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Rank Correlation Plots for Use With Correlated Input Variables in Simulation Studies

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Rank Correlation Plots for Use with Correlated Input Variables in
Simulation Studies

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ABSTRACT

A method for inducing a desired rank correlation matrix on multivariate input vectors for simulation studies has recently been developed by Iman and Conover (1980b). The primary intention of this procedure is to produce correlated input variables for use with computer models. Since this procedure is distribution free and allows the exact marginal distributions to remain intact it can be used with any marginal distributions for which it is reasonable to think in terms of correlation. In this paper we present a series of rank correlation plots based on this procedure when the marginal distributions are normal, lognormal, uniform and loguniform. These plots provide a convenient tool for both aiding the modeler in determining the degree of dependence among input variables rather than guessing and for communicating with the modeler the effect of different correlation assumptions.

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CORRELATION PLOTS

I. INTRODUCTION

Computer models are used widely to simulate the intricate relationships among variables in many areas of applications in order to estimate unknown quantities or predict future events. Investigation of techniques for efficiently selecting input values has led to the development of Latin hypercube sampling (LHS) by McKay, Conover and Beckman (1979) for those situations where it is inappropriate to consider the input as variables with associated distributions. Procedures for looking at the effect of different distributional assumptions on input variables have been examined in Iman and Conover (1980a).

While much effort has been expended toward the development of new statistical techniques for computer modeling, relatively little attention has been given to the problem of incorporating the dependences that may exist among the input variables. Typically the model input variables are assumed to be independent, but the assumption of independence among the input variables may not be appropriate. Many situations exist in the "real world" where dependences or correlations do exist among the input variables. That is, parameters, design variables, control variables, etc. may be dependent upon one another. The design of an experiment, where the objective is to make broad inferences over the entire joint populations of input variables, that ignores these dependences is poorly conceived and certainly not appropriate.

While there exist a few techniques for incorporating dependences among the input variables, within the context of this paper the method

of Iman and Conover (1981) is reviewed and used to induce the dependences among the input variables. This method is based on rank correlations, and the approach is based on the premise that rank correlation is a meaningful way to define dependences among variables that are monotonically related. This method is reviewed in Section III.

There are at least three reasons for using the concept of correlation, specifically rank correlation, as a vehicle to induce dependences among input variables. First, correlations are well understood by even the novice practitioner provided the raw data is being used and these data came from normal probability distributions. Most all practitioners are familiar with the effect of varying values of the correlation coefficient upon bivariate normal scatterplots and probability contours.

Second, a correlation coefficient computed on raw data may lose meaning and interpretation with data from non-normal populations or in the presence of outliers. However, rank correlation coefficients can be quite meaningful in modeling situations where the variables are monotonically related and not necessarily normally distributed. Furthermore, in some situations such as the bivariate normal and bivariate uniform, the values of the rank and Pearson correlation coefficients are essentially the same.

Third, it may make more sense to talk about monotone relationships, and hence rank correlations, because of the unusual behavior of the Pearson correlation coefficient in certain joint probability distributions.

Consider the following case that involves lognormal distributions. Let $X_1 \sim N(\mu_1, \sigma_1^2)$ and $X_2 \sim N(\mu_2, \sigma_2^2)$. Furthermore, let X_1 and X_2 have a bivariate normal distribution with correlation coefficient denoted

by $\rho(X_1, X_2)$. Now consider the two dimensional transformation defined by $Y_1 = \exp(X_1)$ and $Y_2 = \exp(X_2)$. It is natural to think of the joint distribution of Y_1 and Y_2 as a bivariate lognormal distribution. The correlation coefficient for this joint pdf is given by

$$\rho(Y_1, Y_2) = \frac{\exp[\rho(X_1, X_2) \sigma_1 \sigma_2] - 1}{[(\exp(\sigma_1^2) - 1)(\exp(\sigma_2^2) - 1)]^{1/2}}$$

Therefore, it is easy to show that there exists a lower bound on $\rho(Y_1, Y_2)$ which is greater than -1.

For example, let $\rho(X_1, X_2) = -1$ and $\sigma_1 = \sigma_2 = 1$, then $\rho(Y_1, Y_2)$ cannot be less than $-e^{-1} \approx -0.36788$. The fact that $\rho(Y_1, Y_2)$ cannot be less than $-e^{-1}$ makes interpretation of the correlation coefficient difficult for the novice practitioner. Therefore, it seems more appropriate in these cases to use the rank correlation coefficient which still has range from -1 to +1.

