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Consolidation Theory and Its Applicability to the Dewatering and Covering of Uranium Mill Tailings

Prepared by T. E. Gates

Pacific Northwest Laboratory
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Consolidation Theory and Its Applicability to the Dewatering and Covering of Uranium Mill Tailings

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ABSTRACT

This report is a review and evaluation of soil consolidation theories applicable for evaluating settlement during dewatering and subsequent covering of uranium mill tailings. Such theories may be used to predict both consolidation and water flow related effects in uranium mill tailings during drainage, following sluicing into burial pits. A consolidation theory to be useful must consider the effect of time-dependent loads, nonhomogeneous soil mass, non-linear variation of soil properties with the stress-state parameters, large strain, and saturated and unsaturated flow. Constitutive relations linking the stress deformation state variables with void ratio should be adopted for predicting both consolidation and fluid flow interaction in unsaturated uranium mill tailings.

SUMMARY

In 1982 Pacific Northwest Laboratories (PNL) reviewed Terzaghi's linear consolidation theory to determine its applicability to the dewatering and covering of uranium mill tailings. Karl Terzaghi (1925) was the first to propose a mechanism to explain soil consolidation. Terzaghi applied the concepts of linear consolidation to the settlement of a column of saturated soil that was under constant load and prevented from lateral expansion. In uranium mill tailings disposal, however, the tailings mass provides variable load conditions. A layer of fill deposited in a soft, wet condition will consolidate under its own weight and will usually undergo large strains in the process.

A consolidation theory used to predict both consolidation and related fluid flow effects in uranium mill tailings must consider the effect of increasing loads with deposition, nonhomogeneous soil mass, nonlinear variation of soil properties with the stress-state parameters, large strains, and saturated and unsaturated flow. None of these factors can be modeled by Terzaghi's traditional consolidation theory; however, nonlinear variations of Terzaghi's theory have been proposed.

Extensions of Terzaghi's effective stress concept for saturated soils have been suggested for unsaturated soils. One extension is the constitutive relationships proposed for unsaturated porous systems, which reduces to the traditional effective stress concept for saturated soils. In particular, constitutive relations link the stress deformation state variables with the void ratio, and should be adopted for predicting both consolidation and related fluid flow effects in unsaturated uranium mill tailings. Further research is needed to develop a computer model with the above attributes.

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NOTATIONS

- a_a = fraction of total area that is air-air contact
- a_m = area of mineral-mineral contact per unit area
- a_v = coefficient of compressibility
- a_w = area of water-water contact per unit area
- A = electrical attractive force per unit area
- A_t = the percentage of area of the air-water menisci
- A_w = the percentage of area occupied by water in a wavy surface across the soil
- C_c = compression index
- C_F = coefficient of consolidation used by Gibson, England, and Hussey (1967) defined as $\frac{K}{\gamma_w} \frac{1}{(1+e)} \frac{d\sigma'}{de}$
- C_0 = constant
- C_v = coefficient of consolidation
- e = void ratio
- G_s = specific gravity
- h_m = matrix suction
- h_s = solute suction
- K = coefficient of permeability
- K_0 = coefficient of earth pressure at rest
- n = porosity
- p'' = pore-water pressure deficiency
- $P_c = (U_a - U_w)$, matrix suction
- R = electrical repulsive force per unit area
- S_{cr} = critical degree of saturation
- S_r = degree of saturation

T = time
 U = excess hydrostatic pore pressure
 $U^* = [XU_w + (1-X)U_a]$, equivalent pore pressure
 U_a = pore-air pressure
 U_w = pore-water pressure
 V = total volume
 V_a = volume of air in the element
 V_w = volume of water in the element
 Z = elevation above datum
 β = statistical factor of same type as contact area
 β' = holding or bonding factor which is measure of number of bonds under tension effective in contributing to soil strength
 γ_d = dry unit weight of soil
 ϵ = Eulerian (volumetric) strain
 λ = the structure or fabric of the soil
 σ = total stress
 σ' = effective stress
 $\sigma_1, \sigma_2, \sigma_3$ = major, intermediate, and minor principal stresses
 X = parameter highly dependent on degree of saturation, soil type, and hysteresis effects due to wetting, drying, or stress change
 X_m = effective stress parameter for matrix suction
 X_s = effective stress parameter for solute suction
 ψ = parameter with values ranging from zero to one

1.0 INTRODUCTION

The production of wastes (i.e., mill tailings) from the mining and milling of uranium is of concern to the public and the government. Because these tailings contain quantities of potentially toxic components, disposal and management procedures must insure long-term stability. Generally, tailings are piled on the natural land surface after processing. However, the advantages of returning mill tailings to the pit that was excavated during the surface mining operation are being recognized: better control of radon gas emission from the tailings, and a return to the original topography and vegetative habitat of the mined-out site.

If pit disposal is used, deformation of the tailings pile affects the engineering of the covers used for radon emission control. Further, this deformation effects permeability variations within the tailings material. Hence, the distribution of fluid velocities, which is critical for modeling chemical transport, is influenced by the dynamics of the deformation process.

This study examines Terzaghi's classical vertical consolidation theory for estimating the settlement of wet tailings sluiced into burial pits. Terzaghi's linear theory is evaluated in terms of its strengths and weaknesses as applied to unsaturated uranium mill tailings. Because the classical theory was found to be inadequate to account for the in situ nature of uranium mill tailings, a more complete model of consolidation was posed.

Section 2.0 of this report gives conclusions about the applicability of Terzaghi's vertical consolidation theory for estimating the settlement of unsaturated uranium mill tailings. Section 3.0 presents an overview of the nature of unsaturated tailings, of unsaturated soil, and of characteristics of the tailings pile. Section 4.0 describes one-dimensional consolidation as applied to saturated soils and includes the concepts of linear and nonlinear theory. Section 5.0 presents limitations of the principle of effective stress as applied to unsaturated soils. In addition to consolidation and shear strength, the phenomena of shrinkage and swelling are discussed. Section 6.0 presents the more general constitutive relations for volume change in unsaturated soils. These governing equations help determine the settlement behavior of wet tailings sluiced into burial pits and, in turn, the management of covers used for radon emission control.

2.0 CONCLUSIONS AND RECOMMENDATIONS

If pit disposal is used, deformation of the tailings pile affects the engineering of the covers used for radon emission controls. This deformation effects permeability variations within the tailings material. Hence, the distribution of fluid velocities, which is critical for modeling chemical transport, is influenced by the dynamics of the deformation process.

A consolidation theory used to predict both consolidation and related fluid flow effects in uranium mill tailings must consider the effect of increasing loads with deposition, nonhomogeneous soil mass, nonlinear variation of soil properties with the stress-state parameters, large strain, and unsaturated flow. None of these factors can be modeled by Terzaghi's traditional consolidation theory. In particular:

- An air-water interface, resulting from the air phase in unsaturated systems, complicates the soil mechanics and fluid flow properties that must be measured in an analysis of the flow and settlement in unsaturated systems.
- A unique relationship does not exist between void ratio and effective stress for most unsaturated soils below a critical degree of saturation.
- The two stress-state variables $[(\sigma - U_a), (U_a - U_w)]$, needed for unsaturated soils, account for the behavior of volume change.

