ANL/FPP/TM-141

Dr. 2520
ANL/FPP/TM-141

216
4/14/80
T.S.
R-3501
①
MASTER

WAVE-DRIVER OPTIONS FOR LOW-ASPECT-RATIO STEADY-STATE TOKAMAK REACTORS

by

David A. Ehst

FUSION POWER PROGRAM

Argonne National Laboratory
9700 South Cass Avenue
Argonne, Illinois 60439

Prepared for the
U. S. Department of Energy
under Contract W-31-109-Eng-38

WAVE-DRIVER OPTIONS FOR LOW ASPECT RATIO
STEADY-STATE TOKAMAK REACTORS

by

David A. Ehst

ABSTRACT

Low aspect ratio designs are proposed for steady-state tokamak reactors. Benefits stem from reduced major radius and lessened stresses in the toroidal field coils, resulting in possible cost savings in the tokamak construction. In addition, a low aspect ratio ($A \approx 2.6$) permits the application of a bundle divertor capable of diverting 3-T fields to a power reactor using STARFIRE technology. Such a low aspect ratio is possible with the elimination of poloidal field coils in the central hole of the tokamak, which implies a need for noninductive current drive. Several plasma waves are considered for this application, and it appears likely that a candidate can be found which reduces the electric power for current maintenance to an acceptable value.

I. INTRODUCTION

Previous studies have pointed to the advantages of designing tokamaks with small aspect ratios ($A \lesssim 3$), for both ignition test reactors⁽¹⁾ and full power commercial reactors.⁽²⁾ Those studies, which assumed the maximum stable plasma beta scales as $\beta_t \approx 0.2A^{-1}$, showed that the major radius, R , can be significantly reduced without sacrificing reactor performance if A can be reduced to low values. The studies implied that reductions in R result in reductions in the cost of tokamak fabrication. Another attraction of low A designs is the fact that, due to its $1/R$ dependence, the toroidal field may be reduced to a low value at the outboard edge of the plasma, which would ease the engineering requirements for a bundle divertor⁽³⁾, if such an impurity control device were desirable for long pulse or steady state operation.

Conventional tokamaks are limited to designs with $A \gtrsim 3$ due to the space requirements for the ohmic heating (OH) coils in the central hole of the machine. In particular, reductions in A tend to require prohibitively large increases in the magnetic flux density swing, ΔB^{OH} , required to generate the toroidal plasma current. This limitation on reactor design can be removed if noninductive means can be found to drive the toroidal current. As merely two examples, neutral beams⁽¹⁾ and lower hybrid waves^(4, 5) have been proposed as current drive candidates. The objectives of the present study, which focuses only on wave-driven power reactors, are two-fold. First, the reactor design requirements are found as a function of A , and it is shown that the limitation on low A now results from the increasing rf power required to sustain operation at increasing plasma density and toroidal current.

Also, for $A = 2.6$, it is shown that fields as low as $B_s \approx 3T$ occur on the outboard edge even for relatively large fields ($\sim 11T$) at the toroidal field (TF) coils. This is encouraging for bundle divertor engineering since Sanderson and Stott⁽⁶⁾ have already designed a device which can divert a field of this strength and which can be properly buttressed against the large electromagnetic forces in a reactor-sized tokamak. The second goal of this study is a comparison of various plasma waves which have been proposed as current drivers, and an attempt is made to identify the most attractive candidates for low aspect ratio designs.

In the comparison of reactors with different aspect ratios we will hold the fusion power fixed, and we will also fix the neutron wall load, maximum toroidal field strength, inboard blanket/shield thickness and the toroidal field ripple at the values used in the STARFIRE reactor design.^(7,8) In Sec. II we will conclude that an attractive reactor using STARFIRE technology could be designed with $A < 3$. The conclusions regarding wave-driven currents, in Sec. III, are tentative due to the simplified analysis used. However, the wave power requirements are sufficiently small that our low aspect ratio designs do appear attractive and should receive future, more detailed consideration.

II. REACTOR DESIGN AT LOW ASPECT RATIO

A number of MHD equilibria were calculated for elongated plasmas ($\kappa = 1.6$) with moderate triangularity and broad pressure profiles, $p(\psi) = p_0 \hat{\psi}^{1.4}$, where ψ is the poloidal flux and $\hat{\psi} \equiv (\psi_{\text{limiter}} - \psi) / (\psi_{\text{limiter}} - \psi_{\text{axis}})$. The volume-averaged plasma beta ($= 2 \mu_0 \bar{p} / B_0^2$) was scaled approximately as $\beta_t = 0.25/A$; and our study concentrated on three values of the aspect ratio, $A = 3.6, 3.0$, and 2.6 . To minimize the rf power required to drive the toroidal current, I ,

equilibria were selected which have minimal values of I for a given β_t . In all cases the axis safety factor, q_a , exceeds unity and $q(\psi)$ is monotonic from the magnetic axis to the boundary.

