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INFINITE CLOUD MODEL OF ELECTRIFICATION BY THE PRECIPITATION
MECHANISM IN A POSSIBLE RAINOUT SITUATION

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ABSTRACT

This report examines the extent to which the ionization associated with a radioactive aerosol will limit electric field growth in a thundercloud. Favored mechanisms for thundercloud electrification, called precipitation mechanisms, involve charge separation in collisions between falling hydrometeors and cloud droplets followed by separation of the resulting charged objects as the hydrometeors fall through the cloud and the droplets move with the updraft. We have extended a simple, infinite-cloud, precipitation-mechanism model to include effects due to the presence of radioactive aerosol particles. These effects involve ion generation, convection currents due to motions of ions and charged aerosol particles, and collection of ions and aerosol particles by hydrometeors and cloud droplets.

For our model we assume a spatially homogeneous cloud of

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infinite extent. The macroscopic electric field and current density have only vertical components. We consider one size of hydrometeor and one size of cloud droplet in each of three categories: droplets that have had zero, one, and two collisions with hydrometeors. Condensation is represented by the steady appearance of new cloud droplets. The charge separation in rebounding collisions between cloud droplets and hydrometeors is modeled to represent an inductive mechanism involving polarization of the hydrometeors by the ambient electric field, or a non-inductive mechanism involving the thermoelectric effect, or both kinds of mechanism acting simultaneously.

The model includes collection of ions by hydrometeors, cloud droplets, and aerosol particles. A preliminary extension of the model was made to include collection of aerosol particles by hydrometeors and by cloud droplets. However, we concluded that collection of particles at a rate of a few particles $\text{cm}^{-3} \text{s}^{-1}$ will affect the electric-field strength by at most a few percent over a time of several hundred seconds. Therefore, we have ignored collection of particles for most of our computations.

Our model involves a system of twelve coupled first-order ordinary differential equations that govern the time development of the electrical conditions of our model cloud. This report describes the solutions of this system obtained for various choices of parameters and initial conditions. Our major concern is to assess the effect of a radioactive aerosol on the growth of the electric-field strength. Therefore, we have given most attention to

exploring the behavior of the model for various choices of the ion-generation rate. Our basic conclusion, which we do not believe will be altered by refinements in any of the areas of limitation of the model, is that, although there is some variation with choice of parameters and charge-separation mechanism, an ion-generation rate $G > 10^6 \text{ cm}^{-3} \text{ s}^{-1}$ will prevent significant electric-field growth.

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INTRODUCTION

In his survey of the electrical aspects of rainout, Rosenkilde [1976] noted that the most glaring weakness exposed by the survey was the absence of any self-consistent determination of the electric field in the presence of high ionization. The processes of attachment of ions to radioactive aerosol particles and cloud drops of various sizes and of attachment of radioactive particles to drops are certainly dependent on the ambient electric field. The time development of the electric field in turn depends on the currents associated with the motions of the various charged particles that are present. A question of particular importance is the extent to which ionization associated with radioactive debris limits electric field growth.

The observation of lightning attests the presence of large electric fields in natural thunderclouds. No consensus exists as to the electric-field magnitude required for the initiation of lightning, but it seems likely this will be at least as large as the field required for onset of corona. After considering reports by various authors, Griffiths and Phelps [1976] conclude that electric-field magnitudes in the range 2.5×10^5 to

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$9.5 \times 10^5 \text{ Vm}^{-1}$ are required to obtain corona under conditions likely to exist in the storms. There is not agreement on the mechanism by which such fields are generated in thunderstorms. Favored mechanisms, called precipitation mechanisms, involve the charge separation produced by charged hydrometeors falling away from oppositely charged cloud droplets that move with the updraft.* In one popular kind of model, the charging of these two types of object with opposite sign is a consequence of collisions of cloud droplets with hydrometeors that are polarized by the ambient electric field [Latham and Mason, 1962]. The details of various simple models using this inductive mechanism of charge separation have been worked out, for example, by Kamra [1970], Mason [1972, 1973], Paluch and Sartor [1973a], Ziv and Levin [1974], and Scott and Levin [1975]. Starting from a fair-weather field the order of 100 Vm^{-1} , these calculations yield fields greater than 10^5 Vm^{-1} in times of from 50 to 600 seconds, depending on the particular values of various parameters that are used.

Several non-inductive mechanisms for charge separation in ice-ice or ice-water collisions have been proposed. The most important of these depend on the thermoelectric effect, which involves differential charge transport along a thermal gradient. Reviews of various conjectures and experiments are given by Stow [1969, § 5], Mason [1971, § 9.6], and

*We use the term 'hydrometeor' to denote a liquid or solid precipitation particle of radius greater than $5 \times 10^{-2} \text{ cm}$ that falls with a terminal velocity of hundreds of cm s^{-1} in the absence of electrical effects. We use the term 'droplet' to denote a liquid or solid (water) cloud particle of radius less than $5 \times 10^{-2} \text{ cm}$ that falls with a terminal velocity of a few cm s^{-1} in the absence of electrical effects.

Hobbs [1974, § 10.6]. Unfortunately, experimental investigations of these processes have given very divergent results. The efficacies of these mechanisms are uncertain, and there is evidence that they may depend strongly on impurity concentrations, surface topography of the colliding objects, and other highly variable parameters. If the charge separation per rebounding collision due to non-inductive mechanisms approached the largest values reported, it would be an important feature of thunderstorm electrification. We shall consider the possibility of inductive and non-inductive charge-separation mechanisms, both separately and acting simultaneously.

Precipitation mechanisms for cloud electrification have not been universally accepted, and calculations based on them have been criticized on various counts, particularly by Moore [1974, 1975a, 1975b, 1976a]. However, the few quantitative models of cloud electrification by other mechanisms have so far not proved capable of explaining the development of thunderstorm fields in a reasonable amount of time. (Chiu and Klett, 1976.)

Most precipitation-mechanism models of thunderstorm electrification have in common the assumption of an infinite cloud with neither horizontal nor vertical space dependence of its properties. An exception is recent work by Illingworth and Latham [1975, 1977] on a cylindrically symmetric finite-cloud model. The electric field and the droplet and hydrometeor velocities are assumed to have only vertical components. The evolution of the electric field as a function of time is obtained by solving the equation (in Gaussian units)

$$\frac{dE}{dt} = -4\pi J, \quad (1)$$

which follows from Maxwell's equations upon the assumption of spatial

homogeneity and zero macroscopic charge density. The current density J has contributions from the motions of charged droplets, charged hydrometeors, and in some models atmospheric ions. The ionic contribution to the current density for natural thunderclouds is not well understood, primarily because it is not clear how to represent the ion production by corona [Griffiths, Latham, and Myers, 1974; Levin and Scott, 1975]. There has been no attempt to treat ion-production rates as large as might be encountered in a cloud infected by radioactive debris.

DESCRIPTION OF THE PRESENT MODEL

In attempting to assess the effect of high ion-production rates on electric-field growth, our first effort has been to extend a simple, infinite-cloud, precipitation-mechanism model to include effects due to the presence of radioactive aerosol particles. The following effects must be included: (1) Contributions to the current density due to the convection of ions, free electrons, and charged aerosol particles. (2) Contributions to the charging of droplets and hydrometeors due to collection of ions. (3) Collection of ions by aerosol particles. (4) Ion-ion recombination. (5) Augmentation of ion and free electron number densities due to decay of the radioactive aerosol. (6) Attachment of free electrons to air molecules forming ions. (7) Collection of aerosol particles and free electrons by droplets and hydrometeors. In developing our model we have made numerous assumptions about these and other processes. General assumptions are described in the following paragraphs; specific choices of parameters are discussed in a later section.

Our model cloud is infinite in extent and spatially homogeneous. The macroscopic charge density is assumed to be initially zero; it stays zero because all the physical interactions conserve charge. We consider that the macroscopic electric field and current density have only vertical components; downward is taken as positive. The time development of the electric-field strength is governed by Eq. (1).

Cloud droplets are considered in three categories: droplets that have had zero (virgin droplets), one, and two collisions with hydrometeors. We assume that the probability of rebound from a third collision with a hydro-

meteor is zero. We consider one size of droplet for each of the three categories. The model droplets can represent supercooled liquid water or ice, depending on how certain model parameters are chosen. All droplets are assumed spherical. There are losses in the numbers of droplets per unit volume because of the accretion of a fraction of the droplets colliding with hydrometeors. We also include a parameter representing a constant rate of gain in the number of virgin droplets per unit volume because of the condensation of water vapor.

The charge on a droplet rebounding from a collision with a hydrometeor depends on the electrical conditions existing at the time of the collision. Since the electrical conditions evolve in time, to follow the charges on the droplets exactly would require a myriad of charge categories associated with different collision times. There is a similar difficulty with newly condensed droplets, which continuously appear in the model as the electrical conditions evolve. Therefore, we have assumed that those virgin droplets that are newly condensed have zero charge and that the total charge available for each category of droplets at each step of the calculation is uniformly distributed over the droplets in the category [Scott and Levin, 1975]. Thus, all droplets in a given category are assigned the same charge.

We consider only one size of hydrometeor, which is assumed to be spherical. The inductive mechanism should be operative in ice-ice and ice-water collisions, but Paluch and Sartor [1973b] have argued that the inductive process will not be effective in systematically separating charge for water-water collisions. Also, the non-inductive mechanisms that depend on the thermoelectric effect require an ice hydrometeor. Therefore, we have always considered the hydrometeors to be ice. The number of hydrometeors per unit

volume is assumed constant; the size of the hydrometeors grows in time because of the accretion of a fraction of the colliding droplets.

Hydrometeors and droplets are assumed to move vertically with their terminal velocities as determined by electrical, gravitational, and air viscosity effects. We compute the terminal velocities relative to air as suggested by Ziv and Levin [1974], using the empirical equations for Reynolds number in terms of Best number given by Davies [1945]. An empirical formula for the viscosity of air as a function of temperature was taken from Rogers [1976, p.73]. We use a geometrical cross section to describe droplet-hydrometeor collisions. The model allows this to be multiplied by a constant collision-efficiency if desired. Droplet-droplet collisions are ignored.

For the inductive charge-separation mechanism, provided that the collision contact time is sufficiently long, the charges on a droplet and a hydrometeor immediately after a collision depend on their charges just before the collision, their radii, the ambient electric field, and the angle θ between the electric-field direction and the line joining the centers of the spheres through the point of contact (see Fig. 1) according to the equations of A. M. Gordon as reported by Latham and Mason [1962]. (These equations, which can be derived by the method of images, appear in Appendix II.) In order to account for short collision times, we modify the Gordon equations by introducing as a parameter the charge-transfer efficiency [Paluch and Sartor, 1973a] $K = 1 - \exp(-t_c/t_r)$, where t_c is the contact time for the collision and t_r is the relaxation time for redistribution of charge over the spheres.

The probability that a droplet will rebound from a collision with a hydrometeor certainly depends on the angle θ . However, it is not feasible

to follow each collision with its individual value of θ , so we introduce as a parameter applicable to all collisions the average rebound probability $\overline{\alpha(\theta)}$. In order to describe the amount of charge exchanged in a collision via the inductive mechanism we also need a parameter representing the average value of the product of the rebound probability and the cosine of θ , divided by the average value of the rebound probability; $\langle \cos\theta \rangle \equiv \overline{\alpha(\theta)\cos\theta} / \overline{\alpha(\theta)}$.

In order to represent a non-inductive charge-separation mechanism we introduce a parameter Δ representing a constant amount of charge exchanged in each rebounding droplet-hydrometeor collision regardless of the conditions at the time of the collision. The sign of Δ is such that the hydrometeor gains negative charge.

Aufdermaur and Johnson [1972] give experimental evidence that a super-cooled liquid droplet undergoing a rebounding collision with a hail pellet will leave some of its mass behind on the pellet. In order to allow this possibility we introduce a parameter F representing a constant fraction of the droplet mass that is accreted by the hydrometeor in all rebounding collisions.

Radioactive aerosol particles are described by parameters representing their concentration, radii, mass density, and specific activity. Although we made a preliminary extension of the model where these particles are collected by droplets and hydrometeors and particle concentration is a variable (vile infra), for most of our computations particle concentration was held constant. Particle mobilities (velocity divided by applied force) were estimated using the empirical formula given by Fuchs [1964, p. 27] with parameter values appropriate to glass spheres in air. The mean free path for a gas molecule, which appears in this formula, was taken from the U.S. Standard Atmosphere [1962].

We assume that each beta decay of the radioactive aerosol material produces a 0.5 MeV beta particle that generates 1.5×10^4 electron--positive-ion pairs [Rosenkilde, 1976, Appendix C]. Significant processes involving free electrons are attachment to neutrals and detachment from negative ions. We have made estimates of the rates for these processes based on information given by Phelps [1969] and by Biondi [1969]. See also Rosenkilde [1976], Appendix B.

Significant processes involving ions are recombination and collection by hydrometeors, droplets, and aerosol particles. We take the ion-ion recombination coefficient as $1.6 \times 10^{-6} \text{ cm}^3 \text{ s}^{-1}$. The hydrometeors, droplets, and aerosol particles collect ions via convective diffusion with both fluid and electrical forces. To describe the currents of positive and negative ions to the collectors we use the equations of Whipple and Chalmers [1944] or of Klett [1971], depending on whether the parameter eEa/kT has magnitude greater than or less than 4.0, respectively, as suggested by Klett. Here e is the magnitude of the charge of an electron, a is the radius of the collecting object, k is Boltzmann's constant, T is the absolute temperature, and E is the electric-field strength far from the collector.

Ionic mobilities were based on a table given by Rosenkilde [1976, Appendix D]. Electron drift velocities were computed using the formula given by Rosenkilde [1976, Appendix B] based on a tabulation given by Huxley and Crompton [1974].

In an early version of the model free electrons were collected by hydrometeors and droplets following the same equations as used for collection of ions. The results with electron collection included were insignificantly different from the results without collection because the concentration of free electrons is always small compared to the concentration of ions.

Therefore, collection of free electrons was not included in subsequent versions of the model.

It is not simple to make even a crude model of the collection of aerosol particles by hydrometeors and droplets. Theoretical calculations made by Grover [1976] for aerosol radii of from 0.5 to 10 μm indicate that for a droplet-sized collector the collision efficiency will depend strongly on the particular electrical conditions. We have made two preliminary investigations concerning particle collection. (1) For small aerosol particles the collection formulas should not be grossly different from those for collection of ions. In one version of the model we included collection of particles by all categories of droplets and by hydrometeors following the same equations as used to describe collection of ions. (2) In another version of the model we included a constant rate of depletion of the number density of particles. In this computation, as particles are removed their charges are assigned equally to the total charge on hydrometeors and the total charge on virgin droplets so that the model continues to conserve charge.

Our conclusion from these preliminary studies is that collection of aerosol particles at a rate of a few particles $\text{cm}^{-3}\text{s}^{-1}$ will affect the electric-field strength by at most a few percent over a time of several hundred seconds. Therefore, we have ignored collection of aerosol particles for most of our computations and have held the particle number density constant.

Many workers have included in their cloud electrification models some sort of term intended to represent the effect of ion currents arising from the onset of corona. This could significantly inhibit the final stages of field growth. However, experiments by Griffiths and Latham [1974] indicate that fields of 4×10^5 to $5 \times 10^5 \text{Vm}^{-1}$ are required for the onset of corona from ice hydrometeors, perhaps 20% less if the hydrometeors are highly

charged [Griffiths, 1975]. By the time this size field is reached, the question of whether or not the field will grow under the particular conditions assumed has already been answered. Therefore, we have ignored corona in our model.