The following recommendations are based on the unsaturated consolidation theory presented in this report.

- Constitutive relations linking the stress deformation state variables with void ratio should be used to predict both consolidation and the related effects on fluid flow in unsaturated uranium mill tailings.
- A computer model should be developed to consider the effect of increasing loads with deposition, nonhomogenous soil mass, nonlinear variation of soil properties with the stress-state parameters, large strain, and unsaturated and saturated flow.
- Data should be developed, through measurements, to define the variation of void ratio and saturation with reference to the two chosen stress-state variables $[(\sigma - U_a), (U_a - U_w)]$.
- Laboratory and computer model sensitivity studies should be performed to determine whether flow or settlement will be affected most by using the constitutive relations.

3.0 TECHNOLOGY FOR UNSATURATED TAILINGS

The field of soil mechanics can be divided into studies of saturated soils and of unsaturated soils. The differences between unsaturated and saturated soils result from the surface tension forces that are present in unsaturated and multiphase porous systems, but are absent under saturated conditions. The simpler saturated systems involve only two phases, that of the solid or skeleton phase of the porous material, and the wetting liquid phase that fills the pore space. The unsaturated (multiphase) systems involve a third phase in which gas is immiscible with the water in the soil pores. The capillary forces, arising from the continuous air-water interface within the porous material, cause the differences between unsaturated and saturated conditions that affect settlement and flow. The air-water interface (involving surface tension) underlies the "contractile skin" effect for unsaturated soils, discussed in some detail by Fredlund and Morgenstern (1976). The presence of an air-water interface results in more-involved soil mechanics and fluid flow properties that must be measured and considered in an analysis of the flow and settlement in unsaturated systems.

Although saturated conditions have been studied for many years in soil mechanics, the saturated system is increasingly recognized as the reduced, simpler case of the more-involved unsaturated system. Accordingly, the measurements and analysis methods required for the unsaturated condition must reduce to adequately handle the simpler saturated consolidation and flow situations.

Saturated and unsaturated conditions are involved in the uranium mill tailings management evaluation; however, the more-involved unsaturated region is probably of more importance. Soils that commonly fall within the unsaturated category are the natural, desiccated soils; compacted soils; and dewatered uranium mill tailings. These soils generally have negative pore-water pressures and the most prevalent problems in stability considerations are related to their shrinkage and swelling.

3.1 NATURE OF UNSATURATED TAILINGS

Many soil deposits are originally saturated. Tailings material, like lacustrine deposits, is deposited at water contents above its liquid limit and is then consolidated by the weight of the overlaying sediments. As the tailings dry, through dewatering and evaporation, the sediments begin to desiccate. In turn, the pore-water pressure is reduced while the total stress on the sediments remains essentially constant (Figure 3.1). The pore-water pressure becomes negative as a result of the capillary force associated with the air-water interface above the water table, leading to consolidation of the sediments. The tailings are further consolidated and desaturated by evapotranspiration if grasses, plants, or brush are permitted to grow on the tailings pile.

FIGURE 3.1. Stress Distribution During Desiccation of a Soil (Fredlund 1979)

Cracking, a secondary mode of desaturation, can occur if the tension in the water phase exceeds the lateral confining pressure in the soil mass. Desaturation can also occur if the tailings' material structure is subjected to tensile stresses by remolding during the consolidation process.

A tailings deposit is constantly subjected to changing environmental conditions. Before cover placement, these environmental changes cause the tailings deposit to shrink and swell. This is the result of changes in the pore-water pressure distribution, which can take on a wide variety of shapes (Figure 3.1).

3.2 UNSATURATED SOIL

Generally, an unsaturated soil is considered to be a three-phase system. Lambe and Whitman (1969) state that soil contains "three distinct phases; solid (mineral particles), gas, and liquid (usually water)." Fredlund and Morgenstern (1976) postulate that, on the basis of their definition of a phase, the air-water interface or contractile skin should be considered as a fourth and independent phase. They characterize a material as an independent phase if it has properties that differ from the surrounding material and definite bounding surfaces. Fredlund and Morgenstern's (1976) definition of a phase is too limited. The generally acceptable definition of a phase, in terms of physical chemistry, is a part of a system, physically distinct, macroscopically homogeneous, and of fixed or variable composition; it is mechanically separable from the rest of the system. Because the volume of the air-water interface is small and its weight is considered to be part of the weight of water, it is not considered a phase, but rather a surface tension phenomenon. Unsaturated

soil (Figure 3.2) can therefore be considered to involve three phases: one phase comes to equilibrium under applied stress gradients (i.e., soil particles), and two phases flow under applied stress gradients.

3.3 TAILINGS PILES

The physical configuration and properties vary among tailings piles as well as within a single tailings pile. Changes in milling and waste management methods during the active life of the tailings often account for such variability. Other parameters responsible for variability in tailings are the mineralogy of the feed ore, characteristics of the milling operation, method of tailings emplacement, and post-operational treatment. A complex system is developed among the properties inherent to the tailings pile by environmental (hydrographic, geologic, and climatic) conditions.

3.3.1 Physical Configuration

The tailings material, in general, consists of two basic materials: non-plastic silt-size slimes and fine, uniform sands. During tailings emplacement, different tailings zones are formed (Figure 3.3). These zones consist of interbedded layers of the sands and slimes and mixtures of these materials. Most sites contain either large deposits of sands, with few fines, or interbedded sand-slime layers. Zones in which the materials are mixed are difficult to distinguish. Few extensive deposits with only slimes exist. The outlet location governs the deposition pattern. In addition to the layering of the

FIGURE 3.2. Element of Unsaturated Soil (Fredlund and Morgenstern 1977)

FIGURE 3.3. Typical Tailings Pile Cross-Section (CSU 1980)

tailings material in the disposal pit, the percentage of sand, silt, and clay depends on the nature of the parent materials, the characteristics of the milling operation, and the method of the tailings placement.

3.3.2 Physical Properties

Moisture content varies widely in the tailings material, with the sands primarily in the 3 to 9% (dry wt) range and the extended slime deposits in the 30 to 45% (dry wt) range (CSU 1980). Intermediate values of moisture content may be found where sand lenses provide drainage for slime layers, but cannot drain freely themselves.

Because the tailings material is layered, the horizontal permeability is expected to be greater than the vertical permeability (Van Zyl and Robertson 1980). This anisotropy would result from the techniques used for the disposal of tailings into the pit, which create depositional layers of differing permeability. The vertical permeability of slimes probably ranges from 10^{-4} to 10^{-6} cm/sec. (a)

The compression index, C_c , is the functional dependence between effective stress and void ratio for a soil. The compression index of slimes is typical of natural silt material. From tests of seven different tailings disposal embankments, Mittal (1974) reports the compression index for slimes to range from 0.25 to 1.13.

