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THREE-DIMENSIONAL SIMULATIONS
OF
HARMONIC RADIATION AND HARMONIC LASING *

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ABSTRACT

Characteristics of the harmonic emission from free-electron lasers (FELs) are examined in the spontaneous, coherent-spontaneous and stimulated emission regimes. The radiation at both odd and even harmonic frequencies is treated for electron beams with finite emittance and energy spread. In the spontaneous emission regime, the transverse radiation patterns including the transverse frequency dependences, are given. How this expression is modified to include energy spread and emittance is described. In the coherent-spontaneous emission and stimulated emission (lasing) regimes, the interaction of the radiation fields with the electrons must be treated self-consistently. Here, a single-frequency distributed transverse source function for each electron is used in the harmonic version (HELEX) of the 3-D code FELEX to model the harmonic radiation. This code has recently been modified to simultaneously model the fundamental and harmonic interactions for multiple-pass oscillator simulations. These modifications facilitate the examination of FELs under various operating conditions. When the FEL is lasing at the fundamental, the evolution of the harmonic fields can be examined. This evolution is unique in the sense that the electron beam (which is bunched by the fundamental optical field) radiates at the harmonic frequencies in the presence of the harmonic radiation circulating in the cavity. As a result,

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enhancements of the harmonic emission can be observed. Finally, harmonic lasing can occur in cases where there is sufficient gain to overcome cavity losses and lasing at the fundamental can be suppressed. The characteristics and efficiency of these interactions are explored.

I. Introduction

The interest in harmonic radiation from free electron lasers (FELs) has increased in past few years for several important reasons. The level of spontaneous and coherent-spontaneous harmonic power generated during fundamental lasing is of interest from both a mirror damage standpoint and in terms of the use of this radiation for diagnostics and external experimentation. Albeit broadband in frequency, spontaneous emission is the dominant radiation mechanism at the higher harmonics. This radiation diverges faster than the coherent-spontaneous radiation and can suffer significant interception and reflection from the wiggler vacuum tube wall. Its use as an alignment diagnostic can also be important.

The ability to lase at harmonics of the fundamental resonance wavelength would allow the generation of short wavelength radiation with low energy electron beams. Such systems become more important as the quality of electron beams improve and the desire for shorter radiation wavelengths and more compact devices increases. The prediction of the various harmonic power levels for FELs in these various operating regimes is the subject of this paper.

In Section II the mathematical models for the spontaneous emission from a single electron and from an entire electron beam (including emittance effects) are given. Computational examples using these models follow. In Section III a review of the model used to calculate coherent-spontaneous emission is given. Results displaying the effects of harmonic radiation circulating in the cavity are shown. Lasing at the harmonic frequencies is dealt with in Section IV. The energy equation including harmonic force terms is derived and the results used to perform third harmonic lasing simulations. The conclusions are given in Section V.

II. Spontaneous Emission

Electrons radiate spontaneously in a wiggler. Most of the radiation is contained in a forward directed cone of half angle a_w/γ . The far field radiation intensity for a single

electron was analytically derived by Colson¹. In his derivation he correctly included all terms up to order $(1/\gamma^2)$. For a plane-polarized wiggler of length $L_w = N_w \lambda_w$ and peak magnetic field strength B_w , driven by an electron beam of energy γmc^2 , the energy radiated by each electron of charge $q = -e$ per unit solid angle and frequency bandwidth is given by

$$\left. \frac{d^2 I_0}{d\Omega d\omega} \right|_f = \frac{16e^2 N_w^2 \gamma^2}{c} \left(\frac{\sin \nu_f}{\nu_f} \right)^2 \left(\frac{f\xi}{a_w} \right)^2 \left[\left(\frac{\gamma\theta}{a_w} \right)^2 A_{0,f}^2 + 2 \frac{\gamma\theta}{a_w} \cos \phi A_{0,f} A_{1,f} + A_{1,f}^2 \right] \quad (1)$$