Based on these assumptions, we have developed a system of twelve coupled first-order ordinary differential equations that govern the time development of the electrical conditions of our model cloud. These equations are given in words and in symbols in Appendix II; a dictionary of symbols used is given in Appendix I. We have solved these equations for various choices of parameters and initial conditions using the version of the Gear differential equations solving code developed by Hindmarsh [1974]. The computer programs that we have written are listed in Appendix IV. The subroutines of the Gear package are listed by Hindmarsh.

CHOICE OF PARAMETERS AND INITIAL CONDITIONS

There are many parameters that can be varied in our model. Since our major concern is to assess the effect of radioactive aerosol particles on electric-field growth, we have given most attention to considering various values of the ion-generation rate G . We have used G values of from zero up to whatever was required to prevent electric-field growth. In addition to the ion generation from the activity of the particles, we have included a background ion-pair generation rate of $10 \text{ cm}^{-3} \text{ s}^{-1}$. This is significant only for those runs having relatively small G values.

We have investigated three cases for the charge-separation accompanying rebound of droplets colliding with hydrometeors: 1) inductive mechanism only, 2) non-inductive mechanism only, 3) both inductive and non-inductive mechanisms acting simultaneously. In order to investigate the non-inductive mechanism alone, we set the charge-transfer efficiency K of the inductive mechanism equal to zero.

We have noted already that several non-inductive mechanisms that might be operative in ice-ice or ice-water collisions have been proposed, and that the experimental evidence is contradictory. Reynolds, Brook, and Courley [1957] reported a charge separation of the order of $1.7 \times 10^{-13} \text{ C}$ per ice crystal collision when ice crystals of about 50 microns radius and supercooled liquid droplets collided with a simulated graupel pellet of 2mm radius. When we used a value this large for Δ in our model, the charging of the hydrometeors and the growth of the electric field for small G seemed unrealistically abrupt. Most of our runs to investigate non-inductive charge separation have been made with $\Delta = 1.5 \times 10^{-14} \text{ C}$. This still gives a potent electrification mechanism.

As has been pointed out by Illingworth and Latham [1977], the inductive and non-inductive mechanisms acting simultaneously are not simply additive in their effects. Under certain conditions the inductive mechanism can act to discharge a hydrometeor that the non-inductive mechanism is charging. In our runs to investigate the two mechanisms acting simultaneously we have taken $K=1$ and $\Delta = 1.5 \times 10^{-14}$ C.

If the dependence of rebound probability on θ were known, an appropriate average rebound probability could be computed for application to all collisions. Unfortunately, there is not any experimental data directly relating rebound probability and θ for ice-ice or ice-water collisions. Aufdermaur and Johnson [1972] reported that in collisions of supercooled liquid droplets with a simulated hail pellet there were only 1 to 10 rebounds per 1000 collisions. On the basis of these data, Moore [1975b] argued that rebound occurs only for essentially grazing collisions. The report of Scott and Levin [1970] infers a considerably larger rebound probability for collisions of snow crystals with an ice sphere; however, Moore [1975b] suggests that an ice crystal may slide around the surface of the pellet and make its departure from near the equatorial zone.

Values of $\langle \cos\theta \rangle$ and $\overline{\alpha(\theta)}$ used in various models are given in Table 1.

Table 1. Values of $\langle \cos\theta \rangle$ and $\overline{\alpha(\theta)}$ used in various models.

Reference	$\langle \cos\theta \rangle$	$\overline{\alpha(\theta)}$
Kamra [1970]	0.707	1
Mason [1972]	0.707	1
Paluch and Sartor [1973a]	0.5	1
Ziv and Levin [1974]	0.707	0.03 - 0.1

Ziv and Levin used small rebound probabilities because they contemplated water-water collisions.

Moore [1975b, 1976a] has asserted that appropriate values are $\langle \cos \theta \rangle = 0.088$ and $\overline{\alpha(\theta)} = 0.01$, or even smaller. Mason [1976b] has responded that Moore's arguments are "not credible". All in all, the literature permits great freedom in choosing these parameters. The rebound probability for an ice-ice collision is likely greater than that for an ice-water collision, but a unit rebound probability seems too large, especially near the melting point. Moore's argument that rebound from larger values of θ will be favored seems to have merit, but his argument that only grazing collisions need be considered is compelling only for ice-water collisions. The question is confounded because the surface of at least the hydrometeor may not be smoothly spherical. We have made some runs using as representative of ice-water collisions the values $\langle \cos \theta \rangle = 0.088$ and $\overline{\alpha(\theta)} = 0.01$ suggested by Moore; we have made a larger number of runs using $\langle \cos \theta \rangle = 0.2$ and $\overline{\alpha(\theta)} = 0.3$, which we believe are values representative of ice-ice collisions.

Mason [1976a] has asserted that it is unrealistic to assume that the cloud droplets are not, at least in part, replenished by condensation. Most models have in fact assumed enough condensation to keep the droplet concentration roughly constant during field growth. Moore [1975a, 1976a] suggests that this requirement may not be met in many clouds that nevertheless produce lightning.

Because the hydrometeor velocity and hence the collision rate changes in time as the hydrometeor grows and the electrical conditions develop, it is not possible to choose a constant condensation rate that will keep the droplet concentration constant. We have generally chosen a condensation rate of $5 \text{ droplets cm}^{-3} \text{ s}^{-1}$ as giving a compromise between the droplet concentration decrease when the field is small and the droplet concentration

increase when the hydrometeors are nearly levitated.

Choices for other parameters do not require extensive discussion. We have selected values guided by the choices made in earlier models and by what seems representative of actual thunderclouds. Many of the values are taken from the survey by Rosenkilde [1976]. For altitude dependent quantities we have taken values appropriate for an altitude of approximately 4km. The values used for all parameters in the model except those for which many values were tried are given in Appendix III.

We have assumed that at time zero a fully developed cloud exists with typical hydrometeors and droplets. We do not attempt to describe how the cloud became infected with the radioactive aerosol. The initial charges on the hydrometeors, droplets, and particles are assumed to be zero, and the electric-field strength and the ion densities are assigned normal, fair-weather values. We have made a few runs with a negative (i.e. upward directed) initial electric field. A complete list of initial conditions is given in Appendix III.

RESULTS

Results for the electric-field strength vs. time for various ion-generation rates and two hydrometeor concentrations are shown in Figs. 2-4 for the three charge-separation cases. All these curves are for $\bar{\alpha} = 0.3$ and $\langle \cos \theta \rangle = 0.2$. Since the model does not include any dissipation effects arising from corona and the development of lightning, the plots of the electric-field strength are terminated at 10^6 Vm^{-1} . For these curves, the physical event that actually ends electric-field growth is the attainment of an electric-field strength and hydrometeor charge such that the hydrometeors are nearly levitated.

Fig. 2 shows the results when only the inductive mechanism is considered. In this case a rate of production of electron--positive-ion pairs of $G > 10^3 \text{ cm}^{-3} \text{ s}^{-1}$ is required in order to have a significant effect on electric-field growth. Increasing G beyond this value causes increasing delay in the attainment of a large field. If $G > 10^5 \text{ cm}^{-3} \text{ s}^{-1}$ there is no field growth within 600s. (The precise values of G leading to different field-growth conditions here and in the following figures depend somewhat on the hydrometeor number density N .)

Fig. 3 shows that when only the non-inductive mechanism is considered the ultimate field strength is smaller than for the case when only the inductive mechanism was considered (Fig. 2). The smaller field strength obtains because field growth occurs when the hydrometeors are smaller and more readily levitated. The delay in field growth with increasing G is much less than for the inductive mechanism. If the field strength grows beyond

10^5Vm^{-1} , it does so within about 80s. It requires $G \geq 3.2 \times 10^5 \text{cm}^{-3} \text{s}^{-1}$ to have a significant effect on the electric-field growth, and $G \geq 2 \times 10^6 \text{cm}^{-3} \text{s}^{-1}$ prevents field growth; these values are more than an order of magnitude greater than for the inductive mechanism. Consider now the cases where the field strength does not grow beyond approximately 10^4Vm^{-1} ; the steady-state field strength that is attained is more than an order of magnitude greater than non-growth values for the inductive mechanism, which are depressed below the fair-weather value.

The curves of Fig. 4 for the case where the inductive and non-inductive mechanisms are acting simultaneously are very much like those of Fig. 3, indicating that the non-inductive mechanism is dominant for the parameter values we have used. It requires $G \geq 10^5 \text{cm}^{-3} \text{s}^{-1}$ to have a significant effect on the electric-field growth and $G \geq 10^6 \text{cm}^{-3} \text{s}^{-1}$ to prevent field growth; these values are slightly smaller than for the non-inductive mechanism acting alone.

The curves in Figs. 5-13 are presented as supplementary material for the reader who is interested in some of the details of how various quantities develop in time. These curves show the time development for various quantities for each of the three charge-separation classes for three cases:

(1) $G=0$, (2) G near but less than the maximum value consistent with field growth, and (3) G near but greater than the minimum value that prevents field growth. These graphs show that field growth is associated with increase in the magnitudes of the hydrometeor and collided-droplet charges, that field growth terminates when the hydrometeor is levitated, and that field growth occurs for cases where the ion density is held or brought down to reasonably small values.

Fig. 14 shows the time development of the electric-field strength for several cases where we have taken $\overline{\alpha(\theta)} = 0.01$ and $\langle \cos\theta \rangle = 0.088$. For all these runs we have used $N = 10^{-3} \text{ cm}^{-3}$. For the inductive and non-inductive mechanisms operating simultaneously, the shape of the curves is much like those in Fig. 4. However, the value of G required to prevent field growth is about an order of magnitude less than for the situations in Fig. 4, which all have $\overline{\alpha(\theta)} = 0.3$ and $\langle \cos\theta \rangle = 0.2$.

The situation for the inductive mechanism operating alone is markedly different from the curves shown in Fig. 2. In Fig. 14 we see that as G is increased from zero there is at first field growth at earlier times and to greater ultimate field strengths. This trend continues to beyond $G = 10^3 \text{ cm}^{-3} \text{ s}^{-1}$. Further increase in G leads to delay in the onset of field growth, a much more abrupt termination of field growth, and somewhat lower ultimate field strength. (See curves E and L in Fig. 14.) Finally by $G = 10^5 \text{ cm}^{-3} \text{ s}^{-1}$ field growth is prevented. This is essentially the same value of G that prevents field growth under the conditions of Fig. 2.

In an effort to elucidate this behavior we present Figs. 15-21 showing the time development of the ion densities, hydrometeor charge, and droplet charges for the various cases of Fig. 14. These curves should be compared with those of Figs. 5 and 6. The stronger field growth in Fig. 2 seems to be due to greater growth of hydrometeor charge, which we would expect for larger $\overline{\alpha}$ and $\langle \cos\theta \rangle$. The trend to greater field growth as G is increased from zero to $10^3 \text{ cm}^{-3} \text{ s}^{-1}$ in Fig. 14 is associated with the development of significant positive droplet charges, particularly when $(n_1 - n_2)$ becomes a significant fraction of $(n_1 + n_2)$. Here n_1 and n_2 are the number density of positive and of negative ions, respectively. This behavior is also

present in the cases of Fig. 2 (see e.g. Fig. 6), but it is masked by the greater current due to charged hydrometeors. It is of interest to note that the termination of field growth shown by curves E and L in Fig. 14 is not associated with levitation of the hydrometeors, but rather with termination of growth of the magnitudes of the droplet and hydrometeor charges.

We also made some runs with an initial field strength of -100 Vm^{-1} rather than $+100 \text{ Vm}^{-1}$. Consider the equations of the model given in Appendix II under the condition that the initial values of all charges and of the electric field strength are changed in sign. Careful examination reveals the following:

(a) The expressions for the inductive charging mechanism are reversed in sign; i.e., the mechanism will tend to produce positive hydrometeors and negative droplets, and hence to build an electric field of negative sign.

(b) The equations for the collection of ions by various objects show the same kind of symmetry to the extent that $n_1 \approx n_2$ and $w_1 \approx w_2$. Here w_1 and w_2 are the mobility of positive ions and of negative ions, respectively. We have used $w_1/w_2 = 1.84/2.00 = 0.92$, so the second equality is only approximate. Also, situations can arise in the computations where $(n_1 - n_2)$ is a significant fraction of $(n_1 + n_2)$, in certain instances greater than 30% (see Fig. 6 and Fig. 19). Therefore, we expect that the situation if the electrical initial conditions are reversed in sign will be more complex than simply building a negative field in cases where charging by ion collection is important.

(c) The expressions for the charge exchanged in a collision via the non-inductive mechanism are independent of the electrical conditions. Therefore, this mechanism tends to build a positive electric field regardless of the change in sign of the initial electrical conditions.

(d) The expression for the contribution to aerosol particle charging that is directly due to β -decay is not affected by the change in sign of the initial electrical conditions.

Our results are consistent with these observations. The runs we made with $E(t=0) = -100 \text{ Vm}^{-1}$ and with the non-inductive charge-separation mechanism operating produced virtually identical results after the first seconds to those for $E(t=0) = +100 \text{ Vm}^{-1}$. If only the inductive charge-separation mechanism is operating ($\Delta=0$), the results obtained depend on the value of G . With only the background ion production, $G=0$ (and $N = 10^{-3} \text{ cm}^{-3}$), the results for $E(t=0) = -100 \text{ Vm}^{-1}$, except for the virgin droplet charge q and the aerosol particle charge q_p , are essentially identical to those for $E(t=0) = +100 \text{ Vm}^{-1}$ except the charges and the electric field have their signs reversed. Thus, in this case a negative electric field is produced. With the exception of q and q_p , the greatest difference in magnitude of any quantity for the two cases after 125 seconds is about 0.6%. For the negative initial field case, the sign of q changes but its magnitude is less than half as big as for the positive initial field case. The sign of q_p does not change, but its magnitude increases by almost an order of magnitude. Both q and q_p are very small in magnitude compared with the other charges.

With $G = 10^3 \text{ cm}^{-3} \text{ s}^{-1}$, the inductive mechanism with an initial field of $E(t=0) = -100 \text{ Vm}^{-1}$ again results in the building of a large negative field. With the exception of q and q_p , after 600s the greatest difference in the magnitude of any quantity from the case for a positive initial field is about 5% (the difference in the field magnitudes is 0.14%), and the charges and the electric field have opposite signs for the two cases. For this G value, q is the same sign and roughly the same magnitude for the two cases; q_p has opposite sign and roughly the same magnitude for the two cases. Again both q and q_p are very small in magnitude compared with the other charges.

If G is increased to $10^4 \text{ cm}^{-3} \text{ s}^{-1}$, the inductive mechanism with $E(t=0) = -100 \text{ Vm}^{-1}$ produces a positive field. After 600s all quantities have values virtually identical to those produced when $E(t=0) = +100 \text{ Vm}^{-1}$.

Most references describe the electrification of thunderclouds as if only clouds of positive electric field (according to our sign convention of down as positive) exist. Clouds of opposite polarity have been reported in a review by Imyanitov and Shifrin [1962], by Latham and Stow [1969], and by Takeuti and Nakano [1974]. Illingworth and Latham [1977] report measurements by H. Kasemir showing reversal in sign of the vertical electrical-field component within single clouds. If instances of clouds with regions of large negative electric field exist but are rare, this would be an argument favoring the proposition that a non-inductive charge-separation mechanism commonly but not universally operates at least in the early stages of cloud electrification. If negative electric fields are common, this would favor the inductive charge-separation mechanism usually acting alone.

DISCUSSION

Our results for $G=0$ are consistent with what has been obtained with other models of this sort. Both the inductive and the non-inductive charge-separation mechanisms give potent electrification, and acting separately or simultaneously they provide tenable explanation of the origin of large electric-field strength in thunderclouds. Field strengths in excess of $4 \times 10^5 \text{Vm}^{-1}$ are readily produced by the two mechanisms acting simultaneously even for small rebound probability.