The rate of settlement of a porous media is partly controlled by the coefficient of consolidation, C_v , which indicates the rate of compression under a load increment. Coefficient of consolidation values for slimes have been reported to range from 10^{-1} to 10^{-3} cm²/sec (Mittal 1974).

(a) Personal communication from Dr. Irvin S. Dunn and Dr. Loren R. Anderson of Utah State University, Logan, Utah, on January 5, 1982.

4.0 ONE-DIMENSIONAL CONSOLIDATION THEORY: SATURATED SOILS

Consolidation in semipervious, compressible soils can be explained in terms of a transfer of load from an incompressible pore-water to a compressible soil structure. Changes of pressure at the soil boundaries are generally initiated by the consolidating load. These changes set up pressure gradients in the soil, with a consequent flow of water to the boundaries. The resulting loss of soil water from this flow allows the gradual transfer of load to the soil structure and, hence, consolidation.

4.1 LINEAR THEORY

A simple mechanism to explain soil consolidation in a saturated system was first proposed by Karl Terzaghi (1925). He assumed that the grains or particles constituting the soil were more or less bound together by certain molecular forces and constituted a porous material with elastic properties [Figure 4.1(a)]. The voids of the elastic skeleton were assumed to be filled with water. As a load was applied to this system [Figure 4.1(b)], a gradual settlement was produced, dependent on the rate at which the water was being squeezed out of the voids. Whereas the rate of change depends largely on the permeability, the magnitude of movement is determined by the stress-strain response of the soil [Figure 4.1(c)].

FIGURE 4.1. Structural Changes in a Soil Resulting from Increases in Applied Load and Pressure Deficiency (Jennings and Burland 1962).
(a) Loosely packed saturated material; (b) displacement of grains resulting from an increase in applied load; (c) magnitude of movement determined by stress-strain response of the saturated soil.

Terzaghi applied these concepts of linear consolidation to the analysis of the settlement of a column of soil under constant load and prevented from lateral expansion. Additional assumptions made by Terzaghi were:

- isotropy of the material
- reversibility of stress-strain relations, under final equilibrium conditions
- linearity of stress-strain relations
- small strains
- the water contained in the pores is incompressible
- the water flows through the porous skeleton according to Darcy's law.

Terzaghi's fundamental differential equation for linear consolidation of a saturated porous media:

$$C_v \frac{\partial^2 U}{\partial z^2} = \frac{\partial U}{\partial t} \quad (4.1)$$

where

C_v = coefficient of consolidation

U = excess pore-water pressure

z = elevation above datum

describes pore-water pressure as a function of time and position in the soil mass. In addition, his theory depends on the definition of effective stress, σ' :

$$\sigma' = \sigma + U \quad (4.2)$$

where

σ' = effective stress

σ = total stress.

The effective stress in the soil structure and the pore-water pressure carry the loads that are placed on a soil mass. Changes in pore-water pressure are exactly offset by changes in effective stress in the simplest case when the total stress remains constant. This condition would occur after complete deposition of the uranium mill tailings in the saturated state.

Gradients in the potential energy field, made up of the pore pressure and the gravity field, cause flow of water out of the soil mass. The pore space in a soil is reduced by this outflow and the effective stress increases with time.

To simulate the one-dimensional volume change of saturated soil, a confined compression test is often performed. The functional dependence between

vertical stress and void ratio for a soil is given in Figure 4.2. The resulting slope of the curve is defined as the coefficient of compressibility, a_v :

$$a_v = \frac{-\Delta e}{\Delta \sigma'} \quad (4.3)$$

Terzaghi observed that if the results are plotted as void ratio versus logarithm of vertical stress, the curve usually becomes a straight line at large stresses. The slope of this curve is the compression index, C_c :

$$C_c = \frac{\Delta e}{\Delta(\log \sigma')} \quad (4.4)$$

4.2 NONLINEAR THEORY

The classical theory of consolidation is commonly believed to be used extensively because it works reasonably well. More correctly, the theory may be used because (Olson and Ladd 1979):

- It is available and simple to use.
- Uncertainties often result more from poorly defined soil properties, stratigraphy, and loading conditions than from problems in analysis.
- Few well-documented case studies compare predicted versus measured consolidation behavior. In fact, there is a serious lack of experience regarding the magnitude of the final consolidation settlement, let alone the rate of consolidation.

In many field situations, such as in uranium mill tailings deposition, the mass of the soil itself provides the loading. A layer of fill deposited in a soft, wet condition that is normally consolidating under its own weight, will not have uniform soil properties throughout its depth. Further, it usually will undergo comparatively large strains in the process. The variation of the flow and consolidation parameters K , a_v , and C_v can be due to the following factors (Kuppusamy and Anandakrishnan 1971):

- decreasing void ratios at greater depths due to higher consolidation pressures
- finer materials which settle into the intergranular structure
- variation of the soil itself with depth due to nonhomogeneity.

None of these aspects of behavior can be modeled by Terzaghi's traditional consolidation theory.

In recent years interest has increased in the development of more sophisticated models, allowing such possibilities as finite strains; soil parameters

FIGURE 4.2. Idealized Void Ratio-Pressure Diagram
(Spangler and Handy 1973)

that vary with stress level, location, and time; and nonlinear stress-strain relationships to govern soil behavior (Richart 1957; Lo 1960; Davis and Raymond 1965; Barden and Berry 1965; Lee and Sills 1979, 1981).

The variation in soil parameters may be handled in many ways. Davis and Raymond (1965) developed an equation for consolidation considering the coefficient of consolidation constant, but assumed a nonlinear function of void

ratio versus effective stress. Their results closely approximated Terzaghi's results for rate of settlement, but gave a larger variation in the rate of dissipation of pore-water pressure than Terzaghi's.

Strain in a soil causes the successive shortening of the drainage path during consolidation and variation of material properties. The assumption of small strains occurs in the classical method of analysis when the drainage distance is assumed constant during consolidation. Gibson, England, and Hussey (1967) proposed a revised theory that took into account the variation of the coefficient of permeability and the compressibility of the soil mass during consolidation under a static load. Their analysis seems to produce more valid results when strains in the soil mass are large. Mesri and Rokhsar (1974) agreed, stating that the assumption of small strain in the classical theory resulted in slower pore pressure dissipation and rates of compression than would be predicted through large strain analysis.

Terzaghi's theory is founded upon the coefficient of consolidation remaining constant; for small strains this seems to produce reasonable results. However, Gibson, England, and Hussey (1967) stated that Terzaghi's theory overestimated the time rate of consolidation (Figure 4.3) but underestimated the excess pressure at a given time. This leads to an overestimation of the shear strength of a soil deposit.

When the coefficient of consolidation is held constant, consolidation depends solely on the magnitude of strain. However, with large strain, consolidation is accelerated by the further shortening of the drainage path

FIGURE 4.3. Nonlinear Saturated Consolidation (Lloret and Alonso 1980)

[Figure 4.4(a)]. When the coefficient of consolidation varies with strain, the consolidation depends on both the mode of variation and the magnitude of strain [Figure 4.4(b)-(e)]. Eulerian strain and a constant are given as ϵ and C_0 , respectively.

