Presented at the 5th International Conference on
Noise in Physical Systems, Bad Nauheim, Germany,
March 13 - 15, 1978

LBL-8030

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NOISE LIMITATIONS OF THE DC SQUID

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July 1978

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The dc SQUID (Superconducting Quantum Interference Device) [1] is a very sensitive detector of changes in magnetic flux, and is becoming widely used as a magnetometer and as a voltmeter. This paper summarizes the noise limitations of the device, and makes projections of its ultimate performance. A more detailed account of this work is to appear elsewhere [2].

The dc SQUID consists of two Josephson junctions [3] connected in parallel on a superconducting ring. When the current, I , through the junctions is increased from zero, no voltage appears across the SQUID until the current exceeds the critical current, I_c . For $I > I_c$, the voltage, V , increases smoothly with I ; it is essential that the I - V characteristic be non-hysteretic. In operation, the SQUID is biased with a constant current, I_B , greater than I_c . When the magnetic flux, ϕ , threading the ring is changed, the voltage across the SQUID oscillates with a period of one flux quantum, $\phi_0 = h/2e \approx 2 \times 10^{-15}$ Wb. Thus, the SQUID could be used as a digital flux-meter, but, in practice, it is almost invariably used as the null-detector in a feedback system. A voltage change across the SQUID is amplified by suitable room temperature electronics, the output of which is fed back as a current into a coil coupled to the SQUID. Thus, a flux change in the SQUID is cancelled by the flux generated by the feedback current; this current is directly proportional to the applied flux. In this way, changes in flux corresponding to a small fraction of a flux quantum can be detected.

The most sensitive dc SQUID currently available [4] uses two Josephson tunnel junctions resistively shunted to eliminate hysteresis on the I - V characteristic (see Fig. 1). It is fabricated on a 3-mm diameter quartz tube in the following steps: (1) A gold shunt is deposited; (2) a lead band is evaporated around the tube; (3) two niobium strips are sputtered (these make superconducting contacts to the lead band); (4) the niobium films are oxidized; (5) a lead tee is evaporated to form two Josephson junctions with the niobium strips; (6) a slit is made in the lead band; (7) the device is covered with a thin insulating layer, and a superconducting ground plane (lead) is deposited over the narrow strips of the SQUID to lower their inductance; (8) copper leads are attached with pellets of niobium. The inductance of the SQUID, L , is about 1 nH, the resistance of each shunt, R , is about 0.6 Ω , and the area, critical current, I_0 , and capacitance, C , of each junction are about $10^4 \mu\text{m}^2$, $2 \mu\text{A}$, and 500 pF, respectively. The critical current is chosen so that $2LI_0 \geq \phi_0$, the optimum operation condition [5], and the shunt resistance is chosen so that $2\pi I_0 R^2 C \leq \phi_0$, the condition for the I - V characteristic to be non-hysteretic [6]. When the SQUID is operated in the feedback mode, the spectral density, S_ϕ , of the flux noise is white at frequencies above about 10^{-2} Hz, with a value of about $10^{-9} \phi_0^2 \text{Hz}^{-1}$. This cor-

Fig. 1 Tunnel junction dc SQUID

responds to an rms magnetic field noise of about $10^{-14} \text{ T Hz}^{-1/2}$, and an energy resolution per Hz, $S_\phi/2L$, of about $2 \times 10^{-30} \text{ J Hz}^{-1}$. The room temperature electronics is appropriately matched to the SQUID so that its noise temperature, 1K or less, is well below the temperature of the SQUID, usually 4.2K. Thus, the performance of the SQUID is limited by intrinsic noise.

