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PHYSICAL PROCESSES IN SPIN POLARIZED PLASMAS

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ABSTRACT

If the plasma in a nuclear fusion reactor is polarized, the nuclear reactions are modified in such a way as to enhance the reactor performance. We calculate in detail the modification of these nuclear reactions by different modes of polarization of the nuclear fuel. We also consider in detail the various physical processes that can lead to depolarization and show that they are by and large slow enough that a high degree of polarization can be maintained.

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1. INTRODUCTION

The strong dependence of nuclear fusion cross sections on the center of mass energy of two colliding nuclei plays a dominant role in designs of nuclear fusion reactors. The cross section also depends on another important variable, the total center of mass angular momentum J of the colliding nuclei. As will appear, this angular momentum depends on the relative spin angular momentum of the nuclei just prior to collision. In this paper and in the accompanying paper we explore the consequences of this spin dependence and show that if proper use is made of this dependence, decided improvement in nuclear reactor performance will result.

The fact that nuclear cross sections in general depend on angular momentum is well established [1,2]. In fact, this dependence is used routinely as a diagnostic of polarization of nuclei in the case of the D-T reaction [3].

At the kinetic energies usually considered in fusion reactors the orbital angular momentum of two nuclei, colliding with a sufficiently small impact parameter that a nuclear reaction might occur, is essentially zero. Thus, the angular momentum is all in the intrinsic spin of the nuclei ($1/2 \hbar$ for the triton and $\hbar$ for the deuteron). Therefore, by controlling the relative orientation of the spin angular momentum of the nuclei just prior to a collision, the total angular momentum of the system in collision can be controlled, and the nuclear reaction can be modified to some extent. A natural quantization axis for the nuclear spin vector is the direction of the macroscopic magnetic field $\underline{B}$ that confines the plasma. As will be shown in this paper this quantization is strong in the sense that most effects that might tend to depolarize the orientation of the spins along $\underline{B}$ are so weak that the nuclear spins will stay polarized for times much longer than the nuclear

reaction time. Thus, once the nuclei of the fusion plasma are all polarized in the desired manner relative to $\underline{B}$, they may be considered to remain so.

Assume for the moment that the nuclear ions can be polarized at will. There are several choices for the polarization states of the various nuclear spins in the reactor. The consequences of these choices are examined in Sec. 2. Consider a deuterium-tritium (D-T) reactor and let m_2 be the magnetic quantum number of the deuteron (D) and m_3 that for a triton (T). There are three essentially different choices of polarization: (a) Both D and T polarized along $\underline{B}$ ($m_2 = 1$, $m_3 = 1/2$). (b) D polarized across $\underline{B}$ ($m_2 = 0$) and T unpolarized ($m_2 = \pm 1/2$ with equally likelihood). (c) D polarized opposite to $\underline{B}$, and T polarized along $\underline{B}$ ($m_2 = -1$, $m_3 = 1/2$). It is shown in Sec. 2 that for choice (a) the effective nuclear cross section is equal to 1.5 times the averaged cross section for unpolarized nuclei. In addition, the resulting alpha particles and neutrons produced by the reactor are emitted nonisotropically. The differential cross section is proportional to $\sin^2 \theta$ where θ is the angle of emission with respect to $\underline{B}$: there is a preponderance produced perpendicular to $\underline{B}$ and none produced parallel to $\underline{B}$. The emission of the neutron is essentially in the opposite direction to that of the alpha particle because of their large momenta relative to that of the colliding nuclei. For choice (b) the effective nuclear cross section is equal to the unpolarized cross section, but again the alpha particle and neutron emission is nonisotropic. The differential cross section is proportional to $1 + 3 \cos^2 \theta$, so that the emission along $\underline{B}$ is four times as likely (per unit steradian) as that across $\underline{B}$. For choice (c) the effective cross section is one-half the unpolarized case and the angular distribution of the reactor products is the same as for choice (b). Thus, by making use of the intrinsic spin variable, quite a large variety can be introduced into the type of

reactions that occur for a D-T plasma. This variety could be made use of to improve the performance of a fusion reactor. Exactly similar results are valid for the D - He^3 reaction with the He^3 nucleus replacing the T nucleus. (He^3 has a spin of $1/2$ & also.) For the D-D reaction similar changes are also possible. In addition, there is the further possibility of a large suppression of the cross section if the D are all polarized parallel to $\underline{B}$. This suppression itself could have some practical applications which are discussed in Sec. 2.

The main interaction of the spin of a nucleus with its environment is the interaction of its nuclear magnetic moment with the magnetic field in the frame of the moving nucleus. For the deuteron there is also the interaction of its quadrupole electric moment with electric field gradients. Because the main magnetic field $\underline{B}_0$ is strong, the primary effect is a rapid precession of the nuclear spin about $\underline{B}_0$. In this precession the spin vector and the magnetic field can be considered to make an angle θ_g which is closely related to the polarization of the nucleus. The angle θ_g may be made precise for spin $1/2$ particles but is only a qualitative concept for spin 1 particles. (The precession rate Ω_p is comparable to the deuteron gyrofrequency Ω_2 of its cyclical motion about $\underline{B}$. $\Omega_{p2} = 0.86 \Omega_2$ for deuterons and $\Omega_{p3} = 5.94 \Omega_2$ for tritons.) This rapid precession gives considerable stability to angle θ_g . It can be disturbed by (a) large scale variations of $\underline{B}$ due to the geometry of the reactor, (b) sudden abrupt changes in $\underline{B}$ produced by the current due to occasional collisions, (c) effective small scale induced variations produced by the gyromotion of the particle orbit in the field gradients of the main field, (d) and time dependent variations of the magnetic field produced by currents in the plasma. (e) In addition, the nuclei may be in the atomic state during injection into the reactor as fuel or during recycling. Atomic

processes can then depolarize the nuclei. (f) Finally, wall interaction may occur during recycling.

In Sec. 3 we examine the rate of change of θ_g , the polarization angle, produced by all these processes and show that processes (a) - (c) and (e) are very small. Further, in all likelihood the amplitude of time dependent fluctuations to be expected in a quiet steady-state nuclear reactor are so small that process (d) is also a weak process. Process (f) is the most dangerous to polarization, but it may be avoided or reduced by proper choice of wall materials.

Most of these results have been presented in an abbreviated form in Refs. [4] and [5]. In this paper these results are derived in detail. In an accompanying paper [6] a more formal procedure is given for detailed description of plasmas including the spin variables. A generalization of the usual kinetic equation for plasmas is made to nine-dimensional space. The independent variables are $\underline{r}$, $\underline{v}$, and $\underline{p}$, space, velocity, and polarization state vectors. Then moments over spin space are taken to derive equations for these moments in ordinary phase space. It is found that moments up to first order for tritons and up to second order for deuterons are adequate for a complete description of the nuclei including their nuclear reactions. These moment equations close, so that for tritons one arrives at four kinetic equations for four distribution functions. For deuterons one has thirteen distribution functions and thirteen equations. These equations may be solved asymptotically to obtain a more precise treatment of the depolarization results discussed above.

The application of the results of Secs. 2 and 3 to a fusion reactor using polarized fuel are discussed in Sec. 4.

2. NUCLEAR PHYSICS WITH POLARIZED SPIN

2.1. The D-T reaction

We first discuss the effect of spin on the $D(T,n)He^4$ reaction. Its large cross section makes it a natural choice for first generation fusion reactors. The dependence of the reaction on spin at low energies is fairly clear. When a D and a T nuclei collide and penetrate the coulomb barrier, their energy is very close to an excited state of the compound nucleus He^5 which is an intermediate step in the D-T reaction (see Fig. 1). This nucleus is unstable, but its excited states have life times long enough that they may be considered as quantum states with definite angular momentum and parity. As shown in Fig. 1 an excited state of He^5 lies 107 keV above the energy of the unbound D and T at zero kinetic energy. This state has an angular momentum $J = 3/2$ and even parity. If the colliding D-T system has angular momentum $J = 3/2$ and even parity (orbital angular momentum an even multiple of $\hbar$), then the excited He^5 nucleus is formed with high probability and decay to the state $He^4 + n$ with the release of 17 MeV follows. If the colliding nuclei have a different angular momentum or parity, the chances of a reaction are smaller by two orders of magnitude. In fact, it is estimated from experiments that the fraction of reactions occurring through this excited state is from 0.95 to 0.99 [3].

A simple estimate shows that for ion energies typical of a fusion reactor, only collisions with impact parameters small enough that the relative orbital angular momentum is zero can lead to a reaction. For example, consider a collision between a D and T with center of mass energy $E_r = \mu v^2/2 = 100$ keV, where $\mu = m_2 m_3 / (m_2 + m_3)$ is the relative mass and v the relative velocity. Then the relative angular momentum $L = (2\mu E)^{1/2} b = 0.08 \text{ b}_{13} \hbar$ for the impact parameter b in units of 10^{-13} cm (fermi's). Thus, if b is small

enough for a nuclear reaction, L is small compared to M , and the orbital angular momentum quantum number l is zero. Therefore, we may restrict ourselves to collisions in the $l = 0$ state. (In actual fact the nucleus has some chance of penetrating the coulomb barrier with $l = 1$. However, because of the even parity of the resonant state, we may ignore these events.) In the $l = 0$ state the total angular momentum is the result of adding the spin angular momentum of the D and T nuclei. Since the spin of D is M and that of T is $(1/2)M$, the total angular momentum is JM where $J = 1/2$ or $3/2$.

The statistical weight of the $J = 3/2$ state is four and that of the $J = 1/2$ state is two, so if the D and T are both randomly polarized, the probability for the colliding system to be in the $J = 3/2$ state is two thirds. An immediate consequence is that if one ensures that the nuclei come together with $J = 3/2$, the effective cross section is three halves the unpolarized cross section. One way to accomplish this is to align all the spins of both the D and T nuclei along B .

We now consider the detailed dependence of the effective nuclear cross section on the spin orientations of the D and T nuclei. Let $|m_2, m_3\rangle$ represent the eigenstate in which the magnetic quantum number of D is m_2 and T is m_3 . The spins of the D and T are in general uncorrelated so we write

$$|\chi_D\rangle = \sum_{m_2} a_{m_2} |m_2\rangle \quad (1a)$$

$$|\chi_T\rangle = \sum_{m_3} b_{m_3} |m_3\rangle \quad (1b)$$

$$|\chi\rangle = |\chi_D\rangle |\chi_T\rangle = \sum_{m_2 m_3} a_{m_2} b_{m_3} |m_2 m_3\rangle \quad (1c)$$

for an arbitrary D or T. $|a_{m_2}|^2 = d_{m_2}$ denotes the probability the deuteron is in the m_2 state and correspondingly for T, $|b_{m_3}|^2 = t_{m_3}$. The nuclear cross section depends on the J variable and the relative energy only, so the probability of a nuclear reaction may be written as

$$\sigma = \sum_{J=1/2, 3/2} \sum_{m=-J}^J \sigma_J |\langle J, m | \chi \rangle|^2, \quad (2)$$

where $\sigma_{3/2} \gg \sigma_{1/2}$ and $\langle J, m |$ is the J m state of the colliding particles. Substituting Eq. (1c) into (2) we have

$$\sigma = \sum_{J=1/2, 3/2} \sum_{m_2, m_3} d_{m_2} t_{m_3} |\langle J, m_2 + m_3 | m_2, m_3 \rangle|^2 \sigma_J. \quad (3)$$

The quantities $\langle J, m_2 + m_3 | m_2, m_3 \rangle$ are the Clebsch-Gordon coefficients for addition of angular momentum. We require those for which $\underline{J}\hbar = \underline{s}_D + \underline{s}_T = (\underline{J}_1 + \underline{J}_2)\hbar$ where $\underline{s}_D$ and $\underline{s}_T$ are the spin operators for D and T. In Table I we list the relevant values of these coefficients. Making use of these coefficients we find that Eq. (3) reduces to

$$\sigma = (a + \frac{2}{3}b + \frac{1}{3}c)\sigma_{3/2} + (\frac{1}{3}b + \frac{2}{3}c)\sigma_{1/2} \quad (4)$$

where

$$a = d_1 t_{1/2} + d_{-1} t_{-1/2} \quad (5a)$$

$$b = d_0 (t_{1/2} + t_{-1/2}) = d_0 \quad (5b)$$

$$c = d_1 t_{-1/2} + d_{-1} t_{1/2}. \quad (5c)$$

The a represents the probability that the spins of D and T are both aligned (either parallel to $\underline{B}$ or antiparallel to $\underline{B}$). For this case the probability that $J = 3/2$ is 1. The b represents the probability that D is polarized perpendicular to $\underline{B}$. In this case the probability that $J = 3/2$ is $2/3$. The c represents the probability that the spins are antialigned. In this case the probability of $J = 3/2$ is $1/3$. (This is not zero as would be expected on classical grounds, because the spins are not exactly oppositely aligned.)

From Eq. (4) we may verify the conclusions stated in the introduction concerning the effective nuclear cross sections. Neglect $\sigma_{1/2}$. If the ions are unpolarized, the d 's are $1/3$, the t 's are $1/2$, and from Eqs. (5a) - (5c) $a = b = 1/3$ so the unpolarized cross section $\sigma_0 = 2/3 \sigma_{3/2}$. For case (a) we have $\sigma_a = \sigma_{3/2} = 3/2 \sigma_0$. For case (b), $\sigma_b = 2/3 \sigma_{3/2} = \sigma_0$. For case (c), $\sigma_c = 1/3 \sigma_{3/2} = 1/2 \sigma_0$.