To illustrate, consider the coefficient of consolidation increasing monotonically with Eulerian strain (see region above the constant C_v line in Figure 4.5); as the strain develops, consolidation is further accelerated. The reduction of C_v , however, retards the accelerating process because of the shortening of the drainage path that occurs with strain. A strain invariant situation eventually results where the decrease of C_v is just enough to cancel the acceleration from strain. This is the case, $C_F = \text{constant}$ or $C_v \propto (1+\epsilon)^{-2}$ also shown in Figure 4.5. Slower consolidation occurs with increased strain below this curve, because a decrease in the coefficient of consolidation outweighs the acceleration from strain. The coefficient of consolidation, C_F , used by Gibson, England, and Hussey (1967) is defined as

$$C_F = \frac{K}{\gamma_w} \frac{1}{(1+e)} \frac{d\sigma'}{de} .$$

Curves 1 through 4 in Figure 4.5 represent C_v equal to $C_0(1+\epsilon)^2$, $C_0(1+0.5\epsilon)$, $C_0(1-0.5\epsilon)$, and $C_0(1+\epsilon)^{-3}$, respectively.

One of the fundamental assumptions of Terzaghi's theory of consolidation is that the coefficient of permeability, K , is a constant. The errors arising from this assumption depend on the magnitude of the load increment and the resulting void ratio changes. Particularly in hydraulic fills, such as tailings piles, the variation in permeability is extremely important. As has already been mentioned, the rate of dissipation of excess pore-water pressure may be very great as a result of drainage paths formed by the sand component. On the other hand, this rate of dissipation may be restricted because of the heterogenous arrangement of the slimes and clays within a pile. Typically, the finer fractions are expected to settle into the intergranular structure during sedimentation (Figure 3.3), suggesting a greater horizontal than vertical permeability.

FIGURE 4.4. Degree of Settlement Versus Time for Different Forms of C_v and Magnitudes of Strain (Lee and Sills 1979)

FIGURE 4.5. Variations of the Coefficient of Consolidation C_v with the Eulerian Strain (Lee and Sills 1979)

5.0 LIMITATIONS OF THE PRINCIPLE OF EFFECTIVE STRESS AS APPLIED TO UNSATURATED SOILS

Classical soil mechanics have been concerned chiefly with soil in which the voids are completely filled with water. For such soils, Terzaghi formulated his principle of effective stress, on which the classical theory of consolidation depends. This principle is invaluable to the engineer who must predict soil behavior, and is regarded by many foundation engineers as axiomatic. In addition, the principle of effective stress for saturated soils lies at the foundation of most theories and practices of modern soil mechanics.

Uranium mill tailings, like many engineering soils in semi-arid regions, do not conform to the classical pattern because they are not fully saturated. Besides the questions of consolidation and shear strength, the problems of unsaturated soils also include heave caused by swelling of desiccated clays and additional settlement from collapse of grain structure upon wetting the loaded soil. Because these phenomena are not encountered in saturated soils, Terzaghi's theory does not account for them.

5.1 EFFECTIVE STRESS IN SATURATED SOILS

The principle of effective stress, as formulated by Karl Terzaghi (1925, 1936) may be stated in the form of two propositions:

1. All measurable effects in a change of stress such as compression, a change of shearing resistance, and distortion result solely from changes in effective stress.
2. The effective stress, σ' , is defined as the excess of the total applied stress, σ , over the pore pressure, U , i.e.:

$$\sigma' = \sigma - U \quad (5.1)$$

The work of Rendulic (1936), Bishop and Eldin (1950), Henkel (1959, 1960), and Skempton (1960) has adequately verified the validity of the principle of effective stress for saturated soils.

The principle of effective stress has been restated several times since Terzaghi first used it in 1925; however, the assumptions on which it is based and the limitations to which it is subjected are seldom completely discussed. Terzaghi (1936) stated the principle of effective stress in the following terms:

"The stresses in any point of a section through a mass of soil can be computed from the total principal stress $\sigma_1, \sigma_2, \sigma_3$ which act in this point. If the voids of the soil are filled with water under a stress U , the total principal stresses consist of two parts. One part, U , acts in the water and in the solid in every direction with equal intensity. It is called the neutral stress (or pore-water

pressure). The balance $\sigma'_1 = \sigma_1 - U$, $\sigma'_2 = \sigma_2 - U$, $\sigma'_3 = \sigma_3 - U$ represents an excess over the neutral stress U , and it has its seat exclusively in the solid phase of the soil."

"This fraction of the total principal stresses will be called the effective principal stresses..... A change in the neutral stress U produces practically no volume change and has practically no influence on the stress conditions for failure..... Porous material (such as sand, clay, and concrete) react to a change of U as if they were incompressible and as if their internal friction were equal to zero. All the measurable effects of a change in stress, such as compression, distortion, and a change of shearing resistance are exclusively due to changes in the effective stress σ'_1 , σ'_2 , σ'_3 . Hence, every investigation of the stability of a saturated body of soil requires the knowledge of both the total and the neutral stresses."

σ_1 , σ_2 , and σ_3 equals the major, intermediate, and minor principal stresses, respectively.

5.2 EFFECTIVE STRESS IN UNSATURATED SOILS

The principle of effective stress is a hypothesis which attributes certain properties exhibited by a soil to the action of an effective stress in the soil. The success of the effective stress equation for saturated soils has led many researchers to attempt to extend the principle of effective stress to the case of unsaturated soils. Soil moisture suction, or pressure deficiency in the soil water, contributes directly to the effective stress in the soil only for the small range of pressure where the soil remains fully saturated (Aitchison and Donald 1956). However, as soon as air enters the pore space (Figure 3.2) the pressure in the pore-water no longer acts over the whole cross-sectional area and Equation (5.1) no longer applies.

Attempting to account for this behavior in unsaturated soils, Bishop (1959) modified Terzaghi's classical expression for effective stress for saturated soils to the following form:

$$\sigma' = (\sigma - U_a) + \chi(U_a - U_w) \quad (5.2)$$

where

U_a = pore-air pressure

U_w = pore-water pressure

$(U_a - U_w)$ = matrix suction

χ = parameter that depends on the degree of saturation, soil type, and hysteresis effects due to wetting, drying, or stress change.

In addition, Jennings (1961), Croney et al. (1958), and Aitchison (1960) all proposed modified forms of effective stress equations to account for the two-phase nature of the pore fluid in an unsaturated soil. These and other equations are summarized in Table 5.1. The various expressions in Table 5.1 defining effective stress are in agreement. Because the expression proposed by Bishop (1959) included a term for the pressure in the gas phase, it is used most often.

To retain the present understanding of the effective stress law in which the effective stress is considered to be made up of two components, one resulting from total normal pressure and the other from pressures exerted by the fluid in the pores, Equation (5.2) can be rewritten in the form:

$$\begin{aligned}\sigma' &= \sigma - [X U_w + (1-X) U_a] \\ &= \sigma - U^*\end{aligned}\quad (5.3)$$

The quantity $[X U_w + (1-X) U_a]$ may be considered as an equivalent pore-pressure U^* (i.e., that portion of the effective stress in a soil resulting from fluid pressures in the pores). The ordinary Terzaghi law results if the soil is saturated, $X=1$.