Table I lists the three equilibria and the required current, $\tilde{I}$, calculated for a major radius $R = 6.7$ m and an on-axis field of $B_0 = 5.7$ T. It is obvious that lower aspect ratios have larger plasma cross sections for a fixed R , and the consequence is the increase in $\tilde{I}$ in going to lower A . However, when these equilibria are used to size reactors of equal power and wall loading, we find that the increase in β_t at lower A reduces both R and B_0 , such that the actual reactor current, I , does not increase as quickly as given in Table I.

In order to quantitatively determine the effect of reduced aspect ratio, a study was performed using TRAC-II, the plasma subroutine of the ANL Tokamak Systems Code.⁽²⁾ In the study the fusion power, wall load, inboard blanket/shield thickness, vacuum scrapeoff width, and maximum toroidal field are all kept constant, at the following values: $P_f = 3400$ MW, $W = 3.6$ MW/m², $\Delta_{B/S}^i = 1.2$ m, $\Delta_v = 0.2$ m, and $B = 11.1$ T. In these calculations we assumed equal ion and electron containment times, peaked temperature and density profiles, a 4% beryllium impurity concentration, and a high alpha particle wall reflection coefficient ($\mathcal{R}_\alpha = 0.7$). The results are given in Table II. The parameters for $A = 3.6$ are essentially those of the STARFIRE reactor. The relatively high plasma temperature selected in the STARFIRE design ($\bar{T}_i \approx \bar{T}_e = 20$ keV) was based on a compromise between high fusion power density at lower temperatures and low rf current drive power at high temperatures.^(4,9) If we are willing to accept greater rf power requirements the plasma can be operated at lower temperatures. Notice, in going to lower A the major radius must decrease to keep the ratio P_f/W constant. The consequent reduction in

$B_0 \{ = B[1-A^{-1} - (\Delta_{B/S}^i + \Delta_v)/R] \}$ and in the average magnetic field pressure is compensated by the postulated increase in β_t as well as by increases of the plasma volume, V , and of the fusion reactivity $\langle \sigma v \rangle / \bar{T}_i^2$ at lower A , such that P_f is kept constant for the examples in the table. $A = 2.6$ is near the lower limit possible since $\langle \sigma v \rangle / \bar{T}_i^2$ is a maximum in the range around 10 keV. The increase in I is less than 20% as A is reduced from 3.6 to 2.6, but we expect a larger increase in the rf power, P_{rf} , required, since P_{rf} is proportional to both I and $\bar{n}_e$, as will be demonstrated in Sec. III.

First, however, let us examine the effect of lowering A on bundle divertor engineering and on the reactor cost. The toroidal field is to be diverted near the outboard edge of the plasma, at a distance $R_s = R(1 + A^{-1})$ from the symmetry axis. We denote the field strength as B_s at this location, and Fig. 1 displays B_s as a function of aspect ratio. As predicted, the reduction of A greatly reduces B_s ; at $A = 2.6$, $B_s = 3.06$ T, which is comparable to field strengths which appear feasible from structural engineering considerations. (6)

To gauge the dependence of reactor cost on aspect ratio we examine the first wall area and the TF coil size, which are two primary indicators of cost. The variation in Table II and Fig. 1 was performed with both P_f and W held constant. Consequently, the first wall area and, to some extent, the blanket volume are independent of the aspect ratio in this study. The TF coils were sized according to approximate formulas for constant tension magnets. (10)

The thickness of the TF magnet, Δ_m , increases for smaller major radius. The value $\Delta_m = 0.96$ m in Table III corresponds to the 11.1 T design for STARFIRE with $A = 3.6$ and $R = 7$ m. The table shows a 9% increase in Δ_m at $A = 2.6$.

The inboard edge of the inboard leg of the TF coil is located at a radius $R_0 \equiv R - a - \Delta_v - \Delta_{B/S}^i - \Delta_m$ (see Fig. 2), where $a = R/A$ is the plasma minor

radius. The small aspect ratio designs shrink this "hole in the doughnut", but even at $A = 2.6$ there appears to be adequate room left to locate a support cylinder (of thickness $\Delta_{\text{sup}} = 0.32$ m) inside the TF coils. Under the assumption that waves drive the toroidal current there is no need to locate ohmic heating coils inside R_0 ; likewise, the nearly elliptic plasma cross section can likely be formed without equilibrium field (EF) coils down in the central hole.⁽¹¹⁾ If a more triangular cross section were required, the EF coil placement could hamper the design of the $A = 2.6$ reactor. The outboard blanket, shield, and access spacing, Δ^0 , was chosen to keep the peak to average field ripple, δ , at one percent at R_s , the STARFIRE value; the ripple was calculated from the analytic formulas of Söll.⁽¹²⁾ In all cases the ripple on the magnetic axis is much smaller. The inboard and outboard TF coil leg locations as well as the TF coil half-height, R_1 , R_2 , and H_{max} , respectively, are given in Table III. We see that the smaller major radius at lower A reduces R_1 , but the field ripple constraint keeps R_2 nearly constant, and the constant tension magnet shape produces a larger vertical dimension at lower A . Thus, the coil circumference, L , increases at lower A . However, the magnet tension is much lower at the smaller values of A so the cross sectional area, $\Delta_m \times R_1$, decreases significantly at lower A . The magnet volume, which principally determines its cost, is proportional to $\Delta_m \times L \times 2\pi R_1$, and it is evident that the $A = 2.6$ design requires ~25% less material than the $A = 3.6$ TF magnets.

