MASTER

UTILIZATION OF CONTINUOUS SLOWING DOWN THEORY IN
CONJUNCTION WITH CONVENTIONAL SPATIAL CALCULATIONS

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ABSTRACT

The separable kernel analogy is used to develop a mechanism to utilize generalized continuous slowing down theory in conventional types of methods for calculating space-dependent spectra. A particular transport code, DTF-IV is modified to operate efficiently based on continuous slowing down theory. It is demonstrated that major reductions in storage requirements are made possible. The modified code is applied to a set of idealized test problems and adapted benchmark problems that deals with the types of difficulties typically encountered in fast reactor analysis. In all cases, good results are achieved both for spectra and for collapsed few-group constants.

INTRODUCTION

Continuous slowing down theory has proved to be of considerable utility for problems of both thermal and fast reactors. The theory has been applied to problems with space dependence and time dependence.

It will be desirable to incorporate this type of theory into conventional space dependent multigroup computer codes in a manner which capitalizes on all desirable mathematical features of the theory. Previously, multigroup cross section collapse codes, of which 1DX¹ is representative, have required very large increases in data storage commitments as the number of energy groups and the number of materials used increases. Consequently, such codes have used only moderate numbers of groups.

In this paper, a multigroup scheme will be presented which used the CSDT and a separable slowing down kernel in a manner that minimizes the storage requirement while maintaining excellent accuracy. Using these new methods, we demonstrate storage savings of up to 60% in some typical fast reactor problems employing as few as 29 energy groups. When these procedures are applied to problems involving 250 energy groups, (240 is the number of groups in the ANS 19.1 Supergroup standard), the storage savings approaches 96%. Before presenting the new methods, a review of earlier work is in order.

Continuous Slowing Down Theory (CSDT) as described by Weinberg and Wigner² and Greuling and Goertzel³ was developed originally for elastic slowing down of neutrons as applied to thermal reactors. Driscoll and Kaplan⁴ applied CSDT to fast reactors using parameters based on correlations.

Segev⁵ used an approach in which the collision density was expanded in a two term Taylor series (in lethargy or energy) to determine the slowing down parameters. Stacey⁶ separated the elastic and inelastic scattering and applied CSDT to only the elastic scattering. Walti and Grossman⁷ used a synthetic kernel method which Walti later extended to gas cooled reactors. Kamei⁸ proposed a model in which the inelastic scattering was produced in an analytical manner and the slowing down integral resulted when a ratio of the spectrum and a reference spectrum was expanded in a Taylor series.

At the Rensselaer Polytechnic Institute, several researchers have developed methods and models to improve the application of Continuous Slowing Down Theory to fast reactors. Dunn and Becker⁹ produced analytical expressions for fast reactor spectra including the inelastic scattering and neutron resonances in fast reactor materials. In their analysis, the Grueling and Goertzel equations were still employed but two slowing down parameters $\xi(x)$ and γ (gamma) were redefined. Dunn and Becker first solved a zero absorption reference case, to find the slowing down density $q(u)$ at lethargy u . The parameter $\xi(u)$ could be obtained from expressions resulting from the slowing down density and the flux. The other slowing down parameter, γ , was determined through an ad-hoc definition.

Burns and Becker¹⁰ applied these procedures to single material problems involving substantial absorption. Burns¹¹ went on to improve the procedure for calculating γ . Parvez¹² developed a method for calculating infinite medium multigroup (rather than continuous) fluxes and then using the relations between flux and slowing down density to obtain ξ and γ .

Sierra¹³ applied the work of Dunn and Becker, and Burns when he calculated the spectra in his analysis of LMFBR depletion processes involving core compositions. Epstein, Biffer, and Becker¹⁴ utilized CSDT methods coupled with integral transport methods in analyzing source free multiregion problems. Biffer¹⁵ extended this work to the depletion process. Badruzzaman¹⁶ applied this CSDT-Integral Transport methodology to the analysis of space and exposure dependent fast reactor spectra, and to fast reactor fuel management problems

Becker and Burns¹⁰ showed that continuous slowing down theory of the Greuling-Goertzel form is equivalent to slowing down by a separable kernel and vice versa. They also showed how a separable kernel corresponding to generalized continuous slowing down theory including effects of inelastic scattering could be constructed. This separable kernel analogy was utilized to construct efficient space-dependent procedures using integral transport theory.¹⁴⁻¹⁶

Most space-dependent spectrum calculations are performed with multigroup diffusion codes and with multigroup transport codes based on the differential (rather than integral) form of the transport equation. This paper explores the prospects for utilizing the separable kernel analogy to construct efficient procedures together with conventional multigroup techniques. It will be demonstrated that major reductions in data storage requirements can be achieved without significant sacrifice in accuracy.

THEORY

In the analysis that follows, the technique of Dunn and Becker⁹ and Becker and Burns¹⁰ will be used in obtaining a relationship for the infinite medium moderating parameters ξ and γ . The value of ξ is chosen

by Dunn and Becker so as to give the correct solution to an easily solved reference slowing down problem in which there is zero absorption, thereby eliminating γ . Starting with the integral equation for the slowing down density $q(u)$ with limits extending from $u - \ln(\frac{1}{\alpha})$ to 0 and from $u + \ln(\frac{1}{\alpha})$ to ∞ , i.e.,

$$q(u) = \int_0^u du' \int_0^\infty du'' \Sigma_s(u') \phi(u') K(u', u'') \quad (1)$$

where K is the slowing down kernel. The flux is assumed to be given by the Wigner approximation

$$\phi(u) = \frac{S(u)}{\Sigma_t(u)} + \frac{q(u)}{\xi \Sigma_s(u)} \quad (2)$$

where $S(u)$ represents the fission source or external source. In the above expression, the slowing down kernel K can be thought of in terms of the scattering transference kernel p ,

$$K(u', u) = \int_0^\infty p(u', u'') du'' \quad (3)$$

Becker and Burns¹⁰ then introduced the following separable kernel

$$p(u', u) = f(u') h(u) \quad (4)$$

in order to represent the scattering transference probability as the product of two terms; one dependent only on u' , the other only on u .

