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A COMPUTATIONAL MODEL FOR COAL TRANSPORT AND COMBUSTION

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Contract Recipient: Clarkson University

Project Principal Investigator: Goodarz Ahmadi

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SUMMARY

In the period of June 1, 1992 to August 31, 1992 further progress has been made in formulation of the thermodynamically consistent, rate dependent model for turbulent two-phase flows. The thermodynamically admissible constitutive equations were derived, and the case of a simple shear flow was studied.

The kinetic model for rapid flows of granular materials, which includes frictional losses, was used for studying gravity flow down an inclined chute. Some typical velocity profiles were obtained and the results were compared with the data of Savage. The effect of inter-particle friction was also studied.

Progress has been made in developing a computational model for rapid granular and two-phase flows in complex geometries. A discrete element numerical scheme was used for granular flow analysis. A predictor-corrector numerical procedure was developed for analyzing two-phase flows.

Progress has been made in the construction of the experimental simple shear flow device. The design of the device was improved and its main body was assembled.

PROGRESS REPORT

OBJECTIVES

The objective of this project is to develop an accurate model describing turbulent flows of coal slurries, rapid flows of granular coal-air mixtures, and turbulent coal combustion processes. The other main objective is to develop a computer code incorporating the new model. Experimental verification of the foundation of the model is also included in the study.

SIGNIFICANCE TO FOSSIL ENERGY PROGRAM

A completely satisfactory theory describing the bulk coal transport including the interstitial fluid effects does not exist. This is particularly the case for turbulent flows of dense coal particle-liquid mixtures and chemically active coal combustor flows. Coal slurry and bulk transports, and operation of coal combustors accounts for a substantial portion of the cost of coal energy conversion systems. The major increase in cost arises from the need to over-design these facilities to guarantee reliability. Understanding the flow behavior of relatively dense coal slurries and bulk solids in various geometries, including coal combustors, are indispensable to economical design of the needed equipment. This project aims to develop a sound practical model for coal transport and combustion. In addition, a computational predictive capability for analyzing rapid flows of granular coal particles, and reacting and nonreacting turbulent flows of dense or dilute multiphase coal mixtures will be provided.

A THERMODYNAMICALLY CONSISTENT RATE-DEPENDENT MODEL FOR TURBULENT TWO-PHASE FLOWS

The basic conservation laws governing the motion of two-phase solid-fluid are studied. These include phasic mass and linear momentum balance along with transport equations for the fluctuation phasic kinetic energies. Based on the averaged Clausius-Duhem inequality for the mixture in a turbulent state of motion, thermodynamically admissible expressions for the phasic stress tensors, and heat and fluctuation energy flux vectors were derived. The developed stress constitutive equations were rate-dependent and nonlinear. For dense two-phase solid particles-liquid mixtures, the material parameters of the model were evaluated from the limiting conditions of rapid flows of dry spherical granular particles, and single-phase turbulent fluid flow. The special case of simple shear flows of glass beads-water mixtures were studied, and results concerning phasic stresses and normal stress differences were obtained.

Figure 1. Experimental results for restitution coefficient of spheres colliding in water.

The coefficient of restitution, r , plays an important role in the overall behavior of the two-phase flow mixtures. The value of r for glass spheres colliding in a liquid is not available. Therefore, an experimental study for measuring the coefficient of restitution was performed. Glass spheres of two different sizes were dropped in a tank of water. A video camera was used to trace the particle trajectory and its rebound from the glass bottom of the tank. The velocities of the sphere before and after collision with the lower wall were evaluated, and were used to estimate the coefficient of restitution. Figure 1 shows the experimental data for the restitution coefficient for glass spheres in water. It is observed that r varies between 0.51 and 0.62. In the theoretical model, an averaged coefficient of restitution of $r = 0.55$ was used.

Figure 2. Variations of nondimensional particulate and fluid phasic fluctuation energies with solid volume fraction.

Figure 2 shows variations of nondimensional particulate and fluid phasic fluctuation energies with solid volume fraction for 1.1 mm glass beads in water. It is observed that the nondimensional particulate fluctuation energy does not change appreciably with solid volume

fraction, while that of the fluid phase decreases with an increase in the solid volume fraction. At present, experimental confirmation of these results is not possible due to the lack of data for the phasic fluctuation energies. Verification of this result is part of the future experimental phase of this project.