Thus, we conclude that an aspect ratio as low as 2.6 is feasible with STARFIRE technology at no penalty in the cost of the first wall, blanket and shield, and with possible savings in the cost of the toroidal magnets. The resulting tokamak has $B_s \sim 3$ T, as required for a bundle divertor design. The circumferential separation of the TF coils in the equatorial plane is

$L_{\text{sep}} = 2\pi R_0/12 \approx 6.7$ m for the 12-coil reactor envisioned here. This spacing appears generous enough to provide divertor coil support and shielding;⁽⁶⁾ if, however, additional neutron shielding were necessary, the divertor design would be more difficult. The main penalty associated with lower aspect ratio is the increased rf power required for steady-state operation, which will be addressed in the following section.

III. WAVE-DRIVEN CURRENTS AT LOW ASPECT RATIO

Having identified the advantage of operation without ohmic heating coils we now discuss a number of plasma waves which could be utilized to drive the toroidal current. In order to compare different wave drivers we write the ratio of power density dissipated to current density generated as

$$\frac{\hat{p}}{\hat{j}} = \frac{n_e e^3 \ln \Lambda}{\epsilon_0^2 m_e^2 v_e^2} \frac{\hat{p}}{\hat{j}} \approx 1.0 \times 10^{-18} \frac{n_e}{T_e} \frac{\hat{p}}{\hat{j}} \quad (1)$$

where $\hat{p}$ and $\hat{j}$ are the normalized (dimensionless) quantities defined by Fisch and his coworkers; ⁽¹³⁻¹⁶⁾ n_e is the electron density in m^{-3} , T_e is the electron temperature in keV, and we set $\ln \Lambda \approx 20$. In principle, the spatial damping profile, $p(\underline{r})$, for a given wave can be calculated and the local current density, $j(\underline{r})$, can be obtained from Eq. (1). The present study ignores profile effects for the most part, using instead the spatially averaged quantities. Our conclusions must be accepted with some caution, therefore. Factors of two or three arising from profile effects may change the relative attractiveness of different waves.

The local current generation efficiency, $\hat{j}/\hat{p}$, depends on a wave's phase speed and polarization and on the electron and ion distribution functions; these factors determine the interplay of various mechanisms which govern energy

and momentum transfer among the particles and the wave. Generally, a reactor designer selects the wave angular frequency and antenna structure (e.g., wavelength in the toroidal direction), and the plasma dispersion properties determine the wave's structure perpendicular to the magnetic field. Figure 3 schematically depicts this parameter space and identifies the six cold plasma waves which we will consider. In our discussion we will frequently refer to a wave's phase speed parallel to the tokamak magnetic field, $c/k_{\parallel}$, and for simplicity we will ignore the poloidal contribution to $\mathbf{B}$. The parallel phase speed will be examined in comparison with electron and ion thermal speeds, $v_j = \sqrt{T_j/m_j}$, T_j and m_j being the temperature and mass of the j -th species.

To establish a reference point let us first consider lower hybrid wave-driven currents. This is the most thoroughly studied driver to date^(4,5,13,17) and is the only one for which we have made current density profile calculations. The lower hybrid wave exists for $\Omega_j \ll \omega \ll \omega_e$, where Ω_j is the gyrofrequency of the j -th species. To avoid parametric decay and minimize $\hat{p}/j$ we operate at $\omega = 1.8 \omega_{\text{LH}} = 1.8 \omega_{\text{pi}} \left[1 + \omega_{\text{pe}}^2 \Omega_e^{-2} \right]^{-1/2}$, where ω_{pj} is the j -th species' plasma frequency. In this frequency range waveguide arrays are an attractive launching mechanism and breakdown considerations limit us to $k_{\parallel} \equiv 2\pi/\lambda_{\parallel} \leq 1/10 \text{ cm}^{-1}$. Consequently (see Fig. 3) we must operate at $\omega/(k_{\parallel} v_e) \gg 1$. Electron Landau damping directly converts wave momentum into electron momentum in this regime, and consequently $\hat{p}/j$ is evaluated for $\omega/(k_{\parallel} v_e) \sim 3-4$. We have calculated wave damping profiles from WKB theory,⁽¹⁷⁾ adjusting the wave spectrum in order to approximate the equilibrium current densities for the three cases in Table I and to minimize the rf power absorbed in the plasma. A distinguishing feature of lower hybrid waves is the necessity of operating above a minimum parallel index of refraction, $n_{\parallel} = ck_{\parallel}/\omega$, in order to avoid wave reflection. The lower