Using Eq. 4, an expression similar to the Greuling and Goertzel approximation can be derived as follows:

$$\gamma(u) \frac{\partial q(u)}{\partial u} + q(u) = \xi(u) \Sigma_s(u) \phi(u) \quad (5)$$

with parameters given by,

$$\frac{1}{\xi(u)} = \frac{1}{f(u)} \frac{\partial G(u)}{\partial u} \quad (6)$$

$$\frac{1}{\gamma(u)} = \frac{1}{G(u)} \frac{\partial G(u)}{\partial u} \quad (7)$$

where,

$$\frac{1}{G(u)} = \int_u^{\infty} du' h(u') \quad (8)$$

By noting, on the grounds of conservation, that

$$K(u,u) = \int_u^{\infty} p(u,u') = 1, \quad (9)$$

they find that,

$$G(u) = f(u) \quad (10)$$

and that,

$$\gamma(u) = \xi(u) \quad (11)$$

Alternate formulation can admit γ not equal to ξ .

The present analysis makes use of the inverse relationship between the separable kernel parameters f and h , and the parameter ξ . When Eq. (10) is substituted in Eq. (8) and the result differentiated, the following relationship is obtained:

$$h(u) = \frac{1}{f^2(u)} \frac{df}{du} \quad (12)$$

When this result is substituted in Eq. (6) it is found that

$$\frac{1}{\xi(u)} = f(u) h(u). \quad (13)$$

This relationship forms the basis for a link between CSDT and the separable kernel

Combining Eqs. (12) and (13) yields,

$$\xi(u) = \frac{f(u)}{df/du} \quad , \quad (14)$$

which when solved directly gives,

$$f(u) = A \exp \left[\int_a^u \frac{du'}{\xi(u')} \right] \quad (15)$$

and corresponding result for $h(u)$, i.e.,

$$h(u) = \frac{1}{A\xi(u)} \exp \left[- \int_a^u \frac{du'}{\xi(u')} \right] \quad (16)$$

where A is an arbitrary constant and a is an arbitrary limit. Equations (15) and (16) provide the direct coupling between the separable kernel and the slowing down parameter ξ . These relationships will now be incorporated in the multigroup transport formalisms.

Consider the multigroup transport equation with isotropic scattering

$$\Omega \cdot \nabla \phi^g + \Sigma_t^g \phi^g = \sum_{g'}^g \Sigma_s^{g' \rightarrow g} \phi_0^{g'} + S^g \quad (17)$$

where zero subscript denotes scatter flux. The summation can be rearranged as follows;

$$\sum_{g'}^g \Sigma_s^{g' \rightarrow g} \phi_0^{g'} = \Sigma_s^{g \rightarrow g} \phi_0^g + \sum_{g' \neq g} \Sigma_s^{g' \rightarrow g} \phi_0^{g'} \quad (18)$$

Expanding as a function of lethargy leads to

$$\Sigma_s^{g' \rightarrow g} \phi_0^{g'} = \int du \int du' \Sigma_s(u') \phi(u') P(u', u) \quad (19)$$

where from Eq. (4)

$$P(u', u) = f(u')h(u) \quad (20)$$

If Eq. (20) is substituted into Eq. (19)

$$\Sigma_s^{g' \rightarrow g} \phi^{g'} = \int_{u_{g'-1}}^{u_{g'}} du h(u) \int_{u_{g'-1}}^{u_{g'}} du' f(u') \Sigma_s(u') \phi(u') \quad (21)$$

It can be seen that

$$\Sigma_s^{g'} \phi^{g'} = \int_{u_{g'-1}}^{u_{g'}} du' \Sigma_s(u') \phi(u') \quad (22)$$

and,

$$f_s^{g'} \Sigma_s^{g'} \phi^{g'} = \int_{u_{g'-1}}^{u_{g'}} du' f(u') \Sigma_s(u') \phi(u') \quad (23)$$

Dividing Eq. (23) by Eq. (22) yields

$$f_s^{g'} = \frac{\int_{u_{g'-1}}^{u_{g'}} du' f(u') \Sigma_s(u') \phi(u')}{\int_{u_{g'-1}}^{u_{g'}} du' \Sigma_s(u') \phi(u')} \quad (24)$$

Now, if $\phi \sim \frac{1}{\Sigma_s}$, i.e., if the scattering collision density is constant,

then,

$$f_s^{g'} = \frac{\int_{u_{g'-1}}^{u_{g'}} du' f(u')}{\int_{u_{g'-1}}^{u_{g'}} du'} = \frac{1}{u_{g'} - u_{g'-1}} \int_{u_{g'-1}}^{u_{g'}} du' f(u') \quad (25)$$

From Eq. 15

$$f(u) = A \exp \int_a^u \frac{du'}{\xi(u')} \quad (26)$$

Substituting Eq. (26) into Eq. (25) evolves,

$$f_s^{g'} = \frac{A}{u_{g'} - u_{g'-1}} \int_{u_{g'-1}}^{u_{g'}} du \exp \int_a^u \frac{du'}{\xi(u')} \quad (27)$$

Equation (27) can be rewritten as,

$$f^{g'} = \frac{A}{u_{g'} - u_{g'-1}} \int_{u_{g'-1}}^{u_{g'}} du e^{\int_a^u \frac{du'}{\xi(u')}} e^{\int_{u_{g'-1}}^u \frac{du'}{\xi(u')}} \quad (28)$$

Assuming $\frac{1}{\xi(u)}$ is fairly linear over the energy group width and letting $a = 0$, the integral $\int_0^{u_{g'-1}}$ can be represented by sums of integrals over the lethargy group widths. Typically,

$$\int_{u_{\text{top}}}^{u_{\text{bottom}}} \frac{du}{\xi(u)} = \Delta u \left[\frac{1}{\xi(u_{\text{top}})} \right] + \frac{\Delta u}{2} \left[\frac{1}{\xi(u_{\text{top}})} - \frac{1}{\xi(u_{\text{bottom}})} \right] \quad (29)$$

which simplifies to,

$$\int_{u_{\text{top}}}^{u_{\text{bottom}}} \frac{du}{\xi(u)} = \Delta u_{g'} \left(\frac{\xi_{\text{bottom}} + \xi_{\text{top}}}{2 \xi_{\text{bottom}} \xi_{\text{top}}} \right) \quad (30)$$

However, $\frac{\xi_{\text{bottom}} + \xi_{\text{top}}}{2 \xi_{\text{bottom}} \xi_{\text{top}}}$ represents the harmonic mean for $\frac{1}{\xi_{g'}}$ in terms of $\xi(u_{\text{bottom}})$ and $\xi(u_{\text{top}})$. Thus,

$$\int_{u_{\text{top}}}^{u_{\text{bottom}}} \frac{du}{\xi(u)} = \frac{\Delta u_{g'}}{\xi_{g'}} \quad (31)$$

This implies that Eq. (28) can be modified to

$$f^{g'} = \frac{A}{u_{g'} - u_{g'-1}} e^{\sum_{g''=1}^{g'-1} \frac{\Delta u_{g''}}{\xi_{g''}}} \int_{u_{g'-1}}^{u_{g'}} du e^{\int_{u_{g'-1}}^u \frac{du'}{\xi(u')}} \quad (32)$$

$$\text{If } B^{g'-1} = \sum_{g''=1}^{g'-1} \frac{\Delta u_{g''}}{\xi_{g''}} \quad (33)$$

then,

$$f^{g'} = \frac{A}{u_{g'} - u_{g'-1}} e^{B^{g'-1}} \int_{u_{g'-1}}^{u_{g'}} du e^{\int_{u_{g'-1}}^u \frac{du'}{\xi(u')}} \quad (34)$$

Equation (34) can be evaluated using numerical integration or through direct integration and evaluation of error functions.

