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**SIMILARITY CONSIDERATIONS
IN ONE-COMPONENT
TWO-PHASE FLOW**

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1. INTRODUCTION

There are numerous cases where two-phase flow of water substance is encountered in engineering practice. Since a complete theoretical understanding of such phenomena is not yet at hand, recourse is usually made to some sort of experimentation during the design process. Conducting two-phase flow measurements with water for cases of geothermal wells, for instance, can be technically difficult, risky and expensive. The main problems to be faced in such experiments include high temperatures, high pressures, high heat fluxes due to the high latent heat of evaporation of water, and cumbersome geometric dimensions of the experimental facility.

It is, therefore, desirable to investigate whether it is possible to simulate actual field conditions with a scaled laboratory setup. It is hoped that a more convenient working fluid can be found and that the size of the experimental facility will be more manageable.

The idea of similarity, or scaling, in single-phase flow is usually achieved by using appropriate dimensionless numbers that stem from an understanding of the physical phenomena as well as their mathematical formulation. Scaling laws in two-phase flow are more complex. A great deal of attention is required in the choice and definition of the dimensionless numbers necessary for scaling of two-phase flow processes because of:

- i) The diversity of physical phenomena involved;
- ii) An incomplete understanding of the phenomena;
- iii) The lack of a general mathematical formulation of the two-phase flow problem.

In addition to the above, and independent of the physical process itself, the scaling of the thermodynamic variables of the modelling substance plays a very important role [1].

Geometric similarity by itself, apart from thermodynamic similarity, is not sufficient for complete similitude since the characteristic length scales in two-phase flow do not follow the geometric scaling law. The most obvious example of this relates to the lack of proportionality between bubble diameter and well diameter when the latter is altered in accordance with the geometric scaling law: The bubbles retain their size in each case. Furthermore, even if one were able to achieve geometric scaling of a geothermal well, the problem of high pressure and temperature in the fluid would remain. Thus, we realize, along with [1], that fluid scaling is of primary importance in two-phase flow.

In the present study, we will concentrate on the issue of thermodynamic scaling or similarity. By using a simple two-phase fluid model, we will determine dimensionless variables and parameters that are significant for the thermodynamic scaling of a substance. Although the analysis itself is valid to the extent that the real fluids tested for similarity satisfy our simplified model fluid assumptions, we will concentrate on the possibility of scaling water with dichlorotetrafluorethane (R114). Fluorinated hydrocarbons have been found appropriate (using various criteria) for the simulation of water-steam flows [1,2], and a number of two-phase flow experiments have been conducted using refrigerants of this kind [3-5]. It should be pointed out at this point that air-water two-phase flow cannot adequately model single-substance two-phase flow at high mass flow rates (leading to choking) because of the absence of the compressibility effect caused by phase change.

Thus our objective here is to explore the technical feasibility of simulating two-phase geothermal well flow by using R114 in a laboratory facility.

2. MODEL TWO-PHASE FLUID

Our quest for similarity laws to describe the behavior of two-phase liquid-vapor flows, either between substances or within a given flow regime of a single substance, begins with the establishment of a hypothetical, simplified fluid model.

The defining assumptions listed below were arrived at by a study of the properties of actual two-phase fluids and are reasonably good approximations:

- i) Latent heat of evaporation is constant;
- ii) Specific volume of the saturated liquid phase is constant;
- iii) Enthalpy of the saturated liquid phase is a linear function of temperature;
- iv) Saturated vapor phase obeys the perfect gas equation of state.

In mathematical form, these assumptions may be written as follows:

$$h_{fg} \equiv h_g - h_f = \text{constant.} \quad (1)$$

$$v_f = \text{constant.} \quad (2)$$

$$h_f = h_f(T) \quad \text{or} \quad (3a)$$

$$(\partial h_f / \partial T)_{\text{sat}} \equiv c_f = \text{constant.} \quad (3b)$$

$$P v_g = R_a T. \quad (4)$$

It is generally true that $v_g \gg v_f$, leading to $v_g \approx v_{fg}$ since $v_{fg} = v_g - v_f$. Thus eq. (4) may be written approximately as

$$P v_{fg} = R_a T. \quad (5)$$

It can be demonstrated that these assumptions are reasonably valid for water

substance over the range of geothermal applications, from 100 to 300°C; for R114 they apply in the range from -20 to +70°C.

