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SURFACE HEAT FLUX AND
UNIFORM VOLUMETRIC
HEATING ON BLANKET DESIGN
FOR FUSION REACTORS**

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ABSTRACT

An analytical solution for the temperature profile and film temperature drop for fully-developed, laminar flow in a circular tube is provided. The surface heat flux varies circumferentially but is constant along the axis of the tube. The volumetric heat generation is uniform in the fluid. The fully developed laminar velocity profile is approximated by a power velocity profile to represent the flattening effect of a perpendicular magnetic field when the coolant is electrically conductive. The presence of volumetric heat generation in the fluid adds another component to the film temperature drop to that due to the surface heat flux. The reduction of the boundary layer thickness by a perpendicular magnetic field reduces both of these two film temperature drops. The Nusselt number for constant surface heat flux increases from 4.36 for parabolic velocity profile to 8 for nearly flat velocity profile or slug flow. The corresponding increase in the Nusselt number for uniform volumetric heat generation is from 2.46 to 5.33. A strong perpendicular magnetic field can reduce the film temperature drop by a factor of two if the fluid is electrically conducting. The effect of nonuniformity of the surface heat flux, however, is to reduce the Nusselt number or increase the film temperature drop at the location of the maximum heat flux compared to the case of uniform surface heat flux. At the point of maximum surface heat flux with a cosine variation, which is very close to the case of a coolant tube in the first wall and limiter/divertor plate of a fusion reactor, the Nusselt number can be reduced from 4.36 to 2.7 and from 8 to 3 for parabolic and flat velocity profiles, respectively. The effect of perpendicular magnetic field (or the flatness of the velocity profile) is less pronounced on the film temperature drop due to nonuniform surface heat flux than on that due to uniform surface heat flux. An example is provided to show the relative effects of these two film temperature drops in the thermal design of fusion reactors.

I. INTRODUCTION

There are situations where the heat flux on the surface of a coolant channel is nonuniform and, at the same time, volumetric heat is generated in the coolant flowing through a coolant channel. An example of this is a coolant channel in the first wall and limiter/divertor plate of a fusion reactor. The side of the coolant channel which faces the plasma receives radiation heat flux. Fusion neutrons generate volumetric heat in the structural material and coolant. In a fission reactor using liquid fuel, large volumetric heat is generated due to the fission reaction. The heat is removed through the channel wall and/or by the fuel slurry acting also as the coolant. Film temperature drop for such a case was determined by Pependiek [1]. Even with uniform volumetric heat generation, there is a temperature drop across the boundary layer because fluid velocity is smaller in the boundary layer near the wall and hence part of the heat needs to be conducted to the central fluid which then convects away the heat. Reynolds [2] treated the film temperature drop for the case of nonuniform surface heat flux on a circular tube with parabolic velocity profile of the fluid.

Liquid metals, particularly liquid lithium, are considered as coolants in fusion reactor applications. Liquid lithium was proposed as a coolant for fusion reactors by Holroyd and Mitchell [3] and Hunt and Hancox [4]. A recent reversed-field pinch reactor study, TITAN [5], has used liquid lithium as the primary coolant in one version of the design of the fusion reactor core. Since liquid metals are electrically conducting, the presence of any transverse magnetic field will affect the coolant velocity profile and hence the film temperature drop.

In the thermal-hydraulic design for a fusion reactor, the maximum structure temperature of a coolant channel is one of the design limits. Accurate determination of the structure temperature is, therefore, necessary for a safe design. The film temperature drops directly affect the channel-wall temperature. It is highly desirable to quantify the effects of magnetic field, volumetric heat generation and

nonuniformity of surface heat flux on the film temperature drop for liquid-metal as well as nonconducting fluid flow through the coolant channel in a fusion reactor.

The objectives of this work are to find analytical estimates for these effects. The analytical solution uses several assumptions. The coolant channel is taken to be circular tube. The flow is assumed laminar and fully developed. Power velocity profile is used to approximate the flattening of the velocity profile by a perpendicular magnetic field. The magnetic field is transverse to a coolant channel rather than radial in fusion applications and the channel cross section may be non-circular. The analytical results will, therefore, be approximate. In section II, the analytical solutions are derived. The analytical results and examples of application to fusion reactors are discussed in section III. Conclusions and recommendations are given in section IV.