As a further illustration of this unusual behavior of the Pearson correlation coefficient consider the following. Let Y follow a lognormal distribution with range from 0.2 to 6.0, where the lower limit is taken to be the lower .001 quantile and the upper limit is taken to be the upper .001 quantile. This corresponds to a normal distribution with mean = 0.91161 and standard deviation = .55031. Therefore, let $\sigma_1 = \sigma_2 = .55031$ and $\rho(X_1, X_2) = -1$ in the above formula, which produces a lower bound of $\rho(Y_1, Y_2) = .7387$.

Such a bivariate lognormal distribution was sampled using the Iman and Conover method. Some summary data are given in Table B-5 in Appendix B. The fact that some of the sample Pearson correlation coefficients exceed this lower bound could be due to a couple of reasons. The first is simply

round-off error. The second is the fact that the bivariate sample generated using this method may not be a true bivariate lognormal distribution. However, the results obtained here are amazingly close and hence it is felt that the bivariate plots produced and exhibited in this paper accurately reflect what the actual joint pdf is in each case considered.

In conclusion, it is felt that rank correlations are a meaningful and appropriate way to induce dependences among input variables.

In Section II, the problem of setting the value of the rank correlation coefficient that will be used to induce the desired dependence between the two variables is discussed. Section III presents a review of the Iman and Conover technique, while Section IV presents the series of rank correlation plots based on this procedure and which are exhibited in Appendix A. Section V contains an additional application, and the final section contains a discussion.

II. Specifying the Rank Correlations

In any type of computer modeling involving random sampling of the input variables, whether it is simple random sampling, stratified sampling or Latin hypercube sampling, the validity of the model output depends to a great extent on how closely the sampled joint distribution of the input variables agrees with the true joint distribution. That is, if a correlation structure exists among the input variables, but the actual sampling takes place as if the input variables were independent, the theoretical properties of the statistics formed from the output may no longer be valid. Estimators intended to be unbiased or consistent may not be. The

procedure presented in Iman and Conover (1981) can be expected to bring the joint distribution of the input variables closer to the true joint distribution than would be attained under the faulty assumption of independence.

Therefore, it is extremely important that existing correlation structure be specified as accurately as possible. The modeler may find to some degree one of the following three situations. He may have some knowledge of the correlation values and the distributions of the variables. The knowledge is usually based upon his experience as the "subject area specialist," and no other data are available.

The second situation is a slight improvement over the first. The modeler may or may not have knowledge of the correlation coefficient values, but does possess knowledge of the distributions of the variables along with a basic idea of what the bivariate plots should look like.

The third situation is where the modeler has actual bivariate data from which marginal empirical distributions can be derived. These distributions may or may not look like the more common ones (normal, lognormal, uniform, etc.). In any case, a sample must be drawn that closely resembles the joint distribution of the bivariate data available.

There exist other situations of varying degree that the modeler may encounter, but it is felt that at least in the three situations discussed above, a collection of bivariate scatterplots that demonstrate the Iman and Conover method would aid the modeler in specifying the rank correlation values that exist between the input variables. These plots provide a convenient tool for both aiding the modeler in determining the degree of dependence among input variables rather than guessing and for communicating with the modeler the effect of different correlation assumptions.

III. A Review of the Method of Inducing Correlations

Suppose that a random row vector $\tilde{X}$ has a correlation matrix $\tilde{I}$. That is, the elements of $\tilde{X}$ are uncorrelated. Let $\tilde{C}$ be the desired target rank correlation matrix supplied by the user. Since $\tilde{C}$ is positive definite and symmetric, $\tilde{C}$ may be written as $\tilde{C} = \tilde{P}\tilde{P}'$, where $\tilde{P}$ is a lower triangular matrix. The Cholesky factorization scheme used by Scheuer and Stoller (1962) may be used to obtain the lower triangular matrix $\tilde{P}$. Then the transformation $\tilde{X}\tilde{P}'$ has the desired correlation matrix $\tilde{C}$. This is the theoretical basis for this method.

