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NONLINEAR DYNAMICS AND CHAOS IN BOILING WATER REACTORS

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INTRODUCTION

There are currently 72 commercial boiling water reactors (BWRs) in operation or under construction in the Western world, 37 of them in the United States. Consequently, a great effort has been devoted to the study of BWR systems under a wide range of plant operating conditions. This paper represents a contribution to this ongoing effort; its objective is to study the basic dynamic processes in BWR systems, with special emphasis on the physical interpretation of BWR dynamics. The main thrust in this work is the development of phenomenological BWR models suited for analytical studies performed in conjunction with numerical calculations. This approach leads to a deeper understanding of BWR dynamics and facilitates the interpretation of numerical results given by currently available sophisticated BWR codes.

In BWRs, water acts both as moderator and as coolant. During normal operating conditions, both the density reactivity and Doppler reactivity coefficients are negative, so the reactor power is stable (i.e., the power level is maintained constant) without the intervention of the reactor's control system. For example, if the reactor's power level is increased, these negative reactivity coefficients induce negative feedback processes that ultimately reduce the power, thereby stabilizing the reactor.

It is known, though, that systems with negative feedbacks are susceptible to instabilities if the feedback gain or the phase lag is increased beyond some critical value. In the case of a BWR, a reactivity instability manifests itself as a diverging power oscillation. As recent stability experiments^{1,2} have shown, BWRs are susceptible to reactivity instabilities when operated at low flow and relatively high power levels (e.g., 32% flow and 51% power). The importance of understanding reactivity instabilities is underscored by the fact that United States utilities are required by the Nuclear Regulatory Commission to evaluate the reactor stability for every reload core unless plant technical specifications provide for monitoring of neutron flux oscillations in

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so-called limit-cycle "Detect and Suppress" (D&S) region. This region is defined by these specifications and commonly lies below the 40% flow line and above the 80% rod control line. Within this region, the reactor operator must monitor average and local power oscillations to detect instabilities. Should instabilities occur, they should be suppressed either by inserting control rods or by increasing recirculation pump speed.

It has been shown experimentally that BWRs behave as linear systems under normal operating conditions.¹ As discussed in the foregoing, however, BWRs are susceptible to instabilities. When the equilibrium (i.e., operating) point becomes unstable, small oscillations grow large enough so that nonlinearities become important. This phenomenon has been observed in recent experiments in which the reactor power was increased slowly while the flow rate was essentially kept constant. When a certain critical power level was reached, the reactor became unstable and the measured signals began displaying an oscillatory behavior. If the reactor were a linear system, these oscillations would have diverged exponentially. However, these experiments showed that the amplitudes of these oscillations grew only slightly, then stabilized (i.e., remained bounded) due to the appearance of a limit cycle. As is well known, the appearance of a limit cycle is a typical nonlinear phenomenon.

To analyze the effect of nonlinearities on the stability and dynamic behavior of a BWR, it is useful to summarize the main nonlinear process in a BWR. An obvious nonlinearity is associated with the neutron field. For example, in the point kinetics representation of the neutron field, the term "reactivity-times-neutron-density" (ρn) is nonlinear because, due to the inherent reactivity feedback, ρ depends on n . Physically, this means that reactivity perturbations are weighted by the neutron density, a fact that forces the neutron density to be positive at all times. In addition, the reactivity feedback introduces nonlinearities because the cross sections and the associated reactivity coefficients are nonlinear functions of the temperature and moderator density.

In general, the fuel heat-conduction equation is also nonlinear because the heat conductance, density, and heat capacity of the fuel depend on the temperature. The fluid energy and momentum conservation equations are also nonlinear. For example, nonlinearities in the momentum equation appear in both the kinetic-energy and friction terms. Nonlinearities are also found in the dynamics of the recirculation loop. Although the nonlinearities mentioned in the foregoing have little effect on the reactor's stability under small perturbations from steady-state operating conditions, they become the dominant factors governing the reactor's dynamic behavior beyond the regime of linear stability.

