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RESEARCH ON THE
"HYLIFE" LIQUID-FIRST-WALL CONCEPT
FOR FUTURE LASER-FUSION REACTORS

LIQUID SHEET JET EXPERIMENTS:
COMPARISON WITH LINEAR THEORY

September 1980

Principal Investigator:
Professor Myron A. Hoffman

MASTER

UNIVERSITY OF CALIFORNIA, DAVIS
Department of Mechanical Engineering
Davis, California 95616

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Principal Investigator: Professor Myron A. Hoffman

Research Assistants: Henry Asare
Robert Monson

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NOMENCLATURE

A_c	=	Wave envelope amplitude
A_o	=	Amplitude of the nozzle excitation
b	=	Sheet jet thickness (2h)
C_r	=	Wave speed
f	=	Forcing frequency
g	=	Acceleration of gravity
h	=	Sheet jet half-thickness (b/2)
k	=	Wave number, $2\pi/\lambda$
m	=	Mass
p	=	Pressure
r	=	Radius
r_d	=	Ratio of ambient density to liquid density, ρ_a/ρ
t	=	Time
U	=	Vertical component of mean jet velocity
v	=	Transverse component of mean jet velocity
We	=	Weber number, $\rho U_o^2 h_o / \sigma$
w	=	Sheet jet width
x	=	Vertical coordinate
y	=	Transverse coordinate
ξ	=	Coefficient in trajectory theory
δ	=	Differential amount
η	=	Wave surface amplitude in linear theory
λ	=	Wave length
ρ	=	Density
σ	=	Surface tension coefficient
ω	=	Complex frequency in linear temporal theory

Subscripts

a	=	ambient (or aerodynamic)
c	=	critical
i	=	imaginary part
N	=	nozzle
o	=	at the nozzle exit (initial value)
r	=	real part
σ	=	surface tension

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ABSTRACT

It has been proposed to protect the structural walls of a future laser fusion reactor with a curtain or fluid-wall of liquid lithium jets. As part of the investigation of this concept, experiments have been performed on planar sheet water jets issuing vertically downward from slit nozzles. The nozzles were subjected to transverse forced harmonic excitation to simulate the vibrational environment of the laser fusion reactor, and experiments were run at both 1 atm and at lower ambient pressures.

Linear temporal stability theory is shown to predict the onset of the unstable regime and the initial spatial growth rates quite well for the cases where the amplitudes of the nozzle vibration are not too large and the waveform is nearly sinusoidal. In addition, both the linear theory and a simplified trajectory theory are shown to predict the initial wave envelope amplitudes very well. For larger amplitude nozzle excitation, the waveform becomes highly nonlinear and non-sinusoidal and can resemble a sawtooth waveform in some cases; these latter experimental results can only be partially explained by existing theories at the present time.

LIQUID SHEET JET EXPERIMENTS: COMPARISON WITH LINEAR THEORY

H. R. Asaro*, R. K. Takahashi**, and M. A. Hoffman[†]

1. INTRODUCTION

The study of planar or sheet jets of liquid is of interest in atomization studies for droplet production, in fuel injection and combustion studies, in coating and spraying processes and more recently in connection with a new laser fusion reactor design concept [1,2]. In this latter application, which motivated this present research, it has been proposed to use near-vertical liquid lithium jets inside the fusion reactor in front of the inner walls of the reactor vessel. The array of liquid lithium jets would be thick enough (on the order of half a meter to one meter total thickness) to absorb the X-ray and ion energy as well as a good part of the energy of the neutrons released in the pulsed fusion reactions. In this way, the solid structure would be protected from erosion, embrittlement and severe neutron damage, and its useful life increased significantly. The lithium jets act as a "self-healing" liquid first wall of the reactor vessel.

One of the most important potential problem areas in this application of liquid jets arises because the reactor vessel and jet nozzles will probably be subject to significant vibrations due to the "ringing" of the entire structure caused by the pulsed nature of the laser-fusion

* Former Research Assistant

** Research Assistant

† Professor of Mechanical Engineering, University of California, Davis, CA. 95616

microexplosions. Our initial studies of annular liquid jets [3] and thin liquid films on the walls [4] indicated that additional experimentation with sheet jets (and also round jets) would provide a valuable data base for the design of a liquid lithium curtain for a future fusion reactor. The objective of this study was to determine whether transverse forced vibration of the nozzle would cause jet instability and breakup. Breakup might allow neutrons, ions, and X-rays to stream directly through to the solid walls and seriously shorten the useful life of the structure.