We begin by specifying a matrix $\tilde{R}$, of scores. Let the number of input variables be denoted by K , and let N be the sample size. Let $\tilde{R}$ be an $N \times K$ matrix whose columns represent K independent permutations of a set of scores $\{a(i)\}, i = 1, 2, \dots, N$. Each row of $\tilde{R}$, say $\tilde{R}_i$, has K components, where each component assumes one of the values $a(i), i = 1, 2, \dots, N$ with equal probability. Then the row vector $\tilde{R}_i$ has population correlation matrix $\tilde{I}$.

The scores $\{a(i)\}$ are the van der Waerden scores $\Phi^{-1}(1/(N+1))$, where Φ^{-1} is the inverse function of the standard normal distribution function. Hence each column of $\tilde{R}$ is a random permutation of the van der Waerden scores for a sample of size N . The authors have investigated use of other scores and found the van der Waerden scores to have more desirable properties for generating the joint distribution.

While the expected correlation matrix associated with $\tilde{R}$ is $\tilde{I}$, the actual sample correlation matrix of $\tilde{R}$, say $\tilde{T}$, may differ considerably from $\tilde{I}$. Hence, consider only realizations of $\tilde{R}$ which have distinct (non-identical) columns, so that $\tilde{T}$ is positive definite and symmetric.

Then there exists a lower triangular matrix $\tilde{Q}$ such that $\tilde{T} = \tilde{Q}\tilde{Q}'$ and hence $(\tilde{Q}^{-1})' \tilde{T} (\tilde{Q}^{-1}) = \tilde{I}$. Therefore, $\tilde{R}(\tilde{Q}^{-1})'$ will have row vectors that have an exact identity correlation structure. That is, the correlation matrix of $\tilde{R}_1(\tilde{Q}^{-1})'$ is exactly $\tilde{I}$.

It is apparent that $\tilde{R}^* = \tilde{R}(\tilde{Q}^{-1})' \tilde{P}'$ has row vectors that have a correlation matrix exactly equal to $\tilde{C}$. Furthermore, the rank correlation matrix of $\tilde{R}^*$, say $\tilde{M}$, should be close to $\tilde{C}$.

It only remains to generate an $N \times K$ matrix of input vectors, according to any desired method or distribution, as if the K input random variables were independent of each other. Then the values of the variable in each column are arranged so they have the same order (rank) as the corresponding column in $\tilde{R}^*$. Thus the sample rank correlation matrix of the input vectors will be identical to the rank correlation matrix associated with $\tilde{R}^*$. Furthermore, the sample rank correlation matrix of the input vectors should be close to $\tilde{C}$, the desired rank correlation structure.

The interested reader is referred to Iman and Conover (1981) for some further investigation into this method.

This method of inducing rank correlation between variables has the following desirable properties.

- 1) It is distribution free. It may be used with equal facility on all types of input distribution functions.
- 2) It is simple.
- 3) It can be applied to any sampling scheme for which correlated input variables could logically be considered, while preserving the intent of the sampling scheme. That is, the same numbers originally selected as input values are retained; only their pairing is affected to achieve

the desired rank correlation. This means that in Latin hypercube sampling the integrity of the intervals is maintained. If some lattice structure is used for selection of values, that same structure is retained.

- 4) The identity of the marginal distributions on the input variables is maintained, as the pairing of the variables does not change the numbers themselves.

IV. The Rank Correlation Scatterplots

To aid the modeler in specifying the rank correlation values that exist between the variables, a series of scatterplots based on this procedure were produced. These plots are given in Appendix A.

The marginal distributions chosen were the normal, lognormal, uniform and loguniform. It is felt that these are somewhat representative of the general types of distributions that a modeler may use. However, any distribution can be used. The only requirement is that the distribution must be of such a nature that a sample can be drawn from that distribution.

The specific distributions that were sampled are defined by the following ranges. The normal and uniform were given a range from -3.0 to 3.0, and the lognormal and loguniform were given a range of 0.2 to 6.0. In the two situations where the true ranges of the random variables are infinite, the lower limit of the range is taken to be the lower .001 percentile and the upper limit of the range is taken to be the upper .001

percentile. Every possible pairing of these four distributions were considered with the following values of the rank correlation coefficient; 0.0, ± 0.25 , ± 0.50 , ± 0.75 , ± 0.90 , ± 0.95 , ± 0.99 and ± 0.9999 . In every case, a Latin hypercube sample was taken from each marginal distribution with sample size, $N = 1000$. As mentioned earlier, other types of sampling of the marginals could be done. But for this paper only Latin hypercube samples were considered. Simple random samples of the marginals were investigated but are not given, since there were no discernible differences between the scatterplots of these two types of sampling schemes.

