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FILTRATION AND DEWATERING: REVIEW OF LITERATURE

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FILTRATION AND DEWATERING: Review of Literature

by

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ABSTRACT

The unit operations of filtration and dewatering are reviewed. The models employed in studying both these operations are discussed. Cake filtration is discussed in detail. Effect of various parameters on the specific cake resistance and filter medium resistance are also discussed. Methods of improving filtration rate and enhancing dewatering are studied. The filtration and dewatering characteristics of fine coal are also reviewed.

INTRODUCTION

Filtration and dewatering are important aspects of any chemical process involved in producing solid product from a slurry. However, these operations are not studied in as much detail as other industrial operations like distillation, absorption, etc. In the past, the design of filtration equipment was considered more of an art than science. Only in the last few years have attempts been made to understand the fundamental mechanism underlying these operations.

In this review, the present status of filtration and dewatering is discussed, particularly concerning the models used for predicting the rate of filtration and dewatering. Effects of various parameters, size of particles in the slurry, their shape and method of cake formation, etc., on the characteristics of these operations are analyzed. A separate chapter on the use of these processes in coal cleaning is included. Dewatering of fine coal and refuse has attracted a great deal of attention in recent years. Special problems associated with dewatering of fine coal are discussed. Enhanced methods of dewatering are reviewed and finally areas requiring further research are highlighted.

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FILTRATION

Filtration is an operation in which undissolved solid particles are separated from a suspension in liquid, called "slurry," by contacting the slurry with one side of a porous medium. The porous medium is called filter medium. In the presence of some driving force, the filter medium allows the liquid to flow through the capillary channels and retains the solids on its surface. Depending upon the mechanism for arrest and accumulation of particles, the operation of filtration can be divided into two classes (1)¹: Deep Bed Filtration and Cake Filtration.

Deep bed filtration is known by various terms like blocking filtration, surface filtration, clarification and ultrafiltration (when the particles to be removed are very small). Deep bed filtration is normally preferred when the solid content of the slurry is very low (of the order of 0.1 percent). In this operation, a deep bed of porous media like diatomite or fibrous material, supported on a coarse filter, is used to remove the fine solids from the slurry. The particles to be removed will often be considerably smaller than the pores of the filter medium and will penetrate a considerable depth before being captured. The particles can be captured by any of the following mechanisms (2): (1) the direct-sieving action at constrictions in the pore structure, (2) gravity settling, (3) Brownian diffusion, (4) interception at the solid-liquid interfaces, (5) impingement, and (6) electrokinetic forces.

In recent years, the subject of deep bed filtration has developed a rich and varied literature. This has helped to increase the understanding of the mechanism of this operation (3) and attempts to simulate the dynamic behavior of deep bed filters (which is a very complex subject) are also reported in the literature (4). Recently Tien, et al. (5) have reviewed the subject in great depth. Several references can also be found in their article.

Most of the liquid filtration operations follow the mechanism of cake filtration. In this method of operation, the filtered solids are stopped at the surface of the filter medium and then piled upon one another to form a cake of increasing thickness. This cake of the solids forms the "true" filtering medium. In what follows, cake filtration will be examined in some detail. Some general references (1, 6-13) which discuss cake filtration are listed at the end. Also reviews of advances in filtration appear quite regularly in the literature (5, 14-20).

In the filtration operation a cake containing filtrate trapped in the void space between the particles is obtained at the end of the operation. In many cases where the recovery of solids is desirable it is necessary that the moisture content (in most of the cases the liquid under consideration is water, so it is normal practice to use moisture content instead of

¹Underlined numbers in parentheses refer to items in the list of references at the end of this report.

liquid content) of the solids be as low as possible. In order to reduce the moisture content of the cake, the cake is subjected to dewatering. Wakeman (21) classifies dewatering as one of the filtration post-treatment processes. He defines cake dewatering as a process whereby filtrate trapped within the voidage of a filter cake is displaced by the application of desaturating forces to the cake. Desaturating forces can be either mechanical or hydrodynamic.

Filtration and dewatering are distinct (21) in the sense that filtration leads to the formation of a cake containing a relatively low proportion of residual filtrate, while dewatering is used to effect further moisture content reduction of the cake itself. The fundamental principles underlying the two processes are entirely different.

Importance of the dewatering process can be realized in the applications where the final product is the solid particles. Many applications of these solid particles require them to be transported over long distances. It is necessary that they be as dry as possible. In many cases thermal drying is employed to dry the solids. Since thermal drying involves evaporation of water, the energy required for this process is quite high. Even a few percent reduction in moisture content of solids by dewatering can be highly favorable economically.

Mechanism and Theory of Cake Filtration

In cake filtration the slurry is introduced at the upstream side of the filter medium. In most cases the upstream side is at higher pressure (above atmospheric or atmospheric) while the downstream side is at lower pressure (atmospheric or sub-atmospheric). Particles larger than the pores of the filter medium are retained at the surface of the medium, whereas smaller particles enter the pores of the medium. This smaller particle may (1) block the pore opening of the medium completely, or (2) adhere to the walls of the pores thus progressively reducing the internal diameter of the pore, or (3) pass through the filter medium. As the filtration proceeds the particles retained on the filter medium will form a porous structure and the smaller particles which were able to pass through the pores initially will get trapped in this porous cake formed by the deposited particles. This is observed in practice many times as the initial filtrate obtained with a new or washed filter medium is often found to be cloudy, but becomes clearer as time progresses. Hermans and Bredee (22) termed these mechanisms as (1) complete blocking, (2) standard blocking, (3) cake filtration, and (4) intermediate blocking. When the blocking mechanism was neither complete blocking or standard blocking, it was termed intermediate blocking.

When filtering suspensions containing more than 1 percent by volume solids, plugging of the filter medium takes place in the first few seconds during which period a continuous cake covers the surface of the filter medium. This cake immediately becomes the active filter surface and hence plays a very important part in the entire filtration operation.

The structure of the deposited cake resembles the structure of a packed bed so it was but natural to apply the equations of flow through packed beds to cake filtration. When the Kozeny-Carman equation for packed beds is applied to cake filtration one obtains (10):

$$-\frac{dp}{dL} = \frac{k_1 \mu u (1-\epsilon)^2 (s_p/V_p)^2}{\epsilon^3} \quad (1)$$

where

$\frac{dp}{dL}$: pressure gradient across the cake at thickness L

μ : viscosity of the filtrate

u : linear velocity of the filtrate, based on filter area

s_p : surface area of a single particle

V_p : volume of a single particle

ϵ : porosity of cake

k_1 : constant

Porosity ϵ is defined as the ratio of the volume of voids in the cake to the total volume (voids plus particles) of the bed. If V is the volume of filtrate collected from the start of filtration to time t then the linear velocity u is defined as

$$u = \frac{1}{A} \frac{dV}{dt} \quad (2)$$

where A is the surface area of the cake.

The thickness of the cake can be related to the mass of solids deposited by the following equation

$$dm = \rho_p (1-\epsilon) A dL \quad (3)$$

where dm : mass of solids in layer of thickness, dL

ρ_p : density of the solid particles

Eliminating dL from Equations (1) and (3):

$$-dp = \frac{k_1 \mu u (s_p/V_p)^2 (1-\epsilon) dm}{\rho_p A \epsilon^3} \quad (4)$$

In development of Equation (4), it is assumed that the cake is uniform and that the porosity is constant throughout. In deposition of a filter cake this would not apply as ϵ will depend on: (1) the size of particles and their distribution in the slurry, (2) rate of deposition of particles, (3) nature of support, its geometry, and surface structure, and (4) the forces acting on the particles. At this stage the filter cake can be divided into two categories--incompressible and compressible. Incompressible cakes can be defined (23) as those in which the shape or the physical strength of the solids is such that the packing arrangement in the bed can sustain the drag forces imposed on them due to the hydraulic pressure gradient, without any movement. When the solid particles move or change their shape due to the imposed hydraulic pressure gradient, then the cake is compressible. For an incompressible cake, assuming ϵ to be

constant, the quantity $\frac{k_1 (s_p/V_p)^2 (1-\epsilon)}{\epsilon^3 \rho_p}$ becomes a property of the

particles forming the cake. This quantity is defined as the "specific cake resistance," α .

Therefore,

$$\alpha = \frac{\bar{k}_1 (\overline{s_p/V_p})^2 (1-\epsilon)}{\epsilon^3 \rho_p} \quad (5)$$

$\bar{k}_1$ and $(\overline{s_p/V_p})$ represent average value of these quantities. Equation (4) can be rewritten as

$$- dp = \frac{\mu u \alpha}{A} dm \quad (6)$$

which on integration gives the pressure drop, Δp through the filter cake in terms of the mass of the cake m_c as

$$- \Delta p = \frac{\mu u \alpha m_c}{A} \quad (7)$$

Here it is assumed that

- (1) u remains constant throughout the cake
- (2) α is constant over the cake thickness
- (3) m_c is constant

However, this equation does not include the resistance that would be offered by the filter medium. To take into account the resistance offered by the filter medium, Equation (7) is modified to the following form:

$$-\Delta p = \mu u \left(\frac{m_c \alpha}{A} + R_m \right) \quad (8)$$

where R_m is filter medium resistance.

The mass of the deposited solids can be related to the concentration of solids in the slurry by material balance.

$$m_c = \frac{V \cdot C_s}{1 - [(m_f/m_c) - 1] C_s / \rho} = V \cdot C \quad (9)$$

where C : mass of solid per unit volume of filtrate

C_s : mass of solid per unit volume of slurry

m_f : mass of wet cake, including the filtrate retained in its voids

m_c : mass of dry cake after drying

ρ : density of the filtrate

From Equations (2), (8), and (9),

$$\frac{dt}{dV} = \frac{\mu}{A(-\Delta p)} \left[\frac{\alpha C V}{A} + R_m \right] \quad (10)$$

Equation (10) is the basic equation for cake filtration of incompressible cakes.

For compressible cakes the assumption of constant specific cake resistance is not valid. An empirical relation is suggested for specific cake resistance in compressible cakes.

$$\alpha = \alpha_o (-\Delta p)^{s'} \quad (11)$$

where

α_o : constant

$-\Delta p$: pressure gradient across the cake

s' : empirical constant termed as compressibility coefficient of the cake.