Extensive studies of the breakup of fan-shaped sheet jets and sprays have been performed by Dombrowski and his colleagues in connection with spraying and atomization studies [e.g., see Reference 5]. However, for our particular geometry of a nearly-constant-width sheet jet, only two stability analyses have been uncovered in our literature search which are directly applicable. In 1953, Squire [6] published a theoretical study of the linear temporal stability of a horizontal sheet jet. Two years later, Hagerty and Shea [7] independently published a very similar theoretical analysis along with some experimental data directly relevant to our study. Their experiments employed a very thin, horizontal sheet of water with a thickness of only about 1.6 mm and with jet velocities of about 3 to 5 m/s (equivalent to nondimensional thickness parameters, $(kh)_0$ from about 0.25 to 0.44 where k_0 is the initial jet wave number and h_0 is the initial jet half-thickness). The jet nozzle was vibrated transverse to the flow direction at frequencies up to about 200 Hz. Unfortunately, the vibration amplitudes were not given. Measurements of the exponential growth rates were made from flash photographs of the edge view of the sheet jets and their growth rate data agreed very well with the predictions of the linear theory.

In the present experimental study we have extended the work of Hagerty and Shea in several directions in order to better simulate the laser fusion reactor environment. We have employed:

- (a) Thicker sheet water jets such that nondimensional thickness parameters, $(kh)_0$, up to 0.84 (using Nozzle B) were investigated.
- (b) Forced excitation up to several hundred Hz over a wide range of forcing amplitudes to jet half thickness, A_0/h_0 , from about 0.10 to 0.40. The resultant wave envelope amplitude ratio, A_e/A_0 , often reached values as high as 80.
- (c) Vertical (downward) jet orientation in order to include the effect of gravity acceleration.
- (d) Experiments at low ambient pressure in a vacuum vessel as well as at one atmosphere.

In Section 2, we briefly review the key results of the linear temporal stability theories of References [6] and [7]. Then in Sections 3 and 4, we compare the temporal theories to the linear spatial growth rate theory and to a simplified "trajectory theory". The experimental apparatus is described in Section 5, and the results are compared to the various theories in Section 6.

2. REVIEW OF LINEAR TEMPORAL STABILITY THEORY

The theories developed in References [6] and [7] are essentially equivalent and both are basically extensions of the theories of Rayleigh [8] and Lamb [9] for the temporal growth of waves on a single free surface to the present case of two free surfaces. We will henceforth refer to the

linear temporal stability theories of both References [6] and [7] as simply the linear temporal theory. In developing the linear temporal theory, it is assumed that the effect of gravity is negligible so that the mean sheet thickness, $h = 2h_0$, and velocity, U , stay constant at the initial values h_0 and U_0 , respectively. The width of the sheet, w , is assumed to be much greater than h . In addition, it is assumed that both the liquid and the surrounding gas are incompressible and inviscid and that the surface tension between the two fluids is constant. We will assume also that the surrounding fluid, air, is at rest.

The wave amplitude, η , is assumed to be much less than $h_0 = h/2$, so that the interface boundary conditions can be applied at $\pm h_0$ as part of the linearization of the problem. The geometry of the sheet jet with two types of waves on it is shown in Figure 1. For case (a), the waves on the two surfaces are in phase, and this is referred to as the sinuous or "snake" mode. For case (b), the waves are 180° out of phase, and this is referred to as the dilational or "sausage" mode. Reference [7] proves that only these two cases are permitted by the linear theory, and that the sinuous waves of case (a) always have a higher growth rate in the unstable regime and hence will tend to be the dominant waves observed. The linear theory assumes a wave amplitude of the form

$$\eta(x,t) = \text{Real} \left[a_1 e^{i(kx - \omega t)} + a_2 e^{-i(kx - \omega t)} \right] \quad (1)$$

where the wave number, k , is real and ω is complex ($\omega = \omega_r + i\omega_i$) in the case of temporal instabilities. This implies waves which move along the sheet with a wave speed $C_r = \omega_r/k$ and grow in time everywhere along the sheet with a growth rate ω_i .

The solution for the wave amplitude and growth rates is obtained by solving Laplace's Equation for the velocity potential in each region subject to a linearized kinematic constraint at the interface and a linearized pressure boundary condition. As mentioned previously, the boundary conditions are applied at $y = \pm h_0$ as part of the linearization of the problem.

Solution of the linearized equations and boundary conditions leads to the following characteristic equations [6,7]:

$$\left(\frac{c}{k} - U_0\right)^2 \tanh(kh)_0 + \frac{1}{\rho_a} \left(\frac{c}{k}\right)^2 - c^2 k = 0 \quad (2)$$

It can be seen that this is a quadratic equation in $c(k)$ which is easily solved.

