Printed in the United States of America. Available from
National Technical Information Service
U.S. Department of Commerce
5285 Port Royal Road, Springfield, Virginia 22161
NTIS price codes—Printed Copy: A04 Microfiche A01

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Contract No. W-7405-eng-26

MASTER

METALS AND CERAMICS DIVISION

**ANALYSIS OF EFFECT OF TEMPERATURE GRADIENTS ON SURFACE-TENSION
PHENOMENA IN GAS-TUNGSTEN-ARC WELDS**

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Date Published - October 1982

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ABSTRACT

Fluid motion directed by surface tension is considered as a contributor to heat penetration in a weld pool. The potential phenomena at the gas-liquid interface were analyzed, and the dependence of surface motion on temperature in the gas-tungsten-arc (GTA) welding process was examined. An existing heat-transfer model was used and was able to predict weld size to $\pm 50\%$ of the actual value. A momentum-transfer equation was derived by considering the contribution of Lorentz force. The momentum boundary condition was developed and was able to predict the Marangoni effect. The magnitude of surface-tension-driven force is comparable to the gravitational force on one gram. An empirical approach was proposed to couple heat-transfer and momentum-transfer phenomena. A dimensional analysis identified the pertinent dimensionless groups as Reynolds, Weber, Froude, Peclet, and Power numbers and a dimensionless velocity. A simplified form of the correction was developed by combining dimensionless groups to yield a correlation with the Bond, Prandtl, and modified Power numbers. Future experimental work was proposed to test the functionality of the dimensionless groups.

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1. SUMMARY

Surface tension has been suggested as a major means of directing fluid motion in a weld pool. Recently, gas-tungsten-arc (GTA) welding of stainless steel revealed a unique dependence of weld quality (depth/width ratio) on trace quantities of aluminum or sulfur. This variability is currently attributed to the effect of trace compounds on surface tension and its dependence on temperature.

Existing mathematical models for predicting weld quality and weld shape are derived from heat-transfer equations. A common existing model was used, and the assumptions of the model were justified. A calculated arc efficiency was 80%, with the heat loss from the pool surface being less than 2%. The predicted depth-to-width ratio was off by a factor of 2. The major weakness of the model was its inability to predict changes caused by fluid motion. The heat-transfer equation has to be coupled with momentum transfer to characterize the size of a weld pool.

The Lorentz force caused by the interaction between electric current and the induced magnetic field is identified as a major force affecting the fluid motion. The momentum-transfer equation was derived with the effect of Lorentz force, and the pool-surface boundary condition was developed by relating shear stress to surface-tension phenomena. Fluid-flow direction caused by the Marangoni effect can be predicted by this boundary condition. The magnitude of the surface-tension-driven force is about 1000 dynes, which is comparable to the gravitational force on one gram.

An empirical approach was proposed to couple the heat transfer and momentum transfer for the welding process. Important properties were viscosities, densities, surface-tension coefficients, thermal conductivities, and heat capacities. Important welding parameters identified were electrode traveling speed, efficiency, welding current, and voltage. Dimensionless groups such as Reynolds (N_{Re}), Weber (N_{We}), Froude (N_{Fr}), Peclet (N_{Pe}), and Power (N_{Po}) numbers, along with a dimensionless velocity (N_{V*}), were identified by a dimensional analysis. The model was simplified by combining dimensionless groups for future experimental verifications. The final form of the proposed correlation, consisting of the Bond (N_{Bo}), Prandtl (N_{Pr}), and modified Power (N_{Po2}) numbers, was given by:

$$\frac{d}{W} = A \left[1 + \frac{N_{Bo, T_{melt}} - N_{Bo, T_{ref}}}{N_{Bo, T_{ref}}} \right]^B (N_{Pr})^F (N_{Po2})^G$$

Future experiments were outlined to identify the functionality of each dimensionless group. It was also recommended that the effect of trace compounds on the heat distribution in the arc plasma should be examined.

2. INTRODUCTION

2.1 Background

Welding is a complex heat-transfer process where energy (from chemical combustion, lasers, or electric arcs) is used to heat, melt, and fuse together pieces of adjoining metal, which are then allowed to cool and remain bonded. In some cases extra metal is added to the molten weld puddle by melting a rod known as the filler. Two kinds of electrodes are used for electric arcs: consumable and nonconsumable. In the case of consumable electrodes, the electrode is melted as the filler during the welding process. An example of a nonconsumable electrode is a tungsten electrode in the gas-tungsten-arc (GTA) process, shown in Fig. 1. In the GTA process a weld pool, protected by inert gas, is heated by an arc between the tungsten electrode and the work pieces which are to be fused together.

One criterion of the quality of the weld is measured by the extent of melting obtained in the base metal, such that a high weld-depth-to-weld-width ratio occurs. Both heat transfer and fluid motion affect this penetration ability. At present there is no well-defined theory and mathematical model to couple heat transfer and fluid motion in welding processes. Work is proceeding in the Metals and Ceramics Division at ORNL to develop a theory and a mathematical model to characterize the depth-to-width ratio during welding.

2.2 Previous Work

2.2.1 Heat-Transfer Theory Applied to Welding

The general equation for heat transfer comes from an energy balance in the weld pool. Because energy must be conserved, the time rate of internal energy gain must equal the net rate of heat transfer through the surface, plus any sources of energy within the pool such as heat generated by chemical reactions or electric current (23):

$$\begin{array}{l} \text{rate of change of} \\ \text{internal energy} \end{array} = \begin{array}{l} \text{(net rate of heat transfer)} \\ \text{through the faces} \end{array} + \begin{array}{l} \text{(rate of net)} \\ \text{heat generated} \end{array}$$

Mathematically this is described by the Fourier heat-flow equation:

$$\rho C \frac{D\theta}{Dt} = -\nabla \cdot q + \dot{Q}_G \quad (1)$$

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GAS-TUNGSTEN-ARC WELDING PROCESS

DATE 12-3-81	DRAWN BY PSC	FILE NO. CEPS-X-345	FIG. 1
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The terms ρ , C , and k (implicit in q) are the density, specific heat, and thermal conductivity of the material, respectively. θ is the temperature, q is the heat flow, and $\dot{Q}_G$ is the rate of heat generated.

Because heat transfer and temperature distributions during welding are complex, an exact solution is not easily obtained. Several assumptions were made by previous investigators (1):

1. the system is at quasi-steady state;
2. the heat source is concentrated in a point or along a line;
3. there is no Joule heating (I^2R), i.e., $\dot{Q}_G = 0$;
4. no phase changes occur;
5. the pool is isotropic and homogeneous;
6. thermal conductivity, density, and specific heat are constant with temperature;
7. the plane normal to the heat input is infinite; and
8. there are no heat losses by convection or radiation from the pool surface to the atmosphere.

The justification for the above assumptions is that they make it possible to find analytical solutions for the equations.

Since the heat flow q is proportional to the temperature gradient, and the heat generated ($\dot{Q}_G$) in the pool is neglected (assumption 4), Eq. (1) can be further developed into a thermal diffusion equation:

$$\frac{D\theta}{Dt} = \alpha \nabla^2 \theta \quad (2)$$

where

$$\alpha = k/\rho C \quad (3)$$

In the welding process, the electrode is moving with a constant speed in one direction, and a moving coordinate ω is defined with reference to the electrode (see Appendix 9.1). The relationship of this moving coordinate ω to the fixed coordinate x is:

$$\omega = x - vt \quad (4)$$

where v is the constant velocity of the electrode. Therefore, Eq. (2) is transformed to the moving coordinate system:

$$-\frac{v}{\alpha} \frac{\partial \theta}{\partial \omega} = \nabla_m^2 \theta \quad (5)$$

Typical boundary conditions used by previous investigators were:

1. the surface is perfectly insulated (assumption 8);
2. the temperature remains unchanged far from the source;
and
3. if the source is a point source, the heat flux through the surface of the hemisphere drawn around the source must tend to the value of the total heat delivered to the plate.

Mathematical analyses of heat flow in welding were made by Boulton and Lance-Martin (2), Rosenthal and Schmerber (30), Ryalkin (31), and many others. They applied the heat source as either a point source or a line source, along with the above assumptions, to solve the fundamental equation. Investigators, including Thaveri et al. (34), Naka and Masubuchi (24), and Christensen et al. (6), used dimensional expressions to simplify the numerical calculations of heat-flow solutions. From these models, weld depths, widths, or cross-sectional areas were predicted (6, 23) and compared with experimental data. Agreement within a factor of two between theory and experiment was observed for width and depth (6). Due to motion of molten material, heat transfer in the weld puddle cannot be completely described by conduction alone.

2.2.2 Surface-Tension Phenomena in Weld Pools

Recently, GTA welding of stainless steel revealed a unique dependence of weld quality (depth/width ratio) on trace quantities of aluminum or sulfur (12, 14, 29). GTA welds appeared to show radially outward surface fluid flow for weld pools high in aluminum, and radially inward surface fluid flow when sulfur was added to the weld pool. Substantial changes in fusion zone shape were produced by the addition of small amounts of certain elements to a weld pool. For example, the addition of 50 ppm sulfur increased the depth/width ratio of a GTA weld in stainless steel by over 80%, and aluminum additions produced significant decreases in depth/width ratio.