limit, $n_{||}$, increases with $\bar{n}_e$, as shown in Fig. 1. For these waves Eq. (1) becomes (4)

$$\frac{P}{j} \approx 1.1 \cdot 10^{-21} n_e n_{||}^2; \quad (2)$$

the increase of $\bar{n}_e$ and consequently $n_{||}$, with decreasing aspect ratio (Table II) implies less efficient current generation at low aspect ratio. When this is coupled with the larger toroidal current requirements ($I = 13.4$ MA at $A = 2.6$ versus $I = 11.4$ MA at $A = 3.6$) the rf power should increase greatly at low A . This was confirmed by our numerical calculations, labelled P_{LH} in Fig. 4. The total rf power absorbed in the plasma, P_{LH} , is the spatial integral of the rf power density.

P_{LH} almost doubles in going from $A = 3.6$ to $A = 2.6$ in Fig. 4. This was one motivation for selecting $A = 3.6$ in the STARFIRE reactor design of Ref. 7. P_{LH} is already large at $A = 3.6$, and greater circulating power at lower A appeared uneconomical. Moreover, the actual circulating electric power is larger than P_{LH} due to inefficiencies in wave generation, transmission, and coupling. Denoting the circulating electric power as $P_{LH}^{(e)}$, we may write

$$P_{LH}^{(e)} = P_{LH} \epsilon^{-1} \eta_T^{-1} \eta_S^{-1} \eta_{PS}^{-1}, \quad (3)$$

where $\epsilon(<1)$ accounts for plasma heating that does not result in electron current density, η_T is the transmission system efficiency, η_S is the source efficiency, and η_{PS} is the power supply efficiency. The optimized lower hybrid system for STARFIRE⁽¹⁷⁾ has $P_{LH}/P_{LH}^{(e)} = 44\%$. Applying this factor to the $A = 2.6$ case in Fig. 4, about 350 MW would be circulated to sustain operation with lower hybrid waves, which is an unacceptable penalty for a reactor producing only 1400 MW gross electric.

Starting with the high frequency regime in Fig. 3 we will examine alternative waves with a view towards decreasing the current drive power. Electron cyclotron waves were first identified as a candidate once it was realized that momentum input is not always important to current generation.⁽¹⁵⁾ These waves mostly increase the $v_{\perp}$ moment of the electron distribution function, but in so doing they raise the electron energy and decrease the collision frequency. Thus, if toroidally travelling waves are launched, the resonant electrons collide less frequently than their counterparts travelling in the opposite direction. This anisotropic resistivity results in a toroidal current. Assuming the resonant electrons have parallel velocities, $v_{\parallel} \gtrsim v_e$, we would be operating in a regime in which $\hat{p}/\hat{j}$ is predicted to be somewhat larger than the value for lower hybrid wave drive.⁽¹⁵⁾ Moreover, the efficiency of gyrotrons is so poor that we estimate the electric to current drive power conversion efficiency would be $P_{\text{ECR}}/P_{\text{ECR}}^{(e)} \lesssim 30\%$. Hence, it appears these waves do not offer more efficient current generation. The higher costs of gyrotrons and their high voltage, highly regulated power supplies, compared to a lower hybrid system, also detract from this high frequency option.

We focus next on a lower frequency regime, that of the magnetosonic (MS) wave for which $\omega \gtrsim \Omega_i$. This lower frequency permits the generation of low phase speed waves, denoted by LS in Fig. 3, with $\omega/(k_{\parallel}v_e) \ll 1$, which potentially allows higher current generation efficiency than that from lower hybrid waves.⁽¹⁴⁾ For purposes of illustration we consider waves with $\omega = 3.5 \Omega_D$, where the deuteron cyclotron frequency is evaluated on axis, and we ignore ion cyclotron damping. To avoid ion Landau damping we require $\alpha \equiv \omega/(k_{\parallel}v_{\perp}) \gtrsim 6$. Assuming then that wave damping is mainly due to electron Landau damping and transit time magnetic pumping, the following expression applies:⁽¹⁴⁾

$$\frac{\hat{P}_j}{\hat{j}} \approx 0.051 \frac{c}{n_{\parallel\ell} v_e} \left[\frac{1 - (n_{\parallel\ell}/n_{\parallel u})^3}{1 - (n_{\parallel\ell}/n_{\parallel u})^2} \right],$$

provided $n_{\parallel\ell} \gg c/v_e$. Approximating this formula for a narrow range of phase velocities ($n_{\parallel\ell}/n_{\parallel u} \sim 0.8-0.95$) and inserting the result into Eq. (1), we find

$$\frac{P_j}{j} \approx 1.7 \times 10^{-18} \frac{n_e}{n_{\parallel\ell} T_e^{3/2}}, \quad (4)$$

