28
5-31-78

DL-114

LA-7235-MS

Informal Report

UC-66a

Issued: April 1978

MASTER

**Thermal Stress Cracking and the
Enhancement of Heat Extraction from
Fractured Geothermal Reservoirs**

Hugh D. Murphy

**los alamos
scientific laboratory**

of the University of California

LOS ALAMOS, NEW MEXICO 87545

An Affirmative Action/Equal Opportunity Employer

UNITED STATES
DEPARTMENT OF ENERGY
CONTRACT W-7405-ENG. 36

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

Printed in the United States of America. Available from
National Technical Information Service
U.S. Department of Commerce
5285 Port Royal Road
Springfield, VA 22161

Microfilm \$ 3.00

001-025	4.00	126-150	7.25	251-275	10.75	376-400	13.00	501-525	15.25
026-050	4.50	151-175	8.00	276-300	11.00	401-425	13.25	526-550	15.50
051-075	5.25	176-200	9.00	301-325	11.75	426-450	14.00	551-575	16.25
076-100	6.00	201-225	9.25	326-350	12.00	451-475	14.50	576-600	16.50
101-125	6.50	226-250	9.50	351-375	12.50	476-500	15.00	601-up	--1

1. Add \$2.50 for each additional 100-page increment from 601 pages up.

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Department of Energy, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights.

THERMAL STRESS CRACKING AND THE ENHANCEMENT OF HEAT EXTRACTION FROM FRACTURED GEOTHERMAL RESERVOIRS

by

Hugh D. Murphy

ABSTRACT

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Department of Energy, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

Given sufficient time, the extraction of heat from geothermal reservoirs formed by the hydraulic fracturing of competent rock will eventually result in the formation of thermal stress cracks in the reservoir. These cracks penetrate the rock in a manner such that the penetration-to-spacing ratio is approximately one. The penetration depends upon the extent of cooling and the square root of time. Initially then, the cracks are closely spaced and penetrate but little, so that a crazing pattern is apparent; but with increasing time some of these cracks, now more widely spaced, grow deeper. Eventually these larger cracks attain a critical aperture such that significant rates of water flow can be established within them and thus the newly created heat transfer area becomes useful for heat extraction.

At the same time that cracks are forming within the main reservoir, thermal cracking also occurs in the wellbores that communicate with the reservoir. These cracks eventually convey water to and from the reservoir, thus leading to a decrease in the flow impedances that are often concentrated in the wellbore regions.

I. INTRODUCTION

The effects of thermal stress cracking upon heat extraction were first considered by Harlow and Pracht.¹ Their treatment was a numerical one which necessarily lacked the detail required to actually resolve individual cracks. Thus they considered cracking to result in additional porosity which they

distributed homogeneously. This additional porosity, ϕ , was simply related to the volumetric coefficient of thermal contraction of the rock and the cooling of the rock. The new permeability, k , of the thermal stress crack-altered rock was calculated from the Kozeny formula²

$$k = \frac{4D_c^2}{5} \left[\frac{\phi^3}{(1 - \phi)^2} \right] . \quad (1)$$

Thus the permeability of thermally fractured rock was a very sensitive, even critical, function of D_c , the distance between cracks. Harlow and Pracht were not able to calculate D_c a priori, but instead simply assumed various values, and in so doing found that if D_c was 0.01 m or less the power enhancement due to thermal cracking became negligible. Thus the viability of thermal crack enhancement was completely dependent upon the unknown D_c .

To remedy this situation, very fundamental approximate analyses were performed to elucidate the mechanics of thermal stress cracking. Where required, simplifying assumptions were made to make the problem tractable. Most glaring, perhaps, was the omission of a detailed consideration of fracture mechanics. Nevertheless, the results provide a useful guide and furthermore, even after allowing for the uncertainties introduced with these assumptions, they indicate that thermal stress augmentation of heat extraction is a viable process, and in fact, if earth stresses are low enough, can hardly be avoided.

In the first of the sections to follow we consider the thermal cracks that develop in the reservoir itself. The geometry is such that a one-dimensional, linear (slab-like) analysis is appropriate. In the second section we discuss the thermal stresses around the wellbore, for which a cylindrical set of coordinates is more appropriate.