Having defined f^g , Eq. (17) can be rewritten as,

$$\Omega \cdot \nabla \phi^g + \Sigma_t^g \phi^g - \Sigma^{g \rightarrow g} \phi_o^g = h^g \sum_{g'=1}^{g-1} f^{g'} \Sigma_s^{g'} \phi_o^{g'} + S^g \quad (35)$$

Letting $\psi^{g-1} = \sum_{g'=1}^{g-1} f^{g'} \Sigma_s^{g'} \phi_o^{g'}$, Eq. (35) becomes

$$\Omega \cdot \nabla \phi^g + \Sigma_R^g \phi_o^g = h^g \psi^{g-1} + S^g \quad (36)$$

where the removal cross section $\Sigma_R^g = \Sigma_t^g - \Sigma^{g \rightarrow g}$.

Equation (36) represents the multigroup transport equation in the separable kernel sense. The parameters h^g and $\Sigma^{g \rightarrow g}$ will be developed further.

In an attempt to compensate for any minor errors in the present analysis, one of these parameters will take on its exact or analytical value while the other will be adjusted to reproduce an infinite medium (zero leakage) multigroup representation of ϕ^g rather than the continuous function $\phi(u)$. The multigroup flux must first be calculated using the standard slowing down kernel,

$$\phi^g = \frac{\sum_{g' < g} \Sigma^{g' \rightarrow g} \phi^{g'} + S^g}{\Sigma_t^g - \Sigma^{g \rightarrow g}} \quad (37)$$

Since only one adjustment is needed per group, the actual value of $\Sigma^{g \rightarrow g}$ is used and adjustment is made on the h^g parameter. This procedure constitutes a mild departure from a true separable kernel. Thus,

$$h_{\text{adjusted}}^g = \frac{\phi^g(\Sigma_t^g - \Sigma_{\text{actual}}^{g \rightarrow g}) - S^g}{\psi^{g-1}} \quad (38)$$

With the parameters suitably explained, Eq. (36) is illustrative of the motivation for development of the present separable kernel methods. The flux in Eq. (36) can be seen to depend on two flux-like quantities (ϕ_0 and ψ^{g-1}) and a linear array of cross sections. This dependence is much different from multigroup equations using standard slowing down kernels in that a large matrix of cross section transfer probabilities is not required for calculation of the downscatter source to the group.

To review, the downscatter source to any group g is found by multiplying h^g (adjusted previously to reproduce the infinite medium flux) and ψ^{g-1} . The ψ^{g-1} term is evaluated as follows. Using the flux found in group 1, ψ^1 is set equal to

$$\psi^1 = f^1 \Sigma_s^1 \phi^1 \quad (39)$$

and used in calculation of ϕ^2 . Once ϕ^2 is available, ψ^2 can be calculated from

$$\psi^2 = f^2 \Sigma_s^2 \phi^2 + \psi^1 \quad (40)$$

In this manner, the recursion relationship

$$\psi^g = f^g \Sigma_s^g \phi^g + \psi^{g-1} \quad (41)$$

can always be used to calculate ψ^g once ϕ^g has been determined. It thus is found to be sufficient to use a cross section set for each material consisting of six elements per group regardless of the number of groups employed. The first four of the six elements are the normal values of Σ_a , $\nu\Sigma_f$, Σ_t , and $\Sigma_{actual}^{g \rightarrow g}$. The final two elements per group are $h_{adjusted}^g$ and $f_s^g \Sigma_s^g$.

Use of this six element cross section set implies considerable computer storage savings in problems using large numbers of groups. Consider for example, a 250 group data set, which contains 63250 entries for each material while the six element cross section set would only require 1500 entries for material.

An additional implication of Eq. (36) is the potential for further storage savings. Once used to update the parameter ψ , the values of ϕ^g are no longer required and may be written to external file until needed in the next outer iteration. Like the flux ϕ^g , the current J can also be written to external file. Thus, another savings mechanism is possible.

Implantation of the Analysis

The aim of this section is to incorporate the analysis of the previous section in a computer code which solves the multigroup form of the neutron transport equation in one dimensional geometry. DTF-IV¹⁷ was selected for this effort since it is in widespread use, and was already operational at RPI. In addition, a number of other widely used codes such as LDX¹, ANISN¹⁸, and 3DB¹⁹, have data organization very similar to that of DTF-IV. The experiment to be attempted involves modifying DTF-IV to implant the separable kernel, running DTF-IV and the separable kernel modified DTF-IV, and comparing results.

Like many multigroup codes, DTF-IV employs two major subroutines; one called OUTER, to calculate the source of neutrons into a given energy group, and one called INNER, which solves the space dependent problem in one group. Once INNER convergence has been obtained in the first group, calculation proceeds downward in energy to each group, in turn, until converged INNER solutions are obtained in all groups. The downscattering portion of the source to each succeeding group is recomputed as soon as new flux information is available.

The separable kernel parameters f and h will be used in the separable kernel modified DTF-IV to calculate the downscattering terms, f and h will be calculated using an auxiliary program called IDAPRIME.

IDAPRIME first reads a standard DTF-IV format cross section set for the material of interest. This set is dimensioned as a $(G) \times (G+3)$ array where " G " represents the total number of energy groups. For each energy group, the following table of specific cross sections are included;

$$\Sigma_a^g, \nu\Sigma_f^g, \Sigma_t^g, \Sigma^{g \rightarrow g}, \Sigma^{g-1 \rightarrow g}, \dots \Sigma^{1 \rightarrow g} \quad (42)$$

where the superscript g is indicative of the specific energy group.

Next, a source spectrum (usually the material independent χ^g ($g=1, G$) fission spectrum) and the energy group boundaries are read. The cross section input set is multiplied by the number density (if in microscopic form) to convert to macroscopic form and various arrays are initialized.