The experimental evidence as to what maximum field strengths actually occur in thunderclouds, as compared with field strengths inferred from the need to initiate lightning, is surprisingly limited. Gunn [1950] reports fields exceeding $2 \times 10^5 \text{Vm}^{-1}$ were observed several times. Winn, Schwada and Moore [1974] measured fields as large as $4 \times 10^5 \text{Vm}^{-1}$ only two times out of 61 cases. However, their apparatus determines only the field component perpendicular to the axis of their rocket, which will generally be a horizontal component. Illingworth and Latham [1977] report vertical field-component measurements by H. Kasemir showing magnitudes of $1.5\text{--}2 \times 10^5 \text{Vm}^{-1}$, of both signs, in the lower regions of Florida thunderstorms.

A similar situation obtains when we ask whether the hydrometeor and droplet charges we have computed, at least for low ion-generation rates, are in agreement with values found for natural thunderclouds. Most authors report charges of both signs and highly varied magnitudes. Colgate and Romero [1970] report a mean absolute charge fit by the formula $|q| = (1.72 \text{ esu cm}^{-2})r^2$, where r is the droplet radius, for droplets somewhat

smaller than those of our model in the lower few hundred meters of a forming thunderstorm. Griffith and Latham [1975] assert that the data of Gunn [1950] and of Latham and Stow [1969] for the charge Q (esu) on raindrops of radius R (cm) in a field of strength E suggest a relation $Q = -KER^X$ where $0.5 \leq X \leq 3$ (most probably about 2) and $1 \leq K \leq 3$ (most probably 2). However, this requires an imaginative interpretation of incomplete and highly varied data. Gaskell et al. [1977] have recently reported measurements showing no simple relation between charge and radius for hydrometeors near the bases of thunderclouds where the vertical field was generally negative. They also find the charges are often far greater than should be possible for the inductive charge-separation mechanism.

We have determined the coefficient K in the relation $|Q| = KER^2$ for a number of our results for both droplets and hydrometeors either near the maximum field less than 10^6Vm^{-1} for a field growth situation or at the equilibrium field for a non-growth situation. These values do not appear to be a useful way to characterize the computational results. Most of these values could be changed significantly by choosing a different time in the evolution of the model. There is nothing in the literature to make us believe that any of our values for hydrometeor or droplet charges are unreasonable.

We can attempt to make a rough estimate of the amount of ionization that we expect will prevent field growth. Suppose that during its growth in the absence of significant ionization the electric-field strength is roughly an exponential function of the time, $E = E_0 \exp(\xi t)$. Then from Eq. (1) we have

$$-4\pi J = \frac{dE}{dt} = \xi E.$$

Suppose it takes a time $\Delta t = 20-100s$ for the field strength to grow from $100 \text{ Vm}^{-1} = (1/300) \text{ statvolt cm}^{-1}$ to $3 \times 10^5 \text{ Vm}^{-1} = 10 \text{ statvolt cm}^{-1}$. Then $\xi = \Delta \ln E / \Delta t \approx 0.08 - 0.40 s^{-1}$. Thus, the magnitude of the current density that the field-building mechanism can generate is roughly

$$|J| \approx \xi E / 4\pi,$$

with ξ in the range given above.

For the ions, if $n_1 \approx n_2 \approx n$, we have $J_{\text{ions}} \approx ne(w_1 + w_2)E$. If the field is to grow in the presence of the ions we must have

$$\xi / 4\pi > ne(w_1 + w_2),$$

which implies that

$$n < \xi / 4\pi e(w_1 + w_2) \approx 1.2 - 5.8 \times 10^4 \text{ cm}^{-3}$$

An estimate of the ion generation rate associated with this number density can be made using the relations given by Rosenkilde [1976, Appendix C].

If we use Rosenkilde's estimates for the attachment coefficients in his Eq. (C11) we obtain for $E = 3 \times 10^5 \text{ Vm}^{-1}$ and a droplet number density of 100 cm^{-3} that $G \approx 7.2 \times 10^4 - 3.5 \times 10^5 \text{ cm}^{-3} \text{ s}^{-1}$. For the same field and a droplet number density of 200 cm^{-3} we obtain $G \approx 1.4 - 7.0 \times 10^5 \text{ cm}^{-3} \text{ s}^{-1}$. Comparison with the results shown in Figs. 2-4 and Fig. 14 shows that these results predict fairly closely the range of values of G required in our model to prevent field growth.

It would be a mistake to confuse detailed wrinkles in the results of our model with the behavior of real thunderclouds. The model has several important limitations that we shall discuss briefly in the following paragraphs.

The most important limitation of our model is the absence of any space dependence. Paluch and Sartor [1973b] noted the likely importance of including at least variation with altitude in precipitation-mechanism models. Illingworth and Latham [1975, 1977] have begun to develop a pre-

precipitation-mechanism electrification model for a cylindrically symmetric finite cloud; their results indicate a most promising direction for further work on the electrical effects of the presence of radioactive aerosol particles. The problem is to consider space dependence in a meaningful way without having the computation increase enormously.

A second limitation is that our model treats only vertical field components and vertical motions of cloud objects. It is well known that large horizontal electric-field components exist in thunderclouds. Kamra [1977] has qualitatively emphasized the importance of considering non-vertical electric fields for the inductive charge-separation process. However, to note the possible importance of non-vertical electric fields or more realistic cloud dynamics is a long way from seeing how to incorporate them into a computationally manageable model.

We have used only one size for droplets and one size for hydrometeors. Clearly physical reality demands a size distribution for both kinds of objects. Similarly, the droplets should have a distribution of charges, the charge on any droplet depending on the time at which it condensed or underwent collision. If distributions of drop sizes and charges are introduced, it may be necessary to worry about collisions between various sizes of droplets. The difficulty is how to introduce a distribution of properties while keeping the size of the computation within bounds. It is comforting that Levin [1976] and co-workers have not seen significant qualitative changes as they have increased the number of size and charge categories in their model.

The assumption of spherical droplets and hydrometeors is simplistic. The shape of graupel is likely to be not even grossly spherical, and graupel surfaces are certainly rough. This will have definite but not readily calculable effects on rebound probability and charge transfer.

Finally, we have generally ignored the collection of particles by droplets and hydrometeors. We have given reasons why we believe this is reasonable when the goal is to decide whether or not the field strength grows to large values. However, the ultimate concern is rainout, and particle collection is in fact the crucial physical process. As noted earlier, we have done some preliminary work to incorporate particle collection into the model. This would be easier if the theoretical description of the collection process were better understood.

CONCLUSION

We do not believe that refinement of the model in any of the areas of limitation we have mentioned will alter the basic conclusion that, although there is some variation with choice of parameters and charge-separation mechanism, an ion-generation rate $G > 10^6 \text{ cm}^{-3} \text{ s}^{-1}$ will prevent significant electric-field growth.

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APPENDIX I

DICTIONARY:

<u>Symbol Used in Written Notes</u>	<u>Symbol(s) Used in Computer Programs</u>	<u>Definition</u>
A	A	Specific activity of aerosol particles
B	B	Aerosol particle mobility (velocity/force)
C	CONRT	Rate of condensation of virgin droplets per unit volume
C_m	CMASS	Total water mass per unit volume initially present (hydrometeors and droplets) divided by $4\pi/3$
E	YO(5), Y(5), E	Electric field strength*
F	FMASS	Fraction of mass of droplet left behind on hydrometeor after a rebounding collision
G	G	Rate of generation of electron--positive-ion pairs per unit volume due to activity of aerosol particles
G_b	BKGG	Background ion-pair generation rate per unit volume
K	ZK	Charge-transfer efficiency for a droplet-hydrometeor collision.
N	ZN	Number density of hydrometeors
Q	YO(4), Y(4), Q	Charge per hydrometeor
$\bar{Q}$	YO(10), Y(10), Q1TOT	Total charge per unit volume available for once-collided droplets ($\bar{Q} = \bar{e}\bar{q}$)
$\tilde{Q}$	YO(11), Y(11), Q2TOT	Total charge per unit volume available for twice-collided droplets ($\tilde{Q} = \tilde{e}\tilde{q}$)
Q_p	YO(12), Y(12)	Total charge per unit volume on aerosol particles
R	R	Hydrometeor radius

<u>Symbol Used in Written Notes</u>	<u>Symbol(s) Used in Computer Programs</u>	<u>Definitions</u>
T	TEMP	Absolute temperature
V	V	Terminal velocity of hydrometeors relative to air*
a	ALPHA	Ion-ion recombination coefficient
c	YO(6), Y(6), C0	Number density of virgin droplets
c̄	YO(8), Y(8), C1	Number density of once-collided droplets
c̄̄	YO(9), Y(9), C2	Number density of twice-collided droplets
<cosθ>	COSAV	Average value of the product of the re- bound probability and the cosine of the contact angle for a droplet-hydrometeor collision, divided by the average value of the rebound probability.
e	QE	Magnitude of electronic charge
f	COLEF	Collision efficiency for droplet-hydro- meteor collisions
k	BOLTZ	Boltzmann constant
ℓ	ZL	Air molecule mean free path
n _e	YO(7), Y(7), NE	Number density of free electrons
n _p	ZNP	Number density of aerosol particles
n ₁	AN1	Number density of positive ions
n ₂	AN2	Number density of negative ions
q	SQ	Charge per virgin droplet
q̄	SQ1	Charge per once-collided droplet
q̄̄	SQ2	Charge per twice-collided droplet
q _p	QP	Charge per aerosol particle
r	SR	Virgin droplet radius
r̄	SR1	Once-collided droplet radius
r̄̄	SR2	Twice-collided droplet radius

<u>Symbol Used in Written Notes</u>	<u>Symbol(s) Used in Computer Programs</u>	<u>Definition</u>
r_p	RP	Aerosol particle radius
t	T	Time
v	SV	Terminal velocity of virgin droplets relative to air*
$\tilde{v}$	SV1	Terminal velocity of once-collided droplets relative to air*
$\bar{\tilde{v}}$	SV2	Terminal velocity of twice-collided droplets relative to air*
v_e	VE	Electron drift velocity relative to air
v_p	VP	Aerosol particle velocity relative to air
w_e	WE	Mobility of free electrons
w_1	W1	Mobility of positive ions
w_2	W2	Mobility of negative ions
y	Y0(2), Y(2), Y	Difference between number density of positive ions and number density of negative ions ($y = n_1 - n_2$)
z	Y0(1), Y(1), ZZ	Total number density of both positive and negative ions ($z = n_1 + n_2$)
Δ	DELQ	Amount of charge transferred per rebounding droplet-hydrometeor collision via non-inductive mechanism
T	Y0(3), Y(3)	Total charge per unit volume available for virgin droplets ($T = cq$)
$\bar{\alpha}$	ALAVE	Average rebound probability for a droplet-hydrometeor collision
η	ETA	Viscosity of air
ρ_a	ROA	Air mass density
ρ_d	ROS	Droplet mass density
ρ_h	RO	Hydrometeor mass density
ρ_p	ROP	Aerosol particle mass density

<u>Symbol Used in Written Notes</u>	<u>Symbol(s) Used in Computer Programs</u>	<u>Definition</u>
σ	SIG	Cross section for droplet-hyrometeor collisions

- * For these quantities the downward direction is taken as positive, the upward direction as negative.

APPENDIX II

In this appendix we present the mathematical relations used in our model. All equations appearing in this appendix are written assuming Gaussian units for electrical quantities. The symbols used are defined in Appendix I.

The differential equations for the three categories of droplet concentration are as follows:

$$\frac{dc}{dt} = - \left\{ \begin{array}{l} \text{\# of collisions per} \\ \text{unit volume per second} \\ \text{for virgin droplets} \end{array} \right\} + \left\{ \begin{array}{l} \text{\# of new virgin drop-} \\ \text{lets condensed per unit} \\ \text{volume per second} \end{array} \right\} = -\sigma Nc|(V-v)| + C. \quad (1)$$

The collision cross section is given by $\sigma = f\pi(R+r)^2$.

$$\begin{aligned} \frac{d\tilde{c}}{dt} &= \left\{ \begin{array}{l} \text{Rebound} \\ \text{Probability} \end{array} \right\} \left\{ \begin{array}{l} \text{\# of collisions per} \\ \text{unit volume per second} \\ \text{for virgin droplets} \end{array} \right\} - \left\{ \begin{array}{l} \text{\# of collisions per unit} \\ \text{volume per second for} \\ \text{once-collided droplets} \end{array} \right\} \\ &= \tilde{\sigma} Nc|(V-v)| - \tilde{\sigma} N\tilde{c}|(V-\tilde{v})|, \end{aligned} \quad (2)$$

where $\tilde{\sigma} = f\pi(R+\tilde{r})^2$.

$$\begin{aligned} \frac{d\tilde{\tilde{c}}}{dt} &= \left\{ \begin{array}{l} \text{Rebound} \\ \text{Probability} \end{array} \right\} \left\{ \begin{array}{l} \text{\# of collisions per unit} \\ \text{volume per second for} \\ \text{once-collided droplets} \end{array} \right\} - \left\{ \begin{array}{l} \text{\# of collisions per unit} \\ \text{volume per second for} \\ \text{twice-collided droplets} \end{array} \right\} \\ &= \tilde{\tilde{\sigma}} N\tilde{c}|(V-\tilde{v})| - \tilde{\tilde{\sigma}} N\tilde{\tilde{c}}|(V-\tilde{\tilde{v}})|, \end{aligned} \quad (3)$$

where $\tilde{\tilde{\sigma}} = f\pi(R+\tilde{\tilde{r}})^2$.

The differential equation for the number density of positive ions is

$$\begin{aligned} \frac{dn_1}{dt} &= \left\{ \begin{array}{l} \text{Rate of genera-} \\ \text{tion of electron--} \\ \text{pos.-ion pairs per} \\ \text{unit volume} \end{array} \right\} - \left\{ \begin{array}{l} \text{Rate of ion-ion} \\ \text{recombination} \\ \text{per unit volume} \end{array} \right\} - \left\{ \begin{array}{l} \text{Rate of collection} \\ \text{of pos.-ions by droplets} \\ \text{per unit volume} \end{array} \right\} \\ &\quad - \left\{ \begin{array}{l} \text{Rate of collection of} \\ \text{pos. ions by hydrometeors} \\ \text{per unit volume} \end{array} \right\} - \left\{ \begin{array}{l} \text{Rate of collection of} \\ \text{pos. ions by particles} \\ \text{per unit volume} \end{array} \right\} + \left\{ \begin{array}{l} \text{Rate of background} \\ \text{generation of ion-pairs} \\ \text{per unit volume} \end{array} \right\} \\ &= G - an_1n_2 - cR_1(r,q) - \tilde{c}R_1(\tilde{r},\tilde{q}) - \tilde{\tilde{c}}R_1(\tilde{\tilde{r}},\tilde{\tilde{q}}) - NR_1(R,Q) - n_p R_1(r_p,q_p) + G_b. \quad (4) \end{aligned}$$

The ion-collection rates R_1 are discussed below. The equation for the time derivative of the negative-ion number density has a similar structure:

$$\begin{aligned}
 \frac{dn_2}{dt} = & - \left\{ \begin{array}{l} \text{Rate of ion-ion} \\ \text{recombination} \\ \text{per unit volume} \end{array} \right\} - \left\{ \begin{array}{l} \text{Rate of collection of} \\ \text{neg. ions by droplets} \\ \text{per unit volume} \end{array} \right\} - \left\{ \begin{array}{l} \text{Rate of collection of} \\ \text{neg.ions by hydrometeors} \\ \text{per unit volume} \end{array} \right\} \\
 & - \left\{ \begin{array}{l} \text{Rate of collection of} \\ \text{neg.ions by particles} \\ \text{per unit volume} \end{array} \right\} + \left\{ \begin{array}{l} \text{Rate of attachment of} \\ \text{free electrons to neu-} \\ \text{tral air molecules per} \\ \text{unit volume} \end{array} \right\} - \left\{ \begin{array}{l} \text{Rate of detachment of} \\ \text{electrons from neg.ions} \\ \text{per unit volume} \end{array} \right\} \\
 & + \left\{ \begin{array}{l} \text{Rate of background} \\ \text{generation of ion-pairs} \\ \text{per unit volume} \end{array} \right\} \\
 = & -an_1n_2 - cR_2(r,q) - \bar{c}R_1(\bar{r},\bar{q}) - \bar{c}\bar{R}_2(\bar{r},\bar{q}) - NR_2(R,Q) - n_pR_2(r_p,q_p) \\
 & + 2.9 \times 10^7 n_e - 0.51n_2 + G_b.
 \end{aligned} \tag{5}$$

The ion-collection rates R_2 are discussed below.