II. THERMAL CRACKING AND HEAT EXTRACTION ENHANCEMENT IN THE MAIN RESERVOIR

It is assumed that the reservoir was formed by the hydraulic fracturing of previously unfractured, and impermeable, hot dry rock. For simplicity we assume a single vertically oriented hydraulic fracture. However, the following results are also appropriate for multiple fractures, so long as these are spaced far enough apart (tens of meters) so that there is no thermal interaction. The hydraulic fracture or fractures extend horizontally in the direction of S_2 , the

maximum horizontal compressive earth stress. Thus initially heat flows conductively from the surrounding rock in the direction orthogonal to S_2 , that is, coaxial with S_3 , the minimum horizontal compressive earth stress. This heat is conveyed to the surface by a flow of cold water introduced through an injection well. The cold water is gradually heated in its transit through the hydraulic fracture and its heat is delivered to the surface for power generation via a producing well. The thermal gradients set up in the rock by this heat extraction process result in thermal stresses in the S_2 direction which eventually produce thermal cracks that propagate in the S_3 direction.

A. Crack Propagation

The plane strain solution for the S_2 direction thermal stress induced in a homogeneous linearly elastic rock contiguous to a large hydraulic fracture is (Ref. 3, p. 401)

$$\sigma = E\alpha T/(1 - \mu) \quad , \quad (2)$$

where

σ = tensile stress

E = Elastic modulus

α = linear thermal expansion coefficient

T = the cooling of the rock

μ = Poisson's ratio.

If it is assumed that the stress concentration at the root of a crack is very large, or that the tensile strength of the rock is negligible, then cracking will occur when the induced tension just equals S_2 . For numerical examples we will use data appropriate for the site of the first hot dry rock geothermal reservoir located at Fenton Hill, New Mexico. From flow experiments there it has been estimated⁴ that S_2 is 386 bars (5600 psi). Estimates of E , α , and μ for granite are,⁵ respectively, 5×10^5 bars (7×10^6 psi), 8×10^{-6} per °C, and 0.22. Thus, according to Eq. (2), cracking is expected when the rock is cooled by 75°C. When the rock surface next to the main hydraulic fracture is cooled in excess of 75°C, the crack penetrates and stays open to the point where the rock temperature is 75°C lower than its original temperature.

An estimate⁶ of the rock cooling is

$$T = T_s \operatorname{erfc} \left(\frac{z}{\sqrt{4\kappa t}} \right) \quad , \quad (3)$$

where

T = the rock cooling at any point

T_s = the rock cooling at the surface ($z = 0$)

z = distance into the rock, measured parallel to S_3 , from the fracture surface

κ = thermal diffusivity of the rock

t = time

erfc = complimentary error function.

Thus, at the critical degree of cooling, T_c ($= 75^\circ\text{C}$), the crack penetration distance, z_c , becomes

$$z_c = \sqrt{4\kappa t} \operatorname{erfc}^{-1}(T_c/T_s) \quad . \quad (4)$$

Maximum crack penetration occurs at the maximum T_s , which is estimated to be 135°C for a heat extraction experiment at Fenton Hill. No cracking at all occurs when $T_s \leq T_c = 75^\circ\text{C}$. Figure 1 illustrates thermal stress crack penetrations for various values of T_s and time. A value of $1.0 \times 10^{-6} \text{ m}^2/\text{s}$ was used for κ . Note that for the first half year of operation, most cracks are no greater than two or three meters in penetration.

Having estimated the penetration of these cracks, we must next estimate the spacing of the cracks. This is a typical end effects problem, for which it is known that, as a first approximation, crack spacings and penetrations are roughly equal. This claim is supported by both calculations⁷ and observations of thermal stress cracking experiments performed on concrete slabs.⁸ Note that this suggests that at early time, the cracks in a reservoir will be closely spaced and of very shallow penetration, but as time goes on, fewer but deeper penetrating cracks will form. This process relieves the stress field around the shallow cracks. The process is illustrated in Fig. 2.

If the aperture of the small stress-relieved cracks is ignored, then the maximum aperture, $\delta_{\max}$, of the largest cracks at anytime is

$$\delta_{\max} = \alpha T_s z_c \quad . \quad (5)$$

Furthermore, if the aperture is approximated as (this expression ensures smooth closure of the crack),

Fig. 1.
Penetration of thermal cracks.

