CURRENT APPLICATIONS OF OPTIMAL ESTIMATION AND CONTROL THEORY TO THE LOFT REACTOR PLANT

Joseph J. Feeley and J. Louis Tylee
EG&G Idaho, Inc.
P.O. Box 1625
Idaho Falls, Idaho 83415

Abstract

Two advanced estimation and control systems being developed for the LOFT reactor plant are described and evaluated. The advanced protection system, based on a Kalman filter estimator, is capable of providing on-line estimates of such critical variables as fuel and cladding temperature, DNBR, and LHGR. The steam generator LQG control system provides stable, closed-loop, zero steady state error control over a wide power range and also provides on-line estimates of certain unmeasurable variables as steam generator power output and cooling capacity for operator information.

1. Introduction

With the recent incidents at Three Mile Island and other facilities, there is a renewed interest in the protection and control of nuclear power plants. An active area of research is in the application of optimal estimation and control theories to developing protection and control strategies for these plants. The research and implementations, to date, indicate that the on-line digital optimal estimation and prediction of plant variables and unmeasured parameters, coupled with optimal control algorithms, offer increased safety and plant efficiency when compared to the current conventional controls.

Under a Department of Energy sponsored project, two advanced protection and control systems are being developed for the Loss-of-Fluid Test (LOFT) reactor located at the Idaho National Engineering Laboratory. The LOFT plant, shown schematically in Figure 1.1, can produce 50 MW(t) and is used to analyze both operational transients and loss of coolant accidents in pressurized water reactors. The first system being developed for LOFT is an advanced protection system based on a Kalman filter estimator, which will provide optimal estimates of unmeasurable variables which are critical to overall plant integrity. The second system under design specifically addresses the problem of steam generator control. The steam generator controller is basically a digital linear-quadratic-gaussian (LQG) regulator with non-zero setpoint and integral feedback features. Preliminary simulation studies show both of these systems to be superior to the current LOFT protection and control systems.

2. Advanced Plant Protection System

2.1 Background

Many variables crucial to the protection of nuclear power plants cannot be measured directly. For example, the primary purpose of a pressurized water reactor protection system is to prevent the melting of the nuclear core and its cladding under accident conditions. Unfortunately, the harsh environment of the nuclear core precludes direct, reliable measurement of the fuel and cladding temperatures with present instrumentation. Protection system performance must therefore rely on measurement of other variables external to the reactor core. A typical protection system may use neutron flux measurements in the shield tanks surrounding the reactor, the temperature, pressure, and flow rate of the coolant in the primary loop, control rod position, and steam flow and steam generator water level in the secondary coolant loop. Continuous comparison of these measurement values to pre-established setpoints is used to determine the need for plant shutdown.

Because of these measurement constraints, an extensive simulation study is required as part of the protection system design to verify that the available measurements and associated trip setpoints are adequate to maintain the integrity of the plant. The simulation study involves developing a detailed mathematical model of the entire power plant, including the proposed protection system, programming the model on a computer, hypothesizing a set of accidents believed credible for the given plant design, and simulating the plant response under these accident conditions. The simulation results are then analyzed to determine whether or not the limiting values of any plant variable have been exceeded. If they have, the protection system design is modified and the accidents are resimulated. This process is repeated until a satisfactory, although probably not optimum, protection system design evolves. The entire process must then be repeated to account for potential changes in, or uncertainty in the knowledge of, plant parameter values, and some compromise in design reached. Complex as this procedure is, it is further complicated by the fact that it is not possible, or even desirable, to simulate the full spectrum of credible accidents with a single computer code.

So, in practice, protection system design becomes an iterative process involving a set of postulated accidents, whose completeness depends on the skill and experience of the analyst. Design of the deterministic plant models and computer codes and a number of specialists from a variety of disciplines including reactor physics, thermal-hydraulics, instrumentation, and systems analysis. This very complicated process is not only expensive but also burdens with opportunities for serious errors and misunderstandings directly affecting reactor safety. Clearly, some improvement in reactor protection system design methods is desirable.