Figure 3. Variations of nondimensional velocity, solid volume fraction and fluctuation kinetic energy profiles. Comparison with the experimental data of Savage (1979).

ANALYSIS OF RAPID GRANULAR FLOWS INCLUDING FRICTIONAL LOSSES

Rapid granular gravity flows down an inclined chute was studied. The previously developed kinetic model for dry particulate flows, which includes frictional losses, was used in the analysis. The equations of motion were solved using a fourth-order Runge-Kutta numerical scheme. The mean velocity, the fluctuation kinetic energy and the solid volume fraction profiles were evaluated. Figures 3 and 4 display typical velocity, solid volume fraction and fluctuation kinetic energy profiles across the flow. A coefficient of restitution of $r=0.9$ for particles was also

used. These figures show that the mean velocity could take different shapes depending on the boundary conditions. The solid volume fraction generally increases with depth in the sample shown. For different values of energy loss coefficient at the wall, however, the fluctuation kinetic energies have different trends of variation.

Figure 4. Variations of nondimensional velocity, solid volume fraction and fluctuation kinetic energy profiles.

The experimental data of Savage (1979) for the mean velocity profile is also shown in figure 3. It is observed that the model prediction is in reasonable agreement with the data. However, the predicted mean velocity profile somewhat over-estimates the experimental data.

COMPUTATIONAL MODEL DEVELOPMENT

The goal of this phase of the study is to develop an appropriate computational scheme for solving granular and two-phase flows.

Granular Flow

Progress has been made in developing a computational model for granular flows. The FLOWER code was modified for application to granular flow analysis. FLOWER is a code for simulating transient three-dimensional, free surface water flow in rivers, estuaries and coastal regions. It uses the integral form of the governing equation and the so-called discrete element method. The flow regions are represented by rectangular elements of variable sizes, while impermeable boundary elements are constructed from truncated rectangles, thus allowing for accurate representation of complex boundary conditions. The governing equations of granular materials are, of course, far more complex than that of water. Nevertheless, the mathematical structure of the rapid granular flow equations are quite similar to that of water. Therefore, it is possible to modify the code to make it applicable to granular flow analysis. The scheme uses the discrete element procedure, and the composite time and space splitting method. It leads to an explicit scheme of solution.

Two-Phase Flow

As noted in the previous report, for developing an appropriate two-fluid computational code, the starting single phase code needs to be compressible. The MAST code, which was developed for compressible laminar and turbulent flow analysis, is being extended for analyzing two-phase solid-liquid and solid-gas flows.

Formulation of appropriate equations of motion of dense two-phase mixtures and some features of the computational scheme of solution are described in Appendix A. In this section a few sample preliminary results are described.

Simulation is performed for a channel which is 10 cm wide and 180 cm long. For material parameters, $\mu_0^f = 1.84 \times 10^{-4} \text{ N s/m}^2$, $\rho_0^f = 1 \text{ kg/m}^3$ and $\rho_0 = 1000 \text{ kg/m}^3$ are used. A

uniform mean velocity of $V = 0.2$ m/s for both particulate and fluid phases, and solid volume fraction of 0.3 at the channel inlet are also assumed. Particles are assumed to be spherical with a diameter of $100\text{ }\mu\text{m}$. Thus, the flow Reynolds number based on the channel width is 200. A non-staggered grid with 65×35 node points is used. Uniform initial profiles for the mean velocities for both particulate and fluid phases, and solid volume fraction are specified. The no-slip boundary condition for the fluid phase, and the slip condition for particulate phase are used. At the outlet, zero normal gradient conditions are assumed.

Figure 5. Velocity vector plot for the fluid phase.

The fluid phase velocity vector plot at time $t = 20$ s is shown in Figure 5. It is observed that the characteristic parabolic velocity profile of single-phase laminar flow is being replaced by a rather flat profile due to the interaction between the two phases. The simulation results show that the velocity profile of the particulate phase is very close to that of the fluid phase. Therefore, the particle velocity field is not shown here. Figure 6 displays the corresponding pressure field contours. Because of the presence of the particulate phase, the pressure drop is much larger than

that of the corresponding single-phase fluid case with the same inlet conditions.

Figure 6. Pressure field contour plot.

