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SIMULATING THE $SU(2)$ SECTOR OF THE STANDARD MODEL
WITH DYNAMICAL FERMIONS*

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I-HSIU LEE**

Physics Department
Brookhaven National Laboratory
Upton, NY 11973

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ABSTRACT

The two-generation $SU(2)$ sector of the standard model with zero Yukawa couplings is studied on the lattice. The results from analytic studies and simulations with quenched fermions are reviewed. The methods and results of a Langevin simulation with dynamical fermions are presented. Implications for the strongly coupled standard model are mentioned.

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1. INTRODUCTION

Recently, there has been a surge of interest in nonperturbative studies of the standard electroweak theory. There are several reasons why one would like to study this theory nonperturbatively. First, the theory is, in general, not asymptotically free. Although perturbative calculations have so far proved to be adequate in describing the low-energy phenomena up to about 100 GeV, it is possible that there are significant nonperturbative effects due, for example, to a heavy Higgs boson and/or a heavy fermion. Moreover, a scenario involving nonperturbative behavior of the theory has been proposed in the strongly coupled standard model. More generally, it is well known that certain interesting phenomena can occur in a manner which is not evident in any order of perturbation theory. It is desirable to see if any such phenomena occur in the electroweak theory. Furthermore, the theory makes crucial use of a scalar sector, including its self-interactions, and its interactions with the gauge and fermion sectors. On the other hand, there is now strong evidence that the four-dimensional pure $\lambda\phi^4$ theory is a free field theory, contrary to expectation based on naive perturbative considerations. Several questions naturally arise. Does this property of the scalar theory persist in the presence of the full theory with coupled gauge and fermion fields? What are the implications of this for the standard model? Finally, a full understanding of the nonperturbative properties of the standard electroweak theory may provide new insights for the construction of theories beyond the standard model.

Most of the studies in this direction have been devoted to lattice Higgs models which are related to the bosonic sector of the standard electroweak theory. Since fermions do couple to the bosonic sector of the standard model, it is important to study their effects on the lattice Higgs models.

For this purpose, analytic and numerical studies have been carried out on an $SU(2)$ lattice gauge theory with a scalar and a (staggered) fermion fields, both in the fundamental representation.¹⁻⁷ The discretized Euclidean action of this model can be written in usual notation as

$$S = S_G + S_H + S_F \tag{1.1a}$$

where

$$S_G = \beta_g \sum_{\text{plaq.}} [1 - P] \quad (1.1b)$$

$$S_H = -2\beta_h \sum_{n,\mu} \text{Re}(\phi_n^\dagger U_{n,\mu} \phi_{n+e_\mu}) \quad (1.1c)$$

$$S_F = \frac{1}{2} \sum_{n,\mu} \eta_{n,\mu} (\bar{\chi}_n U_{n,\mu} \chi_{n+e_\mu} - \bar{\chi}_{n+e_\mu} U_{n,\mu}^\dagger \chi_n) \quad (1.1d)$$

with $\beta_g = 4/g^2$, $\beta_h = m^2/2\lambda$, and $P = (1/2)\text{Tr}(U_{\text{plaq.}})$. Recall that an $\text{SU}(2)$ chiral theory with $2N$ left-handed Weyl fields can be rewritten as an equivalent vectorlike theory with N left-handed and N right-handed (charge-conjugated) fields.⁸⁻¹¹ This has the important consequence that it allows one to study the $\text{SU}(2)$ sector of the standard model by using fermions which are coupled in a vectorial manner to the gauge fields, and hence, to avoid the problems with attempts to put chiral fermions on the lattice. In particular, the lattice model described by Eq. (1.1) is equivalent to the two-generation $\text{SU}(2)$ sector of the standard model with no Yukawa couplings. (The case with non-zero Yukawa couplings is discussed in Ref. 8.) This model can be regarded as an approximation to the standard electroweak theory in the limit where the $\text{U}(1)$ gauge coupling and the Yukawa couplings are weak. In this limit, the right-handed fermions are free fields which decouple from the rest of the theory and hence can be neglected. Here we shall discuss the results of a numerical simulation of this model with dynamical fermions.