Equations (10) and (11) constitute the basic equations for compressible cakes.

The specific cake resistance must be determined experimentally. In attacking a filtration problem the first step is filtration analysis of the slurry. The next step would be to use this analysis to determine the filtration conditions and the final step would be to select the type of filter to fulfill the needs of the problem.

Analysis of Filtration Data

The most common way of collecting filtration data is to subject the slurry under consideration to constant pressure filtration and measure the filtrate collected as a function of time. For constant pressure filtration, integration of Equation (10) gives

$$t = \frac{\mu}{(-\Delta p)} \left[\frac{C\alpha}{2} \left(\frac{V}{A} \right)^2 + R_m \frac{V}{A} \right] \quad (12)$$

A plot of V vs. t will give a parabola. Ruth (24) recognizes this as the fundamental axiom of constant pressure filtration. If a plot of

$\frac{\Delta t}{\Delta V}$ vs. V is made, then it should yield a straight line (as indicated by

Equation (10)). From the slope of the straight line, α , and from the intercept, R_m , can be obtained. For a compressible cake, by conducting experiments at several different pressures, α_o and s' can be determined with Equation (11) by making a plot of $\ln \alpha$ vs. $\ln (-\Delta p)$. Sperry (25) discusses this method at length and also gives a simple method for characterizing the parabolic time-discharge curve. When dilute slurries were filtered and the data analyzed they gave a parabola for the plot of V vs. t . However, when concentrated slurries were filtered (26)

these relationships broke down. The plot of $\frac{\Delta t}{\Delta V}$ vs. V was no longer a straight line, particularly in the region of the start of the operation. The breakdown of the theory may be due to simplifying assumptions of constant α and R_m in the development of Equation (12). These will be discussed below.

Factors Affecting α and R_m

In a filtration problem it is very difficult to predict the value of specific cake resistance, α and filter medium resistance, R_m . Even when these parameters are determined experimentally their reliability is questionable because a slight variation in process conditions will change their value significantly.

Filter Medium Resistance

The resistance offered by the filter medium will depend upon: (1) the structure of the filter media, (2) the particle size distribution and concentration in the slurry, (3) the rate of deposition of the particles, and (4) the nature of the driving force.

Grace (27) studied the internal structure of a large variety of filter media. He applied the permeability, bubble pressure, and mercury intrusion methods to measure pore radius and pore-size distribution of the media and related these to the construction variables characteristic of the media. The performance of these filter media were then determined by the filtration of very dilute suspensions containing spherical particles of known size distribution (2). The filtration laws proposed by Hermans and Bredee (22) were applied to study the filtration prior to the formation of macroscopic cake and regions of "standard-blocking" filtration were found to occur. As discussed before in standard-blocking filtration, the solid particles adhere to the walls of the pores and progressively reduce the internal diameter of the pore.

These experiments were conducted under rather ideal conditions (low solid concentration in slurry and spherical particles) which are normally not realized. It is generally found that both initial filtration pressure and solids concentration have great influence on the effective resistance of the medium. The greater the concentration of slurry, the smaller will be the average distance between the particles and smaller will be the tendency for the particle to be drawn into the streamlines directed towards open pores. The particles in concentrated suspension tend to distribute themselves fairly evenly over the filter surface and form bridges. As a result, suspensions of high concentration gives rise to cakes of lower resistance than those formed from dilute suspensions. But Smith (28) found that for a given system and filter medium a critical pressure drop exists for each solid concentration above which the mechanism of bridging fails to occur and plugging of the filter media takes place. This may increase the filter media resistance drastically. There is no direct experimental evidence of this phenomena, but this might happen for highly compressible cakes of flocculated suspensions. Plugging might occur at the surface of the filter media due to intense sheer at the pore openings. Once the filter media is plugged, the flow rate of filtrate may in fact decrease by increasing the pressure drop across the slurry.

Sometimes while filtering a slurry, if the initial rate of deposition is very high due to initial low resistance and high driving force

the filter medium will likely be plugged and the resistance of the medium increased. As Nemeth and Sirois (29) point out, the nature of driving force may also affect the rate of filtration. They observed that during continuous vacuum filtration, because of the negative pressure differential imposed on the slurry, gas bubbles evolve from either air dissolved in the liquid or absorbed on solid surfaces or trapped in fine cracks in the solid surface of the cake. The movement of bubbles through the channels of the filter media is restricted and hence it hinders the flow of filtrate through these channels. This phenomenon affects the resistance offered by filter media. Also, in this case, it is difficult to characterize and quantify the effect precisely and hence the filter-medium resistance cannot be predicted accurately. However, the filter medium resistance builds up from a very small value to a constant value over a very short initial period of filtration. If the filtration cycle is long and the filter medium resistance is relatively small compared to the cake resistance, (say 1 percent of the cake resistance), then its contribution to overall resistance would not affect the filtration process drastically.

Selection of a specific filter medium for a particular operation becomes a difficult task because of the variety of filter media available. A useful, qualitative review of the principal forms of filter media is given by Dickey (6) dealing with questions of strength, etc., and also containing much practical information on the performance of media. French (30) reviews woven and nonwoven filter fabrics and discusses the various modes of fabric construction. Koyacs (31) describes media composed of wire cloth, metal edge filters, fused metallic fibers, ceramics, and cellulose. Shoemaker (32) covers a spectrum of filter media and makes practical suggestions on media selection and classification. Purchas (7) suggests criteria to judge filter media and Rushton and Griffiths (33) review filter media with special emphasis on characterization of filter media, mechanism of particle deposition, resistance of used media, effect of medium structure on cake resistance, etc.

Specific Filter Cake Resistance

Specific filter cake resistance is expressed as

$$\alpha = \frac{\bar{k}_1 \left(\frac{s_p}{V_p} \right)^2 (1-\epsilon)}{\epsilon^3 \cdot \rho_p}$$

where $\bar{k}_1$ is an empirical coefficient and its value ranges from 4.2 to 5.0 (11).

As can be observed from the equation, α depends on: (1) particle size, shape, and distribution, and (2) porosity of the filter cake. It is not very difficult to characterize particle properties. Several books (34-36) are published which concentrate in this area and they

discuss the methods of characterizing particles and their advantages and disadvantages. The quantity (s_p/V_p) can be determined with a fair degree of accuracy. However, if the particles are not rigid and if they deform under the applied hydraulic pressure, then (s_p/V_p) could no longer be predicted from measurement of only particle characteristics. But normally the particles do not deform under applied pressure gradient.

Porosity is a quantity which is very difficult to determine. It depends on the structure of the cake which is formed by the deposited solid particles. Some of the factors which will affect the structure of the cake are: (1) particle size distribution in the slurry, (2) shape of the particles and their packing behavior, (3) the manner in which the cake is formed, (4) the geometry and nature of the supporting medium, (5) the force acting on the cake, and (6) concentration of particles in the slurry.

In a filter cake, the porosity will change from layer to layer being maximum at the surface of cake in contact with slurry and minimum at the surface in contact with the filter medium. A fluid pressure gradient exists from the surface of the cake to the surface of filter medium because of the frictional losses arising from the flow of filtrate through the cake. The fluid will exert a compressive force on the particles and therefore tend to consolidate the cake. This force exerted by the fluid on each particle will be transmitted to the particles deeper in the cake. The compressive force will vary from a minimum of zero at the free surface of the cake to a maximum at the filter medium. This compressive force will tend to pack the particle more closely at the filter surface and hence a minimum porosity is observed at the filter medium surface.

Besides the hydraulic pressure gradient, the manner in which the cake is formed also plays an important role in affecting the structure of the cake. In many laboratory experiments, the filtration data are collected either with a Buchner funnel or a filter leaf. The orientation of the cake formation in these cases is exactly opposite. In the Buchner funnel, the solids are deposited in a downward direction whereas in leaf filters the solids are deposited in an upward direction by applied vacuum. There is a marked difference in filter cake resistance determined by these two methods with the same slurry. This difference can be attributed to sedimentation (37) of particles away from the filtration zone in leaf tests. The qualitative explanation for these effects can be given as follows:

Consider a concentrated slurry with narrow particle size distribution. In this situation, all the particles will sediment at approximately the same rate so a more rapid build-up of particles will occur on an upward-facing surface (Buchner funnel) and a correspondingly reduced rate of build-up will take place on a downward facing filter surface. Thus there will be a tendency for accelerated filtration with downward facing filter surface due to higher porosity and vice versa. On the other hand, if the suspension is relatively dilute, so that large particles settle at relatively faster rate, then there will be a preferential deposition of large particles on an upward facing surface

during the initial stages of filtration. This will result in the formation of a porous cake and hence low specific cake resistance. Conversely for a downward facing surface, fine particles will initially be deposited preferentially and the cake resistance will be correspondingly increased. Thus it can be seen that the orientation of filter surface is an important factor in determining the specific resistance of a cake. Rushton and Rushton (37) studied the sedimentation effect and proposed a correlation for specific cake resistance which takes into account the size distribution of deposited particles on the cake. However, in their analysis they assumed that porosity remains constant from layer to layer which is a rather questionable assumption.

As there are so many factors affecting the structure of a filter cake it becomes a very difficult task to characterize it. Ward et al. (38) statistically studied the structure of packed beds formed by various methods in order to find out whether any anisotropy exists and if it does to what extent it can be diagnosed. Their results show that for sedimented bed there is slight evidence that the particles arrange themselves in some definite structure in a direction normal to the direction of sedimentation. However, there is very little evidence that the pore spaces form a regular structure. The same is true for filter cakes at low pressures. But as the filtration pressure is increased, a strong indication of structure in both directions is obtained. So it appears that if experiments are conducted carefully to simulate the operating conditions as far as possible, then it would be possible to predict specific resistance from the properties of the particles.

Experimental Determination of Specific Cake Resistance

Due to the difficulties encountered in determining the value of specific cake resistance by carrying out filtration runs, investigations were directed to precisely define this quantity by some other approaches. Principal approaches to the problem were (1) determination of mean or local specific resistance of filter cake in terms of its permeability and the properties which affect permeability, and (2) description of the resistance of a filter cake by the principle of dimensional similitude as the friction loss experienced by fluid which passes through the cake.

