential impacts on public health and safety [1-1,1-2]. Recently, there has been renewed interest in designing fusion-fission hybrid systems [1-3,1-4] with the objective of mating neutron rich-energy poor fusion with neutron poor-energy rich fission. The objective of the study presented in this report is to examine, in a preliminary way, some potential safety questions for conceptual fusion-fission hybrid reactors.

In this examination, a fusion-fission hybrid reactor is conceptually similar to any pure fusion device, with one major exception. This exception manifests itself with a different blanket design. In a hybrid system, all or part of the blanket may contain fertile material (e.g., thorium 232 or uranium-238), fissile material (e.g., uranium-233, 235, and plutonium-239) and residual actinide material (e.g., americium-241 and 243). The amount and type of these materials depends on many factors, including burn-up, depletion, and the goal of the system itself. It should be noted that hybrid fusion may not appear to be conceptually the same as pure fusion to the plasma physicist or magnet designer because of changes in confinement requirements, among others.

At present, fusion-fission hybrid reactor designs can be optimized in one of three ways: (1) as power producers, (2) as fissile fuel producers and (3) as actinide burners. In addition, these fusion systems will operate on the deuterium-tritium-lithium (D-T-Li) fuel cycle. Hence, tritium must be bred in the blanket via absorption of neutrons produced by the D-T reaction. These neutrons also can induce activity in the first wall via (n,γ) and $(n,2n)$ reactions, can transmute fertile elements to fissile elements (uranium and thorium), can induce fission (in uranium and plutonium) and can burn out residual actinides (via fission and absorption) produced in fission reactors.

As a result, there are four major potential hazards: (1) tritium, (2) activation products (first wall, structural and corrosion), (3) actinide elements, and (4) fission products. The relative amounts and importance of these hazards will depend upon how the system is optimized, i.e., its goal. For example, the Lawrence Livermore Laboratory (LLL) hybrid design [1-3] has three goals: (1) to produce enough tritium to sustain the D-T reaction, (2) to produce enough energy (primarily via fast fission in U^{238} and Pu^{239} , and secondarily via direct conversion) to sustain the plant, and (3) to produce fissile fuel to sustain several Light Water Reactors (LWR's). Hence, the blanket will initially contain depleted uranium (virtually zero enrichment of fissile fuel) and will build up several atom-percent plutonium, as well as an equilibrium fission product inventory. The Westinghouse Actinide Burner, on the other hand [1-4], is optimized to "burn out" residual actinides. Hence, it will initially have a large fissile inventory (of Np-237, Am-241 and Am-243), which will be depleted with time.

The work reported here centered on several major themes: (1) are there significant accidents or accident sequences which can lead to mobility of these hazards, (2) are there mechanisms for breaching containment so that they can be released to the environment, and (3) what if any, are the potential pathways and consequences to man.

In Section 2, an assessment of the radioactive inventory in terms of hazard potential, critical pathways, and dose is presented. Section 3 contains a set of event trees for several significant accident sequences. Section 4 contains an analysis of the potential for blanket criticality. Section 5 presents a discussion and analysis of heat-removal safety-related questions and Section 6 addresses magnet-related safety issues. Section 7 assesses the effects of high magnetic fields on physical properties and the resultant effects on safety calculations. The conclusions, recommendations, and need for further study are presented in Section 8.

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3. Bender, D. J., J. D. Lee, et. al. "A Reference Mirror-Hybrid Fusion-Fission Reactor," Transactions of the American Nuclear Society, Vol. 26 June 1977.
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Section 2

RADIOACTIVE INVENTORIES (HAZARD POTENTIAL)

In a fusion-fission hybrid reactor, there are four major potential radioactive hazards: (1) tritium; (2) activation products (first wall, structural, and corrosion); (3) actinide elements; and (4) fission products. The tritium and first-wall activation products are a result of the deuterium-tritium fuel cycle which produces 14 MeV neutrons and are the principle hazards of fusion systems [2-1]. For fission reactors, the principle hazards are fission products and actinide elements [2-2]. Although tritium is also a fission product, it is a negligible contributor in accident analysis, but may be a major contributor during routine release.

In this section, an attempt is made to examine those effects which enhance the potential hazards of these inventories. In Section 2.1, fission products and actinides are discussed. Although their release and principle health effects have been treated for light-water reactors (LWR's) [2-3], and liquid metal fast breeder reactors (LMFBR's) [2-4], their modified character in a blanket with a 14 MeV neutron source has not been considered. In Section 2.2, the question of tritium pathways has been carried further than in previous work [2-5]. Although tritium inventories are not unique to hybrid systems, there may be a greater potential for their release in a power-producing (fissioning) blanket. Section 2.3 contains a summary and conclusions of this section.

First-wall activation product inventories should not differ significantly from pure fusion systems, because they depend principally on the wall loading. Hence, they are not discussed in this section. However, since the potential for their release may change in a hybrid system, accidents leading to first-wall mobility are discussed in later sections of the report.

2.1 FISSION PRODUCTS AND ACTINIDES

A significant hazard associated with the operation of a hybrid system (or any fissioning system) resides in the accumulated inventory of fission products and

actinides in the fissioning blanket. Potential hazards of radionuclides produced in a fission system have been evaluated for light water reactors (LWR's) [2-3]. Here, a comparison is made of the character of the radioactive inventory in hybrids to that in LWR's to estimate the relative hazard.

This section begins with a definition of radioactive "character" which is the sum of the products of individual nuclide quantities and some factor or index which correlates to their biological hazard. Next, the general procedure is described for the calculation of fission product inventories in terms of energy production and power density. This is followed by a discussion of those parameters which effect the character of the radioactive inventory both in terms of specific isotopes and in terms of overall hazards. The radioactive inventory of the Livermore reference design with uranium carbide fuel [2-6], operating at 60 W/cc and providing a total power of 1000 MWt over a period of three years, is compared to that of an LWR with the same energy output.

2.1.1 Radionuclide Character

The quantities of radioactive nuclides in fissioning systems can be determined by a set of build-up and depletion equations, as described in Section 2.1.2. However, the quantity alone does not provide much information on health effects from releases in normal or accident situations. One kg of tritium released to the biosphere induces different consequences than one kg of Sr^{90} . Some problems would exist even if curies were used as the unit of measurement. Quantitative measurements do not express the relative hazards caused by such factors as beta decay energy, extent of incorporation in food chains, transport characteristics in the atmosphere, and the expected release from fuel under accident situations.

In order to quantify the potential effects of a specific nuclide, the inventory of that nuclide (in grams or curies) has to be multiplied by an index of hazard. A widely-used index of hazard results from the division of the nuclide activity (in curies) by the appropriate maximum permissible concentration (MPC) in air or water, as specified by 10-CFR-20 [2-7]. By including the MPC, the decay mode and biological behavior of a nuclide are taken into account.

Although the hazard index is a good measure of short-term hazards of a nuclide or mixtures of nuclides, it does not explicitly account for their long-term availability. For this reason, Gera and Jacobs [2-8] proposed the use of the

potential hazard index (PHI). As defined, the PHI is a function of the amount of a radionuclide, the radiotoxicity of the nuclide, and the probability that humans will be exposed to radiation emitted by the radionuclide. The PHI is expressed as:

$$PHI = P(T_{1/2}/\ln 2)Q/MPI \quad . \quad (2.1)$$

P is the probability that the activity will reach humans. It is based on: 1) the biological availability of the nuclide after it is dispersed in the environment, and 2) the probability of release. For this study, the biological availability is assumed to be unity, and the probability of release is taken to be the release fraction described in WASH-1400 [2-3]. Q is the radioactive inventory in curies. MPI is the maximum annual intake in curies per year and is based on MPC's. $T_{1/2}$ is the radioactive half-life in years.

2.1.2 Calculation of Nuclide Quantities

In order to determine the quantity of radioactive material in a fissioning system in which radioactive species accumulate and disappear as a result of fission, transmutation, and radioactive decay, it is necessary to solve a set of coupled ordinary nonhomogenous differential equations of the form:

$$\begin{aligned} \frac{dN_i}{dt} = & -\lambda_i N_i + \gamma_i FR - N_i \sigma_{n,r}^i \phi \\ & + \sum_{j=1}^M N_j F_{ji} \sigma_{ji}^i \phi + \sum_{j=1}^M \lambda_{ij} \lambda_j N_j, \end{aligned} \quad (2.2)$$

where

- N_i = number density of species i,
- λ = decay constant of species i,
- γ_i = fission yield of species i,
- FR = fission-rate density,
- $\sigma_{n,r}^i$ = cross section for removal of species i by a neutron interaction, spectrum averaged over the region of interest,
- ϕ = spatially averaged total neutron flux,
- M = total number of isotopes,
- F_{ji} = fraction of neutron interactions in an atom of species j that lead to an atom of species i,
- σ^j = spectrum averaged, total interaction cross-section of species j,

λ_{ij} = fraction of radioactive decays in species j that provide atoms of species i.

Equation (2.2) can be also written as:

$$\frac{d\vec{N}}{dt} = [A] \vec{N} + \vec{\gamma}FR, \quad (2.3)$$

where

$\vec{N}$ = column vector of radionuclide number densities,
 $[A]$ = matrix of transmutation coefficients,
 $\vec{\gamma}$ = column vector of fission yields, and
 FR = fission rate.

Equation (2.3) has the particular solution

$$\vec{N}(t) = \left(\exp\{[A]t\} - [I] \right) [A]^{-1} \vec{\gamma}(FR)t \quad (2.4)$$

and the homogenous solution

$$\vec{N}(t) = \exp\{[A]t\} \vec{N}_0. \quad (2.5)$$

The exponential in Equations (2.4) and (2.5) is the matrix exponential, which is a square matrix defined by the series:

$$\exp\{[A]t\} = [I] + [A]t + \frac{[A]^2}{2!} t^2 + \dots \quad (2.6)$$

This mode of solution of Equation (2.3) is called the matrix exponential method, and has been described by Bell [2-9].

The matrix exponential method has been incorporated in the Oak Ridge Isotope Generation and Depletion Code (ORIGEN), which has provided a useful tool for calculating fission product and actinide inventories [2-9].

ORIGEN was designed to calculate radioactive inventories in thermal and fast spectrum cores. However, the transition matrix, $[A]$, in ORIGEN includes only (n,γ) transmutations for fission products. In addition, the ORIGEN libraries do not have fission-yield data for 14 MeV-induced fissions. Both of these restraints made it necessary to calculate fission-product inventories without the aid of ORIGEN in those neutron spectra that contained a significant amount of 14 MeV neutron population. This required some simplifying assumptions, that added a few percent of error to the lumped-parameter system results.

2.1.3 Governing Parameters of Fission Product Quantities

The quantity of a given fission product nuclide, as stated in Section 2.1.2, is determined by the direct fission yield and the transmutations. The transmutations include all spontaneous or neutron-induced reactions that transform one nuclide into another one.

Fission Yields. Studies of fissioning nuclei [2-10], such as U^{235} , Pu^{239} , and U^{238} , reveal that the compound nucleus can split up in more than 40 different ways, yielding over 80 primary fission products or fragments. The fission yield associated with a given mass number is defined as the percentage ratio of nuclei generated in the process having the given mass number, to the total number of fission products. Since two nuclei result from each fission, the total yield is 200%. As Figure 2-1 shows, the probability of an equal split is low for fissions induced by thermal neutrons. However, the dip in the fission-yield curve increases with increasing neutron energy. It should be noted that the scale of Figure 2-1 is logarithmic. The curves shift to the right as the mass of the fissioning nucleus increases. Thus, the quantity of a given fission product depends on the neutron spectrum and the fissioning material in the system. Ruthenium-106 (a hazardous fission product) has a fission yield that varies by more than an order of magnitude, depending on the fuel type and neutron spectrum.

Transmutations. Nuclides exposed to a neutron flux undergo several interactions. The initial fission fragments are unstable, but get stabilized through a series of beta decays. Each member of this chain has an affinity for neutrons, defined by their cross section. Some important transmuting neutron-nucleon reactions are: (n,γ) , $(n,2n)$, $(n,3n)$, (n,p) , and (n,α) [2-11]. Charged-particle reactions are relatively improbable in almost all fission products because of the large neutron excess and high coulomb barrier. The radiative-capture cross section (n,γ) is significant (≥ 1 barn) in the thermal range. For most fission products, this cross section has a $1/v$ or $1/\sqrt{E}$ behavior, which sharply reduces its effect in fast spectra.

The $(n,2n)$ and $(n,3n)$ reactions involve a threshold energy and plateau range. Below a minimum incident neutron energy, the cross section is zero. Above this minimum, there is a range of energies with nearly-constant cross section (approximately 1 to 2 barns for most fission products). Above this range, the cross section is again zero. The threshold for the $(n,2n)$ reaction in Ru^{106} is 8.07 MeV, while that for the $(n,3n)$ reaction is 14.4 MeV [2-11]. These reactions

Figure 2-1. Fission Yields for Thermal Fissions in U-238 and Fast Spectrum and 14 MeV Fissions in U-238.

convert hazardous long-lived radionuclides into short-lived ones, which pose a lesser threat to the biosphere in terms of their potential hazard index.

Table 2-1 lists the fission yields, the (n,2n) reaction threshold, and the (n,2n) cross section at 14.1 MeV for some of the more abundant and hazardous fission products. The fission yields were taken from Reference [2-10] and the cross section data were calculated by the procedures described in Reference [2-11].

2.1.4 Comparative Risks of Blanket Radioactive Inventory

In order to establish a benchmark for a comparative hazard determination, the quantity and character of the most significant fission products in an LWR are shown in Table 2-2. These values apply after three years of operation at a continuous thermal power of 1000 MWt. This calculation was made using the ORIGEN code. The power density was assumed to be 30 MW/metric ton. The average flux during the irradiation period was 3.0×10^{13} n/cm² sec. The fuel was enriched with 3.3% U²³⁵. Figure 2-2 shows the hazard index as a function of decay time for the fission products of an LWR.

Table 2-3 lists the parameters used for the reference blanket in this study. The power was arbitrarily chosen as 1000 MWt. The other parameters correspond to the uranium carbide blanket described in Reference [2-6].

Table 2-4 lists the quantities and character of the most important radionuclides in the hybrid blanket (defined by Table 2-3) in the absence of (n,2n) reactions. This calculation was carried out by ORIGEN, using a neutron spectrum typical of an LMFBR. The fission yields used were those for a fast spectrum. The amount of Pu²³⁹ was calculated separately. All fissile fuel production was assumed to produce Pu²³⁹ atoms. This calculation was intended to provide an upper limit to the radioactive inventory of the reference blanket and its comparison to an LWR fission product inventory.

Table 2-5 presents nuclide quantities and characteristics in the same blanket; however, these calculations were carried out with the assumption that the mean flux of neutrons in the hybrid blanket between 10 and 14 MeV is 5.0×10^{14} n/cm² sec, causing fission product removal by (n,2n) reactions. Based on the design study considered in Reference [2-6], this is an unrealistic assumption. However, it represents the lower limit of fission-product inventory. It is

Table 2-1

SOME PROPERTIES OF ABUNDANT AND HAZARDOUS FISSION PRODUCTS.

Fission Product	Thermal (U235)	Fission Yield		Threshold (MeV)		14.1 MeV Cross Section	
		Fast Spectrum (U238)	14.1 MeV (U238)	(n,2n)	(n,3n)	$\sigma(n,2n)$	$\sigma(n,3n)$
SR-89	0.0471	0.0267	0.0283	6.60	15.8	1.5 b	0
SR-90	0.0586	0.0351	0.0334	8.26	15.1	1.5 b	0
TC-99	0.0615	0.0633	0.058	9.42	17.0	1.32 b	0
Ru-106	0.00391	0.0283	0.0244	8.07	14.4	1.75 b	0
I-131	0.0287	0.033	0.036	7.48	14.4	2.0 b	0
Cs-137	0.0628	0.0546	0.0513	7.25	13.0	1.6 b	0.4 b
Ce-144	0.0542	0.0484	0.0367	7.37	13.3	1.8 b	0.3 b

Table 2-2

FISSION PRODUCTS AND ACTINIDES IN AN LWR
AFTER 3 YRS OPERATION AT 1 GWt.

Radionuclide	Quantity (Curies)	MPC Ingestion (ci/m ³ water)	MPC Inhalation (ci/m ³ air)	Hazard Index (m ³ water)	Hazard Index (m ³ air)	Release Fraction	Half Life (Days)	PHI
SR-89	2.39x10 ⁷	3.0x10 ⁻⁶	3.0x10 ⁻¹⁰	7.97x10 ¹²	7.97x10 ¹⁶	0.05	52.1	1.12x10 ¹¹
SR-90	2.59x10 ⁶	3.0x10 ⁻⁷	3.0x10 ⁻¹¹	8.63x10 ¹²	8.63x10 ¹⁶	0.05	11,030	2.57x10 ¹³
TC-99	4.77x10 ²	2.0x10 ⁻⁴	2.0x10 ⁻⁹	2.38x10 ⁶	2.38x10 ¹¹	0.4	7.77x10 ⁷	3.88x10 ¹²
Ru-106	1.82x10 ⁷	1.0x10 ⁻⁵	2.0x10 ⁻¹⁰	1.82x10 ¹²	9.1x10 ¹⁶	0.4	366	7.09x10 ¹²
I-131	2.87x10 ⁷	3.0x10 ⁻⁷	1.0x10 ⁻¹⁰	9.6x10 ¹³	2.87x10 ¹⁷	0.7	8.05	8.75x10 ¹¹
Cs-137	3.6x10 ⁶	2.0x10 ⁻⁵	5.0x10 ⁻¹⁰	1.8x10 ¹¹	7.2x10 ¹⁵	0.4	11,000	1.69x 0 ¹³
Ce-144	3.7x10 ⁷	1.0x10 ⁻⁵	2.0x10 ⁻¹⁰	3.7x10 ¹²	1.85x10 ¹⁷	3x10 ⁻³	284	8.40x10 ¹⁰
Pu-239	1.06x10 ⁴	5.0x10 ⁻⁶	6.0x10 ⁻¹⁴	2.12x10 ⁹	1.77x10 ¹⁷	3x10 ⁻³	8.9x10 ⁶	2.55x10 ¹⁵

Figure 2.2. Time-Dependent Behavior of the Hazard Index of Fission Product Nuclides after Discharge from an LWR.

Table 2-3
REFERENCE BLANKET PARAMETERS

Thermal Power (MWt)	1,000
Fuel Type	Uranium Carbide
Average Power Density (W/cm ³)	34
Enrichment (atom-percent)	0.7
Energy/Fusion (MeV)	144
Fusions/sec	4.33×10^{19}
Fissions/sec	2.71×10^{19}
Mean Total Flux (n/cm ² sec)	10^{15}
Flux above 10 MeV (n/cm ² sec)	5.0×10^{14}
Fissile Atom Product (atoms/fusion)	1.4

Table 2-4
FISSION PRODUCT AND ACTINIDE CHARACTER IN A HYBRID BLANKET
WITHOUT (n,2n) FISSION PRODUCT REACTIONS AFTER THREE YEARS OF OPERATION AT 1 GWt

RADIONUCLIDE	QUANTITY (Ci)	HAZARD INDEX (m ³ water)	HAZARD INDEX (m ³ air)	PHI
Sr-89	2.55×10^7	8.50×10^{12}	8.50×10^{16}	1.19×10^{11}
Sr-90	1.97×10^6	6.56×10^{12}	6.56×10^{16}	1.95×10^{13}
Tc-99	5.00×10^2	2.49×10^6	2.49×10^{11}	4.06×10^{12}
Ru-106	2.06×10^7	2.06×10^{12}	1.03×10^{17}	8.02×10^{12}
I-131	2.67×10^7	8.93×10^{13}	2.67×10^{17}	8.14×10^{11}
Cs-137	3.44×10^6	1.72×10^{11}	6.88×10^{15}	1.61×10^{13}
Ce-144	3.22×10^7	3.2×10^{12}	1.61×10^{17}	7.31×10^{10}
Pu-239	1.25×10^5	2.49×10^{10}	2.09×10^{18}	3.01×10^{16}

Table 2-5

FISSION PRODUCT AND ACTINIDE CHARACTER IN A HYBRID BLANKET
AFTER THREE YEARS OF OPERATION AT 1 GWt WITH (n,2n) REACTIONS ACCOUNTED FOR

RADIONUCLIDE	QUANTITY (Ci)	HAZARD INDEX (m ³ water)	HAZARD INDEX (m ³ air)	PHI
Sr-89	2.07×10^7	6.90×10^{12}	6.90×10^{16}	9.66×10^{10}
Sr-90	1.66×10^6	5.53×10^{12}	5.53×10^{16}	1.64×10^{13}
Tc-99	4.75×10^2	2.37×10^6	2.37×10^{11}	3.86×10^{12}
Ru-106	1.69×10^7	1.69×10^{12}	8.45×10^{16}	6.60×10^{12}
I-131	2.6×10^7	8.69×10^{13}	2.60×10^{17}	7.92×10^{11}
Cs-137	2.57×10^6	1.28×10^{11}	5.14×10^{15}	1.20×10^{13}
Ce-144	1.81×10^7	1.80×10^{12}	9.05×10^{17}	4.11×10^{10}
Pu-239	1.25×10^5	2.49×10^{10}	2.09×10^{18}	3.01×10^{16}

interesting to note that the difference between the upper and lower limits is not much greater than the probable error. Moreover, neither the upper nor the lower limit differs significantly from the LWR fission product inventory for the same energy produced. The significant difference between the LWR inventory and that of the hybrid is the amount of Pu^{239} . The Pu^{239} bred in a hybrid blanket represents a greater hazardous inventory than an LWR producing the same energy.

It was found that a spatially averaged flux of at least $10^{16} \text{ n/cm}^2 \text{ sec}$, and in the range of 10-14 MeV would be necessary to alter the character of the fission-product inventory significantly. However, this would most likely result in severe wall loading and design problems.

2.2 TRITIUM HAZARDS

Plasma operation in a hybrid system requires large inventories of tritium. In addition to the fissioning blanket, there are other blanket segments involved in tritium breeding. Tritium release mechanisms from pure fusion systems have been described in the literature [2-1]. Tritium releases are often associated with normal operations or with small off-normal events (such as fires or transportation accidents), which may not provide a sufficient mechanism for fission product release. For this reason, the quantity of tritium stored at a hybrid site could likely represent the most significant radioactive inventory.

To quantify the magnitude of this hazard due to various release mechanisms, a computer program, Tritium man-REM Consequence Model (TREM), was developed and employed. This program works in three phases. First, given a release of tritium and sufficient meteorological data and site-related information, it predicts the available concentration of tritium in the air, surface water, and soil. Secondly, it determines available doses, in rem, that would be incurred from inhalation and ingestion of water, vegetables, fruit, meat, milk, and fish in radial mesh intervals within a 80 km radius from the source. Finally, based on the population distribution, agricultural production, and surface water data, the dose commitment in man-rem is calculated. A detailed description of this model is available in Ref. [2-12]. A summary of the model and results of its application to sample sites is included here.

This section begins with a description of the atmospheric dispersion model employed in the TREM program. This is followed by a summary of the uptake and dose

calculations which convert ground-level concentrations of tritium species into potential health effects. Inhalation and ingestion dose predictions within an 80 km radius from a tritium source are presented for the sample sites.

2.2.1 Source Terms and Atmospheric Dispersion

In order to assess the radiological implications of tritium releases, it is necessary to determine the distribution of tritium species released to the atmosphere and their subsequent transfer to biological systems at ground level. An atmospheric transport model is an important link which connects an accidental or routine release to the resulting consequences. For a fixed release quantity, the magnitude and duration of exposure to a given dose level depends upon the characteristics of the transporting medium — the atmosphere — during the time of release.

Atmospheric dispersion based on the Gaussian plume model is examined for three release categories: 1) instantaneous releases lasting less than one hour, 2) short-term releases lasting from one hour to several days, 3) continuous uniform releases. The continuous release calculation uses the source rate (ci/sec) to establish yearly average ground level concentration (ci/m^3) in radial and angular mesh intervals. Temporal variations in wind speed, stability category, and wind direction are incorporated into the calculation through a three-dimensional frequency matrix (wind rose). Short-term release calculations use source quantities (curies) to determine the exposure ($\text{ci sec}/\text{m}^3$), which is an infinite time integral of concentration. In other respects, this category is similar to continuous releases. Instantaneous releases are a subset of short-term releases, with the exception that their effective exposure duration due to plume spreading is comparable to the release duration. Briggs' correlations [2-13] for standard deviations were employed in each category. The basic diffusion equations are modified to account for cloud depletion due to dry deposition (rain-out), plume rise, and inversion layers. The model also incorporates seasonal and hourly variations in wind speed, wind direction, and atmospheric stability.

2.2.2 Tritium Uptake and Dose Model

Health effects from tritium species released to the atmosphere are determined by converting ground-level concentrations into potential biological effects. Ground-level concentrations are obtained by using the procedures described in

Section 2.2.1. Dose effects to humans from tritium are proportional to their concentration in body water as THO or T_2O . The major task is to establish the amount of tritium dispersed in the air that finds its way into humans.

Tritium represents only a small hazard as an external radiation source, because it is a low-energy beta emitter. However, it represents a significant internal hazard. Tritium can enter the body through several paths. It can be inhaled by the lungs and absorbed through the skin as water vapor in contaminated air. It could also be ingested through contaminated water, fish, vegetables, fruit, milk, or meat in which tritium has been taken up directly or through food-chain transfer.

After a review of previous work, the tritium dose model included in TREM was developed [2-12]. This model is based on the work of Anspaugh et al. [2-14], but has been modified for inclusion in the TREM program. The modifications include allowance for deposition to soil during rain-out, handling of nonaqueous tritium compounds such as LiOT, calculation of drinking water and fish consumption doses, and determination of the dose from food products or water. It also accounts for food produced in, but consumed outside of the contaminated sector. The food intake along a food chain is considered to be a continuous, rather than discrete process. It was shown [2-12] that this probably does not overestimate the short-term dose calculations significantly. Incorporation of tritium from free water into organic molecules in biological systems is accounted for by using the longest reported tritium turn-over periods.

Food-chain doses result mainly from the intake of tritium by plants which receive tritium through enrichment by contaminated atmospheric moisture. During periods of rain, tritium as THO (or LiOT) is transported from atmosphere to soil by wet deposition, and to plants by root uptake. Species, e.g. LiOT, which are not taken up directly by plants are treated as transporting tritium to plants via deposition to and exchange with soil moisture, followed by root uptake.

Each downwind segment represents a separate environment divided into a set of interacting water compartments which exchange THO. Humans are at the top of this transfer chain. Inhalation and food-chain pathways used in the model are shown in Figure 2-3.

Figure 2-3. Schematic Outline of the TREM Model.

For continuous releases, the dose rate to humans is the sum of the dose rates from each pathway

$$\dot{D}_T = \sum_{I=1}^N \dot{D}_I^{\infty} . \quad (2.7)$$

Here, $\dot{D}_T$ = total dose rate to humans from a continuous release

N = number of pathways

$\dot{D}_I^{\infty}$ = equilibrium dose rate from pathway I.

A finite release with duration T provides a dose rate $\dot{D}_I^o$ during the release, which is a different time-dependent function from $\dot{D}_I^i$, which describes conditions after exposure. The total dose from a finite release is:

$$\dot{D}_T = \sum_{I=1}^N \left[\int_0^T \dot{D}_I^o(t) dt + \int_T^{\infty} \dot{D}_I^i(t) dt \right] . \quad (2.8)$$

Here, D_T is the total dose in rem from dose rates $\dot{D}_I^o$ and $\dot{D}_I^i$ in rem/unit time.

This dose model was intended to represent the observed tritium behavior and predict a reasonable upper limit of consequences from various releases. The reported observation that tritium is assimilated as biologically active substances in animals, is accounted for by using the longest reported in-body half times. The half life of tritium in plants is increased with increasing exposure duration.

During periods of rain, when humidity is high and the atmospheric tritium enrichment is low, and during releases of non-aqueous tritium, the principle mechanism for tritium transport to plants is by deposition to soil and uptake by plant roots. If there is no rain, tritium can enter plants by atmospheric moisture. Surface waters as modelled, can receive tritium through deposition from contaminated air.

2.2.3 Application of the Dose Model to Sample Sites

In this section, potential consequences of tritium releases are determined by the application of the TREM computer code to specific sample sites. These calculations serve three purposes. First, they provide a demonstration of the TREM models. Secondly, they present a conservative analysis of the potential doses in rem and the actually incurred doses that could result from normal

operations and from accident sequences in the operation of controlled thermonuclear reactors (CTR's). Finally, they reveal those site-related parameters that can alter the consequences of tritium releases.

The maximum available doses and the actually incurred doses within 80 km are presented as estimated source terms for each sample site. These calculations are applied to continuous normal releases, short-term releases (accidental or normal), and large-scale instantaneous releases.

Tritium releases during normal operations can be classified into three categories [2-15]:

- 1) Leakage from fluid system connections and penetrations into the blanket and shield regions of the facility, with subsequent release within containment and to the environment.
- 2) Releases within containment and to the environment during routine maintenance.
- 3) Permeation of tritium into coolant with releases to the environment as a result of steam generator blowdown, low-grade heat dissipation, and related coolant system operations.

Releases in the first two categories are expected to be T_2 or HT. However, the third category, which involves tritium permeation into the steam system and exchange with protium to form THO, provides a source of tritiated water vapor. Calculated tritium entry to the steam varies from a fraction of a curie per day to as much as 30 curies per day, depending upon the amount of oxide film formation in the steam generator [2-16]. Kabele, et al. [2-15] have estimated as high as 3770 curies/year releases to the environment from the steam system of UWMMAK-I, and an overall release from all systems (excluding storage facilities) of 5000 ci/yr.

The specific form of tritium storage has not been described in major conceptual design studies. It is expected to be in the order of several kg, in order to avoid power outages while the tritium breeding systems are inoperative [2-17]. Releases during accidents are assumed to be either T_2O or $(LiOT)_x$ [2-1]. For this study, accidental releases are assumed to produce 10^8 curies of tritium as THO or LiOT within one-half to one hour time periods.

Available doses and dose rates along with actual dose commitments depend upon the meteorologic, geographic, and demographic characteristics of the release

locations. For this study, the assumption is made that hybrids will have similar siting requirements for cooling water and remoteness to those of the present generation LWR's. Under this premise, data for two LWR sites in the U.S. having contrasting climatic and topographic features were selected for sample dose calculations. Site A is the location of the Indian Point facility in New York State, roughly 25 miles north of New York City. Site B is the location of the Rancho Seco facility in central California. A summary of important site-related parameters for these locations is given in Table 2-6.

- Population data for both sites is that projected for the year 2010. For each site, accessible information sources provided data on the relative amounts of land involved in various agricultural activities. However, no information was available on specific production in kg per unit land area. This was calculated from overall U.S. agricultural statistics.

Continuous Uniform Releases. The yearly average ground-level concentration within 1 km of the site boundary was calculated based on an assumed release rate of 10 curie per day. The frequency distribution for wind speed, stability class, and direction were obtained from the respective environmental impact statements. To calculate the maximum potential dose that would result from this release rate, it was assumed that some individuals reside full time at the site boundary (200m) in the direction of the highest concentration, and eat fruits, vegetables, meats, and milk from plants grown and animals grazed at this same location. It is assumed that all drinking water and fish come from a hypothetical reservoir at the site boundary. The results of the maximum contaminated consumption to one man are given in Table 2-7 for both sites.

The actual dose in man-rem/yr that would be incurred within 80 km of each site from a 10 ci/day release are given in Table 2-8 for all pathways, except for fish consumption for which data was not readily available. Since the maximum dose-to-man calculation reveals only a small contribution from fish consumption, it has been neglected.