A PHENOMENOLOGICAL MODEL OF BWR DYNAMICS

A phenomenological model for studying nonlinear BWR dynamics must obviously contain the essential processes that control the dynamic behavior of a BWR, yet it must be sufficiently simple to allow extensive numerical simulation of a wide range of reactor operating conditions. The development of such a model has been the objective of the work described in Refs. 3 and 4. There it has been shown that the simplest phenomenological model that retains the essential physical processes controlling the dynamic behavior of a BWR contains a one-point representation of the reactor kinetics, a one-node representation of the heat transfer process in the fuel, and a two-node representation of the channel thermal hydraulics to account for the void reactivity feedback. Mathematically, these processes can be represented by the following system of ordinary differential equations:

$$\frac{dn(t)}{dt} = \frac{\rho(t) - \beta}{\Lambda} n(t) + \lambda c + \frac{\rho}{\Lambda} \quad , \quad (1)$$

$$\frac{dc(t)}{dt} = \frac{\beta}{\Lambda} n(t) - \lambda c \quad , \quad (2)$$

$$\frac{dT(t)}{dt} = a_1 n(t) - a_2 T(t) \quad , \quad (3)$$

$$\frac{d^2 \rho_\alpha(t)}{dt^2} + a_3 \frac{d\rho_\alpha(t)}{dt} + a_4 \rho_\alpha = kT(t) \quad , \quad (4)$$

and

$$\rho(t) = \rho_\alpha(t) + DT(t) \quad , \quad (5)$$

where

$n(t)$ = excess neutron density normalized to the steady-state neutron density,

$c(t)$ = excess delayed neutron precursors concentration, also normalized to the steady-state neutron density,

$T(t)$ = excess average fuel temperature,

$\rho_\alpha(t)$ = excess void reactivity feedback.

Note that the nonlinearity appears in the neutronic equation through the parametric feedback introduced by the reactivity.

As detailed in Ref. 4, the parameters in the phenomenological model represented by Eqs. (1) through (5) were obtained by functionally fitting the transfer function, as calculated by the LAPUR code, of the Vermont Yankee reactor with operating conditions equivalent to those of stability test 7N. These test conditions have been chosen for the present nonlinear study because a limit cycle was experimentally observed during this test. The specific numerical values for the resulting model parameters are presented in Table I.

The parameter k , which is proportional to the void reactivity coefficient and the fuel heat transfer coefficient, controls the gain of the feedback and, thus, defines the linear stability of this reactor model. The value of k_0 given in Table I is the critical value above which the model becomes unstable. By artificially increasing the value of k above k_0 , the model can be made unstable and can be used to study BWR dynamic behavior in the nonlinear regime.

The validation of the above model has been detailed in Ref. 4. The model accurately represents the dynamic behavior of the reactor. For example, Fig. 1 shows the remarkable agreement between the model's reactivity-to-power transfer and the LAPUR-calculated transfer function for the conditions of test 7N.²

ANALYTICAL STUDY

Taking advantage of the low order of the present model, analytical work can be performed to understand the behavior of the model's solutions. To start the transient solutions reported in this work, a step perturbation Δ in the neutron population was applied to the heat

Table 1

Model Parameters for Vermont Yankee Test 7N

Parameter	Value	Units
a_1	25.04	$K \cdot s^{-1}$
a_2	0.23	s^{-1}
a_3	2.25	s^{-1}
a_4	6.82	s^{-2}
k_0	-3.70×10^{-3}	$K^{-1} \cdot s^{-2}$
D	-2.52×10^{-5}	K^{-1}
β	0.0056	---
Λ	4.00×10^{-5}	s^{-1}
λ	0.08	s^{-1}

generation term in Eq. (3). (This perturbation has the same effect as changing the reactivity and was chosen to facilitate subsequent computations.) The equilibrium points were obtained by setting the time derivatives in Eqs. (1) through (5) to zero. Straightforward algebra yields two equilibrium points, designated A and B, where

$$A = [n = -\Delta, c = -\beta\Delta/(\lambda\Lambda), T = 0, \rho_\alpha = 0] \quad , \quad (6)$$

and

$$B = [n = -1, c = -\beta/(\lambda\Lambda), T = a_1(\Delta - 1)/a_2, \rho_\alpha = ka_1(\Delta - 1)/(a_2a_4)] \quad . \quad (7)$$