Fig. 2.
Idealized crack patterns.

$$\delta = \delta_{\max} (1 - z/z_c)^2, \quad (6)$$

then the mean aperture, $\bar{\delta}$, is

$$\bar{\delta} = \frac{1}{z_c} \int_0^{z_c} \delta dz = \delta_{\max}/3, \quad (7)$$

and if Eqs. (5) and (4) are substituted in Eq. (7), the result is

$$\bar{\delta} = \frac{2}{3} \alpha T_s \sqrt{\kappa t} \operatorname{erfc}^{-1} (T_c/T_s). \quad (8)$$

Representative values for $\bar{\delta}$ are plotted in Fig. 3. During the first half year of operation, most cracks are less than a millimeter in aperture.

B. Enhanced Heat Extraction

Continued study of the effects of thermal cracking depends upon whether the hydraulic fracture is propped open (either natural, "self" propping as is the case at Fenton Hill,⁴ or artificial propping) or held open in an inflated state by pressurization.

1. Pressurized Fractures. When the hydraulic fracture is held open by pressurization, then its aperture width, w , is also growing due to thermal contraction. Thus

$$w = w_0 + 2\alpha \int_0^{\infty} T \, dz \quad . \quad (9)$$

Substituting Eq. (3) and integrating gives the result

$$w = w_0 + 4\alpha T_s \sqrt{kt/\pi} \quad . \quad (10)$$

The value w_0 is the initial value of the pressurized aperture, obtainable from the theory of elasticity. If it is assumed for simplicity that $w_0 = 0$, the ratio of Eqs. (8) and (10) gives

$$\frac{\bar{\delta}}{w} = \frac{\sqrt{\pi}}{6} \operatorname{erfc}^{-1} (T_c/T_s) \quad . \quad (11)$$

Fig. 3.
Growth of crack apertures.

Since the maximum value of T_s for the Fenton Hill reservoir is expected to be 135°C, we obtain

$$\frac{\bar{\delta}}{w} \leq 0.12 \quad . \quad (12)$$

When viscous effects dominate the flow, the flow rate capacity (flow rate per unit penetration) of any crack is proportional to the cube of its aperture. Thus the ratio of the flow rate

capacity in the thermal cracks to that in the main fracture will be $(\bar{\delta}/w)^3 \leq 0.002$. It is obvious then that, when viscous forces dominate the fluid dynamics in a pressurized hydraulic fracture, the flow rate and thus the augmented heat transfer to be expected from thermal stress cracking is insignificant.

Fortunately this situation changes considerably when natural convection effects due to the heating of the water are considered. It has been shown⁶ that the natural convection term becomes equal, and greater, in magnitude, than the viscous term in the linear momentum equation when the ratio of Grashof and Reynolds numbers is one or greater, or,

$$\frac{\bar{\delta}^3 R g \beta T_s}{\nu \dot{Q}} \geq 1 \quad , \quad (13)$$

where

R = the characteristic radius of the fracture

g = gravitational acceleration

β = volumetric thermal expansion coefficient of water

ν = kinematic viscosity of water

$\dot{Q}$ = volumetric flow rate.

Ordinarily R is taken as the fracture radius, but when the inlet and outlet are close together, as is the case at Fenton Hill, R is more accurately taken as the distance between the inlet and outlet, approximately 100 m. The anticipated flow rate is about $0.008 \text{ m}^3/\text{s}$ (130 gpm) and the water properties β and ν are approximately 2×10^{-4} per $^\circ\text{C}$ and $1.6 \times 10^{-7} \text{ m}^2/\text{s}$, respectively. Solving Eq. (13) for $\bar{\delta}$ results in a critical value $\bar{\delta}_c \geq 0.4 \text{ mm}$ for natural convection to become important insofar as establishing significant flow rates in thermal stress cracks.

From Fig. 3 it is seen that this "critical" aperture would be attained after 25 days of operation for the very small region near the inlet, where T_s is maximum. Unfortunately, there isn't enough space near the inlet for very many critical size cracks to develop; so significant heat transfer enhancement will only occur far from the inlet, where T_s will be less than the maximum. Thus waiting times for significant thermal stress heat enhancement in a pressurized fracture will be several months to a year (Fig. 3).