2.2 Advanced Design Approach

The methods of optimal estimation theory offer a promising new approach to plant protection system design. The advanced protection system, shown in Figure 2.1, is conceptually quite straightforward. A Kalman filter is used to generate estimates of the current plant state vector x, based on a set of available noisy, diverse measurements y. The state vector would include such variables as fuel and cladding temperature, and values of related variables such as DNBR (departure from nucleate boiling ratio) and LHGR (linear heat generation rate) may be readily obtained from the estimated state variables. The state estimates and values of DNBR and LHGR are then directly compared with limiting setpoints and appropriate control action is initiated to avoid violating these limits.

This approach offers a number of advantages over current methods. System modeling efforts will be concentrated on development of a suitable model for the estimator which will lead to a more efficient and organized modeling effort and make model limitations and assumptions more clearly visible. Protective action will be based on a direct comparison of an optimal estimate of a critical variable with its limiting value, not on auxiliary variables whose limits were determined by a complicated analysis involving a myriad of simplifying, often conflicting, assumptions. Changing plant parameter values can be estimated on-line and the effects immediately accounted for in the estimator.
Measurement diversity, an important element in protection system reliability, is inherent in the advanced system. Reactor plant safety will be independent of an analyst's ability to postulate a complete set of potential reactor accidents. Finally, as seen in Figure 2.1, it would be relatively simple to add an optimal state feedback controller to this system since estimates of the full plant state vector are available. The generated optimal control, \( u_d \), could either be used as suggested control information to the primary system for the plant operator or could be used in a closed-loop fashion to provide a complete computer-based advanced plant protection and control system.

2.3 Estimator Design

The mathematics of the Kalman filter estimator are documented in numerous texts [1, 2] and no effort is made here to reproduce the pertinent equations. Instead, attention is paid to describing the development of the mathematical model of the LOFT plant dynamics required by the Kalman filter formulation. The linearized, discrete Kalman filter estimator, upon which the advanced LOFT protection system is currently based, requires knowledge of the plant dynamics in the form:

\[
\begin{align*}
X_{k+1} &= \Phi X_k + \Lambda U_k \\
y_k &= Y X_k + Z U_k
\end{align*}
\]

where \( \Phi, \Lambda, Y, \) and \( Z \) are time-varying matrices of suitable dimension. Unfortunately, the dynamics of a nuclear reactor and its supporting subsystems are highly nonlinear and direct derivation of a linear model in the form of (1) and (2) is difficult. Thus, it is convenient to first derive the nonlinear plant model and then numerically linearize it about some operating point to obtain the four linear system matrices.

The nonlinear model of LOFT used in the Kalman filter is based on a model described in some detail in Reference [3] and has been validated using available test data. The model consists of 22 first-order nonlinear differential equations which describe the dynamics of the core, primary loop, pressurizer, and condenser. Figure 1.1. Standard time-dependent point kinetics with two delayed neutron groups are used to model the power generation within the nuclear fuel. Reactivity sources include changes in fuel temperature, primary coolant density, and control rod position. An average fuel rod with a single fuel node and a single cladding node, separated by a variable width gas gap, constitutes the model describing the transfer of the heat within the fuel to the core coolant.

The primary coolant loop is divided into five nodes to model the transport of heat from the core through the loop. These five nodes include the core, core bypass, hot leg, steam generator primary, and cold leg. Provisions for direct heat deposition into the core and core bypass coolant are made in the model. The pressurizer, which acts as a surge tank to maintain primary loop pressure, is modeled as a homogeneous, saturated fluid system, with surge flow being calculated based on the instantaneous mass change in the primary loop.

The LOFT steam generator is a vertical, U-tube, recirculation type similar to those used in most pressurized water reactors. Its dynamics are modeled by assuming the steam and water in the secondary side are in a homogeneous, saturated mixture, with mass and heat balances on this mixture yielding the desired state equations. A separate mass balance on the water in the downcomer region is used to provide accurate predictions of steam generator water level. The dynamics of both the main steam control valve and feedwater flow valve are modeled.

The steam flow from the steam generator flows immediately into the condenser which is cooled by a bank of six variable pitch fans. A single state equation models the condensation process. A model very similar to that of the pressurizer is used to model the dynamics of the condensate receiver. Finally, a constant amount of feedwater subcooling, needed to maintain the net positive suction head requirements of the feedwater pump, is assumed.