Equilibrium point A corresponds to normal reactor operation, while point B corresponds to a shutdown condition. Note that the definition $n = (N - N_0)/N_0$ implies that point $n = -1$ corresponds to $N = 0$, where N is the absolute neutron density. For a perturbation Δ of this kind, point B is always unstable while point A is stable as long as $k < k_0$. For $k > k_0$, both equilibrium points are unstable.

An examination of Eq. (1) reveals that neglecting the delayed neutron effects gives

$$\frac{dn}{dt} = \frac{\rho(n+1)}{\Lambda} \quad . \quad (8)$$

Fig. 1. Comparison between the reactivity-to-power transfer functions calculated by the code LAPUR (+++) and the phenomenological model (---).

Equation (8) highlights the fact that dn/dt will always tend to zero when n approaches $n = -1$, regardless of the value of ρ . Physically this means that the absolute neutron population N cannot be negative. Therefore, not just point B, but the whole $n = -1$ line in phase space is unstable in the sense that it repels trajectories (i.e., temporal model solutions). This is in contrast to the linear models customarily used for BWR dynamic modeling; in such models, ρ does not induce a parametric reactivity feedback, so the absolute neutron population can become mathematically negative.

It follows from the above discussion that the phase-space trajectories should appear as shown in Fig. 2 when they are locally close to the equilibrium points (A and B). The situation shown in Fig. 2(a) depicts the trajectories when equilibrium point A is stable. All the trajectories spiral toward the stable equilibrium point. This situation represents normal BWR operation. Once the linear stability threshold is crossed, point A becomes unstable, so the trajectories spiral away from it. This scenario is depicted in Fig. 2(b). In both cases, the trajectories are asymptotically tangent to the $n = -1$ because they can neither cross it nor reach the unstable equilibrium point B.

Away from the two equilibrium points, possible behaviors of the phase-space trajectories are presented in Fig. 3. Case (a) depicts a globally unstable system where the trajectory continually spirals away from point A. Case (b) describes a situation where the trajectory departs from the equilibrium point but eventually stabilizes itself due to the nonlinearities in the system; thus the trajectory remains bounded and converges to a closed line. This closed line defines a limit cycle, which corresponds to a periodic solution of fixed amplitude. Case (c) is similar to case (b) insofar as the trajectory is repelled by equilibrium point A and remains bounded due to the nonlinearities. The difference between cases (b) and (c) is that no periodic solution (i.e., a limit cycle or closed line in phase space) exists in case (c); although the trajectory stays within a bounded region, it never converges to a closed curve or to an equilibrium point. This bounded region is called a strange attractor, and the solution of a system of equations with a strange attractor is said to be aperiodic.

DETERMINISTIC NUMERICAL ANALYSIS

For the deterministic numerical analysis presented below, a 10% step-type perturbation was introduced in the neutron population in

Fig. 2. Phase-space trajectory of the solution close to the two equilibrium points.

Fig. 3. Possible behavior of the phase-space trajectories away from the equilibrium points.

Eq. (3) at time $t = 0^+$. The solution was then allowed to converge freely to its final state. This numerical procedure was used because the solution converged quickly and smoothly to its final state.

