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TITLE HYDRO-THERMO-MECHANICAL RESPONSE OF A FRACTURED ROCK BLOCK

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Hydro-Thermo-Mechanical Response of a Fractured Rock Block

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ABSTRACT

Hydro-thermo-mechanical effects in fractured rocks are important in many engineering applications and geophysical processes. Modeling these effects is made difficult by the fact that the governing equations are nonlinear and coupled, and the problems to be solved are three dimensional. In this paper we describe a numerical code developed for this purpose. The code is finite element based to allow for complicated geometries, and the time differencing is implicit, allowing for large time steps. The use of state-of-the-art equation solvers has resulted in a practical code. The code is capable of fully three dimensional simulations, however, in this paper we consider only the case of two dimensional heat and mass flow coupled to one dimensional deformation. Partial verification of the code is obtained by comparison with published semianalytical results. Several examples are presented to demonstrate the effects of matrix expansion, due to pore pressure and heating, on fracture opening due to fluid injection.

INTRODUCTION

Coupled hydro-thermal-mechanical processes in fractured rock masses are important in a variety of engineering applications such as geothermal energy production, underground waste storage, and enhanced oil recovery. The coupled processes are also important in understanding many fundamental geophysical phenomena such as the transport of magma through the crust, the development of hydrothermal systems, and the influence of crustal processes on ground water hydrology.

Due to the importance of the coupled processes, considerable effort has been devoted to solving the mathematical equations representing these processes. The analytical work in this area has been of necessity, limited to the study of one or two of the hydraulic, thermal, and deformational processes at a time (e. g. , Rice and Cleary, 1976; Palciauskas and Domenico, 1989; Perkins and Kern, 1961; and Elsworth, 1989). This is due to the fact that the equations involved are coupled and nonlinear. Solutions have also been obtained using semianalytical techniques, where simplifying assumptions and similarity arguments are used to reduce the set of coupled, nonlinear, multidimensional partial differential equations to a more manageable ordinary differential equation (e. g. , Wijesinghe, 1986). These studies are extremely valuable in providing an insight into the physics of the coupled phenomena and in setting the stage for more involved numerical, computer based, solution techniques.

Over the past few decades several numerical codes have been developed to model various aspects of the coupled hydro-thermo-mechanical processes. Use has been made of different forms and combinations of finite difference (e. g. , Hopkirk and Rybach, 1989; and Hart and St. John, 1986), finite element (e. g. , Noorishad et al. , 1982; and Ohnishi et al. , 1985) and discrete element (e. g. , Cundall, 1985) techniques to discretize the spatial domain of the problem for computational purposes. In most of these approaches, the time dependence of the equations is solved explicitly, typically using successive substitution to solve at least some of the several equations at a given time and then marching explicitly forward in time. This

procedure has the advantage of simplicity and of not having to deal with a large number of nonlinear, simultaneous equations, but suffers from having to use small time steps to get stable solutions, making large simulations, over thousands of years, impractical.

In order to avoid these difficulties, work is under way to develop a robust and flexible numerical code at Los Alamos National Laboratory. A fully implicit, coupled formulation is used to allow for large time steps. This approach requires the solution of nonlinear, coupled algebraic equations at each time step. The Newton-Raphson technique is used to handle nonlinearities. The resulting matrix equations are solved using an ortho-minimization technique accelerated with incomplete factorization (Zyvoloski, 1936). The code is finite element based in order to allow for complicated geometries, however, the technique is modified to facilitate unwinding. In this paper we describe the mathematical model used to represent the coupled processes in a fractured geological medium. A description of the code and the numerical techniques is also given. The code is then applied to the problem of nonisothermal fluid injection into a pre-existing fracture including the poroelastic and thermal effects within the surrounding matrix.