II. TEMPERATURE PROFILE AND FILM TEMPERATURE DROP

The steady-state, non-homogeneous energy equation in cylindrical coordinates (r, θ, z) can be written as

$$k_f \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} \right) = \rho c_p u(r) \frac{\partial T}{\partial z} - q_f''' \quad (1)$$

where k_f is the thermal conductivity of the fluid, T is the temperature, and ρ and c_p are the density and specific heat of the fluid, respectively. The uniform volumetric heat generation is denoted by q_f''' and the fully developed velocity profile is represented by $u(r)$. It is assumed that the surface heat flux and the volumetric heat generation do not vary along the axis of the tube. Therefore, T is a linear function of the axial coordinate, z . The magnetic field is assumed to be uniform and perpendicular to the coolant flow (such as a uniform radial magnetic field). In this case, the velocity profile will be flattened uniformly along the circumference, and there will not be any θ -dependence as is the case with a transverse magnetic field [6]. The velocity profile can then be approximated by the following equation.

$$u(r) = \frac{m+2}{m} U \left\{ 1 - \left(\frac{r}{a} \right)^m \right\}, \quad (2)$$

where U is the mean velocity and a is the inner radius of the tube. The power index, m , increases with the increase of perpendicular Hartmann number. For $m=2$, the velocity profile is parabolic corresponding to zero Hartmann number. The mean velocity is given by

$$U = \frac{2}{a^2} \int_0^a u(r) r dr.$$

Solution to equation (1) will be obtained for surface heat flux which varies arbitrarily along the circumference. The procedure adopted is similar to that in ref.[2]. Figure 1 shows the three steps involved in the solution. The boundary and symmetry conditions are shown in Fig. 1a. First, equation (1), with boundary and symmetry conditions, is solved for the case shown in Fig. 1a where the surface heat flux $q'' > 0$ for $-\beta \leq \theta \leq \beta$, and $q''=0$ on the rest of the circumference. Second, by superposition of the result in the previous step, solution with segmental heating, as shown in Fig. 1b, is obtained. Finally, in the limit, replacing the summation in the previous step by integration, the desired solution for the case of arbitrary surface heat flux, Fig. 1c, is obtained.

Let $T_f(z)$ be the mean fluid temperature and r be normalized by the inner radius of the tube. By energy balance for the case (a) in Fig. 1, we get

$$\frac{dT}{dz} = \frac{dT_f}{dz} = \left(\frac{2\beta q''}{\pi a \rho c_p U} + \frac{q_f'''}{\rho c_p U} \right).$$

Using this axial derivative and the velocity profile, equation (2), in equation (1), we get

$$\frac{\partial^2 T}{\partial r^2} + \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} = \frac{m+2}{m} (1-r^m) \frac{2a\beta q''}{\pi k_f} + \left\{ \frac{m+2}{m} (1-r^m) - 1 \right\} \frac{a^2 q_f'''}{k_f}. \quad (3)$$

Since the quantity we are interested in is the film temperature drop, we define $\Delta T(r, \theta) = T(r, \theta, z) - T_f(z)$. Let

$$\Delta T(r, \theta) = F(r, \theta) + g(r), \quad (4)$$

where $F(r, \theta)$ is the homogeneous solution and $g(r)$ is the particular solution. Inserting equation (4) in equation (3) and equating the right hand side with the particular solution, we get

$$\frac{\partial^2 F}{\partial r^2} + \frac{1}{r} \frac{\partial F}{\partial r} + \frac{1}{r^2} \frac{\partial^2 F}{\partial \theta^2} = 0$$

and

$$\frac{d^2 g}{dr^2} + \frac{1}{r} \frac{dg}{dr} = \frac{m+2}{m} \frac{2a\beta q''}{\pi k_f} (1-r^m) + \frac{a^2 q_f'''}{k_f} \left\{ \frac{m+2}{m} (1-r^m) - 1 \right\}.$$

The solution for the particular component, $g(r)$, is obtained as

$$g(r) = \frac{m+2}{m} \left\{ \frac{r^2}{4} - \frac{r^{m+2}}{(m+2)^2} \right\} (A+B) - B \frac{r^2}{4} + C_1 \ln r + C_2,$$

where $A = \frac{2a\beta q''}{\pi k_f}$, $B = \frac{a^2 q_f'''}{k_f}$, and C_1 and C_2 are constants of integration. $C_1=0$ to make $g(r)$ finite at $r=0$ and C_2 can be absorbed in the homogeneous solution, F . The solution for $g(r)$ then becomes

$$g(r) = \frac{m+2}{m} \left\{ \frac{r^2}{4} - \frac{r^{m+2}}{(m+2)^2} \right\} (A+B) - B \frac{r^2}{4}. \quad (5)$$

The solution for $F(r, \theta)$ is obtained by separation of variables. It is given by

$$F(r, \theta) = c_0 + \sum_{n=1}^{\infty} c_n r^n \cos n\theta, \quad (6)$$

where c 's are constants and n is the eigenvalue. Now adding equations (5) and (6), we get the expression for $\Delta T(r, \theta)$.

$$\Delta T(r, \theta) = (A + B) \frac{m+2}{m} \left\{ \frac{r^2}{4} - \frac{r^{m+2}}{(m+2)^2} \right\} - \frac{r^2}{4} + c_o + \sum_{n=1}^{\infty} c_n r^n \cos n\theta. \quad (7)$$

The constants c_n can be determined by using the boundary conditions on $F(r, \theta)$ and the orthogonality condition.