For the case where the air-to-liquid jet density ratio, ρ_a/ρ_l , is much less than $\tanh(kh)_0$, as is the case here, the solution indicates that the sheet is unstable for frequencies less than a critical frequency:

$$f_c = \frac{1}{2\pi} \frac{U_0}{h_0}$$

We also find that the wave speed, $c_r = U_0$; the corresponding temporal growth rate is then given by:

$$\frac{\omega}{2\pi f} = \sqrt{\left[\frac{r_d U_0}{2\pi f h_0} - \frac{1}{We} \right] \left[\frac{(kh)_0}{\tanh(kh)_0} \right]} \quad (3)$$

where the jet Weber number is based on the sheet half thickness, h_0 :

$$We = \frac{\rho_l U_0^2 h_0}{\sigma} \quad (4)$$

For the special case where $(kh)_0 \ll 1$, the last factor under the radical, $(kh)_0 / \tanh(kh)_0$, is essentially unity; this limiting case defines what we refer to as "thin" jets. The "thick" jets are assumed somewhat arbitrarily to be those for which $(kh)_0 = 0.25$ (where $(kh)_0 / \tanh(kh)_0 = 1.02$).

Some theoretical results are shown by the black dots in Figure 2a for 1.0 atm pressure and by the dashed lines for 0.68 and 0.17 atm. These calculations are for the experimental conditions for the water jets from the thinnest nozzle (Nozzle A with $h_0 = 0.79$ mm) at $U_0 = 5.0$ m/s. We see that both the growth rate at a given frequency and the critical frequency decrease significantly as the ambient air pressure is reduced. In a vacuum the sheet jet should be stable to all disturbance frequencies according to the linear theory. Also shown by the black dots in Figure 2b are the predicted temporal growth rates for the thickest water jet from Nozzle B ($h_0 = 3.94$ mm) at a velocity of 2.05 m/s and 1 atm pressure. The values from these figures will be compared to the experimental results in Section 6.

The solution for the wave amplitude in the unstable regime ($f > f_c$) is:

$$\eta(x,t) = A_c(t) \cos(kx - \omega_r t) = (a_1 e^{\omega_1 t} + a_2 e^{-\omega_1 t}) \cos(kx - \omega_r t) \quad (5)$$

where $A_c(t)$ is the mean envelope amplitude of the waves. We must next interpret this result for our spatially growing waves with a forced nozzle vibration assumed to be given by:

$$y_N(t) = A_0 \sin(2\pi f t) \quad (6)$$

where $f = \omega_c / 2\pi$.

The initial conditions for the temporal problem thus become

$$\eta(x,0) = 0 \quad (7a)$$

$$\dot{\eta}(x,0) = \dot{\eta}_N(0) \cos kx = (2\pi f A_0) \cos kx \quad (7b)$$

The interpretation of these somewhat unusual initial conditions for a temporal instability is that of initially perfectly flat sheet surfaces but with a cosinusoidal initial normal velocity distribution along the two surfaces. (Usually in temporal theory the surface is assumed to have an initial amplitude ripple on it with no initial normal velocity.)

Initial conditions (7a) and (7b) require that:

$$a_1 = -a_2 = \frac{\pi E A_0}{\omega_i} \quad (8)$$

Finally, the mean wave-envelope amplitude takes the spatial form

$$A_e(x) = A_{o(eff)} (e^{\frac{\omega_i}{U} x/U_0} - e^{-\frac{\omega_i}{U} x/U_0}) \quad (9a)$$

where

$$A_{o(eff)} = \frac{\pi f A_0}{\omega_i} \quad (9b)$$

This result implies that the equivalent spatial growth rate using the linear temporal theory can be approximated by:

$$k_i(LT) = \frac{\omega_i}{U_r} = \frac{\omega_i}{U_0} \quad (10)$$

These linear temporal theory results will be compared to those of the linear spatial theory in the next section.

3. LINEAR SPATIAL STABILITY THEORY

A more direct way to obtain the spatial growth rates is to assume that k is complex and ω is real in the characteristic Equation 2, and this time solve the equation for $k(\omega)$. The equation is nonlinear in k and must be solved numerically, in general. The solution for the spatial growth rate divided by the wave speed, $k_i C_r$, for one typical case at 1 atm pressure is shown by the solid curve on Figure 2a for Nozzle A and on Figure 2b for Nozzle B. The spatial growth rates can be seen to be essentially identical to the temporal growth rates, ω_i (and the wave speed was essentially equal to U_0 in all cases). Consequently, in the following discussions we will simply refer to both the linear temporal and the linear spatial theories as the "linear theory".

4. SIMPLIFIED SPATIAL TRAJECTORY THEORY

In our experiments, we purposely imposed rather large amplitude forced excitation on the sheet jet nozzles to simulate the worst situation for the laser-fusion application. The resulting wavy motion of the sheet jets was of very large amplitude, with peak amplitudes often more than an order of magnitude larger than the jet thickness as illustrated in the typical photograph of the edge view of the sheet jet shown in Figure 3. It occurred to us to try to better understand the behavior of such large-amplitude wavy jets accelerating vertically downward by hypothesizing a simple physical model which we called the "spatial trajectory theory".