An interesting observation is that, once thermal cracks begin to reach the critical size, their growth might be speeded up by doing something to improve the crack flow rate capacity ratio resulting from the inequality [Eq. (12)]. One can't decrease w , of course, but if a low permeability proppant could be introduced into the main fracture only, the resistance to flow in the main fracture could be greatly increased, forcing more of the flow into the thermal stress cracks.

2. Self-Propped Fractures. Based upon measurements of roughness asperities on the surface of freshly fractured core specimens, it is estimated that the average aperture of the self-propped, unpressurized, hydraulic fracture at Fenton Hill is 0.2 mm. The flow rate capacity in the thermal cracks would thus be of the same order of magnitude as that of the main fracture when $\bar{\delta}_C = 0.2$ mm. In this case the waiting time for significant thermal crack effects would be two weeks to two months.

C. Estimates of the Steady-State Crack Spacing, D_C

When significant amounts of water finally do start to flow in the larger thermal cracks, the conductive heat transfer will become two-dimensional. This two-dimensional heat transfer results in even greater thermal contractions, leading to greater yet flow rates, so that the rate of propagation of the water-flowing cracks is accelerated. The previously described pattern, whereby only the more widely spaced cracks continued propagating, now becomes inoperative. Heat extraction from these cracks eventually leads to the formation of new orthogonal thermal stress cracks, now parallel to the original hydraulic fracture. These in turn will eventually lead to a third set of fractures, followed by other sets, so that the system "bootstraps" in time. Since only those cracks with $\bar{\delta} > \bar{\delta}_C$ transmit water, thus propagating and forming new cracks, it is clear that these are the cracks that control the effective permeability of the rock, and that these lead to a self-limiting, steady-state value of the crack spacing. Substituting Eq. (7) in Eq. (5) results in $D_C = 3\bar{\delta}_C/\alpha T_S$ as an estimate of the minimum crack spacing to be used for permeability estimates. Values of D_C are tabulated in Table I.

As can be seen, D_C is of the order of 1 m, two orders of magnitude greater than the value that Harlow and Pracht found necessary for significant thermal stress augmentation of heat extraction. Calculating ϕ from $\bar{\delta}_C$ and using the Kozeny formula presented earlier, it can be shown that the thermally fractured rock will have an effective permeability of the order of one darcy (10^{-12} m²).

TABLE I
ESTIMATES OF MINIMUM CRACK SPACING
(m)

T_s (°C)	Self-Proposed Fractures ($\bar{\delta}_c = 0.2$ mm)	Inflated Fractures ($\bar{\delta}_c \approx 0.4$ mm)
95	0.8	1.6
115	0.7	1.3
135	0.6	1.1

This is an extremely large permeability (typically oil and gas reservoirs have values hundreds and thousands times lower) so it is expected that thermally fractured reservoirs will have low flow impedance.

It is important to note that the thermal drawdown of an actual reservoir will initially follow the patterns established in Refs. 1 and 6. However, as T_s becomes sufficiently greater than T_c , and thermal cracks admit significant flow rates, a state of equilibrium will be reached in which any further decline in reservoir temperature will be offset by the development of new heat transfer areas by thermal cracking. While there may be momentary increases and decreases in the fluid production temperature, it is thus expected that during the thermal cracking phase the production temperature will approach a constant value.

III. THERMAL STRESS CRACKING AT THE WELLBORES

The equations for the stresses in the rock surrounding a wellbore are derived in the Appendix. Equations (A-10) and (A-11) indicate that the azimuthal and vertical stresses, σ^T at the rock surface (wellbore radius = $a = 0.1$ m at Fenton Hill) induced by thermal contraction are

$$\sigma_{\theta}^T(a) = \sigma_z^T(a) = \frac{E\alpha T(a)}{(1 - \mu)} \quad (14)$$

When T is positive (cooling) these stresses are also positive (tensile). A typical value of cooling for the Fenton Hill injection well has been 130°C, so that the induced thermal stress has been 640 bars (9300 psi). To be superimposed upon these thermal stresses are the existing earth stresses and the concentration of these stresses caused by the drilling of the wellbore. The effective stress in the azimuthal direction due to these effects is⁹

$$\sigma_{\theta}^E(a) = 3(S_3 + P_{\text{hydro}}) - (S_2 + P_{\text{hydro}}) + P \quad , \quad (15)$$

where S_3 and S_2 are the minimum and maximum tectonic compressive stresses, taken as negative when compressive. The hydrostatic pressure, P_{hydro} and the pressure increment P above hydrostatic, caused for example, by water injection, are always positive. Equation (15) is taken directly from the hydraulic fracturing literature where it is conventional to distinguish between the total and "effective" stresses actually operating on the solid matrix of the porous medium. This concept was first proposed by Terzaghi.¹⁰ The absolute magnitude of effective stress is usually assumed to be the total acting stress minus the pore pressure, which we have assumed to be hydrostatic here. Effective stresses and their effect upon thermal cracking will be an issue for later discussion.