Friedman (8) proposed that fluid flow in the weld pool is the major factor in determining fusion zone shape. Several forces interact to determine the flow pattern in the weld pool, including surface tension gradients, the Lorentz force (38), and thermal convection. When the concentration of surface-active elements such as sulfur or oxygen is high enough in the weld pool to produce a positive surface tension gradient

($dy/d\theta$), then the surface-tension gradient and the Lorentz force combine to produce strong inward-surface fluid flow, providing an efficient heat transfer to the weld root to form a narrow, deep weld pool (Fig. 2a). When surface-active elements such as aluminum oxide are present, the surface-tension gradient will be in the opposite direction and will oppose the Lorentz force, which results in outward-surface fluid flow, providing an efficient heat transfer to the weld edge to form a wide, shallow weld pool (Fig. 2b).

Some analyses of surface-tension phenomena and its effects on welds have been reported (12, 13, 14). Various effects such as humping (27), undercutting (27), and streaming motion (1) were all attributed to surface tension. Surface-tension data on an iron-oxygen-sulfur system have also been collected (13, 14). Measured values of $dy/d\theta$ for pure iron ranges from -0.35 to -0.5 dyne/cm-°C; $dy/d\theta$ for Fe-S ranges from 1.0 to 4.5 dyne/cm-°C with increasing sulfur concentration. This investigation was devoted to the study of the effect of temperature gradient on the surface tension of the weld pool.

2.3 Objectives and Approach

The goal of this project was to analyze the potential phenomena at the gas-liquid interface and to examine the dependence of surface phenomena on temperature for the GTA welding process. The specific objectives were:

1. to assess the surface temperature profile of a GTA weld pool using an existing heat-transfer model;
2. to determine the bead profile as a function of position by performing a force balance; and
3. to evaluate the momentum boundary condition at the top surface for varying surface-tension dependencies on temperature and to quantitatively predict the direction of fluid motion.

The initial approach was to perform a preliminary study of the physics of the fluid motion and to understand the contribution of various forces acting on the weld pool. The surface temperature profile of a weld pool was determined by using an appropriate model, and assumptions were assessed simultaneously. The bead profile as a function of position was studied with a normal force balance on the surface of the pool in which appropriate forces, such as Lorentz force and surface tension, were included. The pool was assumed to be static.

For the third objective, a tangential force balance was made across the surface with appropriate forces. The flow directions were assessed by varying surface-tension dependencies on temperature.

The equation of motion applied on the weld pool was derived in dimensionless form, and an order of magnitude analysis helped to evaluate the sensitivities of these parameters to physical properties and welding conditions.

2a) High-Sulfur-Concentration Welds

2b) High-Aluminum-Concentration Weld.

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APPARENT FLUID FLOW ON WELD-POOL SURFACE
AND PROPOSED SUBSURFACE FLOW PATTERN

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3. MODEL DEVELOPMENT

3.1 Physical Phenomena

A number of features of the arc-welding process cannot be fully explained by conductive heat flow alone. For instance, the shape of the pool and penetration is known to vary with the extensive fluid motion in the liquid pool. Several theories have been proposed to explain the observed fluid motion. The existence of a crater on the weld pool surface has been interpreted as a result of the impingement of high velocity arc plasma (7, 26), which would create flow outwards from the impingement area, causing fluid motion on the weld-pool surface in the same directions. Surface-tension variations alone are known to be capable of causing bulk fluid motion (1, 4). However, the dominance of the surface-tension-driven flow (Marangoni convection) in producing the fluid motion in arc welding is still uncertain (3).

Another cause of fluid motion is the electromagnetic (Lorentz) force, which arises from the interaction of the welding current with its own (or other) magnetic fields. The origin of the forces is shown schematically in Fig. 3. Current enters the pool from the arc or electrode via a small spot on the surface and then rapidly spreads out. The interaction of the current with its own magnetic field in the pool produces forces radially inwards and downwards from the pool surface. Fluid is drawn inwards along the surface and projected into the pool.

Woods and Milner (38) considered both the effects of Lorentz forces and motion induced by arc impingement in a study involving stationary arc-melted pools and current-carrying mercury pools. They concluded that, while both mechanisms could induce fluid motion, the Lorentz force was more important. Lawson and Kerr (19) observed extensive fluid flow in GTA weld pools and a strong dependence of fluid-flow patterns resulting from Lorentz forces on the position and inclination of the electrode and the position of the ground and contact period; this causes fluid to circulate due to magnetic asymmetry in some of the GTA weld pools.

The presence of low concentrations of surface-active impurities can alter the sign of temperature dependence of the surface tension as well as change the magnitude of the surface tension up to 0.5 dyne/cm-°C. The change in magnitude and direction of surface-tension gradients on the surface of the weld pool changes the fluid flow patterns and the fusion-zone geometry. The possible importance of surface-tension-driven flow in weld pools was suggested by Ishizaki (15) and Andersson (1) several years ago, although Andersson concluded that Marangoni convection was probably not as important as the effect of Lorentz force in weld pools. More recently, surface rippling during laser-surface melting has been attributed to surface-tension gradients.

A surface-tension gradient can be imposed on the surface of a fluid for two reasons. First, if there is a temperature gradient along the

—→ Direction of Lorentz force, which can induce fluid motion.
- - → Direction of current flow.

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SCHEMATIC DIAGRAM OF THE
EFFECT OF LORENTZ FORCE

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surface, there will be a surface-tension gradient if the surface tension is temperature-dependent. Second, if there is a composition gradient along the surface, there will be a surface-tension gradient because the surface tension is composition-dependent. However, it is not known if there is a strong composition gradient along the weld-pool surface.

3.2 Mathematical Model

3.2.1 Surface-Temperature Profile

A schematic diagram of the temperature distribution in a plate on which a weld bead is laid at a constant speed v is shown in Fig. 4. Lines 1 to 5 represent isotherms on the surface, while the dotted lines represent isotherms on the traverse section ABCD. The coordinate axis is xyz ; the origin 0 is on the surface underneath the welding arc, the x -axis lies in the direction of welding, and the z -axis is placed in the thickness of the plate downward.

The fundamental expression for heat conduction is given by the Fourier heat-flow equation [Eq. (1)] for a given boundary condition (shape and intensity of the heat source, configuration of the weldment, etc.). Two features of heat flow during arc welding are: 1) the heat source moves at a constant speed near the surface of the work piece, and 2) the size of the heat source (welding arc) is small compared with the size of the work piece.

Most of the mathematical analyses made of heat flow in weldments assumed quasi-steady-state welding with a nonfinite heat source, where the heat-distribution pattern from a local source affects temperature distribution substantially only in the region adjacent to the source. In Fig. 5a, for example, temperature distributions were calculated by assuming that the heat source is a point source at P in a three-dimensional system. In the case of a thin slab, the heat source can be a line source (along line PP' in Fig. 5b), and the temperature distribution becomes a two-dimensional-plane problem. Hemispheres of isotherms are predicted by the point-source models, and cylinders of isotherms are predicted by line-source models. These models were compared with the phenomena described in Sect. 3.1, and the point-source model was shown to be more realistic. The model chosen for heat-transfer analysis had a point heat source in a three-dimensional semi-infinite medium.

After applying the assumptions discussed in Sect. 2.2.1, the heat flow equation was transformed to a moving coordinate system:

$$-\frac{v}{\alpha} \frac{\partial \theta}{\partial \omega} = \nabla_m^2 \theta \quad (5)$$

The boundary conditions were:

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TEMPERATURE DISTRIBUTION IN A PLATE WHEN
 A WELD BEAD IS ON THE SURFACE

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5a. Point Source

5b. Line Source

---- Isotherms

θ Arc temperatures

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TEMPERATURE DISTRIBUTIONS PREDICTED
BY DIFFERENT HEAT SOURCES

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FIG.
5

1. Since the source is a point source, the heat flux through the surface of the hemisphere drawn around the source must tend to the value of the total heat $\dot{Q}$ delivered to the plate because the radius of the sphere tends to zero. Mathematically, if r is the radius of the sphere, then:

$$\lim_{r \rightarrow 0} (-2\pi r^2 k \frac{\partial \theta}{\partial r}) = \dot{Q} \quad (6)$$

where

$$r = \sqrt{x^2 + y^2 + z^2} \quad (7)$$

2. Heat loss through the surface is negligible:

$$\frac{\partial \theta}{\partial z} = 0 \quad \text{for } z = 0 \text{ and } r \neq 0 \quad (8)$$

3. The temperature of the plate remains unchanged at an infinite distance from the source:

$$\frac{\partial \theta}{\partial x} = 0 \quad \text{for } x = \pm \infty \quad (9)$$

$$\frac{\partial \theta}{\partial y} = 0 \quad \text{for } y = \pm \infty \quad (10)$$

$$\frac{\partial \theta}{\partial z} = 0 \quad \text{for } z = \infty \quad (11)$$

The above boundary conditions were applied to Eq. (5), and the solution, as presented by Rosenthal and Schmerber (30), is:

$$\theta - \theta_0 = \frac{\dot{Q}}{2\pi k} e^{(-\frac{v}{2\alpha} x)} \left(\frac{e^{-(v/2\alpha)(r)}}{r} \right) \quad (12)$$

where θ_0 is the initial temperature of the slab. This equation was further simplified by Christensen et al. (6) into the following dimensionless expression:

$$\textcircled{H} = \frac{\exp - (R + \xi)}{R} \quad (13)$$

where

$$\Theta = \frac{\theta - \theta_0}{\dot{Q}v/4\pi k u} \quad (14)$$

$$\xi = \frac{\omega}{2\alpha/v} \quad (15)$$

$$\eta = \frac{y}{2\alpha/v} \quad (16)$$

$$\zeta = \frac{z}{2\alpha/v} \quad (17)$$

$$R = \frac{r}{2\alpha/v} = (\xi^2 + \eta^2 + \zeta^2)^{1/2} \quad (18)$$

3.2.2 Basic Equation of Hydromagnetics

Consider the weld pool as fluid which is electrically conductive, and suppose that magnetic fields are prevalent. This leads to two effects: first, the motion of the electrically conducting fluid across the magnetic lines of force altered by the electric currents; and second, the fluid, which carries currents, traverses the magnetic fields, contributing to additional forces acting on the fluid element (5).