The resulting continuous nonlinear equations are in the form:

\[
\begin{align*}
x'(t) &= f(x, u) \\
y(t) &= g(x, u)
\end{align*}
\]

Using standard linearization techniques [1], i.e., expanding (3) and (4) in a Taylor series, retaining only the linear terms, and then evaluating the resulting derivatives about a nominal operating point, will transform (3) and (4) into linear equations of the form:

\[
\begin{align*}
x'(t) &= Fx + Lu \\
y(t) &= Yx + Zu
\end{align*}
\]

Then applying established discretization methods [4] allows (5) and (6) to be cast in the desired form seen in equations (1) and (2). This model, when implemented in a Kalman filter as shown in Figure 2.1, will allow the optimal estimation of the unmeasurable state vector \( x \).

As previously mentioned, in addition to using the estimate of the state vector, estimates of DNBR and LHGR will also be used to formulate a plant protection scheme. The computation of DNBR and LHGR are essentially based on empirical correlations [5] which require as input data values some of the state estimates, e.g., cladding temperature.

2.4 Simulation Results

The Kalman filter estimator for the advanced protection system has been designed and some preliminary simulation results obtained. The estimator algorithms and simulated plant model are currently implemented on a CYBER 176 system. Implementation of these algorithms on a PDP-11/55 minicomputer to allow real-time estimation is currently in progress. Evaluation of the real-time performance of the algorithms will be made using a large hybrid model of the LOFT plant to provide simulated transient data for the estimator.

Figures 2.2 through 2.5 display the results of the estimator to a simulated rod withdrawal accident with plant measurements taken every second. An uncontrolled rod withdrawal when the reactor is operating could be caused by an equipment failure in the rod control system. This type of accident would cause an increase in reactor power, pressure, and temperature that normally would be detected and terminated by the operator. If, however, the operator fails to take action, the plant protection system should initiate a reactor scram before DNB or excessive fuel temperatures are reached. In this simulation, it was assumed that the plant was at full power (50 MW) and a control rod was withdrawn for 25 seconds before the operator could stop it.

Figures 2.2 and 2.3 show the resulting power, temperature, and pressure increases due to the reactivity insertion. Note that the estimator follows the plant measurements quite well. Figures 2.4 and 2.5 display the corresponding estimates of fuel and cladding temperatures, DNBR and LHGR. Obviously, the fuel and cladding temperatures increase due to increased power production. The increased power increases the heat flux through the clad and hence LHGR rises resulting in a corresponding decrease in DNBR.

Under the existing LOFT plant protection system design, a scram would have occurred when reactor power exceeded 53 MW. Perhaps with the advanced protection
system, a scram could be avoided if the estimates of fuel and clad temperatures, ONBR, and LHGR remain within acceptable limits. What these limits should be must still be decided. The important point to make is that scram decisions in the advanced system will be made on optimal estimates directly related to reactor integrity while the existing system relies on noisy measurements of auxiliary variables.

3. Steam Generator Estimation and Control System

3.1 Background

The control of steam generator water level is a long-standing problem in both fossil and nuclear power plants. Existing control schemes generally provide automatic level control above 20% power but require manual control at lower powers. Automatic control is usually accomplished by positioning the feedwater control valve in response to a control signal generated by some form of the classical "three element" analog controller. (The "three element" controller is so named because it uses three input signals, level, feed flow rate, and steam flow rate, to generate the feed water control valve command.) While considerable attention has been given to the steam generator water level control problem over the years, as typified by References [6] through [9], loss of water level control has been cited [8], [10] as an important cause of reactor plant shutdowns, especially at low (<20%) power, and has been identified as an area requiring additional research [10], [11].

3.2 Physical System

The steam generators used in most pressurized water reactor plants are of the natural circulation type. A cutaway view of the LOFT steam generator is shown in Figure 3.1. The steam generator is divided into primary and secondary sides by the "U" tube walls. On the primary side, hot reactor coolant flows into the inlet plenum, through the "U" tubes where it gives up heat to the secondary side, and back via the outlet plenum to the reactor for reheating. On the secondary side, in the liquid region of the drum, incoming feedwater is mixed with circulating water returned from the separator. The resulting subcooled mixture flows down the annular downcomer and enters the evaporator just above the tube sheet. Heat is added to the secondary coolant continuously as it flows up past the hot evaporator "U" tubes. This results in a well defined subcooled region in the lower part of the evaporator and a two phase mixture in the upper part. The two phase mixture continues to rise through the unheated riser section to the centrifugal separator.