The first approach was based on the modified Darcy's empirical relation for flow through porous beds, which states that

$$u = K \frac{(-\Delta P)}{\mu L} \quad (13)$$

where

K = permeability coefficient

u = linear velocity of filtrate based on filter area, A .

ΔP = applied hydraulic pressure gradient

μ = viscosity of filtrate

L = thickness of filter cake.

Comparing Equation (7) and (13), and substituting for dm from Equation (3), we have

$$\alpha = \frac{1}{K_p (1-\epsilon)} \quad (14)$$

The principle reason to follow this approach is that the methods of determining permeability are standardized (36). Permeability can be determined in compression-permeability cells (16, 39) under varying conditions of stress, making it possible to simulate a filter cake by dividing it into various sections and then determining the permeability of each section. From the information provided by compression-permeability cells, an average value of specific cake resistance can be calculated. Hoffing and Lockhart (40) used this approach to study specific cake resistance quite successfully. Dullien (48) has developed a mathematical model to predict permeability in porous media.

Wakeman (23) has critically reviewed the validity of compression-permeability cell data in prediction and interpretation of filtration data. He concludes that a unique one-to-one correspondence between compression-permeability cell data and filtration data is not possible without a priori knowledge of cake porosity. He stresses the need to develop experimental techniques and theories which would obviate the need for compression-permeability cells and give a further insight into formation and structure of filter cakes.

Another approach to study the filtration characteristics is based on the knowledge that as the fluid flows through a cake the pressure energy of the upstream fluid is dissipated against frictional, eddy and kinetic energy losses encountered in its passage through the cake. In general, the flow is laminar and energy dissipation is one of viscous friction. One would expect, therefore, that relations between flow rates, pressure gradient and fluid properties, could be correlated for a given bed by a modified friction factor chart. Brownell and Katz (13, 41) developed such charts, along with several empirical relations to characterize filtration experiments. However, in their approach the computations required are lengthy and the quantities involved in the calculations are difficult to determine and critical enough to make their rough estimation undesirable. This approach is criticized because of its lack of generality and consequent inaccuracy for high porosity aggregates, particularly in turbulent regions. Due to empirical nature of the approach, it can only be used in the estimation of conventional filtration resistance for special cases.

Modification of Conventional Filtration Equation

As it is, the conventional equation for filtration, Equation (7) does not give accurate values of specific cake resistance. Thus, some modifications were suggested by several workers. Both Cairman (39) and Ruth (42) showed that the specific resistance of a given cake is a function only of the applied pressure. Although the point resistance, α_p , varies throughout the thickness of a compressible cake, an average value can be defined as

$$\alpha_{av} = P_f / \int_0^{P_f} (dP/\alpha_p) \quad (15)$$

where

P_f : applied fluid pressure

α_p : specific cake resistance, point value

α_{av} : average cake resistance.

Kottwitz, et al. (43) suggest that in several cases α_{mean} be defined as

$$\alpha_{mean} = \int_0^{P_f} \alpha_p dP/P_f \quad (16)$$

gave better results than α_{av} .

The specific cake resistance, as evaluated by the classical filtration equation, was much different than experimentally observed values. In order to account for this abnormal resistance offered by the cake, Ruth (16) accounted that a portion of the measured void volume can be considered as unused or "dead." The hypothesis of dead void volume is shown to have physical basis as a region adjoining the capillary wall in which electro-osmosis, induced by streaming potential generation, opposed the flow caused by hydrostatic pressure difference. Ruth (16) tested this hypothesis experimentally and concluded that the electrokinetic phenomena plays an important role in the success or failure of filtration operations. An experimental method to evaluate the dead void volume is also discussed (16).

All these theories discussed are not able to account for all the aspects of filtration, and there are some serious doubts raised about various aspects of these classical filtration theories. However, in recent years, due to better understanding of the mechanism of filtration it has become possible to suggest a more rigorous method of analysis.

For constant pressure filtration, it is generally assumed that (1) the filtration resistance is constant at an average value over the filter cake, as determined by Equation (15), (2) the ratio of the mass of wet to mass of dry cake, m , is constant; where $m = m_f/m_c$, and (3) at any instant, the rate of flow, dV/dt , is constant throughout the cake. It is now known that the first two assumptions are in error during the initial period of filtration and the last assumption may be strikingly in error for thick slurries.

Tiller et al. (44-47) in a series of papers attempts to resolve this problem. Tiller (47) considers the resistance offered by filter media in defining the average specific cake resistance and modifies the definition of average cake resistance as

$$\alpha_R = \frac{P_f - P_l}{P_f - P_l} \int_0^{\frac{dP_s}{\alpha}} \frac{1}{\alpha} dP_s \quad (17)$$

where

α_R : average specific cake resistance

P_f : applied fluid pressure

P_l : pressure at the medium ($= \mu R_m Q$)

Q : volumetric flow rate per unit filter area at the filter medium exit

P_s : total compressive pressure at a distance x from surface of cake

α : specific cake resistance at x

The variation of porosity with distance from the cake surface and time is correlated with various parameters (39). It is observed that at each instant of time, porosity drops throughout the cake until the medium is reached, where it is minimum. As the cake thickness increases, the local porosity increases with time at a given distance from the cake surface. The ratio of mass of wet to mass of dry cake is related to the average porosity by

$$m = 1 + \frac{\rho \epsilon_{av}}{\rho_s (1 - \epsilon_{av})} \quad (18)$$

As ϵ_{av} decreases, m will decrease. Also at any given point, the porosity decreases with time. Consequently, liquid is literally squeezed out of each individual element. Since flow is additive, the flow rate necessarily increases as the liquid approaches the medium.

From the analysis of problem, Tiller et al. (46) arrive at a simplified form of the filtration equation, which allows for the variation of ϵ and flow rate through the cake:

$$Q = \frac{(-\Delta P)}{\mu (J \alpha_R \frac{m_c}{A} + R_m)} \quad (19)$$

where J is a correction factor, defined as

$$J = 1 - \frac{(m-1)s}{\epsilon_{av}(1-ms)} \int_0^1 (\epsilon_x - \epsilon_{av}) \left(\frac{x}{L}\right) d\left(\frac{m_c x}{m}\right) \quad (20)$$

where s = fraction of solids in slurry.

As can be seen, the equation suggests that the conventional filtration resistance must be multiplied by a factor which depends upon porosity variation and slurry concentration. For dilute slurries, s approaches zero and the equation reduces to the conventional form. Also it can be observed that the filtration resistance decreases substantially as the slurry concentration increases. Hence, smaller filter areas would be required for high concentration slurries.

In their analysis, Tiller et al. (44) assumed that in the cake, equilibrium porosity is reached instantaneously with changes in pressure and they assume the validity of the data collected by compressibility-permeability cell as representative of specific cake resistance and porosity in the actual operation. Wakeman (23) raises serious doubts about this assumption. In his analysis of the problem, Wakeman (23) combines Darcy's law applied to the flow of liquid relative to the solid particles with the continuity requirement. He obtains a set of nonlinear partial integro-differential equations which can be solved numerically only if ϵ versus P_f information is known.

In recent years, another novel approach to the theory of filtration has gained popularity. After the pioneering work of Fatt (49) in modelling a porous medium by networks of capillaries, several workers have contributed in developing the model. Chatzis and Dullien (50) applied the model to the flow of fluid through sandstone with reasonable success. Application of these models to filtration is being considered (51).

Improving Filtration Rate

The rate of filtration can be improved in many ways. As can be observed from Equation (10), the rate of filtration is (1) inversely proportional

to the viscosity of filtrate, (2) directly proportional to the applied hydraulic pressure gradient, and (3) inversely proportional to the resistance offered by the cake. The effect of slurry concentration on rate of filtration is complex because it also changes the specific cake resistance. Fuhrmeister (52) discusses methods of changing these parameters in such a way that the rate of filtration increases. Reeves (53) suggests dilution as a means to decrease the viscosity when filtering viscous liquids. Some of these parameters and their effect on filtration is discussed below.

Viscosity Effect

Decreasing the viscosity of the filtrate tends to increase the rate of filtration. This can be achieved by heating the slurry, but this may not be economically feasible when large quantities of slurries are to be treated. For viscous filtrate, dilution by a liquid of lower viscosity will help to improve filtration rate, but the quantity to be filtered increases to a great extent. Reeves (53) discusses how to optimize these contradictory parameters. Reeves developed an equation for calculating the optimum amount of diluent to be added to viscous liquid (which in his case was paraffinic oil) in order to get maximum filtration rate of diluent free liquor. He uses Equation (10) and a correlation for viscosity of mixture to develop this expression. Calculated value of optimum dilution match very well with experimentally observed values. Fuhrmeister (52) increased the temperature of the slurry to reduce viscosity and observed approximately three-fold increase in the rate of filtration for 25 percent CaCO_3 slurry in water by increasing temperature of slurry from 60°F to 180°F.

Hot solution containing surface active agent was also used for increasing filtration rate (54). Because of the hot solution the viscosity is decreased and addition of surface active agent reduces surface tension of the solution. Reduction in viscosity enhances rate of filtration and reduction in surface tension leads to less residual water in the cake.

Steam is also used in some cases to reduce viscosity. The effect of steam on filtration rate is discussed in the later part in detail.

Hydraulic Pressure Gradient

The higher the pressure drop across the filter cake the higher will be the filtration rate. But there is an upper limit on pressure gradient in vacuum filtration because of the nature of the process. And in pressure operation, the upper limit will be fixed by (1) economics and (2) filter cake resistance. As discussed before, as the pressure gradient increases, filter cake resistance also increases and, in some cases, the rate of filtration is observed to decrease with increasing pressure. Also, the cake has a tendency to crack with increasing pressure. Once a crack is formed, any further increase in pressure has no effect on filtration rate.

Reduction of Cake Resistance

The resistance offered by the cake depends upon the structure of the cake and can be decreased by increasing the porosity of the cake. There are several ways in which this can be achieved:

By Using Filter Aids

Filter aids are used in two ways: (1) by precoating the filter media with a pre-coat and (2) by addition of a filter aid such as diatomaceous earth, Kieselghur or other inert porous material to the slurry.