One interesting result occurs when any of the distributions is paired with the uniform distribution and the rank correlation coefficient is given the value of 0.9999. The joint pdf's in these situations are very nearly identical to the cumulative distribution function of the distribution that was paired with the uniform distribution (apart from a scaling of the uniform distribution so that its range is from zero to one). As the rank correlation coefficient approaches negative one, the joint pdf of these two random variables approaches $1 - F(x)$, where $F(x)$ is the cumulative distribution function of the distribution paired with the uniform distribution.

This is a very appealing property of this method of sampling with induced correlation, and in general, would be a necessary requirement of any such sampling scheme.

V. Sampling from Available Bivariate Data

In Section II, the situation of actually having data from the process that is being modeled was discussed. Attention is restricted without loss

of generality to bivariate distributions. The modeler could take this data and estimate the marginal distributions. Sampling would then proceed as described in Section III, using the estimated rank correlation coefficient as the target rank correlation.

However, it may be very difficult to identify the "distributional family" from which these marginals come. Furthermore, it may be equally difficult to sample the estimated distribution function that is based on splines. Therefore, the modeler may wish to sample directly from the marginal empirical distribution functions derived from the bivariate data.

Two such examples are presented to illustrate this technique. The first is a set of $N=24$ pairs of sample values that measure the permeability and porosity of the 3rd Venango Oil Sands in Pennsylvania. The data were taken from Musket (1937, p103 - 106). The sample rank correlation coefficient for this set of data is 0.674. This value was then used as a target rank correlation in this procedure and the marginals were taken to be the empirical distribution functions. The original data and the simulated joint distribution for this set of $N=24$ points are given in Figure 1. This sampling procedure was able to "duplicate" this set of bivariate data very well.

Since the sampling was done from the empirical distribution functions, the simulated sample values are identical to the original data points. What has changed is the pairing of the individual pairs of points. In essence, what this procedure does is change the pairings of data points in such a way as to maintain a rank correlation near the value of the target correlation.

If in the simulated sample we take a sample size larger than the number of original data points, then some (or all) of the original data points will

Figure 1

Scatterplots of the Original Data and the Simulated Joint Distribution of Permeability and Porosity of the 3rd Venango Oil Sands in Pennsylvania (taken from Musket (1937), p. 103-106).

be repeated in the simulation sample. This procedure will pair these observations in such a way as to maintain the rank correlation near the target correlation.

The foregoing "test" of this sampling procedure was repeated on a second set of $N=305$ pairs of sample values that are measures of the flow rate and suspended sediment for the Rio Grande River near Bernalillo, New Mexico. The original data and simulated joint distribution are given as scatterplots in Figure 2. Again, this sampling procedure "duplicated" the observed data quite well.

In summary, it is felt that when sampling from given empirical distribution functions, that the present sampling scheme can adequately reproduce a given set of bivariate observations, provided, of course, that the two variables are monotonically related.

VI. Discussion

Methods for pairing observations on independent random variables in order to induce a rank correlation structure are reviewed in this paper, and a series of scatterplots are presented to illustrate the procedure. Furthermore, the scatterplots can be used by the modeler as an aid in specifying the rank correlations among the input variables. This procedure has several advantages over some of the other procedures, but is not without its limitations.

First, this procedure cannot be used when the range space of the bivariate random variables (X,Y) is not a product space. In such a case,

Figure 2

Scatterplots of the Original Data and the Simulated Joint Distribution of Flow Rate and Suspended Sediment for the Rio Grande River near Bernalillo, New Mexico - (taken from Love (1956), p. 380-381)

sampling from the ranges of the marginals followed by appropriate pairings is almost certainly going to produce pairs of points outside the range space of (X,Y) . In such cases, some other procedure of simulation would need to be used.

Secondly, a multivariate distribution is much more complex than a mere collection of marginal distributions and a variance - covariance matrix. However, it is usually not possible to obtain more rigid specifications than those. In fact, it is more usual to find only the marginal input distributions specified with the correlation matrix defaulted to the identity matrix for simplicity. If correlations are appropriate, the Iman and Conover method of inducing correlations on input variables would be a definite improvement. Of course, if more complete information about the multivariate input distribution is available, it should be used in the sampling scheme if methods are available for incorporating that information.