3. THERMODYNAMIC RELATIONS

To retain thermodynamic consistency, one must satisfy the Clausius-Clapeyron relation

$$(dP/dT)_{\text{sat}} = s_{fg}/v_{fg}, \quad (6)$$

as well as the Gibbs equation

$$Tds = dh - v dP. \quad (7)$$

Applying eq. (7) to a system undergoing a phase change, i.e., one at constant pressure, yields

$$s_{fg} = h_{fg}/T. \quad (8)$$

By substitution of eqs. (5) and (8) into eq. (6) one obtains

$$\frac{dP}{P} = \frac{h_{fg}}{R_a T} \frac{dT}{T}, \quad (9)$$

which exhibits, in differential form, the relationship between pressure and temperature along the saturation line for our model fluid.

4. BASIC EQUATIONS IN DIMENSIONLESS FORM

Any extensive (or specific) thermodynamic property of a two-phase mixture may be expressed as a function of temperature, T (which determines the properties of the individual phases), and the dryness fraction, x . Furthermore, each variable may be made dimensionless with respect to a reference

value of the variable (which we will denote by means of a subscript 0). The dimensionless variable will be denoted by a tilde, i.e., superscript ~.

Thus, the Clausius-Clapeyron equation may be transformed in this way to

$$\frac{d\tilde{P}}{\tilde{P}} = \frac{h_{fg}}{R_a T_0} \frac{d\tilde{T}}{\tilde{T}^2}, \quad (10)$$

where

$$\tilde{P} = P/P_0, \quad (11)$$

and

$$\tilde{T} = T/T_0. \quad (12)$$

The specific volume, v , may be expressed via the lever rule as follows:

$$v(x,T) = v_f(T) + x v_{fg}(T). \quad (13)$$

If we combine this with eq. (5), we obtain

$$v(x,T) = v_f(T) + x \frac{R_a T}{P}. \quad (14)$$

Two additional reference values are needed, v_0 and x_0 , and the corresponding new variables are:

$$\tilde{v} = v/v_0, \quad (15)$$

and

$$\tilde{x} = x/x_0. \quad (16)$$

By a straightforward transformation, eq. (14) becomes

$$v_0 \tilde{v} = v_f + \frac{x_0 \tilde{x} R_a T_0 \tilde{T}}{p_0 \tilde{p}}, \quad (17)$$

or

$$\frac{v_0}{v_f} \tilde{v} = 1 + \left(\frac{R_a}{c_f} x_0 \right) \left(\frac{R_a T_0}{h_{fg}} \right) \left(\frac{h_{fg} c_f}{p_0 v_f R_a} \right) \frac{\tilde{x} \tilde{T}}{\tilde{p}}. \quad (18)$$

A similar approach may be taken for the specific enthalpy, h . Thus,

$$h(x, T) = h_f(T) + x h_{fg}(T) \quad (19)$$

$$dh = dh_f + h_{fg} dx \quad (20)$$

$$dh = c_f dT + h_{fg} dx \quad (21)$$

$$\tilde{h} = h/h_0 \quad (22)$$

$$h_0 d\tilde{h} = c_f T_0 d\tilde{T} + h_{fg} x_0 d\tilde{x} \quad (23)$$

$$\frac{R_a h_0}{c_f h_{fg}} d\tilde{h} = \frac{R_a T_0}{h_{fg}} d\tilde{T} + \frac{R_a}{c_f} x_0 d\tilde{x}. \quad (24)$$

We begin from eq. (7) for the specific entropy. By combining it with eqs. (13) and (21), we obtain

$$Tds = c_f dT + h_{fg} dx - x v_{fg} dP - v_f dP \quad (25)$$

$$T_0 s_0 \tilde{T} d\tilde{s} = c_f T_0 d\tilde{T} + x_0 h_{fg} d\tilde{x} - x_0 \tilde{x} h_{fg} d\tilde{T}/\tilde{T} - v_f P_0 d\tilde{P} \quad (26)$$

$$\left(\frac{R_a T_0}{h_{fg}} \right) \left(\frac{s_0}{c_f} \right) \tilde{T} d\tilde{s} = \frac{R_a T_0}{h_{fg}} d\tilde{T} + \frac{R_a}{c_f} x_0 \left(d\tilde{x} - \tilde{x} \frac{d\tilde{T}}{\tilde{T}} \right) - \frac{v_f P_0}{h_{fg}} \frac{R_a}{c_f} d\tilde{P}, \quad (27)$$