The separator physically diverts the liquid water back to the liquid region of the drum, and allows the water vapor to rise, through an additional steam dryer, to the steam outlet. It will be noted that internal secondary side flow rates are determined by local pressure differences, and that the thermal driving head generated by the density difference between the fluid in the drum-downcomer region and the fluid in the evaporator-riser region are a dominant factor. The magnitude of the circulation flow rate varies widely with power level and is responsible for the increased sensitivity of drum level to steam flow changes at low power, and the delayed effect of feedwater flow changes on drum level. Transient changes in the net flow into the drum section are responsible for the counter-intuitive increase in drum level associated with an increase in steam flow (the "swell" effect) and the corresponding level decrease associated with a decrease in steam flow (the "shrink" effect).

3.3 Objective and Approach

The basic objectives of existing steam generator water level control systems are to maintain drum level at some desired value in spite of changes in other process variables, and to provide for smooth changes in the controlled variables during process setpoint changes. The desired level may be either constant or programmed as a function of steam flow. The objective of this study was to develop a control system capable of smoother, more stable operation over a wider range of power levels than existing systems. To achieve this objective, the new design must account for the fundamental difficulties in process control more completely than current designs. Some of the more significant control concerns are listed below:

1. nonlinear, highly-interactive, multi-variable nature of problem
2. increased sensitivity of steam generator dynamics at low power
3. noisy, inaccurate steam and feed flow measurements at low power
4. counter-intuitive "shrink" and "swell" of drum level
5. varying control valve sensitivity
6. delayed effects of feedwater flow rate changes on level

In view of these concerns and the basic objectives, the design task was formulated as a multivariable, stochastic, optimal estimation and control problem. Anticipating that an analog implementation of the resulting estimation and control algorithm would be impractical, the problem was formulated in discrete time to accommodate on-line digital computer control. The design effort is scheduled to proceed in three distinct phases. The first phase, the results of which are reported here, consist of model development, preliminary estimation and control system design, and closed loop system simulation on a large main frame computer (CYBER 176). The second phase will consist of real-time closed-loop simulation studies with the intended control computer (PDP-11/55) interfaced with a hybrid computer simulation of the process. Finally, the third phase will involve actual experimentation on the LOFT plant.

3.4 Mathematical Model

An early step in estimation and control system design is the development of a dynamic mathematical model of the process. Model development may naturally proceed along one of two lines, either physical or empirical. While an empirical model will generally have the advantages of simplicity and low order, its validity is strictly limited to the region in which the identification data was obtained, and even small excursions from this region may result in serious error. Since it is generally difficult to obtain a sufficient range of reactor plant transient data for adequate empirical identification, the modeling effort proceeded on a physical basis with the tacit assumption that a physically based model will still give reasonable predictions quite far from its region of strict validation.

Anticipating a frequent need for appropriate thermal-hydraulic relationships in modeling other reactor plant components in addition to the steam generator, a general set of equations describing the conservation of mass, energy, and momentum were derived. Expressing these conservation laws in state variable format, with pressure, quality, and mass flow rate as state variables, and using linearized expressions for the water properties, provided a convenient set of
equations for modeling a distributed system discretized into a number of interconnected control volumes.

Applying these expressions to a steam generator control volume arrangement similar to All's [12] Model D, resulted in a 28th order state variable model. Heat transfer and flow coefficients were chosen to force model predictions to agree with steady state LOFT test data and the resulting model termed the "truth" model of the steam generator. The "truth" model was reduced to 7th order by physical reasoning and eigenvalue analysis to simplify the estimation and control system design calculations, and to examine the effects of model mismatching in controlling a 28th order plant simulation with a control system based on a 7th order model. Dynamic performance of the "truth" and reduced order models was compared and found to be similar, but not identical. Full dynamic verification of the models is difficult due to a lack of adequate test data, but comparisons with available transient data and known performance of similar steam generators are favorable.

The increased sensitivity of level to changes in steam flow and, to a more limited degree, hot leg temperature, at low power are demonstrated in the simulation results of Figure 3.2. The increased difficulty of control at low power is also demonstrated by the open loop eigenvalue loci of Figure 3.3, where there is a general trend toward the right half s plane as power level is reduced.