2. ANALYTIC AND QUENCHED RESULTS

Let me first briefly review the results known before the dynamical fermion simulation was performed.

The model under consideration was first studied Ref. 1, where a numerical simulation was performed using the quenched approximation. We chose to work at $\beta_g = 0.5$ where all local observables are analytic functions of β_h in the absence of fermions. We measured the chiral condensate at several values of β_h . The data show evidence for the existence of a chiral transition at a finite β_h . We also found that, in the small- β_h region where the chiral condensate is nonzero, a massless Goldstone boson appears in the spectrum, while

in the large- β_h region where the chiral condensate vanishes, there exist massless fermions. Therefore, the simulation with the quenched approximation suggests that there exists a new transition associated with the fermions.

This finding motivates one to look for a way to demonstrate the existence of the chiral transition and determine its properties analytically. Several analytic studies^{2,4-9,13-17} were carried out for various models in the strong gauge coupling limit (i.e. $\beta_g = 0$), using a mean-field type of approximation. One of the main results of these studies is that the existence of the chiral transition is a very general feature of lattice gauge theories with scalar and fermion fields. Some of these works are discussed in R. Shrock's report at this meeting. I will only mention the results for the model described by Eq. (1.1). With the mean field approximation, we found that² the chiral condensate decreases monotonically and continuously as β_h increases from zero to $\beta_h \approx 2.76$, beyond which the condensate vanishes identically. Thus there is a second-order chiral transition at $\beta_g = 0$, and the critical point is at $\beta_{h,c} \approx 2.76$.

For comparison, a quenched simulation² was performed at $\beta_g = 0$, and the data suggests that a chiral transition occurs at approximately $\beta_{h,c} \approx 2.7$. This is in good agreement with the analytic result, actually to better than the 10 to 20 % level of accuracy typical of the mean field approximation in $d = 4$ spin models.

Although our analytic studies were carried out at the strong gauge coupling limit, the result is actually more general. Since a small- β_g expansion has a finite radius of convergence, it follows that the critical point at $\beta_g = 0$ lies on a line of chiral phase transitions which extends into the interior of the phase diagram at $\beta_g > 0$ (and also to $\beta_g < 0$, but the latter region is not of direct physical interest). However, as was shown in Ref. 13, this line of chiral transitions cannot stop in the interior of the phase diagram, since it separates a region where the chiral condensate is nonzero from another where the condensate vanishes identically, and these two regions cannot be analytically connected to each other. Therefore, the chiral transition found at $\beta_g = 0$ extends into the interior of the phase diagram all the way until it reaches one of the two boundaries, $\beta_h = 0$ or $\beta_g = \infty$ (or the intersection of these boundaries). Furthermore, it is worth noting that the chiral transition is determined to be of second-order at $\beta_g = 0$. From the small- β_g expansion, it

follows that the transition will remain second-order for at least a finite range of β_g adjacent to $\beta_g = 0$.

3. METHOD FOR DYNAMICAL FERMION SIMULATION

It is clearly desirable to upgrade the quenched simulations to include dynamical fermions. We have done this for $I = 1/2$ fermions³ (and recently, for $I = 1$ fermions¹⁹), using a Langevin algorithm.^{20,21} It is well known²⁰ that, after a sufficiently large number of Langevin time steps, the configurations generated by a discrete Langevin process are distributed with a probability density $e^{-\bar{S}}$, where $\bar{S} = S + \Delta\tau S_1 + O((\Delta\tau)^2)$. For the SU(2) model under consideration, we found that, in addition to yielding extra terms in the action, the leading correction $\Delta\tau S_1$ shifts the couplings β_g and β_h to

$$\beta'_g = (1 - \frac{8}{3}\Delta\tau)\beta_g \quad (3.1)$$

and

$$\beta'_h = (1 - \frac{23}{12}\Delta\tau)\beta_h \quad (3.2)$$

This is relevant for comparisons of data obtained from different fermion algorithms. With a sufficiently small step size, $\Delta\tau$, the systematic error will be small, and the action $\bar{S}$ will have the same qualitative features as the original action S .