Figure 7. Solid volume fraction contour plot.

Figure 7 shows the solid volume fraction field contours. It is observed that concentration of particles are quite uniform in the core region of the flow. However, very close to the wall, the particle concentration decreases due to the presence of the walls.

EXPERIMENTAL STUDY

The main objective of the experimental effort is to verify the foundation of the thermodynamical formulation. The other main goals are to provide reliable data for velocity, concentration and fluctuation energy profiles for a simple shear flow of granular materials and/or turbulent two-phase flows in order to verify the theoretical predictions.

Granular Shear Flow Device

Progress has been made in the construction of the granular simple shear flow setup. Various components of the device are fabricated and the setup is assembled. The original design for the experimental apparatus was also further improved. The earlier design had a potential difficulty in providing the needed power transmission to the drive shaft via the twin-sided belt. The reason for this problem was the lack of angle-of-wrap around the drive gear. In order to attain proper power transfer, the design was modified to provide approximately 180 degrees of wrap around the drive gear. This required fabrication of an additional shaft and gear assembly.

Pictures of the assembled setup are shown in figures 8 and 9. The additional assembly which was added to the original design can also be seen from figure 8. The twin-sided belt is driven by the 1.5 Hp motor in a clockwise direction. The belt then wraps around the first drive gear which will rotate the shaft and the second drive gear in the counter-clockwise direction. The second drive gear drives the single sided belt which turns the inner cylinder. The twin-sided belt leaves the first drive gear and wraps around the third drive gear, which turns the outer cylinder in

the clockwise direction. Figure 9 shows the entire assembly and both drive belts.

Figure 8. Top view of the simple shear flow device.

Since the coaxial cylinders were constructed from plexiglass, it is difficult to see from the pictures the separate layers of plexiglass required to maintain the required mono-granular layer. There is a 31 inch outside diameter and 20 inch inside diameter, half inch thick disk affixed to the frame. This is the base of the shearing cell. Directly above the base is a 24 inch outside diameter plate, which is rotated by the single sided belt and the second drive gear. This quarter inch thick plate, which rotates counter-clockwise, is the inner wall of the shearing cell. Above the 24 inch plate are two half inch thick, 29.75 inch outside diameter, spacer, stationary pieces of plexiglass, which accommodate the casing for the mono-granular layer and the upper wall of the shearing cell.

The lowest, bottom piece of the spacer plexiglass, is a disk which allows proper spacing for the bearing assembly of the inner cylinder. The second, upper-most piece of the spacer plexiglass, is a plate which is affixed to the center shaft, and maintains the bottom spacer at the proper level. Above the two stationary, spacer pieces of plexiglass is a 30 inch diameter plate, which is rotated by the twin-sided belt and the third drive gear. Affixed to this half inch thick plexiglass plate, which turns in the clockwise direction, is a 4.5 inch wide piece of Lexan flexible plexiglass around its circumference. This provides the outer shell of the shearing cell.

Figure 9. Side view of the simple shear flow device.

Due to the necessity of the modifications of the original design, assembly of the system has taken longer than originally anticipated. The system will be completed in the near future, so that the experimentation can be initiated.

ARTICLES AND PRESENTATIONS

Journals Publications

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H. Ounis and G. Ahmadi, "Motions of Small Particles in a Turbulent Simple Shear Flow Field Under Microgravity Condition," *Physics of Fluids A* 3, 2559-2570 (1991).

S. Abu-Zaid and G. Ahmadi, "A Thermodynamically Consistent Stress Transport Model for Rotating Turbulent Flows," *Geophys. Astrophys. Fluid Dynamics* 61, 109-121 (1991).

G. Ahmadi, "A Thermodynamically Consistent Rate-Dependent Model for Turbulence, Part I - Formulation," *Int. J. Non-Linear Mech.* 26, 595-607 (1991).

K.A. Elrais, W. Eckerle, G. Ahmadi and A.H. Eraslan, "Simulation of Transient Three-Dimensional Natural Convection and Saturated Pool Boiling, *Int. J. Numerical Methods Heat Fluid Flow* 2, 139-154 (1992).