Short-Term Releases. The variation of the time-integrated dose in rem from each pathway as a function of the release duration, to a hypothetical person at the site boundary, is shown in Figure 2-4. The release quantity was fixed at 3650 curies for this calculation. It can be seen that after approximately 25 days,

Table 2-6
SITE-RELATED PARAMETERS FOR TWO SAMPLE SITES

PARAMETERS	SITE A	SITE B
Total Permanent Population Estimated for the Year 2010	26,000,000	3,500,000
Principal Land Use within 80 km	Residential, Industrial, Some Agriculture	Agricultural, Residential
Yearly Average Absolute Humidity (g/m^3)	7.33	8.45
Annual Rain Frequency (%)	5.5	1.9
Percent of Land Used for Agriculture within 80 km	< 30	<u>≥</u> 60
Principal Wind Directions	SSE, S, N, NNW	E, ESE, SSW
Principal Population Sectors	S, SSE, SSW	NW, WNW, SSW, NNW

Table 2-7

DOSE TO A PERSON AT THE SITE BOUNDARY FROM A 10 Ci/DAY RELEASE OF THO

PATHWAY	DOSE IN MAN-REM/YEAR	
	SITE A	SITE B
Inhalation	3.23	3.47
Vegetables	2.21	2.05
Meat	1.02	0.970
Milk	0.800	0.756
Drinking Water	0.537	0.534
<u>Fish</u>	<u>0.001</u>	<u>0.017</u>
TOTAL	7.798	7.797
Background	100	100

Table 2-8

DOSE COMMITMENT WITHIN 80 km FROM A 10 Ci/YEAR THO RELEASE

PATHWAY	DOSE IN MAN-REM/YEAR WITHIN 80 km	
	SITE A	SITE B
Inhalation	95.4	8.71
Vegetables	2.63	4.78
Meat	0.251	0.43
Milk	0.62	1.06
Drinking Water	9.7	1.36

Figure 2-4. Dependence of the Total Integrated Dose at the Site Boundary on Release Duration for a Fixed Release of 3,650 Curies.

the dose becomes independent of the release duration. For short time periods, up to one day, the dose increases with decreasing duration. For releases less than one day, the half-time of tritium in plants decreases with exposure duration so that the overall dose does not increase. This calculation used the parameters for Site A. The yearly frequency distribution of wind speed, direction, and stability class was scaled to the release duration.

The results of the same calculation applied to a hypothetical human, living at a distance of 5 km from the source, is shown in Figure 2-5. In this case, the dose decreases for very short releases, due to depletion of tritium from the passing cloud by dry deposition upwind. Inhalation and drinking water doses which do not involve plants are essentially independent of release duration.

Instantaneous Releases. Effects of instantaneous releases were evaluated by postulating a 10^8 curie discharge of THO from a height of 10 m. The duration of the release was chosen as 1/2 hour. By our assumptions, the direction of wind did not change in the duration of the release, and a windspeed of 2.5 m/sec and stability condition D (the one for which the highest concentration at release height 10 m and distance 200 m occurs) was used. The resulting time-integrated dose to man at the site boundary for each pathway is given in Table 2-9.

In addition to calculating the consequences of accidents in which the worst assumed atmospheric conditions prevail, the expected consequence was also determined for each site, based on the frequency of occurrence of various atmospheric conditions. The expected consequence for each site was calculated using the expression

$$\langle C \rangle = \sum_{s=A}^F \sum_{v=1}^N \sum_{d=1}^{16} C(s,v,d) F(s,v,d) \quad , \quad (2.9)$$

where

$\langle C \rangle$ = the expected consequence in man-rem for a given pathway,

$C(s,v,d)$ = the incurred dose for a given pathway under stability condition s , wind speed category v , and wind direction d ,

$F(s,v,d)$ = the frequency of the joint occurrence of stability condition s , wind speed category v , and wind direction d (normalized to unity), and

N = the number of wind-speed categories.

Figure 2-5. Dependence of the Total Integrated Dose at 5 km on Duration for a Fixed Release of 3,650 Curies.

Table 2-9
DOSES TO A HYPOTHETICAL HUMAN AT THE SITE BOUNDARY FROM A 10^8
Ci INSTANTANEOUS RELEASE OF THO

PATHWAY	DOSE IN REM	
	Wind Speed = 1 m/s	Wind Speed = 2.5 m/s
Inhalation	2,240	895
Vegetables	5,840	2,340
Meat	2,467	986
Milk	1,295	770
Drinking Water	18.3	7.32

Table 2-10
EXPECTED CONSEQUENCES FROM AN INSTANTANEOUS RELEASE OF 10^8 Ci of THO
WITHIN A RADIUS OF 80 km

PATHWAY	DOSE IN MAN-REM	
	SITE A	SITE B
Inhalation	2.12×10^6	1.67×10^5
Vegetables	1.75×10^5	3.15×10^5
Meat	1.53×10^4	2.57×10^4
Milk	3.87×10^4	6.50×10^4
<u>Drinking Water</u>	<u>2.16×10^5</u>	<u>2.68×10^5</u>
TOTAL	2.56×10^6	8.31×10^5

Table 2-10 (Continued)

EXPECTED CONSEQUENCES FROM AN INSTANTANEOUS RELEASE OF 10^8 Ci OF THO
DURING PERIODS OF RAIN WITHIN A RADIUS OF 80 km

PATHWAY	DOSE IN MAN-REM	
	SITE A	SITE B
Inhalation	4.56×10^5	3.81×10^3
Vegetables	4.43×10^4	9.91×10^4
Meat	3.71×10^3	7.88×10^3
Milk	9.46×10^3	2.01×10^4
<u>Drinking Water</u>	<u>1.67×10^5</u>	<u>1.45×10^3</u>
TOTAL	6.80×10^5	1.32×10^5

EXPECTED DOSES FROM AN INSTANTANEOUS RELEASE OF 10^8 Ci of LiOT
WITHIN A RADIUS OF 80 km

PATHWAY	DOSE IN MAN-REM	
	SITE A	SITE B
Inhalation	1.0×10^6	3.0×10^4
Vegetables	5.16×10^4	1.13×10^5
Meat	4.32×10^3	8.93×10^3
Milk	1.10×10^4	2.28×10^4
<u>Drinking Water</u>	<u>9.70×10^4</u>	<u>7.02×10^3</u>
TOTAL	1.16×10^6	1.82×10^5

The expected consequences for each site from a 10^8 curie release of THO are given in Table 2-10. The expected consequences for this release, assuming a 100% probability of rain with a washout coefficient of 10^{-3} sec^{-1} , are also shown in this table, together with consequences from a 10^8 ci release of LiOT for a dry deposition velocity of 10^{-2} m/sec . In each case, the assumed release height was 10 m.

2.3 CONCLUSIONS

In the hybrid system there are four types of radioactive material — tritium, activation products, actinides, and fission products. In the event of a major accident (considered a low-probability event), the activation products, fission products, and actinides provide the most significant hazard. The character of the fission product and actinide inventory has been examined in this section. The fission product inventory in a hybrid blanket, in terms of a continuous power output of 1 Gwt for a 3 year period, can be expected to differ from that in an LWR as the result of a different fission-yield spectrum and high-energy neutron-induced transmutations. However, it was found here that this difference, which was based on a few abundant and hazardous fission products, is almost insignificant for the reference design [2-6]. An increase of the average neutron flux in the energy range above 10 Mev by 2 orders of magnitude was found to reduce the fission-product hazard of the blanket significantly. It decreased the inventory of long-lived nuclides and increased that of shorter-lived nuclides. However, the occurrence of such an increase in flux seems to be unlikely, based on the wall-loading limitations.

The most significant detriment, in terms of the hazard index and the potential hazard index, resides in the accumulation of plutonium and other actinides in the fission blanket. Even in the conceptual design of a hybrid actinide burner, the inventory of actinides increases to a significant hazard level before consumption through fission takes effect [2-18].

Large inventories of tritium are associated with the plasma in a hybrid, and also with external storage and transportation systems. Unlike fission products and actinides, tritium species can be released in rather large quantities as a result of normal operations or small off-normal events.

To quantify the magnitude of the hazard presented by tritium releases, a computer model (TREM) has been prepared to describe the dispersion, uptake, and dose

commitment in man-rem from tritiated species [2-12].

Calculations carried out with the TREM program indicate that fueling, processing, storage, and transportation operations associated with CTR systems, and involving tritium as THO or in a combustible form such as T_2 as TH, should not be ignored. Dose commitments in the order of 10^5 man-rem/kg released, can be reasonably expected within 80 km of the source. Long-term environmental implications of tritium accumulation in the hydrosphere have been shown to be lower on a per curie basis than consequences within 80 km. However, as the number of facilities releasing small quantities of tritium increases, the unavoidable accumulation of tritium in the world water system will play a more significant role. A more thorough understanding of the dynamics of tritium movement in the hydrosphere is necessary to determine this effect.

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Section 3

EVENT TREES

3.1 INTRODUCTION

Event trees have been used previously in the examination of safety for preliminary fusion power reactor designs as an aid in identifying potential phenomena having a profound effect on accident sequences [3-1,3-2]. The selection of possible initiating events for these event trees depend on two criteria:

Can the initiating event lead to the potential release of biologically hazardous material, either to the environment or to a part of the containment where the material is not present under normal operating conditions, and

Can the initiating event tap large sources of energy which could eventually be used to violate containment integrity, or in some other way release a hazardous substance.

The basic design employed in the construction of the various event trees presented here, is one of the conceptual Mirror-Hybrid Power Reactors, designed at the Lawrence Livermore Laboratory [3-3]. In this design, the DT plasma is magnetically confined by Yin-Yang magnets in a mirror configuration. The blanket consists of a fast-fission zone with an outer tritium breeding zone, and uses high-pressure helium as the blanket coolant. A thorough description of the blanket is given in Section 4.

Two events which have the potential for the ultimate displacement or release of hazardous materials, are the blanket loss-of-flow-accident (LOFA) and the blanket loss-of-coolant accident (LOCA). In both cases, the presence of decay heat (from the fusion products) represents a source of energy that could ultimately melt the fuel.

A potential source of energy is the energy produced by fast-fission in the blanket that surrounds the plasma. This potential source of energy could be released if the blanket became critical. Another large source of potential energy is the plasma. A density-temperature instability could lead to a power

overshoot. Possible consequences are: localized melting of the blanket fuel due to an over-power/flow mismatch as well as a plasma quench on the first wall, causing localized first-wall melting or vaporization.

The magnet section also represents a large source of stored energy. However, the various accidents (event trees) involving magnet failures have already been discussed in detail in a previous fusion safety report [3-1].

3.2 MAJOR SEQUENCES

In this study, event trees have been constructed for the following accident sequences:

- (1) Blanket Loss-of-Flow Accident (LOFA),
- (2) Blanket Loss-of-Coolant Accident (LOCA),
- (3) Steam-Ingress Accident (SIA), and
- (4) Plasma Density-Temperature Transient Accident.

These event trees are not intended to be considered all-inclusive. Instead, they represent some major safety areas of concern which may be unique to a fusion-fission hybrid reactor.

The LOFA and LOCA are presented because such considerations are intrinsic to any power producer in which a heat-transport system is used in conjunction with a steam generator. The Steam-Ingress Accident was studied because of its potential for drastically changing the neutronics of the blanket; i.e. from a fast spectrum to a thermal spectrum, causing a large change in neutron multiplication. Plasma Density-Temperature Transients can potentially lead to over-power situations exceeding the plant design limits.

The event trees constructed for these potential events were then used to plan some of the phenomenological studies presented in Sections 4, 5, and 6.

3.3 BLANKET LOSS-OF-FLOW ACCIDENT

The flow diagram for this accident is shown in Figure 3-1. It is assumed in the blanket loss-of-flow accident (LOFA) that the pumping system for forced convection fails. After the initiation of a LOFA, the plasma reaction will either be shut down, or remain in operation.

Figure 3-1. Blanket Loss-of-Flow Accident.

First, we shall consider the case when the reactor is shut down. A major question that arises is whether or not decay heat can be removed passively. In the LLL conceptual design [3-4] a depressurized condition is considered, in which the pressure of the system drops from 20 atm to 1 atm. The calculations indicated that the decay heat can be removed, as long as the heat exchangers remain operational.

In general, the passive removal of decay heat will depend on natural convection and a sink (i.e. the heat exchangers or steam generators). If the system is unable to cool itself passively (e.g. loss of heat sink), then the decay heat will cause an increased temperature in the fuel, first wall, and structure with ultimate melting. Detailed analysis is required to obtain the melting sequence, as well as subsequent motion. Retained fission gas may act as a driving mechanism, as in LMFBR accidents.

Since the blanket must surround the plasma, the direction in which the molten material will move with respect to the blanket sub-module, will depend on the blanket's orientation. We consider two extreme cases: (a) the sub-module facing downwards, and (b) the sub-module facing upwards. In case (a), the molten clad would flow to the first wall. The fuel would then slump and fall downwards on top of the failed cladding. Without any available means of cooling, eventually all of the fuel will probably melt and reconfigure. If the first wall should fail, then the clad material and fuel would fall onto the other portion of the blanket sub-modules that are facing upwards. In the latter case, the molten cladding will fall onto the tritium breeding zone ($\text{LiAlO}_2 + \text{C}$ canned in SS). The zone may act as a catcher for the stainless steel and molten fuel. Therefore, in the absence of passive cooling, it can be envisioned that all of the fuel and cladding will eventually slump and rearrange itself on the bottom portion of the reactor's containment.

This sequence then leads to two major questions: (1) Is it possible for the fuel and cladding to assume some geometric configuration that would allow criticality to occur, and (2) What type of post-accident heat removal is necessary to containing the fuel? The question of criticality will depend upon burn-up, which in turn will dictate the plutonium enrichment as well as the geometry involved. The problems associated with burn-up, enrichment, and criticality of the fuel are discussed in Section 4. If the fuel could become critical, a power excursion

would result, generating energy and leading to a release of fission products. It would also complicate the problem of trying to contain the molten fuel and the problem of containment overpressurization. There is also the possibility of a lithium aluminate-concrete reaction, should this portion of the blanket melt. If a breach in the primary containment should occur, the possible escape of the fission products and tritium is increased.

Another possible scenario in a LOFA occurs if the plasma is not shut down after the accident is initiated. Adiabatic studies have shown that in this case, the time scale for the LOFA changes from minutes to seconds [3-4]. The fuel would reach its melting point adiabatically in ~ 50 seconds. Again, the sequence of clad, fuel, and first wall melting requires a detailed calculation. One mechanism for initiation of a magnet quench is the failure of the first wall. It may fail by reaching its melting point, or by rupture due to a pressure rise in the helium, as it is being heated. Helium has a low heat capacity which will allow its temperature and pressure to rise quickly. The sequence of the LOFA at this point would follow that of a shutdown, except there will be more heat in the system which must be removed.

In discussing the LOFA, the two extreme cases (with and without plasma shutdown) were considered. In reality, however, there will be a lag time between the shutdown and the initiation of the accident. The effects of this lag time should be taken into account in any detailed LOFA analysis.

3.4 BLANKET LOSS-OF-COOLANT ACCIDENT

In a blanket loss-of-coolant accident (LOCA), it is postulated that the helium coolant is lost from the blanket via a pipe rupture or loss of a seal. After a LOCA is initiated, the plasma will be either shut down or will continue to burn.

In either case, the blanket and first wall thermal responses to the LOCA will depend on the size and location of the break in the cooling system. In fact, there may be a very high initial flow rate, causing the fuel and cladding temperatures to drop at the beginning stages of a LOCA. However, as the LOCA progresses, the density and pressure of the coolant will decrease, causing the heat-transfer ability to decrease also. The blanket will have to rely on the containment for any type of circulation for cooling during a LOCA.

In the case where shutdown occurs, the question is whether or not the blanket is able to remove the decay heat passively. This in turn, will depend on the break size as well as on the particular design parameters. Various heat-up rates (for the LLL design [3-3]) during a LOCA are reported in Section 5. The accident scenario may be basically the same as that for a LOFA under certain circumstances. The various possibilities are shown in Figure 3-2.

In the case without plasma shutdown, the fuel, cladding, and first wall, will heat up as the coolant is lost. First-wall failure is expected to occur by either overpressurization of the system, or by its own internal heating. This should result in plasma quenching. The accident at this point will follow the same general scenario as in a LOFA. The amount of heat generated prior to first-wall failure may or may not be greater than that in the case of a LOFA.

It should be noted that, although the event trees for the LOFA and LOCA appear to be quite similar, the detailed physics of each accident are quite different, and so are their respective consequences.

3.5 STEAM-INGRESS ACCIDENT

The event tree for the Steam-Ingress Accident is shown in Figure 3-3. In this accident, it is assumed that the steam from the steam generator system enters the blanket region of the reactor. Two situations can occur in the Steam-Ingress Accident (SIA): the plasma will either continue to burn, or it will be shut down.

First, we consider the case where the plasma is shut down. Once the steam enters the blanket region, the blanket will either remain intact, or the pressure of the steam may be sufficient to cause a change in the geometry of the blanket. This change may be important, since there are only two modes by which the reactivity of the blanket may change: (1) change in composition, and (2) change in geometry. The composition is changed for example by the addition of steam, and the geometry by the possible disruptive forces of the steam. In either case, the final result will be a change in neutron multiplication of the blanket. For the specific LLL design, several studies were made to investigate the reactivity effects of a SIA; these are reported in Section 4. These studies were performed as a result of constructing the SIA event trees, and are for a specific design. The SIA event tree itself is constructed to be generic for any Gas-Cooled Fission-Fusion reactor.

Figure 3-2. Blanket Loss-of-Coolant Accident.

Figure 3-3a. Steam Ingress Accident (With Shutdown).

Figure 3-3b. Steam Ingress Accident (Without Shutdown).

If the neutron multiplication decreases as a result of steam ingress, then the major problem is the decay-heat removal. If the decay heat is not removed adequately, a chain of events will follow the LOFA. If neutron multiplication increases, two possibilities become apparent: (1) the blanket remains subcritical, or (2) it becomes critical. If it remains sub-critical, the question of adequate decay-heat removal arises, as discussed above. If the blanket becomes critical or super-critical, it will suffer a power rise and will heat up. Eventually, the blanket will be sub-critical by either some temperature-feedback mechanism such as the Doppler or fuel expansion coefficient, or by fuel rearrangement. The extra heat generated in this period must be removed; however, it is unlikely that there will be adequate means to cool the system. Therefore, the accident will become similar to a LOFA when passive heat removal fails. Another likely possibility is an energy release, caused by the blanket becoming super-critical. The damage to the containment would be dependent on the magnitude of such an energy release. Such a disruption could result in the release of fission products and actinides.

In the second scenario, the plasma continues to burn, even though steam has penetrated the blanket region. As in the case of a hypothetical shutdown, the neutron multiplication will either increase, or decrease. In the former case, it is unlikely that the blanket will be coolable as designed, since an over-power situation would occur. First-wall failure would probably cause a plasma quench. At that point, the accident scenario would resemble the shutdown. If the blanket becomes super-critical, the possibility of an energetic dis-assembly would have to be considered.

3.6 PLASMA DENSITY-TEMPERATURE INSTABILITY TRANSIENT ACCIDENT

It has been demonstrated [3-5,3-6,3-7] that fluctuations in the density and/or temperature of a plasma can lead to density-temperature (n-T) instabilities, and to power overshoots for Tokamak systems. To determine the consequences of such plasma instabilities and the resultant overshoots, it has been assumed that the engineered safety-feedback controls fail, i.e. that it is an unprotected accident. The possible scenarios for this accident will depend on the plasma density and temperature. An event tree for the accident is shown in Figure 3-4.

First we consider the case when the plasma temperature decreases below the ignition temperature. The fusion reaction would cease, causing a further drop in plasma

Figure 3-4. Density-Temperature Instability Transient Accident.

temperature, until it became a resistive load, reducing the current. This will cause the collapse of the magnetic field in the blanket, and induce forces in the piping, blanket structure, and first wall. The induced forces have the potential to cause failure of pipes, ducts, and conduits. This can lead to a helium leak into either the vacuum chamber, or into the primary containment. The plasma may also diffuse towards the first wall, but it is unlikely to cause significant damage, since it will be cooled beforehand.

If the plasma temperature increases, a much more complicated accident scenario results. The plasma behavior depends on several factors because the density and temperature are coupled. Consequently, the plasma density and temperature may behave in any one of the following ways: (1) They may increase initially and then decrease to below-ignition conditions, resulting in a quench, (2) They may go to a new steady state, or (3) They may increase and then return to their original steady state.

One concern is that the increase in density and temperature could lead to a violation of the β confinement criteria, which is necessary for plasma containment, causing it to quench on the first wall. The effects of this occurrence has been discussed in a previous report [3-1]. It was shown that a quench can lead to first wall damage and loss of integrity, either by the plasma hitting the first wall directly, or by induced forces.

Another concern in a fission-fusion power reactor is the increase in the first-wall neutron loading, which results from an increase in the plasma density and temperature. This occurs because the neutron source is determined by the product of the ion densities and the reaction rate, which is temperature dependent. The magnitude of the power overshoot is determined by evaluating this product over the time interval of the excursion.

In the presence of fissionable material, the possibility of a power overshoot also becomes an area of interest. This occurs because a sudden increase in the neutron source will increase the power density and the adiabatic heating. Such an event in the fissionable material may cause localized fuel or clad failure. Also, the heat-removal capacity of the helium may be exceeded, causing overpressurization of the system and initiating a blanket LOCA. Such an overshoot can then lead to fission-product and actinide release. The effects of a blanket LOCA are discussed in Section 3.3.

3.7 CONCLUSIONS

In this section, event trees were constructed for some possible accidents in conceptual fission-fusion power reactors. These are not intended to be all inclusive, they only try to demonstrate some areas of concern in the realm of safety. Although a particular design was used as a guide, an attempt was made to discuss them in a generic way.

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Section 4

CRITICALITY CONSIDERATIONS

4.1 INTRODUCTION

One of the significant safety questions for fusion-fission hybrid reactor design concepts concerns the possible criticality of the fissionable blanket. Although such blankets are designed to be far subcritical throughout the blanket lifetime, various mechanisms of reactivity insertion or geometrical rearrangement could be envisioned which might, in an accident situation, lead to criticality. In this work, the results of simple numerical calculations for one such design are reported. These calculations were performed to obtain some insight into the important physical considerations related to criticality.

If blanket criticality can be achieved, the potential for large-scale radioactive release and/or permanent system damage becomes very great. Overpower situations can lead to fuel melting, with subsequent fission-product and/or actinide release. Depending upon the rate of reactivity insertion, blanket disruption is also a possibility. Such energetic disruptions, as postulated for liquid-metal fast breeder reactors, for example, can lead to breach of containment with release of radioactivity to the environment.

Reactivity changes in a subcritical system can be caused by a change in composition and/or a change in geometric configuration. Various changes in the system, which might be associated with accidents in the hybrid blanket, were investigated, and their effect on neutron multiplication was examined. These changes included:

1. The ingress of steam into a fast-spectrum blanket,
2. Complete melt-down of the blanket material, and
3. Steam ingress with resulting blanket expansion.

The first process is an example of a blanket-composition change, which could result in a thermal neutron spectrum, thus leading to criticality. The second change involves a geometric reconfiguration, and could arise due to a loss-of-coolant accident. Steam ingress with blanket expansion is an example of a combined

composition and geometry change, which might be caused, for example, by a metal-water reaction which generates large internal pressures.

4.2 REFERENCE DESIGN DESCRIPTION

The Lawrence Livermore Laboratory (LLL) Mirror Hybrid Reactor [4-1] served as the reference design and basis for the calculations in this work. The design utilizes the Yin-Yang mirror configuration for plasma confinement and the D-T cycle for energy production. The blanket design is a modular concept, composed of uranium carbide fuel pins with stainless steel cladding, and is cooled by helium at a pressure of 20 atmospheres. The blanket consists of 20 modules located inside the magnet coils, each of which contains a collection of about 400 dome-shaped cylindrical pressure vessels, called submodules, containing the fuel assemblies and the tritium-breeding material. The basic submodule design is illustrated in Figures 4-1 and 4-2.

The criticality studies used the computer programs ANISN [4-2] and DEplete [4-3]. ANISN is a one-dimensional (1D), multigroup, discrete ordinates, transport code which solves the Boltzmann transport equation, utilizing the S_N method. DEplete estimates the plutonium buildup and uranium burnup as a function of time in the blanket. This is important, because most blankets breed fissile material, and the reactivity increases with time due to this fissile buildup.

The fusion-fission device was modeled as a spherical annulus with a 3.2 meter radius plasma neutron source in the center, which is surrounded by three spherical blanket shells. The first shell is the stainless steel first wall, 0.5 centimeters thick, with an inner radius of 4 meters. This corresponds to the domed ends of the submodules. The second shell is the 20 centimeter thick fission zone, containing the uranium carbide fuel. The third shell, composed of LiAlO_2 , is the 60 centimeter thick tritium-breeding zone.

The source calculations, performed to obtain the time-dependent plutonium buildup, utilized cross-sections obtained from a 25 group library [4-4]. It included fast groups up to 14 MeV, and had been used for a similar study of a fusion-fission hybrid. The 16 group Hansen-Roach cross-section library [4-5] was used for the criticality calculations. All coded computations were performed on the UCLA IBM 360/91 computer.

Figure 4-1. Fast-Fission Blanket Submodule.

- * Number of fuel rod per submodule ≈ 270
- * Thickness of cladding = 0.02 cm

Figure 4-2. Top View of Fast-Fission Blanket Submodule.

4.3 TIME-DEPENDENT SOURCE CALCULATIONS

The blanket-composition changes throughout reactor lifetime were estimated using the ANISN and DEplete computer codes. The blanket was divided into several intervals, ten of which were in the fission zone, and the flux was computed for each one. These values were then fed into DEplete to compute the plutonium-239 and 240 buildup and the uranium-235 and 238 burnup during each time step. This was accomplished by employing the production-destruction rate equation for each isotope of interest. The general form of this equation is:

$$\frac{dN}{dt} = N'\sigma_c \phi - N\sigma_a \phi,$$

where

$$\frac{dN}{dt} = \text{density change of isotope } N,$$

$$N'\sigma_c \phi = \text{the production of isotope } N, \text{ and}$$

$$N\sigma_a \phi = \text{the destruction of isotope } N.$$

The computed atom densities were then inserted into the ANISN input to compute the fluxes for the next time step.

The flux calculations were accomplished using a P_1 expansion of the scattering cross-section and an S_4 Gauss quadrature set for flux integration. Prompted by the reference design report, the length of a time step was chosen as the time required for a 1% burnup of the initial U-238 atom density, averaged over the fission zone. Figure 4-3 shows the value of K_{eff} as a function of percent burnup, assuming no plutonium recycle in order to achieve the highest possible plutonium concentration and, hence, the highest value of neutron multiplication. This figure also demonstrates that the neutron multiplication peaks at about 29% burnup, or after approximately 20 years of continuous operation with no recycle. This is, of course, unrealistic if material and economic considerations are taken into account. According to Livermore's proposed fuel management scheme*, the maximum attainable burnup of U-238 should not exceed 5% in any fuel bundle. In order to obtain a more realistic idea of the actual neutron multiplication during the blanket lifetime, the calculations were conducted at the time when 5% of the initial U-238 was burned up. This burnup corresponds to approximately 5 1/2 years of continuous operation.

*At the time of this writing, a recent paper [4-6] by the LLL group indicates a trend towards even lower burnups.

Figure 4-3. K_{eff} as a Function of Burnup.

4.4 INNER-RADIUS DECREASE

The potential for criticality was investigated for various geometric changes which insert reactivity. The first one was equivalent to a decrease in the inner radius of the modeled spherical annulus, while the "shells" were maintained at a constant volume. This requires an adjustment of the outer radius, to compensate for volume changes. The U-238 atom density was assumed to be 5% burned out. This assures a plutonium buildup sufficiently high to encompass any normal buildup that will be encountered in this design. As the inner radius decreased, the neutron multiplication was found to increase. However, even when fully compressed ($R_1 = 0.0$ cm), the blanket material was still very subcritical ($K_{eff} \approx 0.76$).

To fully assess the potential for criticality of these geometric changes, calculations were performed to investigate the change in neutron multiplication as the plutonium density changed. For these calculations the blanket was assumed to be fully compressed. It was also assumed that the stainless-steel first wall occupied the center of the sphere, with uranium carbide and cladding on the outside of this, and with lithium aluminate surrounding them. Figure 4-4 shows K_{eff} for this configuration, as a function of burnup. It illustrates that criticality is achieved only after approximately 10% of the initial U-238 concentration has burned out.

4.5 MELT-DOWN ACCIDENT

In accident situations, such as a loss of coolant, blanket melting may cause a geometrical rearrangement, resulting in a more reactive configuration. The loss of structural integrity coupled with the temperature increase of the molten fuel provides a high potential for the large-scale release of very hazardous radioactive materials. Criticality calculations were performed for three different changes associated with a melt-down accident. In each case, the material was modeled as a sphere, which achieves the most reactive geometry.

In the first configuration, the blanket material occupies separate zones within the sphere. The uranium carbide is in the center, with stainless steel surrounding it, and LiAlO_2 on the outside. This achieves a K_{eff} of approximately 0.77 with a uranium burnup of 5%. Figure 4-4 is also representative of the change in K_{eff} as burnup increases. This is due to the fact that the placement of blanket materials within the different zones of the sphere changes K_{eff} very little, as long as the LiAlO_2 surrounds the UC.

Figure 4-4. K_{eff} vs. Burnup, Fully Compressed Blanket (Inner Radius = 0.0 cm).

The second change assumes that the blanket material had melted, slumped, and homogeneously mixed into a bare sphere configuration. Figure 4-5 displays the results of these calculations and shows K_{eff} as the burnup increases. This configuration was found to be very subcritical, as a result of increased moderation into the resonance region.

The last change investigates the effect on neutron multiplication of the removal of carbon from the fuel. The fissile and fertile materials, with no carbon, are assumed to be homogeneously mixed in a bare sphere configuration. The carbon removal effectively increases the heavy-metal density as well as hardens the neutron spectrum due to lost moderation. Figure 4-6 shows the change in K_{eff} for increasing burnup and indicates that criticality will be achieved after approximately 5 1/2% burnup.

4.6 STEAM INGRESS ACCIDENT

The ingress of steam into the fast spectrum blanket has the potential for neutron thermalization and, with a suitable fissile buildup, possible criticality. This study investigates the instantaneous reactivity of the system for various compositions and geometries associated with different stages of the accident. It is assumed that a pipe break in the heat exchanger releases high-pressure steam into the relatively low-pressure helium cooling system of the blanket. The steam begins to fill the submodules, which contain the blanket material, in the first stage of the steam ingress accident. As more steam enters the blanket system, the increased pressure may cause one or more of the submodules to fail, thereby allowing steam to enter the plasma chamber, which comprises the second stage of the accident. The third and last stage involves a blanket expansion, modeled by keeping the blanket thickness constant and increasing the distance between the submodules, allowing more thermalization.

Calculations performed in this study included the steady-state determination of K_{eff} as a function of the amount of steam occupying the system, as well as the resonance-corrected cross sections to be used in the K_{eff} calculations. These corrected cross-sections include both energy and spatial self-shielding effects using methods found in References [4-4] and [4-5]. The K_{eff} calculations assumed an approximately 5% burnup of U-238 to ensure a plutonium buildup representative of the worst-case conditions. The results of the calculations are shown in Figure 4-7. A general increase in K_{eff} is seen, as steam content is increased in the system. Criticality is not achieved until well into the blanket expansion stage of the accident.

Figure 4-5. K_{eff} vs. Burnup, Fully Compressed Blanket, Blanket Material Homogeneously Mixed.

Figure 4-6. K_{eff} vs. Burnup, Fully Compressed Blanket, No Carbon in Fuel.

Figure 4-7. Steam Ingress Accident.