Aitchison (1960) derived a theoretical expression for X that showed the dependence of X on suction as well as on degree of saturation. The parameter X was given by the expression:

$$X = \frac{\sigma'}{p_c} = S_r + \frac{1}{p_c} \sum_0^{p_c} 0.3 p_c \Delta S_r \quad (5.4)$$

where

$$p_c = (U_a - U_w)$$

S_r = degree of saturation.

By assuming various pressure deficiency/degree of saturation relationships, Donald (1960) derived a theoretical $X:S_r$ relationship shown in Figure 5.1.

Bishop et al. (1960) showed that X values obtained from drained triaxial tests on samples of Braehead silt, that was placed as a slurry mixture and drained by suction, compared reasonably well with theoretical values calculated by Donald (1960). A general relationship between compacted soils did not exist. Also, the value of the parameter X was not necessarily identical to the value in the strength-effective stress relationship, as it was in the relationship between volume change and effective stress.

Bishop and Donald (1961) performed triaxial tests to examine the validity of the principle of effective stress as expressed by Equation (5.2). Jennings and Burland (1962) presented experimental data that, while showing Equation (5.2) to be statically correct, indicated that effective stress did not

TABLE 5.1. Effective Stress Equations for Unsaturated Soils
(Fredlund and Morgenstern 1977)

Author	Equation	Description of Variables
Bishop 1959	$\sigma' = \sigma - U_a + X(U_a - U_w)$	X = parameter related to degree of saturation U_a = the pressure in gas and vapor phase
Croney 1958 et al.	$\sigma' = \sigma - \beta' U_w$	β' = holding or bonding factor which is measure of number of bonds under tension effective in contributing to soil strength
Lambe 1960	$\sigma = \sigma_a a_m + U_a a_a + U_w a_w + R - A$	a_a = fraction of total area that is air-air contact
Aitchison 1960	$\sigma' = \sigma + \psi p''$	ψ = parameter with values ranging from zero to one p'' = pore-water pressure deficiency
Jennings 1961	$\sigma' = \sigma + \beta p''$	β = statistical factor of same type as contact area; should be measured experimentally in each case
Richards 1966	$\sigma' = \sigma - U_a + X_m(h_m + U_a) + X_s(h_s + U_a)$	X_m = effective stress parameter for matrix suction h_m = matrix suction X_s = effective stress parameter for solute suction h_s = solute suction

control the behavior of most unsaturated soils, rather, functions of the variables σ and $[XU_w + (1-X)U_a]$ controlled the behavior. Hence, a unique relationship does not exist between void ratio and effective stress as defined by Equation (5.2) for most unsaturated soils below a critical degree of saturation, S_{cr} . Depending largely on grain size, results have shown that the critical degree of saturation might be as low as 20% for coarse granular soils, such as sand; 40 to 50% for silts; and as high as 85% for clayey soils.

- 1 - COMPACTED BOULDER CLAY ($-2\mu=4\%$)
 - 2 - COMPACTED SHALE ($-2\mu=22\%$)
 - 3 - BRAEHEAD SILT BISHOP AND DONALD (1961)
 - 4 - SILT ($-2\mu=3\%$)
 - 5 - SILTY CLAY ($-2\mu=23\%$)
 - 6 - THEORETICAL DONALD (1960)
- } BISHOP
et al (1960).

FIGURE 5.1. Curves of Parameter X Against Degree of Saturation for Various Soils (Jennings and Burland 1962)

Matyas (1963) also found that the parameter χ could not be readily obtained, particularly for volume change, because of the dominating influence of structural effects. Using experimental evidence and an idealized model, Matyas concluded that results from comparison tests were valid only if the structural effects could be minimized. To achieve this the requirement of similar saturation history would have to be satisfied.

Bishop and Blight (1963) re-examined the principle of effective stress for both saturated and unsaturated soils. Earlier Equation (5.2) stated that a change in the term $(U_a - U_w)$ did not relate directly to a change in neutral stress because the term represents a pressure difference that, in general, acts only over a part of the surface area of the soil particles (Bishop 1959). A change in the value of $(U_a - U_w)$ was almost invariably accompanied by a significant change in χ . Further, the existence of large surface tension forces within the soil led to differences in soil structure in specimens following apparently similar effective stress paths. These concepts implied that the value of χ was less unique than usually assumed. For the case of volume change under all-around pressure, Bishop and Blight suggested that plots of void ratio versus $(\sigma - U_a)$ and $(U_a - U_w)$ were more appropriate to describe consolidating behavior of unsaturated soils. They noticed the change in qualitative behavior from swelling to collapse at different stress levels: "The load at which a compacted fill changes over from swelling to decreasing in volume on saturation depends on compaction effort, placement, water content, and soil type, particularly on the clay fractions." They further stated that if the shape of the surface could be obtained for a specified stress path, the following equation could be evaluated:

$$\sigma' = (\sigma - U_a) + f (U_a - U_w) \quad (5.5)$$

Bishop and Blight (1963) stated that "the effective stress is, by definition, that function of total stress and pore pressure which controls the mechanical effects of a change in stress, such as volume change and a change in shear strength. The principle of effective stress is the assertion that such a function exists, with determinate parameters, under a given set of conditions."

This definition is based on Terzaghi's (1936) expression for saturated soils and may be considered in two parts:

1. The changes in volume and shear strength in stress state are entirely caused by the change in effective stresses (i.e., shear strength and void ratio are unique functions of effective stress).
2. The effective stress that is responsible for the mechanical effects in a soil element is uniquely determined by the total stress and pore pressure.

Therefore:

$$\sigma' = f (\sigma, U_a, U_w)$$

6.0 CONSTITUTIVE RELATIONS FOR VOLUME CHANGE IN UNSATURATED SOILS

6.1 STATE OF UNSATURATED SOIL AND STATE PARAMETERS

Numerous effective stress equations have been proposed for unsaturated soils. Common to these proposed equations is the incorporation of a soil parameter to couple together more than one stress variable. However, as previously stated, the description of the stress-state should consist of independent stress variables.

The unsaturated soil is considered to be a three-phase material, composed of solids, liquids, and gases. The solid phase consists of insoluble mineral particles that are individually considered to be incompressible, but through aggregation may form a compressible mass. The void geometry of the soil responds to changes in both external loads and internal fluid conditions as a result of the varying compressibilities of the different phases of the system. Hence, the state of an element can be specified by the totality of quantities that specify the element under physical conditions existing in a soil. The physical variants of the soil that describe the complete state of a soil element, without reference to its previous history, are called state parameters.