The basic equations which express the interactions between the fluid motions of the magnetic fields are contained in Maxwell's equations and in the equations of hydrodynamics which are suitably modified.

With the displacement currents ignored, Maxwell's equations are:

$$\nabla \cdot \underline{B} = 0 \quad (19)$$

$$\nabla \cdot \underline{J} = 0 \quad (20)$$

$$\nabla \times \underline{H} = \underline{J} \quad (21)$$

$$\nabla \times \underline{E} = - \frac{\partial \underline{B}}{\partial t} \quad (22)$$

For a moving fluid, Ohm's law takes the following form:

$$\underline{J} = \sigma (\underline{E} + \underline{u} \times \underline{B}) \quad (23)$$

The equations incorporate the effect of fluid motion on the electromagnetic field through the occurrence of the velocity $\underline{u}$ in Eq. (23).

The force experienced by the fluid element in a magnetic field $\underline{B}$ is the Lorentz force $\underline{L}$:

$$\underline{L} = \underline{J} \times \underline{B} \quad (24)$$

Including the Lorentz force among the other forces acting on the fluid, the equation of motion for laminar flow:

$$\rho \frac{D\underline{u}}{Dt} = \mu \nabla^2 \underline{u} + \rho \underline{g} + \underline{J} \times \underline{B} - \nabla P \quad (25)$$

and for turbulent flow:

$$\rho \frac{D\underline{u}}{Dt} = \nabla(\mu_{eff} \nabla \underline{u}) + \rho \underline{g} + \underline{J} \times \underline{B} - \nabla P \quad (26)$$

Equations (25) and (26) together with the Maxwell equations (19) through (22) and Ohm's law (23) and their appropriate boundary conditions represent a complete statement of the problem to be solved.

The governing equations are quite complex; however, appreciable simplification is possible through the identification of the dominant terms under certain conditions. Equation (23) is commonly rewritten in terms of the magnetic flux intensity. Substituting Eqs. (21) and (22) into Eq. (23), with some manipulation, we have:

$$\frac{\partial \underline{B}}{\partial t} = \nabla \times (\underline{u} \times \underline{B}) + \eta_m \nabla^2 \underline{B} \quad (27)$$

which is called the induction equation. The quantity

$$\eta_m = \frac{1}{\sigma_e \mu_m} \quad (28)$$

is called the magnetic diffusivity.

If Eq. (27) is made dimensionless, the following dimensionless group appears in the dimensionless form (see Appendix 9.2):

$$N_{Re,m} = U_o L_o / \eta_m \quad (29)$$

Here U_0 and L_0 are the characteristic velocity and characteristic length of the system, respectively. $N_{Re,m}$ is called the magnetic Reynolds number and is the ratio of magnetic convection to magnetic diffusion. When the magnetic Reynolds number is small, the induction Eq. (27) may be simplified to:

$$\frac{\partial \mathbf{B}}{\partial t} = \eta_m \nabla^2 \mathbf{B} \quad (30)$$

Under such conditions the magnetic transport equation may be uncoupled from the equation of motion, which renders the computational task much easier.

4. RESULTS AND DISCUSSION

4.1 Surface Temperature Profile

The development of a heat-transfer model for a point heat source in a three-dimensional semi-infinite medium was presented in Sect. 3.2.1. The model was developed by Rosenthal and Schermer (30) and was simplified into a dimensionless expression by Christensen et al. (6):

$$\Theta = \frac{\exp - (R + \xi)}{R} \quad (13)$$

The model was used in this study with a given set of GTA welding parameters (14): welding current, 150 amps; voltage, 9.6 V; electrode traveling speed, 0.212 cm/s (5 in./min); argon gas velocity, 80 m/s (22); and an arc length (the electrode-to-pool distance) of 0.152 cm. Several assumptions for this model need to be justified before the results are presented.

First, the arc efficiency was examined. The energy available during welding is often characterized by the product of the arc current and voltage. Multiplication of this number by some efficiency factor yields the magnitude of the thermal energy transmitted to the pool. Measurements of efficiency are difficult; however, some generalizations can be made (23, 10). A consumable-electrode process would be expected to have a higher efficiency than a nonconsumable-electrode process because more of the nonconsumable electrode is heated to a higher temperature, with consequently greater radiation losses. Different shielding gases could also affect the location at which the electrical energy is converted to internal energy and thus affect efficiency. Christensen et al. (6) measured the heat leaving the bottom and sides of a weld plate by recording the temperature rise of a rapid stream of water used to cool the plate. They reported arc efficiencies of 21 to 48%.

Friedman and Glickstein (9) and others (22, 25) measured temperatures at various points on the metal workpiece by Chromel-Alumel thermocouples and reported arc efficiencies of 30 to 85%.

In our analysis the arc efficiency was affected by the radiative heat loss from the plasma. The calculation assumed radiative heat loss through a cylindrically-shaped arc surface, shown in Fig. 6. The emissivity ϵ of the plasma is 0.012, and the highest plasma temperature reported is 14,200 K (22). The calculated heat loss (see Appendix 9.3) accounted for nearly 20% of the total energy from the electrode, giving an arc efficiency of 80%. This arc efficiency was within the range of efficiencies (30 - 85%) from experiments (22, 25) and was used in all our calculations.

An order-of-magnitude analysis of radiative and convective heat loss at the surface of the weld pool is presented in Appendices 9.4 and 9.5, and the total heat loss from the surface was found to be less than 2%. This result is consistent with calculations by previous investigators (6, 17, 28). Thus the assumption of negligible heat loss at the surface is justified.

The last assumption to be verified pertained to the latent heat of fusion effect. Rosenthal's or Christensen's model assumed that no phase change occurred, thus latent heat effect was ignored. Friedman and Glickstein (9, 11) performed heat-flow calculations with and without latent-heat effect and concluded that the latent-heat effect is important in a weld only when full penetration is approached or met. In our model the plate is semi-infinite in all directions, thus the latent-heat effect was neglected.

The pool dimensions were calculated by applying Eq. (13). The region of interest for temperature, which is between (H) at the melting point for iron (1520°C) and (H) at the boiling point (2740°C) is plotted as a function of the dimensionless y-coordinate η in Fig. 7. The temperature distribution in the ξ - η plane and in the ζ - η plane is shown in Figs. 8a and 8b respectively. (ξ is the dimensionless moving coordinate, and ζ is the dimensionless coordinate of depth.). The isotherms are ellipsoids (shown in Appendix 9.6), and as temperature increases towards the center of the pool, the isotherms become circular at (H) larger than unity (as seen in Fig. 8).

The radius of the weld pool was calculated with the given welding parameters. The radius corresponding to the melting temperature ($(H) = 2.88$) was 0.19 cm. The calculated weld width was 0.38 cm, compared with the average experimentally measured value of 0.65 cm (32); thus the calculated value was off by a factor of 2. The predicted depth-to-width ratio was 0.5 vs the average experimental result of 0.36.

While there is good agreement (within $\pm 30\%$) between experimental and calculated values for width, depth, and cross-sectional area of the molten zone, there are several shortcomings which lead to poor quantitative

8a. ξ - η Plane 8b. ζ - η Plane

Scale: 1 unit = 0.75 cm

$$\xi = \frac{u}{2\alpha/v}$$

$$\zeta = \frac{z}{2\alpha/v}$$

$$\eta = \frac{y}{2\alpha/v}$$

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FIG.

8

predictions. First, the weld pool is calculated to be hemispherical, which does not agree with experimental data (6, 32). The model only predicts a semi-circular pool in the vertical cross section, and the shape does not change with heat input or electrode velocity. However, the shape of an actual weld pool is semi-ellipsoidal, and it changes with heat input or electrode velocity (6, 23). Second, as the radius approaches zero, the model predicts the temperature to increase to infinity, which is physically impossible and is caused by assuming an infinitely small point-heat source. Since the weld bead which acts as the heat source in the semi-infinite medium is of finite size, the equation is valid only at a distance removed from the point source. And finally, because of fluid motion in the molten pool, heat transfer in the weld pool cannot be completely described by conduction alone. The heat-transfer model must be coupled with the equation of motion to characterize the size and shape of the weld pool. Because of the complexity of the system, there is no better choice from the existing models. Investigators are currently modifying the existing model by applying a Gaussian distribution of heat source (8, 9) in place of a point source or line source, but so far no solution has been reported.

Since the model cannot predict the maximum temperature of the weld pool, the maximum temperature has to be determined by the physical properties of the material and by measured values. Measurements with thermocouples were made by Christensen *et al.* (6) over a wide range of temperature. They were dipped into the pool, and the signals from the couples were recorded by a 24-channel loop oscillograph. The surface temperatures immediately beneath the arc could not be measured even approximately because of violent fluctuations arising from a rapid sequence of direct exposures of the hot junction. In addition to the fluctuations caused by the intermittent exposure to the arc, a rapid deterioration of the hot junction, followed by an instantaneous re-establishment of the thermocouple circuit through the liquid metal, gave rise to similar fluctuations of the signal. Less-violent fluctuations were encountered at some distance below the surface, where temperatures in the range of 1600 to 2000°C were observed. Since the temperature of the molten pool should not exceed its boiling point (2740°C), the maximum temperature of 2740°C was chosen to define the domain of interest for our calculations.

