LBL--25024

DE88 011959

Fifth Winter Workshop on Nuclear Dynamics, Sun Valley, Idaho, 22-26 February, 1988

Quasifragments: Hot Nuclei Embedded in a Nucleon Vapor

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In nuclear collisions at intermediate energies, metastable complex nuclear fragments are produced abundantly during the primary stage of the reaction. Therefore, statistical models for nuclear disassembly must incorporate highly excited unstable fragment states in the phase space considered. In our previous treatments of nuclear disassembly,[1] a particular unstable fragment state was included in the final phase space provided its (estimated) half-life exceeded the time characterizing the breakup process. A similar prescription was also employed in the recent exact microcanonical model of nuclear disassembly.[2] Although such life-time arguments are intuitively appealing in the context of a disassembling source, their relevance is less clear for the treatment of static problems, *e.g.* excited infinite nuclear matter at subsaturation densities. It is therefore desirable to seek a better foundation for the description of highly excited nuclear states, applicable to both static and dynamical scenarios. This is also practically important for the implementation of event generators developed to provide samples of multi-fragment final states of medium-energy nuclear collisions. Furthermore, microscopic dynamical simulations also encounter the problem when seeking to give a realistic description of the final nuclear fragments.

A consistent treatment of the metastable states can only be achieved if a nucleon vapor surrounding the fragments is included in the calculation. Therefore, we discuss a hot nucleus embedded in a vapor of nucleons, a *quasifragment*. By the generalized Levinson's Theorem,[3] the total partition function of the entire system is insensitive to the particular prescription used to delineate the hot fragment in relation to the vapor, in the independent-particle idealization. However, as mentioned above, it is of practical importance to calculate the yields of different fragment types, and the *final* species distribution is sensitive to how this separation is made. The separation is also relevant to the question of the liquid-vapor phase transition in nuclear matter.

Our studies are directed towards nuclear matter at densities near (but below) the saturation density ($\approx$0.17 fm^{-3}) and at excitation comparable to the nuclear binding energy ($\approx$10 MeV/N). Under such circumstances, the system will typically appear as an assembly of quasifragments interspersed with a vapor of unbound nucleons. The grand-canonical ensemble is well suited for the statistical description of a quasifragment in equilibrium with a surrounding nucleon vapor. In the single-particle idealization, it is possible to express the partition function for the quasifragment, $\tilde{Z}_A$, in terms of its effective density of single-particle states, $\tilde{g}_A(\epsilon)$,

$$\ln \tilde{Z}_A(\alpha,\beta) = \int \tilde{g}_A(\epsilon)\ \ln[1+e^{-\alpha-\beta\epsilon}]\ d\epsilon\ . \tag{1}$$

Here β is the inverse of the imposed temperature τ and α is related to the chemical potential μ_A by $\alpha = -\beta\mu_A$. The mean number of nucleons in the quasifragment and its mean energy are then given by

$$<A> = -\frac{\partial \ln \tilde{Z}_A}{\partial\alpha} = \int \frac{\tilde{g}_A(\epsilon)}{1+e^{\beta(\epsilon-\mu)}}\ d\epsilon\ , \tag{2}$$

$$<E> = -\frac{\partial \ln \tilde{Z}_A}{\partial\beta} = \int \frac{\tilde{g}_A(\epsilon)\ \epsilon}{1+e^{\beta(\epsilon-\mu)}}\ d\epsilon\ . \tag{3}$$

The requirement that the quasifragment contain a specified number of nucleons on the mean yields an implicit equation for the chemical potential $\mu_A(\tau)$, which can be readily solved by iteration. The second equation then yields the mean excitation energy per nucleon as a function of the imposed temperature τ. (The ground-state energy is obtained by evaluating (3) for $\tau = 0$.)