where we have introduced

$$\tilde{s} = s/s_0. \quad (28)$$

Up to this point, the reference values (P_0, T_0, v_0, x_0, h_0 , and s_0) have yet to be specified. It is clear, however, that proper choice of these quantities can lead to drastic simplifications of the basic equations written in terms of the new, dimensionless variables. For example, by inspection of eq. (10), it may be seen that by choosing

$$T_0 \equiv \frac{h_{fg}}{R_a} \quad (29)$$

we obtain

$$\frac{d\tilde{P}}{\tilde{P}} = \frac{d\tilde{T}}{\tilde{T}^2} \quad (30)$$

$$= -d(1/\tilde{T}). \quad (31)$$

In like manner, we choose the other reference quantities as follows:

$$P_0 \equiv \frac{h_{fg} c_f}{v_f R_a} \quad (32)$$

$$v_0 = v_f \quad (33)$$

$$x_0 \equiv \frac{c_f}{R_a} \quad (34)$$

$$h_0 \equiv \frac{h_{fg} c_f}{R_a} \quad (35)$$

$$s_0 \equiv c_f. \quad (36)$$

It may be convenient to view these relationships in slightly different, but equivalent form:

$$p_0 = \frac{T_0 c_f}{v_f} = \frac{h_0}{v_f} \quad (37)$$

$$x_0 = \frac{T_0 c_f}{h_{fg}} = \frac{h_0}{h_{fg}} \quad (38)$$

$$h_0 = T_0 c_f. \quad (39)$$

Another important thermodynamic property is the two-phase flow choking velocity, a_c , or the choking flow density, ψ_c . This topic was treated by our group in an earlier report [6]. The results will be reworked here to draw out the dimensionless equations and scale factors for these quantities.

By definition,

$$\psi_c^2 = - \left(\frac{\partial P}{\partial v} \right)_s, \quad (40)$$

or

$$1/\psi_c^2 = - \left(\frac{\partial v}{\partial T} \right)_s \left(\frac{dT}{dP} \right)_{sat} \quad (40a)$$

$$= - \left(\frac{\partial v}{\partial T} \right)_s (P')^{-1}. \quad (40b)$$

The partial derivative may be expressed as

$$\left(\frac{\partial v}{\partial T}\right)_s = \frac{dv_f}{dT} + \left(\frac{\partial x}{\partial T}\right)_s v_{fg} + x \frac{dv_{fg}}{dT} \quad (41)$$

$$= \left(\frac{\partial x}{\partial T}\right)_s v_{fg} + x \frac{dv_{fg}}{dT}, \quad (41a)$$

where

$$\left(\frac{\partial x}{\partial T}\right)_s = -\frac{1}{s_{fg}} \frac{ds_f}{dT} + x s_{fg} \frac{d}{dT} \left(\frac{1}{s_{fg}} \right), \quad (41b)$$

and

$$\frac{ds_f}{dT} = \frac{c_f}{T} - v_f \frac{p'}{T}, \quad (41c)$$

since

$$dh_f = c_f dT. \quad (41d)$$

These equations may be combined to yield

$$\left(\frac{\partial x}{\partial T}\right)_s = -\frac{1}{T s_{fg}} (c_f - v_f p') + x s_{fg} \frac{d}{dT} \left(\frac{1}{s_{fg}} \right) \quad (42)$$

and

$$\left(\frac{\partial v}{\partial T}\right)_s = -\frac{1}{T P'} (c_f - v_f P' + x h_{fg} P''/P'). \quad (42a)$$

This last equation may now be used to express the choking flow density by means

of eq. (40b):

$$\frac{1}{\psi_c^2} = \frac{1}{T(P')^2} \left[c_f - v_f P' + x h_{fg} \frac{d}{dT} (\ln P') \right] \quad (43)$$

$$= \frac{R_a^2 T^3}{h_{fg}^2 P^2} \left[c_f - v_f \frac{h_{fg} P}{R_a T^2} + x h_{fg} \left(\frac{h_{fg}}{R_a T^2} - \frac{2}{T} \right) \right] \quad (43a)$$

$$= \frac{R_a v_f^2}{c_f h_{fg}} \frac{\tilde{T}^{-3}}{\tilde{P}^2} [1 - \tilde{P} \tilde{T}^2 + \tilde{x} \tilde{T}(\tilde{T}-2)]. \quad (43b)$$

We have used the definitions of the previously defined dimensionless variables to obtain the last expression.