We now give a brief discussion of the theoretical intrinsic noise limitations of the SQUID. A detailed computer analysis [5] shows that, when the flux in the SQUID is $\sim (2n + 1)\phi_0/4$ and the current bias is optimized, the flux-voltage transfer function has its optimum value

$$\frac{\partial V}{\partial \phi} \approx \frac{R}{L}, \quad (1)$$

$\approx 1 \mu\text{V}/\phi_0$. The voltage noise, $V_N(t)$, across the SQUID has a spectral density $S_V = \gamma_V 4k_B T(R/2)$, where $\gamma_V \approx 8$ for typical SQUID parameters [5]. Thus, the equivalent flux noise has a spectral density $S_\phi = S_V/(\partial V/\partial \phi)^2 \approx \gamma_V 2k_B T L^2/R$. Taking $T = 4.2\text{K}$, $L = \ln H$, and $R = 0.6\Omega$, we estimate an rms flux noise of about $2 \times 10^{-5} \phi_0 \text{ Hz}^{-1/2}$, in reasonable agreement with the measured value. The energy resolution per Hz, $\epsilon/1\text{Hz} = S_\phi/2L \approx \gamma_V k_B T/(R/L)$, is of the order of the thermal noise, $k_B T$, divided by the frequency $R/L \approx 0.6\text{GHz}$. This is approximately the frequency of the ac Josephson supercurrents at the voltage at which the SQUID is biased. The energy resolution can be written in an alternative way using the constraints $2LI_0 = \phi_0$ and $2\pi I_0 R^2 C = \phi_0$:

$$\epsilon/1\text{Hz} = \gamma_V 4\pi^{1/2} k_B T (LC)^{1/2}. \quad (2)$$

From (2), it is evident that the energy resolution can be improved by reducing L , C , and T . In practice, it is undesirable to reduce L significantly because it becomes increasingly difficult to couple external circuits efficiently as L is reduced. If one were to reduce the junction area from $10^4 \mu\text{m}^2$ to $1 \mu\text{m}^2$, the capacitance would drop by a factor of nearly 10^4 , (somewhat less, because the barrier thickness would have to be reduced slightly to maintain the same critical current), and $\epsilon/1\text{Hz}$ should decrease by nearly two orders of magnitude. One could also improve the performance by reducing the temperature, although it would be necessary to use a preamplifier with noise temperature less than the SQUID temperature, for example, a second SQUID. In the limit where $k_B T/2e$ is less than the bias voltage, $\sim I_0 R$, the shot noise in the tunnel junctions becomes greater than the Johnson noise in the resistive

shunts. The spectral density of the voltage noise produced by the shot noise current is $S_V(s) = \gamma_V(s) 2e(2I_0)(R/2)^2 = \gamma_V(s) e I_0 R^2$ (the numerical value of $\gamma_V(s)$ has not yet been computed). Thus, $S_\phi(s) = \gamma_V(s) e I_0 R^2 / (R/L)^2 = \gamma_V(s) e I_0 L^2$, and

$$\epsilon/1\text{Hz} = \gamma_V(s) e I_0 L/2 \sim h, \quad (3)$$

where we have assumed $\gamma_V(s) \sim \gamma_V \approx 8$, and set $2LI_0 = h/2e$. Thus, the ultimate resolution per Hz appears to be of the order of Planck's constant.

In many applications, the SQUID is inductively coupled to an input circuit. It is then necessary to take into account not only the voltage noise across the SQUID, but also the current noise in the SQUID ring which induces noise into the input circuit. As an example, we consider the SQUID voltmeter shown in Fig. 2. The input circuit consists of a voltage source, $E_i(t)$, of zero

Fig. 2 SQUID used as voltmeter

impedance, in series with an input resistance, R_i , and an input inductance, L_i , coupled to the SQUID. In the limit $\omega \ll R/L$, the impedance is $Z_i = R_i + j\omega L_i$; in this limit, the reflected contribution of the SQUID is negligible for all practical circuits. A current, $I(t)$, in the input circuit induces a flux $M_i I(t)$ in the SQUID, where $M_i^2 = \alpha^2 L L_i$ (α^2 is the coupling coefficient), and thus produces a voltage change $V(t) = M_i (\partial V / \partial \phi) I(t) = Z_f I(t)$ across the SQUID, where Z_f is the forward impedance.

