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This report covers the research progress on the referenced DOE Grant, during the calendar year 1992. The new results obtained during this period are described below, keyed to the references listed on the last two pages of the report. These references are listed in three categories:

The *first* category of references ([1] - [8]) comprises papers which either have been published in 1992 or are scheduled to appear in 1993, and which reflect research done primarily prior to 1992, even though in some cases some minor revisions have been made in 1992. Contents of these papers, and their contributions have been discussed in last year's report, and hence will not be included here.

The *second* category of references, namely [9] - [13], describe research which was initiated during the last quarter of 1991, and completed during the first half of 1992. The first two, [9] and [10], were presented at two conferences in 1992, and some preliminary results on their contents were included in the report submitted last year. The other three, all accepted for refereed publications, present results which are the outcome of research during the first half of 1992, as well as some preliminary research conducted during the last part of 1991. Full versions of these papers were included as a part of the first progress report of this year, submitted about three weeks ago.

The *third* category of papers, namely [14] - [18], all present new research initiated and carried out in 1992, primarily during its second half. Two of these, [17] and [18], were in fact completed only a week ago, and are the culmination of our major research efforts during the latter part of 1992.

Following this prelude, we now proceed with a description of our research findings in 1992, keyed to references [9] - [18]:

One general area of research we have concentrated our efforts on is the *robustness* of a controller, designed based on a nominal linear system, to unmodeled fast dynamics and external norm-bounded disturbances. The issue here is not only the robustness analysis of a given controller, but also, and more importantly, the design of an *optimally robust* controller that will show "minimum" sensitivity to such structural and external perturbations. We have cast this problem in the framework of H^∞ -optimal control, and have modeled the unmodeled dynamics using the mathematical set-up of *singular perturbations*. Paper [8], which will appear in the March 1993 issue of *Automatica* had introduced such a formulation, and solved the problem for the case when the controller has access to perfect state measurements. Research initiated in the second half of 1991, and completed in 1992, as reported in [9], [10], [12] and [13], has advanced this theory in two significant directions: the cases of imperfect state measurements and sampled-data measurements. These extensions have required the development of new tools of analysis and use of new lines of thought. In the "imperfect state measurements" case (first discussed for the time-invariant case in [9], and then for the more general time-variant and finite-horizon case in [12]), the measurement channel carrying the state information is noisy, with the noise being due to an unknown norm-bounded disturbance. The system dynamics are described by a vector-valued singularly perturbed linear differential equation, with the precise value of the singular perturbation parameter (ϵ) not known. Furthermore, the system dynamics are also subjected to

an unknown deterministic disturbance, which is again bounded in norm. The cost function adopted for performance evaluation and robustness analysis under different controllers is quadratic (in the state, control and disturbance variables). The objective is to design an ϵ -independent controller, based on the parameters of the unperturbed (nominal) system and using the information received from the imperfect measurement channel, that will deliver the best achievable performance for the perturbed, full-order system. In [12] we present a complete solution to this problem when the time-horizon of interest is not necessarily infinite, thus allowing for designs with good transient properties. For a brief description of the results of [12], let $\gamma^*(\epsilon)$ denote the ϵ -independent H^∞ -optimal performance of the perturbed system, and γ_s, γ_f denote respectively the best (ϵ -independent) performances of the “slow” and “fast” subsystems, under appropriate “slow” and “fast” quadratic cost functions. Then, our first result is that as the parameter ϵ approaches zero, the quantity $\gamma^*(\epsilon)$ is bounded from above by the maximum of γ_s and γ_f . This is a significant result, because it relates the asymptotic performance of the full-order system to performances of two ϵ -independent systems, and also shows that there is generally no continuity at $\epsilon = 0$ (since it is not necessarily true that $\gamma^*(0) = \gamma_s$). We then construct in the paper a time-varying controller, of order equal to the dimension of the slow state, and show that under some conditions (which require the fast subsystem be open-loop stable, and its open-loop performance not exceed γ_s) it achieves a desirable performance ($> \gamma_s$) for the full-order system, for small values of ϵ . If these conditions do not hold, then we have developed a sequential design procedure, that yields again an ϵ -independent, but now a composite controller, that uses some properties of the fast subsystem. This design is quite different from the one developed in [8] for the perfect state information case, and the resulting reduced-order controllers exhibit satisfactory performance, as documented (both analytically, and through simulations) in the paper. The counterpart of these results in the sampled-state case has been obtained in [13], where we have shown that a certain design based on the slow subsystem only leads to controllers yielding any achievable performance for the full-order system. Hence, by sampling the measurements, one can improve robustness of the design in the presence of unmodeled dynamics, of course at the expense of some degradation in the nominal performance.