The primary function of a filter aid is to provide a porous cake structure. Filter aids give cakes of more open texture, which allow higher rates of flow, greater rigidity, and higher pressure of filtration. However, care should be taken to ensure that the good effect of the filter aid is not destroyed by employing too high a filtration pressure, because the cakes formed are usually compressible. The presence of the filter aid reduces the specific resistance of cake and also results in formation of a thicker cake. If too much filter aid is used the increased thickness of cake tends to counterbalance these advantages (55). The action of filter aid and its effect on cake properties, such as particle size and shape, and on rate of filtration are discussed in the literature (56-57).

By Changing the Particle Size

If the particles forming the cake are made as large as possible, the cake formed will have larger pores and hence will offer less resistance. This will improve filtration rate. Particle size can be increased by coagulation. Coagulation of particles will take place by adding some chemicals, namely flocculants (26), or by changing the pH of the slurry. Flocculated suspensions give rise to a more porous cake, but they also increase the compressibility of the cake. Changing the pH would help only in those cases where it leads to precipitation of the solid particles. Fuhrmeister (52) added ferric chloride to change the pH of elutriated sewage sludge. He observed a six-fold increase in filtration rate when percent of ferric chloride in solids was increased from 3 to 7 percent by weight.

By Mechanical Means

(1) Delayed cake filtration. During filtration operation, if the build-up of solids can be reduced the effective cake thickness will be less and the rate of flow of filtrate will be increased. This is achieved in practice by incorporating moving blades in the filter equipment (12) so that the thickness of cake is limited to the clearance between the filter medium and the blades. The solids are retained in suspension and the solid concentration in the feed vessel increases until the particles are in permanent physical contact. At this stage, the flow rate of filtrate reduces significantly. By using this technique, a higher filtration rate than conventional is obtained and also the final cake is significantly drier because the blades effectively break down the bridges or arches, which give rise to a structure in the filter cake.

(2) Variable-chamber filters. In cakes undergoing filtration, only the layers closest to the medium are subject to high mechanical compression whereas the layers farthest from the medium are subject only to light hydraulic forces. If the surface layers of the cake are compacted in some manner so that the pressure distribution throughout the filter cake is uniform, then higher filtration rates and drier cakes can be obtained (20). This is achieved by a variety of filters known as variable-chamber filters.

The effect of solid concentration in the slurry on the filtration rate is complex. As discussed above, increasing slurry concentration may reduce specific cake resistance because of the formation of a more uniform cake due to even distribution of particles. This will help in increasing the filtration rate. While, on the other hand, increased solid concentration means increased particle deposition on the filter media and hence formation of thicker cakes. As the length through which filtrate travel increases, rate of filtration will decrease.

DEWATERING

The removal of water from the void spaces of the filter cake is termed as dewatering. The ease with which this water can be removed will largely depend on characteristics of the porous cake under consideration.

Porous systems may be classified into several categories in terms of the structure of the pores (58). Three kinds of pores are recognized according to their size: (1) voids, which are large enough so that little of any foreign substance contained in them is within the field of attraction of the walls, (2) capillaries, which are small enough for most of the foreign substance to be attracted by walls, and (3) "force spaces" which approach at dimensions within which every molecule or atom of foreign substance is affected by the field of force of the walls. A large number of pores constitute a porous system. Void systems are easily filled and emptied of foreign substances. Capillary systems offer resistance to filling with foreign substances, and on emptying a portion remains behind. "Force space systems" can be filled or emptied only with molecularly dispersed substances and only under great pressure or specific chemical forces.

An aggregate with two or three kinds of pores is a combination pore system, whose pores may be jointly active or partly active and partly passive in a given process. A porous system may be ordered or disordered and may be dispersed or connected in construction. According to the geometric arrangement, the pores can be classified as closed, canals, networks, or perforated structure.

Filtration of a slurry results in a cake in which capillaries are filled with filtrate. In the dewatering step, this water is removed by applying desaturating forces. As stated before, this force could be either mechanical, as in dewatering by compression, or hydrodynamic as in displacement of filtrate with air under a pressure gradient.

Factors affecting displacement dewatering can be suitably grouped as follows (21):

Properties of the cake	Properties of the fluid	Interfacial properties	Other factors
Particle size	Density	Interfacial tension (liquid-liquid)	Temperature
Particle shape	Viscosity		Pressure gradient
Mode of particle packing		Surface tension (gas-liquid)	Rate of displacement
Dimensions of the cake			

Dewatering by Compression

Residual filtrate can be removed from the filter cake by compression, and the matrix of solid particles in liquid continuum is maintained throughout the operation. Although the cake remains fully saturated, liquid is removed and the ratio of the weight of liquid to the weight of solids in the cake is reduced. This is, in fact, the principle of operation of many filters like plate and frame filter, variable chamber filter, belt filters, etc. In plate and frame filter, the slurry is forced into the filter chamber under pressure, thereby increasing the proportion of chamber volume occupied by the solids. In variable chamber filter, the cake is further compressed by an inflatable diaphragm situated within the chamber.

Compression of filter cake results in lowering of the porosity, and hence squeezing of the filtrate out of the pores. However, if a filter cake initially has high porosity (59), it does not necessarily mean that the liquid will be easy to remove from the cake, although it might well mean that liquid can be removed by mechanically changing the structure of the cake.

The ease with which liquid can be removed from a cake will depend upon the diameter of the pores. Pore diameter is related to particle diameter but particularly for irregular shaped particles the relation is difficult to obtain. However, rough calculations (59) show that for a pore size of 2 μm , a pressure gradient of about 2 atmosphere is required to remove the capillary water. For pore size of the order of 0.1 μm , the pressure required would be about 20 atmospheres. This indicates a strong effect due to surface tension in capillaries of small diameter. If surface tension of the filtrate is decreased, then it would be much easier to remove liquid from the small diameter capillaries in the cake. This then will reduce the residual moisture content of the cake.

Effect of Surfactant on Dewatering

Many workers have studied the effect of surface active agents on the filtration characteristics of a slurry (60-63). If the filter cake is considered as a bundle of interconnected capillaries, the lowering of residual moisture with surfactant addition can best be interpreted in terms of capillary rise phenomena. In these circumstances, the capillary rise, h , in a bed of particles of pore radius, r , may be thought of as a measure of the liquid saturation and is given by the equation

$$h = \frac{2\sigma \cos \theta}{r \rho_w g} \quad (21)$$

where σ is the surface tension of the liquid of density ρ_w and θ is the contact angle. Consequently, it would be expected that reduction in σ brought about by surfactant addition should produce corresponding reduction in the moisture content of the filter cake.

Silverblatt and Dahlstrom (60) found that the moisture content of the filter cake decreases sharply with increasing concentration of surfactant, but it approaches an asymptotic constant value for large surfactant concentration. Nicol (62) and Keller et al. (63) report that anionic surfactants give better performance than cationic surfactants when dewatering a cake of coal. Nicol (62) gives an explanation for this in terms of the adsorption at the solid-solution interface. Although no direct measurement of surfactant adsorption on coal has been reported, electrophoretic data suggest that the coal surface is negatively charged at neutral pH. Therefore, the concentration loss from solution arising through adsorption is likely to be greater in the case of the cationic surfactant, and, therefore, higher concentrations are required to produce a given equilibrium surface tension than in case of the anionic detergent.

Keller et al. (63) list the properties of an ideal surfactant for aiding the dewatering of coal as:

1. The surfactant molecule should adsorb on the coal oriented with the hydrocarbon tails extended to the water phase.
2. The hydrocarbon tail should be as hydrophobic as possible and still retain a degree of water solubility of the whole molecule.
3. The head group should be the least hydrophilic group possible and still retain the solubility of the whole molecule.
4. The adsorbed molecule should have a large effective area so that fewer molecules are required to affect the interfaces.
5. The surfactant should decrease the interfacial tension at the liquid-air interface as much as possible at the lowest concentration.

6. The molecules should not be prone to "hemimicellation," i.e., aggregation of two surfactant molecules in opposite orientation close to a solid surface. If this happens it should happen at a high concentration which should be avoided.

In this work, Keller and coworkers studied the effect of surfactants like dodecyl pyridinium chloride (cationic), sodium dodecyl sulfate, aerosol A-196, aerosol-OT (all anionic) and triton X-114 (nonionic) on Bruceton coal and observed as much as 3-4 percent reduction in moisture content of the cake with as little as 10^{-6} mole fraction of the surfactant. However, they recommend that the conclusions generated be used as a function of the rank of coal under investigation and nature of the mineral within the sample.

Dolina (61) studied the influence of rank and degree of oxidation of coal on dewatering and concludes that cationic or nonionic surfactants should be used to promote efficient dewatering of oxidized coals.

Gray (64) added oil to the slurry in presence of surfactants, and found the dewatering to be more effective. The oil was even more effective than some water soluble flocculating agents. He explains that this may be due to the fact that in addition to a flocculating action, oil also increases the receding contact angle of water on the coal. (The contact angle of water with coal is assumed to be zero degree, so that any increment in this angle will decrease the $\cos \theta$ term in Equation (21) and hence it tends to improve dewatering.) As a result of these two phenomena, better dewatering is achieved. He suggests the use of oil soluble surfactants. Furthermore, the addition of oil will increase the calorific value of the coal.

Effect of Temperature Gradient Across the Filter Cake

Another method of decreasing the surface tension of filtrate will be to establish a temperature gradient across the cake, because for most liquids surface tension is a function of temperature. In presence of a small temperature gradient across the filter cake, the flow of liquid in the filter cake may take place by two mechanisms (65): (1) thermal capillary flow, and (2) thermo-osmosis. In thermal capillary flow, the flow of liquid takes place due to a surface tension gradient established as a result of the temperature gradient. Liquid moves in the direction of higher surface tension. Since the surface tension of water decreases with increasing temperature, water in a capillary will move from a warm region to a cold region in the presence of a temperature gradient. Due to the temperature gradient, the vapor pressure of filtrate on two sides of the filter cake will be different. Because of this difference in vapor pressure, liquid flow will be set up from higher pressure to lower pressure. As for the thermo-osmosis, the liquid should also move from higher temperature to lower temperature. However, in very small diameter capillaries, behavior of liquid is not normal, and flow in reverse direction has been observed (65). Haase (66) reports that the direction in which the flow of liquid takes place will depend upon: (1) type of liquid under consideration, (2) the nature of the porous media, and (3) mean temperature of the system.