where

$$A_{\alpha,f} = (-1)^{f+\alpha} \sum_{n=-\infty}^{\infty} (-1)^n J_n(f\xi) [J_{f-2n-\alpha}(f\sigma) + J_{f-2n+\alpha}(f\sigma)] \quad (2)$$

$$\nu_f = \pi N \left[f - \frac{\lambda_w}{2\gamma^2 \lambda_s} (1 + a_w^2/2 + \gamma^2 \theta^2) \right] \quad (3)$$

$$f\sigma = \frac{8(f\xi)(\gamma\theta)}{a_w} \cos \phi \quad (4)$$

$$\xi = \frac{a_w^2}{4(1 + a_w^2/2 + \gamma^2 \theta^2)} \quad (5)$$

and

$$a_w = \frac{|e| B_w \lambda_w}{2\pi mc^2} \quad (6)$$

is the normalized wiggler magnetic vector potential. The radiation wavelength is λ_s , f is the harmonic number and $J_m(z)$ represents the Bessel function of the first kind of order m . As depicted in Figure 1, it has been assumed that the wiggler lies along the z -axis and the observation angles θ and ϕ are formed by the intersection of the line of observation, call it $\hat{n}$, with the z -axis and the angle made by the intersection of the projection of $\hat{n}$ onto the xy -plane with the x -axis, respectively. It should be noted that other derivations^{2,3} omit an axial phase term, thereby obtaining an erroneous azimuthally symmetric result. The harmonic radiation pattern described in Eq.(1) has multiple lobes where the number of lobes is equal to the harmonic number. A cold electron beam (zero energy spread and emittance) has a radiation pattern that is identical in shape to this pattern.

The only frequency dependence in Eq.(1) is in the $\sin \nu_f / \nu_f$ term. For $N_w \gg 1$, this term is sharply peaked about $\nu_f = 0$, where the frequency becomes

$$\omega = \frac{2\gamma^2 f k_w c}{1 + a_w^2/2 + \gamma^2 \theta^2} \quad (7)$$

Figure 1. Relation between the cartesian coordinates, the line of observation and the spherical observation angles.

An on-axis observer ($\theta \Rightarrow 0$) sees the frequency

$$\omega_{\text{axis}} = \frac{2\gamma^2 k_w c f}{1 + a_w^2/2} \quad (8)$$

so that, normalizing with respect to this frequency, the relative frequency shift off-axis is given by

$$\frac{\omega}{\omega_{\text{axis}}} = \frac{1}{1 + \left(\frac{\gamma^2 \theta^2}{1 + a_w^2/2} \right)} \quad (9)$$

From Eq.(9) one sees that at the edge of the $\gamma\theta \simeq 1$ cone, the frequency has changed by roughly a factor of two. It should also be noted that since the frequency depends only on θ (and not ϕ), a specific frequency appears spatially as a “frequency ring”.

Spontaneous emission from an FEL is composed of the radiation from all the electrons in the electron beam. Since the electron beam has a distribution of electrons in energy, space and transverse velocity, some method must be devised to take these distributions into account in order to obtain the correct spontaneous emission pattern for a non-ideal electron beam. As a first step we can show that the spontaneous emission of an electron with a transverse drift (with respect to the wiggler axis) is identical (to order $1/\gamma^2$) in both intensity and frequency to the spontaneous radiation pattern of the on-axis electron. The only difference is that the drifting electron’s radiation pattern is rotated by $\vec{\beta}_{x0} + \vec{\beta}_{y0}$ away from the wiggler axis where $\vec{\beta}_{x0}$ and $\vec{\beta}_{y0}$ correspond to the drift velocities in the $\hat{x}$ and $\hat{y}$ directions, respectively. The velocity of an electron in a sinusoidal wiggler field including

transverse drift motion is given by

$$\vec{\beta}_e = \begin{pmatrix} \beta_{xe} \\ \beta_{ye} \\ \beta_{ze} \end{pmatrix} = \begin{pmatrix} \frac{a_w}{\gamma} \cos k_w z + \beta_{x0} \\ \beta_{y0} \\ \bar{\beta}_z - \frac{a_w}{\gamma} \beta_{x0} \cos k_w z - \frac{a_w^2}{4\gamma^2} \cos 2k_w z \end{pmatrix} \quad (10)$$