MATHEMATICAL MODEL

Fluid flow is modeled using the mass balance within a compressible medium combined with the Darcy law for flux (Bear, 1972). The permeability and porosity of the matrix is allowed to vary as a known function of the local fluid pressure. The permeability of the fractures, on the other hand, is expressed as a power law function of the fracture aperture (Tsang and Witherspoon, 1981), calculated from the displacement equations. Conservation of mass of the pore fluid is expressed by

$$\frac{\partial A_m}{\partial t} + \nabla \cdot f_m + q_m = 0,$$

where the mass of fluid per unit volume is

$$A_m = \phi \rho$$

and the mass flux is

$$f_m = \rho V$$

where ϕ is the porosity of the matrix (void volume/total volume), V is the fluid velocity, ρ is the fluid density, and q_m is the mass source. This equation is augmented with Darcy's law for single phase flow through a porous medium

$$V = - \frac{k}{\mu} \nabla P$$

where k is the permeability, μ is the fluid viscosity, and P is the fluid pressure.

The energy balance performed for the rock-fluid mixture, assuming that the rock and fluid are in equilibrium with each other at every point, is given by

$$\frac{\partial A_e}{\partial t} + \nabla \cdot f_e + q_e = 0$$

where the energy per unit volume is

$$A_e = (1 - \phi)\rho_r U_r + \phi \rho U$$

and the energy flux vector is

$$f_e = \rho h V - K \nabla T$$

Here U_r is the specific internal energy of the rock, U is the specific internal energy of the fluid, h is the specific enthalpy of the fluid, T is the temperature, q_e is the energy source, and K is the bulk thermal conductivity.

Solid displacements are calculated using static stress balance with Biot's poroelastic equations for small displacements (Biot, 1941). Linear thermoelasticity (Timoshenko and Goodier, 1951) is used to include thermal expansion/contraction effects. The static form of the force balance inside the porous solid leads to

$$\frac{\partial \sigma_{ij}}{\partial x_j} + b_i = 0$$

where b_i is the body force, i, j , and k are indices in cyclic order for a Cartesian coordinate system, and σ_{ij} are the components of the stress tensor in the porous solid. For points on the fracture face, an additional external force is present due to the fluid pressure within the fracture. Using the theory of linear poroelasticity for small strains, these are related to the strain tensor ϵ_{ij} by

$$\epsilon_{ii} = \frac{\sigma_{ii}}{E} - \frac{\nu}{E}(\sigma_{jj} + \sigma_{kk}) - \frac{\Delta P}{3H} + \alpha \Delta T$$

for the diagonal components, where no sum is implied over repeated indices, and the offdiagonal components are given by

$$\epsilon_{ij} = \frac{2(1+\nu)}{E} \sigma_{ij} ,$$

where α is the coefficient of thermal expansion, E is Young's modulus, ν is Poisson's ratio and H is Biot's physical constant.

Hydro-thermal coupling occurs through the pressure and temperature dependence of fluid density, viscosity, and enthalpy and through the convection terms in the energy equation. The mass and energy equations are also affected by pressure and temperature through the dependence of rock porosity and permeability on these quantities as well as on rock displacements. Hydro-mechanical coupling occurs through the strong dependence of fracture porosity and permeability on displacements and through the inclusion of pore pressure terms in Biot's equations. Thermo-mechanical coupling is included through the temperature terms in the thermoelasticity equation and through the dependence of porosity and permeability on displacements in the convection terms in the energy equation.

NUMERICAL PROCEDURES

The space derivatives in the governing equations are discretized using the finite element method with the Galerkin formulation. At present only linear elements are used in the coupled code. The primary dependent variables interpolated on each element are the fluid pressure P , fluid temperature T , and the three solid displacements U , V , and W in the x, y , and z directions, respectively. The stress equations are formulated with displacements as unknowns. Although the code is finite element based, all the variables and material properties are defined at the nodes. The nonlinear material properties are factored out of the equation coefficients and interpolated between the nodes, evaluated at each iteration. This results in a considerable computational saving as the shape function integrals need to be evaluated only once at the start of the program. This approximation is consistent for small strains. Permeabilities in the flux terms are harmonically averaged. The shape functions are evaluated with the use of either Lobatto or Gauss quadrature. Lobatto quadrature has the same formal accuracy as the Gauss quadrature while yielding considerably fewer nonzero matrix elements. It has been found to work well in fluid flow and heat problems but leads to inaccurate results for the displacement equations.