3.5 Estimation and Control System

The resulting estimation and control system configuration is shown in Figure 3.4 where the indicated variables are the vector quantities defined below:

state vector, x
x(1) = water level in drum
x(2) = enthalpy of drum water
x(3) = pressure in evaporator
x(4) = average quality in evaporator
x(5) = temperature of primary coolant
x(6) = temperature of "U" tubes
x(7) = position of feedwater control valve

measurement vector, z
z(1) = water level in drum
z(2) = temperature of downcomer water
z(3) = pressure in drum
z(4) = temperature of cold leg water
z(5) = position of feedwater control valve
z(6) = feed flow rate
z(7) = steam flow rate

controlled output estimate vector, y
y(1) = water level in drum
y(2) = power out of steam generator

displayed output vector, d
d(1) = power out of steam generator

control input vector, u
u(1) = desired position of feedwater control valve
u(2) = desired position of steam flow control valve

disturbance input vector, u_d
u_d(1) = temperature of feedwater

u_p(2) = flow rate of primary coolant
u_p(3) = pressure in condenser
u_p(4) = pressure at feed pump discharge
u_p(5) = temperature of hot leg water

Either of two different state estimators may be selected to generate the state estimate, x, depending upon real-time computation constraints. If very rapid state estimates are required, a steady state Kalman filter may be used. The steady state filter minimizes computation time, but at the cost of biased estimates due mainly to model mismatching. If more time is available for estimation an extended Kalman filter may be used which produces virtually unbiased estimates.

The extended filter estimates three parameters, secondary side heat transfer coefficient, circulation flow pressure drop coefficients, and the ratio of exit to average quality in the evaporator, in addition to the seven basic reduced order model states. The output estimator operates on the state estimates to produce controlled output variables for feedback control and displayed output variables for operation information. One of these displayed output variables, cooling capacity, is a nonlinear function of several states and is related to the amount of energy required to vaporize all the liquid water in the steam generator. Cooling capacity may be calibrated in terms of "current power seconds" thereby directly indicating to the operator the expected time required to evaporate all the water in the steam generator at the current power level in the event of a sudden loss of feedwater.

The control input, u, is composed of feedforward, u_ff, and feedback, u_fb, components. The feedforward signal is given by

\[ u_{ff} = (y_1 - \omega + AK)^{-1}y \]  (7)

and the feedback signal is given by

\[ u_{fb} = -K - C_k \frac{q_k}{q_k} - (K_p K_i) \frac{q_k}{q_k} \]  (8)

where y is the desired setpoint vector, and K is the control gain matrix composed of proportional and integral submatrices K_p and K_i. Numerical values for the control gain are obtained by minimizing the performance index

\[ J = \sum_{k=0}^{\infty} \begin{bmatrix} q_k^T & 0 & A_0 \end{bmatrix} \begin{bmatrix} q_k \end{bmatrix} + u_k^T B u_k \]  (9)

subject to the constraint

\[ \begin{bmatrix} x_k+1 \end{bmatrix} = \begin{bmatrix} \phi & 0 & \omega \end{bmatrix} \begin{bmatrix} y_k \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} q_k \end{bmatrix} + u_k \]  (10)

where A_0, A_1, and B are the proportional, integral, and input weightings and \( \omega \) is the integral of \( \omega \) over the control interval.

3.6 Results

A digital computer program was written to facilitate estimation and control system design and check performance of the closed loop steam generation process. The program contains a digital simulation of the 28th order continuous steam generator model as well as the discrete estimation and control system. The low order steam generator model is also simulated to facilitate high and low order model comparisons and to permit convenient linearization at any desired operating point through numerical perturbation. The linear system matrices so generated are then available as input to a library of linear system analysis programs for system discretization and computation of such linear system quantities as controllability, observability, eigenvalues, eigenvectors, and estimator and controller gains. Estimation and control system performance was
studied by analyzing a large number of simulation runs. The results of this study are summarized below. Open loop response characteristics of the high and low order models were similar, but not identical. This introduced a degree of realism in the study by explicitly including a number of unmeasured variables. Due to modeling errors, estimation results were found to be sensitive to power level, with poorer results being obtained at lower power. The extended Kalman filter estimates were considerably less biased than the steady state filter estimates, and process noise covariance was found to be a convenient parameter for tuning filter performance.