The temperature profile given in Eq. (13), which is limited to 40% of the radius (between the melting temperature and boiling temperature), was used to characterize the bead profile and momentum boundary condition. The results are discussed in the following sections.

4.2 Normal Force Balance at the Gas-Liquid Interface

Variations of the bead profiles with the fluid flow in the weld pool are difficult to understand because the bead profiles are determined not only by the fluid motion but also by the solidification process of the

weld pool. However, if steady-state behavior is assumed where the perspective (coordinate system) of an observer atop the electrode is adopted, the equation of motion can be rewritten as:

$$\rho \underline{u} \cdot \nabla \underline{u} = \mu \nabla^2 \underline{u} + \rho \underline{g} + \underline{J} \times \underline{B} - \nabla P \quad (31)$$

If a flat semi-infinite base metal with the surface perpendicular to the Z-axis of a cylindrical coordinate system is assumed, if the electrode coinciding with the Z-axis with positive-Z points downwards, as shown in Fig. 9, and if the molten-phase region is considered, the evolution of the gas-liquid interface can be described by performing the normal force balance at the interface (20):

$$P_{nr}^{(1)} + P_Y = P_{nn}^{(2)} \quad (32)$$

where

$$P_Y = \gamma \left[\frac{\partial^2 h / \partial r^2}{[1 + (\partial h / \partial r)^2]^{3/2}} \right] \quad (33)$$

h = evolution function of gas-liquid interface

If it is assumed that curvature of the gas-liquid interface is very small, $|h| \ll 1$, then Eq. (33) may be linearized to:

$$P_Y = \gamma \frac{\partial^2 h}{\partial r^2} \quad (34)$$

If it is assumed that static pressure at $Z = 0$ is P_0 , then $P^{(1)} = P_0 + \rho gh + P_a$, where P_a is the arc-plasma pressure along the interface as a function of position.

If it is assumed that ground contact is symmetrical with respect to the Z-axis, the current enters the welding pool and immediately spreads out radially from the region. There is no Lorentz force in the θ -direction, so the Lorentz force can be rearranged as:

$$\underline{L} = (\nabla \times \underline{H}) \times \underline{B} \quad (35)$$

or:

$$\underline{L} = \mu_m (\nabla \times \underline{H}) \times \underline{H} \quad (36)$$

Electrode is moving out of plane of paper.

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NORMAL FORCE BALANCE AT
GAS-LIQUID INTERFACE

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since $\underline{B} = \mu_m \underline{H}$. Then:

$$L_r = \mu_m H_z \left(\frac{\partial H_r}{\partial z} - \frac{\partial H_z}{\partial r} \right) - \mu_m H_\theta \left(\frac{1}{r} \frac{\partial(rH_\theta)}{\partial r} - \frac{1}{r} \frac{\partial H_r}{\partial \theta} \right) \quad (37)$$

and

$$L_z = \mu_m H_\theta \left(\frac{1}{r} \frac{\partial H_z}{\partial \theta} - \frac{\partial H_\theta}{\partial z} \right) - \mu_m H_r \left(\frac{\partial H_r}{\partial z} - \frac{\partial H_z}{\partial r} \right) \quad (38)$$

With $u_\theta = 0$, the equation of motion can be written for the r-component:

$$\begin{aligned} \rho \left(u_r \frac{\partial u_r}{\partial r} + u_z \frac{\partial u_r}{\partial z} \right) = & - \frac{\partial P}{\partial r} + \mu \left[\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial}{\partial r} (r u_r) \right) + \frac{\partial^2 u_r}{\partial z^2} \right] \\ & + \mu_m \left[H_z \left(\frac{\partial H_r}{\partial z} - \frac{\partial H_z}{\partial r} \right) - H_\theta \left(\frac{1}{r} \frac{\partial(rH_\theta)}{\partial r} - \frac{1}{r} \frac{\partial H_r}{\partial \theta} \right) \right] \end{aligned} \quad (39)$$

and for the Z-component

$$\begin{aligned} \rho \left(u_r \frac{\partial u_z}{\partial r} + u_z \frac{\partial u_z}{\partial z} \right) = & - \frac{\partial P}{\partial z} + \mu \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u_z}{\partial r} \right) + \frac{\partial^2 u_z}{\partial z^2} \right] \\ & + \mu_m \left[H_\theta \left(\frac{1}{r} \frac{\partial H_z}{\partial \theta} - \frac{\partial H_\theta}{\partial z} \right) - H_r \left(\frac{\partial H_r}{\partial z} - \frac{\partial H_z}{\partial r} \right) \right] + \rho g \end{aligned} \quad (40)$$

It is necessary to solve the pressure function for the liquid phase from the equation of motion as well as from the Maxwell equations with some appropriate boundary conditions. Therefore, the curvature height function of the gas-liquid interface, $h(r)$, can be integrated from the normal force balance at the interface. This calculation requires a complex numerical effort. For simplicity, the calculation of the evolution function with the assumption of zero velocity u in the weld pool (the bead profile at static) was tried.

Since H_θ is the only non-zero component of $\underline{H}$ on the gas-liquid surface, from Eq. (21) we have:

$$\frac{1}{r} \frac{\partial(rH_\theta)}{\partial r} = J_z \quad (41)$$

such that:

$$H_0 = \frac{r}{2} (J_Z) + \frac{\text{const}}{r} \quad (42)$$

From the condition of $H_0 = 0$ at $J_Z = 0$ and $J_Z = I/\pi r^2$, we obtain:

$$H_0 = \frac{I}{2\pi r} \quad (43)$$

Therefore, from the equation of motion [Eqs. (39) and (40)], we have:

$$\frac{\partial p_{nn}^{(2)}}{\partial r} = -\mu_m H_0 \left(\frac{1}{r} \frac{\partial (r H_0)}{\partial r} \right) = 0 \quad (44)$$

and:

$$\frac{\partial p_{nn}^{(2)}}{\partial z} = \rho g \quad (45)$$

The liquid-phase pressure $p_{nn}^{(2)}$ along the gas-liquid interface is not a function of r and is found from solving Eq. (45) to be $P_0 + \rho g h$. Substituting the solution to Eq. (45), the solution to Eq. (34), and the definition for $p_{nn}^{(1)}$ into the normal force balance [Eq. (32)], one finds:

$$-\gamma \frac{\partial^2 h}{\partial r^2} = P_a \quad (46)$$

If both the surface tension and arc plasma pressure are constant, the static bead profile can be integrated (with $\partial h / \partial r = 0$ when $r = 0$) to be:

$$h(r) = -\frac{P_a}{2\gamma} r^2 + \text{constant} \quad (47)$$

Equation (47) indicates that the gas-liquid interface is concave-up. If an incompressible fluid is considered, the liquid volume should be conserved such that the gas-liquid interface must have a concave-down region to conserve the liquid volume. Otherwise the fluid will overflow out of the pool. It becomes obvious that the static assumption is not adequate to estimate the bead profile. It is necessary to solve the equation of motion coupled with the heat transfer equation and the Maxwell equations with the appropriate boundary conditions to obtain a more realistic bead profile.

4.3 Tangential Force Balance Along the Gas-Liquid Interface

Surface tension gradients alone are known to be capable of causing bulk fluid motion. Heiple and Roper (14) observed a dramatic change on depth-to-width ratio of the welding pool by adding trace amounts of impurities to the welds. They proposed that the fluid flow patterns and the fusion zone configuration may depend on the change and the direction of the surface tension gradient along the surface of the weld pool caused by the additives. Consequently we modeled the effect of the surface tension gradients on the surface flow and made an estimation of the order of magnitude of these gradients to justify this argument.

An interface between fluid phases may influence the motion of the liquid in two instances: when the interface has finite curvature and when there is a surface-tension gradient along the interface. In both cases, forces appear on the interface that generally affect the liquid motion. The boundary conditions for the interface between two phases are as follows (20):

1. The velocity components normal to the interface of two immiscible phases equal zero:

$$v_n^{(1)} = v_n^{(2)} = 0 \quad (48)$$

2. The normal force components per unit area of the interface must be continuous:

$$p_{nn}^{(1)} + p_n = p_{nn}^{(2)} \quad (49)$$

3. The tangential force components per unit area must be continuous:

$$p_{tt}^{(1)} + p_t = p_{tt}^{(2)} \quad (50)$$

The normal force balance at the interface (condition 2) was discussed in the previous section. From this boundary condition we should be able to find the evolution function of the interface and relate the evolution function to the bead profile. To justify the direction of rotation caused by surface tension gradients in a confined volume, we examined the boundary condition of the tangential force balance at the free surface.

If it is assumed that the interface is a free surface ($p_{tt}^{(1)} = 0$) the local shear stress in the weld pool at the surface equals the p_{tt} gradient in surface tension, which may be written as (20):

$$-\mu \frac{\partial u_r}{\partial z} = \left(\frac{\partial Y}{\partial \theta}\right) \left(\frac{\partial \theta}{\partial r}\right) \text{ at } z = h \quad (51)$$

From Eq. (51), the circulation patterns are qualitatively predicted and are summarized in Table 1.

Table 1. Estimation of Circulation Patterns

Values of		Movement of Fluid on Pool Surface
$\frac{\partial Y}{\partial \theta}$	$\frac{\partial \theta}{\partial r}$	
+	+	away from center
+	-	toward center
-	+	toward center
-	-	away from center

In the weld pool, $(\partial \theta / \partial r)$ is always negative. However, the surface tension gradient $(\partial Y / \partial \theta)$ may have a positive or negative sign, based on the additives in the weld pool. From Table 1 we can generalize that the surface-tension-driven flow will move toward the weld pool center if $(\partial Y / \partial \theta)$ is positive, and outward if $(\partial Y / \partial \theta)$ is negative.