S.J. Chowdhury and G. Ahmadi, "A Thermodynamically Consistent Rate-Dependent Model for Turbulence, Part II - Numerical Results," *Int. J. Non-Linear Mech.* (In Press)

W.G. Paff and G. Ahmadi, "On Convergence of Karhunen-Loeve Series Expansion for a Brownian Particle," *J. Appl. Mech. Trans. ASME* (In Press).

S. Abu-Zaid and G. Ahmadi, "A Stress Transport Model for Granular Flows in a Rotating Frame, *Int. J. Engng. Sci.* (In Press).

Conference Presentations

G. Ahmadi and S. Abu-Zaid, "Overview of Thermodynamical Approach to Modeling Turbulent Flows of Two-Phase Solid-Liquid Mixtures," *International Conference on Multiphase Flow '91-Tsukuba, Tsukuba, Japan, September 24-27, 1991.*

G. Ahmadi and S. Abu-Zaid, "A Thermodynamically Consistent Model for Turbulent Two-Phase Flows," *28th Annual Technical Meeting of the Society of Engineering Science, Gainesville, FL, November 6-8, 1991.*

G. Ahmadi and S. Abu-Zaid, "Analysis of Rapid Shear Flows of Granular Materials by a Stress Transport Model," *28th Annual Technical Meeting of the Society of Engineering Science, Gainesville, FL, November 6-8, 1991.*

G. Ahmadi, "Overview of Thermodynamical Modeling of Turbulent Flows of Multiphase Mixtures," *Proceedings of the International Conference on Engineering Application of Mechanics, Vol. 3, Sharif University of Technology, Ed. by M.S. Sadeghipour et al., June 9-12, 1992, 386-399.*

G. Ahmadi and S. Abu-Zaid, "A Thermodynamically Consistent Stress Transport Model for Rapid Granular Flows," *The 23rd Annual Meeting of the Fine Particle Society, Las Vegas, Nevada, July 13-17, 1992.*

APPENDIX A

TWO-PHASE FLOW COMPUTATIONAL MODEL

GOVERNING EQUATIONS

For an isothermal, fully-saturated two-phase flow with incompressible fluid and particulate constituents, the governing equations of mass and momentum transport as derived by Ma and Ahmadi (1990) are given as:

Conservation of mass,

$$\frac{\partial \nu}{\partial t} + \frac{\partial}{\partial x_j}(\nu v_j) = 0 , \quad (1)$$

and

$$\frac{\partial \nu^f}{\partial t} + \frac{\partial}{\partial x_j}(\nu^f v_j^f) = 0 ; \quad (2)$$

Balance of linear momentum,

$$\begin{aligned} \rho_0 \nu \frac{dv_i}{dt} = & \rho_0 \nu f_i - \nu \frac{\partial p^f}{\partial x_i} - \frac{\partial}{\partial x_i} [\gamma \rho_0 \nu k + \frac{2}{3}(\mu + \mu^T)v_{m,m}] \\ & + \frac{\partial}{\partial x_j} [(\mu + \mu^T)(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i})] + D_0(v_i^f - v_i) , \end{aligned} \quad (3)$$

and

$$\begin{aligned} \rho_0^f \nu^f \frac{dv_i^f}{dt} = & \rho_0^f \nu^f f_i^f - \nu^f \frac{\partial p^f}{\partial x_i} - \frac{2}{3} \frac{\partial}{\partial x_i} [\rho_0^f \nu^f k^f + (\mu^f + \mu^{fT})v_{m,m}^f] \\ & + \frac{\partial}{\partial x_j} [(\mu + \mu^T)(\frac{\partial v_i^f}{\partial x_j} + \frac{\partial v_j^f}{\partial x_i})] + D_0(v_i - v_i^f) ; \end{aligned} \quad (4)$$

Saturation condition,

$$\nu + \nu^f = 1 . \quad (5)$$

In these equations, ν is the solid volume fraction, ρ_0 is the constituent density, v is the mass-weighted average velocity, k is the fluctuation kinetic energy per unit mass,

f is the body force per unit mass, p^f is the mean pressure in the fluid phase, μ is the coefficient of mean viscosity, μ^T is the coefficient of turbulence viscosity and γ is coefficient of restitution of particles. The superscript f refers to the fluid phase and a symbol without a superscript represents a particulate phase quantity. Here, the drag coefficient is given by

$$D_0 = \frac{18\mu_0^f \nu [1 + 0.1(Re_d)^{0.75}]}{d^2 (1 - \frac{\nu}{\nu_m})^{2.5\nu_m}}, \quad (6)$$

where the particle Reynolds number is defined as

$$Re_d = \frac{\rho_0^f d |v_i^f - v_i|}{\mu_0^f}. \quad (7)$$

Here, μ_0 is the viscosity of the fluid phase. In equation (6), it is assumed that the particles are spheres with a diameter d and the limiting maximum solid volume fraction is $\nu_m = 0.64356$.