The nodes that form the fracture are treated differently than those in the rock matrix. Full upwinding is used in the fluid flow and energy transport equations for the fracture nodes. This is facilitated by using a linear combination of the permeability along the fracture instead of harmonic weighting. The

permeability perpendicular to the fracture is harmonically averaged.

Time discretization is accomplished by using the first order, backward Euler method. This is a fully implicit scheme where all functional dependencies are evaluated at the new time step. This leads to a set of nonlinear, coupled algebraic equations that have to be solved at each time step. The Newton-Raphson technique is used to solve these nonlinear equations. The full Jacobian is formed at each iteration. The use of nonlinear polynomial fits to thermodynamic functions has resulted in considerable computational savings. The resultant linear matrix equations are solved by factoring the coefficient matrix. The matrix solution is performed using an iterative ortho-minimization technique accelerated with incomplete factorization. The Newton-Raphson scheme is iterated until the norm of the residuals of the balance equations decreases by a prescribed factor.

RESULTS

The three dimensional, multiphase, hydrology-thermal parts of the code and the two dimensional uncoupled and one way coupled stress code have been verified previously (Zyvoloski and Kelkar, 1987). In this paper we present results for the case of two dimensional fluid and heat flow, fully coupled with the one dimensional deformation of the rock matrix. Although the numerical code being developed is capable of fully three dimensional simulations, it is desirable to start with simpler problems. Comparison is also made with published semianalytical results as a partial verification of the code.

Description of the example problem: A rectangular block of porous, permeable rock with a fracture running along the lower edge is considered, as shown schematically in Figure 1. The lower left hand node is the injection node. Fluid flow and heat transfer are also allowed into the matrix. Matrix deformation changes the fracture aperture. The physical dimensions of the block are 100 meters along the x-axis with 101 nodes and 10 meters along the y-axis with 11 nodes, resulting in a total of 1111 nodes and 1000 elements in the problem. The values of various parameters for the base case are shown in Table I. These values were selected for the purpose of testing only and are not meant to represent any particular field example. A fixed time step of 10 seconds was used in most examples, although other runs were made to check the effect of time step on the solution.

Partial Verification: Figure 2 shows the comparison of results with published (Wijesinghe, 1986) semianalytical calculations. These semianalytical results were obtained for the case of an impermeable rock matrix and assuming that the fracture aperture varies linearly with the local fluid pressure. This is an isothermal example. The smooth curve is the numerical computation and crosses represent published data. A good agreement is noted.

Effects of time step size: For explicit time stepping schemes, the time step size is limited by the speed of propagation if wave equations are being considered, and by the diffusivity of the processes being investigated. In some approaches, the stress equations are solved including their time dependence. This approach is suitable for dynamic problems, but leads to a time step restricted by the speed of sound, usually on the order of microseconds. Even the use of improvement schemes leave the time step limited to a small fraction of a second. The second limitation on the time step results from the hydraulic diffusivity. For the case of a fracture being opened, using the analysis of Wijesinghe (1986), the time step, for the examples being considered here, is limited to a small fraction of a second. However, the use of an implicit scheme alleviates these restrictions. Figures 3 and 4 show the computed fracture aperture as a function of time for step sizes of 0.1, 1.0 and 10. seconds. Note that the scale of Figure 4 is 10 times that of Figure 3. Good stability is noted, clearly demonstrating the desirability of using implicit computations for the problems being considered.

Results of the base case run: In this example, fluid is injected at a constant rate into the fracture and allowed to flow along the fracture, but not into the matrix perpendicular to the fracture. Fluid injection temperature is the same as the initial uniform rock temperature. The dependence of the fracture permeability on the aperture is modeled with the cubic law. The aperture at the injection point opens rapidly, its growth slowing down in time as the fluid flows forward and the fracture propagates. The aperture as a function of distance from the injection point at various times is shown in Figure 5.