The numerical solution of the model shows that limit cycles appear when feedback gain k is increased past critical value k_0 . A limit cycle corresponds to a periodic and bounded solution of a system of equations that describes a closed trajectory in phase space. The phase space for the present model has five dimensions: the neutron density n , the delayed neutron precursors concentration c , the fuel temperature T , and the void reactivity feedback ρ_α and its derivative $d\rho_\alpha/dt$. The trajectory is parameterized in phase space by time. As an illustrative example, Fig. 4 presents the neutron density time trace when the system is perturbed from the unstable equilibrium point by inserting a $\Delta = 10\%$ perturbation. This time trace shows the development of a typical limit cycle; it also shows that once the limit cycle is reached, the signal becomes periodic and oscillates with an amplitude of $\sim 20\%$. The amplitude of this limit cycle has the same order of magnitude as the one experimentally observed at Vermont Yankee. This fact increases confidence in the fact that this phenomenological model represents the general qualitative dynamic behavior of BWRs not only in the linear but also in the nonlinear regime.

A limit cycle of large amplitude is presented in Figs. 5 and 6. The main characteristics of this limit cycle can be studied in the time trace plot (Fig. 5) and summarized as follows:

1. The neutron density undergoes a series of periodic pulses of large magnitude. Between pulses, the neutron population remains at a low level close to the unstable shutdown equilibrium point.
2. The average void fraction, obtained as the ratio between the void reactivity and the void reactivity coefficient, undergoes a slightly deformed sinusoidal behavior, which defines the frequency of the neutronic pulses. The pulses appear during the negative part of the void fraction oscillation.
3. The fuel temperature rises sharply during the neutron pulse and then decays exponentially with the fuel time constant. The

Fig. 4. The development of a typical limit cycle in the time domain.

Fig. 5. Time domain representation of a large-amplitude limit cycle.

Fig. 6. Phase-space representation of a limit cycle.

relative timing of the neutron, temperature, and void fraction pulses indicates that the void fraction oscillations provide the mechanism for generating the neutron pulses, while the temperature oscillations merely follow the evolution of the neutron pulses.

LIMIT CYCLE STABILITY: BIFURCATIONS

In the linear region, the stability of the system is quantified in terms of an *asymptotic decay ratio*;⁴ however, in the nonlinear regime, the asymptotic decay ratio is always equal to 1.0, due to the appearance of limit cycles. Therefore, the decay ratio is not a good descriptor of the reactor dynamic state in this regime. A better dynamic descriptor in the nonlinear regime is the amplitude of the limit cycle oscillations. These two descriptors (i.e., the decay ratio and the limit cycle amplitude) are mutually complementary. Thus, the decay ratio defines the stability of the reactor in the linear regime where the amplitude of the limit cycle oscillations is zero. In the nonlinear regime, the decay ratio becomes constant and equal to 1.0, but the amplitude of the oscillations defines the dynamic behavior of the reactor.

The previous discussion indicates that the concern in the nonlinear regime is the stability of the amplitude of the oscillations. This fact is highlighted in Fig. 7. This figure shows the development of the limit cycle for three different values of the feedback gain: (a) $k = 1.2$, (b) $k = 1.4$, and (c) $k = 1.5$. We observe clear differences among the ways the limit cycle is reached in the three cases. In case (a), the amplitude of the oscillation (which is equal to the maximum value of the pulses, i.e., the signal envelope) follows a smooth curve and promptly converges to the final amplitude. In case (b), the amplitude oscillates around the final value and eventually converges to it. In case (c), however, the amplitude oscillates but never converges to a final value; the amplitude itself describes an undamped periodic oscillation. Thus the amplitude of the limit cycle in case (c) has become unstable and is following a new limit cycle of its own with twice the original period. This fact causes the original signal to periodically exhibit two pulses

Fig. 7. Development of an instability of the limit cycle amplitude.

of different magnitude. This process is customarily called a "period-doubling pitchfork bifurcation."⁵

The bifurcation process can be seen more clearly in phase space. For instance, in the $n - T$ phase-space plane (Fig. 8), the original limit cycle curve splits (bifurcates) into two new curves. The trajectory described by the solution follows first the inside curve and then the outside curve. The cycle is then repeated periodically. The small and large curves in Fig. 8 correspond to the small and large pulses in Fig. 7c.