By inspection of eq. (43b) we can see that the appropriate scale factor for ψ_c is

$$\psi_{c0} \equiv \frac{1}{v_f} \left(\frac{c_f h_{fg}}{R_a} \right)^{1/2}, \quad (44)$$

and that the dimensionless choking flow density is

$$\tilde{\psi}_c = \psi_c v_f \left(\frac{R_a}{c_f h_{fg}} \right)^{1/2}. \quad (44a)$$

Therefore,

$$\frac{1}{\tilde{\psi}_c^2} = \frac{\tilde{T}^{-3}}{\tilde{P}^2} [1 - \tilde{P} \tilde{T}^2 + \tilde{x} \tilde{T}(\tilde{T}-2)]. \quad (44b)$$

The velocity scale can be obtained easily from the relation

$$\psi_c = \rho a_c = a_c/v. \quad (45)$$

Thus,

$$a_c = \left(\frac{c_f h_{fg}}{R_a} \right)^{1/2} \tilde{v} \tilde{\psi}_c, \quad (46)$$

and the scale factor for velocity is seen to be

$$a_{c0} = \left(\frac{c_f h_{fg}}{R_a} \right)^{1/2}, \quad (46a)$$

the dimensionless velocity being

$$\tilde{a}_c = a_c/a_{c0} \quad (46b)$$

$$= a_c \left(\frac{R_a}{c_f h_{fg}} \right)^{1/2}. \quad (46c)$$

Although this derivation was done for the choking velocity, the scale factor is generally applicable for any velocity:

$$\tilde{w} = w/w_0 \quad (47)$$

$$= w \left(\frac{R_a}{c_f h_{fg}} \right)^{1/2}. \quad (47a)$$

The square of the scale factor for velocity is obviously a scale factor for kinetic energy. Furthermore, it may be convenient to view the velocity scale

factor in an alternate way as

$$w_0 = h_0^{1/2}. \quad (48)$$

With these definitions, eq. (18) becomes simply

$$\tilde{v} = 1 + \tilde{x} \frac{\tilde{T}}{\tilde{p}}; \quad (49)$$

eq. (24) becomes

$$d\tilde{h} = d\tilde{T} + d\tilde{x}; \quad (50)$$

and eq. (27) becomes

$$d\tilde{s} = d(\ln \tilde{T}) + d\left(\frac{\tilde{x}}{\tilde{T}}\right) - \frac{d\tilde{p}}{\tilde{T}}. \quad (51)$$

The integration of eqs. (31) and (50) is simple:

$$\tilde{p} = C e^{-1/\tilde{T}}, \quad (52)$$

where

$$C = \tilde{p}_r e^{-1/\tilde{T}_r} \equiv N_D. \quad (53)$$

The reference terms $\tilde{p}_r$ and $\tilde{T}_r$ must be evaluated for chosen physical conditions at which the fluid is expected to operate, and N_D is a similarity parameter.

Thus,

$$\tilde{p} = N_D e^{-1/\tilde{T}}. \quad (54)$$

For enthalpy, we obtain

$$\tilde{h} = \tilde{T} + \tilde{x}. \quad (55)$$

The specific volume may now be given in terms of N_D as follows:

$$\tilde{v} = 1 + \frac{\tilde{x}\tilde{T}}{N_D} e^{1/\tilde{T}}. \quad (56)$$

The entropy equation, eq. (51), may likewise be integrated to yield

$$\tilde{s} = \ln \tilde{T} + \tilde{x}\tilde{T} - N_D \left(1 + \frac{1}{\tilde{T}} \right) e^{-1/\tilde{T}}. \quad (57)$$

The choking flow density is obtained from eq. (44b):

$$\tilde{\psi}_c = N_D e^{-1/\tilde{T}} \tilde{T}^{3/2} / (1 - N_D e^{-1/\tilde{T}} \tilde{T}^2 + \tilde{x}\tilde{T}(\tilde{T}-2))^{1/2}. \quad (58)$$