When the compound state of He^5 breaks up into He^4 and n , it has a definite angular momentum and parity so that the direction of emission of the alpha particle and the neutron will have a definite distribution. The final state vector for the orbital angular momentum and spin of the neutron relative to the alpha particle is

$$|\psi_f\rangle = |\ell s, m_\ell m_s\rangle \langle \ell s, m_\ell m_s | Jm \rangle (\sigma_{Jm_2} t_{m_3})^{1/2} \langle Jm | m_2, m_3 \rangle \quad (6)$$

where ℓ and s are the orbital angular momentum and spin quantum numbers of the neutron ($s = 1/2$), and m_ℓ and m_s are the magnetic quantum numbers. The alpha particle has zero spin. Because the compound state has even parity, $\ell = 1$ is not allowed. The coefficients $\langle \ell s, m_\ell m_s | Jm \rangle$ may be obtained from Table I. Expressing the final eigenstates in terms of spherical harmonics $Y_{\ell m}$, we have

$$\begin{aligned}
\psi_f = & \left(\frac{\sigma_{3/2}}{5}\right)^{1/2} \left\{ \left| -\frac{1}{2} \right\rangle \left[2 y_{22} (d_1 t_{1/2})^{1/2} + y_{21} (d_1 t_{-1/2})^{1/2} + 2 y_{20} \left(\frac{d_0 t_{-1/2}}{3}\right)^{1/2} \right] \right. \\
& - \left| \frac{1}{2} \right\rangle \left[y_{21} (d_1 t_{1/2})^{1/2} + y_{20} \left(\frac{2d_1 t_{-1/2}}{3}\right)^{1/2} + y_{2,-1} \left(\frac{2d_0 t_{-1/2}}{3}\right)^{1/2} \right] \\
& + \left(\frac{\sigma_{1/2}}{3}\right)^{1/2} \left\{ y_{00} \left| \frac{1}{2} \right\rangle \left[(d_0 t_{-1/2})^{1/2} - (d_0 t_{1/2})^{1/2} \right] \right. \\
& \left. \left. + \left| -\frac{1}{2} \right\rangle \left[(d_0 t_{-1/2})^{1/2} - (2d_{-1} t_{1/2})^{1/2} \right] \right\} \quad (7)
\end{aligned}$$

The spherical harmonics Y_{lm} are given in Table II. The states $|1/2\rangle$, $|-1/2\rangle$ denote the spin eigenstates of the neutron with magnetic quantum number $1/2$, $-1/2$, respectively. The differential cross section for the neutron, independent of its spin state is

$$\begin{aligned}
\frac{d\sigma}{d\Omega} = & \sum_{m_s=\pm(1/2)} |\psi_f|^2 = \frac{1}{2\pi} \left\{ \sigma_{3/2} \left[\frac{3}{4} \sin^2 \theta a + \frac{1}{4} (1 + 3 \cos^2 \theta) \left(\frac{2}{3} b + \frac{2}{3} c \right) \right] \right. \\
& \left. + \sigma_{1/2} \left(\frac{1}{3} c + \frac{1}{6} b \right) \right\} \quad (8)
\end{aligned}$$

Upon integration over $d\Omega$ we obtain Eq. (4). A second quantity of interest is the difference $|\psi_s^+|^2 - |\psi_s^-|^2$ of the differential cross sections for the neutron polarized with $m_s = 1/2$ and $m_s = -1/2$. It is

$$\begin{aligned}
\frac{d\sigma^+}{d\Omega} - \frac{d\sigma^-}{d\Omega} = & |\psi_f^+|^2 - |\psi_f^-|^2 \\
= & \frac{\sigma_{3/2}}{2\pi} \left\{ \frac{3}{4} [d - (2e + f)] \cos^2 \theta \sin^2 \theta - \frac{3}{4} d \sin^4 \theta \right. \\
& \left. + \frac{(2e+f)}{12} (3 \cos^2 \theta - 1)^2 \right\} + \frac{\sigma_{1/2}}{2\pi} \left(\frac{f}{3} + \frac{e}{6} \right) \quad (9)
\end{aligned}$$

where

$$d = d_1 t_{1/2} - d_{-1} t_{-1/2}$$

$$e = d_0(t_{1/2} - t_{-1/2})$$

$$f = d_1 t_{-1/2} - d_{-1} t_{1/2} \quad (10)$$

By forming either the sum or difference of Eqs. (8) and (9), the probability of emitting a neutron into a specified solid angle with definite m_n is determined. For example, if $d_1 = t_{1/2} = 1$ the differential cross section for $m_n = 1/2$ is $(3/16\pi) \sigma_{3/2}(1 - \sin^4\theta)$ and for $m_n = -(1/2)$, $(3/16\pi) \sigma_{3/2}(1 + \sin^2\theta)^2$. At 90° the neutron is completely polarized with $m_n = -1/2$.

2.2. The D-He³ reaction

The D(He³, p)He⁴ reaction is entirely analogous to the D(t,n)He⁴ reaction since He³ and T are mirror nuclei. There is a $3/2^+$ resonant state of Li⁵, 407 keV above the energy of a free D and He³ which contributes very strongly to the reaction at low energies (see Fig. 2). The spin dependences of the nuclear cross sections and angular distributions of the reactor products depend on the existence of such a state. All the results about the enhanced cross sections for different polarization states of the D and T, as well as the angular distribution of the neutron and alpha particle, carry over to the D - He³ reaction. The enhanced cross sections for D - He³ are given by Eq. (4) and the differential cross sections for the proton and alpha particle are given by Eqs. (8) and (9). In these equations $\sigma_{3/2}$ and $\sigma_{1/2}$ now are the D - He³ cross sections and the quantities a through f refer to the spin

distribution of the pairs of D and He^3 spins. There are two significant differences. First, in the energy level diagram of Li^5 there is another energy level much closer to the $3/2^+$ level than for He^5 , so $\sigma_{1/2}/\sigma_{3/2}$ may not be so small for D - He^3 as for D - T. Thus, the enhancements may be somewhat smaller for the case with both nuclei polarized parallel to each other. Also the distribution of emission products could be more isotropic. Second, since the D - He^3 reaction produces no neutrons directly, the neutrons come only from the D-D reactions. ($\text{He}^3 - \text{He}^3$ produces no neutrons at low energy.) As will appear in the next subsection, it may be the case that the D-D reaction itself may be suppressed if both D's are polarized in the $m = 1$ state, in which case one could largely eliminate neutrons in a D - He^3 reactor, a very desirable feature.

2.3. The D-D reaction

The D-D reaction is far more complex with respect to the spin dependence of its reaction cross section than either the D-T reaction or the D- He^3 reaction. It seems not to be a resonant type reaction at low energies since there appears to be no excited state of He^4 near the energy of two free deuterons. Also the number of possible spin states of a pair of deuterons is larger since the spin of the deuteron is one. Finally, there are two possible reactions, $\text{D}(\text{dn})\text{He}^3$ and $\text{D}(\text{dp})\text{T}$. There is thus no easy way to estimate the spin dependence of a D-D reaction cross section, and there is no general agreement concerning it.

A simple argument would seem to indicate that if both deuterons had aligned spins (so the total spin $S = 2$), the reaction would be difficult, since to produce a neutron and a He^3 at least one spin must flip and similarly for p and T. If the spin dependence of nuclear forces is small, this event

should be unlikely and the corresponding cross section small. This has been the generally accepted view [7] until recently. The possible states of the deuteron pair are $S = 0$, the singlet state, $S = 1$, the triplet state, and $S = 2$, the quintuplet state, where S is the total spin angular momentum. It was assumed that main contributions to either of the D-D reactions is from the singlet and triplet states with very little contribution from the quintuplet state. The main information on the D-D reaction comes from measurements of the differential cross section for protons from a polarized deuteron beam striking an unpolarized deuteron target [8]. Analyzing this data in terms of the possible states of the initial pair of deuterons, one can draw conclusions as to the magnitude of the possible matrix elements of the reaction. Then in terms of these matrix elements one can draw conclusions as to possible enhancements to be obtained by polarizing the deuterons in various ways.

This analysis has been carried out by Dodder and Hale [9,10] making use of a large amount of data incorporating elastic cross sections, and other related reactions as well, in a large computer code. The results of their analysis are given in Table III. The ratio of the total cross section is E_{11} for deuterons in an $m_2 = 1$ state to the total cross section averaged over all spin polarization states, similarly, for $E_{10}(m_2 = 1, m_2' = 0)$, $E_{00}(m_2 = 0, m_2' = 0)$, etc. This table shows rather mild dependence of the D-D reaction on spin. This mild dependence can be traced to a sizable contribution from the quintuplet state. A possible criticism of this result is that some of the data analyzed by Dodder and Hale involve some thick target measurements. The slowing down of the deuteron in the target combined with the strong dependence of the D-D reaction on energy may introduce some anisotropy into the differential cross section. These spurious anisotropies might be inferred as due to a contribution from the quintuplet state thereby exaggerating its contribution to the reaction.

Another set of data which is from thinner targets has been analyzed by Ad'yasevich and Fomenko [8]. Their conclusion has been that the contribution from the quintuplet state is very small. On the other hand, they had to make a number of assumptions to simplify the analysis of their data which Dodder and Hale did not make. Unfortunately, neither the data employed by Ad'yasevich and Fomenko nor some more recent data not yet analyzed by them [11] have been available to Dodder and Hale for analysis by their code. It is hoped that a copy of this data can be secured in the near future to try to resolve this disagreement about the contribution of the quintuplet state to the D-D reaction. (It should be noted that for the D - He³ reaction with the spins of the D and He³ aligned all D-D collisions are in the quintuplet state.)

In this paper we consider the D-D reaction employing the matrix elements derived by Ad'yasevich and Fomenko [8] to see what they imply for the enhancement of the D(dp)T reaction. At low energy we assume that only orbital angular momentum $l = 0$ and $l = 1$ need be considered. The results of our analysis, which is described in Appendix A, are given in Table IV. (The spins are taken relative to the z axis and the velocity of the deuteron beam makes an angle θ_0 relative to $\hat{z}$.) They are in striking disagreement with those of Dodder and Hale. They predict strong dependence of the D(d,p)T on spin with both large enhancements and suppressions possible. These results, of course, only apply to the proton channel of the D-D reaction. It is generally assumed that the results for the neutron channel D(d,n)He³ are similar because this is the mirror reaction of the D(dp)T reaction. This assumption is plausible because the total cross sections are similar.

Both sets of data and analyses are indirect and to some extent ambiguous. The best way to settle the question of the D-D reaction is the direct one of employing both a polarized beam and polarized target. It is hoped that this measurement will be carried out in the near future.

3. DEPOLARIZATION MECHANISMS

In this section we suppose the spins of nuclei in a thermonuclear reactor have been prepared in a given polarization state and we calculate the rate at which this polarization will change. For example, we could consider the case in which at every point in a D-T reactor all the D nuclei are in a state $m_2 = 1$ and all the T nuclei are in a state $m_3 = 1/2$, relative to local axes with the z axis along B . As these nuclei move with their thermal motion they will see a changing magnetic field direction and magnetic field strength. If the desired polarization state is to be maintained, they must continue to be in states $m_2 = 1$ or $m_3 = 1/2$, respectively, relative to the instantaneous axes at their position. If the magnetic field is slowly changing in space, this will always be the case. However, abrupt changes either in their motion or in the magnetic field direction will lead to a gradual change in their polarization eventually producing a random distribution of spin states. Once this has occurred, there will be no further modification of the nuclear reactions. We propose to show that under conditions expected in a reactor the time for this is very long compared to the average lifetime an ion spends in the reactor. Thus, as long as fresh ions are supplied in the desired polarization state to replace those either burned up or removed from the reactor, all the ions will be in a sufficiently polarized state to produce the desired modification of the nuclear reactions.

As stated in Sec. 1 we divide the various depolarization mechanisms (a) through (f) into six categories and treat them in six separate corresponding subsections.

3.1. Large scale inhomogeneous fields

In all reactors the equilibrium magnetic field B_0 varies in direction and magnitude over macroscopic lengths. Consider a particle of spin s , and treat

its kinetic motion classically and as unaffected by the interaction of its spin magnetic moment with the equilibrium field. B_0 at the position of the particle may thus be regarded as a known function of time. Then the Schroedinger equation for the spin state vector ψ is

$$i\hbar \frac{\partial \psi}{\partial t} = H\psi \quad (11)$$

where

$$H = -\underline{\mu} \cdot \underline{B}_0 = -g \frac{e}{2m_p c} \underline{s} \cdot \underline{B}_0(t) \quad (12)$$

where $\underline{\mu}$ is the magnetic moment, g is the gyromagnetic ratio, 0.86 for D, 5.44 for T, and $\underline{s}$ is the spin vector operator. At any time t take local cartesian coordinates x, y, z with the z axis along $\underline{B}_0 = B_0 \hat{z}$. The axes $\hat{x}$ and $\hat{y}$ are ambiguous. The rate of rotation of the threefold axes can be written

$$\underline{\omega} = \hat{z} \times \frac{d\hat{z}}{dt} + \omega_z \hat{z} \quad (13)$$

(ω_z is related to the orientation of the ambiguous local $\hat{x}$ and $\hat{y}$ directions and can be chosen in any convenient manner.) The spin state ψ can be transformed to the rotated coordinates at time t by the unitary transformation U

$$|\psi'\rangle = U|\psi\rangle \equiv \exp\left(-\frac{i}{\hbar} \int_0^t \underline{s} \cdot \underline{\omega}(t') dt'\right) |\psi\rangle \quad (14)$$

[By this we mean that if $|\psi\rangle$ is a column vector of the components of the spin state in terms of the eigenstates of $\underline{s} \cdot \underline{p}(0)$, then $|\psi'\rangle$ is a column vector for the components in terms of the eigenstates of $\underline{s} \cdot \underline{p}(t)$.]

We may obtain the equation for $|\psi'\rangle$ by differentiating Eq. (14) with respect to time, and making use of the relation $U_{\tilde{z}} \cdot \tilde{b}(t)U^{-1} = \tilde{s} \cdot \tilde{b}(0)$, we obtain

$$\frac{\partial |\psi'\rangle}{\partial t} = -\frac{i}{\hbar} \tilde{s} \cdot \tilde{\omega} U |\psi\rangle - \frac{i}{\hbar} U H |\psi\rangle = -\frac{i}{\hbar} \tilde{s} \cdot \tilde{\omega} |\psi'\rangle + \frac{i}{\hbar} \Omega_p \tilde{s} \cdot \tilde{z} |\psi'\rangle \quad (15)$$

where $\Omega_p(t) = ge B_0(t)/2m_p c$ is the local precession frequency. Write

$$\tilde{\omega} = \omega_z \hat{z} + \frac{1}{2} \omega_- (\hat{x} + i\hat{y}) + \frac{1}{2} \omega_+ (\hat{x} - i\hat{y}) \quad (16)$$

$$\tilde{s} = s_z \hat{z} + \frac{1}{2} s_- (\hat{x} + i\hat{y}) + \frac{1}{2} s_+ (\hat{x} - i\hat{y})$$

where $\omega_{\pm} \equiv \omega_x \pm i \omega_y$, $s_{\pm} \equiv s_x \pm i s_y$. Equation (15) becomes

$$\left[\frac{\partial}{\partial t} - \frac{is_z}{\hbar} (\Omega_p - \omega_z) \right] |\psi'\rangle = -\frac{i}{2\hbar} (s_+ \omega_- + s_- \omega_+) |\psi'\rangle \quad (17)$$

Write

$$|\psi'\rangle = \sum_m c_m(t) |\psi'_m\rangle \quad (18)$$

where $|\psi'_m\rangle$ are the eigenstates of $\tilde{s} \cdot \hat{z}(t)$ for eigenvalues $m\hbar$. Take for initial condition $c_m(0) = \delta_{m,n}$ and assume for $m' \neq m$, $c_{m'} \ll c_m$. The nonzero matrix elements of $s_{\pm}$ are:

$$\langle m | s_+ | m-1 \rangle = \langle m-1 | s_- | m \rangle = [(s+m)(s-m+1)]^{1/2} \hbar. \quad (19)$$

Equation (17) now reduces to

$$\left[\frac{\partial}{\partial t} - i(m+1)(\Omega_p - \omega_z) \right] c_{m\pm 1} = -\frac{i}{2\hbar} [(s \pm m + 1)(s \mp m)]^{1/2} c_m \quad (20a)$$

$$\left[\frac{\partial}{\partial t} - im(\Omega_p - \omega_z)\right]c_m = 0 \quad (20b)$$