where

$$\bar{\beta}_z = 1 - \frac{1 + a_w^2/2}{2\gamma^2} - \beta_{\perp 0}^2/2 \quad (11)$$

and

$$\beta_{\perp 0}^2 = \beta_{x0}^2 + \beta_{y0}^2 \quad (12)$$

as given in Appendix B of reference [4]. We can use a rotation of coordinates to show that to order $1/\gamma^2$, the drift terms transform away. This can be done by first performing a rotation in the $\hat{x}$ - $\hat{z}$ plane where the transformed velocities (denoted by $\prime$) are given by

$$\begin{pmatrix} \beta'_{xe} \\ \beta'_{ze} \end{pmatrix} = \begin{pmatrix} \cos \theta_x & -\sin \theta_x \\ \sin \theta_x & \cos \theta_x \end{pmatrix} \begin{pmatrix} \beta_{xe} \\ \beta_{ze} \end{pmatrix} \quad (13)$$

followed by a rotation in the $\hat{y}$ - $\hat{z}$ plane with the transformation

$$\begin{pmatrix} \beta'_{ye} \\ \beta'_{ze} \end{pmatrix} = \begin{pmatrix} \cos \theta_y & -\sin \theta_y \\ \sin \theta_y & \cos \theta_y \end{pmatrix} \begin{pmatrix} \beta_{ye} \\ \beta_{ze} \end{pmatrix} \quad (14)$$

where θ_x and θ_y are defined in Figure 2.

Figure 2. Relation between the electron guiding center velocity, $\vec{\beta}$, its transverse drift speeds and the two angles of rotation.

Combining the two transformations into one⁵, the complete transformation can now be written

$$\begin{aligned} \begin{pmatrix} \beta'_{xe} \\ \beta'_{ye} \\ \beta'_{ze} \end{pmatrix} &= \begin{pmatrix} \cos \theta_x & 0 & -\sin \theta_x \\ 0 & 1 & 0 \\ \sin \theta_x & 0 & \cos \theta_x \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_y & -\sin \theta_y \\ 0 & \sin \theta_y & \cos \theta_y \end{pmatrix} \begin{pmatrix} \beta_{xe} \\ \beta_{ye} \\ \beta_{ze} \end{pmatrix} \\ &= \begin{pmatrix} \cos \theta_x & -\sin \theta_x \sin \theta_y & -\sin \theta_x \cos \theta_y \\ 0 & \cos \theta_y & -\sin \theta_y \\ \sin \theta_x & \cos \theta_x \sin \theta_y & \cos \theta_x \cos \theta_y \end{pmatrix} \begin{pmatrix} \beta_{xe} \\ \beta_{ye} \\ \beta_{ze} \end{pmatrix}. \end{aligned} \quad (15)$$

Now, since the electron's velocity is mainly in the $\hat{z}$ -direction, we can approximate

$$\sin \theta_{x,y} \simeq \beta_{x0,y0}/\beta_z \simeq \beta_{x0,y0} \quad (16)$$

and

$$\cos \theta_{x,y} \simeq 1 - \theta_{x,y}^2/2 \simeq 1 - \beta_{x0,y0}^2/2 \quad (17)$$

and denoting $\beta_x = (a_w/\gamma) \cos k_w z$ and

$$\beta_z = 1 - \frac{1 + a_w^2/2}{2\gamma^2} - \frac{a_w^2}{4\gamma^2} \cos 2k_w z \quad (18)$$

the transformation equation becomes

$$\begin{pmatrix} \beta'_{xe} \\ \beta'_{ye} \\ \beta'_{ze} \end{pmatrix} = \begin{pmatrix} 1 - \beta_{x0}^2/2 & -\beta_{x0}\beta_{y0} & -\beta_{x0} \\ 0 & 1 - \beta_{y0}^2/2 & -\beta_{y0} \\ \beta_{x0} & \beta_{y0} & 1 - \beta_{\perp 0}^2/2 \end{pmatrix} \begin{pmatrix} \beta_x + \beta_{x0} \\ \beta_{y0} \\ \beta_z - \beta_{x0}\beta_x - \beta_{\perp 0}^2/2 \end{pmatrix} = \begin{pmatrix} \beta_x \\ 0 \\ \beta_z \end{pmatrix} \quad (19)$$