4.7 CONCLUSION

The general trend of all blanket reconfigurations yielded significant increases in neutron multiplication. However, low blanket residence times and, therefore, low fissile inventories prevented criticality, even for the most reactive geometries attainable. From the results of this study, the following conclusions can be drawn for the LLL blanket [4-1]:

1. A blanket melt-down could not achieve a critical configuration, except under extremely high blanket residence times. This assumes a totally reflected sphere of uranium carbide.
2. If the blanket material were in a compacted configuration and homogeneously mixed, the moderating capability of the non-fissionable blanket material coupled with the high resonance absorption of the U-238 would decrease neutron multiplication considerably.
3. Steam ingress causes a significant increase in neutron multiplication due to the excellent moderating capabilities of hydrogen. However, criticality could not be achieved in this design until extremely high U-238 burnup is realized.
4. Criticality could possibly be achieved in a blanket expansion coupled with steam ingress. However, volume fractions of blanket material and water would need to approach those of normal thermal reactors. It has also been estimated that approximately three times the water inventory of the steam generators is needed to create a critical configuration, even with blanket expansion.

The results of this study indicate that criticality would be very difficult to achieve in this design. It must be emphasized that this analysis was performed for only one design. It does show, however, that blankets can be designed which will remain subcritical under extensive changes in both composition and geometry. The methodology presented here could easily be extended to include many different hybrid blanket designs. More detail on the calculations summarized here can be found in Reference [4-3].

4.8 REFERENCES

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Section 5

LOSS-OF-COOLANT AND STEAM INGRESS ACCIDENTS

5.1 INTRODUCTION

Consideration of the safety of fusion-fission hybrid reactors must include the possibility of a severe disruption in the coolant system. Events such as loss-of-coolant and steam ingress accidents during the operation of fission reactors are thought to be among the most difficult accidents to cope with, and require the most stringent systems design. In some ways, a fusion-fission hybrid may share these potential problems and difficulties, since the power densities are comparable to those of fission reactors. Ameliorating factors, such as the subcriticality of the blanket, exist in hybrids. However, it is not prudent to assume that these or other factors can assure against potential hazards to public safety from accidents without rigorous and extensive analysis.

The basic components of the hybrid system based on the deuterium-tritium (D-T) reaction are the fusion plasma and a multi-zoned blanket. The plasma, which is the source of neutrons, is enclosed by a first wall, which also separates it from the blanket. The basic components of the blanket are a tritium-breeding zone and a fission zone. The fission zone serves both for energy multiplication and breeding. High-energy neutrons, charged species, and photons from the plasma interact with the first-wall materials and produce heat. The neutrons are then captured, either in forming fissile fuel (U-233, Pu-249), or in the breeding zone to produce tritium. Several combinations of fusion and fission reactors have been suggested, and are reviewed in References [5-1 through 5-3]. In principle, any method of generating a controlled thermonuclear reaction is a candidate for a hybrid reactor, including tokamaks, mirrors, theta-pinches, lasers, and relativistic electron and ion beams. The fission blanket designs vary between the limits of a purely fertile blanket, in which the breeding of tritium and fissile materials is emphasized to the exclusion of energy multiplication, to the fast and thermal fission blankets, in which the sole function is power production. Realistic hybrid designs may have to operate in some intermediate condition for optimum economics. The heat generated in the blanket must be removed by coolant loops. In some designs, the coolant loop also serves for tritium transport. Table 5-1 lists some candidate materials for the fissile fuels,

coolants, and structures. Among the coolant candidates, helium gas is frequently favored because it is chemically inert, has no magnetohydrodynamic effects, allows for easy removal of tritium, and has a low neutron capture cross section.

In this section of the report, three aspects of fusion-fission blanket safety are evaluated. While an attempt is made to present a generic analysis, those cases in which specific geometric or other factors appear are noted. The first aspect considered is a scenario initiated by a loss of coolant flow or loss of coolant. In either circumstance, the blanket loses its cooling function. The times required for reactor shutdown and for the actuation of auxiliary cooling systems, in order to prevent fuel element failure, are important parameters for the safety analysis. The heat-up rates in the fuel, the cladding, and the first wall have to be considered. The second aspect involves the possibility of water injection into the fast fission blanket, where criticality becomes a possible event. General methodologies for analyzing the events are established. In view of the preliminary nature of the design, the analytical model attempts to be simple but generic. The results are presented in parametric form so that the designer can easily utilize them for future design modifications. The third aspect considered is the integrity of the first wall during a loss-of-coolant accident, which affects not only blanket cooling but also heat removal from the first wall. This situation may arise in a liquid-metal cooled reactor. For such a reactor, the coolant provides a much greater thermal inertia than in the helium-cooled designs discussed here. This difference changes the regions most vulnerable to failure during a loss-of-cooling accident (LOCA).

Table 5-1
MATERIAL CHOICES FOR THE HYBRID BLANKET

FISSILE AND FERTILE FUELS	TRITIUM-BREEDING MATERIALS	MODERATORS	COOLANTS	STRUCTURAL MATERIALS
U Alloys	Li LiAl	None	He	Fe Alloy
UO ₂	LiAlO ₂	C	Li	Zr Alloy
UC	Molten Salt	LiH	Na	
UN			Molten Salt	Al Alloy

5.2 LOSS-OF-COOLANT ACCIDENT AND BLANKET MELT

Introduction

The overheating and melting of the first wall and the blanket in a hybrid fusion-fission power system is of safety concern, since they contain radioactive materials which, if released to the environment, may be a hazard to the public. Although the hybrid designs are still in a preliminary stage, an assessment of the LOCA and an analysis of the phenomenology involved would provide basic knowledge required for future reference designs concerned with minimizing the probability of a LOCA and maximizing the confinement of hazardous materials during the accident.

Blanket melting can be initiated by a sudden, complete failure of the coolant piping, followed by inadequate decay heat removal. Depending on the plutonium concentration, the slumping of the fuel may lead to a critical configuration. For a gas-cooled reactor, "loss of coolant" means a rapid depressurization of the coolant. The rate of depressurization depends on the size and location of the break. The heat removal capability in both the fission blanket and the first wall is degraded. Even after the reactor is successfully shut down, the decay heat still imposes a threat to the structural integrity of the plant. The percentage of decay heat is slightly higher in the fission zone than in the first wall, as shown in Figure 5-1.

One of the disadvantages of a gas-cooled reactor in the event of a LOCA is its lack of thermal inertia. Before the auxiliary coolant system is initiated, the margin of safety relies on the thermal inertia of the fuel, the cladding, and the first wall materials. The melting points of some of these materials are listed in Table 5-2. Carbide, oxide, and nitride fuels have high melting points, while the melting point of silicate fuel is below that of stainless steel. The thermophysical properties of a particular oxide fuel and of stainless steel are shown in Table 5-3. In estimating the heat generation rate in the first wall and blanket, the loss of helium would have only minor effects on the neutronics of the blanket system. Hence, the spatial distribution of the heating in the blanket would be the same as in the steady state, but the transient power level is determined by the shut-down mode of the plasma and the decay-heat characteristic (Figure 5-1).

Two approaches were adopted in the analysis. (1) An approximate approach, in which the depressurization process is decoupled from the heating of the blanket, is used to obtain closed-form solutions. These solutions show the physics clearly, but do not accurately deal with the fact that depressurization causes degradation of the heat transfer in the fuel and first wall. (2) Then, this coupled effect is

Figure 5-1. Afterheat as a Function of Time after Shutdown.

Table 5-2
MELTING TEMPERATURES OF VARIOUS FUEL AND CLADDING MATERIALS

MATERIAL	MELTING TEMPERATURE (°C)
<u>Fuel</u>	
Uranium	1,404 ±3
Plutonium	913
UC	2,638 ±165
PuC	1,923
(U _{0.8} Pu _{0.2})C	2,743
(U _{0.8} Pu _{0.2})C _{0.95}	2,643
UC ₂	2,773
UN	3,123
U ₃ Si	930
(U _{0.8} Pu _{0.2})O ₂	3,032
<u>Cladding/First Wall</u>	
Stainless Steel	1,538

Table 5-3
THERMOPHYSICAL PROPERTIES OF MATERIALS

MATERIAL	TEMPERATURE (°C)						
(U _{0.8} Pu _{0.2})O ₂ Fuel Melting Point 3,032° C	25	500	1,000	1,500	2,000	2,500	3,000
k (cal/s·cm·°C)	0.015	0.009	0.006	0.005	0.005	0.06	0.007
ρ (g/cm ³)	11	10.4	10.2	10.0	9.9	9.8	8.4
C _p (cal/g·°C)	0.058	0.074	0.078	0.085	0.10	0.13	0.12
<u>Stainless Steel Cladding</u> Melting Point 1,538° C							
k (cal/s·cm·°C)	0.04	0.05	0.07				
ρ (g/cm ³)	8.0	7.7	7.5				
C _p (cal/g·°C)	0.11	0.14	0.16				

investigated by the numerical solution of the governing equations.

Depressurization

In estimating the rate of depressurization, the primary coolant loop is modeled as a single volume, V . The thermodynamical properties are assumed to be uniform within the volume. The mass and the energy of the helium during the transient are given by

$$\frac{dM}{dt} = -\dot{m}A \quad (5.1)$$

and

$$\frac{dU}{dt} = -\dot{m}Ah + Q_{in}, \quad (5.2)$$

where

M = helium mass = ρV ,

U = internal energy = $MC_V T$,

h = enthalpy = $C_p T$,

ρ = density,

Q_{in} = the energy transferred from the fuel and the first wall to the coolant, minus the heat transferred to the steam generator,

C_V = the specific heat at constant volume,

C_p = the specific heat at constant pressure,

T = temperature,

$$\dot{m} = \text{the mass flow rate} = k \left[\left(\frac{2}{k+1} \right)^{\frac{k+1}{k-1}} P \rho \right]^{\frac{1}{2}}, \quad (5.3)$$

k = the isentropic gas constant = C_p/C_V , and

P = pressure.

In the approximate approach, Q_{in} is assumed to be zero. Equations (5.1) and (5.2), together with the ideal gas law, $P/\rho = RT$, give

$$\frac{dP}{dt} = -B \frac{P^{3/2}}{\rho^{1/2}} \quad (5.4)$$

and

$$\frac{d\rho}{dt} = -C(P\rho)^{\frac{1}{2}}, \quad (5.5)$$

where

$$B = \frac{A}{V} k \left[k \left(\frac{2}{k+1} \right)^{\frac{k+1}{k-1}} \right]^{\frac{1}{2}}, \text{ and} \quad (5.6)$$

$$C = \frac{A}{V} \left[k \left(\frac{2}{k+1} \right)^{\frac{k+1}{k-1}} \right]^{\frac{1}{2}}. \quad (5.7)$$

Rearranging Equations (5.4) and (5.5), the depressurization rate is given by

$$\frac{d^2 \rho}{dt^2} = \frac{1}{2\rho} (k+1) \left(\frac{d\rho}{dt} \right)^2 \quad (5.8)$$

and

$$\frac{d^2 P}{dt^2} = \frac{1}{2P} \left(3 - \frac{1}{k} \right) \left(\frac{dP}{dt} \right)^2. \quad (5.9)$$

Equations (5.8) and (5.9) can be expressed in non-dimensional form as

$$\frac{d^2 \rho^*}{dt^{*2}} = \frac{1}{2\rho^*} \left(3 - \frac{1}{k} \right) \left(\frac{d\rho^*}{dt^*} \right)^2 \quad (5.10)$$

and

$$\frac{d^2 P^*}{dt^{*2}} = \frac{1}{2P^*} (k+1) \left(\frac{dP^*}{dt^*} \right)^2, \quad (5.11)$$

where

$$P^* = P/P_0,$$

$$\rho^* = \rho/\rho_0,$$

$$t^* = t/t_0,$$

$$P_0 = \text{the initial system pressure,}$$

$$\rho_0 = \text{the initial helium density, and}$$

$$t_0 \equiv \frac{1}{C} \left(\frac{\rho_0}{P_0 g} \right)^{\frac{1}{2}}.$$

The initial conditions for Equations (5.10) and (5.11) are

$$\rho^* = 1 \quad \text{at } t^* = 0, \quad (5.12)$$

$$\frac{d\rho^*}{dt^*} = -1 \quad \text{at } t^* = 0, \quad (5.13)$$

$$P^* = 1 \quad \text{at } t^* = 0, \text{ and} \quad (5.14)$$

$$\frac{dP^*}{dt^*} = -k \quad \text{at } t^* = 0. \quad (5.15)$$

Equations (5.10) through (5.15) uniquely define the transient behavior of the density and pressure in the helium coolant. The solutions are:

$$\rho^* = \frac{n^n}{(n + t^*)^n} \quad (5.16)$$

and

$$p^* = \rho^{*k}, \quad (5.17)$$

where

$$n = \frac{2}{k-1}. \quad (5.18)$$

Hence, Equations (5.18) and (5.17) give a simple closed-form solution for the pressure and the density variation, if the heat transfer to the coolant is neglected.

First Wall and Fuel Rod Heat-Up

As the system is depressurized, the first wall and the fuel rod start to heat up. An energy balance in the first wall, the fuel, and the cladding would yield

$$C_f \frac{dT_f}{dt} = q_f''' - \frac{T_f - T_\phi}{R_1}, \quad (5.19)$$

$$C_\phi \frac{dT_\phi}{dt} = \frac{T_f - T_\phi}{R_1} - \frac{T_\phi - T_c}{R_2}, \text{ and} \quad (5.20)$$

$$C_w \frac{dT_w}{dt} = q_w''' L_w - \frac{T_w - T_c}{R_3}, \quad (5.21)$$

where

T = temperature,

$$C_f = \pi r_f^2 C_{pf} \rho_f,$$

$$C_\phi = 2\pi r_\phi (\Delta r) C_{p\phi} \rho_\phi,$$

$$C_w = \rho_w C_{pw} L_w,$$

ρ = density,

C_p = heat capacity,

q_f''' = the volumetric heat rate in the fuel,

$$R_1 = \frac{1}{4\pi k_f} + \frac{1}{2\pi r_\phi h_g},$$

r_f = the fuel pellet radius,

r_ϕ = the fuel rod radius,

Δr = the clad thickness,

L_w = the first wall thickness,

k = thermal conductivity,

h_g = the conductivity of the gap,

$$R_2 = \frac{1}{2\pi r_2 h_\phi},$$

h_ϕ = the heat-transfer coefficient of helium at the cladding,

$R_3 = \frac{1}{h_w}$ = the inverse of the heat-transfer coefficient of helium at the wall, and

q_w''' = the volumetric heating rate in the first wall.

Subscripts

f = the fuel region,

ϕ = the cladding region, and

w = the first wall region.

The coupling between the heat up in the blanket and the hydraulic condition is through the heat-transfer coefficients h_w and h_ϕ . These coefficients can be evaluated from the depressurization condition. An approximate approach with $h_\phi = 0$ and $h_w = 0$ is outlined for the investigation of the time available for reactor shutdown. Then, Equations (5.19) through (5.21) become:

$$C_f \frac{dT_f}{dt} = q_f''' - \frac{T_f - T_\phi}{R_1}, \quad (5.22)$$

$$C_\phi \frac{dT_\phi}{dt} = \frac{T_f - T_\phi}{R_1}, \text{ and} \quad (5.23)$$

$$C_w \frac{dT_w}{dt} = q_w''' L_w. \quad (5.24)$$

If t_1 is the time required for reactor shutdown, then q_f''' and q_w''' can be approximated by

$$q_f''' = q_{fo}''' + H(t - t_1)(q_{fd}''' - q_{fo}''') \quad (5.25)$$

and

$$q_w''' = q_{wo}''' + H(t - t_1)(q_{wd}''' - q_{wo}'''), \quad (5.26)$$

where

q_{fo}''' , q_{fd}''' = the steady state and decay heats in the fuel, respectively,

q_{wo}''' , q_{wd}''' = the steady state and decay heats in the first wall, respectively,

$H(t - t_1) = 0$ for $(t - t_1) < 0$, and

$H(t - t_1) = 1$ for $(t - t_1) \geq 0$.

The closed-form solution for the various temperature profiles is given as

$$T_f^* = A_1 + A_2 t + A_3 e^{-(H_f + H_c)t} + A_1' + A_2'(t - t_1) + A_3' e^{-(H_f + H_c)(t - t_1)}, \quad (5.27)$$

where

$$A_1 = \left[\frac{Q_o H_f}{(H_f + H_c)^2} + \frac{H_f \bar{B} + H_c}{H_f + H_c} \right] \frac{T_{fo}}{T_o} \quad (5.28)$$

$$A_2 = \frac{H_c Q_o}{H_f + H_c} \frac{T_{fo}}{T_o}, \quad (5.29)$$

$$A_3 = \left[\frac{H_f \bar{B} + H_c}{H_f + H_c} - \frac{Q_o H_f}{(H_f + H_c)^2} \right] \frac{T_{fo}}{T_o} \quad (5.30)$$

$$A_1' = \frac{H_f (Q_d - Q_o) T_{fo}}{(H_f + H_c)^2 T_o} H(t - t_1), \quad (5.31)$$

$$A_2' = \frac{H_c (Q_d - Q_o) T_{fo}}{(H_f + H_c) T_o} H(t - t_1), \quad (5.32)$$

$$A_3' = - \frac{H_f (Q_d - Q_o) T_{fo}}{(H_f + H_c)^2 T_o} H(t - t_1), \quad (5.33)$$

$$Q_o \equiv \frac{q_{fo}'''}{C_f}, \quad (5.34)$$

$$Q_d \equiv \frac{q_{fd}'''}{C_f}, \quad (5.35)$$

$$H_f = \frac{1}{R_1 C_f}, \quad (5.36)$$

$$H_c = \frac{1}{R_1 C_\phi}, \quad (5.37)$$

$$T_f^* = T_f / T_o, \quad (5.38)$$

$$\bar{B} = \frac{T_{co}}{T_{fo}}, \quad (5.39)$$

$$T_c^* = B_1 + B_2 t + B_3 e^{-(H_f + H_c)t} + B_1 + B_2(t - t_1) + B_3 e^{-(H_f + H_c)(t - t_1)}, \quad (5.40)$$

where

$$T_c^* = T_c / T_o,$$

$$B_1 = \left[\frac{H_c + H_f \bar{B}}{H_f + H_c} - \frac{Q_o H_c}{(H_f + H_c)^2} \right] \frac{T_{fo}}{T_o}, \quad (5.41)$$

$$B_2 = \frac{Q_o H_c}{H_f + H_c} \frac{T_{fo}}{T_o} \quad (5.42)$$

$$B_3 = \left[B - \frac{H_c + H_f \bar{B}}{H_f + H_c} + \frac{Q_o H_c}{(H_f + H_c)^2} \right] \frac{T_{fo}}{T_o}, \quad (5.43)$$

$$B_1' = - \frac{(Q_d - Q_o) H_c T_{fo}}{(H_f + H_c)^2 T_o} H(t - t_1), \quad (5.44)$$

$$B_2' = \frac{(Q_d - Q_o) H_c T_{fo}}{(H_f + H_c) T_o} H(t - t_1), \quad (5.45)$$

$$B_3' = \frac{(Q_d - Q_o) H_c T_{fo}}{(H_f + H_c)^2 T_o} H(t - t_1), \text{ and} \quad (5.46)$$

$$T_w = \frac{q_{wo}''' L_w}{C_w} [t + H(t - t_1)(x - 1)(t - t_1)], \quad (5.47)$$

where x is the decay heat fraction in the first wall.

Numerical Solution

The coupling effects of depressurization on the wall heat-up was studied by the numerical integration of Equations (5.1), (5.2), and (5.19) through (5.21). This set of equations can be represented non-dimensionally as:

$$\frac{d\rho^*}{dt^*} = -(\rho^* P^*)^{\frac{1}{2}}, \quad (5.48)$$

$$\frac{dP^*}{dt^*} = -k \frac{P^{*3/2}}{\rho^{*1/2}} + Q^*, \quad (5.49)$$

$$\frac{dT_f^*}{dt^*} = Q_f^* - H_f^*(T_f^* - T_\phi^*), \quad (5.50)$$

$$\frac{dT_\phi^*}{dt^*} = H_\phi^*(T_f^* - T_\phi^*) - H_c^*(T_c^* - T_\phi^*), \text{ and} \quad (5.51)$$

where

$$Q^* = A_c h_\phi (T_\phi^* - T^*)/Q_o + A_w h_w (T_w^* - T^*)/Q_o, \quad (5.53)$$

$$Q_o = \frac{P_o V C_v}{T_o t_o R}, \quad (5.54)$$

$$H_f^* = H_f t_o, \quad (5.56)$$

$$H_\phi^* = H_\phi t_o, \quad (5.57)$$

$$H_c^* = H_c t_o, \quad (5.58)$$

$$H_w^* = H_w t_o, \quad (5.58)$$

$$Q_w^* = Q_w t_o / T_o, \text{ and} \quad (5.59)$$

$$Q_f^* = Q_f t_o / T_o. \quad (5.60)$$

The Lawrence Livermore hybrid design was chosen as the reference design in the numerical computation. The thermal-hydraulic conditions of their blanket design are summarized in Table 5-4.

Table 5-4
HYBRID BLANKET DESIGN PARAMETERS

Power Level	
Power Level per Submodule (kW)	622
Total Power Level (MWt)	4,976
Power Density	
Power Density in the Fission Blanket (W/cm^3)	68.8
Power Density in the First Wall (W/cm^2)	37.6
Decay Heat Power (see Figure 5-1)	
Steady-State Temperature	
Coolant (K)	723
Fuel (K)	1,521
Cladding (K)	1,003
First Wall (K)	1,003
Helium Pressure (atm)	20
Helium Volume (m^3)	962
Heat Transfer Parameters (see text for notation):	
$H_f^* = 0.03971 \text{ s}^{-1}$	$C_f = 1.339 \text{ cal}/\text{cm}\cdot^\circ\text{C}$
$H_\phi^* = 0.5236 \text{ s}^{-1}$	$C_\phi = 0.1016 \text{ cal}/\text{cm}\cdot^\circ\text{C}$
$H_c^* = 0.973 \text{ s}^{-1}$	$C_w = 0.539 \text{ cal}/\text{cm}\cdot^\circ\text{C}$
$H_w^* = 0.0388 \text{ s}^{-1}$	

Results and Discussion

The estimated consequences of a loss-of-coolant accident in a gas-cooled hybrid blanket depend on several parameters, for example the fuel and cladding temperatures, the first wall temperature, and the depressurization rate.

In accordance with the closed-form solution when the heat transfer to the coolant is neglected, the depressurization process is characterized by a single parameter,

$$t^* = \frac{t}{t_o} = \frac{V}{A} \left[k \left(\frac{2}{k+1} \right)^{\frac{k+1}{k-1}} \left(\frac{\rho_o}{P_o g} \right) \right]^{-\frac{1}{2}} t.$$

Physically, this means that the depressurization is a function of three groups of parameters: (i) the break area-to-volume ratio, A/V ; (ii) the initial density-to-pressure ratio, ρ_o/P_o , and (iii) the isentropic gas constant, k . For the given system, such as the Livermore design, $k = 1.67$ for helium, $P_o = 20$ atm, $T_o = 273$ K, $V = 962$ m³, and for a break area of 0.046 m², the time required for the pressure to be halved is 10 seconds.

Figure 5-2 shows the transient blanket pressure during a LOCA, with no heat transfer. The x's represent the solution of Equations (5.16) and (5.17), while the dotted line illustrates the result given by computer code when $Q^* = 0$ is substituted in Equation (5.49). The good fit between these two curves confirms the accuracy of the numerical scheme.

The adiabatic heat-up in the fuel, the cladding, and the first wall, as given by Equations (5.27), (5.40), and (5.47), was computed for three different shutdown times (1, 5, and 10 seconds). If the reactor can be shut down in 1 second, the time available for the actuation of emergency blanket cooling is 4 minutes. If the reactor cannot be shut down within 10 seconds, the available time is shortened by a factor of 2. In all these cases, the cladding fails first, followed by the first wall, which marks the start of gross fuel motion. Fuel slumping may lead to criticality. The modeling of the various possible fuel motions remains to be done as a follow-up study. The possible core reconfigurations from the spherical shell geometry to a critical mass geometry remains to be estimated.

The coupling effects between coolant depressurization and material heat-up were investigated numerically, and the results are shown in Figures 5-3 through 5-6. In Case I (Figure 5-3), where the heat transfer to the steam generator is neglected, the pressure increases because more energy is transferred to the coolant, although

Figure 5-2. Transient Blanket Pressure and Density with No Heat Transfer during a LOCA.

Figure 5-3. Transient Blanket Pressure and Density with No Heat Transfer to the Steam Generator during a LOCA.

Figure 5-4. Transient Blanket Pressure and Density with Heat Transfer to the Steam Generator during a LOCA.

Figure 5-5. Transient Temperature with No Heat Transfer to the Steam Generator during a LOCA.

Figure 5-6. Transient Temperature with Heat Transfer to the Steam Generator during a LOCA.

the mass inventory is becoming depleted. This phenomenon is physically feasible only if the steam generator fails to act as a heat sink. The pressure rise imposes a threat to the integrity of the first wall. In Case II (Figure 5-4), the heat transfer to the steam generator is included in the calculation. It is seen that the depressurization is slower in comparison with the adiabatical case (Figure 5-2). Hence, the closed form solutions tend to overpredict the depressurization rate. The transient temperature responses are shown in Figure 5-5 (Case I) and 5-6 (Case II). It is noted that Case I experiences a much more rapid temperature excursion, as a result of the absence of the heat sink. In both cases, the shutdown time is assumed to be 5 seconds (or $t/t_0 = 0.215$).

5.3 STEAM INGRESS ACCIDENT

In gas-cooled reactors, the conversion of the thermal energy generated to electrical power is accomplished either by a direct helium cycle, using gas turbines, or indirectly, by a steam cycle through the steam generator. The helium pressure on the primary side of the steam generator is usually lower than the steam pressure. A failure within the steam generator could introduce steam into the blanket. The presence of water in the blanket could change both the thermal-hydraulic and the neutronic characteristics there. A change of the blanket neutron multiplication factor could lead to the insertion of reactivity. On the other hand, the build-up of pressure near the first wall would threaten the structural integrity of the wall. The ingress of steam into the primary coolant system could hamper the performance of the circulator, which would eventually lead to the loss of cooling function for the fuel in the hybrid blanket.

The safety analysis of a steam ingress accident is divided into two phases. In the first phase, the thermal hydraulic response in the blanket resulting from the rupture in the steam generator is analyzed. Since the hybrid design is in its preliminary stage, the main purpose of the analysis is to provide an overall study of the potential hazard of such an incident. In the second phase, the neutronic consequence is studied.

Analytical Model

The steam side of the generator is modeled as a single volume, V_1 , with uniform thermodynamic properties of pressure, P_1 , and enthalpy, h_1 , as shown in Figure 5-7. If at time $t = 0$ a rupture occurs along with an instantaneous closure of the feedwater control valve and the main steam isolation valve, a mass and energy

Figure 5-7. Physical Model for Steam Ingress Accident.

balance of the steam generator would lead to

$$\frac{dM_1}{dt} = -\dot{m}_1 A_1 \quad (5.61)$$

and

$$\frac{dU_1}{dt} = -\dot{m}_1 A_1 h_1, \quad (5.62)$$

where

M_1 = the mass of the steam and water,

$\dot{m}_1$ = the two-phase steam and water leakage rate,

A_1 = the area of the break,

U_1 = the total internal energy,

h_1 = internal enthalpy, and

t = time.

If the relation of the specific volume, v_1 , and the specific internal energy, u_1 , were used, that is if

$$v_1 = \frac{V_1}{M_1} \quad (5.63)$$

and

$$u_1 = \frac{U_1}{M_1}, \quad (5.64)$$

then Equations (5.1) and (5.2) reduce to

$$\frac{dv_1}{dt} = \dot{m}_1 \frac{A_1}{V_1} v_1^2 \quad (5.65)$$

and

$$\frac{du_1}{dt} = -\dot{m}_1 \frac{A_1}{V_1} (P_1 v_1^2). \quad (5.66)$$

For a homogeneous mixture,

$$v_1 = x_1 v_g + (1 - x_1) v_f \quad (5.67)$$

and

$$u_1 = x_1 u_g + (1 - x_1) u_f, \quad (5.68)$$

where

x_1 = the quality of the steam,

v_g, v_f = specific volumes for vapor and liquid, respectively, and

u_g, u_f = specific internal energies for vapor and liquid, respectively.

Since they are functions of the pressures only, Equations (5.65) and (5.66) can be expressed in a more compact and non-dimensional form as:

$$\frac{dx_1^*}{dt^*} = \frac{\beta' r - \beta r'}{\beta' \alpha - \beta \alpha'} \quad (5.69)$$

and

$$\frac{dP_1^*}{dt^*} = \frac{\alpha' r - \alpha r'}{\beta' \alpha - \beta \alpha'}, \quad (5.70)$$

where

$$x_1^* = x_1/x_{10}, \quad (5.71)$$

$$P_1^* = P_1/P_{10}, \quad (5.72)$$

$$\dot{m}_1^* = \dot{m}_1/\dot{m}_{10}, \quad (5.73)$$

$$t^* = t/t_0, \quad (5.74)$$

$$t_0 \equiv \frac{v_1 x_{10}}{\dot{m}_{10} A_1 v_{10}},$$

$$u_1^* = u_1/u_{10}, \quad (5.75)$$

$$u_{10} \equiv P_{10} v_{10}, \text{ and}$$

$$v_1^* = v_1/v_{10}. \quad (5.76)$$

$$\alpha = v_g^* - v_f^*, \quad (5.77)$$

$$\beta = -x^* \frac{dv_{fg}^*}{dp^*}, \quad (5.78)$$

$$\alpha' = \dot{m}^* v^{*2}, \quad (5.79)$$

$$\beta' = -\frac{1}{x_0} \frac{du_f^*}{dp^*} - x^* \frac{du_{fg}^*}{dp^*}, \quad (5.80)$$

$$r = \dot{m}^* v^{*2} \quad (5.81)$$

$$r' = -rp^* \quad (5.82)$$

The subscript 0 refers to the initial steady-state values.

The two-phase mass flow rate from the primary system to the helium system is given by Moody's calculation [5-4]. Basically, $\dot{m}_1$ is a function of P_1 and h_1 or P_1 and x_1 , that is,

$$\dot{m}_1 = f(P_1, x_1). \quad (5.83)$$

Equations (5.69), (5.70), and (5.83) constitute the basic equations for the solution of the pressure and quality transients in the steam region of the generator, and for the steam ingression rate.

In the helium region, the ingress of steam causes an increase in pressure. Safety valves could be installed in the primary coolant loop to mitigate this consequence [5-5]. Further study may not favor this safeguard, since it would provide a direct path for the release of tritium. Taking the safety valve into consideration, the mass and energy changes in the helium system are given by

$$\frac{dM_2}{dt} = \dot{m}_1 A_1 - \dot{m}_2 A_2 \quad (5.84)$$

and

$$\frac{dU_2}{dt} = \dot{m}_1 A_1 h_1 - \dot{m}_2 A_2 h_2 + Q_c + Q_w, \quad (5.85)$$

where

- M_2 = the total mass of fluid in the system, water plus helium,
- $\dot{m}_1$ = the steam-ingression rate,
- A_1 = the area of the rupture,
- $\dot{m}_2$ = the flow rate that can be relieved from the primary system,
- A_2 = the area of the relief valve; its actuation depends on its setpoint pressure,
- U_2 = the total internal energy of both helium and water,
- h_1 = the enthalpy of the influx,
- h_2 = the enthalpy of the outflux,
- Q_c = the heat transferred from the core to the gas region
 $= h_f A_f (T_f - T_2),$ (5.86)
- A_f = the surface area of the fuel,
- h_f = the heat-transfer coefficient from the fuel,
- T_2 = the gas temperature,
- T_f = the fuel temperature, given by an energy balance of the fuel, such that

$$\frac{dT_f}{dt} = \frac{q''' A_f}{\rho_f C_{pf} V_f} - \frac{h_f A_f (T_f - T_2)}{\rho_f C_{pf} V_f}, \quad (5.87)$$

q''' = the volumetric heat generation rate,

ρ_f = the fuel density,

C_{pf} = the heat capacity of the fuel,

v_f = the volume of the fuel,

Q_w = the heat transferred from the blanket wall = $h_w A_w (T_w - T_2)$, (5.88)

A_w = the area of the first wall,

h_w = the heat transfer coefficient from the first wall,

T_w = the first wall temperature, given by the energy balance

$$\frac{dT_w}{dt} = \frac{q_w''}{\rho_w C_{pw} d_w} - \frac{h_w (T_w - T_2)}{\rho_w C_{pw} d_w}, \quad (5.89)$$

q_w'' = the first wall heat flux,

ρ_w = the first wall density,

C_{pw} = the specific heat for the first wall, and

d_w = the thickness of the first wall.