Cary (67) investigated the effect of thermal and suction gradient on a sample of moist soil and obtained very interesting results. He found that the net effect of a temperature gradient of $0.5^{\circ}\text{C}/\text{cm}$ of soil was equivalent to an additional pressure gradient of 2 cm of $\text{H}_2\text{O}/\text{cm}$ of soil, when the experiments were carried out at a suction of 1.5 cm of Hg/cm of soil. Also the same temperature gradient was equivalent to a pressure gradient of 250 cm of water/cm of soil, when the applied suction was 10 cm Hg/cm of soil. However, Deryagin et al. (68) did not observe such promising results. They found that the effect due to the temperature gradient was very small. Derjaguin et al. (69) found the analog of thermo-osmosis in the phenomena of thermomechanical effect observed in Helium II.

Fedosov (70) analyzed from a theoretical standpoint the problem of liquid motion in a flat rectangular reservoir in the presence of a temperature gradient at the free surface. His analysis shows that at the free surface the liquid velocity is in a direction opposite to the temperature gradient.

Silverblatt and Dahlstrom (60) conclude that the effect of changing surface tension to aid dewatering is statistically significant but relatively minor in magnitude.

Modification of surface forces in some cases can lead to cracking of the cake (59). Suppose that a cake is formed such that the average pore size is small, and the capillary pressure across the meniscus is very high, then, if there is one single large diameter pore passing through the cake, such that when displacing air pressure is applied or when suction is applied to the water, this pore immediately empties. Branching into this gas-filled pore, there will be many pores of small diameter at the entrance of which large capillary pressure will apply. This will place the cake under great internal stress and a crack will develop so as to reduce this stress. Thus, cake cracking will be found more frequently where the particle size and, therefore, the pore size is small and where the removal of liquid is attempted by air displacement. Wakeman (71) has studied the role of internal stresses in filter cake cracking and he suggests that in order to determine whether or not a cake will fracture, the location and magnitude of the dewatering stresses must be analyzed together with the elastic or elastic-plastic nature of the cake deformation. Cake cracking can reduce the efficiency of subsequent post-filtration processes like cake-washing drastically. Therefore, as far as possible, cake-cracking should be avoided.

Dewatering of Filter Cake by Displacement With Air

This is a very commonly followed method of dewatering and the equipment used for this purpose are (1) rotary vacuum filters, (2) pressure filters, and (3) centrifuges.

In this case, the filter cake can be considered as a matrix of solid particles in a liquid and gas mixture. Hence, this mode of liquid removal reduces to a problem of two-phase flow through a porous medium. In practice, this method will be useful with cakes having a relatively open structure.

In porous media when two fluids are flowing through it, the wetting fluid flows over the surface of the solids, and thereby prevents the contact of the second fluid, the nonwetting fluid, with the solid. This is true as long as there is a film of liquid over the surface of the particles.

For a specified pressure drop over the filter cake, there is a definite quantity of the wetting fluid retained in the capillaries. The amount of the retained fluid depends on a number of parameters: (1) size of the capillaries, (2) pressure gradient of the nonwetting fluid across the porous medium, and (3) interfacial tension of the fluid.

Three important items associated with retention of wetting fluid in the porous bed are:

Saturation, S , is defined as the ratio of volume of voids filled with wetting fluid to total volume of voids.

Residual saturation, s_{∞} , is the fraction of total void volume occupied by residual fluid, i.e., the fluid which cannot be removed by drainage under the applied pressure gradient.

Reduced saturation, s_r , is the fraction of available void volume occupied by wetting fluid, where available void volume is total void volume less volume occupied by residual wetting fluid.

The knowledge of residual saturation in a given bed under known pressure gradient is important because it gives an upper limit as to how much fluid can be removed from the cake. Also it can be used in predicting relationships for the simultaneous flow of two homogeneous phases through porous media.

Empirical Correlations for Residual Saturation

According to Wakeman (21), the residual saturation can be separated into two components--the capillary moisture and the permanent residual moisture. The permanent residual moisture is due to the liquid retained at the point of contact of two adjacent particles, and it is also termed as pendular liquid. The capillary moisture can be measured in terms of the capillary drain height.

For cakes drained by gravity or centrifugal force, capillary drain height has been correlated to the capillary drain number for several particle beds (72), as

$$h_c = \frac{0.275}{\text{capillary drain number}} \quad (22)$$

where h_c : capillary drain height, ft.

$$\text{capillary drain no.: } K^{1/2} \frac{(\rho_w + \rho_w a)g}{\sigma \cos \theta} \quad (23)$$

K : permeability of the porous bed

ρ_w : density of wetting fluid

a : centrifugal acceleration number

σ : surface tension

θ : contact angle

This form of equation has been confirmed for beds of spherical and near spherical particles (73).

The pendular residual saturation is correlated to a dimensionless capillary number, R_c defined as (73)

$$R_c = Kg \left[\frac{\rho_w + \rho_w a + (\Delta P/gL)}{\sigma \cos \theta} \right] \quad (24)$$

The capillary number expresses the ratio of the desaturation to retention forces, where the desaturation driving forces are given as the sum of the force of gravity, centrifugal force, and pressure gradient. The use of the sum of all these forces implies that all are acting co-linearly in the direction of liquid flow from the bed. Dombrowski and Brownell (72) suggest the use of a component in the direction of the flow of liquid of the vector sum of these forces. The correlation between residual saturation and capillary number is in the form of a log-log graph.

Average residual saturation, s_{av} can be determined from

$$s_{av} = \left(1 - \frac{h_c}{L} \right) s_{\infty} + \frac{h_c}{L} \quad (25)$$

For cakes drained under a pressure gradient, Brownell and Katz (41) suggest the correlation

$$s_{av} = \frac{1}{86.3} (R_c)^{-0.264} \quad (26)$$

This correlation gives value of $s_{av} > 1.0$ for very small values of R_c ($R_c < 10^{-8}$) so it should be applied with care. Dombrowski and Brownell (72) have correlated the average saturation of thin beds, defined as those whose depth is of the order of capillary drain height, with permeability and driving force:

$$s_{av} = \text{function of } K^{1/3} \cdot \left(1 + \frac{\Delta P}{\rho_w g L}\right)^2 \quad (27)$$

This correlation is plotted on a log-log paper (72) with the ratio of cake depth to capillary drain height as a parameter. However, to use any one of these correlations, it is essential that the bed permeability K be known.

Experimental Measurement of Residual Saturation

For a given cake, obtained by applying known forces on the filtered cake, the residual saturation can be determined by measuring the residual moisture content, w_∞ of the cake. Residual saturation, s_∞ is related to w_∞ by the following relationship:

$$s_\infty = \frac{w_\infty}{1 - w_\infty} \frac{\rho_s}{\rho_w} \frac{1 - \epsilon}{\epsilon} \quad (28)$$

where w_∞ : residual moisture content, i.e., weight of filtrate retained per unit weight of wet solids.

ϵ : overall porosity of the cake.

Overall porosity can be determined by knowing the total weight, the weight of the solids forming the cake, particle density of the solids, and final volume of the cake.

Permeability

As discussed above, residual saturation is correlated to the permeability of the cake. The permeability can be measured experimentally by mercury permeameter (36) or compression-permeability cells. In the absence of any experimental value, it can be estimated by correlation suggested by Brown (13) and Millington and Quirk (74). Wakeman et al. (75) evaluated these correlations and found them inadequate. Recently Dullien (48) has suggested a model, consisting of a set of cubic networks of arbitrary orientation; to calculate permeability of porous media. This model requires the knowledge of two pore size distribution functions and porosities of the porous media. Permeability values predicted by this model are in far better agreement with experimental values than any existing model. This, according to the author, is due to a better appreciation of the pore structure.

Models for Predicting Dewatering Characteristics of a Cake

Dewatering models have been developed to predict the kinetics of the processes, to evaluate the way the cake moisture content varies with time and the effects of altering process variables. If the dewatering

time is not limited, then equilibrium will be established and then the residual moisture content or residual saturation of the cake can be calculated from the correlations for residual saturation discussed above. However this is not the case usually, hence prediction of the drainage kinetics becomes important. In order to be able to predict kinetics of dewatering, it is essential to have a realistic description of the relative permeability of the draining cake.

Relative permeability of a porous medium to a fluid is defined as the ratio of flow of that fluid to the corresponding rate of flow if that fluid filled the whole of the void spaces. Relative permeability for a fluid is expressed by

$$K_{Rf} = \frac{K_f}{K} \quad (29)$$

where

K_{Rf} : relative permeability of a porous medium to the fluid f

K_f : permeability of the porous medium to the fluid f

K : permeability of the porous medium

Some theoretical models are proposed to calculate relative permeability of a porous medium. Capillary model with interconnection of pores is discussed below (21).

Capillary Model

The porous medium is assumed to be composed of a bundle of capillary tubes whose radii r have a distribution function $\alpha(r)$. This bundle is assumed to be cut into a large number of thin slices, the short tubes in each slice rearranged randomly, and the slices reassembled. For such a model, the relative permeability of porous media to wetting fluid is estimated from the expression

$$K_{Rw} = s_r^2 \frac{\int_{s_\infty}^S \frac{ds_w}{[p_c(s_w)]^2}}{\int_{s_\infty}^1 \frac{ds_w}{[p_c(s_w)]^2}} \quad (30)$$

where

s_r : reduced saturation

$$= \frac{S - s_\infty}{1 - s_\infty} \quad (31)$$

S : saturation of the filtered cake

$p_c(s_w)$: function relating reduced saturation to capillary pressure, and known as capillary pressure-saturation curve.

Capillary pressure p_c is a pressure at which the non-wetting fluid occupies all pores larger than radius r , defined by

$$r = \frac{2 \sigma \cos \theta}{p_c} \quad (32)$$

The capillary pressure-saturation curve can be determined experimentally in a permeability cell. In these experiments, the cake is formed in a permeability cell. Then a known capillary pressure is applied to the system and the volume of water displaced after a long time (so that equilibrium is established) is measured. The pressure is then increased by a small amount and the additional water displaced is measured. From the water displaced and the applied capillary pressure, capillary pressure-saturation curve can be generated. And from this curve, the pore size distribution $\alpha(r)$ can be obtained using Equation (32).