A second topic we have worked on is concerned with the global stabilization of uncertain *nonlinear* systems, in which context we have studied the further merits of the *backstepping* design procedure introduced in our earlier work reported in [5] and [6]. In this approach, the system uncertainties take the form of unknown functions of the state, having known nonlinear functional bounds, and one constructs appropriate Lyapunov functions using various *integrator backstepping* schemes. The earlier designs had used quadratic Lyapunov functions, since they seemed to be the most natural in terms of the ease of control law construction and stability analysis. In [11], however, we show that the quadratic Lyapunov function leads to unnecessarily “hard” control laws having undesirable high-gain properties. We then present a new (nonquadratic) Lyapunov function and use it to design “softer” control laws which exhibit the high-gain properties to a much lesser extent. We show that the “soft” designs eliminate the chattering exhibited by the “hard” designs and achieve the same or better performance with less control effort. It is anticipated that these results will have applications in many other backstepping procedures.

We now describe the contents and contributions of the five papers in the third category. The first one in the list, [14], deals with generalized algebraic and differential Riccati equations, and obtains some fundamental results on the dependence of their solutions on some key parameters. Among the issues studied are the monotonicity of the dependence of the solution on the parameters, asymptotic stability of the solution and its rate of convergence, and the occurrence of *conjugate points* and their dependence on the given

parameters. These results find applications in different robustness studies, one of which has been in showing that the bound obtained in [8] for a reduced-order controller is in fact tight. Two other applications have been in the robustness analysis of multi-time scale systems [16], and systems with nonlinear perturbations [17], both of which we discuss in the sequel.

In [16], we have considered a class of linear-quadratic zero-sum differential games with state dynamics defined on three time scales, where the objective is to obtain necessary and sufficient conditions for the existence of “approximate” saddle-point solutions. The different time scales are characterized in terms of two small positive parameters, ϵ_1 and ϵ_2 , and the terminology “approximate saddle-point solution” is used to refer to saddle-point policies that do not depend on ϵ_1 and ϵ_2 , while providing cost levels within $O(\epsilon_1)$ of the full-order differential game. It is shown in the paper for both finite and infinite horizon formulations, and under perfect state measurements, that the original game can be decomposed into three subgames – slow, fast and fastest, the composite saddle-point solution of which make up the approximate saddle-point solution of the original game. Specifically, for the minimizing player it is necessary to use a *composite* policy that uses the solutions of all three subgames, whereas for the maximizing player it is sufficient to use a *slow* policy. In the finite-horizon case this slow policy could be a feedback policy, whereas in the infinite-horizon case it has to be chosen as an open-loop policy that is generated from the solution and dynamics of the slow game. The paper also presents a detailed study of a numerical example, and discusses important implications of the formulation and the derived results in the study of the robustness of H^∞ -optimal controllers for linear systems when they are subjected to perturbations on more than two time scales. Extensions to general n time scales (with $n > 3$) seem to be quite plausible, whereas extensions to the imperfect state measurements case require the development of some new techniques, which will be one of our pursuits during the coming term.

Another new research direction initiated in 1992 has been the study of the optimal control of linear stochastic systems under exponentiated quadratic cost [15]. Such a formulation captures risk averse behavior, thus providing some measure of robustness not obtainable using a pure quadratic cost function. We, in fact, introduce in [15] an additional measure of robustness into the problem (on top of *exponentiation*) by considering singularly perturbed open-loop dynamics, with the precise value of the singular perturbation parameter (ϵ) unknown. In line with this objective, the mathematical formulation of the problem involves a singularly perturbed linear stochastic differential equation, driven by an appropriately scaled Wiener process. The performance index is the expected value of an exponentiated quadratic cost, where the exponentiation involves a nonzero scalar parameter. For this parametrized stochastic optimization problem, we construct an ϵ -independent control by solving two separate (slow and fast) optimal control problems, and combining the optimal controls of these two problems. It is shown in the paper that this composite controller achieves a performance level sufficiently close to the optimal one whenever the full-order problem has a solution. The slow controller, on the other hand, achieves a finite performance level, with some degradation from the optimal one, whenever the fast subsystem is open-loop stable. The paper also studies the *large deviation* form of the problem, which is the case when the intensity of the noise in the system dynamics decreases to zero. In this case, the composite controller still provides good performance, and the slow controller also performs well when the fast subsystem is stable. In the paper, we also make the connection between this class of problems, when the cost is positively exponentiated, and the class of deterministic singularly perturbed H^∞ -optimal control problems we had studied earlier in [7] and [8]. The importance of this connection is in providing a direct interpretation of the robustness of controllers in systems with (unknown) deterministic