Thus, it is clear that all dimensionless properties of the model fluid, as given by eqs. (54-58), are functions of two dimensionless thermodynamic variables:

$$\tilde{x} = (R_a/c_f)_r x, \quad (59)$$

and

$$\tilde{T} = (R_a/h_{fg})_r T, \quad (60)$$

as well as one similarity parameter.

$$N_D = (\tilde{P}e^{1/\tilde{T}})_r. \quad (61)$$

The important implication of this derivation is that any dimensionless property $\tilde{\Pi}$ can be expressed as follows:

$$\tilde{\Pi} = \tilde{\Pi}(\tilde{x}, \tilde{T}; N_D). \quad (62)$$

Considering two different substances, A and B, if it is true that

$$\tilde{x}(A) = \tilde{x}(B), \quad (63)$$

$$\tilde{T}(A) = \tilde{T}(B), \quad (64)$$

and

$$N_D(A) = N_D(B), \quad (65)$$

then it must be true that

$$\tilde{\Pi}(A) = \tilde{\Pi}(B), \quad (66)$$

and that complete similarity exists between the two substances. In this case, the common value of N_D defines the reference point in the neighborhood of which the actual substances are completely similar. All scales for the fluid properties are evaluated at this reference point.

5. SIMILARITY BETWEEN WATER AND R114

Since the question facing us deals with the behavior of fluids that have undergone a flashing process into the two-phase liquid-vapor region, it is reasonable to choose the reference state as a saturated liquid, i.e., $x = 0$ or $\tilde{x} = 0$. We impose this condition on both substances under consideration, namely water and R114:

$$\tilde{x}(\text{H}_2\text{O}) = \tilde{x}(\text{R114}) = 0. \quad (67)$$

From eq. (61) it can be seen that N_D is a function of $\tilde{T}$ alone since $\tilde{P}$ is a unique function of $\tilde{T}$ for a two-phase fluid. Thus, we may calculate $N_D(\tilde{T})$ for both substances and seek that value of $\tilde{T}$ for which eq. (65) is satisfied. There is no guarantee that such a point exists for any given combination of substances but if it does exist the two substances would be completely similar at that point and would be reasonably similar in some neighborhood of that point.

Figure 1 shows N_D for water and R114 versus T and $\tilde{T}$ for $\tilde{x} = 0$ over a range of $\tilde{T}$ in the region of geothermal interest for water. The curves do not intersect. Thus perfect similarity cannot be obtained using R114. However, the situation is not as bad as might first appear. The importance of the similarity parameter N_D turns on the relative size of the last term in the entropy equation, eq. (57). If this term is small, then the importance of N_D as a necessary similarity parameter will also be diminished. Then similarity could be inferred, approximately, as long as $\tilde{x}$ and $\tilde{T}$ were made equal for the two substances. That is, only eqs. (63) and (64) would be necessary to infer eq. (66).

This possibility can be tested. Once a suitable set of reference points has been selected, temperature correspondence can be calculated using eqs. (60) and (64):

$$T_{R114} = T_{H_2O} \times \left[\frac{h_{fg,R114}}{h_{fg,H_2O}} \right]_r \times \left[\frac{R_{a,H_2O}}{R_{a,R114}} \right]_r. \quad (68)$$

Figure 2 shows the solution to this equation. Equations (59) and (63) give the dryness fraction correspondence

$$x_{R114} = x_{H_2O} \times \left[\frac{R_{a,H_2O}}{R_{a,R114}} \right]_r \times \left[\frac{c_{f,R114}}{c_{f,H_2O}} \right]_r. \quad (68a)$$

According to eq. (68), water at 200°C, for example, matches up with R114 at 18.3°C. The relation between the dryness fractions of water and R114 for this case is:

$$x_{R114} = 2.03x_{H_2O}, \quad (68b)$$

according to eq. (68a).

It is obvious that the analysis presented here is valid for

$$0 < x_{R114} < 1$$

and

$$0 < x_{H_2O} < 1$$

so that one should ignore the cases where the above limits are violated. It should also be pointed out that this analysis is valid for small x and breaks down in the vicinity of $x = 1$. In order to cover the vicinity of $x = 1$, a similar analysis has to be carried out. In the cases of practical

application that we are interested in studying, the dryness fraction is usually small, so we will not carry out such an analysis at this time.