Next consider the differential equations for the charges carried by droplets and by hydrometeors. For the total charge per unit volume carried by virgin droplets we have the following equation:

$$\begin{aligned}
 \frac{dT}{dt} = & \left\{ \begin{array}{l} \text{Charge per} \\ \text{positive ion} \end{array} \right\} \left\{ \begin{array}{l} \text{Rate of collection} \\ \text{of positive ions by} \\ \text{one virgin droplet} \end{array} \right\} \left\{ \begin{array}{l} \# \text{ of virgin droplets} \\ \text{per unit volume} \end{array} \right\} + \left\{ \begin{array}{l} \text{Charge per} \\ \text{negative ion} \end{array} \right\} \\
 & \left\{ \begin{array}{l} \text{Rate of collection} \\ \text{of neg. ions by} \\ \text{one virgin droplet} \end{array} \right\} \left\{ \begin{array}{l} \# \text{ of virgin droplets} \\ \text{per unit volume} \end{array} \right\} - \left\{ \begin{array}{l} \# \text{ of virgin droplet} \\ \text{collisions per unit} \\ \text{volume per second} \end{array} \right\} \left\{ \begin{array}{l} \text{Charge per} \\ \text{virgin droplet} \end{array} \right\} \\
 = & eR_1(r,q)c - eR_2(r,q)c - \sigma N(V-v)T.
 \end{aligned} \tag{6}$$

The differential equation for the total charge per unit volume carried by once-collided droplets is

$$\begin{aligned}
 \frac{dQ}{dt} = & \left\{ \begin{array}{l} \# \text{ of virgin droplet} \\ \text{collisions per unit} \\ \text{volume per second} \end{array} \right\} \left\{ \begin{array}{l} \text{Rebound} \\ \text{Probability} \end{array} \right\} \left\{ \begin{array}{l} \text{Charge that a virgin} \\ \text{droplet carries away} \\ \text{from a collision} \end{array} \right\} \\
 & - \left\{ \begin{array}{l} \# \text{ of once-collided} \\ \text{droplet collisions} \\ \text{per unit volume per} \\ \text{second} \end{array} \right\} \left\{ \begin{array}{l} \text{Charge per} \\ \text{once-collided} \\ \text{droplet} \end{array} \right\} \\
 & + \left\{ \begin{array}{l} \# \text{ of once-collided} \\ \text{droplets per unit} \\ \text{volume} \end{array} \right\} \left\{ \begin{array}{l} \text{Rate of collection of} \\ \text{positive ions by one} \\ \text{once-collided droplet} \end{array} \right\} \left\{ \begin{array}{l} \text{Charge per} \\ \text{positive ion} \end{array} \right\} \\
 & + \left\{ \begin{array}{l} \# \text{ of once-collided} \\ \text{droplets per unit} \\ \text{volume} \end{array} \right\} \left\{ \begin{array}{l} \text{Rate of collection of} \\ \text{negative ions by one} \\ \text{once-collided droplet} \end{array} \right\} \left\{ \begin{array}{l} \text{Charge per} \\ \text{negative ion} \end{array} \right\}. \quad (7)
 \end{aligned}$$

Before we can express this relation algebraically, we must consider the expression for the charge that a droplet carries away from a collision with a hydrometeor. Let Q and q be, respectively, the hydrometeor and droplet charges before collision and Q' and q' be the corresponding charges after the collision. We have for the inductive mechanism [Latham and Mason, 1962]

$$Q' = -\gamma_1 r^2 E \cos \theta + \frac{(Q+q)}{(1+\gamma_2 r^2/R^2)} \quad (8)$$

and

$$q' = (Q+q) - Q' = \gamma_1 r^2 E \cos \theta + \frac{(Q+q)}{(1+R^2/r^2 \gamma_2)} \quad (9)$$

where γ_1 and γ_2 are involved functions of r and R having the limiting values

$\lim_{r/R \rightarrow 0} \gamma_1 = \pi^2/2$ and $\lim_{r/R \rightarrow 0} \gamma_2 = \pi^2/6$. We have used these limiting values in our

computations. The change in droplet charge as a result of the collision

$$\Delta q = q' - q = \gamma_1 r^2 E \cos \theta + \frac{Q}{(1+R^2/r^2 \gamma_2)} - \frac{q}{(1+\gamma_2 r^2/R^2)} \quad (10)$$

provided the contact time is sufficiently long. In order to represent a collision with possibly incomplete charge equilibration, we multiply Δq by the charge transfer efficiency. This gives for the droplet charge just after collision

$$q' = q + K \Delta q = K \gamma_1 r^2 E \cos \theta + \frac{KQ}{(1+R^2/r^2 \gamma_2)} - \frac{Kq}{(1+\gamma_2 r^2/R^2)} + q. \quad (11)$$

To this we must add the amount of charge separated by the non-inductive mechanism, Δ .

Applying these results to equation (7), we have that

$$\begin{aligned} \frac{dQ}{dt} = & \sigma N c |(V-v)| \bar{\alpha} \left\{ K \gamma_1 r^2 E \langle \cos \theta \rangle + \frac{KQ}{(1+R^2/r^2 \gamma_2)} - \frac{Kq}{(1+\gamma_2 r^2/R^2)} + q + \Delta \right\} \\ & - \sigma N \bar{Q} |(V-v)| + e c R_1(\bar{r}, \bar{q}) - e c R_2(\bar{r}, \bar{q}) \quad (12) \end{aligned}$$

where we have inserted the average values $\bar{\alpha}$ and $\langle \cos \theta \rangle$ as discussed in the main text.

Similar considerations apply for the total charge per unit volume available for twice-collided droplets. We have

$$\frac{d\tilde{Q}}{dt} = \left\{ \begin{array}{l} \# \text{ of once-collided drop-} \\ \text{let collisions per unit} \\ \text{volume per second} \end{array} \right\} \left\{ \begin{array}{l} \text{Rebound} \\ \text{Probability} \end{array} \right\} \left\{ \begin{array}{l} \text{Charge that a once-} \\ \text{collided droplet carries} \\ \text{away from a collision} \end{array} \right\}$$

$$- \left\{ \begin{array}{l} \# \text{ of twice-collided} \\ \text{droplet collisions per} \\ \text{unit volume per second} \end{array} \right\} \left\{ \begin{array}{l} \text{Charge per twice-} \\ \text{collided droplet} \end{array} \right\}$$

$$+ \left\{ \begin{array}{l} \# \text{ of twice-collided} \\ \text{droplets per unit} \\ \text{volume} \end{array} \right\} \left\{ \begin{array}{l} \text{Rate of collection of} \\ \text{positive ions by one} \\ \text{twice-collided droplet} \end{array} \right\} \left\{ \begin{array}{l} \text{Charge per} \\ \text{positive ion} \end{array} \right\}$$

$$+ \left\{ \begin{array}{l} \# \text{ of twice-collided} \\ \text{droplets per unit} \\ \text{volume} \end{array} \right\} \left\{ \begin{array}{l} \text{Rate of collection of} \\ \text{negative ions by one} \\ \text{twice-collided droplet} \end{array} \right\} \left\{ \begin{array}{l} \text{Charge per} \\ \text{negative ion} \end{array} \right\}$$

$$= \partial N |(\mathbf{V}-\tilde{\mathbf{v}})| \tilde{\alpha} \left\{ \tilde{c} K \gamma_1 \tilde{r}^2 \tilde{E} \langle \cos \theta \rangle + \frac{\tilde{c} K Q}{(1+R^2/\tilde{r}^2 \gamma_2)} - \frac{K Q}{(1+\gamma_2 \tilde{r}^2/R^2)} + \tilde{Q} + \tilde{c} \Delta \right\}$$

$$- \partial N \tilde{Q} |(\mathbf{V}-\tilde{\mathbf{v}})| + e \tilde{c} R_1(\tilde{r}, \tilde{q}) - e \tilde{c} R_2(\tilde{r}, \tilde{q}). \quad (13)$$

The differential equation for the charge per hydrometeor has the following structure:

$$\begin{aligned}
 \frac{dQ}{dt} = & \left\{ \begin{array}{l} \text{Rate of collection of} \\ \text{positive ions by one} \\ \text{hydrometeor} \end{array} \right\} \left\{ \begin{array}{l} \text{Charge per} \\ \text{positive ion} \end{array} \right\} \\
 & + \left\{ \begin{array}{l} \text{Rate of collection of} \\ \text{negative ions by one} \\ \text{hydrometeor} \end{array} \right\} \left\{ \begin{array}{l} \text{Charge per} \\ \text{negative ion} \end{array} \right\} \\
 & + \left\{ \begin{array}{l} \# \text{ of collisions with} \\ \text{virgin droplets per} \\ \text{hydrometeor per second} \end{array} \right\} \left[\left\{ \begin{array}{l} \text{Charge per} \\ \text{virgin droplet} \end{array} \right\} \left\{ \begin{array}{l} \text{Accretion} \\ \text{Probability} \end{array} \right\} \right. \\
 & + \left. \left\{ \begin{array}{l} \text{Rebound} \\ \text{Probability} \end{array} \right\} \left\{ \begin{array}{l} \text{Charge gain by hydro-} \\ \text{meteor per virgin drop-} \\ \text{let rebounding collision} \end{array} \right\} \right] \\
 & + \left\{ \begin{array}{l} \# \text{ of collisions with} \\ \text{once-collided droplets} \\ \text{per hydrometeor per second} \end{array} \right\} \left[\left\{ \begin{array}{l} \text{Charge per once-} \\ \text{collided droplet} \end{array} \right\} \left\{ \begin{array}{l} \text{Accretion} \\ \text{Probability} \end{array} \right\} \right. \\
 & + \left. \left\{ \begin{array}{l} \text{Rebound} \\ \text{Probability} \end{array} \right\} \left\{ \begin{array}{l} \text{Charge gain by hydro-} \\ \text{meteor per once-collided} \\ \text{droplet rebounding collision} \end{array} \right\} \right] \\
 & + \left\{ \begin{array}{l} \# \text{ of collisions with} \\ \text{twice-collided droplets} \\ \text{per hydrometeor per second} \end{array} \right\} \left[\left\{ \begin{array}{l} \text{Charge per twice-} \\ \text{collided droplet} \end{array} \right\} \left\{ \begin{array}{l} \text{Accretion} \\ \text{Probability} \end{array} \right\} \right. \\
 & + \left. \left\{ \begin{array}{l} \text{Rebound} \\ \text{Probability} \end{array} \right\} \left\{ \begin{array}{l} \text{Charge gain by hydro-} \\ \text{meteor per twice-collided} \\ \text{droplet rebounding collision} \end{array} \right\} \right] .
 \end{aligned}
 \tag{14}$$

From equation (8) we have for the charge gain via the inductive mechanism by the hydrometeor in a rebounding collision with a droplet of charge q and radius r that

$$\Delta Q = Q' - Q = -\gamma_1 r^2 E \cos \theta + \frac{q}{(1+\gamma_2 r^2/R^2)} - \frac{Q}{(1+R^2/r^2 \gamma_2)} \quad (15)$$

provided the contact time is sufficiently long. For use in Eq. (14) we multiply this quantity by the charge transfer efficiency K . The charge gain via the non-inductive mechanism is $-\Delta$. Applying this relation to Eq. (14), we have

$$\begin{aligned} \frac{dQ}{dt} = & eR_1(R, Q) - eR_2(R, Q) + \sigma c |(\mathbf{V} - \mathbf{v})| \left\{ (1-\bar{\alpha})q - \bar{\alpha} \Delta - \gamma_1 \bar{r}^2 E \langle \cos \theta \rangle \bar{\alpha} K \right. \\ & \left. + \frac{\bar{\alpha} K q}{(1+\gamma_2 \bar{r}^2/R^2)} - \frac{\bar{\alpha} K Q}{(1+R^2/\bar{r}^2 \gamma_2)} \right\} + \bar{\sigma} |(\mathbf{V} - \bar{\mathbf{v}})| \left\{ (1-\bar{\alpha})\bar{Q} - \bar{\alpha} \bar{\Delta} - \gamma_1 \bar{r}^2 E \langle \cos \theta \rangle \bar{\alpha} K \right. \\ & \left. + \frac{\bar{\alpha} K \bar{Q}}{(1+\gamma_2 \bar{r}^2/R^2)} - \frac{\bar{\alpha} K Q \bar{c}}{(1+R^2/\bar{r}^2 \gamma_2)} \right\} + \bar{\sigma} |(\mathbf{V} - \bar{\mathbf{v}})| \bar{Q}. \end{aligned} \quad (16)$$

The differential equation for the electric-field strength has the form

$$\frac{dE}{dt} = -4\pi J. \quad (17)$$

The contributions to J from convection of hydrometeors, droplets of all three categories, ions of both signs, aerosol particles, and free electrons all have the following form:

$$\left\{ \begin{array}{l} \# \text{ of objects per} \\ \text{unit volume} \end{array} \right\} \left\{ \begin{array}{l} \text{Charge per} \\ \text{object} \end{array} \right\} \left\{ \begin{array}{l} \text{Velocity} \\ \text{of object} \end{array} \right\}.$$

Thus, we have

$$\begin{aligned} \frac{dE}{dt} = & -4\pi NQV - 4\pi Tv - 4\pi Q\bar{v} - 4\pi Q\bar{v} - 4\pi n_1 e w_1 E - 4\pi n_2 e w_2 E - 4\pi n_e v_e \operatorname{sgn}(E) \\ & - 4\pi n_p q_p v_p. \end{aligned} \quad (18)$$

The differential equation for the number density of free electrons has the following form:

$$\begin{aligned} \frac{dn_e}{dt} = & \left\{ \begin{array}{l} \text{Rate of generation of} \\ \text{electron--pos.ion pairs} \\ \text{per unit volume} \end{array} \right\} - \left\{ \begin{array}{l} \text{Rate of attachment} \\ \text{of free electrons to} \\ \text{neutral air molecules} \\ \text{per unit volume} \end{array} \right\} + \left\{ \begin{array}{l} \text{Rate of detachment of} \\ \text{electrons from neg ions} \\ \text{per unit volume} \end{array} \right\} \\ & + \left\{ \begin{array}{l} \text{Rate of production of} \\ \text{primary electrons per unit volume due to} \\ \beta\text{-decay of the radioactive aerosol} \end{array} \right\} \\ = & G - 2.9 \times 10^7 n_e + 0.51 n_2 + G/1.5 \times 10^4. \end{aligned} \quad (19)$$

Finally, the differential equation for the total charge per unit volume on aerosol particles has the following form:

$$\begin{aligned} \frac{dQ_p}{dt} = & \left\{ \begin{array}{l} \text{Rate of aerosol charge} \\ \text{generation per unit volume} \\ \text{due to } \beta\text{-decay} \end{array} \right\} \\ & + \left\{ \begin{array}{l} \# \text{ of aerosol particles} \\ \text{per unit volume} \end{array} \right\} \left\{ \begin{array}{l} \text{Rate of collection of} \\ \text{pos. ions by one} \\ \text{aerosol particle} \end{array} \right\} \left\{ \begin{array}{l} \text{Charge per} \\ \text{positive ion} \end{array} \right\} \\ & + \left\{ \begin{array}{l} \# \text{ of aerosol particles} \\ \text{per unit volume} \end{array} \right\} \left\{ \begin{array}{l} \text{Rate of collection of} \\ \text{neg. ions by one} \\ \text{aerosol particle} \end{array} \right\} \left\{ \begin{array}{l} \text{Charge per} \\ \text{negative ion} \end{array} \right\} \\ = & eG/1.5 \times 10^4 + en_p R_1(r_p, q_p) - en_p R_2(r_p, q_p). \end{aligned} \quad (20)$$

The size of the hydrometeors grows because of accretion of not-rebounded droplets. Rather than write an additional differential equation for $\frac{dR}{dt}$, we determine R as a function of time by requiring that the total mass of water per unit volume be conserved. This gives the relation

$$R = [C_m + \rho_d C_{tr}^3 - \rho_d (cr^3 + \tilde{c}r^3 + \tilde{\tilde{c}}r^3) / N\rho_h]^{1/3} \quad (21)$$

where

$$C_m = [c(t=0)r^3 + \tilde{c}(t=0)\tilde{r}^3 + \tilde{\tilde{c}}(t=0)\tilde{\tilde{r}}^3]\rho_d + [R(t=0)]^3 N\rho_h \quad (22)$$

The total charge per unit volume is given by $n_1 e - n_2 e + NQ + T + \tilde{Q} + \tilde{\tilde{Q}} - n_e e + Q_p$. Thus, conservation of charge requires that

$$e \frac{dn_1}{dt} - e \frac{dn_2}{dt} + N \frac{dQ}{dt} + \frac{dT}{dt} + \frac{d\tilde{Q}}{dt} + \frac{d\tilde{\tilde{Q}}}{dt} - e \frac{dn_e}{dt} + \frac{dQ_p}{dt} = 0. \quad (23)$$

It is easy to show that the equations written above do indeed satisfy this requirement. The total charge per unit volume is evaluated at intervals during the computation in order to see how well charge conservation is obeyed. The greatest deviation from exact charge conservation has been 1.3×10^{-13} stat-coulomb cm^{-3} . For most runs the deviation has been several orders of magnitude less than this.