4.1 General Case of Increasing $U(x)$:

The basic idea of this simplified, physical approach to the problem is illustrated in Figure 4. We consider the fluid particle, δm , which is at the crest of the wave shortly after the flow leaves the nozzle. It is then assumed that the wave speed, $C_r(x)$, is approximately equal to the vertical component of the local jet velocity, $U(x)$. This assumption implies that a fluid particle which starts in the wave crest stays in the wave crest, even with gravity acceleration, and that its trajectory traces out the envelope of the waves.

A simple experiment was performed to verify this approximation, since it is one of the key assumptions of the trajectory theory. A dimple was made in the wave crests on one side of the flow with a pointed object as the waves passed a point a few centimeters below the nozzle exit. By viewing this dimple with a strobe it was ascertained that the dimple did stay in the wave crests on the side where the dimple was originally produced, even for very large amplitude waves. (Also, the initial wave lengths, λ_0 , closely agreed with U_0/f , confirming that the wave speed, C_r , was close to U_0 .)

A second basic assumption is that the particle which ends up in the wave crest is the one which leaves the nozzle with the maximum initial transverse velocity, v_0 (as the vibrating nozzle passes through the vertical axis), where $v_0 = 2\pi f A_0$.

The basic equations for the trajectory of the fluid particle in the wave crest are:

$$\ddot{x} = -g \quad (11)$$

$$\ddot{y} = \delta F_a - \delta F_G \quad (12)$$

where $\bar{y}$ is the mean wave crest amplitude measured to the mean jet thickness (as shown on Figure 4). The aerodynamic force, δF_a , due to the air pressure difference across the wavy jet has been estimated using the solution for the linearized pressure distribution on a wavy wall of sinusoidal shape (e.g. Ref. [10]). Taking twice the pressure at the wavy wall peaks we obtain:

$$\delta F_a = \left[\frac{4\pi r_d f U}{b} \right] \bar{y} \Delta m \quad (13)$$

The surface tension force, F_s , on the fluid particle in the wave crest has been evaluated approximately by using the mean radius of curvature at the crest of a sine wave [11]:

$$\delta F_s = \frac{2\sigma}{r} \Delta A_s = \left(\frac{8\pi^2 f_{(0)}^2}{\rho b U^2} \right) \bar{y} \Delta m \quad (14)$$

where $\Delta A_s = \omega \Delta x$ = surface area of one side of the fluid element.

These estimates for δF_a and δF_s could possibly be improved for the larger amplitude, nonlinear waves by taking account of the non-sinusoidal shape of the wavy jet at the wave crests. However, for the present paper, we will use the simple linearized estimates of δF_a and δF_s given by Equations (13) and (14) (assuming a near-sinusoidal waveform). Substituting these results in Equation (12) gives

$$\frac{\ddot{y}}{y} - \beta^2 \frac{\dot{y}}{y} = 0 \quad (15)$$

where

$$\beta(t)^2 \equiv \frac{4\pi r_d f U(t)}{b(t)} - \frac{8\pi^2 f_{(0)}^2}{\rho b(t) U(t)^2} \quad (16)$$

It should be noted that both the vertical jet velocity, U , and the jet thickness, b , are functions of the vertical coordinate x or equivalently functions of the fall time, t . $U(t)$ is evaluated by integrating Eq. (11) and then $b(t)$ can be evaluated using the continuity equation (assuming a constant sheet jet width):

$$b(t) = \frac{b_0 U_0}{U(t)} \quad (17)$$

4.2 Special Case of Constant U

For the special case of high initial jet velocities ($U_0 \geq 5$ m/s), the velocity and jet thickness changes in the fall times of the experiments were small. In the limiting case of constant $U = U_0$ and $b = b_0$, β_0 becomes a constant, designated β_0 :

$$\beta_0 = 2\pi f \sqrt{\frac{r_d}{2\rho} \frac{U_0}{h_0} - \frac{1}{\omega_0}} \quad (18)$$

Comparison of this result with Equation (3) reveals that β_0 equals ω_1 , the growth rate from the linear temporal stability theory when β_0 is real (i.e., when ω is complex) for the case where $r_d \ll 1$ and $(kh)_0 \ll 1$. Thus real values of β_0 correspond to the case of the unstable sheet jet where the aerodynamic destabilizing force exceeds the stabilizing surface tension force ($f > f_c$). The wave envelope equation will be identical to the linear theory Eq. 9 for this limiting case of constant velocity, and we can thus identify $\bar{v}$ with A_0 of the linear theory solution (Eq. 9).

5. EXPERIMENTAL APPARATUS AND PROCEDURE

A schematic diagram of the experimental apparatus is shown in Figure 5. The water source was a 7.62 cm (3 inch) diameter industrial-grade water supply line with a flow rate capability of about 13 litres/s (200 GPM). A 5.08 cm (2 inch) diameter pipe about 10 meters long connected the industrial water supply to the apparatus. A standard ASME-type orifice was installed in this pipe about one meter downstream of the main ball valve. The pipe contained a section of crush-proof, flexible hose for vibration isolation and several sections of clear cast-acrylic pipe to allow viewing of the flow (and, in particular, to view any bubbles present).