where we have kept only terms up to order $\mathcal{O}(1/\gamma^2)$ and assumed $\beta_{x0,y0} \propto 1/\gamma$. As seen from Eq.(19), the drift terms have been transformed away leaving expressions identical to those for an electron on axis. Therefore, the radiation *and frequency* pattern of a drifting electron is identical to an on-axis electron, with only a shift of the pattern by $\vec{\beta}_{\perp 0}$. (An additional assumption $\lambda_w \simeq \lambda_w(1 + \beta_{\perp 0}^2/2)$ has also been made.)

With the previous result in mind, we can postulate that the radiation pattern from an ensemble of electrons is given by

$$S(x', y') = \frac{\int dx \int dy e^{-(x^2+y^2)/\bar{r}^2} \int d\beta_x \int d\beta_y e^{-(\beta_x^2+\beta_y^2)/\bar{\beta}_{\perp}^2} S[x' - (x + \beta_x z), y' - (y + \beta_y z)]}{\int dx \int dy e^{-(x^2+y^2)/\bar{r}^2} \int d\beta_x \int d\beta_y e^{-(\beta_x^2+\beta_y^2)/\bar{\beta}_{\perp}^2}} \quad (20)$$

where S is the single-electron radiation pattern and z is the axial observation point. In Eq.(20), gaussian distributions for the electrons' transverse spatial and velocity dependencies have been assumed. This expression can be manipulated into the form

$$S(x', y') = \mathcal{F}^{-1} \left\{ e^{-.25(\bar{r}^2 + \bar{\beta}_{\perp}^2 z^2)(k_x^2 + k_y^2)} \tilde{S}(k_x, k_y) \right\} \quad (21)$$

where $\mathcal{F}^{-1}\{ \}$ represents the inverse Fourier transform and $\hat{S}$ is the Fourier transform of S given by

$$\hat{S}(k_x, k_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dx dy S(x, y) e^{-i(k_x x + k_y y)} \quad (22)$$

In the far field we can assume

$$S = \left. \frac{dI}{d\Omega d\omega} \right|_f \quad (23)$$

and $\bar{r} = 0$ so that the radiation pattern for the electron beam becomes

$$S(x', y') = \mathcal{F}^{-1} \left\{ e^{-.25\beta_{\perp}^2 z^2 (k_x^2 + k_y^2)} \left. \frac{\widetilde{dI}}{d\Omega d\omega} \right|_f \right\} \quad (24)$$

where again, the tilde represents the transformed quantity. Using Eqs.(1)-(6) and (24) the spontaneous emission for a single electron or an entire electron beam with a given energy spread and emittance can be obtained.

The previously described spontaneous emission model has been incorporated into the code SPEMIT. A finite summation is performed providing an excellent approximation for Eq(2). After Eq(1) has been calculated on an x - y grid, an FFT routine is used to obtain its Fourier transform. Multiplication by the angular divergence factor is then performed, after which the FFT routine is used to perform the inverse transform yielding the radiation pattern for the electron ensemble. An example of the code output is given in Figure 3. Here the far-field single-electron radiation pattern at the sixth harmonic is depicted for the 5.0 meter wiggler on the Argonne Advanced Photon Source (APS). Other parameters used for the simulation were: $\lambda_w = 3.3\text{cm}$, $\gamma = 13700.$, $B_w = 3788.0$ gauss, and $I = 0.1$ Amps. The frequency dependence, as given in Eq.(7), is shown as color variation over the plot. Note that the number of maxima is equal to the harmonic number.