Effects of matrix flow: The next case shown is similar to the previous one except that fluid flow is allowed into the matrix as well, although the poroelastic expansion of the matrix is suppressed. The results for three different values of rock permeability, 10^{-24} m², 10^{-14} m², and 10^{-12} m² are shown in Figure 6. As expected, the pressure buildup and hence the aperture of the fracture decreases with increasing matrix permeability. The case of 10^{-12} m² is interesting, where the initial fracture permeability equals the matrix permeability. The fluid loss is so high that the fracture does not open much. Figure 7 shows the effect of increasing the injection rate, a larger fracture aperture results from the higher injection rate.

Effects of matrix expansion: Figures 8 and 9 show the fracture aperture for different values of Biot's physical constant and matrix permeability. The expansion of the rock matrix due to the pore pressure buildup caused by fluid diffusion leads to reduced fracture aperture. As seen in Figures 8 and 9, this effect is more pronounced for higher values of the permeability and of the expansion coefficient.

Effects of thermal expansion: Figure 10 shows the fracture profile at various times for non isothermal fluid injection allowing for thermal expansion of the rock. The coefficient of thermal expansion used here is two orders of magnitude higher than that expected for real rocks so as to amplify the effect during the small simulation time (0.231 hours) that is considered. Similar effects might be expected in real rocks over larger time spans. An interesting fracture profile results. The aperture increases in the beginning as expected, but as the rock temperature near the injection point builds up, its aperture becomes less than that further down the fracture due to thermal expansion, since the thermal front has not had the time to reach further.

Combined hydro-thermal-mechanical problem: Figure 11 shows the fracture aperture profile that resulted from the combined effects of fracture opening, matrix permeation, matrix expansion due to pore pressure, heat convection, thermal diffusion, and thermal expansion of the rock. For comparison, the profile without matrix diffusion and thermal expansion is also shown.

CONCLUSIONS

A computer code has been developed for solving coupled, and nonlinear, fluid flow - heat transfer - solid deformation problems in porous, fractured, geological media. The code is finite element based with modifications to allow for upwinding the flow and transport equations. Time differencing is done implicitly to allow for large time steps in nonlinear problems. The scheme is shown to be stable for time steps several orders of magnitude larger than those expected for explicit schemes. The code has been partially verified by comparison with published semianalytical results. Example problems show that, for the case of two dimensional fluid flow and heat transfer coupled with one dimensional solid deformation, the code provides reasonable results. Effects of fracture opening, matrix permeability, injection rate, and matrix expansion due to pore pressure and thermal expansion are efficiently handled by the code.

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Table I. Parameter Values for the Base Case Example

Length x	100 m
Length y	10 m
Initial Pressure	20 MPa
Initial Temperature	25 °C
Injection Rate	0.1 kg/s
Injection Temperature	25 °C
Matrix Porosity	0.10
Matrix Permeability x	10-12 m ²
Matrix Permeability y	10-24 m ²
Initial Fracture Permeability	10-12 m ²
Thermal Conductivity	2.7 J/m/K
Young's Modulus	10 ³ MPa
Poisson's Ratio	0.0
Thermal Expansion Coefficient	5*10 ⁻⁵ /°C
Pore Pressure Coefficient	10-10 /MPa

Figure 3. Effect of Time Step Size on Computed Fracture Opening, 0.1 and 1.0 second.

Figure 1. A Schematic of the Coupled Fracture Flow Model.

Figure 4. Effect of Time Step Size on Computed Fracture Opening, 1.0 and 10.0 Seconds.

Figure 2. Comparison of Numerical Computations with Semianalytical Results (Wijesinghe 1986).

Figure 5. Fracture Opening Profile for the Base Case Run.

Figure 6. Fracture Opening Profile for Various Values of Matrix Permeability.

Figure 9. Effect of Biot's Pore Pressure Coefficient, High Matrix Permeability Case.

Figure 7. Effect of Injection Rate on Fracture Opening.

Figure 10. Fracture Opening Profile for Hot Fluid Injection Including Thermal Expansion.

Figure 8. Effect of Biot's Pore Pressure Coefficient, Low Matrix Permeability Case.

Figure 11. Comparison of Fracture Profile for the Fully Coupled Case with the Base Case.