4. DATA AND DISCUSSIONS

The results of our dynamical fermion simulations are summarized as follows. We set $\Delta\tau = 0.01$ for most of our measurements (see below for recent work).

We first study the $\beta_g = 0$ case, since we have analytic results here which can be compared with the simulation. Three quantities are measured: the chiral condensate $\langle \bar{\chi}\chi \rangle$, the average plaquette $\langle P \rangle$ and (minus) the gauge-Higgs interaction energy, or equivalently, the average gauge-invariant nearest-neighbor Higgs-Higgs correlation function, $\langle L \rangle \equiv \langle \sum_{n,\mu} \text{Re}(\phi_n^\dagger U_{n,\mu} \phi_{n+e_\mu}) / N_\ell \rangle$, where N_ℓ is the number of links in the lattice. To measure the spontaneous chiral symmetry breaking condensate, $M \equiv \langle \bar{\chi}\chi \rangle$, we have to add a small bare mass as the source to calculate $M_m \equiv \langle \bar{\chi}\chi \rangle_m$, and then

extrapolate to $m = 0$. Although, as is well known, there is rigorously no spontaneous symmetry breaking or phase transition in a finite volume, a wealth of experience in statistical mechanics shows that one can make accurate inferences about the thermodynamic limit from simulations on finite lattices. Our data (see Ref. 3 for graphs) shows that the chiral condensate decreases as β_h increases, and vanishes for sufficiently large β_h . The critical point of the chiral transition is estimated to be $\beta_h = 2.5 \pm 0.3$, in agreement, to the expected level of 10 – 20 %, with the analytic mean field result, and, moreover, also close to the critical value $\beta_h \approx 2.7$ obtained from our quenched simulation.²

To see the effects of fermions on $\langle L \rangle$, recall that, in the absence of fermions, $\langle L \rangle$ can be calculated exactly:

$$\langle L \rangle_{bos.} (\beta_g = 0) = I_2(2\beta_h)/I_1(2\beta_h) \quad (4.1)$$

We found that the dynamical fermion measurements of $\langle L \rangle$ are very close to $\langle L \rangle_{bos.}$, indicating that the fermion effects on $\langle L \rangle$ are negligibly small for $\beta_g = 0$. Since the (reduced) free energy $f(\beta_h) = \lim_{vol. \rightarrow \infty} N_{sites}^{-1} \ln Z$ is non-analytic at the chiral transition, it follows rigorously that all of its derivatives are also non-analytic at this point. This holds, in particular, for $\langle L \rangle$, since $\langle L \rangle = (1/2)\partial f/\partial \beta_h$. Furthermore, this must be a finite non-analyticity, since $\langle L \rangle$ is (proportional to) an energy. We do not see any clear indication of this non-divergent singularity in our data for $\langle L \rangle$. In the absence of fermions,

$$\langle P \rangle_{bos.} (\beta_g = 0) = [I_2(2\beta_h)/I_1(2\beta_h)]^4 \quad (4.2)$$

The effects of fermions on $\langle P \rangle$ are found to be slightly larger than those on $\langle L \rangle$. This may be due to the fact that, in our model, fermions couple to the gauge fields directly while they interact with the scalars only through gauge fields.