Heat penetration in the weld pool may be affected by the flow patterns. Inward rotation tends to enhance heat penetration and causes deeper pool formation because the temperature gradient of the pool in the radial direction is negative. Outward rotation tends to inhibit heat penetration and causes shallow pool formation. This explanation agrees with the results observed by Heiple and Roper (14).

The temperature gradient in a welding pool is approximately 10^4 °C/cm. Measured values of $\partial Y / \partial \theta$ for pure iron are in the range of -0.23 to -0.50 dyne/cm-°C (14). The magnitude of the surface tension gradient can be as large as 5000 dyne/cm². If a weld pool surface is 5 mm in diameter and has a constant surface-tension gradient, the overall surface-tension force is about 1010 dynes. This is the same order of magnitude as the gravitational force acting on one gram of mass; thus the surface-tension-gradient effect may be significant in the fluid flow of a welding pool.

4.4 Dimensional Analysis

The development of the heat-transfer equation, equation of motion, and the equation of magnetic flux were presented in the previous section

and are summarized, respectively, as

$$\frac{V}{\alpha} \frac{\partial \theta}{\partial \omega} = \nabla_m^2 \theta \quad (5)$$

$$\rho \frac{Du}{Dt} = \nabla(\mu \nabla u) + \rho g + J \times B - \nabla P \quad (25)$$

$$\frac{\partial B}{\partial t} = \eta_m \nabla_m^2 B \quad (30)$$

A rigorous solution to the above coupled equations is difficult because of the complexity of the system and is beyond the scope of this project.

An empirical method is proposed to correlate experimentally the depth-to-width ratio of a weld pool with operating variables. The selection of parameters in the correlation is obtained by dimensionless analysis which combines parameters into meaningful dimensionless groups. The depth of the pool was found to be a function of viscosity μ , density ρ , surface tension γ , fluid velocity u , width of the pool W , gravity g , electrode traveling speed v , thermal properties k and C , and the energy input by the electrode I and V . This functionality is shown by:

$$d = f(\mu, \rho, \gamma, u, W, g, v, k, C, I, V) \quad (52)$$

A power relationship was assumed:

$$d = \beta(\mu)^a(\rho)^b(\gamma)^c(u)^d(W)^e(g)^f(v)^g(k)^h(C)^i(I)^j(V)^k \quad (53)$$

and the dimensional units (in terms of mass, length, time, temperature, and coulombs of each of the parameters were substituted in Eq. (53), which yielded:

$$L = \beta \left(\frac{M}{LT} \right)^a \left(\frac{M}{L^3} \right)^b \left(\frac{M}{T^2} \right)^c \left(\frac{L}{T} \right)^d (L)^e \left(\frac{L}{T^2} \right)^f \left(\frac{L}{T} \right)^g \left(\frac{ML}{T^3\theta} \right)^h \left(\frac{L^2}{T^2\theta} \right)^i \left(\frac{C}{T} \right)^j \left(\frac{ML^2}{T^2C} \right)^k \quad (54)$$

Simplifying this by Buckingham- π analysis (see Appendix 9.7) gives:

$$\frac{d}{W} = \beta \left(\frac{\rho u W}{\mu} \right)^{-a} \left(\frac{\rho u^2 W}{\gamma} \right)^{-b} \left(\frac{u^2}{Wg} \right)^{-c} \left(\frac{u}{V} \right)^{-f} \left(\frac{uW}{K/\rho C} \right)^{-g} \left(\frac{IV}{\rho u^3 W^2} \right)^j \quad (55)$$

If all the dimensionless groups are related:

$$N_{Re} = \frac{\rho u W}{\mu} \quad \text{Reynolds number} \quad (56)$$

$$N_{We} = \frac{\rho u^2 W}{\gamma} \quad \text{Weber number} \quad (57)$$

$$N_{Fr} = \frac{u^2}{Wg} \quad \text{Froude number} \quad (58)$$

$$N_{V*} = \frac{u}{v} \quad \text{Dimensionless velocity} \quad (59)$$

$$N_{Pe} = N_{Re} N_{Pe} = \frac{u W}{k/\rho C} = \left(\frac{\rho u W}{\mu} \right) \left(\frac{u/\rho}{\rho C} \right) \quad \text{Peclet number} \quad (60)$$

$$N_{Po} = \frac{n_a I V}{\rho u^3 W^2} \quad \text{Power number} \quad (61)$$

then Eq. (55) becomes:

$$\frac{d}{W} = B(N_{Re})^A (N_{We})^B (N_{Fr})^C (N_{V*})^E (N_{Pe})^F (N_{Po})^G \quad (55')$$

Physically, the depth-to-width ratio is a function of the following: inertia-to-viscous dissipation forces, inertia-to-surface-tension forces, inertia-to-gravitational forces, fluid velocity to electrode-traveling-speed ratio, inertial forces to thermal diffusivity, and energy-input-to-inertial-energy ratio. Experiments can be conducted by varying parameters of different dimensionless groups, and the powers can be deduced by regression.

5. DISCUSSION OF FUTURE WORK

A generalized correlation [Eq. (55')] for weld-depth-to-width ratio was derived in Sect. 4.4. Experiments are needed to correlate the parameters in each dimensionless group to the depth-to-width ratio. This can be done by varying the material, additives, and welding conditions. Difficulties in analysis will be encountered since most of the dimensionless groups have a fluid-velocity term u . Not only is the fluid velocity difficult to control, it is not easy to measure. The equation was simplified so that experiments could be made without a measured velocity. The fluid-velocity term can be eliminated by combining dimensionless groups, assuming the following relationship of the power terms:

$$A = -F \quad (62)$$

$$C = -B + G \quad (63)$$

$$E = G \quad (64)$$

Thus the equation becomes:

$$\frac{d}{W} = \lambda \left(\frac{N_{We}}{N_{Fr}} \right)^B \left(\frac{N_{Pe}}{N_{Re}} \right)^F (N_{Re} N_V N_{Po})^G \quad (65)$$

since

$$N_{Bo} = \frac{N_{We}}{N_{Fr}} = \frac{\rho g W^2}{\gamma} \quad \text{Bond no. } \left(\frac{\text{gravity}}{\text{surface tension}} \right) \quad (66)$$

$$N_{Pr} = \frac{N_{Pe}}{N_{Re}} = \frac{\mu}{kC} \quad \text{Prandtl no. } \left(\frac{\text{viscosity}}{\text{thermal diffusivity}} \right) \quad (67)$$

and $(N_{Fr} N_V N_{Po})$ can be defined as a modified Power number N_{Po2} :

$$(N_{Fr} N_V N_{Po}) = N_{Po2} = \frac{n_a I V}{\rho g W v}$$

Equation (55') becomes:

$$\frac{d}{W} = \lambda (N_{Bo})^B (N_{Pr})^F (N_{Po2})^G \quad (68)$$

The flow direction induced by surface tension gradient on the pool surface is correlated with the Bond term in Eq. (68), which is separated into two components:

$$(N_{Bo})_{ref} = \frac{\rho g W^2}{\gamma_{ref}} \quad (69)$$

$$(N_{Bo})_{melt} = \frac{\rho g W^2}{\gamma_{melt}} \quad (70)$$

γ_{ref} and γ_{melt} are the surface tensions at a reference point on the pool

surface (where the temperature is maximum) and at the melting point; these are related by a first order Taylor series:

$$\gamma_{\text{melt}} = \gamma_{\text{ref}} + r \left(\frac{\partial \gamma}{\partial \theta} \right) \left(\frac{\partial \theta}{\partial y} \right) \quad (71)$$

The N_{Bo} term in Eq. (68) is replaced by $(1 + \frac{N_{Bo, T_{\text{melt}}} - N_{Bo, T_{\text{ref}}}}{N_{Bo, T_{\text{ref}}}})$.

When $N_{Bo, T_{\text{melt}}}$ is greater than $N_{Bo, T_{\text{ref}}}$, the term is larger than unity; physically, when γ_{melt} is smaller than γ_{ref} , fluid will tend to flow inward. In both cases (mathematically and physically), the depth-to-width ratio increases, and the reverse is also consistent. Thus the correlation becomes:

$$\frac{d}{W} = A \left[1 + \frac{N_{Bo, T_{\text{melt}}} - N_{Bo, T_{\text{ref}}}}{N_{Bo, T_{\text{ref}}}} \right]^B (N_{Pr})^F (N_{Po_2})^G \quad (72)$$

or

$$\frac{d}{W} = A \left[1 + \frac{\gamma_{\text{ref}} - \gamma_{\text{melt}}}{\gamma} \right]^B \left(\frac{\mu}{C} \right)^F \left(\frac{n_a IV}{\rho g W \gamma} \right)^G \quad (73)$$

Experiments can be performed by identifying the functionality of each dimensionless group and varying one dimensionless group at a time.

First the Bond number N_{Bo} will be examined. The important parameters in this group are ρ , W , and γ , since g is constant. If the metal workpiece is the same, the density should be unchanged. The actual width of the pool cannot be controlled but is defined experimentally as the hydraulic width W of the pool, which is the maximum transverse area divided by the depth. The sensitivity of this hydraulic width can be estimated by changing welding conditions such as current, voltage, and electrode speed, and a characteristic hydraulic width can be used for each experiment. The remaining parameter in the Bond number is surface-tension coefficient γ . By applying various additives, such as trace amounts of sulfur, aluminum, or selenium, the surface-tension dependencies on temperature ($d\gamma/d\theta$) will change. Thus the value of γ_{melt} will change significantly. By fixing the welding conditions, by using the same welding material, and by varying additives, the Bond number power term (N_{Bo}) can be determined.