The infinite medium flux in each group is calculated using the standard cross section set and the formula,

$$\phi_g = \frac{\sum_{g'=1}^{g-1} \Sigma^{g' \rightarrow g} \phi^{g'} + S^g}{\Sigma_t^g - \Sigma^{g \rightarrow g}} \quad (43)$$

With the flux information available, the slowing down density at the bottom of each group can be found from

$$SDD(g) = q(g) = \sum_{g'} \left(\sum_{g' < g''} \Sigma^{g' \rightarrow g''} \right) \phi^{g'} \quad (44)$$

Proceeding, the values of ξ_{top}^g are calculated using the slowing down density, the flux and the χ^g source.

$$\xi_{bot}^g = \frac{SDD^g}{\Sigma_t^g \phi^g - S^g} \quad (45)$$

$$\xi_{top}^g = \frac{SDD^{g-1}}{\Sigma_t^g \phi^g - S^g} \quad (46)$$

Knowing the energy boundaries from input data, the lethargy boundaries are calculated according to

$$u_{top}^g = \ln \left(\frac{1.E7}{E_g} \right) \quad (47)$$

$$u_{bot}^g = \ln \left(\frac{1.E7}{E_{g+1}} \right) \quad (48)$$

$$du = \ln \left(\frac{E_g}{E_{g+1}} \right) \quad (49)$$

where E_g denotes the energy value at the top of the group g .

Referring to the Analysis section, Eq. (33) is repeated here and is next to be calculated in IDAPRIME.

$$B^g = \sum_g \frac{du^{g-1}}{\xi^{g-1}} \quad (50)$$

Following the analysis in the previous chapter, values of u_{top} , u_{bot} , ξ_{top} , and ξ_{bot} will be used to find the factors in the equation for f which depend upon the discrete form of these parameters; this factor in the f equation is called FPR.

To compute FPR, a function subprogram was established whereby

$$FCALC = EXP \left[\left(\frac{u - u_{top}}{2} \right) \left(\frac{2}{\xi_{top}} + \frac{(u - u_{top}) \left(\frac{1}{\xi_{bot}} - \frac{1}{\xi_{top}} \right)}{u_{bottom} - u_{top}} \right) \right] \quad (51)$$

FSUB is a subroutine which evaluates the integral of the function subprogram through use of Simpson's rule using the half-Simpson multipliers.²⁰ This subroutine returns the value FPR to the main program. Once FPR has been determined, f^g is calculated as,

$$f^g = EXP (B^g * FPR) / DU_g \quad (52)$$

The value of ψ^{g-1} can be calculated knowing f^g , Σ_s^g , and the infinite medium flux as,

$$\psi^{g-1} = f^{g-1} \Sigma_s^{g-1} \phi^{g-1} + \psi^{g-2} \quad (53)$$

$\Sigma^{g \rightarrow g}$ in the present analysis takes on its actual value insuring Σ_R , the removal cross section is preserved. h^g will be adjusted to reproduce the infinite medium reference flux according to

$$h_{adj}^g = \frac{\Sigma_R^g \phi^g - S^g}{\psi^{g-1}} \quad (54)$$

Finally a list of separable kernel effective cross sections to replace the standard set, which was input to IDAPRIME, is printed for each group, the following separable kernel cross sections are required:

$$\Sigma_a, \nu\Sigma_f, \Sigma_t, \Sigma^{g \rightarrow g}, h^g f \Sigma_s^g.$$

Some coding statements were changed to replace the standard

$$\Sigma^{g \rightarrow n \rightarrow g} \psi^n \quad (55)$$

with the separable kernel downscatter term

$$h^g \psi^{g-1}.$$

The essential advantage of this switch is that the dependence upon both knowledge of all fluxes of higher energies and storage of all the cross section transfer pairs, has been removed.

APPLICATION OF THE ANALYSIS

The aim of this section is to show that the formalism of the previous section can be used to obtain accurate space dependent flux spectra. Three idealized fast reactor configurations²¹ will be analyzed first as a test of the procedure. Two additional fast reactor configurations will illustrate the effectiveness of the separable kernel implantation when using materials and mockups from a published benchmark case.²²

The energy range of interest is from 10 MeV down and is divided into 29 groups with energy lethargy spacing shown in Table 1. The multigroup cross sections for all the reactor type problems were obtained by processing an FTR-300²³ data file for the standard DTF-IV version cross sections. Separable kernel effective cross sections were obtained by running the standard cross sections through IDAPRIME.

Analyses have been made in semi-infinite slab geometry. The slab thicknesses and material combinations are summarized in Figure 1. All runs included reflective boundary conditions on the left face and vacuum boundaries on the right face of the slab.

Table 2 lists multigroup values of the parameters and $f\Sigma_g$ and h for idealized reaction problems.

DISCUSSION OF RESULTS

Core Only-Problem #1

Figure 2 which corresponds to a node positioned at 22 cm from the left edge, is typical of the excellent quantitative agreement which exists between the standard and modified DTF-IV energy spectra at both high and low energies and at all spatial positions as seen by Figure 3. Collapsing to the group structure used in Reference 23 and listed in Table 3 are presented in Table 4 and can be seen to be in good agreement. The criticality, k , for the separable kernel method was 1.50472 as compared to 1.49976 for the standard method; an error of only 0.331%.

Core/Blanket-Problem #2

In this application, a 40 cm slab of core material was placed next to a 20 cm slab of blanket material. Figure 4 shows flux vs. energy at 49 cm or approximately the center of the blanket zone. The excellent qualitative agreement is particularly noteworthy in the region from 10 Mev to 2 Kev. At energies below 2 Kev, there is a slight overprediction by the separable kernel method. Table 5 presents four group collapsed cross sections for the blanket zone. Good agreement between the two methods can, again, be observed. The criticality eigenvalue shows the separable kernel modified version having a value of 1.42788 compared to 1.43974 in the standard or base case; error less than 1%.

Core/Blanket/Core/Blanket-Problem #3

This configuration represents an idealized heterogeneous or "parfait" core. The physical set up included 20 cm of core, 10 cm of blanket, 20 cm of core, followed by another 10 cm region of blanket facing the vacuum

boundary. Figure 5 graphs the two methods at 22 cm (in the first blanket zone). Excellent agreement is demonstrated. Figure 6 depicts the flux variation in a particular group with position. Excellent results achieved in this and in the other idealized problems are indicative of the merit of this procedure.

Reactor Problem (no control)-Problem #4

This problem (and the following one) are based upon a published benchmark case.²² This reference provided the compositions for the various materials and a guide for the physical dimensions of the system.