A recent technical report by Iman, Davenport and Zeigler (1980) at Sandia Laboratories provides a user's manual and computer listings for implementing the methods discussed in this paper. A copy of this report can be obtained from the first author on this paper.

APPENDIX A

Scatterplots of Simulated "Joint" Distributions

On the graphs which follow, the axes are labeled to indicate the distribution associated with the random variables which have been correlated. The numbers on the axes themselves correspond to the ranges used with the respective variables.

Figure A-1

Simulated Scatterplots of "Jointly" Distributed Random Variables with Specified Target Rank Correlations with Sample Size = 1000.
Case 1: Normal Distribution vs. Normal Distribution.

Figure A-1

Simulated Scatterplots of "Jointly" Distributed Random Variables with Specified Target Rank Correlations with Sample Size = 1000.
Case 1: Normal Distributions vs. Normal Distribution.

Figure A-1

Simulated Scatterplots of "Jointly" Distributed Random Variables with Specified Target Rank Correlations with Sample Size = 1000.
Case 1: Normal Distribution vs. Normal Distribution.

Figure A-1

Simulated Scatterplots of "Jointly" Distributed Random Variables with Specified Target Rank Correlations with Sample Size = 1000.
Case 1: Normal Distribution vs. Normal Distribution.

Figure A-2

Simulated Scatterplots of "Jointly" Distributed Random Variables with Specified Target Rank Correlations with Sample Size = 1000.
Case 2: Normal Distribution vs. Lognormal Distribution.

Figure A-2

Simulated Scatterplots of "Jointly" Distributed Random Variables with Specified Target Rank Correlations with Sample Size = 1000.
Case 2: Normal Distribution vs. Lognormal Distribution.

Figure A-2

Simulated Scatterplots of "Jointly" Distributed Random Variables with Specified Target Rank Correlations with Sample Size = 1000. Case 2: Normal Distribution vs. Lognormal Distribution.

Figure A-2

Simulated Scatterplots of "Jointly" Distributed Random Variables with Specified Target Rank Correlations with Sample Size = 1000.
Case 2: Normal Distribution vs. Lognormal Distribution.

Figure A-3

Simulated Scatterplots of "Jointly" Distributed Random Variables with Specified Target Rank Correlations with Sample Size = 1000.
Case 3: Normal Distribution vs. Loguniform Distribution.

Figure A-3

Simulated Scatterplots of "Jointly" Distributed Random Variables with Specified Target Rank Correlations with Sample Size = 1000. Case 3: Normal Distribution vs. Loguniform Distribution.

Figure A-3

Simulated Scatterplots of "Jointly" Distributed Random Variables with Specified Target Rank Correlations with Sample Size = 1000.
Case 3: Normal Distribution vs. Loguniform Distribution.

Figure A-3

Simulated Scatterplots of "Jointly" Distributed Random Variables with Specified Target Rank Correlations with Sample Size = 1000.
Case 3: Normal Distribution vs. Loguniform Distribution.

Figure A-4

Simulated Scatterplots of "Jointly" Distributed Random Variables with Specified Target Tank Correlations with Sample Size = 1000.
Case 4: Normal Distribution vs. Uniform Distribution.

Figure A-4

Simulated Scatterplots of "Jointly" Distributed Random Variables with Specified Target Rank Correlations with Sample Size = 1000.
Case 4: Normal Distribution vs. Uniform Distribution.

Figure A-4

Simulated Scatterplots of "Jointly" Distributed Random Variables with Specified Target Rank Correlations with Sample Size = 1000.
Case 4: Normal Distribution vs. Uniform Distribution.

Figure A-4

Simulated Scatterplots of "Jointly" Distributed Random Variables with Specified Target Rank Correlations with Sample Size = 1000.
Case 4: Normal Distribution vs. Uniform Distribution.

Figure A-5

Simulated Scatterplots of "Jointly" Distributed Random Variables with Specified Target Rank Correlations with Sample Size = 1000.
Case 5: Lognormal Distribution vs. Lognormal Distribution.

Figure A-5

Simulated Scatterplots of "Jointly" Distributed Random Variables with Specified Target Rank Correlations with Sample Size = 1000.
Case 5: Lognormal Distribution vs. Lognormal Distribution.