where units are MKS except for T_e which is in keV. In contrast to lower-hybrid waves [Eq. (2)] we presently desire to maximize $n_{\parallel\ell}$. According to the dispersion relation of these waves,⁽¹⁸⁾ the largest $n_{\parallel}$ occurs for the fundamental radial eigenmode of the torus, which has a corresponding wavelength, $\lambda_{\parallel} = V_A/(0.47 \nu)$, where V_A is the Alfvén speed and ν is the wave frequency. Table IV lists the frequencies and wavelengths for an effective ion mass of 2.5 amu; the wave power requirements were taken by substituting the average $\bar{n}_e$ and $\bar{T}_e$ values into Eq. (4) and multiplying the result by the average current density $\bar{j}$ and the plasma volume from Table II. We see from Fig. 4 that this driver may be competitive with lower hybrid waves, although it is uncertain how large $P_{MS}/P_{MS}^{(e)}$, the electric to current drive power efficiency, can be. For these frequencies above 100 MHz, ridged or dielectric waveguide antennas may be possible. However, the short $\lambda_{\parallel}$ (~ 10 cm) will complicate the design of a phased array which efficiently couples to toroidally travelling eigenmodes. In addition, the choice of phase speeds $\omega/(k_{\parallel} v_e) \ll 1$ means the majority of the resonant electrons are magnetically trapped. This neoclassical effect, discussed in Ref. 14, will strongly influence the radial current density profile.

Reference to Fig. 3 shows another candidate, the long $\lambda_{\parallel}$, high phase

speed (HS) magnetosonic modes at $\omega \geq \Omega_i$ with $\omega/(k_{\parallel} v_e) \approx 3-4$. As with lower hybrid waves, waveguide couplers are appropriate and neoclassical effects play no role. Moreover, the transit time pumping associated with these waves results in a $\hat{p}/\hat{j}$ value about 85% that of lower hybrid waves,⁽¹⁴⁾ and wave accessibility is not a constraint, so broad current profiles appear possible.

(Note, as $n_{\parallel} \rightarrow 1$ relativistic effects arise⁽¹⁹⁾ and should be considered in any more accurate analysis.) These waves, which have demonstrated steady state current drive in collisional tokamaks,⁽²⁰⁾ may ultimately be the most attractive for current drive applications; however, they do result in $\hat{p}/\hat{j}$ significantly larger than for waves with $\omega/(k_{\parallel} v_e) \ll 1$.⁽¹⁴⁾

Passing on to a slightly lower frequency regime, we note the recent suggestion⁽¹⁶⁾ that minority species heating by magnetosonic waves can result in current generation. As with electron cyclotron waves, perpendicular heating occurs, but in this case for minority ions travelling in the preferred direction. The resulting anisotropic ion distribution acts like an ion beam, and, provided the minority species has an atomic charge greater than one, collisional drag on the electrons yields a toroidal current. When $w = (\omega - \Omega_I)/(k_{\parallel} v_I) \gg 1$, where the subscript I denotes the minority species, we can write⁽¹⁶⁾

$$\frac{\hat{p}}{\hat{j}} = \frac{1 + m_I/m_e}{3(1 - Z_I^{-1})G(w)}, \quad (5)$$

with $G(w) = w^2(1 + Yw^3)^{-2}$,

$$Y = 0.266 \sqrt{m_e/m_I} \left[1 + (m_I/m_e) \right]^{-1},$$

and where Z_I is the ratio of minority to majority ion atomic charge. Although Eq. (5) was derived for only one minority and one majority species, we will assume it is approximately valid for a 50% mixture of DT fuel and assign the value $m_i = 2.5$ amu to the "majority" ions.

Consider first the possibility of heating a naturally occurring impurity, e.g., the beryllium sputtered from the first wall coating on a STARFIRE reactor. Good impurity control could keep beryllium and helium levels to a few per cent of the DT density, and we ignore minority-majority ion hybrid resonances in our discussion. Beryllium, with an atomic charge to mass ratio of $4/9$, has its cyclotron resonance surface between the tritium ($\Omega_T \approx 1/3$) and the deuterium/helium ($\Omega_D = \Omega_\alpha \times 1/2$) resonant surfaces. The DT ion hybrid resonance likewise occurs between the deuterium and tritium cyclotron resonances; it may be desirable to adjust the relative deuterium and tritium concentrations to move the DT hybrid resonance relative to the beryllium cyclotron resonance, since there is evidence that the coincidence of an ion hybrid resonance strongly affects minority heating at the fundamental resonance.^(21,22) For a beryllium minority species the maximum value of $G(w)$ occurs at $w \equiv w_0 = 10.4$ and results in $\hat{p}/\hat{j}|_{w_0} = 0.043$, which would be comparable to that for some previously discussed waves. However, the spatial variation of Ω_I and v_I make it unlikely that wave damping will occur at such high values of w . If wave damping predominates for $w \approx 3-4$, the current drive power may be unacceptably large — $\hat{p}/\hat{j}|_{w=4} = 0.14$. Moreover, additional power loss is expected if the launched waves must pass through either majority species cyclotron resonance to reach the beryllium resonance.