Further increasing the feedback gain k produces a cascade of period-doubling bifurcations, which leads to an aperiodic regime. The transition to aperiodicity is governed by a set of universal constants predicted by Feigenbaum's universality theory.⁵ The bifurcation diagram shown in Fig. 9 is constructed by plotting the extrema of the stationary solution $n(t)$ as a function of the heat transfer coefficient k . A cascade of period-doubling pitchfork bifurcations arises as k is increased past k_0 . The critical bifurcation values k_j converge to the accumulation point $k_\infty = 1.61811k_0$; at k_∞ the BWR model enters into an aperiodic self-pulsing mode. Increasing k past k_∞ leads to regions of aperiodic behavior intermixed with windows of periodicity. Since this BWR model is not driven externally, its period is determined by the parameters appearing in Eqs. (1) through (5), and this period varies continuously with k except when $k = k_j$, $j = 0, 1, \dots$.

Table II presents calculated values of k_j/k_0 , the ratios $\delta_j = (k_j - k_{j-1})/(k_{j+1} - k_j)$, and the ratios α_j of the successive pitchfork splittings, for $j = 0$ through 6. Feigenbaum's theory predicts that, as $j \rightarrow \infty$, δ_j and α_j tend to the universal constants $\delta = 4.6692\dots$ (the convergence ratio) and $\alpha = 2.5029\dots$ (the pitchfork scaling parameter) shown in the last row of Table II. Also presented in this row is the value, calculated by extrapolation, of the accumulation point k_∞ . (In the literature, the "accumulation point" is occasionally called the "critical point.") Overall, there is good agreement between these theoretically predicted values for δ and α and the calculated values δ_j and α_j .

Fig. 8. Illustration of a period-doubling bifurcation in phase space.

Fig. 9. Bifurcation diagram for $n(t)$.

Table II

Estimation of Universal Constants δ and α ;
Aperiodic Behavior Commences at k_∞

Cycle j	Period 2^j	Critical Bifurcation Values, k_j/k_0	δ_j	α_j
0	1	1.0	---	---
1	2	1.470 $\pm$ 0.002	---	---
2	4	1.584 $\pm$ 0.001	4.123 $\pm$ 0.126	1.095
3	8	1.6103 $\pm$ 0.0001	4.335 $\pm$ 0.295	2.207 $\pm$ 0.097
4	16	1.6165 $\pm$ 0.0001	4.242 $\pm$ 0.314	2.391 $\pm$ 0.332
5	32	1.61775 $\pm$ 0.00001	4.960 $\pm$ 0.596	2.465 $\pm$ 0.154
6	64	1.618025 $\pm$ 0.000005	4.545 $\pm$ 0.648	2.517 $\pm$ 0.094
∞	∞	1.61811 (extrapolated)	4.6692... (Ref.2)	2.5029... (Ref.2)

Fig. 10. Plot of the $(j+1)$ 'th maximum of the excess neutron population, $n(t)$, versus the j 'th maximum of $n(t)$, for $j = 0, 1, 2, \dots$: (a) periodic region, $k = 1.61803k_0$, (b) aperiodic region, $k = 1.65k_0$, and (c) aperiodic region, $k = 1.80k_0$.

Figure 10 shows one-dimensional mappings obtained when the Poincaré map for this BWR model is reduced by plotting the j 'th maximum of $n(t)$ versus the $(j + 1)$ 'th maximum of $n(t)$. The plot in Fig. 10a is obtained for $k = k_7 = 1.61803k_0$, when the BWR model has just undergone its seventh bifurcation to a 128-cycle; to the numerical accuracy in this work, $k = k_7$ delineates the limits of the periodic region since the slightest increase over k_7 triggers aperiodic behavior. Figure 10a indicates that in the periodic region, the behavior of this BWR model is essentially that of a one-dimensional noninvertible map with a quadratic extremum, a result expected from the previously discussed calculations of δ and α .