Imagine that the system is enclosed in a (large) volume Ω and let $g_0(\epsilon)$ be the corresponding density of single-particle states when no fragment is present, *i.e.* for a constant potential $V \equiv 0$. Now modify the potential V so that it exhibits a well corresponding to the effective nuclear potential and let the corresponding density of single-particle states be $g_V(\epsilon)$. The induced change can be written in the form

$$\Delta g(\epsilon) \equiv g_V(\epsilon) - g_0(\epsilon) = \sum_{\ell n}(2\ell+1)\delta(\epsilon - \epsilon_{n\ell}) \; + \; \frac{1}{\pi}\sum_{\ell}(2\ell+1)\frac{d\delta_\ell(\epsilon)}{d\epsilon} \; . \tag{4}$$

Here the first sum expresses the addition of the bound levels ($\epsilon_{n\ell} < 0$) of the well, each being characterized by its orbital angular momentum ℓ and its radial quantum number n. The second sum expresses the change in the density of unbound states in terms of the phase shifts $\delta_\ell(\epsilon)$. It should be noted that this latter term is negative for most values of the energy ϵ, since the phase shift exhibits an overall decrease as the energy is raised (in fact, $\delta_\ell(\epsilon) \rightarrow 0$ as $\epsilon \rightarrow \infty$). In particular, the total number of single-particle states remains unchanged by the modification of V,

$$\Delta N \equiv \int \Delta g(\epsilon) d\epsilon = \sum_\ell (2\ell+1)N_\ell \; + \; \frac{1}{\pi}\sum_\ell (2\ell+1)(\delta_\ell(\infty) - \delta_\ell(0)) = 0 \; . \tag{5}$$

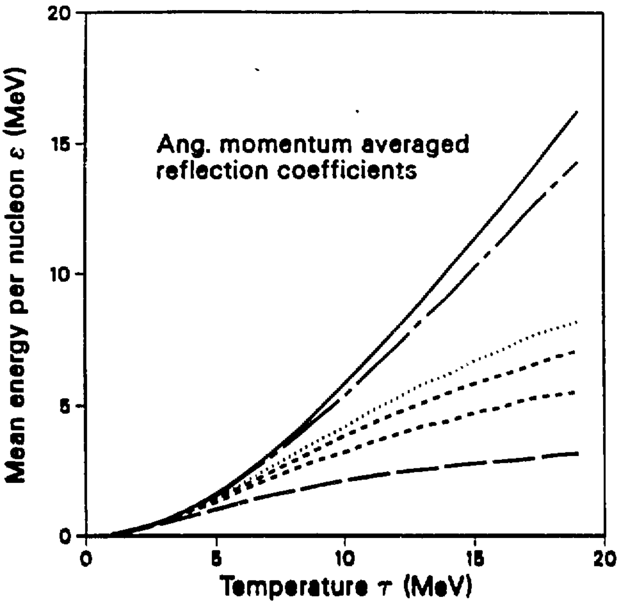
This statement is a generalization of Levinson's Theorem [3,4], which was used in (5) in the form stating that the phase shift at zero energy equals π times the number of bound states, $\delta_\ell(0) = \pi N_\ell$.[5]

A key issue in the present study is how to partition the single-particle states between the quasifragment and the vapor. For the discussion of this problem it is useful to adopt a semiclassical picture in which any single-particle state can be associated with definite regions of space. The modification in the effective single-particle potential V, *i.e.* the formation of the nuclear potential well, is confined to near the domain occupied by the nuclear fragment, $\Omega_A \approx \frac{4\pi}{3}R_A^3$, and leaves the environment unchanged. The same must be the case for the associated density of states. If the dependence of $g(\epsilon)$ on the total volume Ω is known, it is easy to express the contribution $g_>(\epsilon)$ from nucleons located outside the fragment domain,

$$g_>(\epsilon) = \tilde{\Omega}\frac{dg_V(\epsilon)}{d\Omega} \doteq \tilde{\Omega}\frac{dg_0(\epsilon)}{d\Omega} \; , \tag{6}$$

where $\tilde{\Omega} = \Omega - \Omega_A$. Clearly, these states are to be associated with the vapor. The classification of nucleons situated within the domain occupied by the fragment is less clear. At the low-energy end it is obvious that the bound levels are to be associated with the fragment. Conversely, at the highest energies, where the presence of the potential well is barely discernible, it appears most reasonable to associate the states with the vapor. There is probably no unique way to resolve this problem and the specific criterion we have adopted is based on physical arguments.