Figure A-5

Simulated Scatterplots of "Jointly" Distributed Random Variables with Specified Target Rank Correlations with Sample Size = 1000.
Case 5: Lognormal Distribution vs. Lognormal Distribution.

Figure A-5

Simulated Scatterplots of "Jointly" Distributed Random Variables
with Specified Target Rank Correlations with Sample Size = 1000.
Case 5: Lognormal Distribution vs. Lognormal Distribution.

Figure A-6

Simulated Scatterplots of "Jointly" Distributed Random Variables with Specified Target Rank Correlations with Sample Size = 1000. Case 6: Lognormal Distribution vs. Loguniform Distribution.

Figure A-6

Simulated Scatterplots of "Jointly" Distributed Random Variables with Specified Target Rank Correlations with Sample Size = 1000.
Case 6: Lognormal Distribution vs. Loguniform Distribution.

Figure A-6

Simulated Scatterplots of "Jointly" Distributed Random Variables with Specified Target Rank Correlations with Sample Size = 1000.
Case 6: Lognormal Distribution vs. Loguniform Distribution.

Figure A-6

Simulated Scatterplots of "Jointly" Distributed Random Variables with Specified Target Rank Correlations with Sample Size = 1000.
Case 6: Lognormal Distribution vs. Loguniform Distribution.

Figure A-7

Simulated Scatterplots of "Jointly" Distributed Random Variables with Specified Target Rank Correlations with Sample Size = 1000. Case 7: Lognormal Distribution vs. Uniform Distribution.

Figure A-7

Simulated Scatterplots of "Jointly" Distributed Random Variables with Specified Target Rank Correlations with Sample Size = 1000. Case 7: Lognormal Distribution vs. Uniform Distribution.

Figure A-7

Simulated Scatterplots of "Jointly" Distributed Random Variables with Specified Target Rank Correlations with Sample Size = 1000. Case 7: Lognormal Distribution vs. Uniform Distribution.

Simulated Scatterplots of "Jointly" Distributed Random Variables with Specified Target Rank Correlations with Sample Size = 1000.
Case 7: Lognormal Distribution vs. Uniform Distribution.

Figure A-8

Simulated Scatterplots of "Jointly" Distributed Random Variables
with Specified Target Rank Correlations with Sample Size = 1000.
Case 8: Loguniform Distribution vs. Loguniform Distribution.

Figure A-8

Simulated Scatterplots of "Jointly" Distributed Random Variables with Specified Target Rank Correlations with Sample Size = 1000.
Case 8: Loguniform Distribution vs. Loguniform Distribution.

Figure A-8

Simulated Scatterplots of "Jointly" Distributed Random Variables with Specified Target Rank Correlations with Sample Size = 1000.
Case 8: Loguniform Distribution vs. Loguniform Distribution.

Figure A-8

Simulated Scatterplots of "Jointly" Distributed Random Variables with Specified Target Rank Correlations with Sample Size = 1000. Case 8: Loguniform Distribution vs. Loguniform Distribution.

Figure A-9

Simulated Scatterplots of "Jointly" Distributed Random Variables with Specified Target Rank Correlations with Sample Size = 1000.
Case 9: Loguniform Distribution vs. Uniform Distribution.

Figure A-9

Simulated Scatterplots of "Jointly" Distributed Random Variables with Specified Target Rank Correlations with Sample Size = 1000.
Case 9: Loguniform Distribution vs. Uniform Distribution.

Figure A-9

Simulated Scatterplots of "Jointly" Distributed Random Variables with Specified Target Rank Correlations with Sample Size = 1000. Case 9: Loguniform Distribution vs. Uniform Distribution.

Figure A-9

Simulated Scatterplots of "Jointly" Distributed Random Variables with Specified Target Rank Correlations with Sample Size = 1000. Case 9: Loguniform Distribution vs. Uniform Distribution.

Figure A-10

Simulated Scatterplots of "Jointly" Distributed Random Variables with Specified Target Rank Correlations with Sample Size = 1000.
Case 10: Uniform Distribution vs. Uniform Distribution.

Figure A-10

Simulated Scatterplots of "Jointly" Distributed Random Variables with Specified Target Rank Correlations with Sample Size = 1000.
Case 10: Uniform Distribution vs. Uniform Distribution.