The calculations involve the solution of Equations (5.69) through (5.83) for the time variation of the thermodynamical properties and the ingress steam-flow rate. With or without the actuation of the safety valve, the mass and energy inventories in the helium system are then updated (Equations (5.84) and (5.85)). The pressure, temperature, and quality of the mixture are then established by the following iterative procedure.

First, the specific volume of the steam and water mixture is calculated as

$$v_s = \frac{V}{M_s}, \quad (5.90)$$

where M_s is the mass of the steam, given by

$$\frac{dM_s}{dt} = \dot{m}_1 A_1 - \dot{m}_2 y A_2, \quad (5.91)$$

where y is the mass fraction of the steam in the mixture.

Then, a pressure, P_s , is assumed. The steam can be in a superheated or in a saturated state. Supposing a superheated state with the two thermodynamic properties P_s and v_s known, the internal energy, u_s , and the temperature, T , can be determined.

If the helium is at the same temperature as the steam-water mixture, then the total energy, U_2 , is given by

$$U_2 = M_s u_s + M_{He} C_{vHe} T, \quad (5.92)$$

where

M_{He} = the helium mass, and

C_{vHe} = the specific heat of the helium.

By comparing the value of Equation (5.92) with the total energy given by Equation (5.85), P_s can be determined. Then the total pressure in the helium is given by

$$P_2 = P_s + P_{He},$$

where P_{He} is the partial pressure of the helium.

Results and Discussion

A computer code, based on the methodology outlined in the previous section, was developed to analyze the consequences of a steam ingress accident in a hybrid fusion-fission reactor. The numerical technique for solving the set of ordinary differential equations from Equations (5.9) through (5.25) is based on the Runge-Kutta scheme. The data for the computation are outlined in Table 5-5. The system parameters are based on the Lawrence Livermore Laboratory design [5-5]. A parametric study was performed to determine the effects of the area of the rupture on system response. The results are shown in Figures 5-8 through 5-13. In all cases, the safety valve is assumed to be closed.

If the area of the break is small, the pressure increase in the helium system is rather slow (Figure 5-8). The temperature rise is faster because of the superheated state of the steam and the continued influx of heat from the fission zone. As the break area is increased, both the temperature and the pressure experience a drop before rising (Figures 5-10 through 5-13). This is thought to be caused by the mixing of the low-temperature water with the hotter helium. The temperature difference governs the system response during the first few seconds. Because of the different coolant-temperature response, the fuel temperature follows a similar trend for different rupture areas. In all cases, the pressure and the temperature inside the steam generator do not change drastically, mainly because of its fairly large mass inventory.

Table 5-5
SYSTEM PARAMETERS FOR STEAM INGRESSION ANALYSIS

HELIUM PROPERTIES			
Specific Heat of Constant Pressure, C_p (Btu/lb·°F)			1.25
Specific Heat of Constant Volume, C_v (Btu/lb·°F)			0.753
Gas Constant, R (Btu/lb·mol)			1.986
Molecular Weight, M (lb/lb·mol)			4
STEAM PROPERTIES (ASME Steam Table)			
Specific Heat of Constant Pressure, C_p (Btu/lb·°F)			2.445
Specific Heat of Constant Volume, C_v (Btu/lb·°F)			0.335
SYSTEM PARAMETERS			
	HELIUM	STEAM	FUEL
Pressure (atm)	20	170	--
Temperature (°R)	1,302	--	1,900
Quality	--	0.05	--
Mass (lb)	2.86×10^6	--	--
Internal Energy (Btu)	2.8×10^6	--	--
Volume (ft ³)	1.5×10^{10}	6,000	2,700
Break Area (ft ²)	--	1-0.01	--

It is observed that when the break area is reduced by two orders of magnitude, the time required for the pressure to increase from 20 atm to 50 atm increases by only a factor of two.

The amount of steam in the fission blanket is of interest for the criticality calculation. Figure 5-14 shows the steam mass for different rupture areas. The thermodynamical state of the steam is illustrated in Figure 5-15. For small breaks, the steam is in a superheated state.

Figure 5-8. Pressure Response of a 0.01 ft² Break Area in a Steam-Ingress Accident.

Figure 5-9. Temperature Response of a 0.01 ft^2 Break Area in a Steam Ingress Accident.

Figure 5-10. Pressure Response of a 0.1 ft² Break Area in a Steam-Ingress Accident.

Figure 5-11. Temperature Response of a 0.1 ft^2 Break Area in a Steam-Ingress Accident.

Figure 5-12. Pressure Response of a 1 ft² Break Area in a Steam-Ingress Accident.

Figure 5-13. Temperature Response for a 1 ft^2 Break Area in a Steam-Ingress Accident.

Figure 5-14. Steam Mass in Fission Blanket for Different Rupture Areas.

Figure 5-15. Quality of Steam in the Fission Blanket.

5.4 FIRST WALL INTEGRITY DURING A LOSS-OF-COOLANT ACCIDENT

Introduction

In a fusion-fission hybrid reactor, a possible mode of off-normal release of hazardous material during a loss-of-coolant accident (LOCA) is via the failure of the first wall [5-6]. A LOCA in the blanket will certainly have a direct effect on the fissile or fertile fuel, as shown in Section 5.1, but it may also have consequences in other systems than the blanket. A significant portion of the hazard of a LOCA may derive from these exterior systems, if they have fast thermal responses. The hazard may not be easily avoided by secondary cooling systems, nor will it be directly reduced by the subcriticality of the blanket.

Previous studies of loss-of-flow accidents (LOFA's) in wetted-wall laser fusion reactors [5-7] have shown that first wall failure can occur with thermal response times as short as 1 to 10 seconds. This quick response arises from the fact that a significant part of the reactor power is deposited in the first wall, and that an active heat-removal system must be employed for first wall protection. Since one of the more promising means of implementing a fusion-fission hybrid is through laser fusion, which has simpler geometry than devices such as the tokamak, the course of a LOCA in a comparable fusion-fission hybrid reactor has been considered. At the beginning of this study, no suitable reference designs were available, although it was known that the Lawrence Livermore Laboratory and Westinghouse were developing such designs. In this analysis, it is assumed that the blanket is a single coolant loop, and that the coolant is a liquid metal such as lithium. The previous analysis of a LOFA in such a reactor showed that failure of the first wall protection scheme, which is based on the radial inflow of molten lithium, occurs due to thermal overload. In turn, this can lead to first wall failure. By contrast, it will be seen that this factor plays only a small role in the course of a LOCA. Rather, the high power density in the blanket drives the thermal overloading of the first wall. While the following analysis was based on a specific concept for fusion containment and system design, it is possible that similar results would occur with quite different reactor designs.

One of the most difficult questions that must be considered in the event of a LOCA is whether the plasma will shut down on its own. In terms of the thermal loads associated with the reactor components, this question arises in the modeling of the thermal power output of the reactor as a function of time after the initiation of a LOCA. While some measure of conservatism about what constitutes a "worst case" analysis is proper, it must be based on an analysis of the possible physical

phenomena. First, let us consider an unprotected accident, in which the normal means of sensing an accident fail. In laser-initiated fusion, the requirements for evacuation of the plasma cavity are less stringent than in magnetic-confinement devices, such as tokamaks. Consequently, significant disruptions of the first wall cooling system are conceivable without the interruption of pellet delivery and implosion.

In the case of magnetic devices, an accident is likely to disrupt the plasma. Some hold that the plasma will cool rapidly and "promptly" cease to generate neutrons. Such a prompt quench may also create structural problems, since the plasma current must be transferred to other conductors in the system. The question of plasma shut-off was considered in Section 3, where the scenarios presented included an overpower transient at the initiation of the accident. Two of the characteristic times in the transient analysis of such an event are the particle and energy confinement times. These must be on the order of seconds for successful tokamak operation, even as a fusion-fission hybrid. In the consideration of LOCA's, the thermal response times may be of the same order, or even shorter. Hence, it is unrealistic to presume that the plasma ceases operation promptly within the time scale of the accident. In principle, plasma dynamics should be coupled with the LOCA analysis; however, they are clearly very design-dependent.

The following analysis assumes that plasma operation continues during the LOCA. This represents a worst case, and can be justified *ex post facto* by the short thermal response time of the first wall to the LOCA.

Analytical Model of a LOCA

This analysis of the course of a LOCA is based on the geometry shown in Figure 5-16. In this figure, a coolant supply runs parallel to the first wall of the plasma vessel, and splits into flows through the blanket and the first wall. The thermal load at the first wall is a function of the plasma conditions and of the state of the coolant supply at the first wall. There are two modes of coolant removal from the first wall, sputtering and thermal evaporation; these will remove different amounts of heat. Consequently, the thermal load on the first wall may not be a monotonic function of the coolant supply and the wall temperature. The total thermal load in the blanket per unit of first wall area is taken as Q , a fraction of which, η , is normally deposited in the first wall itself. The coolant loss rate from the plasma side of the first wall for the two mechanisms may be derived by an extension of the LOFA analysis previously presented [5-7].

Figure 5-16. Blanket Flow Configuration.

Sequential vaporization occurs when the thermal load which leads to sputtering (e.g., ionization) arrives in short pulses, such as from a laser fusion device, so that sputtering and thermal evaporation alternate as modes of heat removal. Competitive vaporization, with both modes operating simultaneously, occurs when the thermal load is sensibly time-independent. The vaporization rates are:

$$r_S/r_S^0 = 1 - \frac{z + 1 - \alpha}{z + 1} (pz + 1) h(z + 1 + \alpha) + p(z - \alpha) h(z - \alpha)$$

and

$$r_T/r_S^0 = z - \frac{z(z + 1 - \alpha)}{z + 1} (p - 1) - p(z - \alpha) h(z - \alpha), \quad (5.93)$$

where

r = the rate of vaporization,

S = a subscript denoting sputtering,

T = a subscript denoting thermal evaporation,

r_S^0 = the rate of sputtering required to transport the entire thermal load to the first wall, ηQ ,

α = the coolant supply rate-to- r_S^0 ratio,

z = the ratio of the thermal evaporation rate given by kinetic theory to r_S^0 ,

$p = 1$ if thermal evaporation is preferred, and 0 if it is not, and

$h(x) = 0$ for $x < 0$ and 1 for $x > 0$.

The net thermal input to the first wall, q_{net} , is then

$$q_{\text{net}} = \eta Q - \rho_M \Delta H_{\text{vap}} (r_T + Cr_S), \quad (5.94)$$

where

ρ_M = the molar density of the coolant,

ΔH_{vap} = the heat of vaporization, and

C = the ratio of energy transported by sputtering to that by thermal vaporization, for equal vaporization rates.

The ranges of possible values for these parameters are described in Reference [5-7].

The temperature of the first wall can be described by the differential equation

$$\rho C_p w \frac{\partial T}{\partial t} = w k \nabla^2 T + q_{\text{net}} + \bar{B} - B - U_{\text{eff}}(T - T_b), \quad (5.95)$$

where

w = the thickness of the first wall,

ρ = the density of the first wall,

C_p = the heat capacity of the first wall,

K = the thermal conductivity of the first wall,

B = the radiant-heat transfer rate, given by the Stefan-Boltzmann law as

$$\epsilon \sigma T^4,$$

U_{eff} = the effective heat-transfer coefficient from the first wall to the blanket coolant at T_b ,

$$\bar{B} = \frac{1}{4\pi} \iint d\theta d\phi B W(\theta, \phi), \text{ and} \quad (5.96)$$

W = the radiative view factor.

For a spherical geometry W is a constant, and hence $\bar{B}$ is also a constant equal to the average value of B .

U_{eff} can be estimated from a simplified transport model in which conductive and convective transport is considered under volumetric heating of the coolant. The blanket is partitioned into two regions, as described previously, and shown in Figure 5-16. At steady state, the thermal load in the blanket, $(1 - \eta)Q$, will produce a temperature profile described by the equation

$$K_c \nabla^2 T - \rho_c C_{pc} \vec{v} \cdot \nabla T + (1 - \eta)Q f(\vec{r}) = 0, \quad (5.97)$$

where

K_c = the thermal conductivity of the coolant,

ρ_c = the density of the coolant,

C_{pc} = the heat capacity of the coolant,

$f(\vec{r}) d\vec{r}$ = that fraction of the blanket heating which is deposited per unit volume between $\vec{r}$ and $\vec{r} + d\vec{r}$, and

$\vec{v}$ = the velocity of the coolant.

The direction of flow, and hence the velocity vector, will change as a function of position, as shown in Figure 5-16. A boundary condition is applied at the first wall; if x is the coordinate perpendicular to the first wall, then

$$-K \left. \frac{\partial T}{\partial x} \right|_{\text{first wall}} = \eta Q. \quad (5.98)$$

If the temperature is partitioned as

$$T = T^\circ + \eta T', \quad (5.99)$$

then in Equation (5.97), T° is the solution to the temperature profile in the

presence of volumetric heating (but no surface heating) at the first wall, and T' is related only to the surface heating. The equations defining T' are then:

$$\begin{aligned}
 & \kappa_c \nabla^2 T' - \rho_c C_{pc} \vec{v} \cdot \nabla T' = 0 \\
 \text{and} \quad & - \left. \frac{\partial T'}{\partial x} \right|_{\text{first wall}} = Q.
 \end{aligned} \tag{5.100}$$

If the blanket is approximated as being of infinite extent in the direction parallel to the first wall, and having thicknesses H and L in the regions where the flow is parallel and perpendicular to the first wall, respectively, then Equation (5.100) will give the results shown in Figure 5-17. There, U_{eff} is defined as the ratio of the heat flux, Q , to the temperature difference, T' , across the blanket. It can be seen from Figure 5-17 that the maximum value of U_{eff} is k/H , which corresponds to a transport across the gap in which the flow is parallel to the first wall, with no further resistance throughout the rest of the blanket. The effect of the LOCA is to cause the flow in the blanket to be towards the first wall, which reduces the value of U_{eff} significantly. In essence, the first wall is thermally isolated from the blanket under these conditions, with respect to conduction.

If a spherical cavity is considered, the cavity conditions can be evaluated from the solution of Equation (5.97), if a reasonable choice is made for $f(r)$, such as

$$f(r) = \frac{1}{d} \exp(-r/d), \tag{5.101}$$

where d is the characteristic attenuation length for neutrons. While this choice is somewhat arbitrary, it is convenient. Conventional and appropriate values of d may be established readily for various blanket components. Typically, d will range from 0.1 to 0.5 m. A typical solution of Equations (5.97) and (5.101) is shown in Figure 5-18.

The course of drainage of the coolant may be given by the flow rate from the reactor, F , as

$$\frac{dV(x)}{dt} = -F(t), \tag{5.102}$$

where $V(x) dx$ is the volume of the blanket contained within a distance, x to $x + dx$, from the top of the sphere. For a reactor with a spherical plasma chamber of radius R and blanket thickness P , $V(x)$ is

Figure 5-17. Effective First Wall Heat Transfer Coefficient, U_{eff} .

Figure 5-18. Steady-State Temperature Profile.

$$V(x) = \begin{cases} \frac{\pi}{3} x^2 (3R + 3P - x), & x < P \\ \pi x P (2R + P) - \frac{\pi}{3} P^2 (3R + P), & P < x < P + 2R \\ \frac{\pi}{3} x^2 (3R + 3P - x) - \frac{4}{3} \pi R^3, & P + 2R < x < 2P + 2R. \end{cases} \quad (5.103)$$

The drainage of coolant from the reactor will leave the first wall region $x > P$ without a supply of coolant. The exposed region is defined in terms of a polar angle, θ , measured from the top of the reactor cavity, such that $0 < \theta < \theta_{\max}$, where

$$\cos \theta_{\max} = \begin{cases} 0, & x < P \\ \frac{R + P - x}{R}, & P < x < P + 2R \\ 1, & P + 2R < x < 2P + 2R. \end{cases} \quad (5.104)$$

The equation for $\theta_{\max}$ is found by the combination of Equations (5.102) through (5.104),

$$\frac{d \cos \theta_{\max}}{dt} = \begin{cases} 0, & x < P \\ \frac{F(t)}{RP(2R + P)}, & P < x < P + 2R \\ 0, & P + 2R < x < 2P + 2R. \end{cases} \quad (5.105)$$

For a constant flow rate, F° , Equation (5.105) predicts an initial time, t° , before the drainage reaches the level of the first wall,

$$t^\circ = \frac{\pi}{3} \frac{P^2 (3R + 2P)}{F^\circ}. \quad (5.106)$$

Thereafter, $\cos \theta_{\max}$ will decrease linearly until a time bt° , where

$$b = 1 + \frac{6R}{P} \frac{2R + P}{3R + 2P}. \quad (5.107)$$

The estimation of plausible values for t° and b requires assumptions both for a geometric scale for the reactor and for the nature and location of the event which leads to a LOCA. Without undue detail, the pressure of the coolant is likely to be on the order of 10 MPa. If a pipe break occurs at about 1 m from the reactor, and if the diameter of the pipe is about 0.3 m, then this pressure can drive a volumetric flow rate of approximately $500 \text{ m}^3/\text{s}$. This, of course, would last for only a short time, since depressurization would occur. Under a hydrostatic head of about 4 m of

lithium, the flow rate would drop to about $2 \text{ m}^3/\text{s}$. Under the assumption that R and P are approximately 2 and 1.5 m, respectively, then $b \approx 6$ and t° would be in the range of 0.04 to 10 seconds. The smaller value of t° would require that pressurization of the coolant be maintained, which is unlikely without a continuous supply of lithium to the coolant circuit. The larger value is undoubtedly too large, since it ignores the initial pressurization of the coolant circuit. Thus, significant drainage will occur in less than 10 seconds, making it quite difficult to assure reactor shut-down during the drainage time.

Equation (5.97) may then be solved, assuming azimuthal symmetry for the reactor and using the value of U_{eff} given by Equation (5.100). In the region $0 < \theta < \theta_{\text{max}}$, U_{eff} is taken to be 0, since there is no coolant present. It is necessary to solve these equations numerically, since non-linearities enter through the radiative transport terms.

Results and Discussion

An almost unlimited number of cases could be set up and solved. It is not clear how a generalized phenomenology of the time scales and consequences of a LOCA could be established. Perhaps it could be achieved by considering a "typical" reactor design, evaluating the time to failure of the first wall, and then iteratively changing the design until acceptably long times result, to allow confidence that the reactor can be shut down without damage. Of course, design changes could include possibilities such as conversion of the blanket from a homogeneous one to a collection of independent modules, a change which could not be considered in the formalism developed above.

A sample case is shown in Figure 5-19, in which dimensionless times τ and τ° are used by dividing t and t° , respectively, by $\rho C_p w_p^2/K$. The temperature, T , is rendered dimensionless by division by QH/K , the temperature rise if all the heat generated in the blanket were to be transported across the inner zone of the blanket shown in Figure 5-16. For the case shown in Figure 5-18, melting of the first wall occurs at a dimensionless temperature of 0.018. Characteristic times for typical parameter values are in the range of 10 to 50 seconds. Of particular note is that the first wall can respond faster than it would if it were completely adiabatic with respect to the blanket.

5.5 CONCLUSIONS

Three aspects of a LOCA and of a steam ingress accident have been considered. First, the state of the fissile/fertile elements was examined for a helium-cooled reactor.

Figure 5-19. Transient Thermal Response.

Dominating the consideration of such an accident is the question of whether the fuel elements can melt, since this could lead to a condition in which bulk fuel motion could occur. This fuel motion, as discussed in Section 4, might lead to criticality of the blanket. The model presented here considers two cases. In the first, the depressurization and thermal heating processes are decoupled. In the latter, they are treated in a coupled fashion. The decoupled model demonstrates that a characteristic time, estimated to be between 2 and 10 seconds, exists, during which significant depressurization occurs. The latter model, with its ability to consider the temperature history of the fuel, indicates that reactor shut-down must occur within a comparable time period if cladding failure is to be avoided.

The consequences of a rupture between the helium primary coolant loop and a steam generator have been explored. The analysis includes both thermal and neutronic effects. It shows that times on the order of 10 seconds are associated with significant pressure and temperature effects in the blanket, and that while the course of the ingress accident can be quite sensitive to the size of the rupture, the characteristic time, due to compensating effects, is much less so. It is concluded that significant damage could be encountered within the blanket before any appreciable change has occurred in the steam generator.

Lastly, the response of a lithium-cooled reactor has been analyzed to establish the characteristic time to failure of the first wall. The first wall will have the fastest thermal response time in a reactor. A LOCA is found to produce almost complete thermal isolation of the first wall in a homogeneous blanket. Characteristic times associated with the failure of the first wall can be on the order of 10 to 50 seconds, and are sensitive to the characteristic time for the drainage of coolant from the reactor blanket.

In summary, the three related events considered have demonstrated that significant damage to the blanket, the fuel-element containment, and the structural components of the reactor can occur in the range of from 1 to 10 seconds. These times are comparable to the range of times for reactor shut-down. Hence, it may become a difficult problem to halt reactor operation before at least significant damage has occurred to the reactor components. It is not clear whether this damage would lead to significant releases of radioactive or otherwise hazardous materials, although accidents such as considered here have been shown to present pathways for such releases.

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Section 6.

MAGNET CONSIDERATIONS

6.1 SCOPE

Previous safety work in the area of central station fusion power reactors has been concerned primarily with special aspects of toroidal devices, in particular tokamak systems. An attempt has been made to focus attention on generic aspects of all magnets belonging to the class of toroidal confinement schemes. This work has been outlined in Ref. [6-1].

Summarizing some important findings, it is noted that there are two main problem areas; 1) the possibility of encountering a magnet failure due to anomalies in the magnet proper and, 2) the possibility of the initiation of a severe disturbance due to anomalies in the dewar systems and auxiliary components. The special feature of a fusion reactor, in contrast to pure magnet power or storage systems used in power electrotechnology, is the possibility that; 1) the disturbing event may originate in the fusion system (e.g. neutron irradiation of magnet system components) and 2) that the accident sequence at one stage or another may lead to a breach of containment with radioactive material released to the environment. It is the latter aspect which has been identified as one major problem of the fusion reactor power system.

To place the magnet safety discussion into a proper perspective, it is necessary to assess the superconducting magnet experience gained so far and the related accident potential. Since the mid 1960's, the prototype development work on large magnets has been impressive and characterized by an excellent safety record. For instance, it has been pointed out [6-2] that no major serious accident with severe injuries has become known in this development work, which utilizes currents in the order of magnitude from 1000 to 10000 Amps. A recent summary of safety work in magnet technology at Brookhaven National Laboratory (BNL) arrives at similar conclusions regarding the safety outlook for magnet development. Since the consequences of a routine event, such as pipe rupture, may be quite severe in a tritium containing fusion-fission system, all possible

magnet failure causes, or other factors related to dewar systems have been inspected carefully from the safety design point of view.

In this context, questions arise concerning the peculiarities of hybrid systems. One particular design, the Lawrence Livermore Laboratory (LLL) Yin-Yang coil system [6-4], has been sufficiently documented to recognize generic aspects of this particular mirror hybrid system. The work reported here in this area has included, in particular, consideration of forces and possible effects on confining wall integrity due to magnetically accelerated matter. This phase of the study is considered in Section 6.2. Phenomenological aspects of the safety study have been continued on the basis of the preceding work with toroidal devices. Stability-design philosophy is to a large extent safety-design philosophy. In this context, the stabilizer of the composite (normal metal-superconductor) plays a crucial role. Yet, very few clear design numbers have emerged in this area of prime concern for the magnet layout. These phenomenological aspects are dealt with in Section 6.3.

The latter area includes superconductor-coolant interaction, which in turn is affected strongly by coolant-state choice, provided that design criteria of "cryo-static stability" are used. The continued dialogue in these areas between the user of stability information for design, the researcher's evaluation, and the safety inspection, is a contributing factor to a future safety record that equals the excellence of that in the past. Along this line of thought, some relevant issues are raised and recommendations are forwarded in the general area of magnet systems.

6.2 SPECIAL ASPECTS OF MAGNET ACCIDENT SEQUENCES IN HYBRID FISSION-FUSION POWER SYSTEMS.

An exhibition of failure records (mostly photographs) at a recent conference [6-5] shows failure-event evidence primarily for relatively closed configurations. In these geometries, the windings form a nearly-continuous "shell" over an extended region in space. Classical examples are cylindrical solenoids and dipole magnets. In contrast, the Yin-Yang configuration appears to be more "open" from the point of view of magnet conductor package shape. In a simplified view, the main magnet system may be considered as an assembly of two horseshoes. It is convenient to regard this type of configuration as a special category of magnet systems.

A precursor to the Yin-Yang shape is the baseball magnet, whose layout with the entire dewar system apparently resembles closely the conventional magnet systems of the superconducting power technology. A strong clamping element may be used to take care of the forces which try to separate the "seams" of the baseball. The entire assembly may be immersed in a dewar, providing a pool cooling system.

In the Yin-Yang design [6-4], however, this approach has been modified. The clamps have been replaced by a structure on the outside which has to take the large forces that are developed. The truss structure demands careful attention for the minimization of heat leaks. However, for the conductor design, it has the advantage that the windings may be laid into the structural support element without too many asymmetries in the force distributions. Though no detailed dewar design is available for this mirror system, the alternate design of force-cooled "cabled" integrated conductor-dewar envelope appears to be worth the consideration. Accordingly, there are several specific aspects of the Yin-Yang magnet system of importance:

- a) Principal availability of a more open conductor assembly,
- b) Associated with (a), the possibility of forced convection cooling or equivalent schemes which depart from "classical" pool dewars,
- c) Further possible vulnerability in case of loss-of-integrity of components and exposure to magnetic fields with strong field gradients.

The truss elements used may be exposed to the effects of cold helium in case of leakage, and the welded joints may show embrittlements under the combined effect of irradiation and cold temperature. As a consequence, there may be a finite probability of local failure in the structure, with possible quench initiation at a location in the magnet. External disturbances, such as in an earthquake, may trigger vibrations in the same structure. When natural frequencies are excited, fatigue may be accelerated.

As far as the cryo-envelope is concerned, there appears to be no detailed information on rapid quenches of the magnetic field. As the field collapses rapidly, eddy currents may be induced in the metallic dewars. The skin depth varies with purity. Therefore, stainless steel dewars show different effects from aluminum dewars. Although the use of plastic dewars has not been attractive in the presence of radiation, some large plastic dewars were developed for fast

discharges in conjunction with the Scyllac fusion-system developments. [6-6].

Obviously, an extreme case is discharge with the buildup of the magnetic fields by surface currents, which in turn interact with the external field "seen". Another extreme case is strong dissipation with no significant field gradients in the metal shell. Obviously, optimum conditions can be found based on the choice of metallic material for dewar design, the shaping of metal parts, plastic dewars, or combinations of plastic and metals with optimum decay conditions.

Since remotely -controlled handling machines have to be installed for necessary shutdowns and repairs, there is a danger of failure of some parts. In general, "loose matter" may exist in domains which "see" finite magnetic fields. Aside from dust, and nuts and bolts (left from the construction phase), there is a finite probability of breakup of a sizable object, e.g., one end of a remotely-controlled handling machine. Obviously, no severe disturbance may result in most situations, unless there is a strong accelerating force. For the establishment of this force, another concern is the rapid decay of the magnetic field due to a quench with a characteristic decay time t_c . For the coincidence of two initiating events, several simplified field configurations have been considered to assess the magnitude of forces acting on metals of small (paramagnetic) susceptibility, or perfect conductors. The calculations are outlined below. Conservative conditions have been presumed, i.e., the forces are an upper bound of the real forces that exist when eddy-current heating dissipates some of the energy available.

Another configuration considered in great detail is the parallel-plate system of perfect conductors. Again, the upper bounds have been evaluated for the kinetic-energy release. The major details of this study [6-7] are contained in Appendix I.

To examine the salient features of the acceleration process, one of the simplest solenoid geometries has been presumed with a straight, long cylinder with a half length b , which is much larger than the inner radius, a_1 (Figure 6-1). The object exposed to the time-varying magnetic field is idealized as a cylinder of diameter D and height $H = D$. A vertical solenoid axis has been adopted to assess the mechanical action in the light of mechanical potential energies gained during an upward motion acting against gravity.

Figure 6-1. Sketch of the Solenoid Geometry.

A reasonably homogeneous field is assumed to permit a simple relation between the magnetic pressure seen by the body, and the resulting force on the body.

The force may be expressed in terms of the maximum gradient in the magnetic flux density (B), and the magnetic pressure difference ($P_m = B^2/(2\mu_o)$), respectively. The axial field distribution, B_z , may be written in terms of the ratio of the radii $\alpha = a_2/a_1$, the half-length ratio $\beta = b/a_1$, and the dimensionless axial coordinate $\gamma = z/a_1$ (Figure 6-1). The component of B in the z -direction is

$$B_z = f(\gamma) + f(-\gamma) . \quad (6.1)$$

The function f is conveniently expressed in terms of the filling factor and the transport current density, j , as

$$f = \frac{1}{2} \mu_o \lambda j a_1 (\gamma + \beta) \ln \left\{ \frac{\alpha + \sqrt{\alpha^2 + (\gamma + \beta)^2}}{1 + \sqrt{1 + (\gamma + \beta)^2}} \right\} \quad (6.2)$$

The value of $B_z (dB_z/dz)$ is important, because the axial force on the material is proportional to the magnetic pressure difference, which in turn increases as $|\Delta B|$ is enlarged.

$$F \sim \Delta(B_z^2/2 \mu_o) \quad (6.3)$$

From the B_z -distribution [6-1], it is obvious that there is only one maximum of dP_m/dz in the range $0 \leq |\gamma| < |\beta|$. Therefore, it is easy to find the maximum magnetic pressure gradient numerically for specified values of α and β , i.e., for a specified solenoid geometry. Figures 6-2a through 6-2c show the various results obtained from the calculations.

Figure 6-2a shows the point of maximum gradient, $z_m/b = \gamma_m/\beta$, as the function of the ratio of half-width to inner radius, for given α and β values. Since β/α is greater than unity, the point of maximum pressure gradient moves rapidly toward the ends of the solenoid. Figure 6-2b shows the flux density, B_z , at the point of maximum gradient (normalized in terms of the center field value B_{zo}). As it can be seen on the figure, this function becomes very flat at argument values beyond unity. Figure 6-2c shows the pressure gradient in dimensionless form. The reference pressure gradient is the center field pressure, $B_{zo}^2/(2\mu_o)$, divided by the inner radius. Accordingly, Figure 6-2c presents $|\text{grad } P_m|_{\text{max}} / |\text{grad } P_m|_{\text{Ref}}$ as a function of β/α . Again, it can be seen that

a. Dimensionless Location of the Maximum Magnetic Pressure Gradient as a Function of Length-to-Inner Diameter Ratio (β/α).

b. Magnetic Field Intensity (Normalized in Terms of Center-Field Value) versus β/α .

c. Magnetic Pressure Gradient Ratio versus β/α .

Figure 6-2.

there is little variation in the maximum pressure gradient if the solenoid is relatively long.