Figure 7 shows the standard and separable kernel fluxes in the Sodium (position=50 cm). Excellent agreement is again observed. Collapsed four group cross sections are presented in Tables 6 and 7 for the Inner Core and Sodium regions respectively. Criticality of the base case is 1.23790 vs. 1.22563 (less than 1% error) in the modified version.

Reactor Problem (w/control)-Problem #5

Like the no control problem before, the reactor problem with control is based on the benchmark case of reference 22. The physical setup is the same as for the previous problem except the sodium of region four (45-55 cm, nodes 10 and 11) has been replaced with a boron absorber. Again, a reflective boundary exists on the left edge of the slab with a vacuum boundary bordering the right hand edge.

Until this time, a strong absorber had not been treated by the separable kernel method. The techniques used thus far will still be employed to calculate the relevant separable kernel parameters. The reactor problem with control will be a test of this procedure.

Figure 8 shows generally good agreement with some underprediction of the flux spectrum in the neighborhood of the major sodium resonance. Figure 9 shows a spatial dependence. Nodes 10 and 11 are in the boron absorber region while nodes 12, 13, and 14 represent the outer core. There is a modest error in a limited region. Agreement improves in the outer core and is excellent again in the radial blanket.

The criticality, k , in the separable kernel version was 1.08810 compared to the base case value of 1.09843; i.e., less than 1% error. Collapsed cross sections for the Boron Absorber appear in Table 8.

The agreement of the flux spectra, four group collapsed cross sections, and eigenvalues between the two methods can be characterized as excellent overall.

Storage Reduction

The purpose of this section is the development of storage saving techniques and the illustration of the relative independence of storage required upon the number of energy groups used in the separable kernel modified version of DTF-IV. In the discussion that follows, comparisons will be made relative to the storage requirements of DTF-IV. It is recognized that it is possible to improve storage characteristics of DTF-IV somewhat without resorting to separable kernels, and that other transport codes may include more efficient treatment of data storage. However, the comparisons below are deemed to be appropriate in that they represent actual storage changes for a given, commonly-used computer code.

Thus far, the standard cross section table contained $(G+3)$ elements in each of G groups. The separable kernel cross section table uses only six elements in each of the G groups. Therefore, the savings, in computer words, from the reduction of the cross section table can be represented by

$$\text{Storage Savings} = MG(G-3) \quad (57)$$

where M is the number of materials used; including the number of anisotropic components. It should be pointed out that first, downscatter only is assumed, and secondly, in the modified version of DTF-IV, at most six elements can be used for each group in the anisotropic components. In general, anisotropic components usually have not included inelastic scatter transfers (though ENDF/B files now include anisotropic components of inelastic scattering) and thus used only four elements per group. DTF-IV necessarily stores this information in arrays which are the same size as those for the isotropic component cross sections filling the remaining (G-1) elements with zeroes. Therefore, in the standard method

$$I * ISCT * G * (G-1) \quad (58)$$

represents the total number of zeroes held in chargeable storage when I isotropic materials having ISCT scattering (3 for P3, etc.) are used in the run.

Since the modified version uses six elements per group, regardless of the number of groups or the level of anisotropic scatter, a considerable storage savings can be attained when a reactor problem using many materials (isotropic and/or anisotropic) is to be run. Tables 9 and 10 show the savings benefit of the separable kernel method cross section table for various run configurations. This savings is shown opposite "XS Table Reduced" and the percentage storage reduction for this case is shown opposite "% Storage Red. XS." Few element anisotropic components are included in the number of materials where applicable.

Further storage reductions can also be realized when the fluxes of the continuous slowing down theory and the separable kernel formulations are considered. In the modified version of DTF-IV, the flux in any group "g" depends only upon the separable kernel parameters for group g, the flux-like summation term ψ^{g-1} , and the flux in the previous energy group. That is to say, the fluxes ϕ^i ($i < g-1$) are not required to calculate ϕ^g . Therefore, a scheme will be investigated that will enable solution of the computer code without storing arrays dependent upon the number of groups in the computer core memory.

The motivation for this procedure can be illustrated by the following example which assembles 49 energy groups and 53 radial intervals. The Flux Isotropic Component of Angular Flux (XN) is an array, in DTF-IV, dimensioned as the number of intervals by the number of groups.

Standard Method	Separable Kernel	
XN (53,49)	XN (53)	
2597 words required	53 words required	(59)
Savings of 2544 words		

Thus, each time a particular energy group of XN is required, a column vector dimensioned (1 x 53) for that particular group will be accessed by the computer core, used for calculation, updated, and written back out to non-chargeable storage.

Removal of the XN array consists of changing both the array handling pointers and coding statements, and adding the mechanism to perform a random access operation to find the location of the first element for group g of the XN array followed by a sequential read/write of the elements for group g themselves.

The actual coding statements required may vary from computer to computer. In DTF-IV, five arrays have been found candidates for this relocation:

XN (IM,IGM)	Flux Isotropic Component
Q (IM,IGM)	Isotropic Distributed Source
C (IHM,IGM,MT)	Cross Section Table
XJ (IGM,IM,NM)	Currents
B (IGM,MM)	Boundary Fluxes

where,

IM	Number of Intervals
IGM	Number of Groups
IHM	Cross Section Table Length
MT	Total Number of Materials
NM	Number of Currents
MM	Number of Ordinates

Arrays that are singly dimensioned of IGM (such as QG (IGM) Volume Integrated Source) are not candidates for this storage reduction as the potential for savings of 49 computer words is not worth the effort. Further, the cross section array has already been significantly reduced and is accessed so frequently that it will be left in "core".

Tests were conducted to find savings resulting from the removal of these arrays singly and in combinations. The Q array was found to require so many accesses that the time required for execution increased enough to negate any possible storage savings. The arrays which proved to be most practical when removed were XN (flux) and XJ (current).

Tables 9 and 10 also show the savings that can be achieved using these separable kernel techniques. In particular, the additional savings due to the removal of the flux array XN and removal of both the flux array and the current array XJ are shown in these tables opposite

the labels "XS + XN OUT" (for the case of cross section table reduced and flux removed to external file) and "XS + XN + XJ OUT" (for the case of cross section table reduced and both flux and current stored on external file).