Next define the new independent variable ϕ by

$$\phi = \int_0^t [\Omega_p(t') - \omega_z(t')] dt' \quad (21)$$

Equation (20b) yields $c_m = \exp(im\phi)$ and integration of Eq. (20a) yields

$$c_{m\pm 1} = -\frac{i}{2}[(s\pm m+1)(s\mp m)]^{1/2} \exp[(m\pm 1)\phi] \int_0^\phi \frac{d\phi' \omega_{\mp}(\phi')}{\Omega_p - \omega_z} \exp(\mp i\phi') \quad (22)$$

Let us apply Eq. (22) to the motion of a triton initially polarized in the $m_3 = 1/2$ state. Set $\omega_z = 0$. Then Eq. (22) yields

$$c_{-1/2} = \frac{\exp(-i\phi)}{2} \int_0^\phi e^{i\phi'} \frac{\omega_+(\phi')}{\Omega_p(\phi')} d\phi' \quad (23)$$

Note $d\omega_-/d\phi = (d\omega_-/dx)(dx/dt)(dt/d\phi) = \mathbf{x} \cdot \nabla \omega_- / \Omega_p \approx (p_3/9H) \omega_+$ where p_3 is the triton gyroradius and H is the scale size of variation of ω_+ . Thus, if the scale of variation of the field is large compared to p_3 , the rate of variation of $\omega_+(\phi)$ is slow compared to the factor $\exp(i\phi')$ and the net accumulation of $c_{-1/2}$ over a long time is exponentially small. (It is true that the gyromotion produces a variation of ω_+ with ϕ at a frequency of 8.91 but the oscillation in ω_+ is very small, and as shown in subsection 3.3. $c_{-1/2}$ is very small and oscillatory.) As a simple illustration of these results, consider the motion of the triton in a purely toroidal field and let its perpendicular velocity be zero. Take $\hat{x}$ along the toroidal axis. Then $\omega_+ = v/R$ where R is the radius of curvature of the line along which the triton moves. ω_+ and Ω_p are thus constant and Eq. (23) yields

$$c_{-1/2} = (1 - e^{i\Omega_p t}) \left(\frac{v}{4R\omega_p} \right).$$

$c_{-1/2}$ is thus small, but more importantly there is no secular change. The probability that the triton is in the $-1/2$ state at any time is of order $|v/2R\omega_p|^2 = |\rho/2R|^2$ a very small number. Thus, we may conclude that a triton moving in the equilibrium field of the reactor will always be locally in an eigenstate to a high degree of approximation, if it is initially. It will preserve its polarization. Similar considerations apply to a deuteron.

Inspection of Eq. (22) or (23) shows under what conditions we might expect the polarization to change. Replace B_0 by B , an arbitrary given field. If B abruptly changes direction as in a collision, Eq. (23) will yield a change in $c_{-1/2}$. Because of the factor $\exp(i\phi)$, we expect such changes to be random in phase so that only the square of such changes matters. Similarly, if a wave with a magnetic perturbation encounters the particle, it can make a resonant change if $\omega_+(\phi)$ is such as to have a unit harmonic. We now consider such changes.

3.2. Depolarization by collisions

The typical energy change of ions during collisions is of order kT the thermal energy of the ions. For $T = 10$ keV this is about eleven orders of magnitude larger than the energy necessary to flip a nuclear spin in a magnetic field of 50 kilogauss. Intuitively, it is surprising that the nuclear spin orientation could survive collisions so much more energetic than its energy of orientation. However, during the collision the actual torque exerted on the spin of the ion is very small. This together with the short time duration of the collision leads to the conclusion that the change in the spin polarization of the ion is very small. As an example, a typical

collision of an electron with an ion could have an impact parameter of 10^{-10} cm. Typical electron velocities are 10^{10} cm/sec. The duration of the collision is 2×10^{-20} sec. The magnetic field produced at the ion by the electron motion is $ev/r^2c \approx 10^{10}$ gauss. The torque on the ion is $\mu B = 5 \times 10^{-24} \times 10^{10} = 5 \times 10^{-14}$ dyne cm which produces a rate of rotation $\approx 5 \times 10^{-14}/\hbar = 5 \times 10^{13}$ rad/sec. The net rotation is 10^{-6} radians for this collision. These results are typical of the average collision. The mean angle of rotation of the spin is r_p/b where $r_p = e^2/m_p c^2$ is the classical proton radius and b the impact parameter. It is expected that the ion will suffer 10^4 collisions per second with electrons. Since these are incoherent, it takes 10^{12} collisions to depolarize the ion so the polarization would survive for 10^8 seconds (adding long range collisions reduces this to 10^6 seconds). These very rough considerations indicate that the amount of depolarization produced by collisions is negligible by a large factor. Therefore to establish that collisional depolarization is small, it is sufficient to determine upper bounds to the collisional depolarization rates. This is fortunate as there are many cases to consider and a precise calculation of all of them while possible would be quite complicated.

There are three collisional depolarization interactions, spin-orbit spin-spin and, for the deuteron, quadrupole interaction. We consider them in turn.

First, we consider depolarization of a polarized triton by the spin-orbit interaction with an electron. Let the ion be at rest, the electron move with velocity v prior to the interaction and the impact parameter be b . As in subsection 3.1 we treat the electron motion classically. Then the magnetic field at the triton is $\underline{B} = (-e\mathbf{v} \times \underline{r})/(cr^3) = (+e\mathbf{l}/r^3 cm_e)$ where $\underline{l}$ is the angular momentum of the electron about the triton. Thus, by Eqs. (11) and (12) the equation for the state vector ψ is

$$i\hbar \frac{\partial}{\partial t} |\psi\rangle = H\psi = -\frac{g e^2}{2m_p c^2 m_e} \frac{\mathbf{s} \cdot \mathbf{l}}{r^3} |\psi\rangle \quad (24)$$

Write as before

$$|\psi\rangle = c_{1/2} \left| \frac{1}{2} \right\rangle + c_{-1/2} \left| -\frac{1}{2} \right\rangle \quad (25)$$

and assume $c_{-1/2} \ll c_{1/2}$. Since $\mathbf{g} \cdot \mathbf{l} = \{ (s_z l_z + (1/2) s_- (l_x + i l_y) + (1/2) s_+ (l_x - i l_y) \}$, we may write the equation for $c_{-1/2}$ as

$$\frac{dc_{-1/2}}{dt} = \frac{g_3 i \hbar p}{4m_e} \frac{(l_x + i l_y)}{r^3} c_{1/2} \quad (26)$$

(Note that the Thomas precession correction for the nuclear precession can be neglected for electron nuclear collisions, since it is smaller than the precession in the magnetic field of the electron by a mass ratio. It is of order unity however for nucleon-nucleon spin-orbit interactions; see below.)

Thus, the total change of $c_{-1/2}$ during the collision is

$$\Delta c_{-1/2} = \frac{g_3 i \hbar p}{4m_e} (l_x + i l_y) \int \frac{dt}{r^3} = \frac{g_3 i \hbar p}{r^3} \frac{(l_x + i l_y)}{l} \int \frac{d\theta}{r} \quad (27)$$

where θ is the polar angle of the electron relative to the triton (Fig. 3).

But

$$\frac{1}{r} = \frac{1}{b_0} \left[(b_0^2 + b^2)^{1/2} \cos \theta + b_0 \right]$$

where $b_0 = e^2/m_e v^2$ and $\tan \theta_1 = b/b_0$, so

$$\Delta c_{-1/2} = \frac{g_3 i \hbar p}{2b} (\hat{l}_x + i \hat{l}_y) (b + b_0 \tan^{-1} \frac{b}{b_0}) c_{1/2} \quad (28)$$

Now if $b \gg b_0$,

$$\Delta c_{-1/2} = -\frac{g_2 r_p}{2b} (\ell_x + i\ell_y) c_{1/2} \quad (29)$$

On the other hand, if $b = b_0$ and if the electron energy $\gg 50$ eV, then $\ell = e^2/v \ll \hbar$. But ℓ if nonzero cannot be less than $\hbar$, so the smallest impact parameter that makes sense is the de Broglie wavelength $\lambda = \hbar/mv$. (As a matter of fact, if $b \ll \lambda$, it is not appropriate to treat the electron motion classically.) However, the lowest cutoff appears in the final answer only logarithmically and it is shown in Appendix B that taking the classical result with $b = b_{\min} = \lambda$ is sufficiently accurate. [In actual fact, the exact result quantum mechanically corresponds to $b_{\min} = (e^2/mv^2)$ and $\hbar$ does not enter at all.]

Equation (29) gives the change in $\Delta c_{-1/2}$ during a single collision. In between collisions $c_{1/2}$ precesses rapidly so it may be taken at each collision to be of unit magnitude but random phase. Thus, the changes in $\Delta c_{-1/2}$ are random and the average change is zero. The sum of $|\Delta c_{-1/2}|^2$ after a time t is gotten by summing over all collisions. Since there are $2\pi v b db n_e$ collisions per unit time, with b in db we have

$$\begin{aligned} \frac{\sum (\Delta c_{-1/2})^2}{\Delta t} &= \int_{b_{\min}}^{b_{\max}} 2\pi v b db n_e |\Delta c_{-1/2}|^2 \\ &= \frac{\pi}{2} g_2^2 r_p^2 n_e v \langle (\hat{\ell}_x + i\hat{\ell}_y)^2 \rangle \int_{b_{\min}}^{b_{\max}} \frac{b db}{b^2} \\ &= \frac{\pi}{3} g_2^2 r_p^2 v \log\left(\frac{b_{\max}}{b_{\min}}\right) \quad (30) \end{aligned}$$

For the average value of $|\ell_x + i\ell_y|^2/\ell^2 = \sin^2 \phi$, where ϕ is the angle between ℓ and the z axis, we take $2/3$. $b_{\max} = c/\omega_{pe}$ the electromagnetic cutoff and we

take $b_{\min} = \lambda$. Equating Eq. (30) to $n_e g v$ we obtain for the cross section for depolarization by the spin-orbit interaction between tritons and electrons

$$\sigma_{te} = \frac{\pi}{3} g_3^2 r_p^2 \log \left(\frac{c}{\omega_{pe}} \frac{mv}{h} \right) \quad (\text{spin orbit e-t}) \quad (31)$$

(This cross section appears to differ from that quoted in Refs. 4 and 5 because g_3 here is defined so that it is larger by a factor of 2.)

To obtain the cross sections for spin-orbit depolarization of tritons by tritons we carry out the same calculation with the following changes. The equation for $1/r$ is $1/r = [(b^2 + b_0^2)^{1/2} \cos \theta - b_0]$. The parenthesis in Eq. (28) is replaced by $b - b_0 \tan^{-1} b/b_0 \approx -1/3 b^2/b_0^3$ if $b \ll b_0$, so that the integral in Eq. (30) converges at $b = 0$. (In addition, the Thomas precession is of importance which changes the result by a factor of order unity. For this case g_3 is replaced by $g_3^T = g_3 - 4/3$. Since the cross sections are already very small, we need not include this correction.) As a consequence we obtain for the cross section for depolarization by spin-orbit interaction with other tritons

$$\sigma_{tt} = \frac{\pi}{3} g_3^2 r_p^2 \log \left(\frac{c}{\omega_{pe} b_0} \right) \quad (32)$$

which is identical to the cross section for depolarization by electrons except for the logarithmic factor. Similarly, we find that the spin orbit depolarization of the triton by deuterons $\sigma_{td} = \sigma_{tt}$ (except for a slightly different correction for precession which we ignore).

In summary, the spin-orbit depolarization of tritons has comparable cross sections for electrons, tritons, and deuterons. However, when one considers that the relative velocity of the electrons and tritons is much larger than ion-triton relative velocities, it is clear that the major contribution to

depolarization is by electrons. The cross section for depolarization electrons is $1.8 \times 10^{-29} \text{ cm}^2$. Take $v_e = (2T/m)^{1/2} = 6 \times 10^9 \text{ cm/sec}$ at $T = 10^4 \text{ eV}$. If $n_e = 3 \times 10^{14}$, the depolarization rate is $3 \times 10^{-5} \text{ s}^{-1}$.

Now consider polarized deuterons in the $m = 1$ state. The calculation is identical except that the deuteron of the matrix element of s_z for the transition $m = 1$ to $m = 0$ is $\sqrt{2}$. There results

$$\sigma_{De} = \frac{2\pi}{3} g_2^2 r_p^2 \log \left(\frac{c}{\omega_{pe}} \frac{mv}{k} \right) \quad m(1 \rightarrow 0)$$

$$\sigma_{DT} = \sigma_{DD} = \frac{2\pi}{3} g_2^2 r_p^2 \log \left(\frac{c}{\omega_{pe} b_o} \right) \quad . \quad (33)$$

On the other hand, if the deuterons are in the $m = 0$, they can depolarize by changing to $m = 1$ or $m = -1$. Either process represents depolarization and has cross sections equal to Eq. (33). The spin orbit depolarization rates for D are thus $1.25 \times 10^{-6} \text{ s}^{-1}$ for $m = 1$ state and $2.5 \times 10^{-6} \text{ s}^{-1}$ for the $m = 0$.

It is interesting to repeat the calculation of spin-orbit depolarization for electrons, which have much larger magnetic moments ($g \approx 3700$). One finds $\sigma = 2.52 \times 10^{-23} \text{ cm}^2$ and the depolarization rate for the above numbers is 4 s^{-1} . Thus it is difficult to keep electrons polarized.

The spin-spin interactions are considerably more complicated than the spin-orbit interactions. However, the depolarization rates which they produce are smaller than those produced by the spin-orbit interaction by an order of magnitude. Their cross sections as well as those for the quadrupole interaction are considered in Appendix B.