In order to improve upon this unsatisfactory result one could consider injecting a specific minority species. The best candidate appears to be ^3He ,

for which we find $\hat{p}/\hat{j}|_{w_0=6.7} = 0.073$ and $\hat{p}/\hat{j}|_{w=4} = 0.11$. This latter value, while it is smaller than for the beryllium minority, could only compete favorably with other wave drivers if the electric to current drive power conversion were close to perfectly efficient. However, the ^3He fundamental resonance coincides with the harmonic $\omega = 2\Omega_T$ resonance, and it may be expected that some inefficiency is associated with direct tritium heating. Note also that the deliberate injection of a minority species dilutes the DT fuel density (for a fixed β_t) which makes an undesirable reduction in the reactor power compared to, say, a lower hybrid driven reactor. In the face of parasitic heating losses due to the presence of four or five ion species, we must conclude that in a reactor it will be difficult to achieve highly efficient current drive via minority heating.

We turn finally to the lowest frequency range in Fig. 3, that of Alfvén waves with $\omega \ll \Omega_i$. These waves directly transfer momentum to electrons with the desirable characteristic that $\omega/(k_{\parallel} v_e) \ll 1$, and Eq. (4) is valid. Ion damping is negligible provided $\omega < \Omega_T/2$ and $\alpha \gtrsim 6$. The compressional mode propagates throughout the plasma volume, and the fundamental radial eigenmode of the cavity, which is the most attractive for driving current in high β_t plasma, obeys the following approximate dispersion relation:⁽¹⁴⁾

$$\alpha \equiv \frac{\omega}{k_{\parallel} v_i} \approx \frac{2}{\sqrt{\beta_t}} \left[1 + \left(\frac{\pi A}{SN} \right)^2 \right]^{1/2}, \quad (6)$$

where the toroidal mode number is $k_{\parallel} = N/R$, and the shape factor S (≈ 1.32) is used to obtain the approximate radial mode number $k_r \approx \pi A/(SR)$. According to Eq. (4), p/j is minimized by exciting the slowest phase velocities (largest $n_{\parallel\ell}$), subject only to $\alpha \gtrsim 6$. However, α does not decrease significantly for N larger than 20-50, as evidenced by Eq. (6), while the antenna design becomes

more complex at large N . For the present illustration we take $N = 48$, and we compute α , ω , and $\lambda_{\parallel}$ for the three reactors of Table II. The resulting $n_{\parallel}$ is used in Eq. (4), and the Alfvén power, P_A , required to sustain the reactor current is determined as $P_A = (p/j)\bar{j}V$; see Table V.

Figure 4 shows that Alfvén waves require the least power to drive currents among the modes we have considered. In fact, with electric power to Alfvén current drive power efficiencies as low as $P_A/P_A^{(e)} = 25\%$, the electric power $P_A^{(e)}$ for the $A = 2.6$ reactor is less than 200 MW. Although tube and transmission line efficiencies are high at these frequencies we should note some possible limitations. First, these low frequencies may require loops or other internal antennas whose compatibility with a fusion environment is questionable. Antennas may couple to high Q cavity modes, which eases the engineering, but care must be taken to properly account for excitation of the shear Alfvén wave.⁽²³⁾ This mode is a highly localized power sink whose contribution to driven currents is still uncertain. Also, as pointed out earlier, these low phase velocity waves principally supply canonical angular momentum to trapped electrons, and the consequences of this neoclassical effect are unclear.

IV. CONCLUSIONS

We have argued that low aspect ratio tokamak designs are attractive for steady state wave driven reactors. For example, selection of low aspect ratios ($A \sim 2.6$) permits the diversion of ~ 3 T fields (which is considered feasible for the bundle divertor in Ref. 6) in a reactor with the same fusion power, wall load, and TF coil strength (11.1 T) as in the STARFIRE design. An important benefit of the $A = 2.6$ choice is a substantial reduction in the

TF coil volume (and presumably cost) compared to that for an $A = 3.6$ design. This result confirms the independent finding, in Ref. 8, that $A = 2.6$ minimizes the bus bar cost of energy when the current drive power is negligible.

Current drive power generally increases at lower A and is certainly not negligible for lower hybrid waves. Three wave candidates have been identified which could potentially require less power to maintain steady-state operation. We estimate magnetosonic waves to need 85% of the lower hybrid power when the phase speed, $\omega/k_{\parallel}$, exceeds the electron thermal speed, v_e . The wavelengths and frequencies for this regime may permit the use of waveguide couplers to the plasma, which is an attractive feature. If these waves are broadcast at a low phase speed, with $\omega/(k_{\parallel}v_e) \ll 1$, we predict improved performance, $P_{MS} \sim 0.73 P_{LH}$. However, this regime requires $\lambda_{\parallel} \leq 10$ cm, which makes the antenna design difficult. Alfvén waves further reduce the absorbed wave power, $P_A \sim 0.32 P_{LH}$; however, at these low frequencies (5-10 MHz) waveguide launchers are not practical, and internal conductors with $\lambda_{\parallel} \sim 1$ m are indicated. None of these three waves suffers from the accessibility constraint of lower hybrid waves, and broad current density profiles appear possible in principle.