In many cases the value of S_2 is not well known so we investigate the condition for which the development of thermal cracks is least favorable. This occurs when $S_2 = S_3$ so that

$$\sigma_{\theta}^E(a) = 2(S_3 + P_{\text{hydro}}) + P \quad . \quad (16)$$

Once again we examine conditions at Fenton Hill where typically⁴

$$S_3 = -375 \text{ bars } (-5400 \text{ psi})$$

$$P_{\text{hydro}} = 290 \text{ bars } (4200 \text{ psi})$$

$$P = 70 \text{ bars } (1000 \text{ psi}).$$

Thus $\sigma_{\theta}^E(a) = -100 \text{ bars } (-1400 \text{ psi})$. The thermal tensile stress clearly exceeds this and thermal fracturing must occur, even if the strength of the granite was to approach 70 bars (1000 psi), a value* that begins to strain credibility. The effective vertical stress is $S_1 + P_{\text{hydro}}$ or -430 bars (-6200 psi), so it is apparent that even horizontal thermal cracks can occur. It must be concluded then that the Fenton Hill injection well, and even the designated production well, because of several cold water injection experiments, are both extensively thermally fractured.

*Hubbert and Willis¹¹ argue that the presence of joints and imperfections actually reduces the strength of rock to essentially zero.

The penetration of these thermal cracks can be estimated as follows. The stress concentration decays with radius as¹¹

$$\begin{aligned} \sigma_{\theta}^E(r) = & (S_3 + P_{\text{hydro}}) \left[1 + \frac{1}{2} \left(\frac{a}{r} \right)^2 + \frac{3}{2} \left(\frac{a}{r} \right)^4 \right] \\ & + (S_2 + P_{\text{hydro}}) \left[\frac{1}{2} \left(\frac{a}{r} \right)^2 - \frac{3}{2} \left(\frac{a}{r} \right)^4 \right] + P \left(\frac{a}{r} \right)^2 . \end{aligned} \quad (17)$$

Again the least favorable condition for the propagation of thermal cracks occurs when $S_2 = S_3$, or

$$\sigma_{\theta}^E(r) = (S_3 + P_{\text{hydro}}) \left[1 + \left(\frac{a}{r} \right)^2 \right] + P \left(\frac{a}{r} \right)^2 . \quad (18)$$

Taking S_3 , P_{hydro} , and P as before, computed values of $\sigma_{\theta}^E(r)$ are shown in Fig. 4.

The approximate temperature distribution in the rock wall, for a borehole cooling of $T = -130^{\circ}\text{C}$, is given by¹²

$$T(r) = \frac{-130[\ln 4\kappa t/r^2 - 0.5772]}{\ln 4\kappa t/a^2 - 0.5772} . \quad (19)$$

Again κ is thermal diffusivity, $10^{-6} \text{ m}^2/\text{s}$, and t is time, which for the longest experiment to date at Fenton Hill has been 3 weeks. These temperatures are also presented in Fig. 4. In addition are shown the accompanying azimuthal thermal stresses evaluated from Eq. (A-6) of the Appendix. It can be seen that even if the rock tensile strength approaches 70 bars (1000 psi), the induced tensile stress will exceed the absolute sum of $\sigma_{\theta}^E(r)$ and the rock strength for a radial distance of 0.5 m.

The thermal stress in the z-direction is greater than $\sigma_{\theta}^E(r)$, but the effective tectonic stress was shown earlier to be -430 bars. Thus if the strength is 70 bars, it is expected that horizontal thermal cracks have a total radius of only 0.2 m so far. Thus the crack penetration is of the order of 0.1 m, and by the end effect criterion that the crack spacing to penetration ratio is roughly one, we expect these horizontal cracks to be roughly 0.1 m apart. Given

Fig. 4.
Temperatures and stresses near the bore-hole.

long enough times, both azimuthal and horizontal cracks will propagate farther and farther away from the wellbore, eventually intersecting and forming new connections with the main reservoir. This will result in a welcome reduction in flow impedance precisely where it is most needed.