Next, we study the interior of the phase diagram. As was mentioned earlier, the qualitative features established at $\beta_g = 0$ should remain the same for a finite strip adjacent to $\beta_g = 0$. This expectation is borne out by our data for $\beta_g = 0.5$. The chiral condensate at $\beta_g = 0.5$ behaves very similarly to the $\beta_g = 0$ case (except that the critical point of the chiral transition now moves to a lower value of β_h). The same is true of other quantities

we measured. According to our data, the chiral transitions at $\beta_g = 0$ and 0.5 are both consistent with being of second-order, in agreement with the result of our analytic study.

So far, we have been studying the region of the phase diagram where there is no phase transition in the absence of fermions. To see how the chiral transition and the confinement-Higgs transition affect each other, we study the case $\beta_g = 1.9$. There are several reasons for this choice. First, it has been shown^{22,23} that, in the absence of fermions, the confinement-Higgs transition exists for $\beta_g > 1.6$. Second, since the Langevin algorithm introduces an effective shift of the couplings to slightly smaller values, a β_g value too close to 1.6 would be inappropriate for our purpose. On the other hand, the chiral condensate (where it is nonzero) decreases very rapidly as β_g increases. To get a clear signal, β_g cannot be too large. Furthermore, finite size effects are more severe for larger β_g . The choice of $\beta_g = 1.9$ is a compromise between these considerations.

$M_m a^3$ at $\beta_g = 1.9$ for $ma = 0.1$ and 0.05 is plotted in Fig. 4 of Ref. 3 (where a is the lattice spacing). If we extrapolate this data to $ma = 0$, we find that the chiral transition occurs at $\beta_h \approx 0.4$. To see whether there is a separate confinement-Higgs transition, we measure $\langle L \rangle$. The data shows no evidence for any other transition than the one at $\beta_h \approx 0.4$. In contrast, the same simulation for the purely bosonic system shows that the confinement-Higgs transition occurs at $\beta_h \approx 0.52$. This result suggests that, in the presence of fermions (in the fundamental representation), the chiral transition coincides with the confinement-Higgs transition. Furthermore, for a given β_h , the measured value of $\langle L \rangle$ with dynamical fermions is always greater than that in the purely bosonic system. This suggests that fermion effects tend to make the system more ordered. The data also indicates that the behavior of $\langle L \rangle$ at $\beta_g = 1.9$ is more similar to that at $\beta_g = \infty$ than at $\beta_g = 0$. Moreover, there is some indication that the transition at $\beta_g = 1.9$ is weakly first-order.

From our simulation with dynamical fermions, we obtain the phase diagram in the presence of fermions, which is shown in fig. 6 of Ref. 3. Obviously, the location of the chiral phase boundary in the limit $\beta_g \rightarrow \infty$ cannot be definitively determined by simulations at any finite β_g ; for this reason, we did not draw this phase boundary past the point at $\beta_g = 1.9$ where our last measurement was made. Rather, we appended an