Table 11 extended these same storage modification techniques to a two dimensional type code. (A two dimensional code was not run. Instead, the number of intervals used was squared in order to represent the spatial mesh structure of two dimensional approaches). It is interesting to note that the percentage savings realized in cases 1 - 4 of Table 11 are significantly less than the analogous savings observed in Tables 9 and 10. This is due to the increased importance of fluxes on total storage. To fully achieve the benefits of the implied savings, a strategy for accessing the temporary file as efficiently as possible, is needed. Case in point is the array Q that was mentioned before. The overabundance of calls to this array caused a computer time penalty that was greater than the savings in storage. In a problem involving 250 groups, one method, in which information is accessed in groups of 10 rather than individually, might prove useful. An access of more than one group at a time would imply the need for reserving some additional storage, however.

The main concern in this paper has been with storage requirements of one-dimensional codes that are used for space-dependent collapse of multigroup data to few group data. Accordingly, in view of the results of Tables 9 and 10 only limited attention was given to storage reduction associated with flux-like quantities. However, for two-dimensional codes,

more basic restructuring in data handling may lead to the projected storage benefits without associated access penalties. Such restructuring may be based on the characteristics of downscatter problems. Once a flux has been calculated, in the separable kernel method, it is not needed again if the problem is non-multiplying, and may be written directly to an output tape. If the problem is multiplying, then the flux would be needed again only for calculating fission source for the next outer iteration. The fission rate may be updated continuously (i.e., at each iteration). It may be treated as a one-group quantity, with subsequent recognition of multigroup characteristics coming from combination with the fission spectrum. A code may permit only one fission spectrum. Even if multiple fission spectra are permitted, storage would be governed by number of materials rather than number of energy groups.

It may be observed that, while there can be truly major reductions in storage requirements for codes like DTF-IV, significant storage can be required for auxiliary codes, like the IDAPRIME used here, which construct the separable kernel parameters from detailed transfer matrices. However, such auxiliary codes can process one material at a time. Consequently, for realistic fast reactor problems, the auxiliary code should not provide the basic limitation on storage except when very large numbers of groups are involved.

If the number of groups is large, then storage requirements for the auxiliary code can be minimized by reading data only as needed. Referring back to Eq. (37), group cross-sections would be read in only as the flux for that group is evaluated. Thus, to evaluate ϕ^g , it would

be necessary to read Σ_t^g , $\Sigma^{g \rightarrow g}$, and $\Sigma^{g-1-n \rightarrow g}$ for $0 < n < g$. These data would be read into the same array previously occupied by Σ_t^{g-1} , $\Sigma^{g-1 \rightarrow g-1}$, $\Sigma^{g-1-n \rightarrow g-1}$, which data no longer are needed. Thus, data storage would be linearly dependent on number of groups. For an infinite-medium (i.e., one point) problem, any inefficiency of data handling for this purpose should be tolerable.

SUMMARY AND CONCLUSIONS

Research for this paper was begun in order to understand better the mechanics of the separable kernel, to implant the techniques based on separable kernels in a multigroup computer code, and to determine whether benefits realized are sufficient to make the new method computationally attractive. In the theory section, the developments of Continuous Slowing Down Theory were presented. The separable kernel was integrated with CSDT and the parameters ξ , f , and h were derived. Using a flux-like quantity ψ , the multigroup transport equation was rewritten using the separable kernel parameters. It was observed that this equation was not dependent on fluxes of higher energies to find the flux in any given group.

The theory was then used to calculate a new cross section set which unlike the standard set contained only six elements for each group regardless of the number of groups. Coding statements in DTF-IV were changed to accommodate the separable kernel and three idealized fast reactor problems were run to check the accuracy of the method. After noting the excellent agreement between the standard DTF-IV and the modified version, two fast reactor problems were set up based upon a published Benchmark case. Again, very good agreement was observed. It was then concluded that the methods were, indeed, of sufficient accuracy for fast reactor applications.

An analysis was made of the storage benefits of the separable kernel procedure. The primary mechanism for storage savings was found, at least for one-dimensional problems, to be in the required cross section table. Further reductions were achieved by removing arrays dependent on the number of groups from the chargeable storage area and placing them in external files. This reduction was possible due to a basic assumption of the CSDT, namely that slowing down density in any group depends only on the slowing down density in the previous group. Thus, storage reductions of up to 90% were observed.

The analysis itself is considered to be well suited for application to fast reactor problems requiring a large number of energy groups. It is particularly well suited for collapsing cross sections.

Table I. 29 Group Energy Structure

Group	Energy	
1	0.10000E+08	0.60650E+07
2	0.60650E+07	0.36790E+07
3	0.36790E+07	0.22310E+07
4	0.22310E+07	0.13530E+07
5	0.13530E+07	0.82090E+06
6	0.82090E+06	0.49790E+06
7	0.49790E+06	0.38770E+06
8	0.38770E+06	0.30200E+06
9	0.30200E+06	0.18320E+06
10	0.18320E+06	0.11110E+06
11	0.11110E+06	0.67380E+05
12	0.67380E+05	0.40870E+05
13	0.40870E+05	0.25540E+05
14	0.25540E+05	0.19890E+05
15	0.19890E+05	0.15030E+05
16	0.15030E+05	0.91190E+04
17	0.91190E+04	0.55310E+04
18	0.55310E+04	0.33550E+04
19	0.33550E+04	0.28400E+04
20	0.28400E+04	0.24040E+04
21	0.24040E+04	0.20350E+04
22	0.20350E+04	0.12340E+04
23	0.12340E+04	0.74850E+03
24	0.74850E+03	0.45400E+03
25	0.45400E+03	0.27540E+03
26	0.27540E+03	0.13010E+03
27	0.13010E+03	0.61440E+02
28	0.61440E+02	0.13710E+02
29	0.13710E+02	THERMAL

Table II. Separable Kernel Parameters for Core and
Blanket Regions of Test Problems