To determine the dependency of the Prandtl number (N_{Pr}), the welding metal must be varied. Stainless steel, aluminum, or tin can be used because each material has different transport, thermal, and physical properties (μ , k , C , ρ). No additives are necessary. Note that as density varies, both

the Bond number and Prandtl number change; therefore it is important to determine the dependency of Bond number before that of the Prandtl number.

The dependency of the modified Power number is determined by varying the current, voltage, or the electrode traveling speed. The current and electrode traveling speed are varied easily by setting the desired parameters. Voltage is a function of arc length (distance between electrode to pool) and should be measured as the potential difference between the electrode and the pool (9). The only parameter that is hard to control is the arc efficiency η_a . Arc efficiency varies not only with the power input but also with the size of the jet and the distance between the electrode and the workpiece. This value can be determined experimentally (9) by measuring the temperature at various points on the metal by thermocouple or pyrometer and performing an energy balance. The arc efficiency can also be assessed approximately by calculating the radiative heat loss from the plasma, except that the value of emissivity ϵ is still uncertain (22). Experimental data were obtained for depth-to-width ratio with varying welding current and electrode velocity, but holding the ratio IV/v constant, as noted in Eq. (73). The data were not correlated, however, because the arc efficiency was unknown.

Finally, after additives are applied to a weld process, these additives may evaporate and change the energy distribution of the plasma onto the weld pool. This could vary the heat transfer and heat penetration. Work needs to be undertaken to study this effect on the heat-transfer phenomena in the welding process.

6. CONCLUSIONS

1. A three-dimensional heat conduction model with a point source was used to predict the temperature profile of the weld pool by assuming that the base metal was semi-infinite.
2. An arc efficiency was calculated to be 80%; heat loss through the weld pool surface by convection and radiation was less than 2%.
3. This model predicts unrealistic infinite temperatures as the radius of the isotherm goes to zero. This model is valid for the region ($>40\%$ of the radius) with the temperature below the boiling temperature.
4. Prediction of the depth-to-width ratio of the weld pool is always 0.5, which is about 20 to 100% higher than the experimental data.
5. The direction of fluid flow in the weld pool can be determined by the sign of the surface-tension gradient if surface-tension-driven flow is the dominant effect.
6. An empirical formula for depth-to-width ratio was proposed by performing a dimensional analysis:

$$\frac{d}{W} = A \left[1 + \frac{N_{Bo, T_m} - N_{Bo, T_{ref}}}{N_{Bo, T_{ref}}} \right]^B (N_{Pr})^F (N_{Po2})^G$$

A thorough understanding of arc efficiency η_a is required in the modified Power number N_{Po2} .

7. Fundamental equations for the fluid motion in an electromagnetic field were formulated in vector forms. Lorentz force is orthogonal to the direction of current and the magnetic field.

8. The assumption of a static weld pool is invalid for predicting the bead profile due to nonconservation of mass as formulated.

7. RECOMMENDATIONS

1. Effects of additives on the energy distribution of the arc plasma should be examined because the additives may vaporize in the arc plasma and change the energy distribution, thereby changing the configuration of the weld pool.

2. Experiments are identified to correlate the depth-to-width ratio with the proposed dimensionless groups, e.g., the relative Bond number, the Prandtl number, and the modified Power number. The power efficiency of the welding arc must be measured experimentally before variations in welding power variables can be studied.

8. ACKNOWLEDGMENTS

We thank S.A. David for providing good understanding of the problem. We also express our gratitude to S.E. Hanzelka and V.T. Houchin for their assistance. Special thanks go to T.A. Welton for sharing his insight.

9. APPENDIX

9.1 Transformation of Thermal-Diffusion Equation from Fixed-Coordinate System to Moving Coordinate System

From Eq. (2):

$$\frac{D\theta}{Dt} = \alpha \nabla^2 \theta \quad (2)$$

or

$$\frac{\partial \theta}{\partial t} + v_x \frac{\partial \theta}{\partial x} + v_y \frac{\partial \theta}{\partial y} + v_z \frac{\partial \theta}{\partial z} = \alpha \left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} \right) \quad (2)$$

x , y , and z are fixed coordinates with reference to the metal workpiece. The heat source is specified by one of the boundary conditions. This boundary condition is moving at the same speed that the electrode travels. Therefore, for mathematical convenience, the coordinate is transformed to a moving coordinate system (ω , y , z) with reference to the electrode. The relationship of the moving coordinate ω to the fixed coordinate x is:

$$\omega = x - vt \quad (4)$$

where v is the constant velocity of the electrode.

In the fixed coordinate system, Eq. (2) can be simplified to:

$$\frac{\partial \theta}{\partial t} = \alpha \left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} \right) \quad (74)$$

since there is no motion in the weld pool relative to the fixed coordinate system. From Eq. (4),

$$\frac{\partial \omega}{\partial t} = -v \quad (75)$$

and

$$\frac{\partial \omega}{\partial x} = 1 \quad (76)$$

since x and t are independent variables. By the chain rule:

$$\frac{\partial \theta}{\partial t} = \frac{\partial \theta}{\partial \omega} \frac{\partial \omega}{\partial t} = -v \frac{\partial \theta}{\partial \omega} \quad (77)$$

and

$$\frac{\partial^2 \theta}{\partial x^2} = \frac{\partial^2 \theta}{\partial \omega^2} \left(\frac{\partial \omega}{\partial x} \right)^2 = \frac{\partial^2 \theta}{\partial \omega^2} \quad (78)$$

Thus, Eq. (75) can be transformed into the moving coordinate system as:

$$- \frac{v}{\alpha} \frac{\partial \theta}{\partial \omega} = \frac{\partial^2 \theta}{\partial \omega^2} + \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} \quad (79)$$

Another way of doing this coordinate transformation is to perform an energy balance around a differential volume in the moving-coordinate system. The material is then moving at the velocity $-v$ relative to the moving coordinate system.

$$\begin{aligned} \Delta \omega \Delta y \Delta z \frac{\partial}{\partial t}(\rho C \theta) &= \Delta y \Delta z [(-v)\rho C \theta|_{\omega} - (-v)\rho C \theta|_{\omega+\Delta \omega}] \\ &+ \Delta y \Delta z (q_{\omega}|_{\omega} - q_{\omega}|_{\omega+\Delta \omega}) + \Delta \omega \Delta z (q_y|_y - q_y|_{y+\Delta y}) \\ &+ \Delta \omega \Delta y (q_z|_z - q_z|_{z+\Delta z}) \end{aligned} \quad (80)$$

Taking the limit of this expression as $\Delta \omega$, Δy , and Δz approach zero, we obtain:

$$\frac{\partial}{\partial t}(\rho C \theta) = v \frac{\partial}{\partial \omega}(\rho C \theta) - \left(\frac{\partial q_{\omega}}{\partial \omega} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z} \right) \quad (81)$$

Since $q_i = -k(\partial \theta / \partial x_i)$, where x_i represents the coordinate system, then at steady-state Eq. (81) becomes:

$$- \frac{v}{\alpha} \frac{\partial \theta}{\partial \omega} = \frac{\partial^2 \theta}{\partial \omega^2} + \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} \quad (82)$$

where ρ and C are constant and $\alpha = k/\rho C$.

9.2 Magnetic Reynolds Number

The following dimensionless variables are defined:

$$B^* = B/B_0 \quad (83)$$

$$u^* = u/u_0 \quad (84)$$

$$t^* = T/T_0, \quad t_0 = L_0/u_0 \quad (85)(86)$$

$$\nabla^* = \nabla / L_0 \quad (87)$$

where B_0 is the characteristic intensity of the magnetic field, u_0 is the characteristic velocity of the weld pool, and L_0 is the characteristic length. With the above dimensionless variables, Eq. (27) can be rewritten in a dimensionless form as Eq. (27'):

$$\frac{\partial B^*}{\partial t^*} = \nabla^* \times (\underline{u}^* \times \underline{B}^*) + (\eta_m / L_0 u_0) \nabla^{*2} B^* \quad (27')$$

The dimensionless group $(L_0 u_0 / \eta_m)$ is defined as the magnetic Reynolds number:

$$N_{Re,m} = L_0 u_0 / \eta_m = \frac{\text{magnetic convection}}{\text{magnetic diffusion}} \quad (88)$$

If $\eta_m = 1/\sigma_e \mu_m$, then,

$$\sigma_e = 1.4 \times 10^6 \text{ (ohm-m)}^{-1}$$

$$\mu_m = 1.26 \times 10^{-6} \text{ Henry/m}$$

and if

$$u_0 \sim 0.1 \text{ m/s}$$

$$L_0 \sim 0.005 \text{ m,}$$

then $N_{Re,m} \sim 0.0009$.

9.3 Radiative Heat Loss from Plasma

If the following is given:

$$\epsilon_{\text{plasma}} = 0.012 \text{ (22)}$$

$$\theta_{\text{plasma}} = 14,200 \text{ (22)}$$

$$\theta_{\text{amb}} = 300 \text{ K}$$

the radiative heat lost $\dot{Q}_R$ through the side of a cylinder of plasma between the electrode and the surface of the workpiece (Fig. 6) is:

$$\begin{aligned} \dot{Q}_R &= \epsilon \sigma A (\theta_{\text{plasma}}^4 - \theta_{\text{amb}}^4) \text{ cal/s} & (89) \\ &= (0.012)(1.35 \times 10^{-12} \text{ cal/s-cm}^2\text{-K}^4)(2\pi \times 0.1 \times 0.152 \text{ cm}^2) \\ &\quad \times (14,200^4 - 300^4 \text{ cal/s}) \\ &= 62.9 \text{ cal/s} \end{aligned}$$

Total heat available from the electrode $\dot{Q}_{\text{heat}}$:

$$\dot{Q}_{\text{heat}} = (0.24)(VI) = (0.24)(9.6)(150) = 345.6 \text{ cal/s}$$

$$\therefore \% \text{ heat loss from plasma} = \frac{62.9}{345.6}(100) = 18.2 \%$$

Therefore heat loss from plasma (assuming radiation to be the main factor) is roughly 20%. An arc efficiency of 80% can be used.