The equations actually solved by the Gear package are written in terms of the quantities $z = n_1 + n_2$ and $y = n_1 - n_2$ rather than in terms of n_1 and n_2 themselves. Thus, some of the equations appearing in the computer programs are modified from those in this appendix.

As noted in the main text, the ion collection rates are computed using the formulas of Whipple and Chalmers [1944] or of Klett [1971] depending on whether the magnitude of eEa/kT is greater than or less than 4.0, respectively,

where a is the collector radius. The Klett relation can be expressed in the form

$$R_{1,2}(r,q) = 4\pi n_{1,2} \frac{w_{1,2} kT}{e} \left[\frac{1 - \frac{s\Omega}{2} + \frac{\Omega^2}{6} - \frac{s\Omega^3}{24} + \frac{\Omega^4}{120} + \dots + \frac{\gamma}{2}}{1 + \frac{\Omega^2}{12} + \frac{\Omega^4}{360} + \dots} \right], \quad (24)$$

where $\Omega = eq/rkT$, $\gamma = er|E|/kT$, and $s_{1,2} = \pm 1$.

The Whipple-Chalmers relations cannot be expressed by a single formula. A whole array of relations is required, depending on the particular conditions. These relations are displayed in Table II-1.

TABLE II - I

Whipple - Chalmers type Ion-Collection Ratesfor a collector of radius R and charge Q

Collector Motion	Collector Charge	Collection Rate for Positive Ions, $R_1(R, Q)$	Collection Rate for Negative Ions, $R_2(R, Q)$
<u>POSITIVE (Downward) FIELD:</u>			
falling, $w_1 E > V$	$-3ER^2 < Q < +3ER^2$	$\frac{\pi n_1 w_1 (3ER^2 - Q)^2}{3ER^2}$	$\frac{\pi n_2 w_2 (3ER^2 + Q)^2}{3ER^2}$
falling, $w_1 E < V$	$0 < Q < +3ER^2$	0	$\frac{\pi n_2 w_2 (3ER^2 + Q)^2}{3ER^2}$
falling, $w_1 E < V$	$-3ER^2 < Q < 0$	$-4\pi n_1 w_1 Q$	$\frac{\pi n_2 w_2 (3ER^2 + Q)^2}{3ER^2}$
falling	$Q > +3ER^2$	0	$4\pi n_2 w_2 Q$
falling	$Q < -3ER^2$	$-4\pi n_1 w_1 Q$	0
rising, $w_2 E > V $	$-3ER^2 < Q < +3ER^2$	$\frac{\pi n_1 w_1 (3ER^2 - Q)^2}{3ER^2}$	$\frac{\pi n_2 w_2 (3ER^2 + Q)^2}{3ER^2}$
rising, $w_2 E < V $	$0 < Q < +3ER^2$	$\frac{\pi n_1 w_1 (3ER^2 - Q)^2}{3ER^2}$	$4\pi n_2 w_2 Q$
rising, $w_2 E < V $	$-3ER^2 < Q < 0$	$\frac{\pi n_1 w_1 (3ER^2 - Q)^2}{3ER^2}$	0
rising	$Q > +3ER^2$	0	$4\pi n_2 w_2 Q$
rising	$Q < -3ER^2$	$-4\pi n_1 w_1 Q$	0

TABLE II - 1 Continued

NEGATIVE (Upward) FIELD:

falling, $ w_2 E > V$	$-3 E R^2 < Q < +3 E R^2$	$\frac{\pi n_1 w_1 (3 E R^2 - Q)^2}{3 E R^2}$	$\frac{\pi n_2 w_2 (3 E R^2 + Q)^2}{3 E R^2}$
falling, $ w_2 E < V$	$0 < Q < +3 E R^2$	$\frac{\pi n_1 w_1 (3 E R^2 - Q)^2}{3 E R^2}$	$4\pi n_2 w_2 Q$
falling, $ w_2 E < V$	$-3 E R^2 < Q < 0$	$\frac{\pi n_1 w_1 (3 E R^2 - Q)^2}{3 E R^2}$	0
falling	$Q > +3 E R^2$	0	$4\pi n_2 w_2 Q$
falling	$Q < -3 E R^2$	$-4\pi n_1 w_1 Q$	0
rising, $ w_1 E > V $	$-3 E R^2 < Q < +3 E R^2$	$\frac{\pi n_1 w_1 (3 E R^2 - Q)^2}{3 E R^2}$	$\frac{\pi n_2 w_2 (3 E R^2 + Q)^2}{3 E R^2}$
rising, $ w_1 E < V $	$0 < Q < +3 E R^2$	0	$\frac{\pi n_2 w_2 (3 E R^2 + Q)^2}{3 E R^2}$
rising, $ w_1 E < V $	$-3 E R^2 < Q < 0$	$-4\pi n_1 w_1 Q$	$\frac{\pi n_2 w_2 (3 E R^2 + Q)^2}{3 E R^2}$
rising	$Q > +3 E R^2$	0	$4\pi n_2 w_2 Q$
rising	$Q < -3 E R^2$	$-4\pi n_1 w_1 Q$	0

APPENDIX III

PARAMETER VALUES:

$$C = 5.0 \text{ cm}^{-3} \text{ s}^{-1}$$

$$F = 0.1$$

$$G_b = 10 \text{ cm}^{-3} \text{ s}^{-1}$$

$$K = 1.0$$

$$N = 1.0 \times 10^{-3} \text{ cm}^{-3}$$

$$T = 260^\circ \text{K}$$

$$a = 1.6 \times 10^{-6} \text{ cm}^3 \text{ s}^{-1}$$

$$\langle \cos \theta \rangle = 0.2, 0.088$$

$$e = 4.80286 \times 10^{-10} \text{ statcoulombs}$$

$$f = 1.0$$

$$k = 1.38044 \times 10^{-16} \text{ ergs K}^{-1}$$

$$\ell = 1.03 \times 10^{-5} \text{ cm}$$

$$n_p = 1.0 \times 10^4 \text{ cm}^{-3}$$

$$r = 2.0 \times 10^{-3} \text{ cm}$$

$$r_p = 1.0 \times 10^{-5} \text{ cm}$$

$$w_1 = 552 \text{ cm}^2 \text{ s}^{-1} \text{ statvolt}^{-1} = 1.84 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$$

$$w_2 = 600 \text{ cm}^2 \text{ s}^{-1} \text{ statvolt}^{-1} = 2.00 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$$

$$\Delta = 0, 4.5 \times 10^{-5} \text{ statcoulombs}$$

$$\bar{\alpha} = 0.3, 0.01$$

$$\rho_a = 8.2 \times 10^{-4} \text{ g cm}^{-3}$$

$$\rho_d = 1.0 \text{ g cm}^{-3}$$

$$\rho_h = 6.0 \times 10^{-1} \text{ g cm}^{-3}$$

$$\rho_p = 3.0 \text{ g cm}^{-3}$$

INITIAL VALUES:

$$E(t=0) = \pm(1/300) \text{ statvolt cm}^{-1} = \pm 100 \text{ Vm}^{-1}$$

$$Q(t=0) = 0.0 \text{ statcoulombs}$$

$$\bar{Q}(t=0) = 0.0 \text{ statcoulombs}$$

$$\dot{Q}(t=0) = 0.0 \text{ statcoulombs}$$

$$Q_p(t=0) = 0.0 \text{ statcoulombs}$$

$$R(t=0) = 0.1 \text{ cm}$$

$$c(t=0) = 200 \text{ cm}^{-3}$$

$$\bar{c}(t=0) = 0.0 \text{ cm}^{-3}$$

$$\dot{\bar{c}}(t=0) = 0.0 \text{ cm}^{-3}$$

$$n_e(t=0) = 0.0 \text{ cm}^{-3}$$

$$n_i(t=0) = 2.5 \times 10^3 \text{ cm}^{-3}$$

$$q(t=0) = 0.0 \text{ statcoulombs}$$

C*
 C*
 C* VERSION MARK IVA
 C* REVISION -- JUNE 1977 -- FREE ELECTRONS INCLUDED (WITHOUT COLLECTION BY
 C* OTHER OBJECTS). PARTICLES INCLUDED IN INTERMEDIATE WAY; PARTICLES COLLECT
 C* IONS, BUT PARTICLE COLLECTION BY DROPLETS AND HYDROMETEORS IS NEGLECTED.
 C* THIS IS THE MAIN PROGRAM FOR USING THE GEAR PACKAGE TO SOLVE THE D.E. FOR
 C* THE CASE OF ARBITRARY IONIZATION, WITH DROPLET-HYDROMETEOR COLLISIONS, BUT
 C* FIXED HYDROMETEOR CONCENTRATION. A NONE ZERO CONDENSATION RATE FOR NEW
 C* DROPLETS AND FRACTIONAL MASS LOSS DURING COLLISION ARE POSSIBLE OPTIONS.
 C* VIRGIN, ONCE-COLLIDED, AND TWICE-COLLIDED DROPLETS ARE CONSIDERED; IT IS
 C* ASSUMED THAT THE REBOUND PROBABILITY FOR COLLISIONS OF TWICE-COLLIDED
 C* DROPLETS IS ZERO. THE CONDITION OF WATER MASS CONSERVATION IS USED TO
 C* ELIMINATE ONE D. E. FROM THE SYSTEM.

C*
 C* --DICTIONARY--
 C* A = SPECIFIC ACTIVITY OF PARTICLES
 C* ALAVE = AVERAGE REBOUND PROBABILITY FOR A DROPLET/HYDROMETEOR COLLISION.
 C* ALPHA = ION RECOMBINATION COEFFICIENT.
 C* AN1 = NUMBER DENSITY OF POSITIVE IONS.
 C* AN2 = NUMBER DENSITY OF NEGATIVE IONS.
 C* B = PARTICLE MOBILITY (VELOCITY/FORCE)
 C* BKGG = BACKGROUND (COSMIC RAY, ETC.) ION PAIR GENERATION RATE PER UNIT
 C* VOLUME.
 C* BOLTZ = BOLTZMANN CONSTANT.
 C* CMASS = TOTAL WATER MASS PER UNIT VOLUME INITIALLY PRESENT IN DROPLETS
 C* AND HYDROMETEORS DIVIDED BY (4/3)PI.
 C* COLEF = COLLISION EFFICIENCY FOR DROPLET-HYDROMETEOR COLLISIONS.
 C* CONRT = CONDENSATION RATE PER UNIT VOLUME OF VIRGIN DROPLETS.
 C* COSAV = AVERAGE VALUE OF COSINE OF ANGLE BETWEEN ELECTRIC FIELD
 C* DIRECTION AND HYDROMETEOR RADIUS OUT TO POINT OF CONTACT FOR A
 C* DROPLET/HYDROMETEOR COLLISION.
 C* DELO = AMOUNT OF CHARGE TRANSFERRED PER REBOUNDED DROPLET-HYDROMETEOR
 C* COLLISION VIA NON-INDUCTIVE MECHANISM.
 C* ETA = VISCOSITY OF AIR.
 C* FMASS = FRACTION OF MASS OF DROPLET LEFT ON HYDROMETEOR AFTER COLLISION.
 C* G = RATE OF GENERATION OF ELECTRON--POS-ION PAIRS PER UNIT VOLUME.
 C* DUE TO BETA DECAY OF AEROSOL PARTICLES.
 C* IPIC = PARAMETER THAT TELLS WHETHER PLOTTING IS DESIRED OR NOT. FOR
 C* NEGATIVE VALUES OF IPIC THE PLOTTING STEPS ARE SKIPPED; FOR
 C* POSITIVE VALUES PLOTS ARE PROVIDED.
 C* QE = MAGNITUDE OF CHARGE ON ELECTRON.
 C* QP = PARTICLE CHARGE
 C* QSUM = TOTAL CHARGE ON ALL OBJECTS, MONITORED TO SEE HOW WELL THE
 C* COMPUTATION IS MAINTAINING CHARGE CONSERVATION.
 C* R = HYDROMETEOR RADIUS.
 C* RO = MASS DENSITY OF HYDROMETEORS.
 C* ROA = MASS DENSITY OF AIR.
 C* ROP = PARTICLE MASS DENSITY
 C* ROS = MASS DENSITY OF DROPLETS.
 C* RP = PARTICLE RADIUS
 C* SIG = COLLISION CROSS-SECTION.
 C* SQ = CHARGE PER VIRGIN DROPLET.
 C* SQ1 = CHARGE PER ONCE-COLLIDED DROPLET.
 C* SQ2 = CHARGE PER TWICE-COLLIDED DROPLET.
 C* SR = DROPLET RADIUS.
 C* SR1 = ONCE-COLLIDED DROPLET RADIUS