Similarly for nonwetting fluid, the expression can be written as:

$$K_{RN} = (1 - s_r)^2 \int_{s_r}^1 \frac{d s_r}{[p_c(s_r)]^2} \bigg/ \int_0^1 \frac{d s_r}{[p_c(s_r)]^2} \quad (33)$$

Once the capillary-saturation curve is generated experimentally, the relative permeabilities of wetting and nonwetting fluids in the porous media can be determined.

One of the models, based on large number of drainage experiments with various porous media, for reduced saturation is

$$s_r = (p_b/p_c)^\lambda \quad \text{for } p_c \geq p_b \quad (34)$$

where λ is pore size distribution index and p_b is the minimum pressure or threshold pressure needed to initiate displacement of the wetting fluid from porous medium. If these expressions are assumed to be valid, then from Equation (30) and (34)

$$K_{RW} = (p_b/p_c)^{2+3\lambda} \quad (35)$$

and

$$K_{RN} = \left[1 - \left(\frac{p_b}{p_c} \right)^\lambda \right]^2 \left[1 - \left(\frac{p_b}{p_c} \right)^{2+\lambda} \right] \quad (36)$$

The pore size distribution index λ will depend upon particle size distribution and the relative arrangement of the particles with respect to one another.

The capillary model or connectivity model (21) can be extended to calculate the kinetics of dewatering. Applying Darcy's law to a layer of the medium, one obtains (if $p - p_c$ is constant)

$$\frac{\Delta V}{\Delta t} = \frac{K_s A}{\mu L} (p - p_c)_s \quad (37)$$

where ΔV : volume of filtrate drained in time t

K_s : permeability of cake, at saturation, S , which can be related to relative permeability-saturation curve

A : cross-sectional area of porous media

L : thickness of porous media

Due to the filtrate drained, the saturation of the bed changes by an amount ΔS

$$\Delta S = \frac{\Delta V}{AL \epsilon} \quad (38)$$

Eliminating ΔV from Equations (37) and (38), an expression for the change in saturation of the medium with time is obtained as

$$\Delta S = \frac{K_s (p - p_c)_s \Delta t}{L^2 \mu \epsilon} \quad (39)$$

The model shows relative importance of various parameters during the dewatering operation.

Bundle of Capillary Model

Gray (64) assumed the porous media to be made up of a bundle of circular capillaries of known diameter and obtained an equation for dewatering curve as

$$V_t = \sum_{n=1}^n \Delta V_n \left[1 - \frac{2(p - p_c) r_n^2}{3\mu L_e (48 L_e + R_m r_n^2)} \right] \quad (40)$$

where V_t : volume of filtrate held per unit volume of coal after time t from saturation

ΔV_n : volume of water held in capillary of radius r_n , being emptied at suction p_c

L_e : effective length of capillaries

R_m : resistance of the filter medium per unit area

In applying this equation, (1) the knowledge of the pore structure in the filter cake is essential, which can be obtained from capillary pressure curve for the cake, and (2) the effective length of the bed should be known. This length will be greater than the thickness of the bed by a factor, ξ , called the tortuosity factor. It is difficult to obtain a numerical value of ξ . In practice, a value of 1.5 has been suggested.

Film Drainage Model

Another model for dewatering (73) is proposed based on the mechanism by which drainage under gravity takes place in packed beds.

Network Models for Dewatering

As discussed above, a network of capillaries provides a reasonably realistic model for a porous media. Network models were reviewed and their application to dewatering has been considered (51).

Data for kinetics of dewatering are very scarce, so it is difficult to compare the models. However, the network models appear promising and their use in flow through porous media is increasing in the last few years.

Enhancement of Dewatering

Two important aspects of dewatering are (1) rate of dewatering and (2) final moisture content of the discharged solids. It is desirable to have a high rate of dewatering and a moisture content approaching equilibrium moisture content.

From Equation (24) and (37), the following general conclusions can be drawn:

Rate of dewatering increases as:

- (1) driving force increases,
- (2) permeability of the medium increases,
- (3) viscosity of filtrate decreases,
- (4) area of filter media increases, and
- (5) thickness of cake decreases.

Residual saturation will decrease with:

- (1) increasing driving force,
- (2) decreasing surface tension of filtrate,
- (3) increasing contact angle, and
- (4) increasing bed permeability.

Of these factors, driving force will be dictated by process conditions and economics. Area of filter media will also be determined by the economics of process. Thickness of cake will be determined by geometry

of the filter and solid concentration in the slurry. Permeability of the bed can be increased by adding some flocculants to the slurry. Effect of the surface tension on the dewatering is discussed above.

Silverblatt and Dahlstrom (60) have found that viscosity affects dewatering more than surface tension. A significant decrease in residual moisture content of the cake can be observed due to surface tension reduction only when equilibrium conditions are reached. Normally the time of dewatering in commercial units is not long enough to approach equilibrium, so that significant changes in residual moisture content would not be observed. Lloyd and Dodds (59) from a simple analysis come to the conclusion that although viscosity of liquid retained in the cake is important when the time permitted for liquid removal is small, filtrate cannot be displaced by blowing or sucking air through the cake normally for particles below 50 μm , as the air pressure required will exceed one atmosphere. For particles above 50 μm in size displacement of filtrate in pores by air is more likely to lead to a lower moisture content than any attempt to change the structure of cake by compression. For particles below 50 μm in size, the most efficient method of removing liquid will be compression. Viscosity effects will be important for particles above 50 μm in size and surface tension will play an important role for particles below 50 μm in size.

Another factor which will affect dewatering is the receding contact angle of filtrate in capillaries of the cake. Gray (64) while studying the dewatering of fine coal particles observed that the residual moisture content of cake decreased when the coal was coated with a layer of oil. This, he concludes, may not be due entirely to the agglomeration of coal particles, but to the increased contact angle. The effect of agglomeration on coal dewatering was studied by other workers (78-80). In most cases enhancement of dewatering rate is observed.

Methods of Reducing Viscosity to Aid Dewatering

Reduction in viscosity of the filtrate will help in increasing the rate of filtration and dewatering. Heating the cake with hot air, heating the slurry (81), adding low viscosity liquid to viscous filtrate (53), using hot surfactant solution (54, 82), and introducing steam into the cake (83-90) are some of the ways by which viscosity of filtrate is decreased. Some of these are discussed above. From their experiments on pilot-scale filtration, Wilson and Miller (82) found that:

1. Heating the feed slurry reduced the cake moisture by 1.5 to 2 percent and also increased the filtration rate.
2. Steam assisted filtration decreased cake moisture by 3 to 4 percent.
3. A combination of surfactant-assisted filtration and filtration with hot slurry lowered the cake moisture by 1.5 to 2.5 percent. Their experiments were carried out on pilot plant scale with feed containing coal particles of 3/8 inch to 28 mesh size, on a conventional disk filter.

Heating with hot air (84) is not very effective in the case of coal cakes because hot air carries only a relatively small amount of sensible heat and the permeability of most coal filter cakes is low so that the quantity of heat which could be transferred would be of little value. Various types of radiant heating devices have been used for coal dewatering. They can transfer considerable quantities of heat, but particles on the surface of the cake shield those underneath so that only the surface layer is heated. Besides, the use of radiant heat is likely to degrade the coal product and is inherently hazardous.

Among all these processes, steam dewatering appears most promising for reducing the viscosity of filtrate. Some important aspects of steam dewatering are discussed below.

Steam Assisted Dewatering

As compared to radiant heating or heating with air, steam penetrates the filter cake and condenses on the particles, where its heat of condensation is released. This rapid heat transfer in the capillary bed results in the formation of a "condensation front." This front which is made up of condensed steam and residual liquid seals the pores of the cake as it drains toward the filter medium and prevents raw steam from passing directly into the filter drainage passages. If it is assumed that solids are at essentially equilibrium temperature by the time the "condensation front" passes, a temperature gradient varying from the atmospheric boiling point of water on the outside (due to condensation of steam) to the vacuum equivalent boiling point at the filter medium will be established.

Steam filter cakes contain considerable sensible heat. If these cakes are cooled adiabatically (89), this heat can be used to evaporate some of the residual moisture content. It appears that the reduction of cake moisture content depends directly on the permeability of the filter cake. Final moisture content of steam dewatered adiabatically cooled cake is correlated to percentage of -200 mesh particles in the cake (84). With up to about 50 percent of -200 mesh particles in the cake, the reduction in moisture content obtained by steam dewatering followed by heated air evaporation is as much as 20 percent of that contained in an ambient air dried filter cake.

Laboratory and pilot plant tests indicate (84) that steam consumption will probably range from 0.75 to 1.0 kg of steam per kg of water removed. Energy requirement for steam dewatering is nearly half of that needed in thermal drying.

Iwasaki, et al. (90) developed a centrifuge in which steam can be introduced and vacuum can be applied to the system. When they used steam in the centrifuge, they obtained a reduction of about 6 percent in moisture content in dried cake over that obtained by centrifuge.

Comparison of steam dewatering with conventional dewatering aided by surfactants reveals that steam dewatering is more effective than

surfactant aided dewatering. However, as pointed out by Lloyd and Dodds (59) for particles of smaller size (50 μm or less) surfactant aided dewatering will be more effective than steam dewatering.

Ultrasonics and electrofication by direct current are also used for aiding dewatering. The findings of this method of dewatering are discussed below.

Ultrasonics

Otsuka and Fairbanks (91) studied the dewatering of fine coal particles with ultrasonics. They investigated two systems: (1) airborne ultrasonic radiation near the surface of the filter cake, and (2) ultrasonic vibration of the filtration membrane. In both of these cases, they found that ultrasonics assisted dewatering resulted in drier coal cakes. According to the authors the effect of ultrasonics application to the system was to reduce the liquid friction flow through the filter cake pores, as the viscous flow through cake channels was changed to plug type of liquid flow. Kowalska et al. (92) while investigating the dehydration of sludge with ultrasonics in the presence of flocculants observed some positive effect. They suggested that "this positive effect, resulting in improved filtering properties of mineral sludge, should be understood to result from the destructive and dispersive activity occurring in conditions of cavitation. Such an effect of ultrasonic waves decreases the hydration forces, surface tension, adhesion and cohesion forces and electrokinetic potential, and brings about changes in the intermolecular forces of orientation and so on. Ultrasound may, therefore, set free the bound water and facilitate the orientation of dipoles and electrically charged groups. This may influence the packing of particles in the solid phase and cause it to form a more open structure of filter bed. The final effect of all these complex changes is an improvement of the filtering properties." Fairbanks and Hackett (93) and Fairbanks (94) found that application of ultrasonics increases the rate of filtration to a great extent. Fairbanks and Hackett (93) suspect that the observed increase in filtration rate of slurry containing iron and limestone may be due to coagulation of these particles. Ultrasound application is also known to aid in drying of fine coal particles (95-99). However, recently experiments conducted with ultrasonics to aid dewatering (51) show insignificant effect of ultrasonics on dewatering.