The flow pipe entered the vacuum vessel vertically and was attached to the various nozzles used in the experiments by means of a custom-made flexible silicone rubber coupling which provided a smooth transition on the inner surface. The nozzles were fabricated from 5.08 cm (2 inch) cast acrylic tubes heated and pressed at one end to form a constant-width slit exit [12].

The vacuum chamber was constructed from a 30 cm diameter cast-acrylic cylinder about 120 cm high. The flat acrylic end plates had O-ring grooves machined in them as well as holes for the many feedthroughs. Due to the large amount of water vapor produced by the sheet water jets during the subatmospheric experiments, a water-jet ejector pump was selected for the vacuum pumping. With the present jet-pump water supply of about 0.8 litres/s (12 GPM), we were able to pump down to about 0.2 atm.

The nozzles were vibrated approximately transversely by means of an electromechanical shaker. (The large radius to the effective hinge point of the nozzles was about 24 cm; this coupled with the relatively small

maximum vibration amplitudes of about 1.6 mm resulted in negligibly small departures from a purely transverse motion.) The shaver was mounted outside the vacuum tank and was attached to the nozzle by means of a steel rod which passed through a specially designed bellows-type feedthrough into the vacuum chamber (see Ref. [12] for design details).

The water exited the nozzle and fell about 70 cm to a small pool at the bottom of the vacuum tank. For the atmospheric pressure runs, the water then simply flowed out a drain pipe in the bottom end plate to a drainage sump (see Figure 5). For the subatmospheric tests, a helical-rotor water pump was used to pump out the water to the drainage sump and maintain a constant head of about 30 cm of water in the bottom of the vacuum tank. Although the water pump usually cavitated and vibrated severely, the use of a crush-proof, flexible hose between the vacuum tank and the pump isolated the experimental apparatus from these vibrations quite well. This open-loop arrangement was found to be much superior to a closed-loop arrangement originally tried. The main disadvantage of the open-loop arrangement was that we had to use the industrial water supply which had substantial amounts of dissolved gases in it. This tended to create problems with bubble formation in the subatmospheric runs. (See Ref. [12] for more details.)

The mass flow rate was measured by the orifice plate in conjunction with an inclined manometer. All the flow rate calibration data in the range of the experiments fell on a straight line with a standard deviation of about $\pm 1.5\%$, at atmospheric pressure. However, for the subatmospheric runs, air bubbles produced both by a small amount of air leaking in through some of the subatmospheric pipe joints and by the dissolved gas

coming out of solution caused somewhat larger uncertainties in the velocity measurement; we estimate that the jet exit velocities have an overall uncertainty of about $\pm 3\%$ for the 1 atm runs and about $\pm 5\%$ for the subatmospheric runs. (All uncertainties are limited sample values with approximately 20:1 odds.)

The vacuum tank pressure was measured roughly by a pressure gauge installed in the top end plate. It was also measured more precisely by a mercury manometer with an estimated uncertainty in the reading of $\pm 10\%$, due primarily to the fact that the vacuum chamber pressure fluctuated. The water temperature (typically 20°C) was measured to better than $\pm 1^{\circ}\text{C}$ by a mercury/glass thermometer. This was considered to be adequate accuracy for estimating the water density and surface tension coefficient.

The electromechanical shaker amplitude was measured by two independent accelerometers, one placed on the shaker face and the other mounted on the nozzle itself. The accelerometers were calibrated over the range of frequencies from 10 to 1000 Hz on a large electromechanical shaker with a built-in, calibrated accelerometer [12]. Based on these calibrations, the estimated uncertainty in the nozzle amplitude is $\pm 4\%$, due primarily to the limited number of digits on the digital voltmeter available at the time.

The basic wave amplitude data were obtained from photographs of the edge view of the wavy sheet jet such as shown in Figures 3 and 6. These photographs were often the superposition of a relatively long exposure photo used to define the wave envelope and a short flash exposure to show up the wave form.

For the larger vibrational amplitudes, where A_p could also be read from the photos, the values of A_p measured agreed with those calculated from the accelerometer readings to within about $\pm 5\%$. For smaller vibrational amplitudes, only the accelerometer values were used.

In reading the amplitude of the mean wave envelope, A_p , at various vertical positions, it was necessary to subtract out the thickness of any fluid accumulations in the sheet edges as well as any fluid lumps or "blobs" near the edge regions of the wave crests, which were often present at large wave amplitudes and low jet velocities. Reasonably accurate corrections were possible after careful visual study of the jet wave shapes using a stroboscope. The estimated uncertainty of these mean wave envelope amplitude data is ± 0.75 mm for Nozzle A and ± 1.0 mm for Nozzle B. However, this is more like a bias error, since the eye can integrate and smooth the data read off the photos quite well. (The small scatter of the readings shown in figures such as Figs. 7 and 8 illustrates this point.)