arrow to it, indicating that, as noted in Ref. 13, it could not end anywhere in the interior of the diagram. Since our dynamical fermion measurements at $\beta_g = 1.9$ show that the chiral phase boundary coincides with the confinement-Higgs boundary, and since the latter terminates at the $O(4)$ critical point $\beta_h \approx 0.3$ as $\beta_g \rightarrow \infty$, a possible scenario is that the chiral boundary also ends there in this limit. This is the same inference that one would draw from the previous quenched fermion data^{1,18} which also indicated the chiral boundary coincided with the confinement-Higgs boundary where the latter was present. However, the $\beta_g = 0$ and $\beta_g = \infty$ lines are on a qualitatively different footing: the region around the line $\beta_g = 0$, both for positive and negative β_g , with $\beta_h < \beta_{h,crit.}$, is a region of spontaneous chiral symmetry breaking. In contrast, as we have discussed in Ref. 3, for $\beta_g = \infty$, the effective partition function $Z = Z_{O(4)}Z_{ff}$, where the first factor refers to the $O(4)$ spin model, and the second refers to a (massless) free fermion. Hence, at $\beta_g = \infty$, $\langle \bar{\chi}\chi \rangle = 0$ for all β_h . Thus, if the chiral phase boundary does meet the $O(4)$ critical point, the line segment from $(\beta_g, \beta_h) = (\infty, \beta_{h,crit.O(4)})$ to $(\infty, 0)$ constitutes the remaining portion of the boundary. A different scenario, a priori, is one in which the chiral boundary ends at the point $(\infty, 0)$; however, at present, there is no evidence, either from quenched^{1,2,18} or dynamical³ simulations, for this scenario. Two specific features which one would look for to test for this latter picture would be (a) the observation, at a given β_g , of two transitions as a function of β_h , with the chiral transition occurring at a smaller value of β_h than the line of transitions emanating from the $O(4)$ critical point at $\beta_g = \infty$; (b) essentially equivalently, observation, at a given β_g , of the chiral transition occurring at a value of β_h below $\beta_{h,crit.O(4)}$. The opposite behavior is observed in both quenched data^{1,18} at $\beta_g = 2.3$ and our dynamical fermion data³ at $\beta_g = 1.9$: only one transition is observed, with the chiral and confinement-Higgs boundaries appearing to coincide, and the values of β_h at this phase boundary lie above $\beta_{h,crit.O(4)}$.

It is clearly of interest to investigate the chiral boundary further. To go to values of β_g significantly larger than about 2 will be quite difficult for reasons mentioned above. Fortunately, one of our most important findings is independent of the details of the location of the chiral boundary at large β_g : the region with small β_g and small β_h (customarily labeled as the confinement region) is no longer analytically connected to the region with

large β_g and large β_h (the Higgs region) when fermions are included, and consequently, the complementarity of states which existed between these regions in the absence of fermions no longer holds in their presence.

Thus, we have shown that fermions affect the phase diagram of a theory in a fundamental way. In particular, for the SU(2) sector of the standard model, the existence of the chiral phase boundary in the phase diagram implies that it is necessary to approach the continuum limit of the lattice model from within the chirally symmetric phase in order to obtain a spectrum with light fermions. The fact that the chiral transition is continuous for at least a finite range of β_g opens up new possibilities for the continuum limit of the lattice theory to be taken. It remains to be seen if any of these may yield physically interesting continuum theories. In this connection, recall, as we noted before,^{2,5,9} that our mean field methods yield a value $\beta = 1/2$ for the critical exponent for the order parameter, $\langle \bar{\chi}\chi \rangle$, and that this value is consistent with our data at $\beta_g = 0$. If a tricritical point separates the continuous from the first-order segments of the chiral boundary, it might be interesting to investigate the vicinity of this point further.

A particularly important result of our studies of the chiral transition concerns the strongly coupled standard model¹² (SCSM), which again treats the SU(2) sector of the electroweak theory. This model was originally motivated in part by the fact that the confinement and Higgs regions of the SU(2) lattice theory (without fermions) were analytically connected. The model requires that chiral symmetry not be spontaneously broken, since this would produce large dynamically generated fermion masses, the natural scale for which is the electroweak scale of ≈ 250 GeV, and would violate electric charge conservation. At the same time, however, it assumes confinement and a spectrum of excited bound states similar to that in usual QCD. We have shown that with fermions the regions of the phase diagram previously labelled confinement and Higgs are completely separated by the non-analytic chiral phase boundary. Thus, we find no evidence for any region which is chirally symmetric and which is also analytically connected to the confinement phase of the SU(2) gauge-fermion theory at $\beta_h = 0$. Independent of the precise location of the chiral boundary, its existence means that there is no complementarity of states between the chirally symmetric and chirally broken, confinement regions. It remains to be seen

whether in the chirally symmetric phase one can find any evidence for excited, bound states characteristic of SCSM. Of course, the natural place for such a spectrum to occur is the latter confinement region, where, however, chiral symmetry is spontaneously broken.

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