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MASTER

Numerical Simulations of Free-Electron Laser Oscillators*

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ABSTRACT

A numerical simulation capability has been developed to model the physics and realistic design constraints of free electron laser oscillators driven by rf linear accelerators. Two computer codes have been written, FELEX and FELP. The code FELP is a one spatial dimension code with essentially unlimited time or spectral resolution. The codes are complementary and their use is dependent upon the problem being addressed. The code FELP is used to model optical and electron micropulse structure, broadband noise, and the sideband instability. The code FELEX models accelerator generated electron beam distributions, the transport of these distributions through wigglers with misalignments and field errors, self-consistent interaction with the optical field, and propagation of the optical field through resonators with realistically modelled components. FELEX is routinely used to match resonator designs to the optical parameters of the electron beam, and used to investigate the physics of 3-D micropulse effects. Some details of the codes will be presented along with various examples of simulation results.

1. Introduction

Numerical simulation methods have proven to be very useful for the analysis and design of free-electron lasers. In general, the only analytically-tractable problem in this field has been the small-signal gain for untapered-wiggler devices. Analytical formulas are available for the low-gain [1] and high-gain [2], [3] Compton-regime cases for perfect electron beam quality. Gain degradation due to finite energy spread and transverse emittance can only be handled numerically [4]. In general, the small-signal gain of an arbitrarily-tapered wiggler device is not analytically known (except for an optical klystron [5], [6] and a linear taper in $k_w = 2\pi/\lambda_w$, where λ_w is the local wiggler wavelength [7]), even in the low gain regime. High gain effects, and the effects of imperfect electron beam quality in both low and high gain regimes, can only be handled numerically. All of the analytic formulas assume an infinitely long electron pulse, reduction of the

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gain due to short pulse effects has been numerically, but not analytically evaluated. Similarly, gain saturation effects, which involve the nonlinear coupling of the light with the electron-beam, have primarily been evaluated using numerical methods despite the pioneering analytical treatment of Kroll, Morton, and Rosenbluth [8].

In this paper, we will briefly describe two computer codes, FELP and FELEX, which have been developed at Los Alamos for the analysis and design of free-electron lasers. FELP is a 1-D finite pulse code which can be used to study finite bandwidth problems. FELEX is a 3-D code which has been primarily used to study the effects of the transverse variation of the properties of the light and electron beams upon FEL performance. Examples of problems resolved by using each code will be given. The problems all relate to FEL oscillators, since the Los Alamos FEL program has been primarily concerned with such devices, but treatment of FEL amplifier problems with these codes is straightforward.

2. FELP and 1-D Problems

FELP is a 1-D time-dependent FEL simulation code. Physical phenomena such as the start-up time of an oscillator and its small-signal gain can be modeled as well as optical cavity length detuning (desynchronization) effects, i.e. the variation of the laser output power and spectrum with cavity length. The mathematical model is due originally to Colson and Ride [9] and has been frequently discussed in the literature [10]-[12]. FELP can also be used in the periodic-boundary-condition approximation which was invented by Colson [13] to treat optical spectral phenomena for problems in which the electron pulse is many slippage lengths long. In both the finite-pulse and periodic-boundary condition versions, the transverse variation of optical and electron-beam properties is ignored. The on-axis variation of the intensity due to the spatial properties of a resonator mode [14] is however included, and transverse emittance effects can be modeled as an effective energy spread [15].

Recently [16] FELP was modified to treat the problem of suppressing sidebands in the Boerig "burst mode" FEL oscillator experiment by including a pair of diffraction gratings in the ring optical resonator. Table I lists some of the parameter values for this planned experiment for which calculations were performed. Figure 1 is a schematic of a grating rhomb which illustrates how this device acts to suppress sidebands. In an FEL of two optical wavelengths λ_1 and λ_2 are simultaneously present in the light incident

upon the first grating, with $\lambda_s > \lambda_0$, the optical path length through the device is greater for λ_s . Hence, light at the sideband wavelength will be slightly delayed and will not overlap with the next electron pulse at the entrance to the wiggler and, therefore, will not experience gain. The cavity length is adjusted to make the round-trip period of light at wavelength λ_0 , which corresponds to the desired operating wavelength, equal to the interval between successive electron pulses from the accelerator (exact synchronism). The mathematical model in FELP for the effect of the grating rhomb is a frequency-dependent phase shift [16].