One might at first be tempted to identify the change $\Delta g(\epsilon)$ in (4) with the nuclear density of single-particle states. However, this suggestion appears somewhat unphysical, considering the fact exhibited above that the contribution from the positive-energy part of $\Delta g(\epsilon)$ to ΔN is negative in (5). A more attractive (yet simple) possibility is to associate the bound single-particle levels $\epsilon_{n\ell}$ with the fragment and let the second term in (4) represent the compensating dilution of the vapor states. This amounts to saying that the many-body states of a quasifragment are those that are based solely on bound single-particle orbitals. Such many-body states are expected to be fairly long-lived, since some residual configuration mixing is required to promote a single nucleon to an unbound orbital from which it may escape. However, unbound nucleon orbitals may have considerable longevity, due to their containment by the centrifugal barrier and their quantal reflection from the nuclear surface. It is with this feature in mind that we propose that the many-body states to be associated with the quasifragment are those generated by metastable single-particle orbitals, the degree of stability being determined by the associated reflection coefficient $R_{n\ell}$.



The mean energy in a quasifragment as a function of the imposed temperature τ, for various effective single-particle level densities $\bar{g}_A(\epsilon)$. The full Fermi-gas level density corresponds to the "hot" limit and produces the top curve, whereas the bottom curve is the "cool" limit, in which only bound single-particle states are included. Modulating the Fermi-gas level density with the average reflection coefficient $\bar{R}_A(\epsilon)$ yields the second and fourth curves from the bottom, corresponding to A=40 and A=100, respectively. For comparison is shown the result of an exponential modulation using a width of τ_0 = 12 MeV (third curve from the bottom). The second curve from the top corresponds to using the reflection coefficient appropriate to a diffuse, flat potential (for which the centrifugal force is ignored).

The change in level density caused by the potential well of the quasifragment can be written

$$\Delta g(\epsilon) \approx g_A(\epsilon) - \Omega_A \frac{\partial}{\partial \Omega} g_0(\epsilon) \ . \tag{7}$$

The first term, $g_A(\epsilon)$, is the density of single-particle states associated with the fragment when its potential well is artificially extended upwards so that no continuum states occur. The second term subtracts the part of the original level density $g_0(\epsilon)$ stemming from single-particle states located within the domain of the fragment. As mentioned earlier, it is physically clear that the level density $g_A(\epsilon)$ need be split so that only its lower-energy part $\bar{g}_A(\epsilon)$ is to be associated with the fragment while its higher-energy part $\tilde{g}_A(\epsilon)$ should be associated with the vapor. This separation is made in the present work by invoking the average reflection and transmission coefficients for single-particle states at a given energy, $\bar{R}_A(\epsilon)$ and $\bar{T}_A(\epsilon)$,

$$g_A(\epsilon) = g_A(\epsilon)\ \bar{R}_A(\epsilon) + g_A(\epsilon)\ \bar{T}_A(\epsilon) = \bar{g}_A(\epsilon) + \tilde{g}_A(\epsilon) \ . \tag{8}$$

The quantum-mechanical reflection coefficient for a nucleon in a given single-particle orbital can be estimated in the parabolic approximation: For a given orbital angular momentum ℓ, we have $R_{n\ell} = R_\ell(\epsilon_{n\ell}) \approx 1/[1+\exp(2\pi e_\ell)]$, where e_ℓ is the energy in excess of the top of the effective potential barrier, in units of the characteristic barrier energy $\hbar\omega_\ell$. The average reflection coefficient $\bar{R}_A(\epsilon)$ for orbitals with an energy near ϵ can then be obtained as a weighted sum over all those values of the angular momentum ℓ for which the effective radial potential $V_\ell^{\rm eff}(r)$ has a minimum.