IV. DISCUSSION

A. Effective Stresses

In the calculation of the stresses in the reservoir we did not account for effective stresses. It was assumed instead that prior to cracking the rock was impermeable so that the total stress, S_2 , was acting. In fact, core specimens from the Fenton Hill wells show evidence of low permeability, typically¹³ in the nano- to submicro-darcy range (10^{-21} to 10^{-18} m²), and low porosity, typically¹⁴ 0.01 to 0.1%.

If these are sufficient to allow an infiltration of fluid such that the pore pressure is raised to hydrostatic, then the effective compressive stress in the S_2 direction is reduced by 290 bars (4200 psi). Consequently, the critical cooling required to initiate thermal cracks is reduced from 75 to 25°C, and for a given surface cooling, substantially greater penetrations and apertures are obtained for the same cooling time.

B. Reservoir Heterogeneity

The rock, of course, is not a perfectly homogeneous, elastically brittle solid, but is instead a heterogeneous, possibly anisotropic, composite of mineral phases, with pre-existing flaws and joints, which may be sealed to various degrees by chemical alteration and deposition. These flaws and joints no doubt act as stress risers, so that the spacing of the thermal cracks may be governed more by the joint spacing rather than the mathematical considerations given above. Given a reasonably uniform statistical distribution of joints and sufficient

time, however, the cracking pattern will approach the pattern calculated. Certainly these joints cannot hinder the thermal cracking process but only enhance it. Furthermore, the existence of joints will provide in situ permeabilities greater than those measured in cores, and adds credence to the effective stress theory which results in lowered resistance to thermal cracking.

C. Other Uncertainties

The value of S_2 used here is an inferred one, subject to some uncertainty. Carrying conservativeness to an extreme, one could suppose that S_2 is as compressive as the overburden pressure, S_1 , which is 690 bars (10 000 psi). This would require a crack initiation T_c of 135°C, 80% higher than the previous value. Such a large T_c would require almost complete depletion of the Fenton Hill reservoir, since the downhole equilibrium rock temperature is only 200°C.

On the other hand, Simmons and Cooper¹⁴ have estimated that downhole at Fenton Hill the granite should have a volumetric compressibility, β , of 2×10^{-6} bars⁻¹ at zero effective earth stress. From this it is possible to estimate $E = 3(1 - 2\mu)/\beta$ or 8.4×10^5 bars (12×10^6 psi). Based upon their estimates, this modulus should increase even more at higher effective earth stresses. However, even the zero stress value results in 50% reductions in the estimated T_c .

It becomes clear then that accurate estimates of the extent of thermal cracking depend upon accurate estimates of in situ values of the earth stresses and rock properties.

V. CONCLUSIONS

Continued heat extraction from a hydraulically fractured geothermal reservoir eventually results in the formation of thermal cracks. Initially these cracks are shallow and closely spaced but as time proceeds they grow deeper and more widely spaced; the actual spacing may of course be altered somewhat by natural joints. Eventually some of the larger cracks grow in aperture sufficiently to allow significant flow rates within the crack and additional heat extraction. The growth of these critically sized cracks then accelerates and eventually a newer set of orthogonal cracks may initiate. These critically sized cracks are spaced approximately 1 m apart. These cracks control the flow properties of the growing reservoir and lead to permeability estimates of the order of 1 darcy. When these critically sized cracks are developed it is expected that the reservoir can then be produced at constant temperature. Precise

estimates of the speed of these processes and the thermal drawdown required to reach this steady condition requires a better knowledge of the in situ properties and stresses than we presently have.

Concurrent with the development of these reservoir cracks, cracking at the wellbores also occurs. These eventually form new conduits to the reservoir precisely where the water flow had been most constricted.