Figure A-10

Simulated Scatterplots of "Jointly" Distributed Random Variables with Specified Target Rank Correlations with Sample Size = 1000.
Case 10: Uniform Distribution vs. Uniform Distribution.

Figure A-10

Simulated Scatterplots of "Jointly" Distributed Random Variables with Specified Target Rank Correlations with Sample Size = 1000.
Case 10: Uniform Distribution vs. Uniform Distribution.

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APPENDIX B.

Tables of the Target Rank Correlations, Sample Rank Correlations,
and Sample Pearson Correlations for each of the ten pairs of
"joint" distributions.

Table B-1

Target Correlations and Realized Sample Correlations for N = 1000;
Normal Distribution vs. Normal Distribution

Target Rank Correlation	Sample Rank Correlation	Sample Pearson Correlation
0.9999	0.9999	0.9999
0.99	0.9891	0.9901
0.95	0.9444	0.9499
0.90	0.8924	0.8999
0.75	0.7379	0.7491
0.50	0.4873	0.4995
0.25	0.2381	0.2484
0.00	0.0020	-0.0070
-0.25	-0.2289	-0.2478
-0.50	-0.4871	-0.4998
-0.75	-0.7371	0.7482
-0.90	-0.8923	-0.8999
-0.95	-0.9471	-0.9494
-0.99	-0.9890	0.9899
-0.9999	-0.9999	-0.9999

Table B-2

Target Correlations and Realized Sample Correlations for N = 1000;
Normal Distribution vs. Lognormal Distribution

Target Rank Correlation	Sample Rank Correlation	Sample Pearson Correlation
0.9999	0.9999	0.9300
0.99	0.9888	0.9202
0.95	0.9456	0.8827
0.90	0.8958	0.8365
0.75	0.7359	0.7017
0.50	0.4872	0.4791
0.25	0.2281	0.2443
0.00	0.0002	-0.0277
-0.25	-0.2346	-0.2372
-0.50	-0.4908	-0.4717
-0.75	-0.7289	-0.7006
-0.90	-0.8880	-0.8389
-0.95	-0.9454	-0.8820
-0.99	-0.9889	-0.9206
-0.9999	-0.9999	-0.9301

Table B-3

Target Correlations and Realized Sample Correlations for N = 1000;
Normal Distribution vs. Loguniform Distribution

Target Rank Correlation	Sample Rank Correlation	Sample Pearson Correlation
0.9999	0.9999	0.9199
0.99	0.9891	0.9101
0.95	0.9452	0.8758
0.90	0.8861	0.8289
0.75	0.7366	0.6955
0.50	0.4871	0.4585
0.25	0.2246	0.2237
0.00	0.0003	0.0017
-0.25	-0.2435	-0.2297
-0.50	-0.4781	-0.4677
-0.75	-0.7389	-0.7054
-0.90	-0.8880	-0.8241
-0.95	-0.9455	-0.8722
-0.99	-0.9886	-0.9110
-0.9999	-0.9999	-0.9199

Table B-4

Target Correlations and Realized Sample Correlations for N = 1000;
Normal Distribution vs, Uniform Distribution

Target Rank Correlation	Sample Rank Correlation	Sample Pearson Correlation
0.9999	0.9999	0.9778
0.99	0.9891	0.9680
0.95	0.9431	0.9278
0.90	0.8895	0.8799
0.75	0.7329	0.7310
0.50	0.4842	0.4876
0.25	0.2332	0.2452
0.00	0.0001	0.0132
-0.25	-0.2379	-0.2330
-0.50	-0.4818	-0.4864
-0.75	-0.7352	-0.7342
-0.90	-0.8915	-0.8802
-0.95	-0.9481	-0.9309
-0.99	-0.9891	-0.9684
-0.9999	-0.9999	-0.9779

Table B-5

Target Correlations and Realized Sample Correlations for N = 1000;
Lognormal Distribution vs. Lognormal Distribution

Target Rank Correlation	Sample Rank Correlation	Sample Pearson Correlation
0.9999	0.9999	0.9997
0.99	0.9892	0.9879
0.95	0.9472	0.9355
0.90	0.8924	0.8919
0.75	0.7315	0.7045
0.50	0.4733	0.4997
0.25	0.2452	0.2228
0.00	0.0013	-0.0293
-0.25	-0.2461	-0.2072
-0.50	-0.4837	-0.3952
-0.75	-0.7263	-0.5818
-0.90	-0.8952	-0.6871
-0.95	-0.9439	-0.7169
-0.99	-0.9889	-0.7441
-0.9999	-0.9999	-0.7479