The force, due to the magnetic pressure gradient, may be related to the center field, B_{zo} , of the solenoid section under consideration as:

$$F_{\max} = A(\Delta P_m)_{\text{eff}} = \text{const } B_{zo}^2 (H/a_1) (D^2 \pi/4) \mu_o \quad (6.4)$$

(The subscript 'eff' denotes the effective value.) Accordingly, the force is proportional to the ratio of the size of the body to the solenoid bore. However, this is valid only if the value of this ratio is sufficiently far from unity. In the standard gravity field (with gravitational acceleration g), the acting forces are

$$F - m g = m(dv/dt), \quad (6.5)$$

(v = velocity, t = time, m = mass.) The right-hand side is expressed as an order-of-magnitude statement: $m \dot{v} = m(v/t_c)$. Thus, the order of magnitude of the velocity attained is

$$v \sim t_c / m (F - mg) \quad (6.6)$$

Substituting Equation (6.4) into Equation (6.6), the velocity is:

$$v_{\text{eff}} = (t_c / m) \left[(\text{const} / \mu_o) B_{zo}^2 (H/a_1) (D^2 \pi/4) - mg \right] . \quad (6.7)$$

The order of magnitude of the kinetic energy gained under these conservative conditions is an upper-bound to the real kinetic energy attained by the body. The order of magnitude of the gained kinetic energy, (E_k), is:

$$E_k = (m/2) v_{\text{eff}}^2 . \quad (6.8)$$

To illustrate the conditions of this idealized situation, the following numerical values are adopted.

Center field:	$B_{zo} = 5 \text{ T},$
Body density:	$\rho = 2700 \text{ kg/m}^3 \text{ (Al)},$
Half length:	$b = 4 \text{ m},$
Bore radius:	$a_1 = 1 \text{ m},$
Outer radius:	$a_2 = 2 \text{ m},$
Characteristic time:	$t_c = 10^{-2} \text{ sec.}$

The size of the body is assumed to be specified by the ratio $(D/2)a_1 = 0.1$. This corresponds to a mass of 17 kg. The final velocity gained is 16 m/s (36 MPH). This final velocity obtained can be used to assess a "minor" impact of a vehicle with this speed and mass. The final kinetic energy-gain is 2kJ.

It is assumed for the calculation that the skin depth corresponding to the characteristic time of decay is much smaller than the body size ($\delta \ll D$). The skin depth, δ , is:

$$\delta = (\rho_{el} t_c / (\pi \mu_o))^{1/2} . \quad (6.9)$$

For the order of magnitude of an electrical resistivity, $\rho_{el} = 10^{-10} \text{ m}$, $t_c \sim 10^{-2}$ seconds, a skin depth = 1 mm is obtained.

Figure 6-3 shows the ratio of the kinetic energy to the reference energy, E_{Ref} . The latter has been chosen as the product of the maximum magnetic pressure and the volume enclosed by the solenoid ($E_{Ref} = \Delta P_m V$). The size ratio, $(D/2)a_1$, which relates the dimension of the loose object to the bore, is chosen as a parameter. It should be noted that the rather fast increase of the kinetic energy indicated in the figure is an ideal envelope to the real kinetic energy gained. As time proceeds, the skin depth increases and the eddy-current dissipation will be more and more pronounced. The absolute values of E_k/E_{Ref} are quite small. Accordingly, the "efficiency" of the kinetic-energy release is rather limited.

Figure 6-4 shows the velocities gained (at $g \gg 0$) versus t_c for various values of the magnetic flux density. Again, there is a monotonic increase with t_c . As the eddy-current effects are neglected, these velocities constitute the upper bounds again.

In conclusion, in the area of magnetically driven "flying objects", relatively small velocities are reached with low efficiency. Factors contributing to the small velocities are:

- Short times of the applied magnetic pressure gradients,
- Relatively large skin depth, and related dissipation, and
- Lack of guidance and flight propulsion "incentives".

Figure 6-3. Ratio of the Kinetic Energy, E_k , to the Reference Energy (E_{Ref} = Total Magnetic Energy of Ideal Solenoid) versus Characteristic Time, t_c , of Field Decay.

Figure 6-4. Ideal Velocities Gained (without Gravitational Forces during Conservative Energy Release) versus Characteristic Time, t_c , of Field Decay.

The main point is also illustrated by studies of high-speed magnetic rocket systems. [6-8] Special design measures, such as evacuated tubes, are necessary to speed up objects beyond a Mach number of unity. Therefore, very excessive kinetic energy increase does not seem to be possible. However, in some situations, damage may be done and appropriate precautions should be incorporated into the design. Since the magnet system is auxiliary to the core of the fusion-fission plant (as is the classical electro-mechanical power system), we can elucidate some of its features by a comparison with a turbine operation.

A possible severe accident involves the dislodging of a rotating turbine part in combination with penetration through the steel casing of the machine. This accident is extremely unlikely; however, the damage to an individual blade of a cascade, and a resulting "blade salad", is a much more probable possibility during power-plant operation. Similarly, a minor anomalous intrusion into the regular magnet-system operation appears to be a more realistic possibility. This may lead to twisted conductor sections, deformed dewar sections, or other accidents. The important event of arcing, with electro-magnetically blown holes into the pipelines, has been addressed already in the previous study of toroidal systems [6-1].

Design remedies to be incorporated may include field-shaping sections. These components may also be necessary because of the rather "open" Yin-Yang configuration. These covers may be paramagnetic metals or ferromagnets. Since the sections are to be removable, no serious shortcomings may result when a magnetically-accelerated object impinges upon them.

As in the area of dewar design, other cover optimizations may be discussed. In some cases, eddy-current damping sections may be desired, e.g., "braking conductors" underneath the Yin-Yang coil, for emergencies. Other considerations may induce thoughts about complete exclusion of magnetic flux, in particular, when the magnetic field is increased significantly and interferes with outside instruments, devices (such as watches), and electromagnetic instruments.

6.3 PHENOMENOLOGICAL MAGNET SAFETY ASPECTS

The emphasis here is on the issue central to all superconducting magnet systems of high magnetic energy which are needed in power plants (regardless of the nature of the system, such as fusion, fission-fusion, MHD, or others). The stability design criteria center around the combination of type II - normal

metal and coolant. The stability of this complex system includes, to a large extent, safety aspects. Therefore, the stability design criterion chosen is, to a considerable degree, a safety design criterion.

In the course of studies concerned with toroidal devices, it has become obvious from thermalization calculations [6-2] that uniformity in temperatures is highly desirable to prevent excessive, damaging, local temperature excursions. This postulate, however, imposes the requirement of large copper cross sections (or equivalent sizes of other good conductors), which in turn, reduces the overall current density. Thus, a large number of conductors may be selected. However, this choice will enhance the inductance which is kept small in most large power system magnets to reduce the danger of high electric fields and the related arcing potential. Therefore, cabled systems with high currents are essential. The same rule applies for the Yin-Yang magnet. Accordingly, bulky magnet systems are as yet predominant in conceptual designs of fusion power systems, although the general design postulates of compact cryopower electro-technology point in the opposite direction. Thus, in the long run, benefits derived from the experience with "dense" magnets should be incorporated into fusion magnet design. Since the temperature excursion is an important anomaly of operation, the adiabatic thermalization temperature has become a nearly-routine item of magnet design. Usually, temperatures from about 90 K to 150 K are reached within the range envisioned by various designs with NbTi. The Yin-Yang design incorporates rather large fields, therefore, use of Nb₃Sn or an equivalent material may be a requirement for a safe magnet at high field locations.

Since this section is concerned with the complex solid-solid-coolant interaction and safety design constraints, we consider first the possible coolant states in the Yin-Yang system, and turn subsequently to specific questions of stabilizer-type II contact.

Yin-Yang System Coolant States

No detailed design procedure seems to be available for the fission-fusion magnet cooling system, therefore, we inspect possible choices of He⁴ coolant states. In principle, H₂ may be used together with Nb₃Sn, however, the additional hazards of H₂ are avoided in the present conceptual magnet designs. By postulating "cryo-static stability", the designer has the choice to tie in the copper thickness of the composite directly with the coolant boundary condition and the

related thermodynamic state. There are several options for the latter, such as a saturated pool, use of a subcooled liquid (He I), use of pressurized He II, and supercritical pressures for He I. In all cases, different post-quench design numbers and post-quench scenarios may result.

Traditional pool cooling is endangered by large degrees of stratification, as may be encountered readily in the upper sections of a Yin-Yang system. The support system for the transmission of forces may be quite "leaky" for heat flow. This heat input, in turn, may cause localized heating in the bath. When the pool is a saturated liquid, the leak leads to vapor bubble formation and to the associated "induced convection". The established convection pattern causes enhanced stratification toward the upper end of the dewar. This effect lowers the peak nucleate pool boiling heat flux. Thus, there is a larger danger of a local quench, which relates directly to the increased coolant quality on top of the pool.

Pressurized He⁴ operation may include subcooled liquid He I, He II, and supercritical He I. A decision in favor of higher pressures is affected by the concern about penetration of foreign matter (through cracks) into the dewar system. Since all other matter is frozen at liquid He temperatures, solid particles may block the coolant channels. Therefore, an exclusion of impurities appears to be quite desirable. Thus, the pressure inside the dewar should be at $P = P_{\text{atm}} + e$, where P_{atm} is the environmental (atmospheric) pressure, and e is an increment, chosen by the designer.

For subcooled He I and He II, the increment e may be kept quite small. However, for both fluid conditions, an additional heat-exchanger system is required. He I in a subcooled state requires a subcooler, He II demands a pressurization heat exchanger, which is usually kept in contact with an (outer) saturated He II "bath" or refrigeration system. Therefore, several fluid-filled lines may be necessary. Both, subcooled He I and He II, may have sufficiently large cryo-static design values to avoid excessive copper cross-sections.

The range of supercritical pressures is readily available (in contrast to steam), since the critical pressure, P_c , is only 2.3 bar. In He I, for pressures above P_c , the thermal diffusivity is quite small. This requires pumping for forced convection, in order to cut down the copper stabilizer mass. Somewhat better conditions are obtained by using supercritical pressures in conjunction with

He II. This mode of operation combines some of the advantage of high-pressure operation. The latter is characterized by rather minor density changes, resulting from a severe quench. This, in turn, will keep the temperature excursion within limits. Despite He II, however, a severe quench may need time for recovery. As two-phase flow (He II - He I) is established during the temperature excursion, the powerful thermo-osmotic process in He II cannot be utilized to its fullest extent. Therefore, auxiliary measures may have to be incorporated into the design (e.g., emergency pumping).

When forced convection is introduced, reliance on internal convection processes (thermo-osmosis in He II, or boiling in He I) is somewhat restricted. Two possibilities exist: 1) application of high pressure at the entrance, and 2) moderate circulation. In the first case, large pumping power may be required, which results in higher cost for the cooling system. In the second case, the main emphasis may be on the removal of local high entropy fluid "pockets" (e.g., vapor or He I). Steady operation with a limited effort of pumping, and emergency operation with strong pumping rates, appears to be a sound compromise.

Several combinations of the various schemes seem to be possible. We turn now to questions of the stabilizer-type II composite.

Stabilizer Contact Phenomena

The stabilizer cannot function satisfactorily if the metallurgical bonds between type II and normal metal are insufficient. Furthermore, if the materials are exposed directly to neutron irradiation, very drastic deterioration takes place in both type II and normal metal. Therefore, a severe accident in the blanket systems will most likely lead to a failure of the magnet.

During the early evolution of commercial materials for superconductor magnets, the quality of type II filaments, bond properties, and normal metal conditions have improved considerably. There is sufficient experimental data for the most popular composite Cu-NbTi, and no accidents have occurred with serious fatal consequences. Thus, the stabilizer has acted as a safeguard in many situations. However, the existing possibility of magnet failure indicates that stabilizers are not "absolute" safety devices. With the introduction of large magnets in conceptual designs, which demand drastic extrapolations from existing systems, questions for optimum conditions (e.g., Cu-NbTi), diagnostic tools, and composite

conductor characterization have received renewed emphasis. In the present subsection, we consider some constraints for the overall transport current density, j_L , and for conditions which characterize the interfacial resistance between normal metal and type II.

Constraints for the overall current density, j , become obvious from the cryo-static stability equation. For a normal metal dissipation rate of $I^2 R$, limiting heat flux density $q_L(\Delta T_L)$, and a cylindrical composite with single-core type II and Cu-diameter $D_o = 2 D_{II}$, the current density is the following:

$$\bar{j} = (16/3) \pi^{-2} I^2 \rho_{cu} / D_o^3 \quad (6.10)$$

($\bar{j}$ = overall, average current density).

The steady-state equation considered is:

$$I^2 R = A_w q_L \quad (6.11)$$

(A_w = the surface area wetted by the coolant). Using the electrical resistivity, ρ_{cu} , of the stabilizer, and with the assumptions introduced ($I \rightarrow I_L$ = limit of design), we can obtain from Equation (6.10):

$$q_L = (16/3) \pi^{-2} I_L^2 \rho_{cu} / D_o^3 \quad (6.12)$$

Accordingly, $I_L / (\frac{\pi}{4} D_o^2) = \bar{j}_L$ may be written as

$$\bar{j}_L = \left\{ \frac{3 q_L}{\rho_{cu} D_o} \right\}^{1/2} \quad (6.13)$$

The copper resistivity is related to the room temperature and to the residual resistivity ratio ($RRR = \rho_{room} / \rho_{cu}$). Thus, (6.13) becomes

$$\bar{j}_L = \left\{ \frac{3(RRR) q_L}{\rho_{room} D_o} \right\}^{1/2} \quad (6.14)$$

(This implies that the type II resistivity in the quenched normal state is much larger than the value of ρ_{cu} .) According to Equation (6.14), $\bar{j}_L$ will be large when D_o is small, provided that q_L has been fixed by the conditions of coolant operation. Thus, the cable design has the disadvantage of a large D_o leading to a small value of the overall current density. The value of RRR is usually

kept around 100, because the presence of the magnetic fields induces a magneto-resistance which lowers the RRR in the large magnetic flux density regions.

Another extreme case is that of a composite conductor with N filaments and a copper cross section (characterized by an equivalent diameter $\bar{D}_o$), which is much larger than the total type II cross section of the filaments. Again, the average current density can readily be obtained from Equation (6.12) for the special case of complete wetting (by cryo-fluid) over a perimeter $\bar{D}_o$. We obtain:

$$\bar{j}_L = \left\{ \frac{4q_L}{\rho_{cu} \bar{D}_o} \right\}^{1/2} \quad (6.15)$$

The expression corresponding to Equation (6.14) is

$$\bar{j}_L = \left\{ \frac{4q_L (RRR)}{\rho_{room} \bar{D}_o} \right\}^{1/2} \quad (6.16)$$

Aside from a small change of the numerical factor, there are no drastic modifications in the previous conditions. Accordingly, for $q_L \approx 0.25 \text{ W/cm}^2$ (which is typical of some design specifications for pool cooling),

$D_o = 0.01 \text{ cm}$, $RRR \sim 10^2$ (i.e., $\rho_{cu} \sim 10^{-8} \Omega \text{ cm}$), we obtain from Equation (6.16) an average limiting current density of $j_L \sim 10^5 \text{ A/cm}^2$. In contrast, if D_o is raised to the order of magnitude of 1 cm, the value of j_L decreases to the order of 10^4 A/cm^2 . Since this calculation does not include the insulation and fluid-filled cross sections, the actual j_L -values are even smaller, by a factor >1 . Therefore, some tradeoff between inductances (i.e., large electric fields of wire conductors, and large currents of cables) is expected to be required for an optimum design. Additional questions arise in conjunction with the use of triple-material conductors, such as type II, Cu, and resistive barriers (e.g., CuNi). We turn now to the area of contacts between various materials, in particular, Cu and type II.

Emphasis is placed upon the simplest configuration, which is the "junction" between an outer Cu-cylinder and single-core type II cylinder. For an assessment of this situation, recent contact studies [6-9,6-10] are considered. Several investigators have been concerned with metal epoxy, metal-superconductors, and type II-Cu contacts. (A summary of this work is presented in Ref.[6-10].)

The contact-study results are used to assess the effect of changing from the wire geometry of small D_0 to cable configurations with large outer diameters.

When the usual NbTi-manufacturing procedures are used, the initial phase of composite drawing may result in thick diffusion layers between type II and Cu, compared to the final thin layer between the metals. As this design choice retains large core diameters, two effects may be encountered: 1) there may be a loss of intrinsic type II stability, and 2) the thermal contact conductance of the interfacial domain may be low. By avoiding this property combination, the composite may be drawn down more and be attached (e.g., by brazing) to a Cu-substrate. This method, however, creates another contact zone. Therefore, it appears to be useful to introduce a different manufacturing process which assures thin domain zones. Nevertheless, the possibility for deterioration during the long exposure to neutron irradiation, which might cause changes in the interfacial domains, cannot be discarded.

Figure 6-5 shows the order of magnitudes of the overall thermal conductances across the interfaces. The cross-hatched region is representative of numerous systems, e.g., metal-epoxy, commercial Cu-solder junctions, Cu-Nb, and others. The largest values of type II superconductors have been reported [6-9] for a NbTi - sheet, diffusion-welded between Cu pieces. This system may be considered quasi-ideal because of the resulting smooth purified surfaces. The thickness of the thin sheet is smaller than the mean free path of its entropy carrier system. The latter consists of lattice vibrations (phonons) in the superconductor, and electrons in the normal metal. The mean free path of phonons for a very thin sheet is large compared to its thickness, therefore an enhanced heat-transport rate results.

The smallest h -values (dotted curve at the lower boundary of the cross-hatched region) are based on measurements with a partially-drawn composite rod (NbTi-Cu) (shown as squares). The comparison [6-10] with final single-filament wire conditions suggests that h of the product (as drawn) will be close to data reported for many epoxy-metal junctions and similar material combinations.

It is noted that the upper boundary of the cross-hatched region follows closely a T^3 -law. This dependence is representative of the acoustic mismatch arising between two metals of different acoustic properties. The high values of $h(T)$ are representative of the "best" contacts prepared under special lab conditions

Figure 6-5. Solid-Solid Boundary Conductances for Low-Temperature Systems.

reported in the literature. In contrast, most of the h-data are low for the metal-dielectric combinations of commercial materials; their numerical values seem to depend on the impurity content of the junction domain.

Concerning the interfacial diagnostics, we shall examine a simple model of current transfer, which may be regarded as a first-order approximation for post-quench transfer conditions. This transfer is also useful for some contact phenomena of current leads.

The model presumes a local normal zone of a well-stabilized composite, whose current is transferred from type II to the stabilizer. Charge transport through the stabilizer is expressed by the current

$$I_{cu} = \sigma_{cu} A_c dV/dx, \quad (6.17)$$

where A_{cu} = stabilizer cross section, σ_{cu} = electrical conductivity of stabilizer. Furthermore, the interfacial transfer of charges through the area, $C_c dx$, of an ideal interfacial element can be written as

$$I_i = C_c dx H_i (V - V_o), \quad (6.18)$$

where C_c = circumference, and V_o = potential of superconductor. Using $U = V - V_o$, the charge transfer of steady flow is described by

$$\frac{d^2 U}{dx^2} + \frac{H_i C_c}{\sigma_{cu} A_{cu}} U = 0 \quad (6.19)$$

Thus, the characteristic length of this model is

$$L_c = \left\{ \frac{\sigma_{cu} A_{cu}}{H_i C_c} \right\}^{1/2}. \quad (6.20)$$

Initially, at $x = 0$, the potential is assumed to be at $U = U_{ext}$. With this constraint, for a long conductor with its length considerably larger than L_c , we obtain the distribution of V from Eq. (6.19) as:

$$U(x) = U_{ext} \exp(-x/L_c). \quad (6.21)$$

The interfacial conductance, $H_i \left(\text{Ohm}^{-1} \text{ m}^2 \right)$, can be related to the thermal results for h . This approach presumes that there is no significant change in H_i along

the conductor length. The interfacial layer is assumed to have alloy-like properties with a Lorenz ratio, L_i , not far from the free-electron value (i.e., we neglect the paramagnetic impurities)

$$L_i = K_i \rho_i / T \quad . \quad (6.22)$$

Here, K =thermal conductivity, ρ_i = interfacial domain value of the electrical resistivity. With a transition-layer thickness d_i , and heat transfer coefficient h , we get:

$$K_i = \bar{K}_i = d_i h \quad . \quad (6.23)$$

Accordingly, the interfacial resistivity is approximately

$$\rho_i \approx L_i T / K_i = L_i T / (d_i h) \quad . \quad (6.24)$$

Another related quantity is the interfacial resistance

$$R_i = \rho_i d_i / A_i \quad (H_i A_i)^{-1} \quad (6.25)$$

and

$$H_i = (\rho_i d_i)^{-1} = \bar{K}_i / (L_i T d_i) = h / (L_i T) \quad . \quad (6.26)$$

By substituting these quantities into Equation (6.20), we can rewrite the characteristic length of current transfer as

$$L_c \approx \left\{ \frac{L_o T}{h C_c} \quad \frac{A_{cu}}{\rho_{cu}} \right\}^{1/2} \quad . \quad (6.27)$$

The free electron value of the Lorenz number is $\bar{L}_o = 2.45 \times 10^{-8} \left(\frac{V}{K} \right)^2 = \frac{W\Omega}{K^2}$.

Impurities resulting from degradation and irradiation may cause undesirable changes in the Lorenz number. The consequences of these changes are discussed in the safety evaluation of toroidal systems.

For a single-filament (type II core) conductor stabilized by Cu, the ratio of the cross section to the circumference is:

$$(\pi/4) \left[D_o^2 - D_F^2 \right] / \pi D_F \quad , \quad \text{or}$$

$$\frac{A_{cu}}{C_c} = \bar{D} \cdot \Delta/D_F = \frac{(D_o + D_F)}{D_F} \frac{\Delta}{2}, \quad (6.28)$$

where $\bar{D} = (D_o + D_F)/2$. The electrical resistivity of the stabilizer may be expressed in terms of the residual resistivity ratio RRR:

$$\rho_{cu} = \rho_{room} / RRR, \quad (6.29)$$

where ρ_{room} = the stabilizer resistivity at room temperature. The characteristic length, in terms of these quantities, is:

$$L_c \simeq \left[\frac{L_o T}{h} \frac{\bar{D}}{D_F} \Delta \frac{RRR}{\rho_{room}} \right]^{1/2}. \quad (6.30)$$

The characteristic length increases with RRR; therefore, it is desirable to reduce RRR to values that are justified within the specified set of constraints. For low (deteriorated) h-values, L_c will be large. The orders of magnitude of L_o and ρ_{cu} are not adjustable for design purposes.

If we substitute

$$h = 10^4 \text{ W/m}^2 \text{ K} \quad \text{at} \quad T = 4\text{K},$$

$$A_{cu}/C_c = 10^{-4} \text{ m}, \quad \text{and}$$

$$\rho_{room} \simeq 10^{-8} \Omega \text{ m}, \text{ and choose RRR to be in the order of } 10^2, \text{ we obtain}$$

a characteristic length, L_c , in the order of 10^{-3} m or 1 mm. In other words, L_c is indeed small compared to the realistic length of a conductor. We should note that, under these conditions, special voltage taps would have to be introduced to measure this steep potential drop, because at a significantly deteriorated joint an ordinary voltage tap may indicate a lack of good contact.

6.4 CONCLUSIONS AND RECOMMENDATIONS

Improved methods for the design of safe superconducting magnet sections have to focus on a) the superconductor, b) its normal metal stabilizer, c) normal metal barriers, and d) coolant conditions. All of these should have desirable interactions, resulting in an optimum rate of entropy transfer. These are elaborated as follows:

- a) The smallest possible size of type-II filaments guarantees intrinsic stability, also safeguards most distinctly against severe post-quench scenarios. This requirement may not be adequately realizable with the conductor configurations proposed for large fusion power system magnet.
- b) The most conservative criterion for stabilizers is the postulate of cryostatic stability. It may be ill-defined, however, for pool situations with large degrees of stratification. Better coolant conditions, such as pressurized He II, can possibly be used in conjunction with forced flow. The latter possibility requires the largest amount of refrigeration. If forced flow is not used continuously, the relaxation times, after a post-quench excursion in temperature, may be in the order of seconds, while some other power-system time-constants (e.g., electromechanical times) [9-11] are in the order of 100 milliseconds. It can be also noted that pressurized He I without flow may be subject to thermal spikes, which appear about once in every hour [6-12]. Similarly, unexpected quenches have been experienced during long-term operation of solenoids [6-14]. Therefore, forced flow may be necessary from the safety-design point of view.

Forced flow is not an automatic remedy for deficiencies in contact domain conductance, caused by quality control deficiencies. Therefore, a package of safety-design measures may be combined for optimum safeguarding.

- c) Steps in the direction of additional safeguards are further sectioning of magnets, demountable magnet portions (e.g., with polygonal straight segments proposed in Ref. [6-13]), large stabilizer copper cross-sections, optimal resistive barriers (even for D.C. magnets, as they may permit transients with shorter response times), and large mass flow of forced, pressurized cryo-He with large pressure drops (in part due to flow resistors in series, needed for stabilization of fluid flow).

Some of these steps may directly contradict the desired cost-benefit considerations. Also, the energy density of the system is considerably lowered by some of these choices, e.g., by the large Cu-cross section. Instrumentation requirements may also be excessive. Voltage taps at short sections of the windings enhance the danger of dielectric breakdown. Therefore, compromises are also necessary in the safety design.

- d) One possible step to be taken in the future can be the use of non-cryostatically stabilized sections, arranged between liquid-stabilized monitoring stations, where thick Cu-layers are employed with voltage taps, emergency arc paths, and emergency cooling line connections. The latter are needed for the loss of refrigeration, and the subsequent switching to reservoir-cooling from large cryo-He containers. To avoid large residence times of a high-entropy fluid (created by a quench), the independently-cooled sections of a magnet should be kept within reasonable fractions of the total conductor lengths.

Accordingly, the travel times for these "hot fluid elements are expected to be reduced (e.g., by cross-flow-counterflow combinations).

In conclusion, there is a rather large "shopping list" for Research and Development work in the magnet safety design area. However, since many elements have evolved in the past ten years, or are tackled intensely at present, there is a good chance to push the magnet size "barrier" to larger and larger units, as required for the 10^3 MW power technology.

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Section 7.

PROPERTIES OF HIGH-TEMPERATURE MATERIALS IN A MAGNETIC FIELD

7.1 INTRODUCTION

The safety of a fusion-fission hybrid power reactor may depend directly and significantly on the response of its components during an off-normal transient event. The course of events during overpower transients or loss-of-coolant accidents (LOCA's) will be determined in large part by the thermal response of the reactor; to an off-normal heat input in the former case, and to a rapid loss of heat-removal capabilities, in the latter case. If events were to proceed in a fashion analogous to that in a pure fission reactor, it would be necessary to know the pressure produced by the abnormally high temperature materials in the reactor at the later stages of such events to assess the consequences of the off-normal event. Analyses such as those suggested by the foregoing events will require information about the thermal and chemical properties of the materials present in the reactor.

While many of the materials proposed for use in fusion-fission hybrids-- such as coolants and fissile and fertile materials-- are being used in commercial and experimental fission reactors and have been studied extensively, it is believed that at no time have such studies been done in the presence of the large magnetic fields postulated for fusion-fission hybrid reactors. It is, therefore, of concern that inappropriate values of thermal and chemical parameters might be used in the design of such a hybrid reactor. Such designs may cause new modes of reactor failure, or enhance the probability or consequences of such failures. It is thus important to survey the thermal and chemical properties needed for the design and analysis of the operation of a fusion-fission hybrid, and at the same time to make some preliminary analyses of whether such parameters might be significantly affected by the presence of the large magnetic fields in the reactor. In a sense, this survey is not a classical safety study, because at no time has a specific event been postulated for which such parameters, or their deviations due to magnetic fields, might be important. Nonetheless, in a generic sense, safety analyses will have to consider the thermal and chemical properties reviewed here, and hence it is important that a reliable set of data be available.

In the final analysis, there is no substitute for experimental verification of a theoretical result, particularly for one based on incomplete information on and understanding of the system under consideration. Yet, it is believed that the following analysis will show that, even for magnetic fields of 10 to 20 T, little or no effect on the operating behavior can be expected in all but a few cases. Since these analyses are largely thermodynamic in origin, there is a fundamental reliability in the calculations. Several other cases will be noted in which, while the preponderance of evidence is that the magnetic fields will have little or no effect on the behavior of the material in question, there is no choice but to leave the definitive answer to experimental verification, since those involve non-thermodynamic and rate-dependent phenomena. These analyses are restricted to materials at high temperatures, and thus do not include the materials of which the magnets are constructed.

7.2 PHASE EQUILIBRIA IN A MAGNETIC FIELD

The changes encountered in phase equilibria in magnetic fields are due to the effect of the field on the Gibbs free energy of the phases present at equilibrium. Generally, it is expected that fusion and fusion-fission hybrid reactors will be constructed of materials which are para- or dia-magnetic. Hence, the following analyses of the free-energy functions will be formulated in terms of the magnetic susceptibility, χ_m , in a way which is appropriate to these materials. With suitable formulations of this parameter, it will be possible consider ferromagnetic materials as well.

The Gibbs free energy may be written as

$$dG = -S dT + V dP + \mu_0 \chi_0 V H dH, \quad (7.1)$$

where the work of establishing a magnetic field in a vacuum, $dw_{\text{vacuum}} = \mu_0 V H dH$, has been deleted, since it is a constant, independent of the phase or phases present.

The magnetic field strength, H , and the magnetic induction, B , can be related as

$$B = \mu_0 H + \mu_0 \frac{M}{V}, \quad (7.2)$$

where M is the total magnetization. For para- and dia-magnetic materials,

$$\frac{M}{V} = \chi_m H; \quad (7.3)$$

therefore, the magnetic induction is

$$B = \mu_0 (1 + \chi_m) H. \quad (7.4)$$

For para- and dis-magnetic materials, χ_m is generally much less than 1, so

Equations (7.1) and (7.4) may be combined to give

$$dG \approx -S dT + V dP + \frac{\chi_m V}{\mu_0} B dB$$

or

$$dG = -S dT + V dP + \frac{1}{2} \frac{\chi_m V}{\mu_0} d(B^2). \quad (7.5)$$

Let us consider two phases which are in equilibrium if $T = T_0$, $P = P_0$, and $B = 0$. We shall examine the temperature or the pressure change, while the magnetic induction changes, and the other parameter is kept constant. If the pressure is held constant, then for phases 1 and 2 one may write

$$dG_1 = -S_1 dT + \frac{1}{2} \frac{\chi_{m,1} V_1}{\mu_0} d(B^2) \quad (7.6)$$

and

$$dG_2 = -S_2 dT + \frac{1}{2} \frac{\chi_{m,2} V_2}{\mu_0} d(B^2).$$

At equilibrium, $dG_1 = dG_2$; thus,

$$\left(\frac{\delta T}{\delta(B^2)} \right)_{P, \text{ phase equilibrium}} = \frac{1}{2\mu_0} \frac{\Delta(\chi_m V)}{\Delta S}. \quad (7.7)$$

Similarly, if T is held constant while P and B vary,

$$dG_1 = V_1 dP + \frac{1}{2} \frac{\chi_{m,1} V_1}{\mu_0} d(B^2) \quad (7.8)$$

and

$$dG_2 = V_2 dP + \frac{1}{2} \frac{\chi_{m,2} V_2}{\mu_0} d(B^2),$$

and the equilibrium, defined by $dG_1 = dG_2$, requires that

$$\left(\frac{\delta P}{\delta(B^2)} \right)_{T, \text{ phase equilibrium}} = - \frac{1}{2\mu_0} \frac{\Delta(\chi_m V)}{\Delta V} = - \frac{1}{2\mu_0} \left(\bar{\chi}_m + \bar{V} \frac{\Delta \chi_m}{\Delta V} \right), \quad (7.9)$$

where $\bar{f} = (f_1 + f_2)/2$.

Equations (7.7) and (7.9) may be used to define the various thermodynamic changes anticipated in the temperatures and pressures at which phase equilibria occur.