The mean phasic coefficients of viscosity are given by

$$\mu^f = \frac{\nu^f \mu_0^f}{(1 - \nu/\nu_m)^{2.5\nu_m}}, \quad \mu = \frac{\nu \mu_0^f}{(1 - \nu/\nu_m)^{2.5\nu_m}}, \quad (8)$$

which are consistent with those of Ishii and Mishima (1984) are used in this study. These coefficients of viscosity as given by equation (8) are consistent with Einstein's equation for effective viscosity of dilute suspensions.

For laminar flows, the phasic fluctuation energies are negligible, and $\mu^T = \mu^{fT} = 0$.

NUMERICAL SOLUTION OF THE EQUATIONS

The governing equations of fluid phase are solved simultaneously with the aid of a non-iterative implicit finite-difference scheme (called PISO standing for *Pressure-Implicit with Splitting of Operators*) which was developed by Issa (1985). This method utilizes the splitting of operations in the solution of the discretized momentum and pressure equations such that the fields obtained at each time-step are close approximations of the exact solution of the difference equations. The formal order of accuracy of the scheme is proportional to powers of δt depending on the number of operation-splittings used. (Here, two such splittings are used.)

Chen *et al.* (1991) have developed a computer code (MAST) for dilute two-phase flows. The formulation uses a hybrid Eulerian-Lagrangian scheme. That is, the

continuous phase is described by the Navier–Stokes equation (Reynolds equation for the case of turbulent flows), and the suspended phase motion is analyzed by a Lagrangian tracking scheme. The $k - \epsilon$ turbulence model and chemical reaction are included in the MAST code. In this phase of the study, the two-phase solid-fluid mixture is treated as two interacting continua. That is, the particulate phase is modeled as a continuous field. The predictor–corrector procedure is extended and used for solving the equations governing the particulate continuum. The MAST computer code is modified by including the new equations for the particulate phase.

Predictor-Corrector

In this section, the details of the predictor–corrector scheme used are described. Let superscripts n and $n + 1$ represent the time level t^n and t^{n+1} , respectively. The discretized form of equations (3) and (4) are formulated using the Euler implicit difference scheme as

$$\frac{1}{\delta t} \{ (\rho_0 \nu v_i)^{n+1} - (\rho_0 \nu v_i)^n \} = H(u_i^{n+1}) - \nu^n \frac{\partial P^{fn}}{\partial x_i} + S_i^n + D_0^n (v_i^{fn} - v_i^n), \quad (9)$$

$$\frac{1}{\delta t} \{ (\rho_0^f \nu^f v_i^f)^{n+1} - (\rho_0^f \nu^f v_i^f)^n \} = H(u_i^{fn+1}) - \nu^{fn} \frac{\partial P^{fn}}{\partial x_i} + S_i^{fn} + D_0^n (v_i^n - v_i^{fn}). \quad (10)$$

The operators $H(v_i)$ and $H(v_i^f)$ are constructed from the second order upwind differencing scheme for the convection terms and the central differencing scheme for the diffusion terms of particulate phase and fluid phase, respectively. Here, S_i and S_i^f are the source terms associated with the Cartesian mean velocity component.