REFERENCES

1. F. H. Harlow and W. E. Pracht, "A Theoretical Study of Geothermal Energy Extraction," J. Geophys. Res. 77, 7038-7048 (1972).
2. American Institute of Physics Handbook, D. E. Gray, Ed. (McGraw Hill, New York, 1963).
3. S. Timoshenko and J. N. Goodier, Theory of Elasticity 2nd Ed., Engineering Societies Monograph (Maple Press, York, Pennsylvania, 1951) pp. 401 and 408.
4. H. D. Murphy, R. G. Lawton, J. W. Tester, R. M. Potter, D. W. Brown, and R. L. Aamodt, "Preliminary Assessment of a Geothermal Energy Reservoir Formed by Hydraulic Fracturing," Soc. Petr. Engr. J., 317-325 (Aug. 1977).
5. S. P. Clark, Handbook of Physical Constants (Geol. Soc. of Amer., New York, 1966) p. 163.
6. R. D. McFarland and H. D. Murphy, "Extracting Energy from Hydraulically-Fractured Geothermal Reservoirs," in 11th Intersoc. Energy Conversion Engineering Conf., Stateline, Nevada 1, 828-835 (1976).
7. L. M. Keer, S. Nemat-Nasser, and K. S. Parihar, "Growth and Stability of Thermally Induced Cracks in Brittle Solids," Sept. 1976, The Technological Institute, Northwestern University, submitted for publication.
8. R. Potter, R. Pettitt, and R. L. Aamodt, Los Alamos Scientific Laboratory, unpublished data, 1978.
9. R. O. Kehle, "Determination of Tectonic Stresses Through Analysis of Hydraulic Well Fracturing," J. Geophys. Res. 69, 259-273 (1964).
10. K. Terzaghi, Theoretical Soil Mechanics (John Wiley and Sons, New York, 1943) p. 51.
11. M. K. Hubbert and D. G. Willis, "Mechanics of Hydraulic Fracturing," Petr. Trans., A.I.M.E. 210, 153-168 (1957).
12. H. T. Ramey, "Wellbore Heat Transmission," J. Petr. Tech., 427-435 (April 1962).

13. J. M. Potter, J. R. Balagna, and R. W. Charles, Los Alamos Scientific Laboratory, unpublished data, 1978.
14. G. Simmons and H. Cooper, "DSA of the Microcracks in GT-2 Core," Informal MIT report to LASL under subcontract X66-54922-1 (September 20, 1976).

APPENDIX

THERMAL STRESSES IN A ROCK CYLINDER

Once again the rock is treated as an isotropic homogeneous linearly elastic solid with thermal expansion coefficient α , elastic modulus E , and Poisson's ratio μ . The coordinate system originates at the well center line. Depth is z , radius is r , and azimuth is θ . The rock extends to infinity, $r \rightarrow \infty$. The well radius is a . Stresses, strains, and the thermal cooling relative to the initially constant temperature are σ , ϵ , and T respectively. Plane strain and axisymmetric conditions are assumed. Radial and z -direction equilibrium of forces requires

$$\left. \begin{aligned} \sigma_{\theta} &= d(r\sigma_r)/dr \\ \int_a^{\infty} r \sigma_z dr &= 0 \end{aligned} \right\} \quad (A-1)$$

The three strain components are (Ref. 3, p. 408)

$$\left. \begin{aligned} \epsilon_r &= \frac{du}{dr} = \frac{1}{E} [\sigma_r - \mu(\sigma_{\theta} + \sigma_z)] - \alpha T \\ \epsilon_{\theta} &= \frac{u}{r} = \frac{1}{E} [\sigma_{\theta} - \mu(\sigma_z + \sigma_r)] - \alpha T \\ \epsilon_z &= \text{constant} = \frac{1}{E} [\sigma_z - \mu(\sigma_r + \sigma_{\theta})] - \alpha T \end{aligned} \right\} \quad (A-2)$$

From Eq. (A-2) the stress components may be calculated.

$$\left. \begin{aligned} \sigma_r &= \frac{E[\epsilon_r(1 - \mu) + \mu(\epsilon_\theta + \epsilon_z)]}{(1 + \mu)(1 - 2\mu)} + \frac{E\alpha T}{1 - 2\mu} \\ \sigma_\theta &= \frac{E[\epsilon_\theta(1 - \mu) + \mu(\epsilon_z + \epsilon_r)]}{(1 + \mu)(1 - 2\mu)} + \frac{E\alpha T}{1 - 2\mu} \\ \sigma_z &= \frac{E[\epsilon_z(1 - \mu) + \mu(\epsilon_r + \epsilon_\theta)]}{(1 + \mu)(1 - 2\mu)} + \frac{E\alpha T}{1 - 2\mu} \end{aligned} \right\} \quad (A-3)$$