For example, consider the vaporization of a paramagnetic material at a fixed pressure, P_0 . If the paramagnetic material obeys Curie's law, then for each phase

$$\chi_{m,i} = C_i/T, \quad (7.10)$$

where C_i is a positive constant. At sufficiently low pressures the value of $\Delta(\chi_m V)$ will approach the value of $\chi_m V$ for the vapor, and ΔS will approach a constant. At that limit, $V = RT/P_0$ for the gas, and hence Equation (7.7) may be written as

$$\frac{\delta T}{\delta(B^2)} \approx \frac{1}{2\mu_0} \frac{C_g R}{P_0 \Delta S_{\text{vap}}}. \quad (7.11)$$

As an example, the boiling point of uranium at 1 atm may be considered. In the absence of a magnetic field, the boiling point is approximately 4,450 K, and the entropy change on vaporization at that temperature is approximately 175 J/mol·K. The Curie constant for uranium is on the order of 0.2 K; hence, the change in normal boiling point between 0 and 10 Tesla will be about 3.7 K, from Equation (7.11). This change is less than the uncertainty in the boiling point temperature, so it does not appear to be of practical significance.

In a similar fashion, a change in melting point may be computed. For example, if both the solid and liquid phases can be described by Curie's law, then Equation (7.7) may be written as

$$\left(\frac{\delta(T^2)}{\delta(B^2)} \right)_{P, \text{ equilibrium}} = \frac{1}{\mu_0} \frac{\Delta(CV)}{\Delta S}, \quad (7.12)$$

or

$$\Delta(T^2) \approx \frac{1}{\mu_0} \frac{\Delta(CV)}{\Delta S} \Delta(B^2).$$

Again, the melting point of uranium under an inert pressure of 1 atm is about 1,400 K. Under these conditions, $V = 1.7 \times 10^{-5} \text{ m}^3/\text{mol}$ for the solid, and $C = 0.2 \text{ K}$. $\Delta(CV)$ might be on the order of 5% of the value for one of phases. Hence, for a change from 0 to 10 T,

$$\Delta(T^2) \approx 2T\Delta T = 0.073 \text{ K}^2$$

$$\Delta T = 2.6 \times 10^{-5} \text{ K},$$

which is a negligible temperature change.

The changes calculated above appear to present innocuous changes in thermal parameters, and therefore do not influence the safety of hybrid reactors. Even if ferromagnetic materials were introduced, the results would likely prove to be of little consequence. For example, for a ferromagnetic material such as soft iron, $\chi_m \gg 1$ and is a function of the magnetization. It is convenient to reformulate the free energy equations in the form

$$\begin{aligned} dG &= -S dT + V dP + \mu_0 H dM \\ B &= \mu_0 \kappa H; \quad \kappa = 1 + \chi_m \\ M &= V(\kappa - 1)H, \end{aligned} \tag{7.13}$$

hence

$$dG = -S dT + V dP + \frac{\kappa - 1}{\kappa} V H dB. \tag{7.14}$$

For a material such as soft iron (cf. Del Toro, Principles of Electrical Engineering, Prentice-Hall, 1965, p. 587), the magnetization curve of H vs. B is well established, and is characterized by κ values of 500 to 1,000. Thus, the contribution to the free energy due to a magnetic field is precisely equivalent to a pressure effect for

$$dP = \frac{\kappa - 1}{\kappa} H dB \approx H dB. \tag{7.15}$$

For example, for a change in magnetic induction from 0 to 1.5 T in soft iron, the data cited above give

$$\int_0^{1.5 \text{ T}} H dB = 1.3 \text{ kPa}.$$

The hydrostatic pressure needed to produce significant changes in melting points or vapor pressures is generally on the order of 10 MPa. Thus, the effect of a large magnetic field on a good ferromagnet such as soft iron, amounting to only about 1 kPa, is likely to be small indeed.

Perhaps pathological phenomena could be identified in which the effects of magnetic fields on phase equilibrium will be significant. It seems that such cases must be rare, since even those materials that are expected to be affected strongly (e.g., simple ferromagnets) appear to be influenced to only a small extent.

7.3 CHEMICAL EQUILIBRIUM PROPERTIES

Calculations for chemical equilibrium in the presence of a magnetic field depend on the same thermodynamic properties as used for the establishment of single component

phase equilibria. In the form of Equation (7.15), the free energy of a mixture of species could be written as

$$dG = -S dT + V dP + \sum_i \mu_{O,i}^H dM_i + \sum_i \bar{G}_i dn_i, \quad (7.16)$$

where $\bar{G}_i$ is the chemical potential of the i^{th} species. For the equilibration of multicomponent mixtures, it is convenient to replace the chemical potential with the activity, defined as

$$\bar{G}_i = \bar{G}_{i,0} + RT \ln a_i. \quad (7.17)$$

It can then be shown (cf. Lewis and Randall, Thermodynamics, McGraw-Hill, 1960, p. 242) that the activity can be approximated for ideal gases as the partial pressure, and for simple condensed-phase solutions as the mole fraction. Phase equilibria are considered at a fixed temperature and total pressure. Examination of Equation (7.16) shows that under these conditions (e.g., $dG = dP = dT = 0$), the equilibrium composition must be a function of the magnetic properties. If the reacting mixture is composed of para- and dia-magnetic materials, then Equations (7.2), (7.3), (7.16), and (7.17) may be combined to give the free energy for each species in the mixture,

$$G_i = \bar{G}_{i,0} + RT \ln a_i + \frac{1}{2\mu_0} \chi_{m,i} V_i B^2. \quad (7.18)$$

It has been assumed in the integration that $\chi_{m,i}$ is independent of the magnetic induction, as is true for para- and dia-magnetic materials. Thus, for a chemical reaction described as

$$\sum_i \nu_i A_i = 0, \quad (7.19)$$

where ν_i is the stoichiometric coefficient of the species A_i , the constraint of equilibrium requires that

$$\sum_i \nu_i G_i = 0. \quad (7.20)$$

Combining Equations (7.18) and (7.20) gives

$$\prod_i a_i^{\nu_i} = K(T, B) = K_0(T) \exp \left(- \frac{B^2}{2\mu_0 RT} \sum_i \nu_i \chi_{m,i} V_i \right). \quad (7.21)$$

Here, $K_0(T)$ is the equilibrium constant in the absence of a magnetic field, given by

$$K_o(T) = \exp \left(- \sum_i v_i \bar{G}_{i,o} / RT \right). \quad (7.22)$$

Equation (7.21) gives, at least in principle, the expected shifts in the equilibrium conditions in the presence of a magnetic field. Unfortunately, the computation of the activity coefficient in the presence of a magnetic field is by no means trivial, since it must be determined by the use of the equation of state of the material, under corresponding conditions. That is, the activity coefficient can be calculated only from equations of the type

$$\left(\frac{\partial \ln a_i}{\partial P} \right)_{T,B} = \frac{\bar{V}_i}{RT}$$

$$\left(\frac{\partial \ln a_i}{\partial 1/T} \right)_{P,B} = - \frac{\bar{H}_i - H_i^*}{R},$$

where $\bar{V}_i$ and $\bar{H}_i$ are the partial molar volume and enthalpy, respectively, and H_i^* is the enthalpy of species i in an ideal gas state. In general, such information is not available for the species of interest in the required range of magnetic inductions. Therefore, it becomes necessary to make significant simplifications, such as assuming volumetric and thermal behavior to be independent of the magnetic field, even under conditions in which this is, at best, problematical. With that assumption, it becomes equally reasonable to approximate the activities of the species by either the partial pressure in units of the standard state pressure for gases, or either the mole fraction or unity for condensed materials.

For example, let us consider the reaction between UO_2 and zirconium, leading to the formation of uranium metal and ZrO_2 . The data needed for the evaluation of the equilibrium conditions for this reaction are shown in Table 7-1. While information is not available for all of these materials, one might assume that χ_m obeys Curie's law. Then, for the reaction

the equilibrium constant is

$$K(T,B) = \frac{a_{ZrO_2} a_U}{a_{Zr} a_{UO_2}}$$

$$= \exp (\Delta S_o / R) \exp (-\Delta H_o / RT) \exp \left(- \frac{(B/T)^2}{2\mu_o R} T_o \sum_i \chi_{m,i,T_o} v_i \right). \quad (7.24)$$

Table 7-1
THERMODYNAMIC PROPERTIES OF URANIUM AND ZIRCONIUM METALS AND OXIDES
(FROM THE HANDBOOK OF CHEMISTRY AND PHYSICS)

	Zr	ZrO ₂	U	UO ₂
$\Delta H_{f,o,298}$ (kJ/mol)	0.0	-1,080.3	0.0	-1,129.7
$S_{f,o,298}$ (J/mol·K)	38.41	50.3	50.3	77.8
$V \times 10^5$ (m ³ /mol)	1.41	2.09	1.25	2.46
$\chi_m \times 10^6$ (at ~298 K)	122	-13.8	450	2,360

The first two terms on the right are the normal zero-field equilibrium constant, while the last term is the correction for the magnetic field, given in terms of the magnetic susceptibility at temperature T_o . From Table 7-1, this last term is

$$\frac{K(B,T)}{K_o(T)} = \exp \left[- \frac{(B/T)^2}{2\mu_o R} T_o \sum_i \nu_i \chi_{m,i,T_o} \nu_i \right]$$

$$= \exp [-0.776 \text{ K}^2 \text{ m}^4 \text{ Wb}^{-2} (B/T)^2]. \quad (7.25)$$

For example, for $B = 10$ T and $T_o = 300$ K, the ratio in Equation (7.25) is 0.9991, indicating a slight stabilization of $\text{Zr} + \text{UO}_2$ with respect to $\text{ZrO}_2 + \text{U}$, due to the magnetic field. This is certainly not a large effect, even though a strongly paramagnetic material was chosen to produce a large effect. This effect could be still larger if ferromagnetic materials were chosen, but such a choice is questionable for use in a fusion-fission reactor. For example, magnetic iron oxides would be strongly destabilized by the presence of a large field, thereby increasing the chemical potential of oxygen and its tendency to form oxides with the other metals present in the system. In general, though, this possibility is precluded for entirely separate reasons, since these reactions are frequently controlled by kinetics, not thermodynamics. Similarly, reactions such as the oxidation of aluminum, which may be favored by the presence of a magnetic field, is generally kinetically controlled. At this time, there appears to be no facile way to consider the effect of magnetic fields on chemical kinetic properties.

7.4 TRANSPORT PROPERTIES IN MAGNETIC FIELDS

Of the various transport properties of materials in fusion-fission hybrid reactors, the thermal conductivity will have the largest implication for the safety of the reactor. In the solid state, thermal conductivity is generally a result of two independent transport processes; transport by the vibratory motion of the lattice (phonons), and by the conduction of electrons delocalized within the lattice. The former process, which dominates the thermal conductivity of non-metals, is independent of the magnetic field, while the latter process, dominant in metals, is significantly dependent on the presence of a magnetic field.

The thermal and electrical conductivities are proportional in metals in the presence of a magnetic field, since they arise from the same phenomenon of electron scattering in the lattice. The equation of motion for the displacement, $\delta\vec{k}$, of the Fermi sphere is

$$\hbar\left(\frac{d}{dt} + 1/\tau\right)\delta\vec{k} = \vec{F}, \quad (7.26)$$

where $\vec{F}$ is the applied force, τ is the collisional relaxation time, and $\hbar$ is Planck's constant divided by 2π . For example, the force applied by an electric field at a frequency ω is

$$\vec{F} = -e\vec{E} \exp(-j\omega t). \quad (7.27)$$

Substituting the de Broglie relation

$$\hbar\vec{v} = \hbar\vec{k} \quad (7.28)$$

into Equation (7.26) and looking for a periodic solution gives

$$\begin{aligned} \delta\vec{v} &= \delta\vec{v}_0 \exp(-j\omega t) \\ \delta\vec{v}_0 &= -\frac{e\tau/m}{1 - j\omega\tau} \vec{E}. \end{aligned} \quad (7.29)$$

In terms of the current density, $\vec{j} = nq\delta\vec{v}_0$, where n is the electron density and q the electron charge ($-e$), the conductivity $\sigma = \vec{j}/\vec{E}$ is given by

$$\sigma = \frac{ne^2\tau}{m(1 - j\omega\tau)}. \quad (7.30)$$

Equation (7.30) is frequently expressed in terms of a dielectric constant, $\epsilon(\omega) = 1 + 4\pi\vec{P}/\vec{E}$, where $\vec{P}$ is the polarization, given by

$$\vec{P} = -ne\delta\vec{r}; \quad \vec{v} = \frac{d\vec{r}}{dt}.$$

Substitution into the equation of motion gives a dielectric constant

$$\epsilon(\omega) = 1 - \frac{4\pi n e^2 / m}{\omega^2 + j\omega\tau}. \quad (7.31)$$

This is a well-established theory, useful for understanding a variety of phenomena. For example, in the limit of $\tau \rightarrow \infty$, the plasma cutoff frequency occurs when $\omega = 0$, that is $\omega_p = (4\pi n e^2 / m)^{1/2}$. While the foregoing calculation deals with the electrical conductivity, the thermal conductivity may be predicted successfully in terms of the Lorenz number, L , defined as

$$L = \kappa / \sigma T, \quad (7.32)$$

where κ is the thermal conductivity. Simplified theories of thermal transport give (cf. Kittel, Solid State Physics)

$$L = \frac{\pi^2}{3} (k/e)^2, \quad (7.33)$$

where k is Boltzmann's constant.

In the presence of a magnetic field, an additional term appears in the force applied in the equation of motion of the Fermi sphere. In general, we can write

$$\vec{F} = -e(\vec{E} + \vec{v} \times \vec{B}) \quad (7.34)$$

Let us consider the case of static electric and magnetic fields. Let the magnetic field define the z axis. Then, Equations (7.26) and (7.34) reduce to:

$$\begin{aligned} m\left(\frac{d}{dt} + \frac{1}{\tau}\right)\delta v_x &= -e(E_x + B_z \delta v_y), \\ m\left(\frac{d}{dt} + \frac{1}{\tau}\right)\delta v_y &= -e(E_y + B_z \delta v_x), \text{ and} \\ m\left(\frac{d}{dt} + \frac{1}{\tau}\right)\delta v_z &= -eE_z. \end{aligned} \quad (7.35)$$

As an application of this formulation, let us consider the case of static fields. The solution for the velocity components, and hence the current densities, may be found by

$$\begin{aligned} \vec{j} &= \vec{\sigma} \vec{E}, \\ \text{or} \quad \vec{\sigma} &= \frac{\sigma_0}{1 + (\omega_c \tau)^2} \begin{bmatrix} 1 & -\omega_c \tau & 0 \\ \omega_c \tau & 1 & 0 \\ 0 & 0 & 1 + (\omega_c \tau)^2 \end{bmatrix}. \end{aligned} \quad (7.36)$$

Here, σ_0 is the conductivity at zero magnetic field, $ne^2\tau/m$, and ω_c is the cyclotron

frequency, eB/m . This result may be further specialized to the case in which an electric field is oriented along the x axis, with a magnetic field along the z axis. An induced electric field will appear in the y direction, called the Hall effect, due to the impossibility of continuous current flow in that direction within a finite object. From Equation (7.36),

$$j_y = \frac{\sigma_0}{1 + (\omega_c \tau)^2} (\omega_c E_x + E_y).$$

If $j_y = 0$,

$$E_y = -\omega_c \tau E_x = -\frac{eB_z \tau}{m} E_x. \quad (7.37)$$

The Hall coefficient is defined as $-E_y/H_z E_x$, and is $e\tau/mc$. Note that under these circumstances, the conduction along the direction of the electric field, j_x , is

$$j_x = \frac{\sigma_0}{1 + (\omega_c \tau)^2} (E_x - \omega_c \tau E_y) = \sigma_0 E_x \quad (7.38)$$

and

$$j_z = 0.$$

Thus, mutually perpendicular electric and magnetic fields will, at steady state, leave the electrical and, presumably, thermal conductivity unaffected.

The more general considerations of the coupling between magnetic fields and thermal conductivity are not as well established. It has been observed (cf. Ziman, Electrons and Phonons, Oxford Press, 1962, p. 483) that for each material,

$$\frac{\Delta\rho}{\rho_0} = F\left(\frac{H}{T}\right), \quad (7.39)$$

where ρ_0 is the zero-field resistivity, $\Delta\rho$ is the increment in resistivity due to the field, and F is some function of geometry. Generally, the increase in resistivity is considered to be a cryogenic phenomenon, but as Equation (7.39) shows, this will not be the case for sufficiently large values of H . A plot of this function is shown in Figure 7-1. The magnetic field is scaled by s , which is defined as the ratio of the zero-field resistivity to the lattice component of resistivity at a standard temperature, usually room temperature. As can be seen from Figure 7-1, large variations in the resistance are possible at sufficiently large magnetic fields. However, this effect must be considered in light of the relatively high temperatures encountered in a fusion-fission hybrid. For example, for $B \approx 0.05$ T and $T \approx 3$ K, the resistivity of pure aluminum is approximately twice

Figure 7-1. Reduced Kohler Diagram.

that at zero magnetic field. Consequently, one would expect that at 5 T and 300 K, conditions with the same value of B/T as the former, one would also see a doubling of the electrical, and presumably the thermal, resistivity. It must be noted, however, that such measurements are done with extremely pure, nearly-perfect single crystals, conditions which would tend to produce large values of the electron-scattering characteristic time, τ . Clearly, the aluminum used in a fusion-fission reactor will be highly disordered, since it will have been rolled or forged and subjected to intense neutron irradiation. Both effects tend to reduce τ . In fact, the effect of the presence of neutron-induced defects on the variation of resistance with a magnetic field has been used by several workers as a means of measuring the degree of damage in graphite samples at room temperature. On balance, then, there is the possibility that the structural materials associated with the fusion-fission hybrids will have transport properties affected by the magnetic fields. However, both the high temperatures and the high degree of neutron-induced and other forms of damage make it unlikely that changes of more than a few per cent would occur. There may be areas in which even such small changes could be of significance, such as the thermal behavior of the fissile or fertile fuel. Only scanty information exists on the magnetoresistance of uranium compounds, and no large effects have been observed. Again, the extreme temperatures and the degree of imperfection argue against any large effects, but in the absence of experimental evidence this argument is not wholly convincing.

Other phenomena analogous to the Hall effect have been observed. The Ettinghausen effect is the best established of these, and results in a temperature gradient in an object placed in perpendicular electric and magnetic fields. The Ettinghausen coefficient is defined as:

$$p^t = \frac{\partial T / \partial y}{H_z j_x}. \quad (7.40)$$

Consideration of the coupled current and thermal fluxes leads to:

$$\vec{j} = \vec{L}_{EE} \cdot \vec{E} + \vec{L}_{ET} \cdot \nabla T \quad (7.41)$$

$$\vec{q} = \vec{L}_{TE} \cdot \vec{E} + \vec{L}_{TT} \cdot \nabla T,$$

where $\vec{q}$ is the thermal flux, and the tensors $\vec{L}_{ij}$ are coupling terms, dependent on the direction and strength of the magnetic field. The subscripts E and T refer to the electrical and thermal fluxes, respectively. The Onsager relations give:

$$\begin{aligned}\vec{L}_{TE}(H = 0) &= -T \vec{L}_{ET}^t(H = 0) \\ \vec{L}_{ij}(\vec{H}) &= \vec{L}_{ji}(-\vec{H}),\end{aligned}\tag{7.42}$$

where the superscript t denotes the transpose matrix. Even with the simplifications of Equation (7.42), there are still four 3×3 matrices, with 21 independent terms, to be evaluated. Placement of $\vec{H}$ along the z axis introduces sufficient symmetry to reduce the problem to 9 independent components. Further, one may consider a system both thermally and electrically insulated, such that $\vec{j} = \vec{q} = 0$. Then, the problem reduces to the following set of equations:

$$\begin{aligned}\vec{E} &= \vec{\rho}(\vec{H}) \cdot \vec{j} + \vec{Q}(\vec{H}) \cdot \nabla \vec{T} \\ \vec{q} &= \vec{\pi}(\vec{H}) \cdot \vec{j} - \vec{\chi}(\vec{H}) \cdot \nabla \vec{T},\end{aligned}\tag{7.43}$$

where

$$\begin{aligned}\vec{\rho} &= \vec{L}_{EE}^{-1}, \\ \vec{Q} &= -\vec{L}_{EE}^{-1} \vec{L}_{ET} \\ \vec{\pi} &= \vec{L}_{TE} \vec{L}_{EE}^{-1}, \text{ and} \\ \vec{\chi} &= -(\vec{L}_{TT} - \vec{L}_{TE} \vec{L}_{EE}^{-1} \vec{L}_{ET}).\end{aligned}$$

As is virtually always the case in dealing with results of irreversible thermodynamics, the effects are small and difficult to determine. (For example, compare the difficulties of studying Soret diffusion, which is mass transport driven by a thermal gradient, to those of studying diffusion described by Fick's law. The latter case is relatively well understood, with extensive correlations for and theoretical understanding of the diffusion coefficient. In the former case, however, it is not even possible to predict the sign of the Soret coefficient, let alone its magnitude or trend with temperature.) It has been claimed that the Ettinghausen effect has been experimentally observed, although there is still some uncertainty about the validity of the experimental conditions. Such an effect, even if relatively weak compared to conduction in fissile or fertile materials, might produce a means for the redistribution of material within the fuel, and hence may produce problems in the thermal and chemical behavior of the fuel. It is, however,

impossible to make even semiquantitative estimates of the magnitude of the effect.

7.5 CONCLUSIONS

The presence of large magnetic fields in fusion-fission hybrids introduces several means by which the thermal response of the reactor could be perturbed to the detriment of reactor safety. These include variations in phase and chemical equilibria, and transport properties. Consideration of these phenomena for a variety of possible reactor materials has uncovered no effects of magnetic fields in the range currently projected for fusion-fission hybrids, on the order of 10 T, that would produce sufficiently large changes in the thermal or transport properties to grossly affect thermal behavior. However, this conclusion can be considered only to be tentative, since many of the thermodynamic parameters required are unknown under conditions of high magnetic field, and can at best be crudely estimated. Some chemical reactions which could be a significant hazard to safe reactor operation if they were to occur may be found to be favored by the presence of a magnetic field. The reactions are normally limited by kinetics, not by thermodynamic driving forces, and no theory or experiment exists currently which would allow the estimation of the effect of a large magnetic field on such rate processes. Lastly, several exotic effects can be postulated, such as thermal gradients induced by electric and magnetic fields. While potential consequences to reactor operations could be postulated due to such phenomena, understanding of these effects is sufficiently rudimentary that even an intelligent guess about the magnitude of the effect appears to be precluded at this time.

Section 8

SUMMARY AND CONCLUSIONS

The objective of the study presented in this report was to examine, in a preliminary way, some potential safety questions for conceptual fusion-fission hybrid reactors. It should be stated that this study and the subsequent analysis presented is largely based upon reference to one design, a conceptual mirror hybrid fusion-fission reactor. The plasma of this reactor operates on the deuterium-tritium fuel cycle and its fast spectrum blanket is fueled with uranium carbide and cooled by helium. An attempt was made to generalize the results wherever possible to enable knowledgeable designers in the future to avoid areas of reactor hazard, when possible, or to provide adequate engineered safeguards, when appropriate.

It should be further emphasized that no effort was made to determine the probabilities of initiating events and that initiators were chosen as "postulates" to illustrate the potential for hazard.

The work reported here centered on several major themes: (1) What are the potential radioactive hazards in a hybrid, (2) Are there significant accidents or accident sequences, which can lead to mobility of these hazards, (3) Are there mechanisms for breaching of the containment that lead to release to the environment, and (4) What are the potential pathways and consequences of releases to man.

The major potential radioactive hazards in hybrid fusion-fission conceptual reactors are: (1) Tritium, (2) Activation products (first wall, structural, and corrosion), (3) Actinide elements, and (4) Fission products. In this study, the potential hazards of tritium, fission products, and the actinides were studied. First wall activation products should not differ significantly from pure fusion systems, since they depend primarily on wall loading.

The fission product inventory in a hybrid blanket, in terms of continuous power output of 1 GWt for a 3-year period, can be expected to differ from that in an

LWR as the result of a) a different fission product yield, and b) high-energy, neutron-induced transmutations. High-energy neutron-induced fission tends to produce more fission product pairs of near-equal atomic weight. For example, ruthenium-106, a hazardous fission product, has a fission yield that increases by an order of magnitude at 14.1 MeV, when compared to thermal neutron-induced fission. However, the threshold for (n,2n) reactions in ruthenium-106 is 8.07 MeV, so that some of it is "burned out" in the blanket. Because of these two competing effects, the difference between an LWR (operating at the same power) and this reference design, in terms of potential hazard index for the fission products, was almost insignificant. Increasing the average neutron flux in the energy range above 10 MeV by 2 orders of magnitude was found to reduce the fission-product hazard of the blanket significantly. It decreased the inventory of long-lived nuclides and increased that of shorter-lived nuclides. However, the occurrence of such an increase in high-energy flux seems unlikely, based on wall loading considerations.

The most significant detriment, in terms of hazard index, resides in the accumulation of plutonium and other actinides in the fission blanket. The potential hazard of plutonium is between 1 and 3 orders of magnitude greater than a comparable LWR, depending upon the neutron flux spectrum. Even in the conceptual design of a hybrid actinide burner, the inventory of actinides increases significantly before consumption by fission is effective [8-1].

Large inventories of tritium are associated with the plasma in a hybrid, and also with external reprocessing, storage, and transportation. Unlike fission products and actinides, tritium species can be released in significant quantities during normal operation, as a result of small off-normal events, or during a catastrophic accident.

To quantify the magnitude of the hazard presented by tritium releases, a computer model (TREM) was prepared to describe the dispersion, uptake, and dose commitment from tritiated species. Critical pathways to man include: air-soil-fruits and vegetables-man; air-soil-plants-cattle (meat & milk)-man; air-water-man; air-water-fish-man; and air-man (via inhalation). Dose commitments on the order of 10^5 man-rem/kg released, can be expected within 80 km of the source. Long-term environmental implications of tritium accumulation in the hydrosphere have been shown to be lower on a per curie basis than consequences within 80 km. However, as the number of facilities releasing small quantities of tritium increases, the unavoidable accumulation of tritium in the world water system will play a more significant role.

In an effort to uncover physical phenomena which might lead to a release of these potential hazards, four event trees were constructed for the following accident sequences:

- (1) Blanket Loss-of-Flow Accident,
- (2) Blanket Loss-of-Coolant Accident,
- (3) Steam-Ingress Accident, and
- (4) Plasma Density-Temperature Transient.

Based upon these event trees, several important phenomena were studied. In particular, loss of coolant and criticality appear to be the major generic safety considerations unique to fission blankets. For this design, steam-ingress to the blanket is also of concern.

Three aspects of loss-of-coolant and steam-ingress accidents have been considered. Firstly, the problem of the state of the fissile/fertile fuel elements has been examined for a helium-cooled reactor. Dominant in the consideration of such an accident is the question of whether the fuel elements can melt, since this could lead to a condition in which bulk fuel motion could occur. As indicated by the event trees, this fuel motion might lead to criticality of the core. The model employed considers two cases. In the first, the depressurization and thermal heating processes are decoupled. In the second, they are treated in a coupled fashion. The decoupled model demonstrates that a characteristic time exists, estimated to be 10 seconds, during which significant depressurization occurs. The latter model, with its ability to consider the temperature history of the fuel, indicates that reactor shut-down must occur within a comparable time scale if cladding failure is to be avoided. The avoidance of clad failure assumes that decay-heat removal is available.

The consequences of a rupture between the helium primary coolant loop and a steam generator have also been considered. The analysis includes both thermal and neutronic effects. The analysis shows that significant pressure and temperature effects can occur in the blanket, during approximately 10 seconds, and that while the course of the steam-ingress accident can be quite sensitive to the size of the rupture, the characteristic time for pressurization is much less so. It is concluded that significant effects could be encountered within the blanket before any appreciable change has occurred in the steam generator due to the large thermal mass of steam.

Lastly, the characteristic time for failure of the first wall is found to determine the response time of a lithium-cooled blanket, since it will have the

fastest time in such systems. A loss-of-coolant accident is found to produce almost complete thermal isolation of the first wall in a homogeneous blanket. Characteristic times associated with the failure of the first wall can be on the order of several seconds and are appreciably sensitive to the characteristic time for drainage of the coolant from the reactor blanket.

The three related events considered have demonstrated that significant damage to the blanket, the fuel elements, and to the structural components of the reactor can occur in 1 to 10 seconds. These times are comparable to the time required for plasma shut-down, hence, it may become difficult to halt reactor operation before significant damage has occurred to reactor components.

Assuming that reactor shut-down has occurred, the problem of decay-heat removal must be considered. The sources of decay heat are the induced activity of the first wall (usually referred to as 'after heat') and the fission products. Since the power densities in the fissile fuel are comparable to those in LWR's, about 300 - 600 W/cm³, decay-heat removal must be provided. For the helium-cooled blanket, loss of forced-circulation heating causes the fuel to melt in the range of 100-200 seconds, due to decay heat [8-2]. For liquid-metal-cooled blankets, natural convection may cool those fuel elements which have the proper orientation with respect to gravity, providing there is a heat sink [8-3]. For an LMFBR, it has been estimated that with loss of heat sink, the decay heat will cause clad melting after the complete evaporation of the sodium liquid, which takes 32 hours [8-4].

As a result of these studies, it appears that highly reliable and even redundant decay-heat removal must be provided. Loss of ability to remove decay heat results in the melting of fuel, with ultimate release of fission products and actinides to the containment structure. In addition, the question of criticality must be addressed.

Although fusion-fission hybrid blankets are designed to be far subcritical throughout blanket life, multiplication can change as a result of:

- (1) Material composition changes, and
- (2) Geometric reconfiguration.

Three changes, which might occur due to an accident, were examined in detail:

- (1) The ingress of steam into a fast-spectrum blanket,
- (2) Complete meltdown of the blanket material, and
- (3) Steam ingress with resulting blanket expansion.

The general trend of all blanket reconfigurations yielded significant increases in neutron multiplication. However, low blanket residence times and, therefore, low fissile inventories prevented criticality, even for the most reactive geometries attainable. From the results of this study, the following specific conclusions can be drawn for reference design:

- (1) A blanket meltdown could not achieve a critical configuration, except under extremely high blanket residence times.
- (2) If the blanket material were in a compacted configuration and homogeneously mixed, the moderating capability of the non-fissionable blanket material coupled with the high resonance absorption of the U-238 would decrease neutron multiplication considerably.
- (3) Steam ingress causes a significant increase in neutron multiplication due to the excellent moderating capabilities of hydrogen. However, criticality could not be achieved in this design until extremely high U-238 burnup is realized.
- (4) Criticality could possibly be achieved in a blanket expansion coupled with steam ingress. However, volume fractions of blanket material and water would need to approach those of conventional thermal reactors. It has also been estimated that approximately three times the water inventory of the steam generators is needed to create a critical configuration, even with blanket expansion.

The results of this study indicate that criticality would be very difficult to achieve in this design. It must be emphasized that the analysis was performed for only one design. It does show, however, that blankets can be designed which will remain subcritical under extensive changes in both composition and geometry.

Work initiated on the safety aspects of large superconducting toroidal magnets [8-5] was extended to include the Yin-Yang coil used in the mirror hybrid system considered in this project. This work included, in particular, consideration of forces and possible effects on containment wall integrity. Safety-design philosophy was also treated in detail. With respect to magnetically-driven missiles, preliminary results indicate that relatively small velocities are reached with low efficiency. Factors contributing to these small velocities are: 1) Short times of the applied magnetic pressure gradients, 2) Relatively large skin depth and related dissipation, and 3) Lack of guidance and flight propulsion "incentives". In the area of missile hazards only a few special cases have been

considered (e.g., the sudden release of sheet, possibly by exploding local charges at the ends of the sheet). These results do not include all possible cases, in particular at high magnetic fields. At the design fields the data indicate relatively small effects due to kinetic energy release. However, the methods used in the Appendix are general enough to permit extension to other situations.