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C*   SR2 = TWICE-COLLIDED DROPLET RADIUS
C*   SV = TERMINAL VELOCITY OF VIRGIN DROPLETS RELATIVE TO AIR.
C*   SV1 = TERMINAL VELOCITY OF ONCE-COLLIDED DROPLETS RELATIVE TO AIR.
C*   SV2 = TERMINAL VELOCITY OF TWICE-COLLIDED DROPLETS RELATIVE TO AIR.
C*   TEMP = ABSOLUTE TEMPERATURE.
C*   V = TERMINAL VELOCITY OF HYDROMETEOR RELATIVE TO AIR. (DOWN IS POSITIVE)
C*   VE = ELECTRON DRIFT VELOCITY RELATIVE TO AIR.
C*   VP = PARTICLE VELOCITY
C*   WE = MOBILITY OF FREE ELECTRONS.
C*   W1 = MOBILITY OF POSITIVE IONS.
C*   W2 = MOBILITY OF NEGATIVE IONS.
C*   Y(1) = AN1 + AN2
C*   Y(2) = AN1 - AN2
C*   Y(3) = CHARGE AVAILABLE FOR VIRGIN DROPLETS PER UNIT VOLUME
C*   Y(4) = CHARGE PER HYDROMETEOR = Q.
C*   Y(5) = ELECTRIC FIELD STRENGTH = E. (DOWN IS POSITIVE.)
C*   Y(6) = NUMBER DENSITY OF VIRGIN DROPLETS = CO.
C*   Y(7) = NUMBER DENSITY OF FREE ELECTRONS = NE.
C*   Y(8) = NUMBER DENSITY OF ONCE-COLLIDED DROPLETS = C1.
C*   Y(9) = NUMBER DENSITY OF TWICE-COLLIDED DROPLETS = C2.
C*   Y(10) = TOTAL CHARGE AVAILABLE FOR ONCE-COLLIDED DROPLETS PER UNIT VOL.
C*           = Q1TOT.
C*   Y(11) = TOTAL CHARGE AVAILABLE FOR TWICE-COLLIDED DROPLETS PER UNIT VOL.
C*           = Q2TOT.
C*   Y(12) = TOTAL CHARGE PER UNIT VOLUME ON PARTICLES
C*   ZK = CHARGE-TRANSFER EFFICIENCY FOR A DROPLET/HYDROMETEOR COLLISION.
C*   ZL = AIR MOLECULE MEAN FREE PATH
C*   ZN = NUMBER DENSITY OF HYDROMETEORS.
C*   ZNP = NUMBER DENSITY OF PARTICLES

```

C

```

IMPLICIT REAL*8 (A-H,O-Z)
COMMON /GEAR9/ HUSED,NQUSED,NSTEP,NFE,NJE
COMMON /TUPN1/G, ALPHA,ZN,TEMP,W1,W2,SR,SR1,SR2,ALAVE,COSAV,ROS,RO,
1ROA,ETA,ZK,R,CMASS,CENRT,RP,ZNP,B,GFP,COLEF,DELQ,BKGG
DIMENSION YO(12),VN1(100),VN2(100),VSQ(100),VQ(100),VE(100),VC(10
10),VC1(100),VC2(100),VR(100),VSQ1(100),VSQ2(100),VMAX(6),VMIN(6),S
2SCALE(6)
1 FORMAT(1H0,5X,'T = ',D17.10,2X,'N1+N2 = ',D17.10,2X,'N1-N2 = ',D17
1.10,5X,'SQ = ',D17.10)
2 FORMAT(1H,5X,'Q = ',D17.10,6X,'E = ',D17.10,5X,'CO = ',D17.10,5X,
1'C1 = ',D17.10)
3 FORMAT(1H,4X,'C2 = ',D17.10,6X,'R = ',D17.10,2X,'Q1TOT = ',D17.10
1,2X,'Q2TOT = ',D17.10)
4 FORMAT(1H,3X,'SQ1 = ',D17.10,4X,'SQ2 = ',D17.10,4X,' V = ',D17.1
10,3X,'QSUM = ',D17.10)
5 FORMAT(1H,5X,'HUSED = ',D17.10,6X,'NQUSED = ',I6,8X,'NSTEP = ',I6,8X
1,'NFE = ',I6,8X,'NJE = ',I6)
6 FORMAT(1H,5X,'INDEX = ',I3)
7 FORMAT(1H1,25X,'POSITIVE ION DENSITY = X',26X,'NEGATIVE ION DENSIT
1Y = +')
8 FORMAT(1H1,25X,'VIRGIN DROPLET CHARGE = X',5X,'ONCE-COLLIDED DROPL
1ET CHARGE = +',5X,'TWICE-COLLIDED DROPLET CHARGE = 0')
9 FORMAT(1H1,25X,'HYDROMETEOR CHARGE')
11 FORMAT(1H1,5X,'VIRGIN DROPLET CONCENTRATION = X',5X,'ONCE-COLLIDED
1DROPLET CONCENTRATION = +',5X,'TWICE-COLLIDED DROPLET CONCENTRATIO
2N = 0')
12 FORMAT(1H1,25X,'ELECTRIC FIELD INTENSITY')
13 FORMAT(1H1,25X,'HYDROMETEOR TERMINAL VELOCITY RELATIVE TO AIR')

```

```

14  FORMAT(1H,4X,'NE = ',D17.10,5X,'QP = ',D17.10)
200  FORMAT(D9.2)
250  FORMAT(I3)
300  FORMAT(1H1,58X,'MODEL PARAMETERS')
301  FORMAT(1H0,12X,'ALAVE = ',D13.5,26X,'ALPHA = ',D13.5,26X,'CONRT =
1',D13.5)
302  FORMAT(13X,'COSAV = ',D13.5,26X,'FMASS = ',D13.5,30X,'G = ',D13.5)
303  FORMAT(16X,'RO = ',D13.5,28X,'ROA = ',D13.5,28X,'ROS = ',D13.5)
304  FORMAT(16X,'SR = ',D13.5,28X,'SR1 = ',D13.5,28X,'SR2 = ',D13.5)
305  FORMAT(14X,'TEMP = ',D13.5,29X,'W1 = ',D13.5,29X,'W2 = ',D13.5)
306  FORMAT(16X,'ZK = ',D13.5,29X,'ZN = ',D13.5,28X,'ZNP = ',D13.5)
601  FORMAT(17X,'A = ',D13.5,29X,'QP = ',D13.5,28X,'ROP = ',D13.5)
602  FORMAT(16X,'RP = ',D13.5,29X,'ZL = ',D13.5)
603  FORMAT(14X,'DELG = ',D13.5,26X,'COLEF = ',D13.5,27X,'8KGG = ',D13.
15)
400  FORMAT(1H-,55X,'INTEGRATOR PARAMETERS')
401  FORMAT(1H0,17X,'EPS = ',D13.5,29X,'H0 = ',D13.5)
402  FORMAT(1H0,18X,'MF = ',I3,39X,'N = ',I3)
500  FORMAT(1H-,56X,'PLOTING PARAMETERS')
501  FORMAT(1H0,42X,'FOR GRAPH OF POSITIVE AND NEGATIVE ION DENSITIES')
502  FORMAT(11X,'VMAX(1) = ',D13.5,24X,'VMIN(1) = ',D13.5,23X,'SCALE(1)
1 = ',D13.5)
503  FORMAT(1H0,52X,'FOR GRAPH OF DROPLET CHARGE')
504  FORMAT(11X,'VMAX(2) = ',D13.5,24X,'VMIN(2) = ',D13.5,23X,'SCALE(2)
1 = ',D13.5)
505  FORMAT(1H0,50X,'FOR GRAPH OF HYDROMETEOR CHARGE')
506  FORMAT(11X,'VMAX(3) = ',D13.5,24X,'VMIN(3) = ',D13.5,23X,'SCALE(3)
1 = ',D13.5)
507  FORMAT(1H0,49X,'FOR GRAPH OF DROPLET CONCENTRATION')
508  FORMAT(11X,'VMAX(4) = ',D13.5,24X,'VMIN(4) = ',D13.5,23X,'SCALE(4)
1 = ',D13.5)
509  FORMAT(1H0,52X,'FOR GRAPH OF ELECTRIC FIELD')
510  FORMAT(11X,'VMAX(5) = ',D13.5,24X,'VMIN(5) = ',D13.5,23X,'SCALE(5)
1 = ',D13.5)
511  FORMAT(1H0,48X,'FOR GRAPH OF HYDROMETEOR VELOCITY')
512  FORMAT(11X,'VMAX(6) = ',D13.5,24X,'VMIN(6) = ',D13.5,23X,'SCALE(6)
1 = ',D13.5)
513  FORMAT(1H0)
C
      CALL ERRSET(208,256,-1,1)

```

```

C-----
C MODEL PARAMETERS INPUT:
C

```

```

PI=3.14159265358979
QE=4.80286D-10
READ(5,200)A
READ(5,200)ALAVE
READ(5,200)ALPHA
READ(5,200)8KGG
READ(5,200)COLEF
READ(5,200)CONRT
READ(5,200)COSAV
READ(5,200)DELG
READ(5,200)FMASS
READ(5,200)QP
READ(5,200)RC
READ(5,200)ROA
READ(5,200)ROP
READ(5,200)ROS
READ(5,200)RP

```

```

      READ(5,200)SR
      READ(5,200)TEMP
      READ(5,200)W1
      READ(5,200)W2
      READ(5,200)ZK
      READ(5,200)ZL
      READ(5,200)ZN
      READ(5,200)ZNP
      ETA=1.72D-4*(3.93D2/(TEMP+1.2D2))*(TEMP/2.73D2)**1.500
      SR2=DLOG(1.D0-FMASS)/3.D0
      SR2=DEXP(SR2)
      SR1=SR2*SR
      SR2=SR2*SR1
      G=A*(4.D0*PI*RP**3/3.D0)*ROP*ZNP*1.5D4
      B=(1.D0+0.77D0*ZL/RP+0.4D0*(ZL/RP)*DEXP(-1.62D0*RP/ZL))/(6.D0*PI*E
1TA*RP)
      GFP=4.D0*PI*RP**3*(RCP-RCA)*9.8D2/3.D0

```

C

C

C INITIAL CONDITIONS AND INTEGRATOR PARAMETERS INPUT:

C

C***** WARNING *****

C TMAX MUST BE CHOSEN SO THAT THE NUMBER OF DATA POINTS (NP) IS NOT GREATER THAN

C 100 OR ELSE THE STORAGE VECTORS FOR PLOTTING MUST BE REDIMENSIONED.

C

C

```

      READ(5,200)AN1
      READ(5,200)R
      READ(5,200)SQ
      READ(5,200)SQ1
      READ(5,200)SQ2
      READ(5,200)TMAX
      READ(5,200)T0
      READ(5,200)TCUT
      READ(5,200)TSTEP
      READ(5,200)Y0(3)
      READ(5,200)Y0(4)
      READ(5,200)Y0(5)
      READ(5,200)Y0(6)
      READ(5,200)Y0(7)
      READ(5,200)Y0(8)
      READ(5,200)Y0(9)
      READ(5,200)Y0(10)
      READ(5,200)Y0(11)
      AN2=AN1-Y0(7)+QP*ZNP/QE
      Y0(1)=AN1+AN2
      Y0(2)=AN1-AN2
      Y0(12)=QP*ZHP
      CALL VELY(RDA,RD,RDS,R,SR,SQ,Y0(4),Y0(5),ETA,SQ1,SQ2,V,SV,SV1,SV2,
1SR1,SR2)
      CMASS=Y0(6)*RDS*SR**3+ZN*RD*R**3
      READ(5,200)EPS
      READ(5,200)H0
      READ(5,250)MF
      READ(5,250)N
      INDEX=1

```

C

C

C PLOTTING PARAMETERS INPUT. (FOR DEFINITIONS SEE COMMENTS IN SUBROUTINE PLOT):

C

READ(5,250)IPIC
IF(IPIC.LT.0) GO TO 197

C

READ(5,250)IDAT
READ(5,200)HU
READ(5,200)VMAX(1)
READ(5,200)VMIN(1)
READ(5,200)SCALE(1)
READ(5,200)VMAX(2)
READ(5,200)VMIN(2)
READ(5,200)SCALE(2)
READ(5,200)VMAX(3)
READ(5,200)VMIN(3)
READ(5,200)SCALE(3)
READ(5,200)VMAX(4)
READ(5,200)VMIN(4)
READ(5,200)SCALE(4)
READ(5,200)VMAX(5)
READ(5,200)VMIN(5)
READ(5,200)SCALE(5)
READ(5,200)VMAX(6)
READ(5,200)VMIN(6)
READ(5,200)SCALE(6)
HMIN=TOUT
197 CONTINUE

C

C

WRITE(6,300)
WRITE(6,301)ALAVE,ALPHA,CONRT
WRITE(6,302)CESAV,FMASS,G
WRITE(6,303)RO,RCA,RCS
WRITE(6,304)SR,SR1,SR2
WRITE(6,305)TEMP,W1,W2
WRITE(6,306)ZK,ZN,ZNP
WRITE(6,601)A,GF,ROP
WRITE(6,602)RP,ZL
WRITE(6,603)DELQ,COLEF,BKGG
WRITE(6,400)
WRITE(6,401)EPS,HQ
WRITE(6,402)MF,N
IF(IPIC.LT.0) GO TO 333
WRITE(6,500)
WRITE(6,501)
WRITE(6,502)VMAX(1),VMIN(1),SCALE(1)
WRITE(6,503)
WRITE(6,504)VMAX(2),VMIN(2),SCALE(2)
WRITE(6,505)
WRITE(6,506)VMAX(3),VMIN(3),SCALE(3)
WRITE(6,507)
WRITE(6,508)VMAX(4),VMIN(4),SCALE(4)
WRITE(6,509)
WRITE(6,510)VMAX(5),VMIN(5),SCALE(5)
WRITE(6,511)
WRITE(6,512)VMAX(6),VMIN(6),SCALE(6)
333 CONTINUE
WRITE(6,513)
NP=1
KIWI=1
GO TO 100
101 KIWI=2

```

10 TOUT=TOUT+TSTEP
   CALL DRIVE(N,TO,H0,Y0,TCUT,EPS,MF,INDEX)
   AN1=(Y0(1)+Y0(2))/2.00
   AN2=(Y0(1)-Y0(2))/2.00
   IF (Y0(6).LT.1.D-6) GO TC 102
   IF (DABS(Y0(3)).LT.1.D-15) GO TO 102
   SQ=Y0(3)/Y0(6)
   GO TO 103
102 SQ=0.00
103 IF(Y0(8).LT.1.D-6)GO TC 99
   IF(DABS(Y0(10)).LT.1.D-15) GO TO 99
   SQ1=Y0(10)/Y0(8)
   GO TO 98
99 SQ1=0.00
98 IF(Y0(9).LT.1.D-6)GO TC 97
   IF(DABS(Y0(11)).LT.1.D-15)GO TO 97
   SQ2=Y0(11)/Y0(9)
   GO TO 96
97 SQ2=0.00
96 CONTINUE
   CALL VELY(RCA,RC,RCS,R,SR,SQ,Y0(4),Y0(5),ETA,SQ1,SQ2,V,SV,SV1,SV2,
1SR1,SR2)
100 CONTINUE
   WRITE(6,1)TOUT,Y0(1),Y0(2),SQ
   WRITE(6,2)Y0(4),Y0(5),Y0(6),Y0(8)
   WRITE(6,3)Y0(9),R,Y0(10),Y0(11)
   QSUM=QE*Y0(2)+ZN*Y0(4)+Y0(3)+Y0(10)+Y0(11)
   QSUM=QSUM-Y0(7)*QE
   QSUM=QSUM+Y0(12)
   CP=Y0(12)/ZNP
   WRITE(6,4)SQ1,SQ2,V,QSUM
   WRITE(6,14)Y0(7),CP

```

C IF(IPIC.LT.0) GO TO 199

C-----
C THE STEPS FROM HERE THROUGH STEP 199 ARE CONCERNED WITH LOADING THE
C STORAGE VECTORS FOR PLOTTING AND INSURING THAT THE PLOTTED POINTS WILL BE
C ON-SCALE.

C
VN1(NP)=AN1
VN2(NP)=AN2
VSQ(NP)=SQ
VQ(NP)=Y0(4)
VE(NP)=Y0(5)
VC0(NP)=Y0(6)
VC1(NP)=Y0(8)
VC2(NP)=Y0(9)
VR(NP)=V
VSQ1(NP)=SQ1
VSQ2(NP)=SQ2
BOX=DMAX1(AN1,AN2,VMAX(1))
IF(BOX.EQ.VMAX(1))GO TC 81
INK=IDINT((BOX-VMAX(1))/SCALE(1))
AUK=DFLOAT(INK+1)
VMAX(1)=VMAX(1)+AUK*SCALE(1)
81 BOX=DMIN1(AN1,AN2,VMIN(1))
IF(BOX.EQ.VMIN(1))GO TO 82
INK=IDINT((VMIN(1)-BOX)/SCALE(1))
AUK=DFLOAT(INK+1)
VMIN(1)=VMIN(1)-AUK*SCALE(1)