Thus, the density of single-particle states for the total system is decomposed as $g(\epsilon) = g_{\text{vapor}}(\epsilon) + \tilde{g}_A(\epsilon)$, where the part associated with the nucleon vapor is $g_{\text{vapor}}(\epsilon) = g_{>}(\epsilon) + \bar{g}_A(\epsilon)$. The partition function Z for the total system then factorizes correspondingly, $Z = Z_{\text{vapor}}\tilde{Z}_A$, where $\tilde{Z}_A$ is given by eq. (1) and Z_{vapor} is given analogously in terms of $g_{\text{vapor}}(\epsilon)$. It should be noted that the total partition function Z is independent of the particular way in which the partition of $g(\epsilon)$ is made, as long as the single-particle idealization is maintained. However, the partition of $g(\epsilon)$ *is* significant when the transition-state approximation is invoked in order to describe the disassembly of the system into distinct, real fragments.

The figure illustrates the result of the proposed reflection-coefficient prescription. The use of the reflection coefficient to truncate the level density leads to a natural A-dependence: a larger nucleus (having a smaller surface-to-volume ratio) holds more energy per nucleon at a given temperature, until this behavior saturates in the limit $A \to \infty$. It is interesting to note that for the relatively adundant lighter nuclei the limiting temperature comes out to be around 8 MeV, so for the practical applications the relatively complicated (but parameter-free) reflection-coefficient prescription can be well approximated by *e.g.* an exponential cutoff with $\tau_0 \approx 8$ MeV.

The present study is motivated by our interest in the description of hot nuclear matter at subsaturation densities, as may be produced in nuclear collisions at medium energies. At the high temperatures of interest, for which the excitation of the system is comparable to its binding energy, the metastable fragments must be considered in conjunction with a surrounding nucleon vapor. (Similar scenarios occur in astrophysical systems, albeit typically at temperatures and densities that are considerably lower, and the proper inclusion of the vapor in those situations have been discussed.[6,7]) The statistical treatment of such systems is conveniently formulated in terms of excitable quasifragments embedded in a nucleon vapor. We have formulated a conceptually simple, parameter-free method for making the formal split of the partition function into a factor associated with the vapor and one factor for each quasifragment. The method can be characterized roughly by saying that a particular nucleon, situated within the interior of a fragment, is considered as part of that fragment if it is reflected back into the fragment's interior when reaching its surface. This prescription yields a mass-dependent limiting nuclear temperature, decreasing from 10-12 MeV for heavy nuclei to around 8 MeV for $A \approx 10$.

We wish to acknowledge instructive discussions with S.E. Koonin, S. Pratt, and P.J. Siemens. One of us (GF) is on leave of absence from Department of Physics, Kent State University, Kent, Ohio 44242. This work was supported by the Director, Office of Energy Research, Division of Nuclear Physics of the Office of High-Energy and Nuclear Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098 and Grant DE-FG02-86ER40251.

References

[1] G. Fai and J. Randrup, Nucl. Phys. **A404** (1983) 551; Comp. Phys. Comm. **42** (1986) 385

[2] S.E. Koonin and J. Randrup, Nucl. Phys. **A474** (1987) 183

[3] H.A. Weidenmüller, Ann. of Phys. **28** (1964) 60

[4] S. Pratt, P.J. Siemens, and Q.N. Usmani, Phys. Lett. **189B** (1987) 1

[5] N. Levinson, Kgl. Danske Videnskab. Selskab. **25** (1949) 9

[6] D.L. Tubbs and S.E. Koonin, Astrophys. J. **232** (1979) L59

[7] T.J. Mazurek, J.M. Lattimer, and G.E. Brown, Astrophys. J. **229** (1979) 713