Our conclusions should be accepted only with certain qualifications. Incorporation of relativistic effects is warranted when $n_{\parallel} \rightarrow 1$, for waves with $\omega/(k_{\parallel}v_e) \gg 1$. For waves with $\omega/(k_{\parallel}v_e) \ll 1$, the canonical angular momentum input to trapped electrons requires additional study. Coupling to the shear Alfvén resonance when $\omega < \Omega_i$ is another concern. Of course, an accurate knowledge of the wave spectral energy density is necessary, since $\hat{p}/\hat{j}$ is sensitive to $n_{\parallel u} - n_{\parallel \ell}$. A high antenna transmission coefficient, which is crucial to efficient power transmission, is expected since cavity modes can be ex-

cited for low frequency waves. The electric to rf power conversion efficiency must also be large for economical operation, and we are optimistic that this will be the case for low frequency systems. Hence, we conclude that steady state tokamak reactor operation at low aspect ratios is a viable option, and the electric power required (for low frequency waves) may be less than that for the lower hybrid wave-driven STARFIRE reactor. Finally we recall that profile effects and detailed engineering analysis of the wave-current drive systems can ultimately modify our crude estimates of required electric power by factors of two or three. Consequently, it is still difficult at this point to eliminate particular waves from future consideration for current drive applications.

ACKNOWLEDGEMENT

The author appreciates discussions with K. Evans, Jr., G. A. Emmert and N. J. Fisch.

REFERENCE

1. D. L. Jassby, Comments Plasma Phys. Cont. Fusion 3 (1978) 151.
2. D. A. Ehst, et al., A Systematic Survey of Tokamak Reactor Physics Design Parameters, Argonne National Laboratory ANL/FPP/TM-95 (1979);
D. A. Ehst, et al., Nucl. Technol. 43 (1979) 28.
3. P. E. Stott, C. M. Wilson, A. Gibson, Nucl. Fusion 17 (1977) 481;
P. E. Stott, C. M. Wilson, A. Gibson, Nucl. Fusion 18 (1978) 475.
4. D. A. Ehst, Nucl. Fusion 19 (1979) 1369.
5. S. Y. Yuen, et al., Nucl. Fusion 20 (1980) 159.
6. A. D. Sanderson, P. E. Stott, J. Nucl. Mater. 76 and 77 (1978) 530.
7. M. A. Abdou, et al., 8th Intern. Conf. on Plasma Physics and Controlled Nuclear Fusion Research (Brussels, 1980); paper IAEA-CN-38/E-1 (preprint).
8. C. C. Baker, et al., STARFIRE - A Commercial Tokamak Fusion Power Plant Study, Argonne National Laboratory, ANL/FPP-80-1 (1980).
9. D. A. Ehst, M. A. Abdou, Trans. Am. Nucl. Soc. 34 (1980) 41.
10. L. R. Turner, M. A. Abdou, Computational Model for Superconducting Toroidal-Field Magnets for a Tokamak Reactor, Argonne National Laboratory ANL/FPP/TM-88 (1977); L. R. Turner, M. A. Abdou, Proc. 7th Symp. on Engineering Problems of Fusion Research, Knoxville, Tenn. (1977) 762.
11. K. Evans, Jr., et al., Proc. 3rd ANS Top. Mtg. on Technology of Controlled Nuclear Fusion, Santa Fe, New Mexico (1978) 1084.
12. M. Söll, An Analytical Approach for Calculating the Magnetic Field Ripple of Toroidal D-Coil Magnet Systems, Max-Planck Institut für Plasmaphysik, Interner Bericht No. 24 (1979).
13. C. F. F. Karney, N. J. Fisch, Phys. Fluids 22 (1979) 7.
14. N. J. Fisch, C. F. F. Karney, Phys. Fluids 24 (1981) 27.

15. N. J. Fisch, A. H. Boozer, Phys. Rev. Lett. 45 (1980) 720.
16. N. J. Fisch, Current Generation by Minority Species Heating, Princeton Plasma Physics Laboratory, PPPL-1684 (1980).
17. D. A. Ehst, et al., Lower-Hybrid Heating and Current Drive System for the STARFIRE Tokamak, Proc. 4th ANS Top. Mtg. on Technology of Controlled Nuclear Fusion, King of Prussia, Pa. (1980) (to be published).
18. T. H. Stix, Nucl. Fusion 15 (1975) 737.
19. N. J. Fisch, Current Generation in a Relativistic Plasma, Princeton Plasma Physics Laboratory, PPPL-1739 (1980).
20. M. Fukuda, K. Matsuura, J. Phys. Soc. Japan 44 (1978) 1344.
21. F. W. Perkins, Nucl. Fusion 17 (1977) 1197.
22. J. Hosea, et al., Phys. Rev. Lett. 43 (1979) 1802.
23. A. Hasegawa, Nucl. Fusion 20 (1980) 1158.