A true test of the substance similarity concept can be drawn up by solving a two-phase flow problem using the two substances and comparing the results in appropriate dimensionless form. The problem involves the flow of the substance up a vertical pipe from the flashing point onward. The analytical solution is obtained from the following set of governing equations that are based on a simplified two-fluid model [7,8].

Conservation of mass

$$\psi x_f = \rho_f \alpha_f w_f \quad (69)$$

$$\psi x_g = \rho_g \alpha_g w_g \quad (69a)$$

or

$$\frac{1-x}{x} = \frac{\gamma}{\sigma} \cdot \frac{1-\alpha}{\alpha} \quad (70)$$

Conservation of momentum

$$\frac{d}{dz} (\psi w_{cm}) = - \frac{dP}{dz} + \frac{\bar{\tau}_{gw} + \bar{\tau}_{fw}}{R_h} - \rho_m g, \quad (71)$$

where w_{cm} is the center-of-mass velocity

$$w_{cm} = \sum x_p w_p, \quad (72)$$

ρ_m is a volume-weighted mixture density

$$\rho_m = \sum \alpha_p \rho_p, \quad (73)$$

and $\bar{\tau}_{pw}$ is surface-weighted wall shear stress for each phase

$$\bar{\tau}_{pw} = \tau_{pw} \times \frac{L_p}{C}. \quad (74)$$

Conservation of Energy

$$h + \frac{1}{2} w_{RMS}^2 + gz = h_0, \quad (75)$$

where w_{RMS} is the root-mean-square velocity

$$w_{RMS} = (\sum \alpha_p w_p^2)^{1/2}. \quad (76)$$

Dissipation Equation

$$\psi T \frac{ds}{dz} = (\bar{\tau}_{wg} w_g + \bar{\tau}_{wf} w_f + \bar{\tau}_i (w_g - w_f)) / R_h, \quad (77)$$

where $\bar{\tau}_i$ is a surface-weighted interfacial shear stress

$$\bar{\tau}_i = \tau_i \times \frac{L_i}{C}. \quad (78)$$

In the above eqs. (71-78), C is the inner perimeter of the pipe, L_p is

that portion of C wetted by phase p , and z_i is the interface length. Since the right-hand-side of the dissipation equation is not well understood, we replace it by a dissipation factor, f , multiplied by the RMS kinetic energy as a primitive approximation. Thus,

$$T \frac{ds}{dz} = \frac{f}{D} \left(\frac{1}{2} w_{RMS}^2 \right), \quad (79)$$

where we arbitrarily adopt a value $f = 0.015$ for the sample calculations.

To simplify the calculations for the purpose of testing the similarity concept, we also assume no slip between the phases, i.e., $w_g = w_f$, or

$$\sigma = w_g/w_f = 1. \quad (80)$$

The solution is obtained by an iterative calculation procedure. The results may be expressed, for example, by giving the relationship between fluid temperature and dryness fraction as functions of the distance downstream of the flashing point.

To express the results in a form that allows a test of the similarity notion we must express the distance in proper dimensionless form. The scale factor for distance emerges from the energy equation, eq. (75). In dimensionless form, eq. (75) becomes

$$\tilde{h} + \frac{1}{2} \tilde{w}_{RMS}^2 + \tilde{z} = \tilde{h}_0, \quad (81)$$

where

$$\tilde{w}_{RMS} = w_{RMS} \left(\frac{R_a}{c_f h_{fg}} \right)^{1/2}. \quad (82)$$

in accordance with our previously-derived scale for velocity, and

$$\tilde{z} = z \left(\frac{gR_a}{c_f h_{fg}} \right), \quad (83)$$

where the new length scale, z_0 , has been defined as

$$z_0 \equiv \frac{c_f h_{fg}}{gR_a} \quad (84)$$

or

$$z_0 = h_0/g. \quad (84a)$$

The dissipation equation then has the following dimensionless form

$$\tilde{T} \frac{d\tilde{s}}{d\tilde{z}} = f \frac{1}{2} \frac{\tilde{w}_{RMS}^2}{\tilde{D}}. \quad (84b)$$