In the aperiodic region, the one-dimensional maps shown in Fig. 10b ($k = 1.650$) and in Fig. 3c ($k = 1.80$) display unexpected foldings, indicating that a double-valued relation might exist between successive maxima. This double valuedness, though, is illusory: An examination of the dynamic evolution of the relation between successive maxima reveals that the BWR model evolves either on the lower or on the upper branch according to whether the magnitudes of preceding maxima form an increasing or a decreasing sequence. This hysteresis-like folding displayed by the reduced Poincaré map indicates that a many-term recursion relation is needed to investigate the behavior of this BWR model in its aperiodic region.

STOCHASTIC NONLINEAR ANALYSIS

In contrast with the previous section, which dealt exclusively with a deterministic analysis of the nonlinear behavior of BWRs, this section analyzes the effects of nonlinearities on the BWR behavior under stochastic (random) excitations (sources). For this purpose, the phenomenological model was externally driven with a band-limited Gaussian white noise, and the equations were solved numerically in the time domain.

Two parameters were varied for this stochastic analysis: the feedback gain k and the variance of the driving noise source. The traces generated for $n(t)$ were fast-Fourier transformed to obtain power spectral densities (PSDs). The development of a limit cycle in the presence of stochastic sources is shown in Fig. 11, where the envelopes (maxima and minima) of the oscillation are plotted as a function of time for three different values of the noise-source variance. To generate these plots, the system was held originally at the unstable equilibrium point. At time $t = 0$, a zero-mean white noise was applied. The amplitude of the oscillations increased until it reached a limit cycle. As expected, the amplitude of these limit cycle oscillations was found to be independent of the magnitude of the driving noise variance.

Studying the effects of the feedback gain on the system behavior shows that, as predicted by linear studies, for stable systems (when $k < k_0$) the neutron PSD exhibits a single peak at the reactor characteristic frequency of oscillation. However, as k approaches k_0 while maintaining the driving-source variance constant, the PSD develops peaks at the harmonics of this fundamental frequency. For $k > k_0$, the power oscillations increase in time and eventually reach a limit cycle, with an enhancement of the harmonic components of the PSD as seen in Fig. 12. This figure shows three PSDs for different values of k . In case (a) the model is barely stable and only the fundamental peak is clearly discernible at ~ 0.4 Hz. Case (b) represents a small amplitude limit cycle for which the value of k is only slightly above the critical value k_0 . Case (c) corresponds to a fully developed large amplitude limit cycle. The main difference between the stable and the unstable

Fig. 11. Envelopes of the development of a limit cycle in the presence of noise.

PSDs is the appearance of higher harmonics. These harmonics have a large magnitude, so they should be measurable in real-life experiments even in the presence of measurement and process noise.

One of the consequences of the appearance of a limit cycle in a reactor is an increase in the variance (noisiness) of the neutron density as seen by the in-core neutron detectors. An increase in neutron noise variance, however, could also be due to an increase in the noise of other variables, such as flow, that influence the neutronics. It is of current interest to be able to distinguish between these two kinds of noise increases because different corrective action might need to be taken, depending on the cause. For example, if the increase in noise is due to an instability (i.e., the appearance of a limit cycle), the reactor can be made more stable by increasing the flow rate through the core; thus, a flow increase eliminates the extra noise. If the increase in noise is due to a pump malfunction, however, an increase in flow would probably only worsen the problem.

Fig. 12. The PSDs before and after the development of a limit cycle: (a) slightly stable, (b) slightly unstable, and (c) fully developed limit cycle.

Distinguishing between different causes leading to a noise increase is a current concern regarding BWR single-loop operation (SLO):⁶ Operating some reactors with a single recirculation pump above 40% of rated flow causes neutron noise increases of 300%. This increase could be caused by a hypothetical instability induced by the special mode of operation, or it could be caused by increased flow noise due to crossflow in the downcomer between the inactive and active jet pump banks. This highlights the need to develop a technique for differentiating between these two scenarios.