Figure 2 shows that the rhomb is indeed effective at suppressing sidebands. The upper graph shows the spectrum of the optical pulse after 200 passes through the resonator with no rhomb; the lower graph shows the optical spectrum with a grating rhomb in the cavity. This calculation was done with a rhomb dispersion such that light at $\lambda_s = 1.02 \lambda_0$ would have a path length larger by 1 cm than light at λ_0 . From Table I, the electron pulse length is 0.6 cm, so a 1 cm delay would shift light at λ_s completely out of overlap with the next electron pulse. The net result of these two calculations is that, with no rhomb, the extraction efficiency with sidebands present reaches about 2.5% whereas the extraction efficiency with the rhomb reaches about 5.5% in steady state.

The FEL interaction produces a phase shift of the optical pulse, the magnitude of which is proportional to the gain. For large gain, the rhomb can compress the optical pulse and reduce the extraction efficiency if the rhomb dispersion is not large enough. Details and parametric studies can be found in [16].

3. FELIX And 3-D Problems

FELIX [17] is a 3-D FEL oscillator simulation code which is used to examine transverse optical mode structure effects and optical interactions with finite-transverse emittance electron-beams. Within the wiggler magnet, 3-D trajectories of individual simulation electrons are followed. The optical field is driven by macroscopic currents constructed from the electron phase space. The electron dynamics includes beams with arbitrary phase space distributions. Optical gain and refractive effects are self-consistently calculated as the light interacts with the electrons inside the wiggler. At the exit of the wiggler, the optical field is propagated through the optical

resonator and back to the entrance of the wiggler where it interacts with the next electron pulse: the steady-state reached after many passes therefore represents a complete self-consistent FEL oscillator solution. The code can be run in three different modes: single wave-front (no pulse effects, cw solution); multiple wave-fronts with periodic-boundary-conditions [13], [17]; and multiple wavefronts with finite optical and electron pulses.

Figure 3 exhibits the equation of motion for the optical electric field which is numerically solved in FELEX. Plane polarization is assumed for light of carrier wavevector $k_0 = 2\pi/\lambda_0$. Also shown in Fig. 3 is the form of the magnetic scalar potential χ_m for a curved-pole-face wiggler [18]. The rectangular coordinate system takes the xz plane to be the plane of the electrons' wiggler motion so that a flat-pole-face wiggler would be represented by $k_{wy} = 0$ and $k_{wx} = k_w(z)$. The current density J that appears in the source term S is obtained by numerical solution of the electrons' equations of motion which are exhibited in Fig. 4. Here γmc^2 is the total energy of an electron, β_x (β_y) is the scaled transverse x-velocity v_x/c (y-velocity v_y/c), and B_i^e are the components of an externally-imposed static magnetic field. The quantity A_s which appears in the equation for ψ is defined by $A_s = (ef_0/mc^2 k_0)$ and represents the dimensionless vector potential of the optical field in the same way that A_w , defined in Fig. 3, is the dimensionless vector potential of the wiggler magnetic field. The electron equations are generalizations to 3-D of the usual 1-D equations [9], and the 1-D optical evolution equation can be deduced from Fig. 3 by projection of the total electric field onto one mode [14].

An example of a problem in which the light is substantially distorted from a single transverse mode, and therefore transverse effects are very important, is given in [19]. This is a 3-D treatment of the same problem that was discussed in [16]. Fig. 5 is a schematic of the optical resonator for the Boeing "burst mode" experiment. From Table I, one notes that the saturated gain is expected to be in the range 15% - 30% per pass. Fig. 6 is a plot of the steady state amplitude vs. transverse position at the entrance to the wiggler for the resonator of Fig. 5 with 25% loss per pass. The wiggler is tapered and described in [19]. The large single pass gain distorts the mode from the shape of the lowest order empty-cavity Gaussian mode. The subsequent

resonator mirrors further distort the shape and produce a very large spot at the wiggler entrance: this large spot is strongly vignetted by the wiggler itself (note the truncated appearance in Fig. 6) and is much larger than the electron-beam. The result of the poor transverse spatial overlap between the light and electron beams is a low saturated power and extraction efficiency.