From Figs. 3-5 it can be seen that the two solution curves for R114 and water are nearly identical. The flashing temperatures were 18.3 C and 200 C for R114 and water, respectively, and the corresponding values for N_D were 0.75 and 1.6. The pipe diameters were scaled in accordance with the requirement that

$$\tilde{D}_{H_2O} = \tilde{D}_{R114} \quad (85)$$

or

$$D_{H_2O} = D_{R114} \times \left(\frac{R_{a,R114}}{R_{a,H_2O}} \right) \left(\frac{C_{f,H_2O}}{C_{f,R114}} \right) \left(\frac{h_{fg,H_2O}}{h_{fg,R114}} \right). \quad (86)$$

The mass flow rates were also scaled in accordance with the similarity requirements on $\tilde{\psi}_c$, eq. (44a). Apart from the substance similarity requirements, it is necessary to make sure that the flow similarity conditions are met. The most important dimensionless numbers associated with the flow are:

$$\text{Mach number: } Ma = \psi/\psi_c, \quad (87)$$

$$\text{Froude number: } Fr = \frac{w_{RMS}}{\sqrt{gD}}, \text{ and} \quad (88)$$

$$\text{Reynolds number: } Re = \frac{Dw_{CM}}{\nu}, \quad (89)$$

where $\nu = \sum x_p \nu_p$.

The form of the Froude number derives from its physical meaning, namely

$$\text{Froude number} = \left[\frac{\text{kinetic energy}}{\text{potential energy}} \right]^{1/2}, \quad (90)$$

and the observation that the numerator is proportional to w_{RMS}^2 , from eq. (74), and the denominator is proportional to gD .

The form of the Reynolds number derives from its physical meaning, namely

$$\text{Reynolds number} = \frac{\text{inertia force}}{\text{viscous force}}, \quad (91)$$

and is consistent with the momentum equation in which the center-of-mass velocity appears.

The parameter of primary importance for choking of the flow is the Mach number [7,8,9]. We therefore kept the same Mach number at flashing for both cases. This condition together with the length scale requirement from the substance similarity analysis will yield different Froude and Reynolds numbers for each case. The values of these parameters which went into Figs. 3-5 are as follows:

<u>Quantity</u>	<u>Water</u>	<u>R114</u>
Mass flow rate, $\dot{m}/(\text{kg/s})$	448	2.5
Pipe diameter, D/m	0.271	0.051
Flash Mach no., Ma^*	0.39	0.39
Flash Froude no., Fr^*	2.51	1.17
Flash Reynolds no., Re^*	11.4×10^6	16.8×10^4

Using eq. (58), the definition of the Froude number at flashing, $Fr^* = \frac{\psi v_f^*}{\sqrt{gd}}$,

and the fact that the Mach number at flashing is the same for both cases, it can be shown that the relation between the flash Froude number for water flow and R114 flow is the following:

$$Fr_{H_2O}^*/Fr_{R114}^* = N_{D,H_2O}/N_{D,R114},$$

i.e., the ratio of the flash Froude numbers is equal to the ratio of the similarity parameters N_D for the two substances.

The distance from the flash level to the point where the flow chokes (choking length) is very important in the design of geothermal wells because of its influence on well productivity. It can be seen from Figs. 3-5

that calculations based on R114 (under the specified conditions) allow us to predict the calculated choking length for water (under its corresponding similar conditions) to within 6%. Because the two curves are so close together all the way from the flash point to the point of choking (the point of maximum $\tilde{z}$ along the curve), we can conclude that the state of the geofluid in the well can be accurately predicted using the results calculated for R114.

The role of the similarity parameter N_D can be seen from Fig. 5 wherein $\tilde{s}$ is plotted versus $\tilde{z}$. It will be recalled that N_D enters the expression for $\tilde{s}$, eq. (58), in a direct way. Since the N_D values were not made equal for the two fluids, one might expect that the $\tilde{s}$ correspondence might be poor. However, the closeness of the curves indicates that N_D plays only a minor role in the similarity problem for the process under study.

Mass flow rates are reduced by a factor of 180 and all linear dimensions are scaled down by approximately a factor of 7.6 in going from water to R114. Thus, the 114m length from flashing to choking for water substance is reduced to 15m for R114.