Group	<u>Core Cross Sections</u>		<u>BLANKET Cross Sections</u>	
	<u>h</u>	<u>f*SIGS</u>	<u>h</u>	<u>f*SIGS</u>
1	0.0	0.19608E+00	0.0	0.20404E+00
2	0.70092E-01	0.18141E+01	0.69373E-01	0.19156E+01
3	0.11142E-01	0.91901E+01	0.10554E-01	0.90541E+01
4	0.28798E-02	0.33655E+02	0.28674E-02	0.29256E+02
5	0.80973E-03	0.15915E+03	0.99921E-03	0.12538E+03
6	0.21727E-03	0.13940E+04	0.32246E-03	0.11589E+04
7	0.16904E-04	0.78332E+04	0.22983E-04	0.69989E+04
8	0.75598E-05	0.10299E+05	0.10281E-04	0.91430E+04
9	0.38971E-05	0.13176E+06	0.52270E-05	0.11762E+06
10	0.22233E-06	0.53095E+07	0.28769E-06	0.49928E+07
11	0.53030E-08	0.34034E+09	0.56167E-08	0.35287E+09
12	0.82208E-10	0.24717E+11	0.89281E-10	0.30148E+11
13	0.12108E-11	0.18022E+13	0.10714E-11	0.24297E+13
14	0.20971E-13	0.10052E+14	0.16154E-13	0.15080E+14
15	0.40409E-14	0.55749E+14	0.30426E-14	0.80014E+14
16	0.82688E-15	0.23082E+16	0.60943E-15	0.36385E+16
17	0.13872E-16	0.37218E+18	0.92426E-17	0.59489E+18
18	0.11344E-18	0.33244E+20	0.70074E-19	0.60779E+20
19	0.17676E-20	0.45834E+21	0.89698E-21	0.77480E+21
20	0.84136E-21	0.57237E+21	0.41264E-21	0.99835E+21
21	0.47586E-21	0.43499E+21	0.22910E-21	0.81904E+21
22	0.23244E-21	0.49233E+22	0.11254E-21	0.10878E+23
23	0.72612E-23	0.34113E+24	0.33460E-23	0.77159E+24
24	0.96548E-25	0.23547E+26	0.44436E-25	0.61262E+26
25	0.13639E-26	0.21849E+28	0.55208E-27	0.66512E+28
26	0.18580E-28	0.51680E+31	0.61418E-29	0.23847E+32
27	0.44148E-32	0.25518E+35	0.98428E-33	0.95101E+35
28	0.89548E-36	0.15662E+48	0.24580E-36	0.45749E+47
29	0.72483E-49	0.15932E+51	0.25404E-48	0.38066E+49

Table III. Four Group Energy Structure for
Collapsed Cross-Sections

GROUP	ENERGY	
1	0.10000E+08	0.80000E+06
2	0.80000E+06	0.10000E+05
3	0.10000E+05	0.10000E+04
4	0.10000E+04	THERMAL

Table IV. Collapsed Core Cross-Sections

	g=1	g=2	g=3	g=4
A. From STANDARD METHOD				
Σ_a	0.0047856	0.0077228	0.0238216	0.0573901
$v\Sigma_f$	0.0114449	0.0094248	0.0223989	0.0709218
Σ_t	0.1956792	0.3109079	0.4898520	0.4320714
$\Sigma_{g \rightarrow g}$	0.1709980	0.2952702	0.4562485	0.3746352
$\Sigma_{g \rightarrow n \rightarrow g}$	0.0	0.0198972	0.0079147	0.0097783
	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0
B. From SEPARABLE KERNEL METHOD				
Σ_a	0.0047967	0.0077007	0.0237909	0.0571792
$v\Sigma_t$	0.0114835	0.0094180	0.0223659	0.0706077
Σ_t	0.1955563	0.3105202	0.4903084	0.4320775
$\Sigma_{g \rightarrow g}$	0.1709403	0.2949665	0.4567746	0.3748536
$\Sigma_{g \rightarrow n \rightarrow g}$	0.0	0.0198210	0.0078528	0.0097390
	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0

Table V. Collapsed Cross-Sections for BLANKET

	g=1	g=2	g=3	g=4
A. From STANDARD METHOD for BLANKET				
Σ_a	0.0020538	0.0055017	0.0135577	0.0198128
$v\Sigma_f$	0.0018881	0.0001512	0.0004290	0.0012275
Σ_t	0.2165844	0.3350948	0.4521475	0.3797361
$\Sigma_{g \rightarrow g}$	0.1914383	0.3204803	0.4279748	0.3597872
$\Sigma_{g \rightarrow n \rightarrow g}$	0.0	0.0230881	0.0091128	0.0106165
	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0
B. From SEPARABLE KERNEL for BLANKET				
Σ_a	0.0020497	0.0054946	0.0135732	0.0199152
$v\Sigma_t$	0.0018761	0.0001511	0.0004299	0.0012377
Σ_t	0.2166156	0.3349989	0.4510403	0.3795505
$\Sigma_{g \rightarrow g}$	0.1914644	0.3204182	0.4267673	0.3594894
$\Sigma_{g \rightarrow n \rightarrow g}$	0.0	0.0230972	0.0090862	0.0107014
	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0

Table VI. Collapsed Cross-Sections: Inner Core, Reactor Problem

	g=1	g=2	g=3	g=4
A. From STANDARD METHOD				
Σ_a	0.0033446	0.0055985	0.0176420	0.0407462
$\nu\Sigma_f$	0.0076092	0.0057303	0.0143606	0.0455670
Σ_t	0.1774132	0.2823636	0.4349785	0.3687579
$\Sigma_{g \rightarrow g}$	0.1556156	0.2685001	0.4066903	0.3279650
$\Sigma_{g \rightarrow n \rightarrow g}$	0.0	0.0184539	0.0082644	0.0106464
	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0
B. From SEPARABLE KERNEL				
Σ_a	0.0033423	0.0055985	0.0176280	0.0407052
$\nu\Sigma_t$	0.0076027	0.0057303	0.0143426	0.0455189
Σ_t	0.1774335	0.2823633	0.4360664	0.3687378
$\Sigma_{g \rightarrow g}$	0.1556304	0.2684926	0.4078277	0.3279860
$\Sigma_{g \rightarrow n \rightarrow g}$	0.0	0.0184616	0.0082717	0.0106110
	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0

Table VII. Collapsed Cross-Sections: Sodium Follower, Reactor Problem

	g=1	g=2	g=3	g=4
	From STANDARD METHOD			
Σ_a	0.0016535	0.0055685	0.0136180	0.0232761
$\nu\Sigma_f$	0.0009096	0.0001438	0.0004109	0.0014270
Σ_t	0.2252731	0.3665530	0.4804658	0.4083952
$\Sigma_{g \rightarrow g}$	0.1988512	0.3491423	0.4533181	0.3844931
$\Sigma_{g \rightarrow n \rightarrow g}$	0.0	0.0247751	0.0118404	0.0135290
	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0
	From SEPARABLE KERNEL			
Σ_a	0.0016415	0.0055451	0.0136238	0.0233005
$\nu\Sigma_t$	0.0008740	0.0001435	0.0004113	0.0014299
Σ_t	0.2253862	0.3660825	0.4800383	0.4084047
$\Sigma_{g \rightarrow g}$	0.1989255	0.3488505	0.4528584	0.3844680
$\Sigma_{g \rightarrow n \rightarrow g}$	0.0	0.0248260	0.0116852	0.0135554
	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0