3.1. The combined effects of field inhomogeneities and collisions

In our discussion of the motion of an ion in an inhomogeneous field B_0 we temporarily deferred discussion of the gyromotion of the ion. If the magnetic field has a shear, the ion in its gyromotion will see a small change in the direction of the magnetic field of order $\delta\theta = \rho_i |\hat{b} \cdot \nabla \mathbf{b}|$ where $\hat{b}$ is the unit vector in the direction B_0 and ρ_i the gyroradius of the ion. A simple application of Eq. (13) shows that such a gyromotion oscillation leads to an oscillation in $\omega_{\pm}$ of order $\Omega\delta\theta$ at the gyrofrequency rate Ω_i . This oscillation is nonresonant with the precession frequency Ω_p , so by Eq. (22) it produces an oscillation in the amplitude $c_{m\pm 1}$ of order $1/2 [(s\pm m+1)(s\pm m)]^{1/2} \delta\theta \times \Omega_i / (\Omega_i - \Omega_{pr})$. For deuterons in the state $m = 1$, c_0 will oscillate by $1/2 \delta c_0 \sqrt{2} \delta\theta / (1 - 0.88) = 5\delta\theta$. To estimate $\delta\theta$ let us consider that for a $B = 5 \times 10^4$ gauss, $T_i = 10^4$ eV a thermal ion has a gyration radius of 0.45 cm. If B rotates by 0.1 rad over 10^2 cm, we have $\delta\theta = 0.00045$ rad and c_0 oscillates by $\delta c_0 = 2 \times 10^{-3}$. This in itself corresponds to a very small depolarization. (The probability that the D is in the $m = 0$ state is 4×10^{-6} at any time.) But a sudden collision could dephase the change in δc_0 so that many collisions could gradually change c_0 , $(\Delta c_0)^2 = N(\delta c_0)^2$ where N is the number of collisions. Then $(\Delta c_0)^2 = v_i t 4 \times 10^{-6} \approx 4 \times 10^{-4} t$, if we take the collision rate as $v_i \approx 10^2$ sec. This is much larger than the depolarization rate estimated for the deuteron by collisions in Sec. 3.2. but it is still very small. For tritons the same estimate may be made. If a triton is initially in the $+1/2$ state, one finds that $\delta c_{1/2} = 1/2 \delta\theta / (8.91 - 1) = 0.066\delta\theta$. Taking the same estimate for the gyration radius and collision frequency as for deuterons, one finds that the depolarization rate is smaller by $(0.013)^2$ which yields $(\Delta c_{1/2})^2 \approx 10^{-7} t$. This rate is smaller than the collisional depolarization rate. A similar calculation for electrons with $\Omega_p = 1.002 \Omega_{pe}$, $\rho_e = \rho_i / 60$, and $v_e = 60 v_i$ leads to $(\Delta c_{-1/2})^2 = 0.042 t$ a rate also smaller than that obtained in Sec. 3.2. for direct collisional depolarization.

To calculate quantitatively the depolarization rate by this process, which we may term gyro-collisional depolarization, a kinetic calculation is necessary. This is carried out in the accompanying paper [6]. In this present paper we content ourselves with a calculation based on the Langevin equations of motion

$$\frac{d\tilde{r}}{dt} = \tilde{v} \quad (34a)$$

$$\frac{d\tilde{v}}{dt} + \tilde{v} \times \tilde{v} = \frac{e}{mc} \tilde{v} \times \tilde{B}_0 + \tilde{A}(t) \quad (34b)$$

We take a model for the sheared field

$$\tilde{b} = \tilde{z} \left[1 - \left(\frac{x}{L} \right)^2 \right]^{1/2} + \tilde{y} \frac{x}{L} \quad (35)$$

where L is the scale length for shear. Then, applying the formalism of the beginning of this section, and assuming $x \ll L$, we have from Eq. (13)

$$\tilde{\omega} = \tilde{b} \times \frac{d\tilde{b}}{dt} = \frac{\tilde{z} \times \tilde{v} \times \tilde{y}}{L} = - \frac{v_x}{L} \tilde{x} \quad (36)$$

We have set $\omega_z = 0$.

Let us consider a deuteron originally in the $m = 1$ state, then Eq. (20a) yields

$$\frac{dc_0}{dt} = - \frac{1}{2} \omega_+ c_1 = - \frac{i\omega_x}{2} c_1 = \frac{iv_x}{2L} c_1 \quad (37)$$

From Eq. (20b) $c_1 = \exp(i\Omega_p t)$ so that

$$c_0 = \frac{i}{2L} \int_0^t e^{i\Omega_p t'} v_x(t') dt' \quad (38)$$

The ensemble averaged, rate of secular increase in the polarization state $m = 0$ is then given by

$$\frac{|c_0^2|}{t} = \frac{2}{4L^2} \operatorname{Re} \int_0^\infty d\tau e^{i\Omega_p \tau} \langle v_x(0) v_x(\tau) \rangle \quad (39)$$

The solution of Eq. (34b) for $v_x(t)$ is

$$v_x(t) = \int_0^t A(t') e^{-\nu(t-t')} \cos \Omega_2(t-t') dt' \quad (40)$$

so

$$\langle v_x(t) v_x(0) \rangle = \frac{v_1^2}{2} e^{-\nu t} \cos(\Omega_2 t) \quad (41)$$

We have taken the ensemble mean of A to be zero and used the fact that $\langle v_x^2 \rangle = v_1^2/2$. Inserting Eq. (41) into Eq. (39) and taking $\nu \ll \Omega_p$, we have

$$\frac{|c_0^2|}{t} = \nu \frac{v_1^2}{4L^2} \left(\frac{\Omega_2}{\Omega_p - \Omega_2} \right)^2 \quad (42)$$

This agrees with our intuitive estimate at the beginning of this subsection. We have derived this result for the case of greatest interest to us. To extend it to other nuclear species or states we need only multiply Eq. (42) by $(l \pm m + 1)(l \pm m)/2$ and employ the appropriate values for Ω_p and Ω_2 .

3.4. Depolarization by waves

Inspection of Eq. (22) shows that the polarization state of the nucleus is most susceptible to variations in the direction of the magnetic field $\hat{b}$ which the nucleus sees as resonant with its natural precession frequency (in the local magnetic field). If the magnetic field is uniform plus variations due to small amplitude waves, $B_0 + \delta B$, the condition is that the Doppler shifted frequency of the wave as seen by the ion be equal to the precession frequency. For a nucleus spiraling about the magnetic field B_0 , there will also be a contribution to depolarization if the frequency of the wave ω satisfies

$$\omega - k_{\parallel} v_{\parallel} + n\Omega_i = \Omega_p$$

where $k_{\parallel}$ is the component of the wave vector along B and n is any integer. This is because in Eq. (22), $\omega_{\pm}$ has frequency components $\omega \pm n\Omega$ due to the gyromotion of the particle (the higher harmonics n are generally weaker than the fundamental $n = 0$ if $k_{\perp} \rho \ll 1$).

Before writing down the general expression for depolarization by waves, let us estimate its magnitude. To be specific let us consider a moving triton in a $m = 1/2$ state and let $B_0 + \delta B$ be the field it sees in its moving frame. Then from Eqs. (11) and (12) we have

$$\begin{aligned} \frac{dc_{-1/2}}{dt} - \frac{i\Omega_p}{2} c_{-1/2} &= \frac{i\Omega_p}{2} \frac{\delta B_x - i\delta B_y}{B_0} c_{1/2} \\ &= \frac{i\Omega_p}{2} \left(\frac{\delta B_x - i\delta B_y}{B_0} \right) \exp\left(-\frac{i\Omega_p}{2} t\right) \end{aligned}$$

or

$$c_{-1/2} = \exp(i\Omega_p t) \int_0^t \frac{i\Omega_p}{2} \left(\frac{\delta B_x - i\delta B_y}{B_p} \right) \exp(i\Omega_p t') dt' \quad (43)$$

Several conclusions can be drawn from Eq. (43). First, it is only the left circular part of the wave that causes a change in $c_{1/2}$ as would be expected from the fact that the triton precesses in the left-handed direction about B_0 . Second, only waves with harmonics near Ω_p (or that differ from Ω_p by an integral multiple of Ω_3) affect polarization. Third, the amount of depolarization produced by such a wave in a time t is proportional to the amount of precession that would be produced by $\delta B_{\perp}$ alone in the absence of B_0 . Using these facts we can estimate the amount of depolarization produced by left circular polarized waves with mean-square amplitudes $\delta B_{\perp}^2$ in frequency range $1/2 \Omega_p - 2\Omega_p$. Such waves are incoherent over a time Ω_p^{-1} and produce a change $\delta c_{-1/2} \approx 1/2 (\delta B_{\perp}/B_0) \Omega_p t_c$ during the decorrelation time $t_c \approx \Omega_p^{-1}$. A simple estimate then shows that

$$\frac{(\delta c_{-1/2})^2}{t} \approx \frac{1}{8} \left\langle \left(\frac{\delta B_{\perp}}{B} \right)^2 \right\rangle \Omega_p \quad (44)$$

We have inserted another factor of $1/2$ to take care of some cancellation in the integral of Eq. (43). Estimate (44) can be extended to deuterons by insertion of the proper matrix element.

As an example of the importance of wave depolarization, let us consider left circular polarized waves with amplitude $\delta B \approx 1$ gauss and $B_0 = 5 \times 10^4$ gauss. Since $g_3 = 5.94$ for a triton $\Omega_p = g_e B / 2m_p c \approx 1.5 \times 10^9 \text{ s}^{-1}$ and Eq. (44) yields

$$\frac{(\delta c_{-1/2})^2}{t} \approx 0.075 \text{ s}^{-1} \quad (45)$$

Thus, even an extremely small amplitude wave can lead to significant depolarization over times of interest. It is easy to show that a thermal level of waves produces negligible depolarization. However, if any waves should be unstable in the range of the triton precession frequency, it is very likely that their amplitude would exceed $\delta B = 1$ gauss and polarization could not be maintained. A similar calculation for deuterons at rest and in the $m = 1$ state shows that for left circular polarized waves in the range of the deuteron frequency, $(\delta c_0)^2/t$ is smaller by $8(g_2/g_3)^2 = 0.17$, so $(\delta c_0)^2/t = 0.013 \text{ s}^{-1}$. This is somewhat slower so δB need be of order 10 gauss to produce significant depolarization. This is still a small amplitude. Therefore almost any waves near the deuteron cyclotron frequency at a nonthermal level would produce depolarization of the deuterons. This means that it would not be possible to heat the deuterons by waves at the deuteron cyclotron frequency and still maintain polarization. On the other hand, in the absence of such heating, the presence of ordinary cyclotron damping should make the existence of unstable waves unlikely.

In order to make these ideas more quantitative consider a general spectrum of waves in a uniform magnetic field and plasma. Let the intensity of the magnetic field be given by

$$\langle \delta \underline{B} \delta \underline{B} \rangle = \sum_j \int d^3k \, \underline{r}^j(\underline{k}) \quad (46)$$

where j indicates the particular mode at wave number $\underline{k}$.

Then it is shown in Appendix C that the rate of depolarization of a nuclear species i is

$$\frac{(\Delta \epsilon_{-1/2})^2}{t} = \alpha_i \sum_{n,j} \pi \Omega_p^2 \int d\mathbf{k}_\perp d\mathbf{k}_\parallel J_n^2 \left(\frac{k_\perp v_\perp}{\Omega} \right) (\hat{x} - i\hat{y}) \cdot \hat{\mathbf{I}} \cdot (\hat{x} + i\hat{y}) \times \delta(\omega_k - k_\parallel v_\parallel - n\Omega_i - \Omega_{pj}) \quad (47)$$

where $\alpha_i = 1$ for tritons, 2 for deuterons in the $m = 1$ state, and 4 for deuterons with $m = 0$ state. The operation $(\hat{x} - i\hat{y}) \cdot \hat{\mathbf{I}} \cdot (\hat{x} + i\hat{y})$ selects waves of the proper polarization.

3.5 Waves of relevance to polarization

The precession frequency of the triton is $5.94 \Omega_2$, where Ω_2 is the deuteron cyclotron frequency. The triton cyclotron frequency is $\Omega_3 = 0.67 \Omega_2$ so waves with $\omega \approx (5.94 \pm 0.67n) \Omega_2$ are of interest. Because of thermal motion, any waves near these frequencies are also of interest. However, if $k_\perp v_\perp / \Omega_3$ is small, only small values of n are of importance because of the Bessel function factor in Eq. (47). Consider $n = 0$, $\omega = 5.94 \Omega_2$. Such waves are in the whistler region with $\omega \approx k v_A$. There is no indication that such waves are unstable, since there are no ions directly resonant with such waves.

Next consider deuterons. The deuteron precession frequency is $0.86 \Omega_2$ so waves with $\omega \approx (0.86 \pm n) \Omega_2$ are of interest. For $n = 0$ the dispersion is rather complicated as this frequency lies between the deuteron cyclotron frequency Ω_2 and the triton cyclotron frequency $0.67 \Omega_2$. As a matter of fact, if the triton and deuteron densities are equal, there is a cutoff at $\omega \approx 0.83 \Omega_2$ very close to Ω_p .

The dispersion relation for these waves for a homogenous magnetic field has been derived in Ref. 12. The results are rather sensitive to changes in B_0 so that the inhomogeneity of the equilibrium magnetic field in an actual device will introduce important modification to these results. However, if we

restrict ourselves to homogeneous plasma, we find for propagation perpendicular to the magnetic field

$$\omega^2 = k^2 v_A^2 \frac{\bar{\Omega}}{\Omega_2} \frac{(\omega^2 - \bar{\Omega}^2)}{(\omega^2 - \bar{\Omega}^2)} \quad (48)$$

where $\bar{\Omega} = \alpha_2 \Omega_2 + \alpha_3 \Omega_3$, $\bar{\bar{\Omega}} = \alpha_2 \Omega_3 + \alpha_3 \Omega_2$, $\Omega_h = (\Omega_2 \Omega_3)^{1/2}$ and $v_A^2 = B^2 / (4\pi n_e m_2)^{1/2}$, where α_2 and α_3 are the fractions of deuterons and tritons by number. For $\alpha_2 = \alpha_3 = 0.5$, $\bar{\Omega} = \bar{\bar{\Omega}} = 0.833 \Omega_2$, and $\Omega_h = 0.816 \Omega_2$. These waves have been discussed by Coppi et al. [12] who show that if propagation is not strictly perpendicular, these waves will resonate with the alpha particles produced by D-T reactions. They show that an anisotropic distribution of the alpha particles such as those discussed in Sec. 2 will tend to drive these modes unstable. However, there is a stabilizing influence on these waves due to transit time damping of electrons which are also resonant with these waves [13]. Under practical conditions the latter is always more important and the waves are damped.

For parallel propagating left circular polarized waves one has

$$\omega^2 = k^2 v_A^2 \frac{(\Omega_2 - \omega)(\omega - \Omega_3)}{\Omega_2 (\omega - \bar{\Omega})} \quad (49)$$

Propagation can only occur for $\omega < \Omega_2$ or for $\bar{\Omega} < \omega < \Omega_2$. The second region includes the precession frequency. However, except for ω very near $\bar{\Omega}$ these waves are slow and strongly damped by ordinary cyclotron damping of the thermal deuterons. If $\omega - \bar{\Omega} \approx 0.01 \Omega_2$, the wave is fast and could be driven unstable by an anisotropic distribution of alpha particles. However, the wave length of such waves is quite large so that a small inhomogeneity in the magnetic field will make $\omega - \bar{\Omega} > 0.01 \Omega_2$ and the wave will be damped.

In summary, waves in the region of the deuteron frequency are potentially dangerous to polarization. A careful theoretical consideration indicates that damping processes are stronger than amplification processes. An experimental test is necessary to be absolutely sure of this.

3.6. Atomic processes

Up to this point we have only considered depolarization processes which occur in a fully ionized plasma. In fact, most nuclei which are present in plasmas go to the cooler outside parts of the plasma where they may recombine to form atoms. They may even enter a short distance into the walls, leave the walls, ionize, and reenter the plasmas. If depolarization occurs during these stages, the net polarization of the plasma will be reduced and the effect of polarization on nuclear reaction rates will be smaller.