Let the superscripts *, **, and *** denote intermediate field values obtained during the splitting process. The equations can hence be factorized as follows:

(a) *Predictor Step.* Using the pressure, solid volume fraction at time step t^n , the discretized equation for velocity components v_i^* and v_i^{f*} are

$$\frac{1}{\delta t} \{ \rho_0 \nu^n v_i^* - \rho_0 \nu^n v_i^n \} = H(v_i^*) - \nu^n \frac{\partial P^{fn}}{\partial x_i} + S_i^n + D_0^n (v_i^{fn} - v_i^*), \quad (11)$$

and

$$\frac{1}{\delta t} \{ \rho_0^f \nu^{fn} v_i^{f*} - \rho_0^f \nu^{fn} v_i^{fn} \} = H(v_i^{f*}) - \nu^{fn} \frac{\partial P^{fn}}{\partial x_i} + S_i^{fn} + D_0^n (v_i^n - v_i^{f*}). \quad (12)$$

With operator splitting such that

$$H(v_i^*) = H'(v_i^*) + A_p v_i^* , \quad (13)$$

and

$$H(v_i^{f*}) = H'(v_i^{f*}) + A_p^f v_i^{f*} , \quad (14)$$

in which A_p and A_p^f are the diagonal elements of the operators $H(v_i^f)$ and $H(v_i^{f*})$, the implicit momentum predictor step becomes

$$\left(\frac{1}{\delta t} - \frac{A_p - D_0^n}{\rho_0 \nu^n}\right) \rho_0 \nu^n v_i^* = H'(v_i^*) - \nu^n \frac{\partial P^{fn}}{\partial x_i} + S_i^n + D_0^n v_i^{fn} + \frac{\rho_0 \nu^n v_i^n}{\delta t} , \quad (15)$$

and

$$\left(\frac{1}{\delta t} - \frac{A_p^f - D_0^n}{\rho_0^f \nu^{fn}}\right) \rho_0^f \nu^{fn} v_i^{f*} = H'(v_i^{f*}) - \nu^{fn} \frac{\partial P^{fn}}{\partial x_i} + S_i^{fn} + D_0^n v_i^* + \frac{\rho_0^f \nu^{fn} v_i^{fn}}{\delta t} . \quad (16)$$

The solutions of these two equations yield v_i^* and v_i^{f*} .

(b) *First Corrector Step.* The mass fluxes calculated from equation (15) and (16) do not necessarily satisfy continuity. The first corrector step is established by splitting the operators

$$H(v_i^{**}) = H'(v_i^*) + A_p \frac{\nu^*}{\nu^n} v_i^{**} , \quad (17)$$

and

$$H(v_i^{f**}) = H'(v_i^{f*}) + A_p^f \frac{\nu^{f*}}{\nu^{fn}} v_i^{f**} , \quad (18)$$

and discretizing (3) and (4) once more

$$\frac{1}{\delta t} \{ \rho_0 \nu^* v_i^{**} - \rho_0 \nu^n v_i^n \} = H'(v_i^*) + A_p \frac{\nu^*}{\nu^n} v_i^{**} - \nu^n \frac{\partial P^{f*}}{\partial x_i} + S_i^n + D_0^n (v_i^{fn} - v_i^{**}) , \quad (19)$$

and

$$\frac{1}{\delta t} \{ \rho_0^f \nu^{f*} v_i^{f**} - \rho_0^f \nu^{fn} v_i^{fn} \} = H'(v_i^{f*}) + A_p^f \frac{\nu^{f*}}{\nu^{fn}} v_i^{f**} - \nu^{fn} \frac{\partial P^{f*}}{\partial x_i} + S_i^{fn} + D_0^n (v_i^* - v_i^{f**}) . \quad (20)$$

Rearranging (19) and (20), we find,

$$\left(\frac{1}{\delta t} - \frac{A_p}{\rho_0 \nu^n} + \frac{D_0^n}{\rho_0 \nu^*}\right) \rho_0 \nu^* v_i^{**} = H'(v_i^*) - \nu^n \frac{\partial P^{f*}}{\partial x_i} + S_i^n + D_0^n v_i^{fn} + \frac{\rho_0 \nu^n v_i^n}{\delta t} , \quad (21)$$

and

$$\left(\frac{1}{\delta t} - \frac{A_p^f}{\rho_0^f \nu^{fn}} + \frac{D_0^n}{\rho_0^f \nu^{f*}}\right) \rho_0^f \nu^{f*} v_i^{f**} = H'(v_i^{f*}) - \nu^{fn} \frac{\partial P^{f*}}{\partial x_i} + S_i^{fn} + D_0^n v_i^* + \frac{\rho_0^f \nu^{fn} v_i^{fn}}{\delta t} . \quad (22)$$