Combining Eqs. (A-1) and (A-2) results in an equation for the radial displacement

$$\frac{d^2u}{dr^2} + \frac{1}{r} \frac{du}{dr} - \frac{u}{r^2} + \frac{1 + \mu}{1 - \mu} \frac{\alpha dT}{dr} = 0 \quad (A-4)$$

This can be integrated to yield

$$u = - \frac{1 + \mu}{1 - \mu} \frac{\alpha}{r} \int_a^r r T dr + C_1 r + C_2/r \quad (A-5)$$

Thermal diffusion processes always result in temperature distributions such that the change in temperature $T \rightarrow 0$ as $r \rightarrow \infty$. Thus as $r \rightarrow \infty$, the integral term in Eq. (A-5) goes to zero. Since $u(r \rightarrow \infty)$ is at least finite, Eq. (A-5) requires $C_1 = 0$.

When the strains ϵ_r and ϵ_θ are evaluated from Eq. (A-5) (with $C_1 = 0$) and substituted in Eq. (A-3) the resulting stresses are

$$\left. \begin{aligned} \frac{\sigma_r}{E} &= \frac{\alpha}{r^2(1 - \mu)} \int_a^r r T dr - \frac{C_2}{(1 + \mu)r^2} + \frac{\mu \epsilon_z}{(1 + \mu)(1 - 2\mu)} \\ \frac{\sigma_\theta}{E} &= \frac{-\alpha}{r^2(1 - \mu)} \int_a^r r T dr + \frac{C_2}{(1 - \mu)r^2} + \frac{\mu \epsilon_z}{(1 + \mu)(1 - 2\mu)} + \frac{\alpha T}{(1 - \mu)} \\ \frac{\sigma_z}{E} &= \frac{\epsilon_z}{(1 - 2\mu)} \frac{1 - \mu}{1 + \mu} + \frac{\alpha T}{1 - \mu} \end{aligned} \right\} \quad (A-6)$$

Applying the axial equilibrium condition, Eq. (A-1) results in

$$\epsilon_z = 0 \quad . \quad (A-7)$$

The constant of integration C_2 is found from the stress, if any, imposed at the rock surface, $r = a$. Thus

$$C_2 = \frac{-(1 + \mu)a^2}{E} \sigma_r(a) \quad . \quad (A-8)$$

The resulting displacement at $r = a$ is

$$u(a) = (1 + \mu) a \sigma_r(a)/E \quad . \quad (A-9)$$

The surprising conclusion is that thermal stresses have absolutely no effect on $u(a)$ - the wellbore surface displacement is zero in the absence of externally applied loads. Substituting Eqs. (A-7) and (A-8) in (A-6) results in the following borehole surface stresses.

$$\sigma_\theta(a) = -(1 + \mu) \sigma_r(a) + E\alpha T(a)/(1 - \mu) \quad . \quad (A-10)$$

$$\sigma_z(a) = \frac{E\alpha T(a)}{(1 - \mu)} \quad . \quad (A-11)$$

When T is positive (cooling) σ_z and σ_θ become tensile. Note that in the absence of cooling Eq. (A-10) predicts $\sigma_\theta(a) = -(1 + \mu) \sigma_r(a)$ whereas the hydraulic fracturing literature^{9,11} would predict in these circumstances that $\sigma_\theta(a) = -\sigma_r(a) = P$. The factor $(1 + \mu)$ arises because here we have assumed a plane, constant axial strain condition whereas the fracturing literature assumes a constant axial stress condition.

The first is appropriate for long cylinders while the second is appropriate for short, wafer-like cylinders. Although we have used the boundary condition $r \rightarrow \infty$ in this thermal stress analysis, in practical effect the outer boundary is located where the temperature change approaches zero. This is located at $r \approx \sqrt{3\kappa \cdot \text{time}}$. The thermal diffusivity κ for rock is $10^{-6} \text{ m}^2/\text{s}$, so even for a 30-day experiment, $r \approx 8 \text{ m}$ is very small compared to the well length of 3000 m. Therefore, plane strain conditions are really more appropriate.