Usually, for at least saturated, normally consolidated clays and cohesionless granular soils, the state of the soil can be specified by the state of stress and void ratio. In the case of unsaturated soils, where the air-water interface exists in the system, additional parameters are needed to describe the state of unsaturation and soil fabric or soil structure. Therefore, the state of an unsaturated soil element can be specified by the parameters, σ , e , S_r , and λ . Where σ is the stress tensor, e is void ratio, S_r is degree of saturation, and λ is the structure or fabric of the soil. The soil structure, which refers to the geometric configuration of the particles in a soil aggregate, has a profound effect on the physical properties of the soil. The parameter λ probably will not be a simple scalar quantity, and no satisfactory quantitative measure has been devised to describe the structure.

Recently, two independent stress variables have been proposed for unsaturated soils (Fredlund and Morgenstern 1976). These can be supported by a stress analysis consistent with that used in multiphase continuum mechanics. The assumptions in the stress analysis (i.e., that the solids are incompressible and the soil is chemically inert) are comparable to those associated with the stress-state description for a saturated soil.

The stress analysis showed that any two of three possible stress variables could be used to describe the stress-state of an unsaturated soil. Possible combinations are: 1) $(\sigma - U_a)$ and $(U_a - U_w)$; 2) $(\sigma - U_w)$ and $(U_a - U_w)$; and 3) $(\sigma - U_a)$ and $(\sigma - U_w)$. In the above, σ is total stress, U_a is pore-air pressure, and U_w is the pore-water pressure. Fredlund and Morgenstern (1976), after attempts to apply the stress-state variables to earth pressure problems, concluded that the $(\sigma - U_a)$ and $(U_a - U_w)$ combination was the most satisfactory because the effects of total stress changes and pore-water pressure

changes can be separated, which is advantageous from a conceptual and analytical standpoint. For many practical problems, the air pressure may be considered atmospheric.

6.2 STATE SURFACE

In Biot's (1941) considerations of three-dimensional consolidation, the soil structure was assumed to behave as a linear, reversible, isotropic material. In addition, the soil was considered to be unsaturated, such that the pore-water contained occluded air bubbles. The effective stress-state variables, $(\sigma - U_w)$, and the pore fluid pressure, U_w , were used to describe the deformation. Further, to describe how the water content changes depended on the stress-state variables, Biot also proposed a second constitutive relationship. Four distinct physical soil properties in total were required to link the stress and deformation state variables.

Accordingly, Bishop and Blight (1963) concluded that the stress path of the components of the total stress, $(\sigma - U_a)$, and of the matrix suction, $(U_a - U_w)$, must be taken into account. This places a severe restriction on the type of prediction that could be made. Component $(\sigma - U_a)$ involves both normal and shear stresses at interparticle contacts; the component $(U_a - U_w)$ involves only normal stresses, and thus generates intergranular stress by different mechanisms. Bishop and Blight (1963) suggested plotting volume change data versus the stress-state variables to form a three-dimensional plot to observe this differing mechanism.

Following the work of Bishop and Blight (1963) and Aitchison (1967), Matyas and Radhakrishna (1968) provided some of the earliest results expressing void ratio and degree of saturation as functions of $(\sigma - U_a)$ and $(U_a - U_w)$ (see Figures 6.1 and 6.2). They performed tests on identically prepared mixtures of 80% flint powder and 20% kaolin, in which the total air and water pressures were controlled in isotropic and K_0 compression tests. To test the sample's uniqueness, the constitutive surface of void ratio and degree of saturation versus the stress-state variables were traced out, using different stress paths. Matyas and Radhakrishna observed that when the $(\sigma - U_a)$ stress was increased or the $(U_a - U_w)$ stress decreased, the results of the void ratio traced out a single warped surface, with the soil structure always decreasing in volume. In addition, they found the following expression for the intergranular stress in unsaturated soil, based on the idealized soil model shown in Figure 6.3:

$$\sigma' = \sigma - U_a + (A_w + A_t) (U_a - U_w) \quad (6.1)$$

where

A_w = the percentage of area occupied by water in a wavy surface across the soil

A_t = the percentage of area of the air-water menisci.

FIGURE 6.1. State Surface for Void Ratio (Matyas and Radhakrishna 1968)

FIGURE 6.2. State Surface for Degree of Saturation (Matyas and Radhakrishna 1968)

FIGURE 6.3. Model of Unsaturated Soil (Matyas and Radhakrishna 1968)

A_t and A_w depend on factors such as particle arrangement and composition, degree of saturation, and stress and strain history. Virtually all of these factors controlling deformation and strength are presented in X.

Barden, Madedor, and Sides (1969) performed tests on low to high plasticity illite clay samples. They, too, controlled the total air and water pressures while investigating the effect of various stress paths during K_0 loading conditions. In all cases, the applied stress, $(\sigma - U_a)$, was increased subsequent to the initial conditions. Except in a few cases, the matrix suction or capillary pressure, $(U_a - U_w)$, was also increased subsequent to the initial conditions. Their results indicated that volume change of the soil structure depends on the direction of saturation.

Fredlund and Morgenstern (1976) executed a detailed analysis of the stress-state variables for unsaturated soil. They demonstrated that the combination, $(\sigma - U_a)$ and $(U_a - U_w)$, was an acceptable fundamental set of stress-state variables for the unsaturated soil. Further, their experimental results showed that the functional relation between void ratio and the two stress-state variables was unique, at least locally. Current evidence shows that the two stress-state variables account reasonably well for the unknown behavior of volume change in an unsaturated soil, including swelling and shrinkage of the soil upon wetting.

6.3 VOLUME CHANGE BEHAVIOR

A qualitative explanation for the volume change behavior of unsaturated soils, using an idealized model of the soil structure, was given by Burland (1965). A similar concept was proposed by Newland (1965), suggesting that there were two parts to the effective stress in unsaturated soils. The first part was caused by forces arising at the points of contact between particles and acting normal to the tangent at the contact point (i.e., endogenic effective stress), and a second part resulted from forces external to the points of contact between particles and related with tangential or shear forces (i.e., exogenic effective stress). Newland postulated that if the endogenic effective stresses in a soil were reduced in any manner, the soil would increase or decrease in volume, depending on whether the applied stress was respectively smaller or larger than the equilibrium-applied stress, which could be supported by the shear resistance arising from the endogenic stresses.

Another assumption made concerning the use of effective stresses to predict volume change behavior is that the compressibility of the soil, in terms of effective stresses, is independent of water content and degree of saturation. However, such an assumption has not been justified through experimentation for a wide range of degree of saturations. It has been shown that if the compressibility of the soil is a variable quantity that depends on the physical state of the soil, then the volume changes of two identical solid elements subjected to different stress and saturation histories, but apparently undergoing the same change in effective stress, could differ widely in nature and magnitude.

To satisfy the principle of effective stress, when determining equivalents of changes in pore pressure and in total stress, the same stress paths must be followed and the same time rates must be used. This is because soils are not ideal elastic materials. Otherwise, errors will occur similar to those that result when the principle of superposition is applied to a structure not obeying Hooke's law. Further, when the permeability is not equal to unity and its value for volume change depends on the ratio of compressibilities, the magnitude and sign of the stress changes influence the value of the permeability itself.