By subtracting (15) and (16) from (21) and (22), respectively, the first momentum increment equations are given as

$$\rho_0 \nu^* v_i^{**} - \rho_0 \nu^n v_i^* = -D_p^* \nu^n \frac{\partial}{\partial x_i} (P^{f*} - P^{fn}) + D_p^* D_0^n (1 - \frac{\nu^n}{\nu^*}) v_i^* , \quad (23)$$

and

$$\rho_0^f \nu^{f*} v_i^{f**} - \rho_0^f \nu^{fn} v_i^{f*} = -D_p^{f*} \nu^{fn} \frac{\partial}{\partial x_i} (P^{f*} - P^{fn}) + D_p^{f*} D_0^n (1 - \frac{\nu^{fn}}{\nu^{f*}}) v_i^{f*} , \quad (24)$$

where

$$D_p^* = \frac{1}{\frac{1}{\delta t} - \frac{A_p}{\rho_0 \nu^n} + \frac{D_0^n}{\rho_0 \nu^*}} . \quad (25)$$

and

$$D_p^{f*} = \frac{1}{\frac{1}{\delta t} - \frac{A_p^f}{\rho_0^f \nu^{fn}} + \frac{D_0^n}{\rho_0^f \nu^{f*}}} . \quad (26)$$

Taking divergence of (23) and (24), respectively, it follows that

$$\begin{aligned} \frac{\partial}{\partial x_i} (\rho_0 \nu^* v_i^{**}) - \frac{\partial}{\partial x_i} (\rho_0 \nu^n v_i^*) &= -\frac{\partial}{\partial x_i} [D_p^* \nu^n \frac{\partial}{\partial x_i} (P^{f*} - P^{fn})] \\ &\quad + \frac{\partial}{\partial x_i} [D_p^* D_0^n (1 - \frac{\nu^n}{\nu^*}) v_i^*] , \end{aligned} \quad (27)$$

and

$$\begin{aligned} \frac{\partial}{\partial x_i} (\rho_0^f \nu^{f*} v_i^{f**}) - \frac{\partial}{\partial x_i} (\rho_0^f \nu^{fn} v_i^{f*}) &= -\frac{\partial}{\partial x_i} [D_p^{f*} \nu^{fn} \frac{\partial}{\partial x_i} (P^{f*} - P^{fn})] \\ &\quad + \frac{\partial}{\partial x_i} [D_p^{f*} D_0^n (1 - \frac{\nu^{fn}}{\nu^{f*}}) v_i^{f*}] , \end{aligned} \quad (28)$$

Rearranging these two equations and imposing the continuity equations

$$\frac{\partial}{\partial x_i} (\rho_0 \nu^* v_i^{**}) = -\frac{\partial}{\partial t} (\rho_0 \nu^*) , \quad (29)$$

and

$$\frac{\partial}{\partial x_i} (\rho_0^f \nu^{f*} v_i^{f**}) = -\frac{\partial}{\partial t} (\rho_0^f \nu^{f*}) , \quad (30)$$

the solid volume fraction equation and the first pressure increment equation become

$$\begin{aligned} \frac{\rho_0 \nu^* - \rho_0 \nu^n}{\delta t} &= -\frac{\partial}{\partial x_i} (\rho_0 \nu^n v_i^*) + \frac{\partial}{\partial x_i} [D_p^* \nu^n \frac{\partial}{\partial x_i} (P^{f*} - P^{fn})] \\ &\quad - \frac{\partial}{\partial x_i} [D_p^* D_0^n (1 - \frac{\nu^n}{\nu^*}) v_i^*] , \end{aligned} \quad (31)$$

and

$$\begin{aligned} \frac{\partial}{\partial x_i} [D_p^{f*} \nu^{fn} \frac{\partial}{\partial x_i} + \frac{\rho_0^f}{\rho_0} D_p^* \nu^n \frac{\partial}{\partial x_i}] (P^{f*} - P^{fn}) &= \frac{\partial}{\partial x_i} [\rho_0^f \nu^{fn} v_i^{f*} + \frac{\rho_0^f}{\rho_0} (\rho_0 \nu^n v_i^*)] \\ &+ \frac{\partial}{\partial x_i} \{ [D_p^{f*} D_0^n (1 - \frac{\nu^{fn}}{\nu^{f*}}) v_i^{f*}] + \frac{\rho_0^f}{\rho_0} [D_p^{f*} D_0^{fn} (1 - \frac{\nu^{fn}}{\nu^{f*}}) v_i^{f*}] \} , \end{aligned} \quad (32)$$

The saturation condition as given by (5) is used to calculate ν^{f*} in the derivation of equation (32). The new pressure field P^* is obtained by solving equation (32). The new solid volume fraction field ν^* may then be evaluated from equation (31). Equations (23) and (24) are then used to determine v_i^{**} and v_i^{f**} , respectively.