9.4 Heat Loss by Forced Convection from the Pool Surface

From Jacob (16), the forced-convection heat transfer coefficient can be calculated with:

$$h_f = \frac{k}{r} (0.0356) \left(\frac{vr}{\alpha} \right)^{0.8} \bigg|_{\theta_{\text{film}}} \quad (90)$$

where

k = thermal conductivity of the gas

$$\alpha = \frac{k}{\rho C} = \text{thermal diffusivity of the gas}$$

$$v_g = \text{velocity of shield gas}$$

$$r = \text{radius of the weld pool}$$

$$\theta_{\text{film}} = \text{film temperature}$$

$$\theta_{\text{film}} = \frac{\theta_{\text{pool}} + \theta_{\text{gas}}}{2} \quad (91)$$

θ_{pool} is the temperature of the melt pool, and θ_{gas} is the temperature of the shield gas.

If it is assumed that the argon gas temperature is higher than the ambient temperature but less than the average pool temperature (between 27 and 2000°C), and that the gas has the same thermal properties as air, then the gas properties are (18):

θ_{pool} (°C)	θ_{gas} (°C)	θ_{film} (°C)	ρ (lb _m /ft ³)	C (Btu/lb _m -°F)	k (Btu/hr-°F)	α (ft ² /hr)
2000	27	1014	0.0173	0.283	0.0451	9.21
2000	1800	1900	0.0100	0.300	0.056	18.7

If the gas velocity is:

$$v_g = 80 \text{ m/s} = 8000 \text{ cm/s}$$

then, considering two cases:

$$1) \text{ at } \theta_{\text{gas}} = 27^\circ\text{C} \text{ or } \theta_{\text{film}} = 1014^\circ\text{C}$$

$$\begin{aligned} h_f &= \frac{(0.0451 \times 4.13 \times 10^{-3})}{r} (0.0356) (8000 \frac{r}{9.21 \times 0.258})^{0.8} \\ &= \frac{4.40 \times 10^{-3}}{r^{0.2}} \text{ cal/s-K-cm}^2 \end{aligned}$$

$$2) \text{ at } \theta_{\text{gas}} = 1800^\circ\text{C} \text{ or } \theta_{\text{film}} = 1900^\circ\text{C}$$

$$\begin{aligned}
 h_f &= \frac{(0.056 \times 4.13 \times 10^{-3})}{r} (0.0356)(8000 \frac{r}{18.7 \times 0.258})^{0.8} \\
 &= \frac{3.10 \times 10^{-3}}{r} \text{ cal/s-K-cm}^2
 \end{aligned}$$

The highest forced-convection flux will be at the lower argon gas temperature; at this extreme case:

$$\theta_{\text{pool}} = 2000^\circ\text{C} = 2273 \text{ K}$$

$$\theta_{\text{gas}} = 27^\circ\text{C} = 300 \text{ K}$$

$$\begin{aligned}
 \dot{Q}_f &= h_f A (\theta_s - \theta_{\text{gas}}) \quad (92) \\
 &= \left(\frac{4.40 \times 10^{-3}}{r^{0.2}} \right) (\pi r^2) (2273 - 300) = 27.3 r^{1.8} \text{ cal/s}
 \end{aligned}$$

From experimental results (32), r for this condition was 0.33 cm. Therefore,

$$\dot{Q}_f = 27.3(0.33)^{1.8} = 3.71 \text{ cal/s}$$

Compared with the heat input $\dot{Q}$ on the workpiece:

$$\dot{Q} = \eta \dot{Q}_{\text{heat}} = (0.8)(345.6) = 276.5 \text{ cal/s}$$

The fraction of heat loss is:

$$\frac{3.71}{276.5}(100\%) = 1.34\%$$

Since the above calculation is for an extreme case, the shield gas temperature is expected to be much higher than 27°C (ambient temperature); the fraction of total heat lost by forced convection should be less than 1.34%.

9.5 Heat Loss by Radiation at the Surface

Radiative heat loss is given by:

$$\dot{Q}_R = \epsilon \sigma A(\theta_s^4 - \theta_{\text{gas}}^4) \quad (89)$$

where

σ = Stefan-Boltzmann constant = 1.35×10^{-12} cal/s-cm²-K

θ_s = temperature at the surface, K

θ_{gas} = temperature of the shield gas above the surface, K

A = area of the surface, cm²

ϵ = emittance

If the maximum average temperature (2300 K) is used as reference, the minimum temperature on the metal workpiece where radiative loss occurs [Eq. (89)] is calculated as 10% of the maximum temperature (raised to the fourth power):

$$\frac{\theta_{\min}^4}{\theta_{\max}^4} = 10\%$$

$$\theta_{\min} = (0.1)^{0.25} (\theta_{\max}) = 1293 \text{ K}$$

Substituting into Eq. (13) with the given conditions, one obtains:

$$\textcircled{H} = \frac{\theta_{\min} - \theta_0}{qv/2\pi k\alpha} = 1.92$$

From Fig. 11, the isotherm is circular. From the temperature profile:

$$\textcircled{H} = \frac{\exp - (R + \xi)}{R} \quad (13)$$

At the surface, under the electrode, $\xi = 0$ and $\zeta = 0$. Thus, $\eta = R$ by Eq. (18). Solving Eq. (13), one obtains $R = 0.363$. Therefore,

$$r = \frac{2\alpha}{v} R = 0.257 \text{ cm}$$

For stainless steel or iron, emissivities of 0.07 - 0.17 are obtained. At the maximum extreme case:

$$\epsilon = 0.17$$

$$\theta_s = \theta_{\max} = 2300 \text{ K}$$

$$\theta_{\text{gas}} = \theta_{\text{amb}} = 300 \text{ K}$$

If the above values are substituted into Eq. (89):

$$\begin{aligned} \dot{Q}_R &= (0.17)(1.35 \times 10^{-12} \text{ cal/s-cm}^2\text{-K}^4) \pi (0.257 \text{ cm})^2 (2300^4 - 300^4 \text{ K}^4) \\ &= 1.33 \text{ cal/s} \end{aligned}$$

The percentage of maximum heat loss is:

$$\frac{\dot{Q}_F}{n\dot{Q}_{\text{heat}}}(100\%) = \frac{1.33}{(0.8)(345.6)}(100\%) = 0.48\%$$

Since this is the maximum extreme case, the heat loss by radiation should be less than 0.48%.

9.6 Calculation of Temperature Profile in a GTA Weld Pool

Physical properties (7, 35, 36, 37) for iron and mild steel (in molten form) are:

$$\text{melting point} = 1520^\circ\text{C}$$

$$\text{boiling point} = 2740^\circ\text{C}$$

$$k \text{ (thermal conductivity)} = 0.12 \text{ cal/cm-s-}^\circ\text{C}$$

$$\alpha \text{ (thermal diffusivity)} = 0.075 \text{ cm}^2/\text{s}$$

k and α are not sensitive to temperature variations within the domain of interest. The Rosenthal equation in a moving coordinate is:

$$\theta - \theta_0 = \frac{\dot{Q}}{2\pi k} \frac{1}{r} e^{-(v/2\alpha)(r+\omega)} \quad (12)$$

From the dimensionless expression:

$$\xi = \frac{v}{2\alpha} \omega \quad (15)$$

$$\eta = \frac{v}{2\alpha} y \quad (16)$$

$$\zeta = \frac{v}{2\alpha} z \quad (17)$$

$$R = \frac{v}{2\alpha} r = (\xi^2 + \eta^2 + \zeta^2)^{\frac{1}{2}} \quad (18)$$

and $(H) = \frac{\theta - \theta_0}{\dot{Q}v/4\pi\alpha k} \quad (14)$

Christensen derived the dimensionless equation:

$$(H) = \frac{1}{R} e^{-(R + \xi)} \quad (13)$$

From the welding parameters:

$$I = 150 \text{ amp}$$

$$V = 9.6 \text{ volt}$$

$$v = 0.212 \text{ cm/s}$$

Values of η and ξ were used to calculate (H) at the surface ($\zeta = 0$). The results for the dimensionless temperature distribution were plotted in Fig. 10a and 10b. The isotherms on the top surface are shown in Fig. 11.