The mode shape shown in Fig. 6 was calculated for resonator properties which would produce a Gaussian mode in an unloaded cavity (electron-beam not present). If the radius of curvature of mirror 1 is adjusted to give the same spot size for the loaded cavity on mirror 2 as the empty-cavity mode, and the radius of curvature of mirror 2 is adjusted to give the same wavefront curvature for the loaded cavity on hyperboloid 2 as the empty-cavity mode (a procedure we refer to as mode-matching), then the steady-state mode profile at the wiggler entrance is as shown in Fig. 7. A Gaussian-like shape has been recovered for the loaded-cavity mode which leads to a much better transverse overlap with the electron-beam and, therefore, higher saturated power and extraction efficiency. For this example, proper mode matching increased the extraction efficiency by 35% over the unmatched case. Further details can be found in [19] which also discusses effects due to finite optical and electron pulses.

4. Other Applications of FELIX

Another application of FELIX is to integrated numerical modeling of rf-linac/FEL systems, as discussed in [20]. The accelerator code PARMELA is used to numerically generate an electron pulse from a complete model of the linac and beam transport system. That electron pulse is then used FELIX to derive the expected FEL performance. In this way, a fully integrated numerical model of the complete FEL system is constructed and used for the analysis of present experiments and the design of new devices.

Another development of FELIX is an extension to model coherent spontaneous emission of harmonics in an FEL oscillator operating at high power at the fundamental wavelength with a plane polarized wiggler [21]. Fig. 8 shows calculated transverse mode profiles for the second and third harmonic at the exit of the wiggler in the Stanford MK III FEL. Coherent spontaneous emission for harmonic number up to 11 has been calculated in 3-D for this device. The results are in good agreement with measurements [22].

Yet another modification to FELIX has been implemented to calculate the effects of wiggler field errors (due to manufacturing flaws in the magnetic material or imperfect alignment during construction) upon the performance of

FEL devices [23]. Fig. 9 is a contour plot of $\log(P(\sigma_b, \Delta x)/P_0)$, where σ_b is the rms field error level, Δx is the electron-beam position measurement error, and P is the performance (small-signal gain) of an XUV SASE amplifier in the presence of errors relative to that of a perfect wiggler P_0 [24]. These studies provide important wiggler tolerances which must be achieved to avoid serious loss of FEL performance.

Since FELEX is fully 3-D, many misalignment problems can be studied, i.e., the effects upon FEL performance of misalignments of the optical axis with the wiggler axis or the electron-beam. A study of such alignment problems has shown [25] experimentally and theoretically that it is possible to excite the second cavity mode under some conditions. Fig. 10 shows experimental and theoretical data from FELEX simulations of the mode shape for different axis offsets for the Los Alamos $10\mu\text{m}$ FEL oscillator.

Finally, we mention briefly another recent addition to FELEX to extend its FEL modeling capability to still more realistic conditions. FELEX models optical components of a resonator as thin lenses: it is now possible to include imperfections of such components by adding the appropriate phase aberrations to the optical field. A comparison of the calculated round-trip loss for the Boeing burst-mode resonator, Fig. 1, using measured mirror surface imperfections showed excellent agreement between results obtained with FELEX and with the completely independently developed physical optics code GLAD [26]. These were empty-cavity calculations; work is in progress to determine the effects of mirror aberrations upon FEL performance.

5 Summary

Two FEL simulation codes have been developed at Los Alamos to model FEL behavior: the 1-D code FELP is used to study the interaction of finite optical and electron micropulses and includes the effects of slippage, oscillator start-up and evolution from spontaneous emission, arbitrarily-tapered wigglers, electron energy spread, sideband generation, and wavelength-dependent optical filters and grating rhombs. The 3-D code FELEX treats FEL problems with transverse variations of the optical and electron beams: it has been used to study the effects of optical mode distortion due to gain and refractive effects of the electron beam as well as apertures and optical elements of the resonator upon FEL performance. FELEX has been extended to use numerically generated electron beams for integrated modeling studies, to calculate coherent spontaneous emission of harmonics in 3-D, and to study

effects of misalignments. Future extensions will include simulations of MOPA systems. These two codes provide a powerful capability for modeling the physics and realistic design constraints of free-electron lasers.