6. DISCUSSION OF RESULTS

Extensive experimental results using three different nozzles were obtained in the course of this research [12,13]. Due to space limitations only a small, but representative sample for two nozzles can be given here.

6.1 Results for Nozzle A

Figure 6 shows typical photographs of the wave growth using Nozzle A ($b_0 = 2b_0 = 1.59$ mm, $w_0 = 78$ mm) for four forcing frequencies [13]. For $U_0 = 5$ m/s, the dimensionless parameter $(kh)_0$ ranged from about

0.03 for 30 Hz to about 0.1 for 100 Hz. Since $(kh)_0 \ll 1$ for these runs, approximations based on assuming $\tanh(kh)_0 \approx (kh)_0$ will be valid. Consequently, these Nozzle A runs are considered to be representative of "thin-jet" cases. The Weber number was about 274.

The wave envelope data read from the photographs of Fig. 6 are plotted in Figure 7 for three forcing frequencies along with the predictions of the linear theory (Eq. 9). The experimental growth rates (based on the slopes of the dashed lines through the data points on Figure 7) are within about 3.2% of the theoretical values for forcing frequencies of 30 and 100 Hz and within about 10% for 70 Hz. The intercepts of the dashed lines with the $x = 0$ axis agree with the predictions for $A_{0(\text{eff})}$ of Eq. (9b) within about $\pm 4\%$.

As can be seen from Figure 7, the experimental results deviate from the linear theory at small fall distances, near $x = 0$. This is partly because it is virtually impossible to read the wave amplitudes accurately very close to the nozzle. The experimental results also deviate from the linear theory at large x when nonlinear effects begin to become important. It can be seen that the nonlinear effects do not come in until a fall distance of about 35 cm for 30 Hz but become important at only about $x = 15$ cm for 100 Hz; this may be due to the increase of $A_{0(\text{eff})}$ with forcing frequency (Eq. 9b).

Shown on Figure 8 is the effect of forcing amplitude, A_0 , on the wave envelope shape for a fixed forcing frequency of 70 Hz. As A_0 (and hence $A_{0(\text{eff})}$) increases, the nonlinear effects can be seen to appear sooner. The linear theory of Eq. (9) (solid line) can be seen to explain the data reasonably well in the initial development region when the amplitudes are not too large and when a near-sinusoidal wave profile is evident on the photographs.

Due to the acceleration of gravity, the sheet jet velocity increases somewhat with fall distance x and the sheet thickness, h , decreases. This causes the local growth rate predicted by the linear theory (Eq. 7) to increase somewhat with fall distance, as shown on Figure 7. Using the variable velocity form of the trajectory theory (Eqs. 15 and 16), we can estimate the effect of these changes on the predictions of the linear theory. For the relatively small changes in the growth rates for Nozzle A flows, we have found these results to be essentially the same as an approximate, variable-velocity linear theory where the local values of $U(x)$, $h(x)$ and $\rho_l(x)$ are used at each step of the numerical solution of Eq. (9). The dashed curve on Figure 8 shows that these variable velocity effects are not large for these run conditions, and they do not explain the experimental results at large fall distances. (As can be seen from the photographs of Figure 6, the wave envelope amplitude appears to saturate at large fall distances. Close examination of the photographs on a light table revealed that the sheet jets actually are beginning to breakup where the wave sheet envelope appears to saturate.)

Data from runs made at two subatmospheric pressures as well as at 1 atm are shown on Figure 10. The predictions of linear theory can be seen to be in reasonably good agreement with the data for the initial development region. This verifies that the growth rate does indeed decrease as the ambient pressure decreases, as predicted by the linear theory (Figure 2a).

In summary, the linear theory appears to predict the initial behavior of the sheet jets from Nozzle A quite well up to ratios of A_o/A_o of between 15 and 30 (depending on A_o , f and p_a) for forcing amplitudes, A_o up to almost $0.4 h_o$ (i.e. $A_o \approx 0.3$ mm).

6.2 Results for Nozzle B

For the runs with the largest initial sheet jet thickness ($h_0 = 2h_{0c} = 7.9$ mm, $w_0 = 75$ mm), the maximum initial jet velocity we could conveniently achieve was about 2.05 m/s; this low initial velocity resulted in substantial change in velocity due to the gravity acceleration. For forcing frequencies from 20 to 70 Hz, $(kh)_0$ ranged from 0.24 to 0.84. Since $(kh)_0 = 0.25$ for most of these runs, we can consider the Nozzle B results as representative of fairly "thick jet" cases. The initial Weber number was about 230.