It is fairly certain that no depolarization will occur during ionization of the atoms as this happens on such a rapid time scale, $\sim 10^{-15}$ seconds, and the nuclei will have no time to depolarize. It is also the case that if nuclei recombine in a strong field, coupling to the spin magnetic moment of the electron will be so strong that little depolarization occurs. To see this let us consider the state of a tritium atom in a very strong field. The Zeeman splitting of the atom is shown in Fig. 4. At zero B the atom has two states which are separated by the hyperfine interaction energy $E = \mu B_0$ where $\mu = e\hbar/m_e c$ is the magnetic moment of the electron. The upper state is threefold degenerate and splits under the influence of the external field. For strong fields $B \gg B_0$ the Zeeman splitting is larger than the hyperfine interaction energy, which may be treated as a perturbation.

We calculate the depolarization per recombination as follows: The spin dependent part of the Hamiltonian (Landau and Lifshitz [14]) is

$$\begin{aligned}
 H &= \frac{e}{m_e c} \mathbf{s}_e \cdot \mathbf{B} + \frac{16\pi}{3i} g \frac{e^2}{2m m_p c^2} \mathbf{s}_e \cdot \mathbf{s}_i \psi^2(0) \\
 &= \frac{e}{m_e c} (\mathbf{s}_e \cdot \mathbf{B} + \frac{2\mathbf{s}_e \cdot \mathbf{s}_i B_c}{(2i+1)})
 \end{aligned} \quad (50)$$

where i is the spin of the nucleus, ($1/2$ in this case) and where the contribution of the interaction of the nucleus has been neglected. Write the eigenstate of (50) as

$$|\psi\rangle = \sum_{m_3, m_e} c_{m_3, m_e} |m_3, m_e\rangle \quad (51)$$

Then the various eigenstates of (50) are

$$\psi_1 = |1/2, 1/2\rangle, \quad \psi_2 = |1/2, -1/2\rangle + c|-1/2, 1/2\rangle$$

$$\psi_3 = |-1/2, 1/2\rangle - c|1/2, -1/2\rangle, \quad \psi_4 = |-1/2, -1/2\rangle.$$

If $B \gg B_c$, $c \ll 1$ and may be determined by perturbation theory with the result

$$c = \frac{2}{2i+1} \frac{\langle 1/2, -1/2 | \mathbf{s}_e \cdot \mathbf{s}_i | -1/2, 1/2 \rangle}{\Delta E} \frac{e}{mc} \frac{B_c}{H} \quad (52)$$

where

$$\Delta E = \frac{e\hbar B}{mc} \quad (53)$$

Evaluation of the matrix element gives $\mu^2/2$ so that

$$c = \frac{1}{2i + 1} \frac{B_c}{B} = \frac{1}{2} \frac{B_c}{B} \quad (54)$$

since $i = 1/2$.

Now imagine a recombination in which the triton is in the $+1/2$ state. If the electron is also in the $+1/2$ state, then the state ψ_1 , is formed with probability 1. After reionization the triton will still be in the $+1/2$ state. On the other hand, if the electron is in the $-1/2$ state the ψ_2 state will be formed with probability essentially unity while the ψ_3 state will be formed with probability c^2 . After reionization the ψ_2 state will reionize with the triton in the $-1/2$ state, with probability c^2 , while the ψ_3 state will reionize with $m_3 = -1/2$ with probability essentially one. Thus, if the electron is in the $-1/2$ state, the chance that the triton will flip into the $-1/2$ state is $2c^2$. Assuming that the electron has random polarization we may average the chances that the flipping occurs with the electron initially in the $+1/2$ state, 0, and the $-1/2$ state, $2c^2$ to obtain

$$P_{\text{recomb}}^T = c^2 = \frac{1}{4} \left(\frac{B_c^T}{B} \right)^2 \quad (55)$$

for the total probability of depolarization of the triton per recombination time. (For triton $B_c^T = 500$ gauss.)

A very similar calculation for deuterons taking into account the different spin and matrix elements yields

$$P_{\text{recomb}}^D (m = 1 \Rightarrow m = 0) = \frac{2}{9} \left(\frac{B_c^D}{B} \right)^2 \quad (56)$$

($B_G^D = 100$ gauss).

As a numerical example, let $B=50$ kilogauss, then $p^T = 2.5 \times 10^{-5}$ and $p^D = 10^{-6}$ for $m = 1$ and 2×10^{-6} for $m = 0$, respectively per recombination. Thus, the amount of depolarization produced by recombination of either ion species is negligible.

In calculating depolarization during recombination we have only taken into account depolarization during the time the recombined atom is in the $1s$ state. This is plausible since the atom spends most of its time in this ground state. However, in the ground state the only way to flip the nuclear spin is to flip the electron spin at the same time and this is restricted by the relatively strong interaction of magnetic moment of the electron with the external field. However, in the $2p$ state the nuclear spin can flip and conserve angular momentum of the electron leaving the electron spin unchanged. This flipping occurs very rapidly and would be quite important if it were not for the very short time the atom spends in the $2p$ state $\sim 10^{-9}s$. Even so it is shown in Appendix D that the probability of spin flip during the time the atom might spend in the $2p$ state during the recombination process is actually an order of magnitude larger than that estimated above. However, it is of negligible significance as a depolarization process as far as fusion ions are concerned, since one would not expect recombination to occur more than ten times.

3.7. Depolarization on walls

A much more serious depolarization process can occur during interaction of the nuclei with the solid walls enclosing the plasma. It is expected that during recycling processes the nuclei will leave the plasma and because of

sheath potentials near the wall will be driven deeply into these walls. It is estimated that it will reside in the wall many tens of milliseconds before leaving and reentering the plasma. During this time it will be subject to all the fluctuating magnetic fields existing in the wall. If the wall is metallic, these fluctuations are known to depolarize the ion in times of order of a few milliseconds [15]. Thus, the presence of metallic walls will certainly depolarize the nuclei, and if they are recycled to the plasma, the whole plasma will lose its polarization. On the other hand, it is suspected that fluctuating magnetic fields in amorphous graphite or similar substances will be greatly reduced. Greenside et al. [16] have investigated depolarization times in such substances and have shown them to be as long as 5 seconds. If this is so, then it should be possible to make the portions of the wall in actual contact with the plasma out of amorphous graphite and avoid this depolarization process. However, because of the unknown nature of many of the processes involved in plasma wall interactions, it will be only possible to settle this question by experimental tests.

4. SUMMARY AND CONCLUSIONS

The possibility of improving the performance of a nuclear reactor by having the nuclei polarized in a given manner depends on a number of factors.

- (1) A sufficient source of polarized nuclei must be available. We have not discussed this to any extent but it is clear that sources exceeding 100 amps equivalent must be developed. Although these are not presently available, prospects of their realization seems promising. Techniques involving either optical pumping [17] or cryogenic techniques [18,19] are being considered and there appears to be no real barrier to either technique. Further, the estimated cost in energy per polarized nucleus is much less than the resulting gain in fusion power.

- (2) Once the nuclei are produced they must be transported into the plasma either by gas puffing, energetic neutral beams, or frozen pellets. The production of polarized energetic neutral beams seems problematic but the other two methods seem feasible. The transport is to be carried through a magnetic field changing in magnitude and direction. In order to avoid depolarization during transport a restriction related to Eq. (22) must be satisfied but simple model fields indicate this to be relatively easy.
- (3) Once the fuel is in the plasma it must remain polarized to a high degree. The considerations of Sec. 3 indicate that the two major dangers to this are wave-nuclear spin interactions and wall interactions during any recycling. (All other depolarization mechanisms appear to be satisfactorily small.) A fairly convincing case can be made that there will be no internally generated waves that could affect the polarization. With less confidence one could believe proper wall materials can be introduced in the region of plasma-wall interaction to remove the second danger to depolarization.

Thus it can be stated that there is no strongly convincing physical argument that polarization cannot be maintained in a fusion reactor.

The mode of polarization that is an optimum for a reactor depends on what restrictions are most important for its economic performance. If stability considerations are uppermost, one can employ the mode of polarization [mode (a) in Sec. 1] that enhances the reactivity. This makes it possible to reduce the pressure of the plasma by a factor 1.22 while maintaining the same nuclear power. On the other hand, if the disintegration of materials or auxiliary equipment is the primary consideration, one could employ the mode of polarization [mode (a) or (b)] that directs the neutrons in an optimum

direction to reduce this disintegration. Finally, it could be possible to improve the stability of the plasma by creating a population of energetic alpha particles with a particular anisotropy.

Perhaps the most exciting consequence of the employment of nuclear fuel is the possibility of a nearly neutron-free D - He³ reactor. This of course depends on the assumption that the D - D reaction can be strongly suppressed which in turn depends on some yet unexplored nuclear physics. It is hoped that the open question of the spin dependence of the D-D reaction can be settled fairly soon.

APPENDIX A

THE SPIN DEPENDENCE OF THE D-D CROSS SECTION

Let us denote the spin states of the deuteron by α ($m_2 = 1$), β ($m_2 = 0$) and γ ($m_2 = -1$). Let us consider as an example the collision of two deuterons 1 and 2 in states α_1 and β_2 , respectively, and with relative velocity along the z' axis making an angle θ_0 with the z axis. Let the $z' = z'_1 - z'_2$ be the relative coordinate. (See Fig. 5.) Then ignoring the coulomb and nuclear potentials we may write for their combined wave function

$$\psi = \frac{\alpha_1 \beta_2 \exp(ikz') + \alpha_2 \beta_1 \exp(-ikz')}{\sqrt{2}} \quad (\text{A-1})$$

since the wave function must be symmetric. We normalize to unit density of the incoming deuteron. We invoke the plane wave expansion

$$\exp(ikz') = \sum_{\ell} a_{\ell}(r) P_{\ell}(\cos \theta') ; \quad (\text{A-2})$$

$a_{\ell}(r)$ are well known for a plane wave. We now include the coulomb interaction and let a_{ℓ} be the solution of the radial wave equation with coulomb repulsion. Further, we terminate the sum after $\ell = 1$ because the centrifugal repulsion will reduce the wave function at small r for low energies. Then employing Table 1 we can combine the spin states $\alpha_1 \beta_2$ into total spin states $|sm_s\rangle$. We make use of the relation $P_{\ell}(-\cos \theta') = (-1)^{\ell} P_{\ell}(\cos \theta')$ to obtain

$$\psi = |2, 1\rangle a_0 P_0 + |1, 1\rangle a_1 P_1(\cos \theta') \quad (\text{A3})$$

Next using the addition formula $\cos \theta' = \cos \theta \cos \theta_0 + \sin \theta \sin \theta_0 \cos \phi$, we may express $P_0(\cos \theta')$ and $P_1(\cos \theta')$ in terms of spherical harmonics $Y_0(\theta, \phi)$, $Y_{1,m}(\theta, \phi)$. Let $b_0 = (4\pi)^{1/2} a_0$, $b_1 = (4\pi/3)^{1/2} a_1$. Then

$$\psi = |2,1\rangle b_0 Y_{00} + |1,1\rangle b_1 \left[\cos \theta_0 Y_{1,0} + \frac{1}{\sqrt{2}} \sin \theta_0 (Y_{1,1} + Y_{1,-1}) \right]. \quad (A4)$$

The Y 's are the normalized wave functions for orbital angular momentum, given in Table 2. We may now employ Table 1 to add the spin and orbital angular momenta to express ψ in terms of the total angular momentum states $|S, L, J; m_S\rangle$

$$\begin{aligned} \psi = & b_0 |2,0,2;1\rangle + \left[\frac{1}{2} b_1 \cos \theta_0 (|112;1\rangle - |111;1\rangle) \right] \\ & + b_1 \sin \theta_0 \left[\left(\frac{1}{2} \right)^{1/2} |112;2\rangle + \left(\frac{1}{12} \right)^{1/2} |112;0\rangle \right. \\ & \left. - \frac{1}{2} |111;0\rangle + \left(\frac{1}{6} \right)^{1/2} |110;0\rangle \right]. \quad (A5) \end{aligned}$$

According to Ad'yasevich and Fomenko [8] the chance of a nuclear reaction depends on the probability that the deuterons are in a particular angular momentum state S, L, J , and are a distance r_0 apart where $r_0 = 7 \times 10^{-13}$ cm, the radius of a deuteron. For example, the chance the deuterons starting in spin state $\alpha_1 \beta_2$ are in a state $|112\rangle$ at r_0 is from Eq. (A5)

$$\begin{aligned} & \left[\left(\frac{1}{2} \right)^{1/2} b_1 \cos \theta_0 \right]^2 + \left[\left(\frac{1}{2} \right)^{1/2} b_1 \sin^2 \theta_0 \right]^2 + \left[\left(\frac{1}{12} \right)^{1/2} b_1 \sin^2 \theta \right]^2 \\ & = \left(\frac{1}{2} + \frac{1}{12} \sin^2 \theta_0 \right) |b_1(r_0)|^2. \end{aligned}$$

In a similar way we may analyze all the possible spin states $\alpha_1\alpha_2$, $\alpha_1\beta_2$, ... $\gamma_1\gamma_2$. We find the only states contributing (for $l = 1, 2$) are $|S, L, J\rangle = |202\rangle$, $|112\rangle$, $|111\rangle$, $|110\rangle$, $|000\rangle$. We may construct Table 5 for probabilities of these states where the arguments of b_0 and b_1 are to be taken at r_0 .

Let us denote the matrix element for a nuclear reaction $D(d, p)T$ to occur in state $|S, L, J\rangle$ by M_{SLJ} . Then for any state ψ the probability of this reaction is

$$\sum_{SLJ} \text{Probability of } (S, L, J) \times |M_{SLJ}|^2 \quad (A6)$$

where the probabilities are to be obtained from Table 5.