Table VIII. Collapsed Cross-Sections: Boron Absorber, Reactor Problem

	g=1	g=2	g=3	g=4
A. STANDARD for BORON				
Σ_a	0.0031158	0.0232607	0.0857364	0.2294559
$\nu\Sigma_f$	0.0000000	0.0000000	0.0000000	0.0000000
Σ_t	0.1969883	0.3793274	0.6078826	0.6512619
$\Sigma_{g \rightarrow g}$	0.1650590	0.3344802	0.4990710	0.4217725
$\Sigma_{g \rightarrow n \rightarrow g}$	0.0	0.0288162	0.0215876	0.0230764
	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0
B. SEPKL for BORON				
Σ_a	0.0031176	0.0231373	0.0853591	0.2305058
$\nu\Sigma_f$	0.0000000	0.0000000	0.0000000	0.0000000
Σ_t	0.1971034	0.3784723	0.6090167	0.6521896
$\Sigma_{g \rightarrow g}$	0.1650432	0.3341237	0.5012028	0.4216470
$\Sigma_{g \rightarrow n \rightarrow g}$	0.0	0.0289453	0.0212121	0.0224562
	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0

Table IX. Storage Savings: Fast Reactor Problems
with S₂ Quadrature

CASE No.	1	2
SCATTERING Order	0	0
QUADRATURE	2	2
GROUPS	49	100
MATERIALS	6	6
ZONES	6	6
INTERVALS	20	20
STANDARD Storage	20414	70802
MODIFIED Storage		
XS table reduced	6890	12602
XS + SN out	5930	10622
% STORAGE Red. XS	66.24	82.20
% STORAGE Red. XS+XN	70.95	84.99

CASE No.	3	4
SCATTERING Order	0	0
QUADRATURE	2	2
GROUPS	150	250
MATERIALS	6	6
ZONES	6	6
INTERVALS	20	20
STANDARD Storage	150502	399902
MODIFIED Storage		
XS table reduced	18202	29402
XS + XN out	15222	24422
% STORAGE Red. XS	87.91	92.65
% STORAGE Red. XS+XN	89.89	93.89

Table X. Storage Savings: Fast Reactor Problems
with S_{16} Quadrature

CASE No.	1	2
SCATTERING Order	0	0
QUADRATURE	16	16
GROUPS	100	250
MATERIALS	6	6
ZONES	6	6
INTERVALS	20	20
STANDARD Storage	73224	402324
MODIFIED Storage		
XS table reduced	15024	31824
XS + XN out	13044	26844
% STORAGE Red. XS	79.48	92.09
% STORAGE Red. XS+XN	82.19	93.33
CASE No.	3	4
SCATTERING Order	3	3
QUADRATURE	16	16
GROUPS	100	250
MATERIALS	18	18
ZONES	6	6
INTERVALS	20	20
STANDARD Storage	200988	1171488
MODIFIED Storage		
XS table reduced	26388	59988
XS + XN out	24408	55008
XS + XN + XJ out	20448	45048
% STORAGE Red. XS	86.87	94.88
% STORAGE Red. XS+XN	87.86	95.30
% STOR. Red. XS+XN+XJ	89.82	96.15

Table XI. Storage Savings: 2D Fast Reactor Problems
with S_2 Quadrature (Diffusion Theory)

CASE No.	1	2
SCATTERING Order	0	0
QUADRATURE	2	2
GROUPS	49	100
MATERIALS	6	6
ZONES	6	6
INTERVALS	20	20
STANDARD Storage	71874	160002
MODIFIED Storage		
XS table reduced	58350	101802
XS + SN out	39150	62202
% STORAGE Red. XS	18.81	36.37
% STORAGE Red. XS+XN	45.53	61.12
CASE No.	3	4
SCATTERING Order	0	0
QUADRATURE	2	2
GROUPS	150	250
MATERIALS	6	6
ZONES	6	6
INTERVALS	20	20
STANDARD Storage	276702	600102
MODIFIED Storage	144402	229602
XS table reduced	84802	130002
XS + XN out		
% STORAGE Red. XS	47.81	61.74
% STORAGE Red. XS+XN	69.35	78.33

FIGURE 1

PROBLEM #	← MATERIAL CONFIGURATION	COMPOSITION	REF. #
1	<div> <div>Position</div> <div>Nodes</div> <div> <div>0</div> <div>1-60</div> <div>60 cm</div> </div> </div> <div>Outer Core</div>	Clinch River	21
2	<div> <div>Position</div> <div>Nodes</div> <div> <div>0</div> <div>1-40</div> <div>40</div> <div>41-60</div> <div>60 cm</div> </div> </div> <div>Outer Core</div> <div>Inner Core</div>	Clinch River	21
3	<div> <div>Position</div> <div>Nodes</div> <div> <div>0</div> <div>1-5</div> <div>20</div> <div>21-30</div> <div>30</div> <div>31-50</div> <div>50</div> <div>51-60</div> <div>60 cm</div> </div> </div> <div>Outer Core</div> <div>Inner Core</div> <div>Outer Core</div> <div>Inner Core</div>	Clinch River	21
4	<div> <div>Position</div> <div>Node</div> <div> <div>0</div> <div>25</div> <div>35</div> <div>45</div> <div>55</div> <div>70</div> <div>100 cm</div> </div> </div> <div>Inner Core</div> <div>Sodium Follower</div> <div>Inner Core</div> <div>Sodium Follower</div> <div>Outer Core</div> <div>Radial Blanket</div>	Buckel, et al. Benchmark	22
5	<div> <div>Position</div> <div>Node</div> <div> <div>0</div> <div>25</div> <div>35</div> <div>45</div> <div>55</div> <div>70</div> <div>100 cm</div> </div> </div> <div>Inner Core</div> <div>Sodium Follower</div> <div>Inner Core</div> <div>Boron Absorber</div> <div>Outer Core</div> <div>Radial Blanket</div>	Buckel, et al. Benchmark	22

FIGURE 2

FLUX/E VS ENERGY

CORE PROBLEM

FIGURE 3

FLUX/E VS POSITION

CORE PROBLEM

FIGURE 4

FLUX/E VS ENERGY

CORE BLANKET PROBLEM

FIGURE 5

FLUX/E VS ENERGY

CORE-BLANKET-CORE-BLANKET PROBLEM

FIGURE 6

FIGURE 7

FLUX/E VS ENERGY

REACTOR (NO CONTROL)

FIGURE 8

FLUX/E VS ENERGY

REACTOR PROBLEM WITH CONTROL

FIGURE 9

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