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Figure Captions

- Fig. 1: Schematic of a grating rhomb illustrating its dispersive delay characteristics.
- Fig. 2: Steady-state calculated optical spectrum for Boeing burst mode experiment without grating rhomb (upper) and with grating rhomb (lower).
- Fig. 3: Optical electric field equation solved in FELEX.
- Fig. 4: Electron equations of motion solved in FELEX.
- Fig. 5: Schematic of Boeing burst mode ring resonator.
- Fig. 6: Transverse variation of optical field amplitude at wiggler entrance without mode-matching.
- Fig. 7: Transverse variation of optical field amplitude at wiggler entrance after mode-matching.
- Fig. 8a: Transverse variation of the amplitude of the second harmonic field at wiggler exit for MK. III FEL
- Fig. 8b: Transverse variation of the amplitude of the third harmonic field at wiggler exit for MK. III FEL.
- Fig. 9: Contour plot of FEL gain vs. rms wiggler field error and beam-position error.
- Fig. 10: Experimental and theoretical mode distortion due to displacement of wiggler and resonator axes.

Table I: Burst Mode Parameters

Electron Beam Parameters

current	300.00 A
gamma	210.50
energy spread	0.50 percent
pulse length	0.60 cm
pulse spatial profile	parabolic

Wiggler Parameters

length	560.00 cm
wavelength	2.18 cm
magnetic field at entrance	8.70 kG
constant section	50.00 cm
energy taper	12.50 percent

Optical Field Parameters

wavelength	0.6328 μm
Rayleigh length	240.00 cm
cavity loss	15, 20, 25, and 30 percent

Fig. 1: Schematic of a grating rhomb illustrating its dispersive delay characteristics.

Fig. 2: Steady-state calculated optical spectrum for Boeing burst mode experiment without grating rhomb (upper) and with grating rhomb (lower).

$$E_x(\vec{r}, t) = E_o(\vec{r}) \cos [k_o z - \omega t + \theta_o(\vec{r})]$$

$$\mathbf{E} = E_o(\vec{r}) \exp[i\theta_o(\vec{r})]$$

$$2ik_o \frac{\partial \mathbf{E}}{\partial z} + \frac{\partial^2 \mathbf{E}}{\partial x^2} + \frac{\partial^2 \mathbf{E}}{\partial y^2} = S(\vec{r})$$

$$S(\vec{r}) = \left\langle -\frac{4\pi k_o}{c} J \frac{a_w(\vec{r})}{\gamma} F_\mu e^{-i(\psi)} \right\rangle$$

where,

$$F_\mu = J_0(\mu) - J_1(\mu) \quad , \quad \mu = \frac{k_o a_w^2(\vec{r})}{8\gamma^2 k_w} \quad , \quad a_w(\vec{r}) = (eB_y(\vec{r})) / (mc^2 k_w)$$

$$\chi_m = -\frac{B(z)}{k_{wy}} \cosh(k_{wx}x) \sinh(k_{wy}y) \sin \left[\int k_w(z) dz \right]$$

$$\vec{H}_w = -\nabla \chi_m \quad , \quad k_w^2 = k_{wx}^2 + k_{wy}^2$$

Fig. 3: Optical electric field equation solved in FLEX.

$$\frac{d\gamma}{d(ct)} = -\frac{e}{mc^2} \frac{a_w(\vec{r})}{2\gamma} F_\mu E_o(\vec{r}) \sin [\psi + \theta_o(\vec{r})]$$

$$\frac{d\psi}{d(ct)} = k_w(z) - \frac{k_o}{2\gamma^2} \left[1 + \frac{a_w(\vec{r})^2}{2} + (\gamma\beta_x)^2 + (\gamma\beta_y)^2 - a_w a_o \cos [\psi + \theta_o] \right]$$

$$\frac{d(\gamma\beta_x)}{d(ct)} = -\frac{a_w^2 k_{wx}^2}{2\gamma} x - \frac{e}{mc^2} (\beta_y H_x^o - \beta_x H_y^o)$$

$$\frac{d(\gamma\beta_y)}{d(ct)} = -\frac{a_w^2 k_{wy}^2}{2\gamma} y - \frac{e}{mc^2} (\beta_x H_x^o - \beta_y H_y^o)$$

$$\frac{dx}{d(ct)} = \beta_x \quad , \quad \frac{dy}{d(ct)} = \beta_y$$

Fig. 4: Electron equations of motion solved in FELEX.

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