TABLE I
Parameters of D-Shaped Equilibria
with $R = 6.7$ m, $B_0 = 5.7$ T

A	ϵ_T	q_a	q_b	$\bar{I}$ (MA)
3.6	0.068	1.64	3.55	10.8
3.0	0.082	1.91	3.87	15.3
2.6	0.098	2.14	4.18	20.4

TABLE II

Reactor Parameters at Fixed $B_0 = 5.7$ T, $n_{e0} = 3400$ MA, $W = 3.6$ MA/m²,
 $r_{B/S} = 1.2$ m, $r_V = 0.7$ m, $R_{t0} = 0.7$, $n_{Be}/n_{DT} = 0.04$

A	ϵ_T	R (m)	ϵ (T)	$\bar{I}_e$ (keV)	I (MA)	$\bar{n}_e$ (10^{20} m ⁻³)
3.6	0.068	6.94	5.80	20.0	11.4	1.19
3.0	0.082	6.37	4.95	17.0	12.6	1.28
2.6	0.098	5.96	4.23	12.0	13.4	1.67

A	$\bar{n}_e/\bar{n}_{DT}$	B_s (T)	$\bar{I}_e$ (keV)	ϵ (m ²)	$\bar{j}$ (MA/m ²)
3.6	0.09	4.54	20.5	797	0.616
3.0	0.07	3.71	16.8	886	0.563
2.6	0.05	3.06	11.7	962	0.513

TABLE III

TF Coil Geometry with $A_V = 0.2$ m, $A_{B/S}^i = 1.2$ m;12 TF Coils with $\epsilon(R_S) = 1\%$

A	A_m^i (m)	R_C (m)	R_i (m)	R (m)	R_s (m)	R_C (m)	ϵ (m)	A_{sup} (m)
3.6	0.96	2.65	3.15	6.94	8.87	12.89	3.36	0.40
3.0	0.99	1.86	2.37	6.37	8.49	12.90	3.63	0.36
2.6	1.05	1.22	1.76	5.96	8.25	12.88	3.92	0.32

A	$H_{m^i \epsilon}$ (m)	L (m)	$2 \cdot R_C \cdot A_m^i \cdot L$ (m ³)	L_{sep} (m)
3.6	7.26	40.01	760.2	6.75
3.0	8.26	44.66	658.4	6.70
2.6	9.24	49.22	571.5	6.74

TABLE IV
Parameters for Low Speed ($\omega/k_{\parallel} < v_e$) Magnetosonic
Wave-Driven Currents at $\alpha = 3.5 \times 10^{-3}$

A	ω (MHz)	V_A (10^3 m/s)	λ (m)	n_e	ω/v_e	α	P_{MS} (MW)
3.6	155	7.4	0.101	19	5.1	18	58
3.0	132	6.1	0.098	23	5.5	16	67
2.6	113	4.5	0.085	31	6.5	14	109

TABLE V
Parameters for Compressional Alfvén Wave Driven Currents
at $\omega < \omega_{UH}$ and $\omega/k_{\parallel} < v_e$; $N = 48$, and ω_T is Evaluated at $B = B_0$

A	α	λ_g (m)	ω (MHz)	n_e	$\omega_T/(2\pi)$ (MHz)	P_A (MW)
3.6	7.7	0.91	7.5	44	29	25
3.0	7.0	0.83	6.7	53	25	29
2.6	6.5	0.78	5.6	69	21	48

Fig. 1. Reactor variation with aspect ratio for fixed fusion power, $P_f = 3400 \text{ MW}$, neutron wall load, $W = 3.6 \text{ MW/m}^2$, maximum toroidal field strength, $B = 11.1 \text{ T}$, inboard blanket/shield thickness, $\Delta_{B/S}^i = 1.2 \text{ m}$, scrapeoff width, $\Delta_v = 0.2 \text{ m}$, and outboard field ripple; $\delta(R_s) = 1\%$, $r_t = 0.25/A$. Lower graph includes lower limit of parallel index of refraction, $n_{\parallel 2}$, to avoid lower hybrid wave reflection.

Fig. 2. Toroidal field (TF) coil geometry with typical equilibrium field (EF) coil placement required to form mildly triangular, elongated plasma.

Fig. 3. Angular frequency, ω , and parallel wave vector, $k_{||}$, for six wave-current-drive possibilities: electron cyclotron heating (ECRH), lower hybrid (LH), low phase speed (LS) magnetosonic (MS), high phase speed (HS) magnetosonic (MS), ion cyclotron heating (ICRH), and Alfvén wave. Ion and electron thermal speeds are designated v_i and v_e , respectively.

Fig. 4. Power absorbed in plasma for the reactors of Fig. 1, for three drivers, lower hybrid, low phase speed magnetosonic, and Alfvén wave.