Consider the chance of a reaction if both deuterons are unpolarized. Then each of our states $\alpha_1\alpha_2, \alpha_1\beta_2, \beta_1\alpha_2, \dots$ has probability $1/9$. Summing over all the probabilities for each state (summing the columns in Table 5), we have

$$\begin{aligned} 9\sigma_{av} = & 5 \cdot 2 b_0^2 |M_{202}|^2 + 5 \cdot \frac{2}{3} b_1^2 |M_{112}|^2 + 3 \cdot b_1^2 |M_{111}|^2 \\ & + 1 \cdot \frac{2}{3} b_1^2 |M_{110}|^2 + 2b_0^2 |M_{000}|^2 \end{aligned} \quad (A7)$$

We may now identify our results with those of Ad'yasevich and Fomenko. They give for the cross section

$$\begin{aligned} \frac{9\sigma_{av}}{\pi\lambda^2} = & 5(|\gamma_1|^2 + |\delta_1|^2) + 5(|\alpha_{12}|^2 + |\alpha_3|^2) \\ & + 3(|\alpha_{11}|^2 + |\beta_{11}|^2) + 1 \cdot |\alpha_{10}|^2 + 1 \cdot |\alpha_0|^2 \end{aligned} \quad (A8)$$

where λ is the de Broglie wavelength of the incident deuteron divided by 2π , and where $|\gamma_1|^2 + |\delta_1|^2$ corresponds to the state $S = 2, L = 0, J = 2$, etc, the corresponding terms relating to the same S, L, J in these two equations. They evaluate these terms at 290 keV as a result of their analysis. Combining with our results we find the matrix elements M .

$$(2|b_0|^2 |M_{202}|^2) = (|\gamma_1|^2 + |\delta_1|^2) \pi \lambda^2 = 0.012 \pi \lambda^2 \quad (A9)$$

$$\frac{2}{3} |b_1|^2 |M_{112}|^2 = (|\alpha_{12}|^2 + |\alpha_3|^2) \pi \lambda^2 = 0.07 \pi \lambda^2 \quad (A10)$$

$$|b_1|^2 |M_{111}|^2 = (|\alpha_{11}|^2 + |\beta_{11}|^2) \pi \lambda^2 = 0.62 \pi \lambda^2 \quad (A11)$$

$$\frac{2}{3} |b_1|^2 |M_{110}|^2 = |\alpha_{10}|^2 \pi \lambda^2 = 0.15 \pi \lambda^2 \quad (A12)$$

$$2|b_0|^2 |M_{000}|^2 = |\alpha_0|^2 \pi \lambda^2 = 1.25 \pi \lambda^2 \quad (A13)$$

We may obtain all the relative matrix elements from Eqs. (A9) through (A13) and hence by Eq. (A6) determine the relative probability for all the possible (initial) spin states that a reaction occurs (at 290 keV). To extend their results to other energies we make the assumption that the main energy dependence of the cross section occurs in the factor $\pi \lambda^2$ and in the coefficients $|b_0|^2$ and $|b_1|^2$. These are essentially the gamow penetration factors. We take the matrix elements M to be independent of energy.

To calculate ratio of the cross section to the average cross section we only need $|b_1|^2/|b_0|^2$. As an example let us calculate the ratio of $\sigma_{\beta\beta}$ the cross section with initial spin state $\beta\beta$ to σ_{av} .

$$\begin{aligned}
\sigma_{\beta\beta} &= \frac{4}{3} |b_o(E)|^2 |M_{202}|^2 + \frac{2}{3} |b_o(E)|^2 |M_{000}|^2 \\
&= \pi\chi^2 \frac{|b_o(E)|^2}{|b_o(290)|^2} \left[\frac{2}{3} (|\gamma_1|^2 + |\delta_1|^2) + \frac{1}{3} |\alpha_o|^2 \right] \\
&= \pi\chi^2 \times 0.425 \frac{|b_o(E)|^2}{|b_o(290)|^2} \quad . \quad (A14)
\end{aligned}$$

Similarly, we may write

$$\begin{aligned}
\sigma_{av} &= \frac{\pi\chi^2}{9} [1.31 b_o^2(E) + 2.36 b_1^2(E)] \\
&= \pi\chi^2 (0.1456 + 0.2622P) \frac{|b_o(E)|^2}{|b_o(290)|^2} \quad (A15)
\end{aligned}$$

where

$$P = \frac{|b_1(E)|^2 |b_o(290)|^2}{|b_1(290)|^2 |b_o(E)|^2} \quad (A16)$$

so for the enhancement of the cross section we have

$$E_{\beta\beta} = \frac{\sigma_{\beta\beta}}{\sigma_{av}} = \frac{0.4247}{0.1456 + 0.2622P} = \frac{2.92}{1+1.8P} \quad . \quad (A17)$$

P can be evaluated by numerically integrating Schroedinger's equation into r_o for various values of E. We find $P(50 \text{ keV}) = 0.2$, $P(100 \text{ keV}) = 0.4$, $P(290 \text{ keV}) = 1$.

For the other spin states we obtain in the same way

$$E_{\alpha\alpha} = \frac{0.0824}{1+1.80P} \quad (A18)$$

$$E_{\beta\alpha} = \frac{0.0412 + (3.55 - 0.0206 \sin^2 \theta_0)P}{1 + 1.80P} \quad (A19)$$

$$E_{\alpha\gamma} = \frac{2.88 + (1.00 - 1.49 \sin^2 \theta_0)P}{1 + 1.80P} \quad (A20)$$

Making use of the above values of P we obtain Table 4 where E_{1-1} corresponds to $E_{\alpha\gamma}$, etc.

It may be remarked that the matrix elements of Ad'yasevich and Fomenko lead one to the conclusion that the quintuplet state which only contributes through $(M_{202})^2$ makes a very small contribution to the averaged cross section. Since $\alpha\alpha$ only contributes through this state, its cross section is greatly reduced from σ_{av} . This is the suppression referred to in the discussion of the D-He³ cross section. As a consequence σ for the other spin states is enhanced, e.g., $E_{\beta\beta}$.

If one is concerned with a reactor, it is important to distinguish which of the various spin states are possible. For $\alpha\alpha$ or $\beta\beta$ all of the deuterons are in the same spin state. For $\alpha\gamma$ or $\alpha\beta$ one requires different spin states for the two colliding deuterons, so that these results apply strictly only to beam-plasma interactions. This allows the deuterons to be distinguishable.

APPENDIX B

DEPOLARIZATION BY COLLISIONS

a. Depolarization of Tritons by Electrons (Spin-Orbit)

In Sec. 3.2 it was remarked that the contribution to σ_{te} from collisions with impact parameter b less than $\lambda = \hbar/mv$, the de Broglie wavelength, had to be treated quantum mechanically. We calculate the entire cross section σ by the Born approximation. This is legitimate because the various partial waves in a coulomb field are near to those for a free wave. From Landau and Lifshitz Quantum Mechanics Eq. (125.7) [14] we have

$$\begin{aligned} d\sigma &= \frac{m_e^2}{4\pi\hbar^4} \left| \int U \exp(-iq \cdot \tilde{r}) dV \right|^2 d\Omega \\ &= \frac{m_e^2}{4\pi\hbar^4} \left| \int \left\langle -\frac{1}{2} ; \exp(-ik' \cdot \tilde{r}) \right| \frac{g_3^2 e^2}{2m_p c m_e} \frac{\tilde{r} \cdot \tilde{k}}{r^3} \left| \frac{1}{2} ; \exp(ik \cdot \tilde{r}) \right\rangle dV \right|^2 d\Omega \end{aligned} \quad (B1)$$

where $|1/2; \exp(ikr)\rangle$ is the state with $m_s = 1/2$, and the electron in a plane wave state $\underline{k}$. $d\sigma$ is the differential cross section for the electron scattered into $\underline{k}'$ in solid angle $d\Omega$ and $m_s = -1/2$ and $\underline{q} \equiv \underline{k}' - \underline{k}$. Now $\underline{k} \cdot \exp(i\underline{k} \cdot \underline{r}) = \hbar(\underline{k} \times \underline{k}) \exp(i\underline{k} \cdot \underline{r})$ and $\langle -1/2 | \underline{g} | 1/2 \rangle = \hbar/2(\hat{x} + i\hat{y})$ so

$$d\sigma = \frac{g_3^2 e^2}{16\pi\hbar^2} \left| \frac{\underline{k} \times (\hat{x} + i\hat{y})}{2} \cdot \int \frac{\underline{r} \exp(-iq \cdot \underline{r})}{r^3} dV \right|^2 d\Omega \quad (B2)$$

But it is easy to show that

$$\int \frac{\tilde{r} \exp(-iq \cdot \tilde{r})}{r^3} dV = \frac{4\pi q}{q^2} \quad (B3)$$

so

$$d\sigma = \frac{g_3^2 r_p^2}{4} \left| \frac{\tilde{k} \times (\tilde{x} + i\tilde{y}) \cdot \tilde{q}}{q^2} \right|^2 d\Omega \quad (B4)$$

Averaging Eq. (B4) over all orientations for the collision we find the bracket averages to $2/3 |\tilde{k} \times \tilde{q}/q^2|^2 = 2/3 [\sin^2\theta/4(\sin^2\theta/2)]^2$. Thus with $d\Omega = 2\pi \sin\theta d\theta$ we get for the total cross section

$$\sigma = \frac{2}{3} \frac{g_3^2 r_p^2}{4} \frac{2\pi}{16} \int_0^{\pi} \frac{\sin^3\theta d\theta}{\sin^4\theta/2} \quad (B5)$$

The integral diverges for small θ corresponding to the usual divergence at large impact parameters. We cut it off at $\theta_{\min} = 2e^2/mv^2$ $b_{\max} = b_0 \omega_p/c$ and obtain

$$\sigma = \frac{\pi}{3} g_3^2 r_p^2 \log \left(\frac{c}{\sqrt{e} b_0 \omega_p} \right) \quad (B6)$$

This more accurate result should replace Eq. (31). Nevertheless Eq. (31) and Eq. (B6) agree up to the logarithmic factor which in turn only differs by replacing the lower cutoff of $\hbar/mv$ in Eq. (31) by $b_0 = e^2/mv^2$. The same modification applies to the electron spin-orbit cross sections for deuterium Eqs. (33).

b. Spin - Spin

We now consider depolarization that can result from the spin-spin interaction. There are a large number of separate cases to consider. First

let the triton be in the $m_3 = 1/2$ state. It can be depolarized by interaction with the electron, assumed unpolarized, with another triton assumed to be in the $m_3 = 1/2$ state or with a deuteron that can be taken in any of the spin states $m_2 = \pm 1$ or 0 depending on the mode of polarization. The depolarization probability for the $m_3 = -1/2$ state can be obtained from any of these.

c. Depolarization of the Triton by Electrons (Spin-Spin)

We treat the interaction by the Born approximation since the minimum impact parameter is λ . The interaction is different according to whether $\ell = 0$ or $\ell = 1$. For $\ell \neq 0$ we have

$$d\sigma_i = \frac{m_e^2}{4\pi \hbar^4} \left| \int U_i \exp(-i\mathbf{q} \cdot \mathbf{r}) d\mathbf{v} \right|^2 d\mathbf{q} \quad (B7)$$

where

$$U_i = \left\langle -\frac{1}{2} m_{ei}' \left| \frac{g_3 e^2}{2m_e m_p c^2} \hat{s}_e \cdot \frac{\hat{\mathbf{I}} - 3\hat{\mathbf{r}}\hat{\mathbf{r}}}{r^3} \cdot \hat{s}_3 \right| \frac{1}{2} m_{ei} \right\rangle \quad (B8)$$

and $\hat{\mathbf{r}} = \mathbf{r}/r$, where i refers to the particular initial and final states of the electron $m_{e1} = m_{ei}' = 1/2$, $m_{e2} = 1/2$, $m_{e2}' = -1/2$ etc., $\mathbf{q} = \mathbf{k}' - \mathbf{k}$. Thus,

$$\begin{aligned} U_1 &= \frac{g_3 e^2}{2m_e m_p c^2} \left\langle -\frac{1}{2} \frac{1}{2} \left| \hat{s}_{ez} \hat{z} \cdot \frac{\hat{\mathbf{I}} - 3\hat{\mathbf{r}}\hat{\mathbf{r}}}{r^3} (\hat{x} + i\hat{y}) \hat{s}_3 \right| \frac{1}{2} \frac{1}{2} \right\rangle \\ &= \frac{g_3 e^2}{2m_e} \frac{\hbar^2}{4} \hat{z} \cdot \frac{\hat{\mathbf{I}} - \hat{\mathbf{r}}\hat{\mathbf{r}}}{r^3} \cdot (\hat{x} + i\hat{y}) \quad (B9) \end{aligned}$$

But

$$\int \frac{\hat{\mathbf{I}} - 3\hat{\mathbf{r}}\hat{\mathbf{r}}}{r^3} d\mathbf{v} = 4\pi \hat{\mathbf{g}} \quad (B10)$$

where $\hat{q}$ is a unit vector in the q direction. Consequently,

$$d\sigma_1 = \frac{g_3^2}{16} r_p^2 \left| \hat{z} \cdot \hat{q} \hat{q} \cdot (\hat{x} + i\hat{y}) \right|^2 d\Omega \quad . \quad (B11)$$

Now $|\hat{z} \cdot \hat{q} \hat{q} \cdot (\hat{x} + i\hat{y})| < 1/2$ for all $\hat{q}$ so

$$\sigma_1 = \int d\sigma_1 d\Omega < \frac{\pi g_3^2}{16} r_p^2 \quad . \quad (B12)$$

In an exactly similar way

$$d\sigma_2 = \frac{g_3^2}{16} r_p^2 \left| (\hat{x} + i\hat{y}) \cdot \hat{q} \hat{q} \cdot (\hat{x} + i\hat{y}) \right| d\Omega < \frac{g_3^2}{16} r_p^2 d\Omega \quad (B13)$$

and

$$\sigma_2 < \frac{g_3^2}{4} \pi r_p^2 \quad . \quad (B14)$$

Again

$$\sigma_3 < \frac{g_3^2}{16} \pi r_p^2 \quad (B15)$$

$$\sigma_4 < \frac{g_3^2}{4} \pi r_p^2 \quad . \quad (B16)$$

Summing over final states of the electron and averaging over initial states we find for the depolarization cross section of the triton by an $l \neq 0$ electron

$$\sigma_{av} < \frac{5\pi}{16} g_3^2 r_p^2 \quad . \quad (B17)$$

Now consider the $l = 0$ electrons. The interaction is

$$\frac{8\pi}{3} \frac{g_3 e^2}{2m_p m_e c^2} \delta(x) \vec{s}_e \cdot \vec{s}_1 \quad . \quad (B18)$$

Since angular momentum is conserved, the only transition to consider is from $m_3 = 1/2$, $m_e = -1/2$ to $m_3 = -1/2$, $m_e = 1/2$. Hence U_3 in Eq. (B7) is replaced by

$$\begin{aligned} U_3 &= \frac{8\pi}{3} \left(\frac{g_3 e^2}{2m_p m_e c^2} \right) \left\langle -\frac{1}{2} \frac{1}{2} \left| \vec{s}_e \cdot \vec{s}_3 \right| \frac{1}{2} - \frac{1}{2} \right\rangle \delta(x) \\ &= \frac{8\pi}{3} \left(\frac{g_3 e^2}{2m_e m_p c^2} \right) \frac{\hbar^2}{4} \delta(x) \end{aligned} \quad (B19)$$

and

$$d\sigma_3 = \frac{g_3^2 r_p^2}{9} d\omega \quad (B20)$$

$$\sigma_3 = \frac{4\pi}{9} g_3^2 r_p^2 \quad . \quad (B21)$$

Again averaging over initial states

$$\sigma_{av} = \frac{2\pi}{9} g_3^2 r_p^2 \quad . \quad (B22)$$

Combining the cross sections for $l = 0$ and $l \neq 0$ we obtain

$$\sigma_{tot} < \left(\frac{2}{9} + \frac{5}{16} \right) \pi g_3^2 r_p^2 = 0.53 g_3^2 \pi r_p^2 \quad . \quad (B23)$$

d. Depolarization of Triton by Tritons (Spin-Spin)