The spontaneous emission pattern for the entire electron beam can also be calculated. When emittance effects are included ($\epsilon = 4.6 \times 10^{-6}\text{cm-rad}$ gaussian $1/e$), the radiation pattern evolves from that given in Figure 3 for the single electron to that given in Figure 4 for the electron ensemble. Note that the fine structure has been washed out by the electron's transverse velocity distribution. The spontaneous emission power for an FEL should always be calculated since it may exceed the coherent-spontaneous power in devices with poor emittance.

III. Coherent-Spontaneous Emission

Spontaneous Emission (bandwidth integrated/cold beam)

Figure 3. Plot of the single-electron spontaneous emission pattern at the sixth harmonic for the Argonne APS. Frequency changes are depicted in color.

Figure 4. Spatial plot of the sixth harmonic spontaneous emission intensity for the Argonne APS wiggler. The distribution of transverse velocities due to emittance has smeared the single-electron pattern.

As previously mentioned, coherent-spontaneous emission arises due to the bunching of the electron beam. In an oscillator or amplifier this bunching is normally produced by the interaction of the electrons with radiation at the fundamental frequency. Since the bunching is a non-linear process, small axial density components are produced at the harmonics of the fundamental bunching wavelength. Coherent harmonic emission can then take place, where the strength of the radiation source is proportional to the amplitude of the density component at the harmonic of interest, and the sum over transverse space of the distributed sources from each electron. The analytical form of the distributed electron sources has been previously derived⁴ and only the results of the formalism are of interest here. The equation describing the evolution of the optical electric field E_f with wavenumber, $f k_s$, is given by

$$\left[2i f k_s \frac{d}{dz} + \frac{\partial^2}{\partial \bar{r}_1^2} \right] E^f = \sum_j^{1,2,3} S_j(y) \quad (25)$$

where the source terms on the right-hand side are given in cgs units by:

- Mono-pole term

$$S_1(y) = \left\langle \frac{i 4 \pi I f k_s a_{wi}}{c} \mathcal{K}_f^{(1)}(\xi, \sigma) \delta(y - y_{0i}) \frac{e^{-i f \psi}}{\gamma} \right\rangle_e \quad (26)$$

- Di-pole term

$$\mathcal{S}_2(y) = \left\langle \frac{4\pi I f k_s a_{wi}}{c} \mathcal{K}_f^{(2)}(\xi, \sigma) \frac{1}{2} [\delta(y - y_{0i} - \chi/2) - \delta(y - y_{0i} + \chi/2)] \frac{e^{-if\sigma}}{\gamma} \right\rangle_e \quad (27)$$

where

$$\mathcal{K}_f^{(m)}(\xi, \sigma) = (-1)^f \sum_{n=-\infty}^{\infty} J_n(f\xi) [(-1)^m J_{2n+f-m}(f\sigma) - J_{2n+f+m}(f\sigma)] \quad (28)$$

and χ is the electron wiggler oscillation amplitude, given by

$$\chi = \frac{a_w}{\gamma k_w} \quad (29)$$

for $\gamma \gg 1$.

Including higher order source terms in the sum in Eq.(25) will not significantly improve the accuracy of the calculation since these terms are effectively averaged out by the electron ensemble, assuming the transverse density profile is smoothly varying.

In the past, single-pass calculations were performed to calculate the coherent spontaneous emission. Those calculations are accurate for FEL amplifiers and for FEL oscillators where the mirror reflectivity at the harmonic frequencies is poor or the resonator harmonic circulation time, τ_f , is significantly different from the fundamental circulation time, τ_1 (due to some dispersive element), i.e. $|\tau_f - \tau_1| \geq \tau_{\text{micropulse}}$. For oscillators with good harmonic mirror reflectivity and no dispersion, the coherent spontaneous emission can interfere with itself on successive passes. To model these interference effects, the harmonic version of the 3-D code FELEX⁶ has been modified so that the fundamental and the harmonic fields can propagate simultaneously around the resonator.

Multipass simulations of the Mark III⁷ oscillator were performed to investigate the evolution of the harmonic fields in the resonator. For simplicity, the mirror reflectivity for the harmonics was assumed to be equal to the fundamental mirror reflectivity. Also, the cavity was assumed to have no dispersion (which is not the case with the calcium fluoride outcoupling plate in place). Equal cavity outcoupling (7%) was imposed for all the harmonics. The harmonic power levels with and without pass-to-pass interference are given in Table 1.