uncertainties in terms of robustness of similar controllers in systems where the uncertainties have a known probabilistic description, and *vice versa*. Such a correspondence is valid when the former class uses quadratic performance and the latter uses an exponentiated quadratic cost function. The development of [15] has been restricted to the perfect state measurement case; our next challenge would be to develop its counterpart for the case when the controller has access to noise-corrupted measurements, where the noise is a Wiener process.

During the last couple of months, we have been studying the robustness of minimax controllers designed for nominally linear or nonlinear systems to intrinsic nonlinear perturbations in the system dynamics and/or measurement equations. We have recently completed a major work on the topic, [17], the contents of which we now describe. Let us first consider the case of nominally linear systems: Here we have systems described by linear state dynamics, which are perturbed by unknown terms that are nonlinear in the state, but linear in the control and the unknown disturbance. A similar nonlinear perturbation is also applied to the measurement equation. The nominal cost function is quadratic, but it is also perturbed by unknown terms that are nonquadratic in the state and quadratic in the control. All nonlinear/nonquadratic perturbation terms satisfy some growth properties, and are multiplied by some small parameter, say ϵ . We undertake in [17] a detailed study on the performance of a parametrized H^∞ -controller designed for the nominal system, when applied to the nonlinearly perturbed system, and with respect to the perturbed performance measure. One of the conclusions drawn from this study is that if the time horizon is infinite, then the H^∞ -controller is robust to such nonlinear perturbations, in the sense that the performance it delivers for the perturbed system asymptotically approaches that for the nominal linear system as the parameter ϵ approaches zero. This is true under both perfect and imperfect state measurements. In the finite-horizon case, however, when transient effects dominate, such a result no longer holds for the imperfect state measurement case. What we show, though, is that one can still obtain a robust controller for this case, provided that the one designed for the nominal system is modified in a particular way. The paper discusses the philosophy behind such a design scheme, and also presents a numerical example to illustrate the theoretical results. The results are subsequently extended to nominally nonlinear systems, which are subjected to similarly structured unknown nonlinear perturbations. For this class, only the perfect state information case is treated, and sufficient conditions are obtained for the nominal minimax controller to be robust and yield asymptotically acceptable performance. This has involved the study of some nonlinear partial differential equations and inequalities in terms of robustness to some small perturbations, which has led to a characterization of the class of uncertainties tolerable when the design is based on a nominal nonlinear system.

Another recent research activity [18] has involved the stabilization of nonlinear systems when the controller does not have direct access to the state of the system, but instead an estimate $\hat{x}$ of the system state x is available for feedback. It is assumed that $\hat{x} = x + d_2$, where $d_2(t)$ is an unknown locally bounded disturbance. Conditions are presented in [18], under which one can design a smooth (estimator-based) feedback law which renders the mapping from the disturbance to the state globally input/output stable. For any initial condition, such a feedback law will guarantee that no finite escapes occur, that bounded disturbances produce bounded signals, and that the system state converges to zero as the disturbance vanishes. It is further shown in [18] that the class of systems for which such feedback laws exist include systems in strict feedback form. One important application is in the problem of output feedback stabilization, where the disturbance comes from a separately designed observer.

List of papers published or submitted, since January 1992

A single asterisk "*" indicates that the paper is based on research initiated during the last quarter of 1991, and completed during the first half of 1992, while a double asterisk "**" indicates that the paper is based on research initiated and carried out in 1992, primarily during its second half.