For determination of this cross section we may not employ the Born approximation, since the minimum impact parameter $b_0 > \lambda$. Let s_3 denote the spin of the test triton and s'_3 of the depolarizing field triton. Let $m_3 = m'_3 = 1/2$. The interaction Hamiltonian is

$$H = \frac{g_3^2 e^2}{4m_p^2 c^2} s_3 \cdot \left(\frac{\hat{I} - 3\hat{r}\hat{r}}{r^3} \right) \cdot s'_3 \quad (B24)$$

Denote by c the amplitude of the state $m_3 = -1/2$, $m_3 = 1/2$. Then

$$\begin{aligned} \frac{dc}{dt} &= \frac{g_3^2 r_p}{4m_p i\hbar} \left\langle -\frac{1}{2} \frac{1}{2} \left| s_3 \cdot \frac{\hat{I} - 3\hat{r}\hat{r}}{r^3} \cdot s'_3 \right| \frac{1}{2} \frac{1}{2} \right\rangle \\ &= \frac{g_3^2 r_p}{16m_p i\hbar} (\hat{x} + i\hat{y}) \cdot \left(\frac{\hat{I} - 3\hat{r}\hat{r}}{r^3} \right) \cdot \hat{z} \quad (B25) \end{aligned}$$

Now assume straight line orbits. Then

$$\int \frac{\hat{I} - 3\hat{r}\hat{r}}{r^3} dt = \int \frac{(\hat{I} - 3\hat{r}\hat{r})}{r(r^2 \dot{\theta})} d\theta = \frac{1}{r^2 \dot{\theta}} \int_{-\pi/2}^{\pi/2} \frac{\cos \theta}{b} (\hat{I} - 3 \cos^2 \theta \hat{b}\hat{b} - 3 \sin^2 \theta \hat{v}\hat{v}) d\theta \quad (B26)$$

where $\hat{b}$ is a unit vector in the direction of the line of apsides (see Fig. 3), $\hat{v}$ a unit vector along y , and $\hat{z}$ a unit vector perpendicular to the plane of motion. $r^2 \dot{\theta} = v b$ is a constant and $1/r = \cos \theta / b$. Carrying out the integration yields

$$\int \frac{\hat{I} - 3\hat{r}\hat{r}}{r^3} dt = \frac{2}{b} (\hat{z}\hat{z} - \hat{b}\hat{b}) \quad (B27)$$

Making use of this we find the change in c over the orbit

$$\Delta c = \frac{ig_3^2 r_p^2 \hbar}{8m_p b^2 v} (\hat{x} + i\hat{y}) \cdot (\hat{\ell}\hat{\ell} - \hat{b}\hat{b}) \cdot \hat{z} \quad (B28)$$

But for any perpendicular unit vectors $\hat{\ell}$ and $\hat{b}$

$$|(\hat{x} + i\hat{y}) \cdot (\hat{\ell}\hat{\ell} - \hat{b}\hat{b}) \cdot \hat{z}| < 1 \quad (B29)$$

so

$$|\Delta c| < \frac{g_3^2 r_p^2 \chi}{8 b^2} \left| \frac{1}{2} \frac{1}{2} \right\rangle + \left| -\frac{1}{2} \frac{1}{2} \right\rangle \quad (B30)$$

where $\chi = \hbar/m_p v$. For the other possible transition m_3 goes to $-1/2$ we have

$$\left\langle -\frac{1}{2} - \frac{1}{2} \left| s_3 \cdot \frac{\hat{I} - \hat{r}\hat{r}}{r^3} \right| \frac{1}{2} \frac{1}{2} \right\rangle = \frac{1}{4} (\hat{x} + i\hat{y}) \cdot \frac{\hat{I} - 3\hat{r}\hat{r}}{r^3} \cdot (\hat{x} + i\hat{y}) \quad (B31)$$

and

$$\Delta c = \frac{-igr_p^2 \chi}{8b^2} (\hat{x} + i\hat{y}) \cdot (\hat{\ell}\hat{\ell} - \hat{b}\hat{b}) \cdot (\hat{x} + i\hat{y}) \quad (B32)$$

But $|(\hat{x} + i\hat{y}) \cdot (\hat{\ell}\hat{\ell} - \hat{b}\hat{b}) \cdot (\hat{x} + i\hat{y})| \leq 2$ so

$$|\Delta c| < \frac{g_3^2 r_p^2 \chi}{4b^2} \quad (B33)$$

Summing the squares of these two transitions and then summing over all b we get for a bound on the total depolarization cross section of triton by triton through the spin spin interaction

$$\sigma_{\text{tot}} < \frac{5\pi g_3^4 r_p^2 \chi^2}{64 b_{\text{min}}^2} \quad . \quad (\text{B34})$$

We take $b_{\text{min}} = b_0$. The contact interaction $l = 0$ is negligible because the probability of the tritons being zero distance apart is so small due to the repulsive coulomb force.

e. Depolarization of Tritons by Deuterons

In order to calculate the cross section for depolarization of tritons by deuterons, we must prescribe on the polarization state of the deuterons. Assuming the deuterons to be completely polarized they may be in the spin state $m_2 = 1$ or $m_2 = 0$ or $m_2 = -1$. There is a different cross section corresponding to each one of these states. Assume first that the field deuteron is in the $m_2 = 1$ state initially. There are two transitions one to $m_2 = 1$, the other to $m_2 = 0$, as the triton makes the transition to $m_3 = -1/2$. The calculation of these transition probabilities is identical to that for triton - triton collisions. We need one further inequality

$$|(\hat{x} - i\hat{y}) \cdot (\hat{\ell}\hat{\ell} - \hat{b}\hat{b}) \cdot (\hat{x} + i\hat{y})| \leq 1 \quad . \quad (\text{B35})$$

If $m_2 = 1$ initially, the result is

$$\sigma < \frac{3\pi}{16} g_2^2 g_3^2 \left(\frac{\chi}{b_0}\right)^2 r_p^2 \quad . \quad (\text{B36})$$

If $m_2 = 0$ initially, there are three transitions to $m_2 = 1, 0$ or -1 . The total cross section satisfies

$$\sigma < \frac{5\pi}{16} g_2^2 g_3^2 \left(\frac{\chi}{b_0}\right)^2 r_p^2 \quad . \quad (\text{B37})$$

Finally, if $m_2 = -1$ initially, there are two transitions and the total cross section satisfies

$$\sigma < \frac{3\pi}{32} g_2^2 g_3^2 \left(\frac{\lambda}{b_0}\right)^2 r_p^2 \quad . \quad (B38)$$

We may make a table of these results. Let us express the cross sections as multiples of $\pi r_p^2 = 7.2 \times 10^{-32} \text{cm}^2$. We take $g_2 = 0.86$, $g_3 = 5.94$, and $\lambda/b_0 = 137 \text{ v/c} = 0.46$ at 10 keV. An asterisk indicates an upper bound

TABLE B-1

Depolarization of T in $m_3 = 1/2$ state

Spin-Spin

By unpolarized electrons	$19.6 \pi r_p^2 *$
By tritons $m_3 = 1/2$	$12.9 \pi r_p^2 *$
By deuterons $m_2 = 1$	$1.0 \pi r_p^2 *$
$m_2 = 0$	$0.83 \pi r_p^2 *$
$m_2 = -1$	$0.5 \pi r_p^2 *$

Spin-Orbit

By electrons	$255 \pi r_p^2$
By ions	$255 \pi r_p^2$

The largest cross sections are those for depolarization by electrons and nuclei by spin orbit interaction. Recall that there are many more electron collisions. Neglect depolarization by ions. Then the cross section for depolarization of a triton is

$$\sigma < (255 + 19.6) \pi r_p^2 = 275 \pi r_p^2 = 2.0 \times 10^{-29} \text{ cm}^2 \quad . \quad (\text{B39})$$

f. Deuterons in the $m_2 = 1$ State

Let us now consider the cross sections for depolarization of deuterons. There are three cases, depolarization of deuterons in the $m = 1$, $m = 0$, or $m = -1$ state. The first and last are essentially identical by symmetry so we consider only the first two cases.

Take a deuteron in the $m = 1$ state. The spin-orbit and spin-spin depolarizations by electrons must be calculated by the Born approximation. Other than slightly different matrix elements for the transitions, the calculation is identical with that for tritons. The spin-orbit and spin-spin interactions with tritons can be carried out classically. An assumption about the polarization state of the triton must be made for the spin-spin interaction and two different cross sections are obtained for the two cases. For the interaction with other deuterons we assume both deuterons in the $m = 1$ state.

In addition, for a deuteron we have an electric quadrupole interaction with electrons and other nuclei. To calculate this cross section for electrons we again employ the Born approximation. The Hamiltonian for the quadrupole interaction is

$$H = \frac{e^2}{\mu^2} Q \cdot \underline{\underline{s}} \cdot \left(\frac{I - 3\hat{r}\hat{r}}{r^3} \right) \cdot \underline{\underline{s}} \quad (\text{B40})$$

where Q is the quadrupole moment of the deuteron. It can be written

$$Q = q_0 \left(\frac{\hbar}{m_p c} \right)^2 = 2.74 \times 10^{-27} \text{ cm}^2$$

where $q_0 = 6.2$. The deuterons can depolarize from the $m = 1$ state by making a transition to $m_2 = 0$ or to $m_2 = -1$. Consider the first transition. Then the expression for U to be employed in Eq. (B7) is

$$U = \langle 0 \left| s_2 \cdot \frac{I-3\hat{r}\hat{r}}{r^3} \cdot s_2 \right| 1 \rangle q_0 \frac{e^2}{m_p^2 c^2}$$

$$= \frac{\sqrt{2}}{2} (\hat{x} + i\hat{y}) \cdot \left(\frac{I-3\hat{r}\hat{r}}{r^3} \right) \cdot \hat{z} q_0 \frac{e^2}{m_p^2 c^2} \quad (B41)$$

With this expression the remainder of the calculation proceeds as before and we obtain

$$\sigma < \frac{\pi q_0^2}{4} r_p^2 \left(\frac{m_e}{m_p} \right)^2 \quad (B42)$$

For the transition to $m = -1$

$$\langle -1 \left| s_2 \cdot \frac{I-3\hat{r}\hat{r}}{r^3} \cdot s_2 \right| 1 \rangle = \frac{1}{2} (\hat{x} + i\hat{y}) \cdot \left(\frac{I-3\hat{r}\hat{r}}{r^3} \right) \cdot (\hat{x} + i\hat{y}) \hbar^2 \quad (B43)$$

and

$$\sigma < \frac{\pi}{2} q_0^2 r_p^2 \left(\frac{m_e}{m_p} \right)^2 \quad (B44)$$

Adding these cross sections we find the cross section for depolarization of the $m = 1$ state by electrons by the electric quadrupole interaction satisfies

$$\sigma < \frac{3\pi}{4} q_0^2 \left(\frac{m_e}{m_p} \right)^2 r_p^2 \quad (B45)$$

The result for depolarization by ions through electric quadrupole interaction is obtained by a classical calculation and satisfies

$$\sigma < 6\pi q_o^2 \left(\frac{\lambda}{b_o}\right)^2 r_p^2 \quad . \quad (B46)$$

We may summarize our results for depolarization of the $m = 1$ deuteron by another table

TABLE B-2

Depolarization of D in $m_2 = 1$ State

Spin Orbit

By electrons	$10.8\pi r_p^2$
By ions	$10.8\pi r_p^2$

Spin Spin

By unpolarized electrons	$0.8\pi r_p^2 *$
By tritons ($m_3 = 1/2$)	$0.84\pi r_p^2 *$
($m_3 = -1/2$)	$0.33\pi r_p^2 *$
By deuterons ($m_2 = 1$)	$0.04\pi r_p^2 *$

Quadrupole

By electrons	$6.6 \times 10^{-5} \pi r_p^2 *$
By ions	$48\pi r_p^2 *$

The largest cross section is that for the electric quadrupole interaction with ions. However again the electron spin-orbit interaction dominates. The total cross section for depolarization of $m_2 = 1$ deuterons by electrons is

$$\sigma = (10.8 + 0.8)\pi r_p^2 = 11.6\pi r_p^2 = 0.84 \times 10^{-30} \text{ cm}^2 \quad . \quad (B47)$$

g. Depolarization of Deuterons in the $m_2 = 0$ State

Finally, we consider the depolarization of deuterons with $m_2 = 0$. They may depolarize into $m_2 = 1$ or $m_2 = -1$ so that in general the cross section is twice as large as that for $m_2 = 1$. The calculations are almost identical to those for $m = 1$ and we find the following table

Table B-3

Depolarization of D in $m = 0$ State

Spin Orbit

By electrons	$21.6\pi r_p^2$
By ions	$21.6\pi r_p^2$

Spin-Spin

By unpolarized electrons	$1.6\pi r_p^2 *$
By tritons ($m_3 = 1/2$)	$1.2\pi r_p^2 *$
($m_3 = -1/2$)	$1.2\pi r_p^2 *$
By deuterons ($m_2 = 0$)	$0.07\pi r_p^2 *$

Quadrupole

By electrons	$0.0004\pi r_p^2 *$
By ions	$32.2\pi r_p^2 *$

Again the spin-orbit interaction with electrons is most important. The total cross section for depolarization of $m_2 = 0$ deuterons by electrons is

$$\sigma = (21.6 + 1.6) \pi r_p^2 = 23.2 \pi r_p^2 = 1.7 \times 10^{-30} \text{ cm}^2 \quad . \quad (\text{B48})$$

APPENDIX C

Depolarization by Waves

Let us consider a spectrum of waves whose magnetic field

$$\underline{\delta B}(\underline{r}, t) = \iiint d\underline{k} d\omega \underline{\delta B}_{\underline{k}, \omega} \exp(-i\omega t + ik_x x + ik_y y + ik_z z) \quad . \quad (\text{C1})$$

In the random phase approximation we form the ensemble average

$$\langle \underline{\delta B}_{\underline{k}, \omega}^* \underline{\delta B}_{\underline{k}', \omega'} \rangle = \sum_j \underline{I}_{\underline{k}}^j \delta(\omega' - \omega) \delta(\underline{k}' - \underline{k}) \delta(\omega - \omega_{\underline{k}}^j) \quad . \quad (\text{C2})$$

The tensor $\underline{I}_{\underline{k}}^j$ represents the intensity of the magnetic field in the j th plasma mode. The mean-square average of δB^2 is

$$\langle \delta B^2 \rangle = \sum_j \int d\underline{k} \text{trace} (\underline{I}_{\underline{k}\omega}^j) \quad . \quad (\text{C3})$$

(Note that if $\underline{\omega}_{\underline{k}}^j = -\underline{\omega}_{\underline{k}}^j$, then $\underline{I}_{\underline{k}}^j = \text{transpose} (\underline{I}_{-\underline{k}}^j)^*$ so each wave is counted twice in I.)