For the operating parameters listed at the bottom of Table 1, constructive interference in the multiple pass simulation gives rise to a factor of 2 to 5 enhancement of the harmonic powers. Therefore, multiple pass simulations should be performed when pass-to-pass interference of the harmonic radiation is expected.

Harmonic	$P_{\text{single pass}}$	$P_{\text{multiple pass}}$
1	1.0×10^8	1.0×10^8
2	2.5×10^4	6.8×10^4
3	5.6×10^4	2.3×10^5
4	2.2×10^3	8.9×10^3
5	2.6×10^3	1.3×10^4
6	2.0×10^2	8.0×10^2
7	1.4×10^2	3.4×10^2

$\lambda_w = 2.3$, $\lambda_s = 3.16 \mu\text{m}$, $\gamma = 86.11$, $\Delta\gamma/\gamma = .3\%$,
 $B_w = 6570$ gauss, $z_r = 73.6\text{cm}$, $I = 30.0$ Amps,
 $\epsilon_x = 2.7 \times 10^{-5}$, $\epsilon_y = 1.0 \times 10^{-5}$ (gaussian $1/e$)

Table 1. Watts of coherent spontaneous power for the second through seventh harmonic for a single pass calculation and for a multiple pass simulation.

IV. Harmonic Lasing

The investigation of lasing at the harmonic frequencies requires that the electron and electromagnetic equations be self-consistent. The electric field equations have already been given in Eqs.(25)-(28). We now need an equation for the electron dynamics that includes the forces due to the harmonic fields. Starting with the force equation, the change in an electron's energy caused by the fundamental *and* the harmonic fields is given by

$$\frac{d\gamma}{dz} = -\frac{e}{mc^2} \vec{v}_\perp \cdot \vec{E} \quad (30)$$

where $\vec{v}_\perp$ is the transverse electron velocity and $\vec{E}$ is the electric field of the optical waves. Assuming that the wiggler magnetic field is in the $\hat{x}$ direction with the dependence $\vec{B} = \hat{x} B_w \sin(k_w z)$, the electrons will wiggle in the $\hat{y}$ direction with velocity

$$\vec{v}_\perp = \hat{y} \frac{a_w c}{\gamma} \cos(k_w z) \quad (31)$$

such that their positions are given by

$$y = y_0 + \chi \sin(k_w z) \quad (32)$$

where a_w is given in Eq.(6) and y_0 is the electron's guiding center position. The physical electric field is given by

$$\vec{E} = \hat{x} E = \hat{x} \sum_{f=1}^{\infty} \text{Re} \left\{ \mathcal{E}_f e^{i f(k_w z - \omega_f t)} \right\} \quad (33)$$

where $\mathcal{E}_f$ is the slowly-varying complex electric field amplitude of the f th harmonic. When the electron's wobble oscillation amplitude is much smaller than the optical mode spotsize, the field seen by the electron can be expanded about its guiding center position so that

$$E \simeq E(y_0) + (y - y_0) \left. \frac{dE}{dy} \right|_{y=y_0} + \dots \quad (34)$$

and using this expression along with Eq.(33) in Eq.(30) gives

$$\frac{d\gamma}{dz} = -\frac{e}{mc^2} \frac{a_w}{\gamma} \sum_{f=1}^{\infty} \text{Re} \left\{ \left[\cos(k_w z) \mathcal{E}_f + \frac{\chi}{2} \sin(2k_w z) \frac{d\mathcal{E}_f}{dy} \right] e^{i\alpha_f} \right\} \quad (35)$$

where

$$\alpha_f = f(k_s z + \omega_s t) + f\psi - f[\xi \sin(2k_w z) + k_w z] + f\sigma \sin(k_w z) \quad (36)$$

is the harmonic optical phase⁸ and $\psi = (k_s + k_w)z - \omega_s t$ is the ponderomotive phase⁹ which is slowly varying near resonance. Averaging this expression over a wiggler wavelength gives