The current noise in the SQUID, $I_N(t)$, has a spectral density $S_I(f) = \gamma_I 2k_B T / R$, where γ_I is a constant that has been computed numerically [2]. $I_N(t)$ produces a real flux noise, and hence a voltage noise across the SQUID: Part of $V_N(t)$ arises from this source [4], and $I_N(t)$ and $V_N(t)$ are thus partially correlated. We take their correlation spectral density to be $S_{VI}(f) = \gamma_{VI} 2k_B T$, where γ_{VI} has also been computed [2]. $I_N(t)$ induces a voltage noise in the input circuit, $E_N(t) = M_i dI_N/dt$, with a spectral density $S_E(f) = \omega^2 M_i^2 S_I(f)$. This voltage noise induces an additional voltage noise, $V'_N(t) = E_N(t)(Z_f/Z_i)$ across the SQUID. These results are summarized in Fig. 3, which shows the equivalent circuit of the input circuit and SQUID. The output impedance of the SQUID, Z , is approximately $R/2$. It has been assumed that the amplifier does not load the SQUID, and that the noise of this amplifier is negligible.

The signal-to-noise ratio of the voltmeter is calculated as follows. An input signal, E_i , produces a mean square voltage output

$$\langle V_o^2 \rangle = \langle E_i^2 \rangle Z_f^2 / |Z_i|^2. \quad (4)$$

The total noise at the SQUID output has a spectral density

$$S_T(f) = S_V(f) + \omega^2 M_i^2 Z_f^2 S_I(f) / |Z_i|^2 + 2\omega^2 M_i L_i Z_f S_{VI}(f) / |Z_i|^2. \quad (5)$$

Fig. 3 Equivalent circuit of SQUID voltmeter showing signal and noise voltages

The signal-to-noise ratio is thus

$$\frac{\langle V_0^2 \rangle}{S_T(f)} = \frac{\langle E_i^2 \rangle Z_f^2}{S_V(f) |Z_i|^2 + \omega^2 M_i^2 Z_f^2 S_I(f) + 2\omega^2 M_i L_i Z_f S_{VI}(f)}. \quad (6)$$

We can define a noise temperature, T_N , by setting $V_0^2 / S_T(f) B = 1$ with $\langle E_i^2 \rangle = 4k_B T_N R_i B$, where B is the bandwidth. This leads to

$$T_N = [S_V(f)(R_i^2 + \omega^2 L_i^2) + \omega^2 M_i^2 Z_f^2 S_I(f) + 2\omega^2 M_i L_i Z_f S_{VI}(f)] / 4k_B R_i Z_f^2. \quad (7)$$

T_N is optimized by setting $\partial T_N / \partial R_i = 0$. Substituting for the spectral densities and for Z_f , we find

$$R_i^{(opt)} = \omega L_i (1 + 2\alpha^2 \gamma_{VI} / \gamma_V + \alpha^4 \gamma_I / \gamma_V)^{1/2}, \quad (8)$$

$$\text{and } T_N^{(opt)} = \frac{\gamma_V}{\alpha^2} \left(\frac{\omega T}{R/L} \right) \left(1 + 2\alpha^2 \frac{\gamma_{VI}}{\gamma_V} + \frac{\alpha^4 \gamma_I}{\gamma_V} \right)^{1/2}. \quad (9)$$

For the dc SQUID described above, we compute [2] $\gamma_V \approx 8$, $\gamma_I \approx 5.5$, and $\gamma_{VI} \approx 6$. With these values, and taking $\alpha^2 = 0.5$, we find $T_N^{(opt)} \approx 22[\omega / (R/L)] T \approx 1 \mu K$ at 1 Hz with $T = 4.2$ K. If one regards R/L as the pump frequency, the expression for $T_N^{(opt)}$ is of the form expected for a parametric amplifier.

One can also estimate $T_N^{s(opt)}$ for the shot noise-limited SQUID by substituting $eI_0 R^2$ for $2k_B T R$ (assuming the γ 's are unchanged). Setting $2LI_0 = \phi_0$, $T_N^{s(opt)} \approx 17\hbar\omega / k_B \approx 10^{-9} K$ at 1 Hz.

I am indebted to C. D. Tesche for performing the computer calculations. This work was supported by the Division of Materials Sciences, Office of Basic Energy Sciences, U. S. Department of Energy.

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This report was done with support from the Department of Energy. Any conclusions or opinions expressed in this report represent solely those of the author(s) and not necessarily those of The Regents of the University of California, the Lawrence Berkeley Laboratory or the Department of Energy.

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