```

82 BOX=DMAX1(SQ,SQ1,SQ2,VMAX(2))
   IF(BOX.EQ.VMAX(2))GO TO 83
   INK=IDINT((BOX-VMAX(2))/SCALE(2))
   AUK=DFLOAT(INK+1)
   VMAX(2)=VMAX(2)+AUK*SCALE(2)
83 BOX=DMIN1(SQ,SQ1,SQ2,VMIN(2))
   IF(BOX.EQ.VMIN(2))GO TO 84
   INK=IDINT((VMIN(2)-BOX)/SCALE(2))
   AUK=DFLOAT(INK+1)
   VMIN(2)=VMIN(2)-AUK*SCALE(2)
84 IF(YO(4).LT.VMAX(3))GO TO 85
   INK=IDINT((YO(4)-VMAX(3))/SCALE(3))
   AUK=DFLOAT(INK+1)
   VMAX(3)=VMAX(3)+AUK*SCALE(3)
85 IF(YO(4).GT.VMIN(3))GO TO 86
   INK=IDINT((VMIN(3)-YO(4))/SCALE(3))
   AUK=DFLOAT(INK+1)
   VMIN(3)=VMIN(3)-AUK*SCALE(3)
86 BOX=DMAX1(YO(6),YO(8),YO(9),VMAX(4))
   IF(BOX.EQ.VMAX(4))GO TO 87
   INK=IDINT((BOX-VMAX(4))/SCALE(4))
   AUK=DFLOAT(INK+1)
   VMAX(4)=VMAX(4)+AUK*SCALE(4)
87 BOX=DMIN1(YO(6),YO(8),YO(9),VMIN(4))
   IF(BOX.EQ.VMIN(4))GO TO 88
   INK=IDINT((VMIN(4)-BOX)/SCALE(4))
   AUK=DFLOAT(INK+1)
   VMIN(4)=VMIN(4)-AUK*SCALE(4)
88 IF(YO(5).LT.VMAX(5))GO TO 89
   INK=IDINT((YO(5)-VMAX(5))/SCALE(5))
   AUK=DFLOAT(INK+1)
   VMAX(5)=VMAX(5)+AUK*SCALE(5)
89 IF(YO(5).GT.VMIN(5))GO TO 78
   INK=IDINT((VMIN(5)-YO(5))/SCALE(5))
   AUK=DFLOAT(INK+1)
   VMIN(5)=VMIN(5)-AUK*SCALE(5)
78 IF (V.LT.VMAX(6)) GO TO 77
   INK=IDINT((V-VMAX(6))/SCALE(6))
   AUK=DFLOAT(INK+1)
   VMAX(6)=VMAX(6)+AUK*SCALE(6)
77 IF (V.GT.VMIN(6)) GO TO 76
   INK=IDINT((VMIN(6)-V)/SCALE(6))
   AUK=DFLOAT(INK+1)
   VMIN(6)=VMIN(6)-AUK*SCALE(6)
199 CONTINUE
76 NP=NP+1

```

C

```

79 IF(KIWI.EQ.1)GO TO 101
   WRITE(6,5)MUSED,NCUSED,NSTEP,NFE,NJE
   IF(INDEX.NE.3)GO TO 20
   IF(TOUT.GE.TMAX)GO TO 30
   GO TO 10
20 WRITE(6,6)INDEX
30 CONTINUE

```

C

```

IF(IPIC.LT.0) GO TO 198

```

C

THE FOLLOWING STEPS CALL THE PLOT SUBROUTINE FOR THE VARIOUS GRAPHS THAT ARE DESIRED.

C

```

      NP=NP-1
      WRITE(6,7)
      CALL PLOT(VN1,VN2,VN2,NP,HMIN,HU,IDAT,VMIN(1),VMAX(1),2)
      WRITE(6,8)
      CALL PLOT(VSQ,VSC1,VSQ2,NP,HMIN,HU,IDAT,VMIN(2),VMAX(2),3)
      WRITE(6,9)
      CALL PLOT(VQ,VQ,VQ,NP,HMIN,HU,IDAT,VMIN(3),VMAX(3),1)
      WRITE(6,11)
      CALL PLOT(VC0,VC1,VC2,NP,HMIN,HU,IDAT,VMIN(4),VMAX(4),3)
      WRITE(6,13)
      CALL PLCT(VR,VR,VR,NP,HMIN,HU,IDAT,VMIN(6),VMAX(6),1)
      WRITE(6,12)
      CALL PLCT(VE,VE,VE,NP,HMIN,HU,IDAT,VMIN(5),VMAX(5),1)
198 CONTINUE
      STOP
      END

```

```

      SUBROUTINE PEDERV(N,T,Y,PC,NO)
      RETURN
      END

```

```

      SUBROUTINE DIFFUN(N,T,Y,YDOT)

```

```

C*****
C*
C* VERSION MARK IVA
C* REVISION -- JUNE 1977 -- FREE ELECTRONS INCLUDED (WITHOUT COLLECTION BY
C* OTHER OBJECTS). PARTICLES INCLUDED IN INTERMEDIATE WAY; PARTICLES COLLECT
C* IONS, BUT PARTICLE COLLECTION BY DROPLETS AND HYDROMETEORS IS NEGLECTED.
C* THE CASE OF ARBITRARY IONIZATION, WITH DROPLET-HYDROMETEOR COLLISIONS, BUT
C* FIXED HYDROMETEOR CONCENTRATION. A NONE ZERO CONDENSATION RATE FOR NEW
C* DROPLETS AND FRACTIONAL MASS LOSS DURING COLLISION ARE POSSIBLE OPTIONS.
C* VIRGIN, ONCE-COLLIDED, AND TWICE-COLLIDED DROPLETS ARE CONSIDERED; IT IS
C* ASSUMED THAT THE REBOUND PROBABILITY FOR COLLISIONS OF TWICE-COLLIDED
C* DROPLETS IS ZERO. THE CONDITION OF WATER MASS CONSERVATION IS USED TO
C* ELIMINATE ONE D. E. FROM THE SYSTEM.
C*
C*****

```

```

      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/TUPNI/G,ALPHA,ZN,TEMP,W1,W2,SR,SR1,SR2,ALAVE,COSAV,ROS,RO,
      IRCA,ETA,ZK,R,CMASS,CENRT,RP,ZNP,B,GFP,CLEF,DELG,BKGG
      DIMENSION Y(N),YDOT(N)
      BOLTZ=1.38044D-16
      QE=4.8G286D-10
      AKT=BOLTZ*TEMP
      PI=3.14159265358979
      R=(CMASS+ROS*CENRT*T*SR**3-ROS*(Y(6)*SR**3+Y(8)*SR1**3+Y(9)*SR2**3
      1))/(ZN*RC)
      R=DLOG(R)/3.00
      R=DEXP(R)
      AN1=(Y(1)+Y(2))/2.00
      AN2=(Y(1)-Y(2))/2.00
      IF (Y(6).LT.1.D-6) GO TO 100
      IF (QABS(Y(3)).LT.1.D-15) GO TO 100
      SQ=Y(3)/Y(6)

```

```

100 GO TO 101
101 SQ = 0.00
    IF(Y(8).LT.1.0-6)GO TO 99
    IF(DABS(Y(10)).LT.1.0-15)GO TC 99
    SQ1=Y(10)/Y(8)
    GO TO 98
99 SQ1=0.00
98 IF(Y(9).LT.1.0-6)GO TO 97
    IF(DABS(Y(11)).LT.1.0-15)GO TC 97
    SQ2=Y(11)/Y(9)
    GO TO 96
97 SQ2=0.00
96 CONTINUE
    CALL VELY(ROA,RO,RQS,R,SR,SQ,Y(4),Y(5),ETA,SQ1,SQ2,V,SV,SV1,SV2,SR
11,SR2)
    Z=QE*Y(4)/(R*AKT)
    EF=DABS(Y(5))
    SIG=PI*(R+SR)**2
    SIG = SIG*COLEF
    SIG1 = PI*(R+SR1)**2
    SIG1 = SIG1*COLEF
    SIG2 = PI*(R+SR2)**2
    SIG2 = SIG2*COLEF
    BGAM=QE*R*EF/AKT
    IF(BGAM.LT.4.00)GO TC 20
    CALL WC(Y(4),Y(5),BGAM,Z,V,AN1,AN2,W1,W2,R,AKT,RS,RD)
    GO TO 30
20 CALL CKLETT(R,Y(4),AKT,Z,BGAM,W1,W2,Y(1),Y(2),RS,RD)
30 CONTINUE
    ZO=QE*SQ/(SR*AKT)
    SGAM=QE*SR*EF/AKT
    IF(SGAM.LT.4.00)GO TO 40
    CALL WC(SQ,Y(5),SGAM,ZO,SV,AN1,AN2,W1,W2,SR,AKT,RSO,RDO)
    GO TO 50
40 CONTINUE
    CALL CKLETT(SR,SQ,AKT,ZO,SGAM,W1,W2,Y(1),Y(2),RSO,RDO)
50 CONTINUE
60 Z1=QE*SQ1/(SR1*AKT)
    Z2=QE*SQ2/(SR2*AKT)
    SGAM1 =QE*SR1*EF/AKT
    SGAM2 =QE*SR2*EF/AKT
    IF (SGAM1.LT.4.00) GO TO 65
    CALL WC(SQ1,Y(5),SGAM1,Z1,SV1,AN1,AN2,W1,W2,SR1,AKT,RS1,RD1)
    GO TO 70
65 CALL CKLETT(SR1,SQ1,AKT,Z1,SGAM1,W1,W2,Y(1),Y(2),RS1,RD1)
70 IF (SGAM2.LT.4.00) GO TO 75
    CALL WC(SQ2,Y(5),SGAM2,Z2,SV2,AN1,AN2,W1,W2,SR2,AKT,RS2,RD2)
    GO TO 80
75 CALL CKLETT(SR2,SQ2,AKT,Z2,SGAM2,W1,W2,Y(1),Y(2),RS2,RD2)
80 CONTINUE
    QP=Y(12)/ZNP
    VP=B*(QP*Y(5)+GFP)
    ZP=QE*QP/(RP*AKT)
    PGAM=QE*RP*EF/AKT
    IF(PGAM.LT.4.00)GO TC 21
    CALL WC(QP,Y(5),PGAM,ZP,VP,AN1,AN2,W1,W2,RP,AKT,RSP,RDP)
    GO TO 22
21 CALL CKLETT(RP,QP,AKT,ZP,PGAM,W1,W2,Y(1),Y(2),RSP,RDP)
22 CONTINUE
    VE=1.05*DSQRT(7.09401*EF+4.17300*EF*EF)

```



```

* AV=DABS(V-SV)
AV1=DABS(V-SV1)
AV2=DABS(V-SV2)
YDOT(6)=-SIG*ZN*Y(6)*AV
YDOT(6)=YDOT(6)+CONRT
YDOT(8)=ALAVE*SIG*ZN*Y(6)*AV-SIG1*ZN*Y(8)*AV1
YDOT(9)=ALAVE*SIG1*ZN*Y(8)*AV1-SIG2*ZN*Y(9)*AV2
YDOT(3)=QE*RD0*Y(6)-SIG*ZN*AV*Y(3)
YDOT(5)=ZN*Y(4)*V+Y(3)*SV+Y(10)*SV1+Y(11)*SV2+((W1-W2)*Y(2)+(W1+W2
1)*Y(1))*QE*Y(5)/2.00
YDOT(5)=-4.00*PI*YDOT(5)
YDOT(5)=YDOT(5)-4.00*PI*Y(7)*QE*VE*DSIGN(1.00,Y(5))
YDOT(5)=YDOT(5)-4.00*PI*ZNP*QP*VP
GM1=PI*PI/2.00
GM2=PI*PI/6.00
YDOT(10)=SIG*ZN*Y(6)*AV*ALAVE*(ZK*GM1*SR1*SR1*Y(5)*COSAV+ZK*Y(4)/
1(1.00+R*R/(GM2*SR1*SR1))-ZK*SQ/(1.00+GM2*SR1*SR1/R**2)+SQ)-SIG1*ZN
2*Y(10)*AV1+QE*Y(8)*RD1
YDOT(10)=YDOT(10)+SIG*Y(6)*AV*ALAVE*ZN*DELQ
YDOT(4)=QE*RD+SIG*Y(6)*AV*((1.00-ALAVE)*SQ+ALAVE*ZK*(-GM1*SR1*SR1*
1Y(5)*CCSAV+SQ/(1.00+GM2*SR1*SR1/R**2)-Y(4)/(1.00+R*R/(GM2*SR1*SR1)
2)))+SIG1*AV1*((1.00-ALAVE)*Y(10)+ALAVE*ZK*(-GM1*SR2*SR2*Y(5)*COSAV
3*Y(8)+Y(10)/(1.00+GM2*SR2*SR2/R**2)-Y(8)*Y(4)/(1.00+R*R/(GM2*SR2*S
4R2)))+SIG2*Y(11)*AV2
YDOT(4)=YDOT(4)-SIG*Y(6)*AV*ALAVE*DELQ-SIG1*Y(8)*AV1*ALAVE*DELQ
YDOT(11)=2.00*G-ALPHA*(Y(1)**2-Y(2)**2)/2.00-Y(6)*RS0-Y(8)*RS1-Y(9)
1*RS2-ZN*RS
YDOT(1)=YDOT(1)-G+2.907*Y(7)+5.10-1*AN2
YDOT(1)=YDOT(1)-ZNP*RSF
YDOT(1)=YDOT(1)+2.00*8KGG
YDOT(2)=-Y(6)*RD0-Y(8)*RD1-Y(9)*RD2-ZN*RD
YDOT(2)=YDOT(2)+G-2.907*Y(7)+5.10-1*AN2
YDOT(2)=YDOT(2)-ZNP*ROP
YDOT(11)=SIG1*ZN*AV1*ALAVE*(Y(8)*ZK*GM1*SR2*SR2*Y(5)*COSAV+Y(8)*ZK
1*Y(4)/(1.00+R*R/(GM2*SR2*SR2))-ZK*Y(10)/(1.00+GM2*SR2*SR2/R**2)+Y(
210))-SIG2*ZN*AV2*Y(11)+Y(9)*QE*RD2
YDOT(11)=YDOT(11)+SIG1*Y(8)*AV1*ALAVE*ZN*CELO
YDOT(7)=G-2.907*Y(7)+5.10-1*AN2
YDOT(7)=YDOT(7)+G/1.504
YDOT(12)=QE*G/1.504+QE*ZNP*ROP
RETURN

```

```

SUBROUTINE VELY(ROA,RO,RGS,R,SR,SQ,BQ,E,ETA,SQ1,SQ2,V,SV,SV1,SV2,S
1R1,SR2)

```

C SUBROUTINE COMPUTES TERMINAL VELOCITIES OF DROPLETS AND OF HYDROMETEORS
C RELATIVE TO AIR USING FORMULAS OF ZIV AND LEVIN AFTER DAVIES. THIS VERSION
C GOES WITH THE PROGRAM 6HERE COLLISIONS ARE CONSIDERED AND THEREFORE COMPUTES
C VELOCITIES FOR VIRGIN, ONCE-COLLIDED, AND TWICE-COLLIDED DROPLETS.
C -----