For the 1 atm pressure runs at $U_0 = 2.05$ m/s, the initial critical frequency was $f_{(crit)0} \approx 23.0$ Hz (see Figure 7b). Typical photographs for runs made for forcing frequencies both below and above $f_{(crit)0}$ are shown on Figure 11 [13]. The dramatic change in the waveform from near-sinusoidal to a sawtooth or zig-zag shape with fall distance can be seen clearly on these photographs for the higher forcing frequencies. For $f = 50$ Hz an additional phenomenon is visible. Using a stroboscope, fluid was observed to accumulate near the wave crests at the front and back edges of the sheet. (The locations of the fluid accumulations are not obvious from the edge-view photographs.) The fluid accumulations show up on photographs such as Figure 11 as "blobs" or "globules" of fluid at the wave crests which appear to be trying to break free of the main wavy sheet. In reading the mean wave envelope data from such figures, we always excluded the fluid blobs and read the amplitude of the zig-zag waveform of the main sheet jet, as mentioned previously.

Typical data for Nozzle B are plotted in Figure 12 along with the predictions of the linear theory. At the lower frequency of 20 Hz, the linear theory predicts that the sheet jet will always be unstable. For

the 20 Hz run at $A_0 = 1$ mm, the constant-velocity linear theory agrees very well with the data even though the velocity doubles in a fall distance of 60 cm, and the temporal growth rate increases by about a factor of five. (The resulting spatial growth rate increases by about a factor of 2.5.) The dashed curve shows the effect of variable velocity (using the approximate linear theory where the local values of $U(x)$, $b(x)$ and $\rho_1(x)$ were used at each computation step, as mentioned previously). For the 20 Hz run, it appears that the variable velocity effects account for at least a part of the difference between the experimental data and the constant velocity linear theory. However, it appears from Figure 12 that non-linear effects become important at fall distances of about 30 cm for a forcing amplitude of 1.4 mm at 20 Hz forcing frequency.

For the higher forcing frequencies above the critical frequency of 23 Hz (Fig. 2b), the sheet jet is initially stable, but as it accelerates downward, the critical frequency increases until the jet becomes unstable at a transition point indicated by x_t (where $f_{crit}(x) = f$) on Figure 11. Even though the sheet jet becomes unstable at $x_t = 24$ cm for the 20 Hz run shown on Figure 12, the data points show that the wave envelope amplitude continues to decrease. This behavior might be incorrectly identified with the behavior of sheet jets in the stable regime ($f < f_{crit}$) as shown by the half-sinusoidal linear theory curve for constant U_0 , b_0 and ρ_1 on Figure 12. In the stable regime, linear theory predicts that the wave envelope amplitude will increase initially due to the initial transverse momentum given the fluid particles as they leave the vibrating nozzle (not due to any instability). This can also be seen from Eq. (15) of the trajectory theory, since κ^2 is negative in the stable regime (i.e. $\delta F_0 > \delta F_a$).

After reaching a maximum, the wave envelope in the stable regime begins to decrease toward zero as the dominant surface tension forces overcome the initial transverse momentum and the weaker aerodynamic forces. For our inviscid theory, the wave envelope amplitude is predicted to continue to oscillate indefinitely due to the absence of any damping forces.

Including the effects of variable $U(x)$, $b(x)$, and $\rho(x)$ in the linear theory actually results in poorer agreement with the data for 70 B as can be seen from the dashed curve on Figure 12. We can tentatively conclude from this that the nonlinear effects are more important than the variable velocity effects for this initially stable case. We can also surmise that the nonlinear effects must be stabilizing for this case in order to counteract the tendency of the variable velocity effects to cause the wave envelope to continue to grow with fall distance, contrary to the experimental results.

In summary, it appears that the linear theory does explain the behavior of the thicker jets from Nozzle B almost as well as for the thinner jets from Nozzle A in the initial region where the waveform is nearly sinusoidal; i.e., up to ratios of A_0/A_0 of about 15 to 20 for values of the forcing amplitude A_0 up to about $0.25 h_0$ (i.e., $A_0 = 1 \text{ mm}$).

Work is in progress on a paper comparing a nonlinear theory developed in Ref. [13] to the data. This theory appears to explain the trends in the data beyond the initial linear region up to the point where the sheet jet amplitude appears to saturate (and the jets begin to breakup) [14].

7. CONCLUSIONS

Sheet jet experiments have verified that the predictions of the linear theory explain the data well for both our thin (small $(kh)_0$) and thick (larger $(kh)_0$) sheet jets in the initial region where the waveform is nearly-sinusoidal and the wave amplitude is not too large. The transformed linear temporal stability theory used to predict the amplitude of the spatially growing waves is compared to the linear spatial theory and is shown to yield the correct growth rates and wave amplitudes provided that the correct initial conditions are used. A simple spatial trajectory theory proposed agrees very well with the approximate linear theory for thin jets and provides additional physical insight into the forces which control the spatial development of the sheet jets in both the unstable and stable regimes.