- [1] R. Srikant and T. Başar, Sequential decomposition and policy iteration schemes for M-player games with partial weak coupling, *Automatica*, vol. 28, no.1, pp. 95-106, January 1992.
- [2] R. Srikant and T. Başar, Asymptotic solutions to weakly coupled stochastic teams with nonclassical information, *IEEE Trans. Automatic Control*, vol. AC-37, no.2, pp. 163-173, February 1992.
- [3] T. Başar and R. Srikant, Stochastic Differential games with weak spatial and strong informational coupling, *Elektrik: An International Journal on Electrical Sciences*, to appear, 1993.
- [4] T. Başar and R. Bansal, Optimum design of measurement channels and control policies for linear-quadratic stochastic systems, *European Journal of Operations Research*, to appear, 1993.
- [5] R.A. Freeman and P.V. Kokotović, Design and comparison of globally stabilizing controllers for an uncertain nonlinear system, in *Systems, Models, and Feedback: Theory and Applications*, A. Isidori and T.J. Tarn, Edts., Birkhäuser, Boston, pp. 249-264, 1992.
- [6] R.A. Freeman, Backstepping design of robust globally stabilizing feedback compensators for nonlinear systems, *Tech. Rep. CCEC-92-0121, Center for Control Engineering and Computation*, ECE, Univ. California, Santa Barbara, 1992.
- [7] Z. Pan and T. Başar, H^∞ -optimal control for singularly perturbed systems. Part I: Perfect state measurements, *Proc. 1992 Automatic Control Conference*, Chicago, IL, pp. 1850-1854, June 1992.
- [8] Z. Pan and T. Başar, H^∞ -optimal control for singularly perturbed systems. Part I: Perfect state measurements, *Automatica*, vol. 29, no. 2, March 1993.
Note: Though carrying the same title as [7], this is a much more expanded version, addressing systems with time-varying parameters, whereas [7] was devoted to the time-invariant case.
- *[9] Z. Pan and T. Başar, H^∞ -optimal control for singularly perturbed systems. Part II: Imperfect state measurements, *Proc. 31st IEEE Conference on Decision and Control*, Tucson, AZ, pp. 943-948, December 1992.
- *[10] Z. Pan and T. Başar, H^∞ -optimal control for singularly perturbed systems. Part III: Sampled state measurements, *Preprints of 5th International Symposium on Dynamic Games and Applications*, Grimentz, Switzerland, July 1992.
Note: This is a preliminary version of [13].
- *[11] R.A. Freeman and P.V. Kokotović, Design of "softer" robust nonlinear control laws, *Automatica*, to appear, 1993; a preliminary version in *Proc. IFAC Symposium on Nonlinear Control System Design*, June 1992, Bordeaux, France.
- *[12] Z. Pan and T. Başar, H^∞ -optimal control for singularly perturbed systems. Part II: Imperfect state measurements, *IEEE Trans. Automatic Control*, to appear, vol. AC-38, late 1993.
Note: Though carrying the same title as [9], this version includes some major new results not included in [9], among which are the complete analysis of the case of time-varying parameters, and a study of the transient behavior under reduced-order controllers.

- *[13] Z. Pan and T. Başar, H^∞ -optimal control for singularly perturbed systems with sampled state measurements, in *Advances in Dynamic Games and Applications*, T. Başar and A. Haurie, Edts., Birkhäuser, Boston, May 1993.
- **[14] Z. Pan and T. Başar, A tight bound for the H^∞ performance of singularly perturbed systems, *CSL Report, University of Illinois*, 1992.
- **[15] Z. Pan and T. Başar, Optimal control of stochastic singularly perturbed systems with exponentiated quadratic cost, *Proc. 12th World IFAC Congress*, Sydney, Australia, July 1993.
- **[16] Z. Pan and T. Başar, Multi-time scale zero-sum differential games with perfect state measurements, Submitted on February 17, 1993 to the *32nd IEEE Conference on Decision and Control*, to be held in San Antonio, TX, December 1993.
- **[17] Z. Pan and T. Başar, Robustness of H^∞ controllers to nonlinear perturbations, Submitted on February 17, 1993 to the *32nd IEEE Conference on Decision and Control*, to be held in San Antonio, TX, December 1993.
- **[18] R.A. Freeman and P.V. Kokotović, Global Robustness of Nonlinear Systems to State Measurement Disturbances, *Completed February 15, 1993. To be presented at the IMA Meetings on Nonlinear Feedback Design*, University of Minnesota, March 8-12, 1993.

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