```

C IMPLICIT REAL*8 (A-H,O-Z)
G=9.802
PI=3.14159265358979
P=ROS
RR=SR
Q=SQ
KK=1

```

```

1 CONTINUE
  AX=(3.2D1/3.D0)*ROA*(P-RCA)*G*RR**3/ETA**2+8.D0*ROA*Q*E/(PI*ETA**2
1)
  S=DSIGN(1.D0,AX)
  X=DABS(AX)
  IF(X.LE.1.2D2)GO TO 10
  Y=DLQGI0(X)
  ALOGRE=-1.29536D0+0.986D0*Y-4.6677D-2*Y*Y+1.1235D-3*Y**3
  RE=DEXP(ALOGRE*2.302585093D0)
  GO TO 20
10 RE=X/2.4C1-2.3363D-4*X*X+2.0154D-6*X**3-6.9105D-9*X**4
20 CV=RE*ETA*S/(2.D0*ROA*RR)
  GO TO(51,52,53,54),KK
51 SV=CV
  KK=2
  P=RO
  RR=R
  Q=BQ
  GO TO 1
52 V=CV
  KK=3
  P=RQS
  RR=SR1
  Q=SQ1
  GO TO 1
53 SV1=CV
  KK=4
  P=RQS
  RR=SR2
  Q=SQ2
  GO TO 1
54 SV2=CV
  RETURN
  END

```

```

C
C
C
C
SUBROUTINE WC(Q,E,GAM,Z,V,AN1,AN2,W1,W2,R,AKT,RWCS,RWCD)
C

```

```

C-----
C SUBROUTINE COMPUTES RATES OF ION COLLECTION FOR POS AND FOR NEG IONS
C USING THE FORMULAS OF WHIPPLE AND CHALMERS, CONSIDERING ALL POSSIBLE
C SIGN COMBINATIONS FOR THE ELECTRIC FIELD AND THE PARTICLE VELOCITY
C RELATIVE TO THE AIR.
C-----
C
C
C

```

```

  IMPLICIT REAL*8 (A-H,O-Z)
  PI=3.14159265358979
  QE=4.80286D-10
  D1N=W1*AKT*AN1/QE
  D2N=W2*AKT*AN2/QE
  X=3.D0*DABS(E)*R*R
  IF(Q.GT.X) GO TO 100
  IF(Q.LT.-X) GO TO 200
  IF(E.GT.0.D0) GO TO 300
  GO TO 400
100 R1P=0.D0
  R2P=PI*D2N*R*2*4.D0
  GO TO 10

```

```

200 RIP=-4.00*PI*D1N*R*Z
   R2P=0.00
   GO TO 10
300 IF(V.GE.0.00) GO TO 500
   IF(W2*E.GT.DABS(V)) GO TO 600
   1 IF(Q.GT.0.00) GO TO 700
   RIP=PI*R*D1N*(3.00*GAM-2.00*Z+(1.00/3.00)*Z**2/GAM)
   R2P=0.00
   GO TO 10
700 RIP=PI*R*D1N*(3.00*GAM-2.00*Z+(1.00/3.00)*Z**2/GAM)
   R2P=PI*D2N*R*Z*4.00
   GO TO 10
600 RIP=PI*R*D1N*(3.00*GAM-2.00*Z+(1.00/3.00)*Z**2/GAM)
   R2P=PI*R*D2N*(3.00*GAM+2.00*Z+(1.00/3.00)*Z**2/GAM)
   GO TO 10
500 IF(W1*E.GT.V) GO TO 600
   2 IF(Q.GE.0.00) GO TO 800
   RIP=-4.00*PI*D1N*R*Z
   R2P=PI*R*D2N*(3.00*GAM+2.00*Z+(1.00/3.00)*Z**2/GAM)
   GO TO 10
800 RIP=0.00
   R2P=PI*R*D2N*(3.00*GAM+2.00*Z+(1.00/3.00)*Z**2/GAM)
   GO TO 10
400 IF(V.GE.0.00) GO TO 500
   IF(W1*CABS(E).GT.DABS(V)) GO TO 600
   GO TO 2
900 IF(W2*CABS(E).GT.V) GO TO 600
   GO TO 1
10 RWCS=RIP+R2P
   RWCD=RIP-R2P
   RETURN
END

```

SUBROUTINE CKLETT(R,G,AKT,Z,GAM,W1,W2,ZZ,Y,RKS,RKD)

SUBROUTINE COMPUTES SUM AND DIFFERENCE OF ICM COLLECTION RATES USING THE
 CKLETT FORMULA. SEE NOTES FOR 23 DECEMBER 1976.

```

IMPLICIT REAL*8 (A-H,O-Z)
PI=3.14159265358979
QE=4.602860-10
B=PI*R*AKT/QE
ANUM=1.00+Z*Z/6.00+Z**4/1.202+Z**6/5.0403+GAM/2.00
DENOM=1.00+Z*Z/1.201+Z**4/3.602+Z**6/2.01604
A=(ANUM/DENOM)*B*2.00
B=B*Z
F=(W1-W2)*ZZ+(W1+W2)*Y
FF=(W1+W2)*ZZ+(W1-W2)*Y
RKD=F*A-FF*B
RKS=FF*A-F*B
RETURN
END

```

SUBROUTINE PLOT(Y1,Y2,Y3,NP,HMIN,HU,IDAT,VMIN,VMAX,NN)

```

C
C   SUBROUTINE GRAPHS THE FUNCTION Y1 IF NN=1, BOTH Y1 AND Y2 IF NN=2, AND
C   Y1,Y2, AND Y3 IF NN=3.
C
C   NP = NUMBER OF POINTS WHERE FUNCTION VALUES ARE GIVEN.
C   HMIN = MINIMUM VALUE OF THE INDEPENDENT VARIABLE (E.G. TIME)
C   HU = INCREMENT OF INDEPENDENT VARIABLE BETWEEN TIC MARKS. (THE
C       PROGRAM PUTS TIC MARKS EVERY TENTH STEP.)
C   IDAT = NUMBER OF INDEPENDENT VARIABLE STEPS PER DATA POINT.
C   VMIN = MINIMUM DEPENDENT VARIABLE VALUE OFR THE PLOT. (THIS DOES NOT
C       NECESSARILY CORRESPOND TO THE MINIMUM VALUE ACTUALLY ATTAINED
C       BY ANY OF THE FUNCTIONS.)
C   VMAX = MAXIMUM DEPENDENT VARIABLE VALUE FOR THE PLOT. (DITTO)
C
C   IT IS ASSUMED THAT THE CALLING PROGRAM HAS THE FUNCTION VALUES IN SEQUENCE
C   IN THE VECTORS Y1, Y2, Y3 AND HAS SPECIFIED ALL THE OTHER PARAMETERS LISTED.
C   THE PLOTTED VALUE IS  $100*(Y(X)-VMIN)/(VMAX-VMIN)$ , AND THUS REPRESENTS THE PER
C   CENT OF FULL SCALE BY WHICH THE FUNCTION VALUE RISES ABOVE THE SPECIFIED
C   MINIMUM.

```

C*****

```

C
C   IMPLICIT REAL*8(A-H,C-Z)
C   DIMENSION Y1(100),Y2(100),Y3(100),IGRAPH(101)
C   DATA NPT,NSP,NAR,NX,NY,NPT2,NPT3/'X',' ','|','|','-','+','0'/
3  FORMAT(1H0,15X,D10.3,33X,'(SCALE UNIT =',C10.3,')',33X,D10.3)
4  FORMAT(1H ,25X,101A1)
5  FORMAT(1H+,25X,101A1)
6  FORMAT(1H+,17X,F6.2)
7  FORMAT(1H )
C   UNIT=(VMAX-VMIN)/10.00
C   WRITE(6,3)VMIN,UNIT,VMAX
C   DO 601 I=1,101
C     IGRAPH(I)=NY
601 CONTINUE
C   DO 602 I=1,11
C     IY=10*I-9
C     IGRAPH(IY)=NAR
602 CONTINUE
C   WRITE(6,4)(IGRAPH(K),K=1,101)
C   WRITE(6,7)
C   JJ=0
C   INCK=1
C   JOY=1
C   NEND=NP*IDAT
C   DO 606 J=1,NEND
C     DO 603 K=1,101
C       IGRAPH(K)=NSP
603 CONTINUE
C     NK=JJ*10+1
C     IF(J.NE.NK)GO TO 604
C     AJ=DFLCAT(JJ)
C     ALABEL=AJ*HU+HMIN
C     IGRAPH(1)=NY
C     IGRAPH(101)=NY
C     JJ=JJ+1
C     GO TO 605
604 IGRAPH(1)=NX
C     IGRAPH(101)=NX
605 CONTINUE

```

```

IF(J.EQ.INCK)GO TO 201
WRITE(6,4)(IGRAPH(K),K=1,101)
GO TO 399
201 INCK=INCK+10AT
IY=100.0*(Y1(JCY)-VMIN)/(VMAX-VMIN)+1.5
IF(IY.LT.1) IY=1
IF(IY.GT.101) IY=101
IGRAPH(IY)=NPT
WRITE(6,4)(IGRAPH(K),K=1,101)
IF(NN.EQ.1)GO TO 398
IGRAPH(IY)=NSP
IGRAPH(1)=NSP
IGRAPH(101)=NSP
IY=100.0*(Y2(JCY)-VMIN)/(VMAX-VMIN)+1.5
IF(IY.LT.1) IY=1
IF(IY.GT.101) IY=101
IGRAPH(IY)=NPT2
WRITE(6,5)(IGRAPH(K),K=1,101)
IF(NN.EQ.2)GO TO 398
IGRAPH(IY)=NSP
IY=100.0*(Y3(JCY)-VMIN)/(VMAX-VMIN)+1.5
IF(IY.LT.1) IY=1
IF(IY.GT.101) IY=101
IGRAPH(IY)=NPT3
WRITE(6,5)(IGRAPH(K),K=1,101)
398 JCY=JCY+1
399 IF(J.EQ.NK) WRITE(6,6)ALABEL
606 CONTINUE
DO 608 JJ=1,101
IGRAPH(JJ)=NY
608 CONTINUE
DO 609 JJ=1,11
IY=10*JJ-9
IGRAPH(IY)=NAR
609 CONTINUE
WRITE(6,4)(IGRAPH(K),K=1,101)
RETURN
END

```

NOTICE

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Fig. 1. A hydrometeor of radius R at the instant of collision with a droplet of radius r . The angle θ is the angle between the vertically downward electric-field direction and the line joining the centers of the spheres through the point of contact.

Fig. 2 Development of the electric-field strength with time when only the inductive charge-separation mechanism is operative. All curves are for $\bar{\alpha} = 0.3$ and $\langle \cos \theta \rangle = 0.2$.

The ion-generation rates and hydrometeor number densities for the various curves are given in the following table; values of all other parameters are given in Appendix III.

Curve	Ion-Generation Rate $G(\text{cm}^{-3} \text{s}^{-1})$	Number Density of Hydrometeors $N(\text{cm}^{-3})$
3	0	10^{-3}
e	9.99×10^2	10^{-3}
d	9.99×10^3	10^{-3}
c	9.99×10^3	10^{-4}
1	3.17×10^4	10^{-3}
b	5.64×10^4	10^{-3}
a	9.99×10^4	10^{-4}
2	9.99×10^4	10^{-3}

FIG. 2

Fig. 3 Development of the electric-field strength with time when only the non-inductive charge-separation mechanism is operative. All curves are for $\bar{\alpha} = 0.3$ and $\langle \cos \theta \rangle = 0.2$.

The ion-generation rates and hydrometeor number densities for the various curves are given in the following table; values of all other parameters are given in Appendix III.

Curve	Ion-Generation Rate $G(\text{cm}^{-3}\text{s}^{-1})$	Number Density of Hydrometeors $N(\text{cm}^{-3})$
7	0	10^{-3}
k	9.99×10^4	10^{-3}
l	3.17×10^5	10^{-3}
m	5.64×10^5	10^{-3}
9	9.99×10^5	10^{-3}
n	9.99×10^5	10^{-4}
s	1.78×10^6	10^{-4}
u	2.38×10^6	10^{-4}
r	3.17×10^6	10^{-4}
o	9.99×10^6	10^{-4}
8	9.99×10^6	10^{-3}
p	9.99×10^8	10^{-3}

FIG. 3

Fig. 4 Development of the electric-field strength with time when the inductive and non-inductive charge-separation mechanisms operate simultaneously. All curves are for $\bar{\alpha} = 0.3$ and $\langle \cos \theta \rangle = 0.2$. The ion-generation rates and hydrometeor number densities for the various curves are given in the following table; values of all other parameters are given in Appendix III.

Curve	Ion-Generation Rate $G(\text{cm}^{-3} \text{s}^{-1})$	Number Density of Hydrometeors $N(\text{cm}^{-3})$
4	0	10^{-3}
j	9.99×10^2	10^{-3}
6	9.99×10^4	10^{-3}
i	9.99×10^4	10^{-4}
h	9.99×10^5	10^{-4}
5	9.99×10^5	10^{-3}
w	1.07×10^6	10^{-4}
v	1.15×10^6	10^{-4}
t	1.33×10^6	10^{-4}
q	1.78×10^6	10^{-4}
g	3.17×10^6	10^{-4}
f	9.99×10^8	10^{-3}

FIG. 4

Figs. 5-13 These figures illustrate the time development of the quantities listed below for each of the three charge-separation classes-- inductive, non-inductive, and simultaneous inductive and non-inductive-- for the cases $G=0$, G near but less than the maximum value consistent with field growth, and G near but greater than the minimum value that prevents field growth. The quantities shown are (1) electric-field strength, (2) hydrometeor velocity, (3) droplet number density, (4) droplet charge, (5) hydrometeor charge, and (6) the sum and the difference of the positive and negative ion number densities. All curves are for $\bar{\alpha} = 0.3$, $\langle \cos\theta \rangle = 0.2$, and $N = 10^{-3} \text{ cm}^{-3}$. Ion-generation rates are given in the following table; values of all other parameters are given in Appendix III. In the droplet number density and droplet charge graphs, solid lines are for virgin droplets, lines of short dashes are for once-collided droplets, and lines of long dashes are for twice-collided droplets.

	Ion-Generation Rate $G(\text{cm}^{-3} \text{ s}^{-1})$	Charge-Separation Mechanism
Fig. 5	0	inductive
Fig. 6	3.14×10^4	inductive
Fig. 7	9.99×10^4	inductive
Fig. 8	0	non-inductive
Fig. 9	9.99×10^5	non-inductive
Fig. 10	9.99×10^6	non-inductive
Fig. 11	0	both
Fig. 12	9.99×10^4	both
Fig. 13	9.99×10^5	both

FIG. 5

FIG. 6

FIG. 7

FIG. 8

FIG. 9

FIG. 10

FIG. 11

FIG. 12

FIG. 13

Fig. 14 Development of the electric-field strength with time when $\bar{\alpha} = 0.01$ and $\langle \cos \theta \rangle = 0.088$. All curves are for $N = 10^{-3} \text{ cm}^{-3}$. Ion-generation rates and charge-separation mechanisms are given in the following table; values of all other parameters are given in Appendix III.

Curve	Ion-Generation Rate $G(\text{cm}^{-3} \text{ s}^{-1})$	Charge-Separation Mechanism
B	0	both
F	9.99×10^3	both
D	9.99×10^4	both
C	9.99×10^5	both
A	0	inductive
J	9.99	inductive
I	99.9	inductive
G	9.99×10^2	inductive
E	9.99×10^3	inductive
L	3.17×10^4	inductive
H	9.99×10^4	inductive

FIG. 14

Fig. 15-21 These figures show the time development of the hydrometeor charge, the droplet charges, and the sum and the difference of the positive and negative ion number densities for the inductive-mechanism cases of Fig. 14. Each figure is labeled with the designation of the curve in Fig. 14 to which it applies. Values of the model parameters are given in the caption to Fig. 14 and in Appendix III. In the droplet number density and droplet charge graphs, solid lines are for virgin droplets, lines of short dashes are for once-collided droplets, and lines of long dashes are for twice-collided droplets.

FIG. 16
Case J

FIG. 15
Case A

Case 17

Case 18

FIG. 17

FIG. 18

FIG. 20

FIG. 19

FIG. 21