With regard to the fluid dynamic aspect of the flow, only the Mach number was considered to be a significant parameter since the slip ratio was assumed to be unity for the preceding example. The importance of slip as a possible additional fluid dynamic similarity parameter was investigated by repeating the previous calculation with a value of slip equal to 2.0, i.e., $\sigma = 2.0$. The results are shown in Figs. 6-8. Since the agreement between the solutions for water and R114 is also very good in this case (except near the point where the flow chokes), we can see that the slip does not need to be scaled when water is being simulated by R114, the same value for each fluid being sufficient to allow similar results.

6. CONCLUSIONS

The simplified model fluid presented here for two-phase flow can serve as a basis for the similarity analysis of a variety of substance flows. For the special case of water and R114, we have seen that exact similarity does not exist in the range of interest for geothermal applications, but that conditions can be found for reasonable similarity which permit one to replace water with R114 in laboratory-size apparatus. Thus experimental data and results obtained using R114 in a properly scaled laboratory setup can be converted with reasonable accuracy to those for water.

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NOMENCLATURE

Latin Letters

a	Speed of propagation of a small disturbance
C	Constant; circumference
c	Specific heat
D	Diameter
f	Friction factor
g	Gravitational acceleration
h	Specific enthalpy
Ma	Mach number
N_D	Similarity parameter
P	Pressure
R_a	Apparent gas constant
R_h	Hydraulic radius
s	Specific entropy
T	Temperature
v	Specific volume
w	Velocity
x	Dryness fraction

Greek Letters

α	Generalized "void" fraction when used with subscript; void fraction
γ	Density ratio, ρ_f/ρ_g
Π	Generalized property
ρ	Density
σ	Slip ratio

Greek Letters (cont.)

τ	Shear stress
ψ	Mass flux, ρw

Subscripts

c	Choking
CM	Center of mass
f	Liquid phase
fg	Evaporation/condensation
g	Vapor phase
i	Interface
m	Mixture
0	Reference variable
p	Phase, either f or g
r	Reference value
RMS	Root mean square
sat	Saturation
w	Wall

Note: Switch in order of subscripts on τ indicates a sign change

Superscripts

'	d/dT
"	d^2/dT^2
-	Average value
~	Dimensionless variable

FIGURE CAPTIONS

- Fig. 1 Similarity parameter N_D versus temperature T and dimensionless temperature $\bar{T}$.
- Fig. 2 Correspondence between R114 and water temperatures.
- Fig. 3 Scaled temperature difference with respect to the flash point versus scaled distance along a vertical pipe: slip ratio = 1.0.
- Fig. 4 Scaled dryness fraction versus scaled distance along a vertical pipe: slip ratio = 1.0.
- Fig. 5 Scaled entropy difference versus scaled distance along a vertical pipe: slip ratio = 1.0.
- Fig. 6 Scaled temperature difference with respect to the flash point versus scaled distance along a vertical pipe: slip ratio = 2.0.
- Fig. 7 Scaled dryness fraction versus scaled distance along a vertical pipe: slip ratio = 2.0.
- Fig. 8 Scaled entropy difference versus scaled distance along a vertical pipe: slip ratio = 2.0.

FIG. 1. SIMILARITY PARAMETER N_D VERSUS TEMPERATURE T AND DIMENSIONLESS TEMPERATURE $\tilde{T}$

FIG. 2. CORRESPONDENCE BETWEEN R114 AND WATER TEMPERATURE

FIG. 3. SCALED TEMPERATURE DIFFERENCE WITH RESPECT TO THE FLASH POINT VERSUS SCALED DISTANCE ALONG A VERTICAL PIPE: SLIP RATIO = 1.0

FIG. 4. SCALED DRYNESS FRACTION VERSUS SCALED DISTANCE ALONG A VERTICAL PIPE:
SLIP RATIO = 1.0

FIG. 5. SCALED ENTROPY DIFFERENCE VERSUS SCALED DISTANCE ALONG A VERTICAL PIPE: SLIP RATIO = 1.0

FIG. 6. SCALED TEMPERATURE DIFFERENCE WITH RESPECT TO THE FLASH POINT VERSUS SCALED DISTANCE ALONG A VERTICAL PIPE: SLIP RATIO = 2.0

FIG. 7. SCALED DRYNESS FRACTION VERSUS SCALED DISTANCE
ALONG A VERTICAL PIPE: SLIP RATIO = 2.0

FIG. 8. SCALED ENTROPY DIFFERENCE VERSUS SCALED DISTANCE
ALONG A VERTICAL PIPE: SLIP RATIO = 2.0

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