The present model was used to generate neutron density time traces for both stable and unstable conditions. The variance of the input noise source was adjusted so that the variance of the output neutron noise was of the same order of magnitude in both cases. The resulting time traces are presented in Fig. 13. Although there are obvious differences between the unstable reactor condition (a) and the stable one (b), it is not easy to determine if case (a) is really a limit cycle or not. A more sensitive technique is required to differentiate these two cases. Figure 14 presents the PSDs of the time traces for the above two cases.

Fig. 13. Comparison between limit cycle oscillations and externally induced noises in the time domain.

Fig. 14. Comparison between limit cycle oscillations and externally induced noise in the frequency domain.

Here, the differences are more obvious. In case (a), where the reactor is unstable, the characteristic peak at ~ 0.4 Hz is very sharp. The main difference, however, is the appearance of higher harmonics in the neutron PSD. Case (b), i.e., the stable condition, also shows harmonic contamination due to the large amplitude of the noise, but since the peak is wider, this contamination is not as obvious as in case (a). Furthermore, in a real-life measurement it would be impossible to distinguish this small harmonic contamination from the measurement and process noise. Therefore, if large-amplitude oscillations occur, nonlinearities appear as a harmonic contamination in the neutron PSD. If the oscillations are due to the appearance of limit cycles, the harmonic contamination is easily distinguishable from the background noise because it appears as sharp peaks at harmonic frequencies of the characteristic peak.

SUMMARY AND CONCLUSIONS

A phenomenological nonlinear model of BWR dynamics has been developed. In the linear regime, the results given by this model closely agree with the results from large, detailed BWR codes. This model also predicts the appearance of experimentally observed limit cycle behavior in the nonlinear regime.

The analytical study of this model has revealed the presence of two equilibrium points. In the linear regime, one equilibrium point is stable, while the other one is associated with an unstable shutdown reactor configuration. Above certain critical values of the model parameters, both equilibrium points become unstable, and the interaction of the phase-space trajectories with two equilibrium points leads to a variety of nonlinear phenomena, such as the appearance of limit cycles and period-doubling bifurcations.

The predictions of the phenomenological model have been analyzed under both deterministic and stochastic stimulations. The deterministic analysis was performed by increasing the feedback gain. In the linear sense, the effect of this increase would be to make the reactor more unstable. In the nonlinear sense, however, the effect is to change the amplitude of the limit cycle. Additional increases in the feedback gain induce limit cycle instabilities leading to period-doubling pitchfork bifurcations and the onset of aperiodic pulsing. This transition to aperiodicity has been found to follow Feigenbaum's scenario, with the remarkable result the BWR dynamic behavior in the nonlinear regime appears to depend on a set of universal constants.

The analysis of the phenomenological model developed in this work has also shown that the void fraction oscillations arising from the BWR's thermal hydraulics provide the mechanism for the onset of the predicted neutron density pulses. The Doppler (fuel temperature) feedback has been shown to play only a minor role.

The nonlinear behavior of a BWR system stimulated by stochastic inputs has also been analyzed in this work. This is, to the author's knowledge, the first analysis of this kind. It has been found that the amplitude of the limit cycle that appears under stochastic excitation of the system is not affected by the strength of the driving stochastic source. The analysis of stochastic descriptors, such as the PSD function, revealed the appearance and enhancement of higher harmonics of the 0.5-Hz resonance that characterizes the BWR dynamic behavior in the linear regime. This finding has led to the development of diagnostic

rules for differentiating between limit cycle oscillations and externally induced increases in the system's noise level. These diagnostic rules have been successfully applied to analyze the problem of abnormal noise increases that have been observed in BWRs under single-loop operating conditions.

To conclude, the work reported in this paper, in addition to its academic value, provides a basis for scoping calculations to determine the viability of limit cycle operation of commercial BWRs.

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