(c) *Second Corrector Step.* Further splitting the operator, i.e.,

$$H(v_i^{***}) = H'(v_i^{**}) + A_p \frac{\nu^{**}}{\nu^n} v_i^{***} , \quad (33)$$

$$H(v_i^{f***}) = H'(v_i^{f**}) + A_p^f \frac{\nu^{f**}}{\nu^{fn}} v_i^{f***} ; \quad (34)$$

and following the same procedure as done in step (b), the second momentum increment equations are may be obtained. i.e.,

$$\begin{aligned} \rho_0 \nu^{**} v_i^{***} - \rho_0 \nu^* v_i^{**} &= D_p^{**} H'(v_i^{**} - v_i^*) - D_p^{**} \nu^n \frac{\partial}{\partial x_i} (P^{f**} - P^{f*}) \\ &+ D_p^{**} D_0^n (1 - \frac{\nu^n}{\nu^{**}}) v_i^{**} , \end{aligned} \quad (35)$$

and

$$\begin{aligned} \rho_0^f \nu^{f**} v_i^{f***} - \rho_0^f \nu^{f*} v_i^{f**} &= D_p^{f**} H'(v_i^{f**} - v_i^{f*}) - D_p^{f**} \nu^{fn} \frac{\partial}{\partial x_i} (P^{f*} - P^{fn}) \\ &+ D_p^{f**} D_0^n (1 - \frac{\nu^{fn}}{\nu^{f**}}) v_i^{f**} , \end{aligned} \quad (36)$$

Similarly, the solid volume fraction equation and the second pressure corrector equation

$$\begin{aligned} \frac{\rho_0 \nu^{**} - \rho_0 \nu^*}{\delta t} &= - \frac{\partial}{\partial x_i} [D_p^{**} H'(v_i^{**} - v_i^*)] + \frac{\partial}{\partial x_i} [D_p^{**} \nu^n \frac{\partial}{\partial x_i} (P^{f**} - P^{f*})] \\ &- \frac{\partial}{\partial x_i} [D_p^{**} D_0^n (1 - \frac{\nu^n}{\nu^{**}}) v_i^{**}] , \end{aligned} \quad (37)$$

and

$$\frac{\partial}{\partial x_i} [D_p^{f**} \nu^{fn} \frac{\partial}{\partial x_i} + \frac{\rho_0^f}{\rho_0} D_p^{**} \nu^n \frac{\partial}{\partial x_i}] (P^{f**} - P^{f*}) = \frac{\partial}{\partial x_i} \{ D_p^{f**} H'(v_i^{f**} - v_i^{f*})$$

$$+ \frac{\rho_0^f}{\rho_0} [D_p^{**} H'(v_i^{**} - v_i^*)] + D_p^{f**} D_0^n (1 - \frac{\nu^{fn}}{\nu^{f**}}) v_i^{f**}] + \frac{\rho_0^f}{\rho_0} [D_p^{**} D_0^n (1 - \frac{\nu^n}{\nu^{**}}) v_i^{**}] \} , \quad (38)$$

Equation (38) is solved for evaluating P^{**} fields. Based on this new pressure field the new solid volume fraction field ν^{**} is obtained by solving equation (37). Equations (35) and (36) may then be used for determining v_i^{***} and v_i^{f***} , respectively. The values of v_i^{***} , v_i^{f***} , ν^{**} and P^{f**} are considered to be the final field values at t^{n+1} time step. A new predictor-corrector step is then repeated for evaluating the flow conditions at $n + 2$ time step.

This algorithm is for simulation of general transient flow. A steady flow is obtained as the time step becomes sufficiently large. Issa (1985) has reported that the two-stage predictor-corrector method gives velocity and pressure fields which differ from the exact solution by terms of $O(\delta t^2)$.

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