Stable closed loop control was demonstrated over a wide range of power levels, from 25% to 100% rated power, using a control update interval of 0.2 seconds. A wide variety of system responses were obtained by changing only the integral state and control input weighting matrices. The simulation results clearly indicated the difficulty of automatic control at low power levels, and stable operation required smaller feedback gains, resulting in slower system response. Reduction of the sampling interval to 0.1 second also improved system performance at low power. Integral feedback was found to be very effective in eliminating steady state errors due to model mismatching. Changes in the process setpoint were made smoothly, quickly, and with little or no overshoot. Setpoint changes were best made by introducing the desired change in the feedback loop and setting the feedforward gain to zero, since the feedforward term tends to speed up system response at the cost of undesirable overshoots in the controlled variables. A typical setpoint change, transient increasing power level from 25 to 30 MW and water level from 121 to 123 inches is shown in Figure 3.5. Introduction of process and measurement noise in the simulation degraded system performance somewhat, but satisfactory response was readily achieved by tuning filter covariances and controller weightings.

4. Conclusions

The potential capabilities of the advanced plant protection system are promising. The next phase in the system development is to test the estimator's real-time performance using a hybrid LOFT model for simulated plant data. Using this simulation, an advanced protection system philosophy will be developed and the performance of the advanced system will be compared to the existing protection system.

Other additions to the plant protection system foreseen include the capability of on-line trend predictions based on the current plant state, the ability to run the LOFT model in the Kalman filter faster than real-time to allow operator inquiries regarding consequences of proposed control actions, and the implementation of an optimal feedback controller. It is planned that this advanced protection system will be installed at the LOFT plant once its performance has been validated using simulated data.

The LQG approach to steam generator control system design was found to be quite practical and efficient. Simulation studies indicate the resulting control system will provide stable automatic control over the entire range of steam generator power operation. Key elements in providing this wide operating range are variable control gains that account for varying process dynamics, and variable estimate gains that account for varying measurement accuracy. Simultaneous automatic control of both feed flow and steam flow is a new concept in LOFT steam generator operation, but simulation results support its feasibility. Estimation and display of such unmeasured variables as power output and cooling capacity should provide valuable information to the operator and may be more meaningful inputs to the plant protection system than, say, steam flow and water level.

In closing it should be noted that, although the work described here is specific to the LOFT reactor, because of the similarity of LOFT to a large PWR the concepts developed should carry over, with some modification, to commercial power generation plants. There are obviously a number of areas, such as state and parameter estimation, distributed control, failure detection, and future state prediction, where the results of modern estimation and control theory and modern digital computing equipment can be brought together to improve the safety and efficiency of today's nuclear power plants. While this observation has undoubtedly been made before, LOFT provides a unique test facility to experiment with these advanced techniques under controlled conditions.

5. References

Manually controlled steam flow control valve
Spray nozzle
Outlet
Air-cooled condenser
•
Condensate receiver
Quick opening slowdown valves
- Blowdown suppression tank
Main coolant controlled pumps
Main feedwater pump
Subcooler
Figure 1.1. LOFT reactor plant schematic diagram

Inputs, u
LOFT plant states, x
Outputs Y
Kalman filter
State estimates, ŷ
Optimal controller
Control Inputs, u*

Figure 2.1. Advanced LOFT Protection System

Figure 2.2. LOFT Rod Withdrawal Accident, Core Power and Hot Leg Temperature Estimates

Figure 2.3. LOFT Rod Withdrawal Accident, Primary and Secondary Pressure Estimates
Figure 2.4. LOFT Rod Withdrawal Accident, Fuel and Cladding Temperature Estimates

Figure 2.5. LOFT Rod Withdrawal Accident, DNBR and LHGR Estimates

Figure 3.1. Steam Generator Schematic

Figure 3.2. Level Sensitivity to Steam Flow and Hot Leg Temperature Changes

Figure 3.3. Steam Generator Open Loop Eigenvalue Loci
Figure 3.4. Steam Generator Estimation and Control System

Figure 3.5. Steam Generator Power and Water Level Increase Transient