In the design philosophy, improvements have to focus on a) the superconductor, b) its normal-metal stabilizer, c) normal-metal barriers, and d) coolant conditions. All of these should have desirable interactions, resulting in an optimum rate of entropy transfer.

The presence of large magnetic fields in fusion-fission hybrids also introduces several means by which the thermal response of the high-temperature reactor materials could be perturbed to the potential detriment of reactor safety. These include variations in phase and chemical equilibria, and transport properties. Consideration of these phenomena for a variety of possible reactor materials uncovered no effects of magnetic fields in the range currently projected for fusion-fission hybrids, on the order of 10 T, that would produce sufficiently large changes in the thermal or transport properties to grossly affect thermal behavior. However, this conclusion should be considered only tentative, since many of the needed thermodynamic parameters are unknown under conditions of high magnetic field, and can be at best only crudely estimated. Some chemical reactions which could be a significant hazard to safe reactor operation if they were to occur may be found to be favored by the presence of a magnetic field. The reactions are normally limited by kinetics, not by thermodynamic driving forces, and no theory or experiment exists currently which would allow one to estimate the effect of a large magnetic field on such rate processes. Lastly, several exotic effects can be postulated, such as thermal gradients induced by electric and magnetic fields. While potential consequences to reactor operations could be postulated due to such phenomena, understanding of these effects is sufficiently rudimentary that even an intelligent guess about the magnitude of the effect appears to be precluded at this time.

Future work on fusion-fission hybrid systems should be carried out in parallel to future design efforts. The type of fissile/fertile fuel, blanket coolant, and the type of tritium breeding material all play an important role in safety. These factors, along with other parametric changes, will enable the designer and the safety analyst to move towards an acceptable design.

8.1 REFERENCES

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APPENDIX I. FRAGMENTING MAGNETS
PART A. THE BASIC INVARIANT

We fix attention upon the idealized, wafer-thin plate geometry of Figure I-1. The magnetic missile is taken here to be the infinite ribbon $-\infty < y < \infty$ of half-width b_- and growing elevation $z = \eta(t)$, with $\eta(0) = a$. Ribbon release is made possible by a lateral, plate-to-missile gap in the amount $\Delta = b_+ - b_-$. Later on, in Part C, we shall have occasion to exploit the evident smallness of Δ relative to b_- , and of b_- itself against magnet depth a , but, for the time being, our developments run forward without any appeal to perturbational approximation.

The succeeding computations are considerably streamlined by the symmetry which abounds. This symmetry is grafted upon a quasi-static (QS) field viewpoint wherein it indicates that electric current $\underline{J}(x,z)$ is purely $\hat{e}_2$ - directed. Thus:

$$\underline{J}(x,z) = \hat{e}_2 I_2(x) \begin{cases} \delta(z-\eta) & ; \quad 0 \leq x^2 < b_-^2 \\ \delta(z-a) & ; \quad x^2 > b_+^2 \end{cases}, \quad (I.1)$$

with $I_2(-x) = I_2(x)$ {units of Ampère per $\hat{e}_1$ - directed meter} and δ standing for Dirac's point distribution.* Restriction (I.1) allows one next to build up his magnetic vector potential $\underline{A}(x,z)$ as:

$$\underline{A} = \hat{e}_2 \psi(x,z), \quad (I.2)$$

with ψ being indifferent to change in the sign of x . Going with Equation (I.2) is a magnetic field:

$$\underline{B} = \nabla \times \underline{A} = \nabla \psi \times \hat{e}_2 = -\hat{e}_1 \left(\frac{\partial \psi}{\partial z} \right) + \hat{e}_3 \left(\frac{\partial \psi}{\partial x} \right) \quad (I.3)$$

whose $\hat{e}_1$ - component shares with ψ a symmetry against $x \rightarrow -x$ replacement, whereas B_3 itself is then antisymmetric.

* Accompanying (I.1), of course, is an $\hat{e}_2$ - directed return flow along the geometrically undisturbed plate $z = 0$. One may as well acknowledge its existence at this point, even though, later on, our choice of a Green's function will be such as to obviate any need to describe its structure.

Figure I-1. (x,z) Section through Ribbon Missile at Growing Elevation $z = \eta(t)$.

From (I.3) one comes to perceive difference $\{\psi(x,z) - \psi(x,0)\}$ in the rôle of an aggregate, negatively $\hat{e}_1$ -directed magnetic flux {i.e., "stream" function per unit, $\hat{e}_2$ -directed meter} at any given elevation $z \geq 0$. Moreover, as $B_z = 0$ along the idealized, perfect conductors at $\{0 \leq x^2 < \infty, 0 \leq y^2 < \infty, z = 0 + \epsilon\}$, $\{0 \leq x^2 < b_-^2, 0 \leq y^2 < \infty, z = \eta \pm \epsilon\}$, and $\{b_+^2 < x^2 < \infty, 0 \leq y^2 < \infty, z = a \pm \epsilon\}$, it follows also from the last item in (I.3) that ψ adopts a constant, segment-specific value upon each of the four plate elements depicted by Figure (I-1). These constants are arbitrary up to a common platform by virtue of the second entry beneath Equation (I.3). We agree henceforth, as we always can, to fix this reference level by setting $\psi = 0$ across the entire undisturbed plate $\{0 \leq x^2 < \infty, 0 \leq y^2 < \infty, z = 0\}$. The solenoidal nature of $\underline{B}$, in the meanwhile, is reflected by a uniform value against x for the limiting flux $\psi(x,\infty)$. Altogether then:

$$\psi(x,\infty) = \mu_0 I_0 a, \quad (I.4)$$

with I_0 being the initial plate current, and the content of (I.4) remaining unchanged throughout the history of ribbon escape as a concession to the circumstance that remote magnet stretches must forever remain undisturbed by a localized expulsion of their mass. Identically the same reasoning will then show that it is a self-consistent gesture to set also $\psi(x,a) = \mu_0 I_0 a$ whenever $x^2 > b_+^2$, and that the (presently null) reference level assigned to ψ at $z = 0$ likewise cannot in any way respond to the evolving missile separation $\eta(t) - a$.

One infers the invariance of $\mu_0 I_0 a = \psi(x,a)$ for $x^2 > b_+^2$ and $t \geq 0$ virtually gratis, simply on the strength of general arguments alone. We proceed next to show in addition that ribbon values $\psi_s = \psi(x,\eta(t))$, $0 \leq x^2 < b_-^2$, too, are wholly insensitive to growth in missile separation $\eta(t) - a$. This invariance being granted, we may tether our subsequent thoughts around a soap-film analogy wherein the receding sliver carries with it a rigidly prescribed amount of magnetic flux.

Further along our work will show that information about ψ offers a complete input regarding the dynamics of missile escape $\eta(t)$. The determination of ψ itself will moreover be anchored around a three-region, boundary-value solution to Laplace's equation, one indeed that is controlled by knowledge in advance about the invariants $\{\psi_s = \psi(x,\eta(t)) \text{ for } 0 \leq x^2 < b_-^2 \text{ and } t \geq 0\}$ and $\{\mu_0 I_0 a = \psi(x,a) \text{ for } x^2 > b_+^2 \text{ and } t \geq 0\}$.

Now, even though our primary aim becomes narrowed thus upon potential ψ , we find it convenient here to use for a logical stepping stone its Green's function connection to current source $\underline{J}$ as detailed in (I.1). In particular, it is in the context of this auxiliary connection, when further augmented by a collateral assessment of electric field $\underline{E}$ as a partner in the moving boundary condition (MBC) later to be described, that the invariance of $\{\psi_s = \psi(x, \eta(t)), 0 \leq x^2 < b_-^2\}$ against time $t \geq 0$ comes at length into a proper focus. The preoccupation with current distribution $\underline{J}$ which now ensues is temporary and, beyond Part B, we shall take a voluntary leave from all effort to scrutinize the structure of $\underline{J}$ versus x .

The connection at issue is set up by noting first from (I.2) the solenoidal feature:

$$\begin{aligned}\nabla \cdot \underline{A} &= \frac{\partial}{\partial y} \psi(x, z) \\ &= 0 ,\end{aligned}\tag{I.5}$$

whence indeed:

$$\nabla^2 \psi = - \mu_0 J_2 .\tag{I.6}$$

Equation (I.6) is further supplemented by stipulating that ψ be continuous even in transit across the plate elements $\{0 \leq x^2 < b_-^2, 0 \leq y^2 < \infty, z = \eta(t)\}$ and $\{x^2 > b_+^2, 0 \leq y^2 < \infty, z = a\}$, where, however, its z -derivative is permitted to jump. This discontinuity in $\hat{e}_3$ -directed slope $\partial\psi/\partial z$ evidently amounts to $-\mu_0 I_2(x)$. As a companion to ψ we introduce also the kernel or else Green's function:

$$G(x-x'; z, z') = \frac{1}{4\pi} \ln \left\{ \frac{(x-x')^2 + (z-z')^2}{(x-x')^2 + (z+z')^2} \right\}\tag{I.7}$$

about which we note at once that:

$$G(x-x'; z, 0) = 0 ,\tag{I.8}$$

a circumstance destined soon to absolve us from any need to specify current upon the (geometrically) undisturbed plane $z = 0$. We readily discover that:

$$\nabla^2 G(x-x'; z, z') = \delta(x-x') \{ \delta(z-z') - \delta(z+z') \} ,\tag{I.9}$$

and hence go on to compute:

$$\psi(x, z) = \int_{\mathcal{D}} dx' dz' \{ \psi(x', z') \nabla'^2 G(x' - x; z', z) - G(x' - x; z', z) \nabla'^2 \psi(x', z') \} \quad (I.10)$$

relative to the domain $\mathcal{D}$ depicted by Figure I-2. Remark that here $(x, z) \in \mathcal{D}$ and that $\mathcal{D}$ itself lies exterior to the current source which (I.1) and (I.6) provide for ψ . The right-hand member from (I.10) submits next to a standard application of the divergence theorem, one that winds its final contour $\partial \mathcal{D}$ around the upper plate fragments in a way suited to promote the appearance of current density $I_2(x)$. The result of all this is symmetric in x , and reads:

$$\begin{aligned} \psi(x, z) = & - \frac{\mu_0}{4\pi} \int_0^{b_-} dx' I_2(x') \cdot \\ & \cdot \left[\ln \left\{ \frac{(x-x')^2 + (z-\eta)^2}{(x-x')^2 + (z+\eta)^2} \right\} + \ln \left\{ \frac{(x+x')^2 + (z-\eta)^2}{(x+x')^2 + (z+\eta)^2} \right\} \right] \\ & - \frac{\mu_0}{4\pi} \int_{b_+}^{\infty} dx' I_2(x') \cdot \\ & \cdot \left[\ln \left\{ \frac{(x-x')^2 + (z-a)^2}{(x-x')^2 + (z+a)^2} \right\} + \ln \left\{ \frac{(x+x')^2 + (z-a)^2}{(x+x')^2 + (z+a)^2} \right\} \right]. \end{aligned} \quad (I.11)$$

The buildup behind (I.11) proceeds without any contribution from distant portions of $\partial \mathcal{D}$, a feature that we now abandon for confirmation in private. In the course of so transforming (I.10) one verifies that a shift of ψ through any given constant upon reference plane $z = 0$ is indeed properly imposed also for all other $z > 0$.

Vanishing B_z along the fixed half-planes $\{x^2 > b_+^2, 0 \leq y^2 < \infty, z = a\}$ has revealed to us a constant value there for ψ . The escaping ribbon $\{0 \leq x^2 < b_-^2, 0 \leq y^2 < \infty, z = \eta(t)\}$, however, undergoes motion as $\dot{\eta} \hat{e}_3$, a circumstance which necessitates for it a boundary condition more rich in structure. About this condition one should remark in advance that we do expect to see it cope successfully with the inevitable discontinuity of laboratory-frame fields $\underline{E}$ and $\underline{B}$ vis-à-vis ribbon

Figure I-2. Domain D and its Boundary ∂D (---) for the Integration from Equation (I.10).

crossing $z = \eta(t) - \epsilon \rightarrow \eta(t) + \epsilon$ throughout its transverse domain $\{0 \leq x^2 < b_-^2, 0 \leq y^2 < \infty\}$. The condition in question is conveniently gotten by passing first to a momentary rest-frame wherein such motion becomes annulled, and then setting there to zero the tangential component $\sim \hat{e}_3 \times \{E + \dot{\eta} \hat{e}_3 \times B\}$ of the electric field actually experienced. We easily convince ourselves that laboratory-frame E is equipped with but a single, $\hat{e}_2$ -directed component E_2 , one indeed which enjoys a symmetric dependence upon x . This amount of field symmetry is found to be consistent here with the Faraday connection whereby E can be produced from B . In fact, the flux-change entry from this connection, when written as $\nabla \times \{-\hat{e}_2 \partial\psi/\partial t\}$, immediately provides us with the anticipated result:

$$E = -\hat{e}_2 \partial\psi/\partial t, \quad (I.12)$$

save for an $\hat{e}_2$ -directed gradient whose constant value can be dismissed out of hand by virtue of the requirement $\{\psi \rightarrow 0+, E_2 \rightarrow 0 \text{ as } z \rightarrow 0+\}$ or else $\{\psi \rightarrow \mu_0 I_0 a-, E_2 \rightarrow 0 \text{ as } z \rightarrow \infty\}$.

We note in passing that, for $\{x^2 > b_+^2, z = a\}$, one confronts as always the standard, zero-velocity condition of null E_2 , whence it is that we revert via (I.12) to our former invariance of ψ , invariance which, of course, must still be supplemented through an asymptotic estimate regarding its specific constant $\mu_0 I_0 a$. More to the point, however, we calculate now for $0 \leq x^2 < b_-^2, z = \eta(t) \pm \epsilon$ the discontinuous result:

$$E_2(x, \eta \pm \epsilon) = \frac{\mu_0 \dot{\eta}}{4\pi} \{\alpha + \beta + \gamma_{\pm}\}; \quad (I.13a)$$

$$\alpha = \int_0^{b_-} dx' \frac{\partial I_2(x', \eta)}{\partial \eta} \cdot \quad (I.13b)$$

$$\cdot \left[\ell n \left\{ \frac{(x-x')^2 + \epsilon^2}{(x-x')^2 + 4\eta^2} \right\} + \ell n \left\{ \frac{(x+x')^2 + \epsilon^2}{(x+x')^2 + 4\eta^2} \right\} \right];$$

$$\beta = \int_{b_+}^{\infty} dx' \frac{\partial I_2(x', \eta)}{\partial \eta} \cdot \quad (I.13c)$$

$$\cdot \left[\ell n \left\{ \frac{(x-x')^2 + (\eta-a)^2}{(x-x')^2 + (\eta+a)^2} \right\} + \ell n \left\{ \frac{(x+x')^2 + (\eta-a)^2}{(x+x')^2 + (\eta+a)^2} \right\} \right];$$

and:

$$\begin{aligned}
 \gamma_{\pm} = & -2 \int_0^{b_-} dx' I_2(x') \cdot \\
 & \cdot \left[2\eta \left\{ \frac{1}{(x-x')^2 + 4\eta^2} + \frac{1}{(x+x')^2 + 4\eta^2} \right\} \right. \\
 & \left. \pm \varepsilon \left\{ \frac{1}{(x-x')^2 + \varepsilon^2} + \frac{1}{(x+x')^2 + \varepsilon^2} \right\} \right] ,
 \end{aligned} \tag{I.13d}$$

wherein an overdot conveys as always a derivative with respect to time, and where we have underscored the essence of our QS viewpoint by writing:

$$\dot{\mathbf{i}}_2 = \dot{\eta} \frac{\partial I_2}{\partial \eta} \quad \{\text{i.e., } I_2(x) \equiv I_2(x, \eta)\} . \tag{I.14}$$

As a complement to (I.13a) - (I.13d), we compute also that:

$$\dot{\eta} \hat{\mathbf{e}}_3 \times \underline{\mathbf{B}} = - \hat{\mathbf{e}}_2 \dot{\eta} \frac{\partial \psi}{\partial z} , \tag{I.15}$$

and hence encounter along the missile faces at $z = \eta(t) \pm \varepsilon$ the following explicit representations:

$$\begin{aligned}
 \dot{\eta} \hat{\mathbf{e}}_3 \times \underline{\mathbf{B}} \Big|_{z=\eta \pm \varepsilon} = & - \hat{\mathbf{e}}_2 \frac{\mu_0 \dot{\eta}}{2\pi} \int_0^{b_-} dx' I_2(x') \cdot \\
 & \cdot \left[2\eta \left\{ \frac{1}{(x-x')^2 + 4\eta^2} + \frac{1}{(x+x')^2 + 4\eta^2} \right\} \right. \\
 & \left. \mp \varepsilon \left\{ \frac{1}{(x-x')^2 + \varepsilon^2} + \frac{1}{(x+x')^2 + \varepsilon^2} \right\} \right] + \hat{\mathbf{e}}_2 \frac{\mu_0 \dot{\eta}}{2\pi} \int_{b_+}^{\infty} dx' I_2(x') \cdot \\
 & \cdot \left[(\eta-a) \left\{ \frac{1}{(x-x')^2 + (\eta-a)^2} + \frac{1}{(x+x')^2 + (\eta-a)^2} \right\} \right. \\
 & \left. - (\eta+a) \left\{ \frac{1}{(x-x')^2 + (\eta+a)^2} + \frac{1}{(x+x')^2 + (\eta+a)^2} \right\} \right] .
 \end{aligned} \tag{I.16}$$

And lastly, a survey across Eqs. (I.12), (I.13a) - (I.13d), and (I.16) unveils the remarkable conclusion that effective field $\sim \{\underline{E} + \dot{\eta} \hat{e}_3 \times \underline{B}\}$ as such is continuous during transit through missile plane $z = \eta(t)$. The surviving pieces from Eqs. (I.13a) - (I.13d) and (I.16), in the meanwhile, collaborate to assemble for $0 \leq x^2 < b_-^2$ a derivative representation:

$$\left. \{\underline{E} + \dot{\eta} \hat{e}_3 \times \underline{B}\} \right|_{z=\eta \pm \epsilon} = \hat{e}_2 \frac{\mu_0 \dot{\eta}}{4\pi} \frac{d}{d\eta} \{\Omega_- + \Omega_+\}; \quad (I.17a)$$

$$\begin{aligned} \Omega_- = \int_0^{b_-} dx' I_2(x') & \left[\ln \left\{ \frac{(x-x')^2 + \epsilon^2}{(x-x')^2 + 4\eta^2} \right\} \right. \\ & \left. + \ln \left\{ \frac{(x+x')^2 + \epsilon^2}{(x+x')^2 + 4\eta^2} \right\} \right]; \end{aligned} \quad (I.17b)$$

and:

$$\begin{aligned} \Omega_+ = \int_{b_+}^{\infty} dx' I_2(x') & \left[\ln \left\{ \frac{(x-x')^2 + (\eta-a)^2}{(x-x')^2 + (\eta+a)^2} \right\} \right. \\ & \left. + \ln \left\{ \frac{(x+x')^2 + (\eta-a)^2}{(x+x')^2 + (\eta+a)^2} \right\} \right], \end{aligned} \quad (I.17c)$$

whose obligatory disappearance there is tantamount to having structure (I.11) made a constant then vis-à-vis missile excursions $\eta-a$. These observations clinch our contention regarding a comprehensive pattern of QS boundary-value invariance for ψ .

PART B. MISSILE DRIVE

Vanishing B_z for $\{0 \leq x^2 < b_-^2, 0 \leq y^2 < \infty, z = \eta(t) \pm \epsilon\}$ assures that magnetic force density $\sim \underline{J} \times \underline{B}$ is purely $\hat{e}_3$ -directed; there is, in other words, no tendency whatsoever toward transverse tear. In assessing this load we must take care of course to filter out the ribbon self-field $\underline{B}^{\text{self}}$; this latter, at any rate, is incapable, on a *priori*, theoretical grounds, of making a net drive contribution.

It is formula (I.16) which offers now a convenient basis for external (i.e., residual plate along $\{x^2 > b_+^2, 0 \leq y^2 < \infty, z = a\}$ and $\{0 \leq x^2 < \infty, 0 \leq y^2 < \infty, z = 0\}$) field computation. In fact, we recognize in (I.16) a familiar, limit representation for Dirac's delta, one that brings into view the discontinuous connection $B_1^{\text{self}} \Big|_{z=\eta \pm \epsilon} = \pm \mu_0 I_2 / 2$ between surface density I_2 and its own, adjacent field B^{self} . This being so, we recover the external field $B_1^{\text{ext}} \Big|_{z=\eta}$ simply as one half the sum of its total values - $\partial\psi/\partial z \Big|_{z=\eta \pm \epsilon}$ at conjugate stations $\{0 \leq x^2 < b_-^2, 0 \leq y^2 < \infty, z = \eta(t) \pm \epsilon\}$. Difference $-\mu_0^{-1} \{ \partial\psi/\partial z \Big|_{z=\eta+\epsilon} - \partial\psi/\partial z \Big|_{z=\eta-\epsilon} \}$, at the same time, figures then as a representation of current density I_2 . And finally, on putting all these diverse items together, with a due regard also to field symmetry,* we get the following, *a posteriori* coupling to missile dynamics $\dot{\eta}$:

$$\frac{d}{d\eta} \{ \dot{\eta}^2 \} = \frac{2}{\rho \mu_0} \int_0^b -dx \left\{ \left(\frac{\partial\psi}{\partial z} \right)^2_{z=\eta-\epsilon} - \left(\frac{\partial\psi}{\partial z} \right)^2_{z=\eta+\epsilon} \right\}, \quad (\text{I.18})$$

provided only that ρ stands now for the constant mass density per unit ribbon length. Equation (I.18) is written in a way to emphasize that its right-hand member should ultimately be viewed as a function of missile displacement η -a.

* Motion symmetry is, strictly speaking, preserved here only when gravity points along $\mp \hat{e}_3$. In that case, and that case alone, one faces the afterthought need to augment Eq. (I.18) on its right with $\mp 2g$. Such need will of course never intrude in the event that magnetic drive, as now written in Eq. (I.18), proves to be the dominant force.

In addition to gravity, we ignore also across the board all other incidental complications of the type posed by geometric irregularity, imperfect conductivity, the edge effects due to finite plate extent and thickness, etc., etc. Worthy of record here is the underlying fact that our QS viewpoint itself implies a small-velocity approximation, one wherein the speed of light is held to be infinite and which is hence coarse-grained as regards the rates of field relaxation.

And finally, the ejection mode now under study is free from all missile-to-residual plate current leakage. An immediate consequence of this omission is to thwart arcing drain upon both energy and its underlying current reservoir. Our present force estimate is therefore to be reckoned as both a conservative overreach and an invitation to further work on missiles that are released across current flow. Field symmetry then must accommodate the rôle also of components E_1 , E_3 , and B_2 , in addition to those already tackled.

Concerning (I.18) we note at once the following features. Its right-hand, driving member is, to begin with, inherently non-negative, an attribute that one infers without any need for detailed calculation, simply by reference to field line dilation/flattening upon upper/lower ribbon faces. The speed of missile release, moreover, is governed by a time scale:

$$t_o = \sqrt{ap/2b_- \mu_o I_o^2}, \quad (I.19)$$

and this by virtue of a persisting proportionality between ψ and the prevailing current background I_o .

And finally, we may remark that, even though (I.18) has been gotten through a canonical process, its driving term exhibits nothing other than a balance between upper/lower magnetic pressure loads. The pressure afterthought leads indeed here to an analogue from steady, incompressible, inviscid and irrotational flow past a baffle offset from a guiding trough into an otherwise quiescent fluid. This analogue carries over practically verbatim up to the point of pressure computation, which becomes a suction then on account of Bernoulli's relation, and the inability to make a blanket claim regarding the invariance of intercepted flow ψ_s .

PART C. THE ZERO-TIME BOUNDARY VALUE PROBLEM

In this part we get down to the business of computing ψ_s , an exercise that is carried out as a preliminary obligation just at the moment of lamina release. We expect of course to witness in ψ_s a certain defect beneath $\mu_o I_o a$, defect which is forever made up by the magnetic flux that sweeps above the receding strip, and, relative to the zero-time configuration, first escapes and then re-enters across the gaps $\{b_- < x < b_+, 0 \leq y^2 < \infty, z = a\}$ and $\{-b_+ < x < -b_-, 0 \leq y^2 < \infty, z = a\}$, respectively.

Our ensuing computations make reference next to Regions I and II on Figure I-3, and begin with the following spectral representations for $\psi(x,z)$:

Region I: $0 \leq z \leq a$

$$\left\{ \begin{array}{l} \psi(x,z) = \int_{-\infty}^{\infty} dk e^{ikx} \mathcal{A}(k) \sinh(kz); \\ \mathcal{A}(-k) = -\mathcal{A}(k); \end{array} \right. \quad (I.20a)$$

$$\mathcal{A}^*(k) = \mathcal{A}(k), \quad (I.20b)$$

$$\mathcal{A}^*(k) = \mathcal{A}(k), \quad (I.20c)$$

Figure I-3. Broken Plate Geometry just past the Moment of Missile Release.

and:

Region II: $z \geq a$

$$\left\{ \begin{array}{l} \psi(x, z) = \mu_0 I_0 a + \int_{-\infty}^{\infty} dk e^{ikx - |k|(z-a)} \mathcal{B}(k); \\ \mathcal{B}(-k) = \mathcal{B}(k); \\ \mathcal{B}^*(k) = \mathcal{B}(k), \end{array} \right. \quad \begin{array}{l} (I.21a) \\ (I.21b) \\ (I.21c) \end{array}$$

structures which respect the homogeneous (i.e., Laplace) version of (I.6), conform to the requirement of symmetry vis-à-vis interchange $x \leftrightarrow -x$, and, by deliberate choice, have been tailored to adopt values preassigned for $z \rightarrow 0^+$ and $z \rightarrow \infty$. The boundary-value match as $z \rightarrow \infty$, in particular, has made it convenient for us to siphon off, and hence explicitly acknowledge the asymptotic, long-wavelength mode.*

Soon enough we shall proceed to link up the Fourier amplitudes $\mathcal{A}$ and $\mathcal{B}$ through a demand of tangential (i.e., B_1) field continuity along the slits $\{b_-^2 < x^2 < b_+^2, 0 \leq y^2 < \infty, z = a\}$. A continuity for B_3 must also be present, but here this feature is amply guaranteed by Eq. (I.3) and the *a priori* continuity there of ψ itself. Before imposing B_1 continuity along $\{b_-^2 < x^2 < b_+^2, 0 \leq y^2 < \infty, z = a\}$, however, it is useful to bring in a temporary appearance by ψ_s . This one does by forcing each of Eqs. (I.20a) and (I.21a) to reproduce the pattern of ψ behavior along $z = a$. And if, interior to:

$$\begin{aligned} \mathcal{J} &= \{b_-^2 < x^2 < b_+^2\} \\ &= \{-b_+ < x < -b_-\} \cup \{b_- < x < b_+\} \end{aligned} \quad (I.22)$$

we write:

$$\begin{aligned} \Gamma(x) &= \Gamma(-x) \\ &= \psi(x, a), \end{aligned} \quad (I.23)$$

* The analogous gesture vis-à-vis Region I representation (I.20a) would be to split off from $\mathcal{A}(k) \sinh(kz)$ the (now linear) mode $\mu_0 I_0 z$. Such isolation is in any event implicit to Eq. (I.24) below by virtue of delta function presence there.

this common demand is found to entail the following pair of inverse Fourier statements:

$$\begin{aligned} \mathcal{A}(k) \sinh(ka) &= \frac{\psi_s}{\pi k} \sin(kb_-) \\ &+ \frac{1}{\pi} \int_{b_-}^{b_+} dx \Gamma(x) \cos(kx) \\ &+ \frac{\mu_o I_o a}{\pi} \left[\pi \delta(k) - \frac{\sin(kb_+)}{k} \right] \end{aligned} \quad (I.24)$$

and, evidently:

$$\mathcal{B}(k) = \mathcal{A}(k) \sinh(ka) - \mu_o I_o a \delta(k) . \quad (I.25)$$

Tangential field continuity for $x \in \mathcal{J}$, in the meanwhile, takes the form:

$$\int_{-\infty}^{\infty} dk e^{ikx} \mathcal{A}(k) k \cosh(ka) = - \int_{-\infty}^{\infty} dk e^{ikx} |k| \mathcal{B}(k) . \quad (I.26)$$

We undertake now a reduction of each side from Eq. (I.26). Its right-hand member, it so happens, is the simpler of the two, and hence that is where we begin. And so, for any $0 \leq x^2 < \infty$ we encounter now on the strength of representations (I.24) and (I.25) the result:

$$\begin{aligned} &- \int_{-\infty}^{\infty} dk e^{ikx} |k| \mathcal{B}(k) = \\ &\left. \begin{aligned} &- \frac{1}{\pi} \int_{b_-}^{b_+} dx_1 \Gamma(x_1) \frac{\partial}{\partial x_1} \int_{-\infty}^{\infty} dk e^{ikx} \sin(|k|x_1) \\ &+ \frac{\mu_o I_o a}{\pi} \int_{-\infty}^{\infty} dk e^{ikx} \sin(|k|b_+) \\ &- \frac{\psi_s}{\pi} \int_{-\infty}^{\infty} dk e^{ikx} \sin(|k|b_-) \end{aligned} \right\} \quad (I.27) \\ &= \frac{1}{\pi} \int_{b_-}^{b_+} dx_1 \Gamma'(x_1) \int_{-\infty}^{\infty} dk e^{ikx} \sin(|k|x_1) , \end{aligned}$$

with superscript prime affixed to $\Gamma(x_1)$ denoting its derivative with respect to x_1 , and a due advantage having been taken of the terminal evaluations:

$$\Gamma(b_-) = \psi_s \quad (I.28)$$

and:

$$\Gamma(b_+) = \mu_0 I_0 a . \quad (I.29)$$

The kernel from the last entry beneath Eq. (I.27) is easily processed through temporary adjunction of an exponential convergence factor (which is to say, via Euler summability), sleight of hand which calls forth Cauchy's principal value filter P and gives:

$$\int_{-\infty}^{\infty} dk e^{ikx} \sin(|k|x_1) = \frac{P}{x_1 - x} + \frac{P}{x_1 + x} . \quad (I.30)$$

Altogether then we get:

$$-\int_{-\infty}^{\infty} dk e^{ikx} |k| \mathcal{B}(k) = \frac{1}{\pi} \int_{\mathcal{I}} dx_1 \Gamma'(x_1) \frac{P}{x_1 - x} \quad (I.31)$$

with a view also to the antisymmetry:

$$\Gamma'(-x) = -\Gamma'(x) \quad (I.32)$$

($x \in \mathcal{I}$) which is a concomitant to Eq. (I.23).

The sinistral member from Eq. (I.26) submits to manipulations of a comparable sort, although, of course, their detail is a shade more intricate. We begin by writing:

$$\left. \begin{aligned} & \int_{-\infty}^{\infty} dk e^{ikx} \mathcal{A}(k) k \cosh(ka) = \mu_0 I_0 \\ & + \frac{1}{\pi} \int_{b_-}^{b_+} dx_1 \Gamma(x_1) \frac{\partial}{\partial x_1} \int_{-\infty}^{\infty} dk e^{ikx} \sin(kx_1) \coth(ka) \\ & - \frac{\mu_0 I_0 a}{\pi} \int_{-\infty}^{\infty} dk e^{ikx} \sin(kb_+) \coth(ka) \\ & + \frac{\psi_s}{\pi} \int_{-\infty}^{\infty} dk e^{ikx} \sin(kb_-) \coth(ka) \end{aligned} \right\} \quad (I.33)$$

$$= \left. \begin{aligned} & \mu_0 I_0 \\ & - \frac{1}{\pi} \int_{b_-}^{b_+} dx_1 \Gamma'(x_1) \int_{-\infty}^{\infty} dk e^{ikx} \sin(kx_1) \coth(ka) . \end{aligned} \right\}$$

Reduction (I.33), just like (I.27) before it, holds unrestrictedly for all x , $0 \leq x^2 < \infty$. Its final kernel, moreover, is readily seen to recapture the former, more simple structure from Eq. (I.30) in the limit of infinite magnet depth $a \rightarrow \infty$. And, of course, such coalescence of kernels is to be expected on *a priori* grounds.