Dewatering by Direct Current

Laboratory tests using direct current to dewater and densify fine-grained coal sludge were performed in order to alleviate disposal problems, and to recover high energy value fuel (100). A slurry of very fine particles (65 percent less than 0.1 mm in diameter) was reduced from 55 percent (dry-weight) moisture content to 20 percent (dry-weight) moisture content by application of dc current for three hours. The dewatering in presence of dc current takes place due to electro-osmosis.

Any material which carries an electrostatic charge relative to the retained water in the cake can be dewatered and consolidated by applying dc current. Consider a negatively charged surface with respect to the water. There will be an electrical double layer formed near the interface of the solid particle and the water. If a potential field is applied between two surfaces of the cake, ions closer to the surface near the particle (in this case positive ions) will migrate towards one of the electrodes (cathode in this case). These ions will carry along the water with them. This is the principle of electro-osmosis, and this principle is used in practice to accomplish dewatering by creating a drainage area at or near the cathode. Quantitative prediction of dewatering rates in a cake is limited by the non-ideal character of the cake.

One big advantage of this technique is that in contrast to hydraulic flow, electro-osmotic flow is essentially independent of pore size, thus, it may be a superior method for dewatering fine size coal cakes.

Electro-osmotic permeability of a cake has been observed to decrease by an order of magnitude as water content decreases (100). As dewatering proceeds the rate of dewatering will decrease approaching zero in the limit. Another important factor in electro-osmotic dewatering is the energy consumption. Before applying this technique to dewater any material, economics of the process should be examined critically.

Equipment Used for Dewatering

Normally vacuum filters and centrifuges are used for dewatering of a cake. Occasionally pressure filters are also used. The factors which are important in selection of filtration equipment of various kinds are discussed at length in several sources (1, 6-8, 11-13, 19, 20, 101, 102). Bosley (103) and Osborne (104) have discussed the various types of vacuum filters and their uses in great detail. Recently Rothwell (105) and Svarvosky (20) have reviewed some of the new equipment which has gained popularity in filtration and dewatering. Some of this equipment is described below.

Vacuum filters

Two types of vacuum filters which are very widely used for dewatering are (1) rotary (drum type) vacuum filter and (2) disk type vacuum filter. In these filters, filtration, washing, dewatering, and discharge of the cake all take place automatically.

In the drum filter, the drum rotates with its circumference dipping into a tank of slurry from which the filtrate is drawn and the cake deposited on the drum surface. As the drum rotates, the cake is dewatered by displacing the water by air and is eventually discharged. The method of discharging the cake is of great importance in maintaining a clean cloth. Methods used to remove the cake are (1) scrapping it off with a knife, (2) discharging it by means of endless strings, and (3) blowing it off by air pressure.

The performance of drum filters is controlled by adjustments in three main variables--drum speed, vacuum, and submergence. The time for dewatering will depend upon the drum speed and it is of the order of 0.5 to 5 minutes. Though the initial cost of the drum filter is high, operating costs are low and it is clean in operation.

A rotary disc filter consists of a number of flat filter elements mounted on a central shaft and connected to a normal vacuum filter valve. As the panels rotate they go through pick-up and dewatering operations similar to those carried out on drum filters and at the discharge point the cake is removed by means of wires or knives. These filters are less costly than the drum filters. However, they do not give as dry a cake as the drum filters.

To design a drum filter, the basic filtration equation, Equation (12) is used. In most cases, the cloth resistance is neglected in comparison to cake resistance. The total cycle time is divided into several periods according to the location of various zones. Very few people (13) have tried to design and predict the behavior of the filter in the dewatering zone. Much of the design is yet done empirically.

Centrifuges

Recently Zeitsch (106) has reviewed centrifugal filtration in great detail. In centrifugal filtration, slurry is fed to a rotating basket having a slotted or perforated wall covered with a filter medium such as canvas or metal cloth. Pressure resulting from the centrifugal action forces the liquid through the filter medium, leaving the solids behind. If the feed to the basket is then shut off and the cake of solids spun for a short time, much of the residual liquid in the cake drains off the particles, leaving the solids much "drier" than those from a filter press or vacuum filter. When the filtered material must subsequently be dried by thermal means considerable savings may result from the use of a centrifuge.

The better performance of centrifugal filtration can be understood by considering the principal modes of filtration (106).

Modes of Filtration (106)

In most cases, the direction of the filtrate flow is perpendicular to the filter medium. Conventional filtration corresponds to the case of vector 1 which implies that the movement of the dispersed solid phase is in the same direction as the filtrate flow. In general, processes of this type become slow and uneconomic whenever a sizable fraction of the particles are smaller than 10 μm . In such cases, the principal resistance opposing the flow of the liquid is caused by the conglomerate of particles representing the cake and not by the filter medium. Consequently, better filtration will result if the particle is forced to move in vectors of direction 2 and 3 (figure 1). When the particle movement is in the direction of vector 2, as in ultra-filtration, cake formation is prevented and the liquid phase is permitted to permeate through the filter medium. The disadvantage of this process is its inability to yield the solids in a drained state. The suspension undergoes a mere thickening.

If drainage of the solids is essential in spite of the particles being very small, the only recourse is the bypass concept. Its principal feature is the delayed cake formation by subjecting the system to forces which would cause the solid particles to move away from the filter medium while inducing the liquid (filtrate) to pass through the medium.

Since a large force is required to produce rapid movement of small particles relative to the surrounding liquid, the gravitational force is considered too weak to cause the required relative motion. Besides, the direction of gravity cannot be manipulated. The only practical application of the by-pass concept is limited to systems based on centrifugal field.

Zeitsch (106) describes a side-filter centrifuge that combines conventional centrifugal filtration in a cylindrical basket with filtration through baskets walls. From his theoretical analysis, he claims a 35 percent reduction in filtration time in such a filter.

He also described a planetary filter centrifuge in which the particle moves in the direction of vector 3. This filter has two or more baskets rotating about its axes, both of which spin on arms connected to a parallel central axis. This arrangement exerts centrifugal force on particles, keeping a portion of the medium clear and allowing the filtrate to bypass the cake.

The flow of fluid in a centrifugal filter can be correlated to the various forces acting on it as follows:

$$Q = \frac{K (F_p + F_c)}{\epsilon L} \quad (41)$$

where Q = flow of filtrate through the cake

F_p = pressure force

F_c = centrifugal force

ϵ = porosity of cake

L = thickness of cake

K = permeability of cake

Equation (41) is a modified form of Darcy's equation for flow through packed beds, with

$$F_p = \frac{\Delta P \cdot A \cdot \epsilon}{\mu} \quad (42)$$

ΔP = pressure gradient across the cake

μ = viscosity of filtrate

The centrifugal force acting on the liquid in the pores from the rotation at the angular velocity ω becomes

$$dF_c = 2\pi r W \epsilon \rho_w \epsilon \omega^2 dr / \mu \quad (43)$$

Also for this case

$$P = 0 \quad \text{at } r = r_B \quad (44)$$

$$P = P_s \quad \text{at } r = r_s$$

From Equations (41-43), using (44) one obtains

$$Q = \frac{2\pi KW}{\mu \ln(r_B/r_s)} \left[P_s + \frac{\rho_w \omega^2 r_B^2}{2} \left(1 - \frac{r_s^2}{r_B^2} \right) \right] \quad (45)$$

Iwasaki, et al. (90) modified the design of conventional centrifuges and provided for vacuuming the centrifuge or introducing steam into the centrifuge. With these modifications they were able to obtain better dewatering.

Continuous Pressure Filters

Usually a pressure filter is used in a batch operation so that the filter can be dismantled and the cake removed. Thus, the use of pressure filters in cake dewatering was limited. However, nowadays continuous pressure filters are also available (107-108). This permits their use in dewatering. In these filters (which are very similar to a plate and frame filter), cake of constant thickness is discharged continuously. Bagdasarín (108) claims that this filter would give better performance than conventional plate and frame filters and rotary vacuum filters.

Recently variable chamber plate (20) and frame filters, belt filters (109-111), and various modifications of these filters (20) have been used for dewatering.

Variable Chamber Filters

The principle behind these types of filters is to compress the slurry uniformly so that more or less uniform porosity is maintained in the cake. As is known the pressure exerted varies from a minimum at the surface of the slurry to a maximum at the filter medium. By exerting a pressure on the surface of the slurry, it can be contracted to give a uniform porous structure. Then drier cakes can be obtained by simply raising the feed pressure. In most designs, the compression is accomplished by inflating a diaphragm, which presses the freshly filtered cake toward the medium. In another design, compression is achieved by a hydraulic ram (20).

Belt Filters

In belt filters the resistance of the cake is controlled by limiting the growth of the cake. This is known as "delayed cake filtration," and its theory is discussed in detail by Tiller and Cheng (112). Methods for limiting cake growth can be classified as follows (20):

1. Removal of cake by gravitational or centrifugal forces,
2. Removal of cake mechanically,
3. Dislodging of the cake by intermittent reversal of flow,
4. Prevention of cake deposition by vibration, and
5. Cross-flow filtration in which slurry moves tangentially to filter medium, causing the cake to be continuously sheared off.

In belt filters the growth of cake is limited mechanically. The cake is also compressed by passing it between rollers. This aids in dewatering. In Europe, belt filters are said to have shut out virtually all other mechanical dewatering methods for sludge treatment (109). Various problems arising in the construction of belt filters are discussed at length by Blenduff and Bond (111). One problem with belt filters is the cost of the belts. In many cases it is found that the wear and tear of belts is quite substantial, which calls for replacement of the belt. The cost of a new belt prohibits its frequent replacement. This factor should be given consideration when a belt filter is to be used for an operation.