We now consider a triton in the $m = 1/2$ state. By Eq. (43)

$$c_{-1/2} = \exp(i\Omega_p t) \int_0^t \frac{i\Omega_p}{2} \frac{\underline{\delta B}_0(t')}{B_0} \cdot (\hat{x} - i\hat{y}) \exp(-i\Omega_p t') dt' \quad (\text{C4})$$

where $\delta B(t)$ is the magnetic field "seen" by the triton. To find $\delta B(t)$ let the motion of the triton in the uniform field $B_0 \hat{z}$ be

$$\begin{aligned}x &= \rho \sin(\Omega_3 t - \phi) \\y &= \rho \cos(\Omega_3 t - \phi) \\z &= v_z t\end{aligned}\quad (C5)$$

where $\rho = v_\perp / \Omega$. Also introduce cylindrical coordinates $k_\perp$, α , k_z in k space. Then substituting Eq. (C5) into Eq. (C1), we have

$$\delta \tilde{B}(t) = \delta \tilde{B}[\underline{r}(t), t] = \int_n \int d\underline{k} d\omega \delta B_{\underline{k}, \omega} J_n(k_\perp \rho) \exp[-i(\omega - k_z v_z - i n \Omega_3)t + i n(\alpha - \phi)] . \quad (C6)$$

Substituting (C6) into (C4) and carrying out the integration in t , we have

$$\begin{aligned}c_{-1/2} &= \exp(-i \Omega_p t) \frac{i \Omega_p}{2} \sum_n \int \int d\underline{k} d\omega \frac{\delta B}{B_0} \cdot (\hat{x} + i \hat{y}) J_n(k_\perp \rho) \exp\left(-\frac{i \Delta \omega t}{2}\right) \frac{\sin(\Delta \omega_n t/2)}{\Delta \omega_n/2} \\&\quad \cdot \exp i n(\alpha - \phi)\end{aligned}\quad (C7)$$

where

$$\Delta \omega_n = \omega - k_z v_z - n \Omega - \Omega_p . \quad (C8)$$

Squaring, taking the ensemble average and making use of Eq. (C2), we obtain

$$\begin{aligned}|c_{-1/2}|^2 &= \frac{\Omega_p^2}{4} \sum_{nm} \int d\underline{k} (\hat{x} - i \hat{y}) \cdot \frac{\underline{r}}{k} \cdot (\hat{x} + i \hat{y}) J_n(k_\perp \rho) J_m(k_\perp \rho) \\&\quad \cdot \exp((n-m)(\alpha - \phi)) \cdot \frac{\sin(\Delta \omega_n t/2)}{\Delta \omega_n/2} \sin \frac{(\Delta \omega_m t/2)}{\Delta \omega_m/2} .\end{aligned}\quad (C9)$$

For large Ωt the sine factors are quite peaked and only the term $n = m$ contributes. Setting

$$\frac{\sin^2(\Delta\omega t/2)}{(\Delta\omega/2)^2} = 2\pi t \delta(\Delta\omega) \quad (C10)$$

we obtain finally the rate of depolarization

$$\begin{aligned} \left\langle \frac{|c_{-1/2}|^2}{t} \right\rangle = & \frac{\pi}{2} \frac{\Omega_p^2}{B_0^2} \sum_{n,j} \int d\mathbf{k} (\hat{x} - i\hat{y}) \cdot \hat{\mathbf{I}}_{\mathbf{k}\omega}^j \cdot (\hat{x} + i\hat{y}) J_n^2(k_\perp \rho) \\ & \cdot \delta(\omega_k^j - k_z v_z - n\Omega_3 - \Omega_p), \quad (C11) \end{aligned}$$

the result quoted in Eq. (47).

The depolarization rate for a triton in the $m_3 = -1/2$ state is also given by Eq. (C11) since the corresponding expression for $c_{1/2}$ is just the complex conjugate of (C4).

Consider next the depolarization of a deuteron in the $m_2 = 1$ state. The expression for c_0 the amplitude of the $m = 0$ state is derived just as in Eq. (43) and is

$$c_0 = \exp(i\Omega_p t) \int_0^t \frac{i\Omega_p}{\sqrt{2}} \frac{[\delta B(t) \cdot (\hat{x} + i\hat{y})]}{B_0} \exp(-i\Omega_p t') dt'.$$

The result for $|c_0|^2/t$ is formally the same as (C11) except for a factor of 2.

Finally, consider the depolarization of a deuteron in state $m_2 = 0$. It can depolarize to either $m_2 = 1$ or $m_2 = -1$ each with probability equal to the depolarization rate of an $m_2 = 1$ deuteron. Thus, the depolarization rate is four times that given by (C11) with Ω_p the deuteron precession frequency and

with the replacement of Ω_3 by Ω_2 . These remarks justify the values given for α in the discussion following Eq. (47).

Let us consider the effective polarization of the waves. As is well known the transverse components of a wave can be decomposed into right and left circularly polarized components (which are in general correlated). We are only interested in such a decomposition of the parts of $\delta \underline{B}$ perpendicular to $\hat{z}$. It is easy to show that

$$(\hat{x} - i\hat{y}) \cdot \underline{I}_{k\omega}^j = (\hat{x} + i\hat{y})$$

represents the left circularly polarized component if $\omega > 0$, and the right circularly polarized component if $\omega < 0$. For $n = 0$ the resonance condition in Eq. (C11) is just such that when $\omega > 0$, the ion sees a left circular polarized wave as left circularly polarized, while if $\omega < 0$, the particle sees a right circular polarized wave as left circularly polarized. This is not the case if $n \neq 0$ as far as the parallel motion is concerned. But when account of the perpendicular gyromotion is considered, the effective fields rotating in the left circular direction is picked out by the Bessel function factor.

APPENDIX D

DEPOLARIZATION IN THE 2-P STATE

Consider a tritium atom in the 2p state. Let the triton nucleus have $m_3 = 1/2$, and let the electron be in the $n = 0$, $m_e = 1/2$ state where m_e is the electron magnetic quantum number. We assume the electron spin does not change because it is anchored by the large external magnetic field B_0 . Then one possible transition is to $m = 1$, $m_3 = -1/2$, $m_e = 1/2$. This is the only

transition preserving angular momentum. The spin-spin interaction Hamiltonian is

$$\delta H = \frac{g_3 e^2}{2m_e m_p c^2} \vec{s}_e \cdot \frac{\vec{I} - 3\vec{r}\vec{r}}{r^3} \cdot \vec{s}_3 \quad (D1)$$

The amplitude c for the transition satisfies

$$\begin{aligned} i\hbar \dot{c} &= \frac{g_3 e^2}{2m_e m_p c^2} \langle 1 \frac{1}{2} - \frac{1}{2} | \vec{s}_e \cdot \frac{\vec{I} - 3\vec{r}\vec{r}}{r^3} \cdot \vec{s}_3 | 0 \frac{1}{2} \frac{1}{2} \rangle \\ &= - \frac{3g_3 e^2 \hbar^2}{8m_e m_p c^2} \int \psi_{211}^* \frac{\hat{z} \cdot \vec{r}\vec{r} \cdot (\hat{x} + i\hat{y})}{r^3} \psi_{210} d^3r \end{aligned} \quad (D2)$$

But

$$\psi_{211}^* = \left(\frac{3}{3\pi}\right)^{1/2} R_{21} \sin\theta \exp(-i\phi)$$

$$\psi_{210} = i\left(\frac{3}{4\pi}\right)^{1/2} R_{21} \cos\theta \quad (D3)$$

$$R_{21} = \frac{1}{2\sqrt{6}} \frac{r}{a^{5/2}} \exp\left(-\frac{r}{2a}\right),$$

where a is the Bohr radius, so

$$\frac{dc}{dt} = - \frac{1}{160\sqrt{2}} \frac{e^2}{m_e m_p c^2 a^3} \frac{\hbar}{a} = - 3.1 \times 10^7 s^{-1} \quad (D4)$$

But the Einstein A value for transition $1p \rightarrow 2p$ is $1.5 \times 10^9 s^{-1}$, so

$$\Delta c = - \frac{3.1 \times 10^7}{1.5 \times 10^9} = 2 \times 10^{-2} \quad (D5)$$

Conversion can also occur from the $m = -1$ orbital state with the same probability but not from the $m = 1$ state. Hence squaring the c's summing and averaging we find the probability of depolarizing from a general 2p state is

$$P = \frac{2}{3} |\Delta c|^2 = 2.8 \times 10^{-4} \quad . \quad (D6)$$

This should be multiplied by the probability that a recombination passes through the 2p state to obtain the depolarization per recombination. A similar result can be obtained for $m = 1$ deuterons

$$P = \frac{1}{3} |\Delta c|^2 = 0.12 \times 10^{-4} \quad (D7)$$

and for $m = 0$ deuterons

$$P = 2 \times \frac{2}{3} |\Delta c|^2 = 0.24 \times 10^{-4} \quad . \quad (D8)$$

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TABLE 1.

Clebsch-Gordon Coefficients

Add angular momentum $1/2$ to angular momentum j_1 . The resulting wave function is

$$a(j_1 + \frac{1}{2})|j_1 + \frac{1}{2}, m\rangle + a(j_1 - \frac{1}{2})|j_1 - \frac{1}{2}, m\rangle$$

where the a's are

$j =$	$m_2 = \frac{1}{2}$	$m_2 = -\frac{1}{2}$
$j_1 + \frac{1}{2}$	$[\frac{j_1 + m + (1/2)}{2j_1 + 1}]^{1/2}$	$[\frac{j_1 - m + (1/2)}{2j_1 + 1}]^{1/2}$
$j_1 - \frac{1}{2}$	$-[\frac{j_1 - m + (1/2)}{2j_1 + 1}]^{1/2}$	$[\frac{j_1 + m + (1/2)}{2j_1 + 1}]^{1/2}$

Add angular momentum 1 to angular momentum j_1 . The resulting wave function is $b(j_1 + 1)|j_1 + 1, m\rangle + b(j_1)|j_1, m\rangle + b(j_1 - 1)|j_1 - 1, m\rangle$ where the b's are

$j =$	$m_2 = 1$	$m_2 = 0$	$m_2 = -1$
$j_1 + 1$	$[\frac{(j_1 + m)(j_1 + m + 1)}{(2j_1 + 1)(2j_1 + 2)}]^{1/2}$	$[\frac{(j_1 - m + 1)(j_1 + m + 1)}{(2j_1 + 1)(j_1 + 1)}]^{1/2}$	$[\frac{(j_1 - m)(j_1 - m + 1)}{(2j_1 + 1)(2j_1 + 2)}]^{1/2}$
j_1	$-[\frac{(j_1 + m)(j_1 - m + 1)}{2j_1(j_1 + 1)}]^{1/2}$	$\frac{m}{[j_1(j_1 + 1)]^{1/2}}$	$[\frac{(j_1 - m)(j_1 + m + 1)}{2j_1(j_1 + 1)}]^{1/2}$
$j_1 - 1$	$[\frac{(j_1 - m)(j_1 - m + 1)}{2j_1(2j_1 + 1)}]^{1/2}$	$-[\frac{(j_1 - m)(j_1 + m)}{j_1(2j_1 + 1)}]^{1/2}$	$[\frac{(j_1 + m + 1)(j_1 + m)}{2j_1(2j_1 + 1)}]^{1/2}$

TABLE II
Spherical Harmonics $Y_{lm}(\theta, \phi)$

$$Y_{00} = \frac{1}{(4\pi)^{1/2}}$$

$$Y_{10} = i\left(\frac{3}{4\pi}\right)^{1/2} \cos \theta ; \quad Y_{1\pm 1} = \mp i\left(\frac{3}{8\pi}\right)^{1/2} \sin \theta \exp(\pm i\phi)$$

$$Y_{20} = \left(\frac{5}{16\pi}\right)^{1/2} (1 - 3 \cos^2 \theta)$$

$$Y_{2\pm 1} = \pm \left(\frac{15}{8\pi}\right)^{1/2} \cos \theta \sin \theta \exp(\pm i\phi)$$

$$Y_{2\pm 2} = -\left(\frac{15}{32\pi}\right)^{1/2} \sin^2 \theta \exp(\pm 2i\phi)$$

TABLE III
D-D Enhancements σ/σ_{av} for $D(dp)T$
(Dodder and Hale)

$E(\text{keV})$	E_{11}	E_{10}	E_{1-1}	E_{00}
100	0.949	1.146	0.672	1.175
200	0.776	1.334	0.550	1.011
300	0.672	1.468	0.488	0.889

TABLE IV (Appendix A)
D-D Enhancements σ/σ_{av} for D(dp)T
(This paper)

E(keV)	E_{11}	E_{10}	E_{1-1}	E_{00}
50	0.061	0.55	$2.26 + 0.21 \sin^2 \theta_0$	2.15
100	0.048	0.85	$1.90 + 0.34 \sin^2 \theta_0$	1.70
290	0.029	1.28	$1.39 + 0.53 \sin^2 \theta_0$	1.04

TABLE V
Probability of State $|SLJ\rangle$

$ SLJ\rangle$	$ 202\rangle$	$ 112\rangle$	$ 111\rangle$	$ 110\rangle$	$ 000\rangle$
$\alpha\alpha$ $\gamma\gamma$	$2b_o^2$	0	0	0	0
$\alpha\beta, \beta\alpha$ $\beta\gamma, \gamma\beta$	b_o^2	$\frac{b_1^2}{2} \left(1 + \frac{\sin^2 \theta_o}{6}\right)$	$\frac{1}{2} b_1^2 \left(1 + \frac{\cos^2 \theta_o}{2}\right)$	$\frac{b_1^2}{6} \sin^2 \theta_o$	0
$\beta\beta$	$\frac{4}{3} b_o^2$	0	0	0	$\frac{2}{3} b_o^2$
$\alpha\gamma$ $\gamma\alpha$	$\frac{1}{3} b_o^2$	$\frac{b_1^2}{2} \left(1 + \frac{\cos^2 \theta_o}{6}\right)$	$\frac{b_1^2}{2} \sin^2 \theta_o$	$\frac{b_1^2}{3} \cos^2 \theta_o$	$\frac{2}{3} b_o^2$

Figure Captions

- Fig. 1. The energy level diagram of He^5 illustrating the near coincidence of $J = 3/2 +$ level with the level corresponding to a free D and T.
- Fig. 2. The energy level diagram of Li^5 .
- Fig. 3. An electron triton collision. The angle θ_1 , and the direction of the line of apsides b is indicated.
- Fig. 4. The energy level diagram indicating the Zeeman splitting of the tritium atom. The levels in increasing energy correspond to the states $\psi_2, \psi_4, \psi_1, \psi_3$ after Eq. (51).
- Fig. 5. A collision of two deuterons in the β α state indicating the definition of the angle θ_0 .

ENERGY LEVEL DIAGRAM FOR He^5

Fig. 1

ENERGY LEVEL DIAGRAM FOR Li^5

Fig. 2

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Fig. 3

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Fig. 4

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Fig. 5

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