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FIGURE TITLES

1. Edge view of the two modes of instability predicted by linear temporal stability theory for a planar sheet jet subject to small perturbations, where (a) is the sinuous or "snake" mode and (b) is the dilational or "sausage" mode.
- 2a. Comparison of growth rates for the water jets from Nozzle A predicted by the linear temporal and linear spatial stability theories for three representative experimental conditions.
- 2b. Comparison of the growth rates from Nozzle B predicted by linear temporal and linear spatial stability theories for one representative set of experimental conditions.
3. Edge view of a typical near-sinusoidal, exponentially-growing waveform of the sheet jet from Nozzle C with forced excitation of the nozzle at $f=70$ ($h_0=1.25$ mm, $U_0=4.75$ m/s., $A_0=1.06$ mm, $p_a=1.0$ atm).
4. Model used in the development of the simplified trajectory theory showing the motion of the fluid particle which is hypothesized to stay in the crest of the falling wave as it grows spatially to large amplitudes.
5. Schematic diagram of the water flow system, vacuum tank apparatus and forced excitation instrumentation used in the sheet jet experiments.
6. Edge view of the sheet jets from Nozzle A ($h_0=0.79$ mm, $U_0=5.0$ m/s., $A_0=0.2$ mm, $p_a=1.0$ atm) for four nozzle forcing frequencies: 30, 50, 70, and 100 Hz (going from left to right).
7. Comparison of the linear theory (solid curves) to the experimental results for Nozzle A of Fig. 6. Dashed lines are best fits to the linear portion of the data.
8. Comparison of the linear theory to experimental data for Nozzle A for four nozzle forcing amplitudes.
9. Local growth rate predictions of the linear theory for Nozzle A taking account of the variable sheet jet velocity and thickness with fall distance due to the acceleration of gravity.
10. Comparison of the linear theory to the experimental data for Nozzle A for three ambient pressures.
11. Edge views of the sheet jets from Nozzle B ($h_0=3.94$ mm, $U_0=2.05$ m/s., $A_0=1.0$ mm, $p_a=1.0$ atm) for three nozzle forcing frequencies: 20, 30, and 50 Hz (going from left to right).
12. Comparison of the linear theory to experimental data for Nozzle B at two typical forcing frequencies.

Figure 1. Edge view of the two modes of instability predicted by linear temporal stability theory for a planar sheet jet subject to small perturbations, where (a) is the sinuous or "snake" mode and (b) is the dilational or "sausage" mode.

Figure 2a. Comparison of growth rates for the water jets from Nozzle A predicted by the linear temporal and linear spatial stability theories for three representative experimental conditions.

Figure 2b. Comparison of the growth rates from Nozzle B predicted by linear temporal and linear spatial stability theories for one representative set of experimental conditions.

Figure 3. Edge view of a typical near-sinusoidal, exponentially-growing waveform of the sheet jet from Nozzle C with forced excitation of the nozzle at $f=70$ ($h_0=1.25$ mm, $U_0=1.75$ m/s, $\lambda_0=1.06$ mm, $p_a=1.0$ atm).

Figure 4. Model used in the development of the simplified trajectory theory showing the motion of the fluid particle which is hypothesized to stay in the crest of the falling wave as it grows spatially to large amplitudes.

Figure 5. Schematic diagram of the water flow system, vacuum tank apparatus and forced excitation instrumentation used in the sheet jet experiments.

Figure 6. Edge views of the sheet jets from Nozzle A ($h_0 = 0.79$ mm, $U_0 = 5.0$ m/s, $A_0 = 0.2$ mm, $p_a = 1.0$ atm) for four nozzle forcing frequencies: 30, 50, 70, and 100 Hz (going from left to right).

Figure 7. Comparison of the linear theory (solid curves) to the experimental results for Nozzle A of Fig. 6. Dashed lines are best fits to the linear portion of the data.

Figure 8. Comparison of the linear theory with the experimental data for Nozzle A for four nozzle forcing amplitudes.

Figure 9. Local growth rate predictions of the linear theory for Nozzle A taking account of the variable sheet jet velocity and thickness with fall distance due to the acceleration of gravity.

Figure 10. Comparison of the linear theory to the experimental data for Nozzle A for three ambient pressures.

Figure 11. Edge views of the sheet jets from Nozzle B ($h_0 = 1.01$ mm, $U_0 = 2.05$ m/s, $\Lambda_0 = 1.0$ mm, $p_a = 1.0$ atm) for three nozzle forcing frequencies: 20, 30, and 50 Hz (going from left to right).

Figure 12. Comparison of the linear theory with the experimental data for Nozzle B at two typical forcing frequencies.