$$\left\langle \frac{d\gamma}{dz} \right\rangle_{\lambda_w} = -\frac{e}{mc^2} \frac{a_w}{\gamma} \sum_{f=1}^{\infty} \text{Re} \left\{ \frac{1}{2\pi} \int_0^{2\pi} d\theta \cos \theta \mathcal{E}_f e^{if\psi} + \frac{\chi}{2} \frac{1}{2\pi} \int_0^{2\pi} d\theta \sin(2\theta) \frac{d\mathcal{E}_f}{dy} e^{if\psi} \right\} \quad (37)$$

and evaluating the integrals in the limit of constant $\mathcal{E}_f$ and ψ over this interval gives

$$\left\langle \frac{d\gamma}{dz} \right\rangle_{\lambda_w} = (-1)^{f-1} \frac{e}{mc^2} \frac{a_w}{2\gamma} \sum_{f=1}^{\infty} \text{Re} \left\{ \mathcal{K}_f^{(1)}(\xi, \sigma) \mathcal{E}_f e^{if\psi} + i \frac{\chi}{2} \mathcal{K}_f^{(2)}(\xi, \sigma) \frac{d\mathcal{E}_f}{dy} e^{if\psi} \right\} \quad (38)$$

where $\mathcal{K}_f^{(m)}(\xi, \sigma)$ is defined in Eq.(28). The optical source equation along with Eq.(38) form a self-consistent set of equations from which harmonic lasing can be modeled.

Small signal gain curves for the fundamental and harmonic frequencies of the Mark II oscillator are plotted in Figure 5 as a function of optical wavelength. The parameters used in the calculation are identical to those listed under Table 1. Only the first, third and fifth harmonics have enough gain to overcome the resonator losses ($\simeq 8\%$). The gain for the second harmonic was roughly 1%. This is consistent with a single mode analytic small signal gain estimate¹⁰. In the region of Figure 5 where the third harmonic gain is greater than the fundamental gain, multiple pass simulations were conducted to observe third harmonic lasing. The electron phase space at saturation is shown in Figure 6. As one would expect, the electrons have clumped together to form 3 bunches. Plots of the third harmonic

gain and power as a function of pass number are given in Figure 7 and 8, respectively. The corresponding plot for the fundamental gain is given in Figure 9. Note that as the third harmonic field grows, the gain in the fundamental is strongly suppressed. The power in the sixth harmonic was observed to grow due to harmonics of the 3rd harmonic bunch density. Approximately 0.15% of the electron beam power was converted to 3rd harmonic optical power. In contrast, when lasing at the fundamental occurs, a conversion efficiency of 0.8% is achieved. Therefore, an efficiency ratio of $\eta_3/\eta_1 = 0.19$ was obtained which is higher than the experimentally measured result of .12¹¹. Better agreement could be achieved if wiggler field errors, pulse effects and the proper mirror reflectivity for the third harmonic were included.

Figure 5. Small signal gain curves for the fundamental and first two odd harmonics for the Mark III oscillator. Note the region where the third harmonic gain exceeds the fundamental gain.

V. Conclusions

The ability to model the harmonic radiation from FELs has been demonstrated. The models are three dimensional in nature and include the effects of energy spread and emittance. Simulation capabilities now exist which span the various operating regimes including

Figure 6. Plot of electron phase space at saturation during third harmonic lasing. Only a small fraction of the electrons used in the simulation are shown.

Figure 7. Third harmonic gain at the wiggler exit from small signal through saturation. The residual gain at saturation is equal to the resonator losses.

Figure 8. Third harmonic optical power as a function of pass number from small signal through saturation.

Figure 9. Gain for the fundamental wavelength during third harmonic lasing.

spontaneous emission, coherent-spontaneous emission and harmonic lasing. Under normal fundamental lasing conditions, enhancements of the coherent-spontaneous harmonic emission of factors of 2 to 5 have been observed due to pass-to-pass interference effects. For conditions where the harmonic gain exceeds the fundamental gain, harmonic lasing occurs with an efficiency consistent with that measured in experiments.

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