Table B-6

Target Correlations and Realized Sample Correlations for N = 1000;
Lognormal Distribution vs. Loguniform Distribution

Target Rank Correlation	Sample Rank Correlation	Sample Pearson Correlation
0.9999	0.9999	0.9592
0.99	0.9888	0.9478
0.95	0.9474	0.9080
0.90	0.8912	0.8579
0.75	0.7363	0.6984
0.50	0.4898	0.4625
0.25	0.2423	0.2290
0.00	0.0019	0.0147
-0.25	-0.2556	-0.2005
-0.50	-0.4895	-0.3950
-0.75	-0.7394	-0.5657
-0.90	-0.8928	-0.6644
-0.95	-0.9457	-0.6985
-0.99	-0.9886	-0.7231
-0.9999	-0.9999	-0.7257

Table B-7

Target Correlations and Realized Sample Correlations for $N = 1000$;
Lognormal Distribution vs. Uniform Distribution

Target Rank Correlation	Sample Rank Correlation	Sample Pearson Correlation
0.9999	0.9999	0.8883
0.99	0.9887	0.8793
0.95	0.9474	0.8477
0.90	0.8935	0.8044
0.75	0.7363	0.6788
0.50	0.4859	0.4583
0.25	0.2386	0.2147
0.00	-0.0034	0.0136
-0.25	-0.2511	-0.2281
-0.50	-0.4719	-0.4356
-0.75	-0.7373	-0.6722
-0.90	-0.8953	-0.8060
-0.95	-0.9454	-0.8454
-0.99	-0.9893	-0.8819
-0.9999	-0.9999	-0.8867

Table B-8

Target Correlations and Realized Sample Correlations for N = 1000;
Loguniform Distribution vs. Loguniform Distribution

Target Rank Correlation	Sample Rank Correlation	Sample Pearson Correlation
0.9999	0.9999	0.9998
0.99	0.9885	0.9881
0.95	0.9435	0.9405
0.90	0.8894	0.8905
0.75	0.7280	0.7169
0.50	0.4793	0.4508
0.25	0.2362	0.2068
0.00	0.0023	-0.0303
-0.25	-0.2465	-0.2105
-0.50	-0.4736	-0.3871
-0.75	-0.7261	-0.5674
-0.90	-0.8875	-0.6595
-0.95	-0.9460	-0.6909
-0.99	-0.9887	-0.7126
-0.9999	-0.9999	-0.7180

Table B-9

Target Correlations and Realized Sample Correlations for N = 1000;
Loguniform Distribution vs. Uniform Distribution

Target Rank Correlation	Sample Rank Correlation	Sample Pearson Correlation
0.9999	0.9999	0.9210
0.99	0.9890	0.9110
0.95	0.9454	0.8728
0.90	0.8955	0.8251
0.75	0.7371	0.6860
0.50	0.4817	0.4422
0.25	0.2119	0.2327
0.00	-0.0002	-0.0123
-0.25	-0.2443	-0.2407
-0.50	-0.4904	-0.4733
-0.75	-0.7252	-0.6823
-0.90	-0.8918	-0.8253
-0.95	-0.9463	-0.8731
-0.99	-0.9893	-0.9119
-0.9999	-0.9999	-0.9210

Table 10

Target Correlations and Realized Sample Correlations for N = 1000;
Uniform Distribution vs. Uniform Distribution

Target Rank Correlation	Sample Rank Correlation	Sample Pearson Correlation
0.9999	0.9999	0.9999
0.99	0.9884	0.9884
0.95	0.9458	0.9457
0.90	0.8965	0.8965
0.75	0.7205	0.7205
0.50	0.4769	0.4769
0.25	0.2305	0.2304
0.00	0.0016	0.0015
-0.25	-0.2272	-0.2272
-0.50	-0.4885	-0.4885
-0.75	-0.7371	-0.7371
-0.90	-0.8940	-0.8940
-0.95	-0.9461	-0.9461
-0.99	-0.9886	-0.9886
-0.9999	-0.9999	-0.9999

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