6.4 CONSTITUTIVE RELATIONS FOR VOLUME CHANGE

The three most commonly used volume-weight variables are the void ratio, degree of saturation, and moisture content. Although relations between these basic variables are fixed for saturated soils, this is not the case for unsaturated soils. A change in volume-weight conditions can involve changes anywhere below the 100% saturation line for an unsaturated soil. Commonly used volume-weight relations, for a selected specific gravity of solids, are shown in Figure 6.4. To describe all volume-weight relations for an unsaturated soil any two of the basic volume-weight variables must be known. Therefore, where only one constitutive relation is required to describe the volume change associated with all phases of a saturated soil, two are required for an unsaturated soil.

FIGURE 6.4. Volume-Weight Relations for Unsaturated Soils (Fredlund 1979)

The deformation variables needed to describe the changes in each phase of an unsaturated element can be derived from the continuity requirement for a multiphase continuum. If the soil particles are assumed incompressible and the volume change of the air-water interface assumed internal to the element, then the continuity requirement for an element reduces to:

$$\epsilon = \frac{\Delta V}{V} = \frac{\Delta V_w}{V} + \frac{\Delta V_a}{V} \quad (6.2)$$

where

- ϵ = volumetric strain
- V = total volume
- V_w = volume of water in the element
- V_a = volume of air in the element.

Fredlund and Morgenstern (1976) proposed and tested constitutive relations to link the stress deformation state variables. Beginning with Equation 8 of Fredlund and Morgenstern (1976) and considering nonlinear variations of the soil properties, the proposed constitutive relationship for volume change in an unsaturated soil is given by:

$$\epsilon = \frac{1}{V} \frac{\partial V}{\partial(\sigma - U_a)} \cdot d(\sigma - U_a) + \frac{1}{V} \frac{\partial V}{\partial(U_a - U_w)} \cdot d(U_a - U_w) \quad (6.3)$$

Assuming that $\epsilon \approx dV/V$ and multiplying through Equation (6.3) by the expression $V/(V - V_v)$, where V_v is the volume of voids, gives:

$$\begin{aligned} \frac{dV}{(V - V_v)} &= \frac{1}{(V - V_v)} \frac{\partial V}{\partial(\sigma - U_a)} \cdot d(\sigma - U_a) \\ &+ \frac{1}{(V - V_v)} \frac{\partial V}{\partial(U_a - U_w)} \cdot d(U_a - U_w) \end{aligned} \quad (6.4)$$

The void ratio, e , is defined as:

$$e = \frac{\text{volume of voids}}{\text{volume of solids}} = \frac{V_v}{V_s} = \frac{V_v}{V - V_v} \quad (6.5)$$

Since the solid particles of the soil are effectively incompressible, the volume of the solids, V_s , is constant. Hence, the differential of Equation (6.5) is:

$$de = \frac{dV_v}{V_s} \quad (6.6)$$

Using $V_s = V - V_v$ and Equation (6.6), the partial derivative terms in Equation (6.4) can be written as:

$$\frac{1}{V - V_v} \frac{\partial V}{\partial(\sigma - U_a)} = \frac{1}{V_s} \frac{\partial(V_v + V_s)}{\partial(\sigma - U_a)} = \frac{1}{V_s} \frac{\partial V_v}{\partial(\sigma - U_a)} = \frac{\partial e}{\partial(\sigma - U_a)} \quad (6.7)$$

and

$$\frac{1}{V - V_v} \frac{\partial V}{\partial(U_a - U_w)} = \frac{1}{V_s} \frac{\partial(V_v + V_s)}{\partial(U_a - U_w)} = \frac{1}{V_s} \frac{\partial V_s}{\partial(U_a - U_w)} = \frac{\partial e}{\partial(U_a - U_w)} \quad (6.8)$$

Substitution of Equations (6.6), (6.7), and (6.8) into Equation (6.4) gives:

$$de = \frac{dV_v}{V_s} = \frac{\partial e}{\partial(\sigma - U_a)} \cdot d(\sigma - U_a) + \frac{\partial e}{\partial(U_a - U_w)} \cdot d(U_a - U_w) \quad (6.9)$$

Equation (6.9) is recognized as the total differential of the void ratio e ; hence, the constitutive relationship for the void ratio as a function of the two stress-state variables is:

$$e = f[(\sigma - U_a), (U_a - U_w)] \quad (6.10)$$

where

f = the constitutive functional relationship that will be measured as a function of the two stress-state variables $[(\sigma - U_a), (U_a - U_w)]$.

From Equation (6.9) two coefficients may be defined:

$$m_1^e = \frac{\partial e}{\partial (\sigma - U_a)} \quad (6.11)$$

and

$$m_2^e = \frac{\partial e}{\partial (U_a - U_w)} \quad (6.12)$$

as the slope (i.e., partial derivative) of the measured relationship defined by Equation (6.10); hence, Equation (6.9) may be simplified to:

$$de = \frac{dV}{V_s} = m_1^e \cdot d(\sigma - U_a) + m_2^e \cdot d(U_a - U_w) \quad (6.13)$$

The volume change for an unsaturated soil, assuming incompressible soil particles, is given by:

$$dV = dV_v = V_s \cdot de \quad (6.14)$$

REFERENCES

- Aitchison, G. D., and I. B. Donald. 1956. "Effective Stresses in Unsaturated Soils." In Proce. 2nd Aust. N. Z. Conf. Soil Mech., pp. 192-199.
- Aitchison, G. D. 1960. "Relationships of Moisture Stress and Effective Stress Functions in Unsaturated Soils." In Conference on Pore Pressure and Suction in Soils, pp. 47-52. Butterworths, London.
- Aitchison, G. D. 1967. "Separate Roles of Site Investigation Quantification of Soils, Properties, and Selection of Operational Environment in the Determination of Foundation Design on Expansive Soils." In Proceedings 3rd Asian Regional Conference on Soil Mechanics and Foundation Engineering, Vol. 2, Haifa, Israel.
- Barden, L., and P. L. Berry. 1965. "Consolidation of Normally Consolidated Clay." Proc. Am. Soc. Civ. Engrs. SM5(4481):15-35.
- Barden, L., A. O. Madedor and G. R. Sides. 1969. "Volume Change Characteristics of Unsaturated Clay." J. Soil Mech. Fdns. Div. Am. Soc. Civ. Engrs., 95, SM1:33-51.
- Biot, M. A. 1941. "General Theory of Three Dimensional Consolidation." J. Appl. Phys. 12:155-164.
- Bishop, A. W., and G. Eldin. 1950. "Undrained Triaxial Tests on Saturated Sands and Their Significance in the General Theory of Shear Strength." Geotechnique 2(1):13-32.
- Bishop, A. W. 1959. "The Principle of Effective Stress." Teknik. Ukelbad (Norway) 39:859-863.
- Bishop, A. W., et al. 1960. "Factors Controlling the Strength of Partly Saturated Cohesive Soils." In ASCE Research Conference on Shear Strength of Cohesive Soils, pp. 503-532, University of Colorado, Boulder, Colorado.
- Bishop, A. W., and I. B. Donald. 1961. "The Experimental Study of Partly Saturated Soil in the Triaxial Apparatus." In Proceedings of the Fifth International Conference on Soil Mechanics and Foundation Engineering, pp. 13-21.
- Bishop, A. W., and G. E. Blight. 1963. "Some Aspects of Effective Stress in Saturated and Partly Saturated Soils." Geotechnique 13(3):177-197.
- Blight, G. E. 1969. "Shear Stability of Dumps and Dams of Gold Mining Waste." The Civil Engineer in South Africa, ASCE, pp. 49.54.