Concerning this more general kernel from (I.33), we proceed by writing now:

$$\begin{aligned}
& \int_{-\infty}^{\infty} dk e^{ikx} \sin(kx_1) \coth(ka) = \\
& \lim_{\omega \rightarrow 1-} \int_{-\infty}^{\infty} dk e^{ikx} \sin(|k|x_1) \left[-1 + \frac{2}{1 - \omega \exp\{-2|k|a\}} \right] \\
& = \lim_{\omega \rightarrow 1-} \int_{-\infty}^{\infty} dk e^{ikx} \sin(|k|x_1) \left[1 + 2 \sum_{n=1}^{\infty} \omega^n \exp\{-2na|k|\} \right] \\
& = \frac{P}{x_1 - x} + \frac{P}{x_1 + x} \cdot \left. \begin{aligned} & + 2 \lim_{\omega \rightarrow 1-} \sum_{n=1}^{\infty} \omega^n \left\{ \frac{x_1 - x}{(x_1 - x)^2 + (2na)^2} + \frac{x_1 + x}{(x_1 + x)^2 + (2na)^2} \right\} \\ & = \frac{P}{x_1 - x} + \frac{P}{x_1 + x} \\ & + \frac{\pi}{2a} \left[\coth \left\{ \frac{\pi(x_1 - x)}{2a} \right\} - \frac{2a}{\pi(x_1 - x)} \right. \\ & \quad \left. + \coth \left\{ \frac{\pi(x_1 + x)}{2a} \right\} - \frac{2a}{\pi(x_1 + x)} \right] \end{aligned} \right\} \quad (I.34)
\end{aligned}$$

after a retrospective appeal also to Eq. (I.30) and to a standard, Mittag-Leffler, partial fraction development for the hyperbolic cotangent $\coth$. In fact, we recognize at once in the last entry from (I.34) a symmetrical version (with respect to x) of the Langevin function:

$$\mathcal{L}(q) = \coth[q] - q^{-1} \quad (I.35)$$

familiar from the (orientational) thermodynamics of the diatomic molecule and about which we anticipate for imminent use the following result:

$$\mathcal{L}(q) \xrightarrow{q \rightarrow 0+} q/3 . \quad (\text{I.36})$$

On putting together now the several diverse pieces (I.26), (I.31), (I.33), (I.34), (I.35), and (I.32), in that order, we get, for $x \in \mathcal{J}$, the following basic integral equation:

$$\int_{\mathcal{J}} dx_1 \Gamma'(x_1) \frac{p}{x_1 - x} = \frac{\pi \mu_o I_o}{2} - \frac{\pi}{4a} \int_{\mathcal{J}} dx_1 \Gamma'(x_1) \mathcal{L}\left(\frac{\pi(x_1 - x)}{2a}\right) \quad (\text{I.37})$$

for the normal magnetic field $B_3(x, a) = \Gamma'(x)$ threading across the slit interval $\mathcal{J}$.

It is amusing perhaps that, even though with (I.26) we had set out to match for $x \in \mathcal{J}$ the tangential fields $B_1(x, a \pm \epsilon)$, its later echo (I.37) features instead vector partner B_3 . Absent also from (I.37) is any visible trace of ψ_s which, however, we subsequently resurrect by setting:

$$\psi_s = \mu_o I_o a - \int_{b_-}^{b_+} dx \Gamma'(x) \quad (\text{I.38})$$

as a self-evident assessment of flux defect beneath $\mu_o I_o a$.

Equation (I.37) is of the singular, Carleman variety, and we shall naturally make no pretense here about solving it exactly. That which we can nevertheless do is to seek an approximation ordered by the ratio of missile half-width b_- to magnet cavity depth a . A convenient access point to this perturbation program is found in (I.36), which permits one to render Eq. (I.37) approximately as:

$$\left. \begin{aligned} \int_{\mathcal{J}} dx_1 \Gamma'_a(x_1) \frac{p}{x_1 - x} &\cong \frac{\pi \mu_o I_o}{2} \\ &- \frac{\pi^2}{24a^2} \int_{\mathcal{J}} dx_1 \Gamma'_\infty(x_1) (x_1 - x) \end{aligned} \right\} \quad (\text{I.39})$$

$$= \frac{\pi \mu_o}{2} \left\{ I_o - \frac{\pi}{6a^2 \mu_o} \int_{b_-}^{b_+} dx_1 x_1 \Gamma'_\infty(x_1) \right\},$$

wherein a subscript newly affixed to Γ' serves henceforth to convey the state of magnet cavity depth, and where:

$$\tilde{I}_o = I_o - \frac{\pi}{6a^2 \mu_o} \int_{b_-}^{b_+} dx_1 x_1 \Gamma'_\infty(x_1) \quad (I.40)$$

sets up on the right a type of low-order current renormalization.

A routine centering of variables leads us to confront next for $x \in \mathcal{J}$ the following connection:

$$\int_{-\left\{\frac{b_+^2 - b_-^2}{2}\right\}}^{\left\{\frac{b_+^2 - b_-^2}{2}\right\}} d\lambda \Gamma'_\infty \left(\sqrt{\lambda + \left\{\frac{b_-^2 + b_+^2}{2}\right\}} \right) \frac{p}{\lambda - \left(x^2 - \left\{\frac{b_-^2 + b_+^2}{2}\right\}\right)} = \frac{\pi \mu_o I_o}{2} , \quad (I.41)$$

together with its approximate counterpart gotten under the replacements $\Gamma'_\infty \rightarrow \Gamma'_a$ and $I_o \rightarrow \tilde{I}_o$. An equation of the type (I.41) occurs in the theory of aerodynamic lift (Reference I-1) and yields in the present instance a solution:

$$\begin{aligned} \Gamma'_\infty(x) &= \text{sign}(x) \left\{ \frac{\frac{\mu_o I_o}{2} \left(x^2 - \left\{\frac{b_-^2 + b_+^2}{2}\right\} \right) + \tilde{\mathcal{D}}}{\sqrt{\left(\frac{b_+^2 - b_-^2}{2}\right)^2 - \left(x^2 - \left\{\frac{b_-^2 + b_+^2}{2}\right\}\right)^2}} \right\} \\ &= \text{sign}(x) \left\{ \frac{\frac{\mu_o I_o}{2}}{\sqrt{(b_+^2 - x^2)(x^2 - b_-^2)}} \right. \\ &\quad \left. + \frac{C_\infty}{\sqrt{(b_+^2 - x^2)(x^2 - b_-^2)}} \right\} \end{aligned} \quad (I.42)$$

built up through a superposition of particular {i.e., $\sim \mu_o I_o x^2/2$ } and homogeneous { $\sim C_\infty = \tilde{\mathcal{D}} - \mu_o I_o (b_-^2 + b_+^2)/4 = \text{constant}$ } components. And, as a companion to (I.42), we also have:

$$\Gamma'_a(x) \cong \text{sign}(x) \left\{ \frac{\mu_0 \tilde{I}_0}{2} \frac{x^2}{\sqrt{(b_+^2 - x^2)(x^2 - b_-^2)}} + \frac{C_a}{\sqrt{(b_+^2 - x^2)(x^2 - b_-^2)}} \right\} \quad (\text{I.43})$$

with C_a being yet another constant.

Our bootstrap procedure for the computation of ψ_s is perceived now to thread its way across the logical chain (I.38) $\rightarrow$ (I.43) $\rightarrow$ (I.40) $\rightarrow$ (I.42), and to necessitate hence a knowledge regarding constants C_a and C_∞ . These auxiliary determinations will in fact engross us for a number of pages hence, but, before embarking upon this task, we pause to note here parenthetically that both (I.42) and (I.43) do indeed possess the right sort of inverse-square-root distance involvement which befits magnetostatic field components adjacent to knife edges.

The determinations of C_a and C_∞ that we now undertake rest upon the observation that missile presence renders our field region multiply connected and places us at liberty hence to specify arbitrarily {i.e., without any impact whatsoever upon asymptotic current density I_0 } the net magnetostatic circulation around any finite, source-filled contour, even though locally the field itself remains of course quite irrotational. In particular, we find it physically plausible to insist here upon current preservation at $2b_+$ I_0 relative to the uniform, quiescent, pre-release configuration.

A detailed implementation of this current invariance is once again most easily taken up in the reference context of infinite magnet depth $a \rightarrow \infty$. A convenient contour for evaluating the magnetic circulation is gotten then as a limiting rectangle of infinite height and width $2b_- < 2x < 2b_+$. We indicate this directed path as a quartet of dashed lines on Figure I-4, whose origin of coordinate z , let it be noted here, has been usefully displaced through the finite distance a relative to that from Figure I-3 preceding. Record is made also of the obvious facts that $B_1^{(\infty)}(x, z) \rightarrow 0, -\mu_0 I_0$ respectively as $z \rightarrow \pm \infty$. Considerations of symmetry evidently enable one to write then:

$$\int_0^\infty dz B_3^{(\infty)}(x, z) = \frac{\mu_0 I_0}{2} (x - b_+) \quad (\text{I.44})$$

and hence to underscore a need for relating $B_3^{(\infty)}(x, z), |z| > 0$, with its aperture {i.e., $z = 0$ } values $\Gamma'_\infty(x)$, $x \in \mathcal{A}$.

Such a connection is easily produced via Fourier's spectral method, one that mimics closely our former, two-region developments (I.20a) - (I.21c) et seq. for ψ . Here, of course, $B_3^{(\infty)}(x, z)$ enjoys a strictly symmetric dependence upon z ,^{*} a feature that we convey by setting:

$$\left\{ \begin{array}{l} B_3^{(\infty)}(x, z) = \int_{-\infty}^{\infty} dk e^{ikx - |kz|} Q(k) \quad ; \end{array} \right. \quad (I.45a)$$

$$\left\{ \begin{array}{l} Q(-k) = -Q(k) \quad ; \end{array} \right. \quad (I.45b)$$

$$\left\{ \begin{array}{l} Q^*(-k) = Q(k) \quad . \end{array} \right. \quad (I.45c)$$

Clearly $B_3^{(\infty)}(x, 0 \pm \epsilon) = 0$ for all $x \notin \mathcal{A}$, whence it is that Fourier inversion along $z = 0$ next gives:

$$Q(k) = \frac{1}{\pi i} \int_{b_-}^{b_+} dx \sin(kx) \Gamma'_{\infty}(x), \quad (I.46)$$

and so:

$$\begin{aligned} B_3^{(\infty)}(x, z) &= \frac{1}{\pi i} \int_{b_-}^{b_+} dx_1 \Gamma'_{\infty}(x_1) \int_{-\infty}^{\infty} dk e^{-|kz| + ikx} \sin(kx_1) \\ &= \frac{|z|}{\pi} \int_{b_-}^{b_+} dx_1 \Gamma'_{\infty}(x_1) \left\{ \frac{1}{(x-x_1)^2 + z^2} - \frac{1}{(x+x_1)^2 + z^2} \right\}, \end{aligned} \quad (I.47)$$

something which causes Eq. (I.44) to read in turn:

$$\frac{1}{\pi} \int_{b_-}^{b_+} dx_1 \Gamma'_{\infty}(x_1) \ln \left\{ \frac{x+x_1}{|x-x_1|} \right\} = \frac{\mu_0 I_0}{2} (x-b_+) \quad . \quad (I.48)$$

We note in passing that, by integrating the $a \rightarrow \infty$ version of Eq. (I.39) between, say, extreme right-hand edge b_+ and an aperture location x , $b_- < x < b_+$, inferior to it, we produce the following structure:

^{*}Symmetry of $B_3^{(\infty)}(x, z)$ vis-à-vis z follows from the mere circumstance of having its entire current source confined to the plane $z = 0$. The requisite symmetry of this flow against x has the impact then of making $B_3^{(\infty)}(x, z)$ asymmetric qua function of x . It is this latter asymmetry which amplitude constraint (I.45b) serves to assure.

$$\int_{b_-}^{b_+} dx_1 \Gamma'_\infty(x_1) \left[\ln \left\{ \frac{x+x_1}{|x-x_1|} \right\} - \ln \left\{ \frac{b_++x_1}{b_+-x_1} \right\} \right] = \frac{\pi \mu_0 I_0}{2} (x-b_+) \quad (I.49)$$

resembling (I.48) but one which, of course, is perforce satisfied identically now by form (I.42) for $\Gamma'_\infty(x)$ and is hence intrinsically incapable of shedding any light about C_∞ . Equations (I.48) and (I.49), however, are not inconsistent as regards a restricted evaluation at $x = b_+ - \epsilon$, agreement expressed by setting:

$$\int_{b_-}^{b_+} dx_1 \Gamma'_\infty(x_1) \ln \left\{ \frac{b_++x_1}{b_+-x_1} \right\} = 0 \quad (I.50)$$

whence follows the ratio:

$$C_\infty = - \frac{\mu_0 I_0}{2} \left\{ I_{(+)} / I_{(-)} \right\} \quad (I.51)$$

of integrals: *

$$I_{(+)} = \int_{b_-}^{b_+} d\lambda \frac{\lambda^2 \ln \left\{ \frac{b_++\lambda}{b_+-\lambda} \right\}}{\sqrt{(b_+^2-\lambda^2)(\lambda^2-b_-^2)}} \quad (I.52)$$

and:

$$I_{(-)} = \int_{b_-}^{b_+} d\lambda \frac{\ln \left\{ \frac{b_++\lambda}{b_+-\lambda} \right\}}{\sqrt{(b_+^2-\lambda^2)(\lambda^2-b_-^2)}} \quad (I.53)$$

* Subscript notation $I_{(\pm)}$ is well deserved by virtue of our ability to write:

$$I_{(\pm)} = \frac{1}{2} \int_{-1}^1 d\lambda \frac{(\alpha_+ + \alpha_- \lambda)^{\pm 1/2}}{\sqrt{1-\lambda^2}} \ln \left\{ \frac{b_+ + \sqrt{\alpha_+ + \alpha_- \lambda}}{b_+ - \sqrt{\alpha_+ + \alpha_- \lambda}} \right\} ,$$

with:

$$\alpha_\pm = \frac{b_+^2 \pm b_-^2}{2} ,$$

as one easily verifies. Evidently one should distinguish the present use of symbol α from its rather more ambitious rôle in Equations (I.13a) and (I.13b) foregoing.

Structures $I_{(\pm)}$ being assuredly not commonplace, it behooves us to enter at this point into the following technical aside regarding how it is that one may best evaluate them. This digression is begun by noting:

$$I_{(+)} = b_+^2 I_{(-)} - 2(b_+ - b_-) \int_0^1 d\lambda \ln \left\{ \frac{\left(\frac{b_+ + b_-}{b_+ - b_-} \right) + \lambda^2}{1 - \lambda^2} \right\} \cdot$$

$$\cdot \sqrt{\frac{\left(\frac{b_+ + b_-}{b_+ - b_-} \right) - 2 \left(\frac{b_-}{b_+ - b_-} \right) \lambda^2 - \lambda^4}{\lambda^2 + 2 \left(\frac{b_-}{b_+ - b_-} \right)}} \quad (I.54)$$

as an elementary, dimensionally balanced reduction, one that is well suited in fact to numerical quadrature provided only that $I_{(-)}$ can be reckoned as a known quantity. Such knowledge is gotten from Reference I-2, which indicates for $I_{(-)}$ a connection to a standard, elliptic-type integral, together with its subsequent approximation in closed form. We find:

$$I_{(-)} = \frac{\pi}{b_+} \int_0^{\pi/2} \frac{d\lambda}{\sqrt{1 - \left(\frac{b_-}{b_+} \right)^2 \sin^2 \lambda}}$$

$$\cong \frac{\pi}{b_+} \ln \left\{ \frac{4b_+}{\sqrt{b_+^2 - b_-^2}} \right\} \cdot \quad (I.55)$$

In this way both ingredients $I_{(\pm)}$ which Equation (I.51) stipulates for the make-up of C_∞ may be deemed to have fallen into their respective places.

Computation of C_a runs along similar, albeit rather more involved lines, complications that we handle once more in a perturbation framework. For a finite magnet cavity, and with a due regard to symmetry, the circuit counterpart

to Equation (I.44) is found to read:*

$$\int_0^x dx' B_1^{(a)}(x', -a) + \int_{-a}^0 dz B_3^{(a)}(x, z) + \int_0^\infty dz B_3^{(a)}(x, z) = -\mu_0 b_+ I_0 \quad (I.56)$$

for any x , $b_- < x < b_+$. The semi-infinite circuit itself appears on Figure I-5.

As before so also here we have dismissed out of hand the horizontal field component $B_1^{(a)}$ across the distant magnet exterior with $z \rightarrow \infty$. The lower horizontal stretch, however, runs now at a finite depression $z = -a + \epsilon$, the result of this aperture-to-circuit proximity being to endow $B_1^{(a)}$ there with more structure than merely that of the former constant $-\mu_0 I_0$. Such structure is usefully related to the spatial behavior of $B_3^{(a)}$ by writing, on the strength of the field being solenoidal, the following integral connection:

$$B_1^{(a)}(x, z) = -\mu_0 I_0 - \frac{\partial}{\partial z} \left\{ \begin{array}{l} \int_{-\infty}^x dx' B_3^{(a)}(x', z) \\ - \int_x^\infty dx' B_3^{(a)}(x', z) \end{array} \right. \quad (I.57)$$

valid across the magnet slot $0 \geq z \geq -a$. For $z \geq 0$ a corresponding relationship is gotten simply by suppressing the asymptotic contribution $-\mu_0 I_0$.

At any rate, as regards $B_3^{(a)}(x, z)$ for $z \geq 0$, we can evidently imitate the previous work (I.45a) - (I.47) by setting now:

*Circuit analogues to Equations (I.44) and (I.56) can evidently be written likewise throughout the subsequent course of missile ejection. Their right-hand members, however, begin then to depart from the values now assigned to them by virtue of current change, change indeed which proceeds in accordance with the demonstrated invariance for ψ_s . We comment here upon all this, but hasten at the same time to remark that our later work will in fact be free from any additional need for circuit evaluation. Such evaluation, leading as it does to an assessment of net ribbon current, has besides an alternative that runs via the normal gradient of magnetic stream function ψ . We shan't pause in the following to flesh out either one of these independent assessments for the maturing history of aggregate ribbon current.

Figure I-5. Ampère Circuit (----) for Normal Magnet
Depth $a < \infty$.

$$B_3^{(a)}(x, z) = \frac{z}{\pi} \int_{b_-}^{b_+} dx_1 \Gamma'_a(x_1) \left\{ \frac{1}{(x-x_1)^2 + z^2} - \frac{1}{(x+x_1)^2 + z^2} \right\}, \quad (I.58)$$

whereas, for $0 \geq z \geq -a$, a comparable spectral development involves writing first:

$$B_3^{(a)}(x, z) = \int_{-\infty}^{\infty} dk e^{ikx} \sinh(k\{a+z\}) Q_-(k), \quad (I.59)$$

with:

$$\begin{aligned} Q_-(k) &= Q_-(-k) \\ &= \frac{1}{\pi i \sinh(ka)} \int_{b_-}^{b_+} dx \sin(kx) \Gamma'_a(x), \end{aligned} \quad (I.60)$$

and then:

$$\begin{aligned} B_3^{(a)}(x, z) &= \frac{1}{\pi i} \int_{b_-}^{b_+} dx_1 \Gamma'_a(x_1) \cdot \\ &\cdot \int_{-\infty}^{\infty} dk e^{ikx} \sin(kx_1) \left\{ \frac{\sinh(k\{a+z\})}{\sinh(ka)} \right\}. \end{aligned} \quad (I.61)$$

A collaboration between Equations (I.57) and (I.61) enables one finally to write, for $0 \geq z \geq -a$:

$$\begin{aligned} B_1^{(a)}(x, z) &= -\mu_0 I_0 + \frac{1}{\pi} \int_{b_-}^{b_+} dx_1 \Gamma'_a(x_1) \cdot \\ &\cdot \int_{-\infty}^{\infty} dk e^{ikx} \sin(kx_1) \left\{ \frac{\cosh(k\{a+z\})}{\sinh(ka)} \right\}. \end{aligned} \quad (I.62)$$

With Equations (I.62), (I.61), and (I.58) we have fully assembled before us all of the ingredients sequentially spelled out along the contour sum (I.56). In particular, with a view to representation (I.58), the semi-infinite integration there of $B_3^{(a)}$ above $z = 0$ unavoidably mimics now the previous evolution (I.44) $\rightarrow$ (I.47) $\rightarrow$ (I.48), the upshot of it all being to have the latter replaced by:

$$\int_{b_-}^{b_+} dx_1 \Gamma'_a(x_1) \ln \left\{ \frac{x+x_1}{|x-x_1|} \right\} \quad (I.63)$$

$$-i \int_{b_-}^{b_+} dx_1 \Gamma'_a(x_1) \int_{-\infty}^{\infty} dk \frac{P}{k} e^{ikx} \sin(kx_1) \coth(ka) = \pi \mu_0 I_0(x-b_+) .$$

We observe now that in Equation (I.63) there occurs a kernel:

$$\Lambda \equiv \Lambda(x, x_1; a) = \int_{-\infty}^{\infty} dk \frac{P}{k} e^{ikx} \sin(kx_1) \coth(ka) \quad (I.64)$$

which sports a principal value injunction Pk^{-1} ; this latter arises most naturally in response to the first pair of spatial integrations designated beneath Equation (I.56). In any event, we note about this kernel that:

$$\Lambda(0, x_1; a) = 0 \quad (I.65)$$

whereas:

$$\begin{aligned} \frac{\partial}{\partial x} \Lambda(x, x_1; a) &= i \int_{-\infty}^{\infty} dk e^{ikx} \sin(kx_1) \coth(ka) = i \left\{ \frac{P}{x_1-x} + \frac{P}{x_1+x} \right\} \\ &+ 2i \sum_{n=1}^{\infty} \left\{ \frac{x_1-x}{(x_1-x)^2 + (2na)^2} \right. \\ &\left. + \frac{x_1+x}{(x_1+x)^2 + (2na)^2} \right\} \end{aligned} \quad (I.66)$$

by virtue of Equation (I.34) preceding. So:

$$\begin{aligned} \Lambda(x, x_1; a) &= \int_0^x dx' \frac{\partial}{\partial x'} \Lambda(x', x_1; a) = i \ln \left\{ \frac{x+x_1}{|x-x_1|} \right\} \\ &+ i \ln \prod_{n=1}^{\infty} \left\{ \frac{1 + \left(\frac{x+x_1}{2na} \right)^2}{1 + \left(\frac{x-x_1}{2na} \right)^2} \right\} . \end{aligned} \quad (I.67)$$

Kernel development (I.67) holds unrestrictedly for all $x, x' > 0$. In Equation (I.63), however, our interest becomes focused solely upon the right-hand slit $b_- < x, x' < b_+$, for which it is adequate to write:

$$\ln \prod_{n=1}^{\infty} \left\{ \frac{1 + \left(\frac{x+x_1}{2na} \right)^2}{1 + \left(\frac{x-x_1}{2na} \right)^2} \right\} \approx \frac{xx_1}{a^2} \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2 xx_1}{6a^2} . \quad (\text{I.68})$$

This simplification as to Λ allows one next to record the following approximate reduction of Equation (I.63):

$$\begin{aligned} \int_{b_-}^{b_+} dx_1 \Gamma'_a(x_1) \ln \left\{ \frac{x+x_1}{|x-x_1|} \right\} \\ + \frac{\pi^2 x}{12a^2} \int_{b_-}^{b_+} dx_1 x_1 \Gamma'_\infty(x_1) \approx \frac{\pi \mu_o I_o}{2} (x-b_+) . \end{aligned} \quad (\text{I.69})$$

Reference to definition (I.40) for renormalized current $\tilde{I}_o$, to representations (I.42) and (I.43) for Γ'_∞ and Γ'_a , and to integral abbreviations (I.52) - (I.53), followed by a specific evaluation at, say, $x = b_+ - \epsilon$, yields finally for C_a from Equation (I.43) the simple, if perhaps unsightly result:

$$C_a \approx \frac{\mu_o}{2 I_{(-)}} \left[\{ \pi b_+ - I_{(+)} \} \tilde{I}_o - \pi b_+ I_o \right] . \quad (\text{I.70})$$

We become obliged now to evaluate $\tilde{I}_o$ according to prescription (I.40). The required use there of Γ'_∞ from Equation (I.42) brings in the following integral reductions:

$$\int_{b_-}^{b_+} d\lambda \frac{\lambda^3}{\sqrt{(b_+^2 - \lambda^2)(\lambda^2 - b_-^2)}} = \frac{\pi}{4} (b_+^2 + b_-^2) \quad (\text{I.71})$$

and:

$$\int_{b_-}^{b_+} d\lambda \frac{\lambda}{\sqrt{(b_+^2 - \lambda^2)(\lambda^2 - b_-^2)}} = \frac{\pi}{2} , \quad (\text{I.72})$$

which are exact and give:

$$\tilde{I}_o = I_o \left[1 - \frac{\pi^2}{48 a^2} \left\{ b_+^2 + b_-^2 + \frac{4 C_\infty}{\mu_o I_o} \right\} \right] \quad (I.73)$$

Similar integrals are thrust upon us by the construction for ψ_s indicated beneath Equation (I.38). Now, however, we encounter elliptic forms approximated as:

$$\begin{aligned} & \int_{b_-}^{b_+} d\lambda \frac{\lambda^2}{\sqrt{(b_+^2 - \lambda^2)(\lambda^2 - b_-^2)}} \\ &= b_+ \int_0^{\pi/2} d\lambda \sqrt{1 - \left(\frac{b_+^2 - b_-^2}{b_+^2} \right) \sin^2 \lambda} \\ &\cong \frac{\pi b_+}{2} \left[1 - \left(\frac{b_+^2 - b_-^2}{4b_+^2} \right) \right], \end{aligned} \quad (I.74)$$

and:

$$\begin{aligned} & \int_{b_-}^{b_+} d\lambda \frac{1}{\sqrt{(b_+^2 - \lambda^2)(\lambda^2 - b_-^2)}} \\ &= \frac{1}{b_+} \int_0^{\pi/2} \frac{d\lambda}{\sqrt{1 - \left(\frac{b_+^2 - b_-^2}{b_+^2} \right) \sin^2 \lambda}} \\ &\cong \frac{\pi}{2b_+} \left[1 + \left(\frac{b_+^2 - b_-^2}{4b_+^2} \right) \right], \end{aligned} \quad (I.75)$$

whence at length it follows from Equations (I.38) and (I.43) combined that:

$$\begin{aligned} \psi_s \cong \mu_o I_o a - \frac{\pi \mu_o \tilde{I}_o b_+}{4} \left[1 - \left(\frac{b_+^2 - b_-^2}{4b_+^2} \right) \right] \\ - \frac{\pi C_a}{2b_+} \left[1 + \left(\frac{b_+^2 - b_-^2}{4b_+^2} \right) \right]. \end{aligned} \quad (I.76)$$

This concludes our approximate assessment of flux defect $\mu_0 I_0 a - \psi_s$.

PART D. STREAM FUNCTION ψ VERSUS MISSILE EXCURSION η -a

We face next a symmetric, three-region boundary-value problem for ψ , its geometry being schematized on Figure I-6. On this diagram, let it be noted, we revert to our earlier coordinate system, from Figure I-1, with z being taken as null along the lower, undisturbed magnet plate. Ribbon value ψ_s is now known by virtue of the work in Part C preceding, but a need arises to grapple here with altogether two connecting aperture distributions. There is of course our former Γ from Equation (I.23), but defined instead across the single perforation $-b_+ < x < b_+$. In addition to it, however, we become obliged to cope also with:

$$\begin{aligned}\theta(x) &= \theta(-x) \\ &= \psi(x, \eta),\end{aligned}\tag{I.77}$$

considered this time over all $x^2 > b_-^2$. In particular, we expect to see the inverse-square-root distance singularities of Equation (I.43) at $x = \pm b_-$ and $x = \pm b_+$ being somehow shared and split up among the respective Γ and θ values there. For us, of course, the interest in Γ and θ will amount to concern about a mere stepping stone toward the dynamical content of Equation (I.18).

As analogues to our previous spectral developments (I.20a) - (I.20c) and (I.21a) - (I.21c), we write now:

Region I: $0 \leq z \leq a$

$$\left\{ \begin{array}{l} \psi(x, z) = \int_{-\infty}^{\infty} dk e^{ikx} \mathcal{A}(k) \sinh(kz); \end{array} \right. \tag{I.78a}$$

$$\left\{ \begin{array}{l} \mathcal{A}(-k) = -\mathcal{A}(k); \end{array} \right. \tag{I.78b}$$

$$\left\{ \begin{array}{l} \mathcal{A}^*(k) = \mathcal{A}(k), \end{array} \right. \tag{I.78c}$$

exactly as before;

Region II: $a \leq z \leq \eta$

$$\left\{ \begin{array}{l} \psi(x, z) = \mu_0 I_0 a + \int_{-\infty}^{\infty} dk e^{ikx} \left\{ e^{kz} \mathcal{F}_{(+)}(k) + e^{-kz} \mathcal{F}_{(-)}(k) \right\}; \end{array} \right. \tag{I.79a}$$

$$\left\{ \begin{array}{l} \mathcal{F}_{(\pm)}(-k) = \mathcal{F}_{(\mp)}(k); \end{array} \right. \tag{I.79b}$$

$$\left\{ \begin{array}{l} \mathcal{F}_{(\pm)}^*(k) = \mathcal{F}_{(\pm)}(k), \end{array} \right. \tag{I.79c}$$

Figure I-6. Three-Region Geometry for a Mature Expulsion.

and:

Region III: $z \geq \eta$

$$\begin{cases} \psi(x, z) = \mu_0 I_0 a + \int_{-\infty}^{\infty} dk e^{ikx - |k|(z-\eta)} \mathcal{B}(k); & (I.80a) \\ \mathcal{B}(-k) = \mathcal{B}(k); & (I.80b) \\ \mathcal{B}^*(k) = \mathcal{B}(k). & (I.80c) \end{cases}$$

We retain here the amplitude symbols $\mathcal{A}$ and $\mathcal{B}$ as in our former, two-region representations (I.20a) - (I.20c) and (I.21a) - (I.21c), but, it goes without saying, their values will be forced now to depart from determinations (I.24) and (I.25) as η moves beyond its initial value a .

In fact, as a replacement to (I.24) and (I.25) we encounter now the following schedule of Fourier-transformed boundary values respectively at:

$z = a$:

$$\begin{aligned} \mathcal{A}(k) \sinh(ka) &= \mu_0 I_0 a \delta(k) + e^{ka} \mathcal{F}_{(+)}(k) + e^{-ka} \mathcal{F}_{(-)}(k) \\ &= \frac{1}{\pi} \int_0^{b_+} dx \Gamma(x) \cos(kx) \\ &\quad + \frac{\mu_0 I_0 a}{\pi} \left[\pi \delta(k) - \frac{\sin(kb_+)}{k} \right] \end{aligned} \quad (I.81)$$

and:

We stop here in mid-climb. Three plateaus have already been scaled, and two more still remain (which is to say, a completion of the section at hand, and computer programming for the missile dynamics outlined beneath Section B). But the path would seem now to be clearly mapped out, and we leave its traversal to a future navigator at another time.

REFERENCES

1. Tricomi, F. G., Integral Equations, Interscience Publishers, Inc., New York, 1957; pp. 166-197, and the original sources therein cited.
2. Gradshteyn, I. S., and I. M. Ryzhik, Table of Integrals, Series, and Products, Academic Press, New York, 1965; p. 564. (There also will one find the elliptic connections whereby are gotten the approximate forms (I.74) and (I.75).)