9.7 Development of Empirical Correlation by Buckingham- π Analysis

From Eq. (54):

$$L = B \left(\frac{M}{LT}\right)^a \left(\frac{M}{L^3}\right)^b \left(\frac{M}{T^2}\right)^c \left(\frac{L}{T}\right)^d (L)^e \left(\frac{L}{T^2}\right)^f \left(\frac{1}{T}\right)^g \left(\frac{ML}{T^3\theta}\right)^h \left(\frac{L^2}{T^2\theta}\right)^i \left(\frac{C}{T}\right)^j \left(\frac{ML^2}{T^2C}\right)^k \quad (54)$$

10a. Vertical Sections Parallel to the ξ axis

$$(H) = \frac{\theta - \theta_0}{Qv/4\pi\alpha k}$$

$$\xi = \frac{z}{2\alpha/v}$$

$$n = \frac{y}{2\alpha/v}$$

$$\xi = \frac{z}{2\alpha/v} = 0$$

10b. Vertical Section Parallel to the η axis

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DISTRIBUTION FOR POINT SOURCES

DATE 12-13-81	DRAWN BY AL	FILE NO. CEPS-X-345	FIG. 10
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$$H = \frac{\theta - \theta_0}{Qv/4\pi k}$$

$$\eta = \frac{y}{2\sqrt{x/v}}$$

$$\xi = \frac{y}{2\sqrt{x/v}} = 0$$

$$\xi = \frac{z}{2\sqrt{x/v}} = 0$$

Curves are isotherms in the η -plane at $\xi = 0$

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Equating exponents of like terms gives:

$$L = 1 = -a - 3b + d + e + f + g + h + 2i + 2k \quad (93)$$

$$M = 0 = a + b + c + h + k \quad (94)$$

$$T = 0 = -a - 2c - d - 2f - g - 3h - 2i - j - 2k \quad (95)$$

$$\theta = 0 = -h - i \quad (96)$$

$$C = 0 = j - k \quad (97)$$

Equations (96) and (97) give:

$$i = -h \quad (98)$$

$$k = j \quad (99)$$

Substituting Eqs. (98) and (99) into Eqs. (93) through (97), one obtains:

$$1 = -a - 3b + d + e + f + g - h + 2j \quad (100)$$

$$0 = a + b + c + h + j \quad (101)$$

$$0 = -a - 2c - d - 2f - g - h - 3j \quad (102)$$

Since there were nine unknowns but only three equations, some assumptions were made to solve the system. Since we expect μ , γ , g , V , K , and I in Eq. (53) to appear in separate numbers, we will derive the equations without eliminating their exponents a , c , f , g , h , and j . Then from Eq. (101):

$$b = -a - c - h - j \quad (103)$$

and from Eq. (102):

$$d = -a - 2c - 2f - g - h - 3j \quad (104)$$

Substituting the above results into Eq. (100) gives:

$$e = 1 + f - a - c - h - 2j \quad (105)$$

Applying the appropriate substitutions into Eq. (53) yields:

$$d = \beta(\mu)^a(\rho)^{-a-c-h-j}(\gamma)^c(u)^{-a-2c-2f-g-h-3j}(W)^{1+f-a-c-h-2j} \\ \times (g)^f(v)^g\left(\frac{K}{C}\right)^h(IV)^j \quad (106)$$

Grouping terms with like exponents gives:

$$\frac{d}{W} = \beta\left(\frac{\mu}{\rho u W}\right)^a\left(\frac{\gamma}{\rho u^2 W}\right)^c\left(\frac{Wg}{u^2}\right)^f\left(\frac{v}{u}\right)^g\left(\frac{K/\rho C}{uW}\right)^h\left(\frac{IV}{\rho u^3 W^2}\right)^j \quad (107)$$

or

$$\frac{d}{W} = \beta\left(\frac{\rho u W}{\mu}\right)^{-a}\left(\frac{\rho u^2 W}{\gamma}\right)^{-c}\left(\frac{u^2}{Wg}\right)^{-f}\left(\frac{u}{v}\right)^{-g}\left(\frac{uW}{K/\rho C}\right)^{-h}\left(\frac{IV}{\rho u^3 W^2}\right)^j \quad (55)$$

9.8 Experiment on GTA Welds

An experiment was performed as an example for future reference. Stainless steel, shielded by argon gas, was heated by a tungsten electrode. Voltage was set to 9.6 V at the digital controller, and current was varied for each test. The welding speed was adjusted so that the total heat input per unit bead length (IV/v) was constant (assuming that the arc efficiency was constant). Pictures of the final products were taken, and the maximum depth and width of each weld were measured. The pictures are shown in Fig. 12, and the conditions and results are summarized in Table 2.

Since the arc efficiency was unknown because it varied with current and voltage, the voltage was fixed at the controller. However, it might not be the actual potential drop across the arc. [Smartt and Key (32) performed experiments and showed that there is a minimum threshold voltage corresponding to various currents.] Measured values of the potential drop across the arc are necessary. Since in this experiment, the ratio of IV/v is constant, from Eq. (73), no variation in depth to width is expected if η is constant. However, from Table 2, the depth-to-width ratio does vary; the only uncontrolled parameter is the unknown arc efficiency η , which is dependent on current and voltage. This dependency needs to be known before the data in Table 2 can be explored further.

a

b

c

d

e

- a) 100 amp
- b) 150 amp
- c) 200 amp
- d) 250 amp
- e) 300 amp

Scale: 1 mm

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EXPERIMENTAL RESULTS OF GTA WELD

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Table 2. Experimental Results of GTA Weld

<u>Weld Number</u>	<u>η_a</u>	<u>Current (amp)</u>	<u>Set Point Voltage (V)</u>	<u>Welding Speed (mm/s)</u>	<u>Heat Input (J/mm)</u>	<u>Depth/Width</u>
1	*	100	9.6	1.41	681	0.24
2	*	150	9.6	2.12	679	0.23
3	*	200	9.6	2.83	678	0.21
4	*	250	9.6	3.53	680	0.39
5	*	300	9.6	4.24	679	0.37

¹ Not known.

9.9 Nomenclature

<u>B</u>	magnetic flux density, Weber/m ²
<u>C</u>	heat capacity, cal/g-K
<u>d</u>	weld depth, cm
<u>E</u>	electric field intensity, V/m
<u>g</u>	gravity, cm/s ²
<u>h</u>	evolution function of the gas-liquid interface
<u>H</u>	magnetic field intensity, amp/m
<u>I</u>	current, amp
<u>J</u>	conduction current density, amp/m ²
<u>k</u>	thermal conductivity, cal/s-cm-K
<u>L</u>	Lorentz force, dyne
<u>L₀</u>	characteristic length, cm
<u>N_{Re}</u>	$\frac{\rho u W}{\mu}$, Reynolds number
<u>N_{Re,m}</u>	$\frac{V_0 L_0}{\eta_m}$, magnetic Reynolds number

$$N_{We} \quad \frac{\rho u^2 W}{\gamma}, \text{ Weber number}$$

$$N_{Fr} \quad \frac{u^2}{Wg}, \text{ Froude number}$$

$$N_{V*} \quad \frac{u}{V}, \text{ dimensionless velocity}$$

$$N_{Pe} \quad \frac{uW}{k/\rho C}, \text{ Peclet number}$$

$$N_{Po} \quad \frac{\eta_a IV}{\rho u^3 W}, \text{ Power number}$$

$$N_{Bo} \quad \frac{\rho g W^2}{\gamma}, \text{ Bond number}$$

$$N_{Pr} \quad \frac{\mu}{kC}, \text{ Prandtl number}$$

$$N_{Po2} \quad \frac{\eta_a IV}{\rho g W v}, \text{ modified Power number}$$

$$P \quad \text{pressure, } N/m^2$$

$$P_a \quad \text{arc pressure, } N/m^2$$

$$P_{nn} \quad \text{normal force per unit area, } N/m^2$$

$$P_n \quad \text{normal force by surface-tension gradients, } N/m^2$$

$$P_o \quad \text{static pressure, } N/m^2$$

$$P_t \quad \text{shear stress by surface-tension gradient, } N/m^2$$

$$P_{tt} \quad \text{shear stress per unit area, } N/m^2$$

$$P_\gamma \quad \text{capillary pressure, } N/m^2$$

$$q \quad \text{heat flux, } \text{cal/cm}^2\text{-s}$$

$$\dot{Q}_f \quad \text{heat loss by forced convection, cal/s}$$

$$\dot{Q}_G \quad \text{heat generation, cal/s}$$

$\dot{Q}_{\text{heat}}$	heat supplied by electricity, cal/s
$\dot{Q}_R$	heat supplied by radiation, cal/s
r	r-coordinate in cylindrical coordinate system
R	dimensionless radius
t	time, s
u	fluid velocity, cm/s
U_0	characteristic velocity, cm/s
v	electrode velocity, cm/s
V	voltage, volts
W	width of weld pool
x	x-coordinate in the fixed Cartesian coordinate system
y	y-coordinate in the Cartesian coordinate system
z	z-coordinate in the Cartesian coordinate system

Greek Symbols

α	thermal diffusivity, cm^2/s
β	dimensionless constant
γ	surface tension coefficient, dyne/cm
ϵ	emissivity
λ	dimensionless constant
Λ	dimensionless constant
ζ	dimensionless z
η	dimensionless y
η_a	thermal efficiency of the electrode
η_m	magnetic diffusivity, cm^2/s
θ	temperature, K; or θ -coordinate in cylindrical coordinates
θ_{amb}	ambient temperature, K

θ_{film}	average of ambient plus surface temperature, K
θ_{gas}	shield-gas temperature, K
θ_{max}	maximum temperature, K
θ_{min}	minimum temperature, K
θ_0	initial temperature, K
θ_{plasma}	plasma temperature, K
θ_{pool}	molten-weld-pool temperature, K
θ_s	surface temperature, K
(H)	dimensionless temperature
μ	viscosity, g/cm-s
μ_{eff}	effective viscosity, g/cm-s
μ_m	magnetic permeability, Henry/m
ν	kinematic viscosity, cm ² /s
ξ	dimensionless ω -coordinate
ρ	fluid density, g/cm ³
σ	Stefan-Boltzmann constant, cal/s-cm ² -K ⁴
σ_e	electrical conductivity, ohm/m
ω	ω -coordinate in the moving Cartesian coordinate system
∇	gradient operator in fixed coordinate
∇_m	gradient operator in moving coordinate

Superscripts

- (1) gas phase above weld pool
- (2) weld-pool liquid phase

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