- Burland, J. B. 1965. "Some Aspects of the Mechanical Behavior of Partly Saturated Soils." In Moisture Equilibrium and Moisture Changes in Soils Beneath Covered Areas, pp. 270-278, Butterworths, Sydney.
- CSU (Colorado State University). 1980. "Inactive Uranium Mill Tailings Sites Analysis of Stability." Quarterly Report July 1980-September 1980, Civil Engineering Department, Colorado State University, Ft. Collins, Colorado.
- Croney, D., J. D. Coleman, and W. P. M. Black. 1958. "The Movement and Distribution of Water in Soil in Relation to Highway Design and Performance." Highway Research Board, Sp, Report No. 40, Washington, D.C.
- Davis, E. H., and G. P. Raymond. 1965. "A Non-Linear Theory of Consolidation." Geotechnique 15(2):161-173.
- Donald, I. B. 1960. "Discussion-Proceedings of Conference on Pore Pressure and Suction in Soils." Butterworth, London.
- Fredlund, D. G., and N. R. Morgenstern. 1976. "Constitutive Relations for Volume Change in Unsaturated Soils." Can. Geotech. J. 13(3):261-276.
- Fredlund, D. G., and N. R. Morgenstern. 1977. "Stress State Variables for Unsaturated Soils." ASCE J. Geotech. Engr. Div. 103 (GT5):447-466.
- Fredlund, D. G. 1979. "Secqnd Canadian Geotechnical Colloquim: Appropriate Concepts and Technology for Unsaturated Soils." Can. Geotechn. J. 16:121-139.
- Gibson, R. E., G. L. England and M. J. L. Hussey. 1967. "The Theory of One-Dimensional Consolidation of Saturated Clays. I. Finite Non-Linear Consolidation of Thin Homogenous Layers." Geotechnique 17:261-273.
- Henkel, D. J. 1959. "The Relationship Between the Strength, Pore-Water Pressure, and Volume Change Characteristics of Saturated Clays." Geotechnique 9(3):119.
- Henkel, D. J. 1960. "The Relationship Between the Effective Stresses and Water Content in Saturated Clays." Geotechnique 10(2):41.
- Jennings, J. E. B. 1961. "A Revised Effective Stress Law for Use in the Prediction of the Behavior of Unsaturated Soils." In Pore Pressure and Suction in Soils, pp. 26-30, Butterworths, London.
- Jennings, J. E., and J. B. Burland. 1962. "Limitations to the Use of Effective Stresses in Partially Saturated Soils." Geotechnique 12(2):125-144.
- Kuppusamy, T., and M. Anandakrishnan. 1971. "Non-Linear Consolidation Characteristics of Thick Clay Layer with General Variation of Parameters." Indian Geotech. J. 1(3):237-248.

- Lambe, T. W. 1960. "A Mechanistic Picture of Shear Strength in Clay." In Proceedings of the ASCE Research Conference on Shear Strength of Cohesive Soils, pp. 555-580, University of Colorado, Boulder, Colorado.
- Lambe, T. W., and R. V. Whitman. 1969. Soil Mechanics. John Wiley, New York.
- Lee, K., and G. C. Sills. 1979. "A Moving Boundary Approach to Large Strain Consolidation of a Thin Soil Layer." In Proc. 3rd Int. Conf. on Num. Methods in Geomech., pp. 163-173, Balkema, Rotterdam.
- Lee, K., and G. C. Sills. 1981. "The Consolidation of a Soil Stratum, Including Self-Weight Effects and Large Strains." International J. for Num. and Anal. Methods in Geomech. 5:405-428.
- Lloret, A., and E. E. Alonso. 1980. "Consolidation of Unsaturated Soils Including Swelling and Collapse Behavior." Geotechnique 30(4):449-477.
- Lo, K. Y. 1960. "Discussion on Rowe, Measurement of the Coefficient of Consolidation of Lacustrine Clay." Geotechnique 10(1):36-39.
- Matyas, E. L. 1963. Compressibility and Shear Strength of Compacted Soils. Ph.D. Thesis, University of London.
- Matyas, E. L., and H. S. Radhakrishna. 1968. "Volume Change Characteristics of Partially Saturated Soils." Geotechnique 18:432-448.
- Mesri, G., and A. J. Rokhsar. 1974. "Theory of Consolidation for Clay." ASCE J. of the Geotech. Eng. Div. 100(GT8):889-904.
- Mittal, H. K. 1974. Design and Performance of Tailings Dams. Ph.D. Thesis, Department of Civil Engineering, The University of Alberta, Edmonton, Alberta.
- Newland, P. L. 1965. "The Behavior of Soils in Terms of Two Kinds of Effective Stress." In Proceedings of a Conference on Engineering Effects of Moisture Changes in Soils., pp. 78-92, Texas A and M Press.
- Olson, R. E., and C. C. Ladd. 1979. "One-Dimensional Consolidation Problems." ASCE J. of the Geotech. Eng. Div. 105 (GT1):11-30.
- Rendulic, L. 1936. "Relation Between Void Ratio and Effective Principal Stresses for a Remolded Silty Clay." In Proc. 1st Int. Conf. Soil Mech., 3:48-51.
- Richards, B. G.. 1966. "The Significance of Moisture Flow and Equilibrium in Unsaturated Soils in Relation to the Design of Engineering Structures Built on Shallow Foundations in Australia." Symposium on Permeability and Capillarity, American Society for Testing Materials, Atlantic City, New Jersey.

- Richart, F. E. 1957. "A Review of the Theories for Sand Drains." Proc. Am. Soc. Civ. Engr. SM3 (1301).
- Skempton, A. W. 1960. "Effective Stress in Soils, Concrete, and Rocks." In Proc. Conf. Pore Pressure, pp. 4-16, Butterworths, London.
- Spangler, M. G., and R. L. Handy. 1973. Soil Engineering. Intext Educational Publishers, New York.
- Terzaghi, K. 1925. Erdbaumechanik auf Bodenphysikalischer Grundlage. Deuticke, Vienna.
- Terzaghi, K. 1936. "The Shearing Resistance of Saturated Soils." In Proceedings of the First International Conference on Soil Mechanics, Vol. 1.
- Van Zyl, D., and A. MacG. Robertson. 1980. "Subsurface Drainage of Tailings Impoundments: Some Design, Construction, and Management Considerations." In Proceedings of the Third Symposium Uranium Mill Tailings Management, pp. 153-177, Colorado State University, Fort Collins, Colorado.

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