DEWATERING OF FINE COAL/REFUSE

Changes in mining methods, environmental regulations, and market requirements as well as the mining and cleaning of poorer grades of coal have focused attention on the need to develop more effective dewatering operations in coal preparation plants. This section presents a brief review of current development and problem areas in connection with the dewatering of fine coal/refuse.

Filtration and Dewatering of Fine Coal

The general principles of filtration and dewatering discussed in previous sections are equally applicable to the filtration and dewatering of fine coal. However, the heterogeneous nature of coal and the size of particles in fine coal distinguish it from any other filtration or dewatering problem.

Filtration Characteristics of Fine Coal

As discussed before, one of the important parameters in any filtration operation is the size distribution of particles in the feed slurry. The presence of very small particles (less than 5 microns), even in small amount, would significantly affect the structure of the cake. The

presence of very small particles leads to a cake containing a large number of small pores. These small pores offer more resistance to the flow of filtrate and also tend to trap water in the cake. As a result, the rate of filtration decreases significantly and a cake with high moisture content is obtained. For a well characterized coal slurry, the filtration rate under a given pressure drop can be calculated by using the design equations (19-20).

Methods of increasing filtration rate, as discussed in the previous chapter, can also be applied to filtration of fine coal. However, some of these methods are not feasible for filtration of coal. Addition of flocculating agents to increase the filtration rate needs careful consideration. A few points which should be considered in this respect are (1) cost of the flocculating agents, and (2) effect of flocculants on downstream processes. Heating of feed slurry to increase filtration rate needs careful economic evaluation. Quantitative information on these methods of increasing filtration rate, for filtration of fine coal slurry, is scarce.

Some preliminary experiments carried out on the measurement of overall filtration rate for coal slurry containing fine coal (-28 mesh) indicates that the cake formed by these small coal particles can be considered incompressible (107).

Dewatering of Fine Coal

The presence of small particles makes fine-coal dewatering a difficult problem. As discussed previously, higher desaturation forces would be necessary to obtain satisfactory dewatering rate from the cake formed by fine coal.

Dewatering rate can be improved by adding steam to the feed slurry or by modifying the surface characteristics of the coal cake. Steam dewatering of the coal cake is studied by several investigators (76-82). Addition of steam to coal cake heats up the water retained in the cake and hence decreases the viscosity of water considerably. Reduction in viscosity increases the rate of dewatering. The important parameters in steam dewatering are determined to be particle size in slurry, solids concentration of coal in slurry, operating vacuum and the steaming rate (105). The energy requirement in steam dewatering is very much less than that needed in thermal drying. However, from the available data on steam dewatering (77), it is observed that up to 40 percent of total operating cost, is due to steam requirements in many cases. Thus, the economics of steam dewatering need to be examined critically.

As pointed out earlier, methods to modify surface characteristics of coal cakes might play an important role in dewatering of coal cake containing a large proportion of small particles. One approach in modifying the coal surface is to add surfactant to the coal. Surface characteristics of coal will largely depend upon the nature of the coal, its rank, degree of oxidation, etc. (55). Some electrophoretic data (56) suggest that the coal surface is negatively charged at

neutral pH. When anionic surfactant in very small amounts is added to a coal cake, it will be adsorbed on the coal easily. This makes the coal surface hydrophobic and hence helps in dewatering.

Dewatering of Refuse

Disposal of refuse in a coal preparation plant poses a unique problem. The refuse does not have any end uses, so in most cases it is dumped into the ponds. Refuse contains harder coarser particles (rocks, minerals, etc.), which are separated from the ROM coal in the initial separation stages, and very fine particles (pyritic sulfur, ash, tailings from flotation cells, etc.), which are separated from the coal as it is being treated in the preparation plant. The quantity of refuse produced in a large plant is quite substantial; for example, in a large complex plant of 1,600 tph capacity, as much as 700 tph of refuse is produced (10%).

The hard coarse particles do not present any problems and they can be easily dewatered and disposed of by dumping. The fine clay type particles (particularly of submicron size) are very difficult to treat. These particles are often obtained as a slurry from a number of wet processing units such as froth flotation cells, hydrocyclones, centrifuges, etc. The concentration of solids in such a slurry can be as low as 2 to 5 percent by weight. The need to conserve water and to meet environmental regulations requires that the solids be removed from this slurry.

The normal practice is to treat this slurry with some flocculating agents. The addition of a flocculating agent promotes agglomeration of particles and helps in the formation of large flocs. Effects of various flocculants on agglomeration and dewatering of fine coal/refuse particles have been studied by several investigators (70-73, 110). A wide variety of polymers are used as flocculants. In some cases, oil is also used as an agglomerating agent, but high costs of oil prevents its use on a widely spread basis. Another cost factor in the use of flocculants is an efficient mixing chamber to mix flocculant with the slurry (70). This operation also requires considerable energy.

After treating the refuse with flocculants it is allowed to settle in thickeners. In the thickeners, a concentrated slurry of flocculated refuse is separated from water. The water separated can be recycled in the plant. The concentrated slurry is then dewatered in settling ponds and vacuum/pressure filters. Normally, filter presses are used because a large driving force can be applied in the filter press. Rotary vacuum filters and solid bowl centrifuges are also used, but to a lesser extent.

The partially dewatered refuse contains about 40 percent by weight of water and its disposal at this stage is still a problem. The solid refuse does not have any end use so further dewatering in thermal driers cannot be justified. In most cases, this partially dewatered refuse is dumped into ponds, (this requires substantially large pondage area), and in some cases incineration has been suggested.

In a coal preparation plant, expensive operations for treating refuse cannot be justified because there is no economic incentive to dewater refuse. Thus, further research in devising more effective and less expensive dewatering technique for refuse dewatering is essential. Currently, horizontal belt filters (102) and pressure filter presses have been used for dewatering refuse with some success, yet further study is required in this area.

CONCLUSIONS

In conclusion, the following points can be noted:

1. The established theory of filtration is of a semi-empirical nature. At present, characteristics of filtration operation, such as rate of filtration, final moisture content of the cake, etc., cannot be obtained from the fundamental properties of the slurry, such as size distribution of particles in the slurry, solid concentration, etc., for a given driving force.
2. Treatment of dewatering operations is also semi-empirical in nature. However, in the last few years attempts to understand the mechanism of filtration and dewatering have increased, and some attempts to correlate filtration and dewatering characteristics with the properties of slurry and cake have appeared in the literature. Three dimensional network models appear to be very promising in dewatering studies.
3. In recent years, new filtration equipment, such as horizontal belt filters, variable chamber filters, steam and vacuum centrifuges, etc., have been introduced to the industry for achieving better filtration and dewatering. Some of these are quite successful.
4. Some problems of modern coal preparation plants can be listed as:
 - (a) Increased production of fine (-28 mesh) coal.
 - (b) Need to dewater fine coal/refuse.
 - (c) Need to reduce or eliminate thermal drying of fine coal.
 - (d) Disposal of refuse.

As far as coal dewatering is concerned, the following research areas are suggested:

1. Fundamental study of dewatering operation.
 - (a) Use of three dimensional network models.
 - (b) Characterization of coal particles, i.e., to determine particle size distribution, particle shape factor, etc.
 - (c) Equilibrium properties of a coal-cake, i.e., determination of residual moisture content, breakthrough pressure, desaturation curve, etc.

- (d) Rate of dewatering and effect of various parameters such as particle size distribution, concentration, driving force, etc., on the rate.
2. Enhanced dewatering of coal cake.
- (a) Application of a temperature gradient across the cake to establish a surface tension gradient.
 - (b) Addition of surfactant to the slurry and its effect on the dynamics of dewatering.
 - (c) Steam dewatering and its economics.
 - (d) Use of new filter equipment and evaluation of equipment performance.
3. Dewatering and disposal of refuse.
- (a) Effect of flocculants on agglomeration of refuse.
 - (b) Settling characteristics of flocculated refuse.
 - (c) Characterization of flocculated refuse.
 - (d) Dewatering of concentrated refuse in filter presses.
 - (e) Application of three dimensional network models to dewatering of refuse.

NOMENCLATURE

A	: Area of filter
a	: Ratio of centrifugal to gravitational acceleration
C	: Concentration of solids
C _s	: Concentration of solids in slurry
F _c	: Force on particle due to centrifugal field
F _p	: Force exerted due to pressure gradient
g	: Gravitational constant
h	: Capillary rise height
h _c	: Capillary drain height
J	: Correction factor for specific cake resistance
K	: Permeability of filter cake
K _F	: Permeability of porous bed to the fluid F
K _{RF}	: Relative permeability of porous bed to fluid F
K _{RN}	: Relative permeability to non-wetting fluid
K _{RW}	: Relative permeability to wetting fluid
K _s	: Permeability of bed at saturation S
k ₁	: Numerical constant in Kozeny-Carman equation
L	: Thickness of cake
Le	: Effective length of capillaries
m	: Mass of solids in layer of thickness L
m _c	: Total mass of solids in dry cake
m _f	: Mass of wet cake
P _b	: Threshold pressure

P_c	: Capillary pressure
P_f	: Applied fluid pressure
Q	: Flow rate of filtrate through cake
R_c	: Capillary number
R_m	: Filter medium resistance
r	: Radius of capillary
r_B, r_o	: Radius of cake in centrifuge at filter medium and surface, respectively
S	: Saturation
S_{av}	: Average residual saturation
s_r	: Reduced saturation
s	: Fraction of solids in slurry
s_p	: Surface area of single particle
s'	: Compressibility coefficient for cake
s_∞	: Residual equilibrium saturation of cake
t	: Time
u	: Linear velocity of filtrate, based on filter area
V	: Volume of filtrate collected from time zero to time t
V_p	: Volume of a single particle
V_t	: Volume of filtrate held per unit volume of coal after time t
W	: Height of centrifuge
W_∞	: Residual moisture content
α	: Specific cake resistance
α_o	: Numerical constant

$\alpha_{av}, \alpha_{mean}$	: Average value of specific cake resistance
ϵ	: Porosity of filter cake
ϵ_{av}	: Average porosity
ϵ_x	: Porosity at a distance x
μ	: Viscosity of filtrate
ρ_p	: Density of solid particles
ρ_w	: Density of water
σ	: Surface tension
ξ	: Tortuosity factor
θ	: Contact angle
λ	: Pore size distribution index, constant
ω	: Angular velocity

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