The boundary conditions on T for the case (a) in Fig. 1 are:

$$\frac{\partial T}{\partial r} \Big|_{r=1} = \begin{cases} \frac{aq''}{k_f} & ; -\beta \leq \theta \leq \beta \\ 0 & ; \beta \leq \theta \leq 2\pi - \beta \end{cases}.$$

The boundary conditions on $F(r, \theta)$ can be obtained from the equations (4), (5) and (7).

$$\frac{\partial F}{\partial r} \Big|_{r=1} = \frac{\partial \Delta T}{\partial r} \Big|_{r=1} - \frac{dg}{dr} \Big|_{r=1}.$$

This gives

$$\frac{\partial F}{\partial r} \Big|_{r=1} = \begin{cases} \frac{aq''}{k_f} \left(1 - \frac{\beta}{\pi}\right) & ; -\beta \leq \theta \leq \beta \\ -\frac{aq''\beta}{\pi k_f} & ; \beta \leq \theta \leq 2\pi - \beta \end{cases}.$$

Now we differentiate equation (6) with respect to r , multiply it by $\cos m\theta$ and then integrate over θ from 0 to 2π using orthogonality and the boundary conditions to obtain

$$c_n = \frac{2aq'' \sin n\beta}{\pi n^2 k_f}; \quad n \geq 1.$$

Equation (7) now becomes

$$\Delta T(r, \theta) = c_o + \frac{2aq''}{\pi k_f} \sum_{n=1}^{\infty} \frac{r^n}{n} \sin n\beta \cos n\theta + (A + B) \frac{m+2}{m} \left\{ \frac{r^2}{4} - \frac{r^{m+2}}{(m+2)^2} \right\} - B \frac{r^2}{4}. \quad (8)$$

The remaining constant c_o can be determined by equating the average of the difference of the fluid temperature from the mean fluid temperature to zero. That is, from

$$\int_0^1 \int_0^{2\pi} u(r) \Delta T(r, \theta) r dr d\theta = 0. \quad (9)$$

From equation (9), using equations (2) and (8), c_o is obtained as

$$c_o = -(A + B) \frac{m^2 + 6m + 12}{8(m+4)(m+2)} + B \frac{(m+2)(m+3)}{2m(m+4)}.$$

The film temperature drop, $\Delta T_f(\theta)$, is obtained by putting $r=1$ in equation (8) and using the expressions for c_o , A and B . This gives

$$\begin{aligned} \Delta T_f(\theta) = & \frac{2aq''\beta}{k_f\pi} \left(\frac{m^2 + 10m + 20}{8m^2 + 48m + 64} \right) + \frac{2aq''}{k_f\pi} \sum_{n=1}^{\infty} \frac{1}{n^2} \sin n\beta \cos n\theta \\ & + \frac{a^2q_f'''}{k_f} \left(\frac{3m^3 + 26m^2 + 68m + 48}{8m^3 + 48m^2 + 64m} \right). \end{aligned} \quad (10)$$

Now if the heat flux is only on a small segment of the surface of the tube as shown in case (b) of Fig. 1, the film temperature drop due to this small segmental heat flux, $\delta T_f(\theta)$, can be obtained by replacing β by $\Delta\omega/2$ and θ by $(\theta - \omega - \Delta\omega/2)$ in equation (10). Thus we obtain

$$\begin{aligned} \delta T_f(\theta) = & \frac{2aq''}{k_f} \frac{\Delta\omega}{2\pi} \left(\frac{m^2 + 10m + 20}{8m^2 + 48m + 64} \right) + \frac{2aq''}{k_f\pi} \sum_{n=1}^{\infty} \frac{1}{n^2} \sin \frac{n\Delta\omega}{2} \\ & \cos n(\theta - \omega - \frac{\Delta\omega}{2}) + \frac{aq_f'''}{k_f} \left(\frac{3m^3 + 26m^2 + 68m + 48}{8m^3 + 48m^2 + 64m} \right). \end{aligned} \quad (11)$$

If there are L small segments with surface heat flux, then the total film temperature drop can be obtained by superposition of the result in equation (11).

$$\Delta T_f(\theta) = \sum_{l=1}^L \delta T_f(\theta). \quad (12)$$

The summation is only over the first two terms in equation (11). For the case of continuous variation of surface heat flux, case (c) in Fig. 1, the film temperature drop can be obtained by replacing $\sum_{l=1}^L$ by $\int_0^{2\pi} d\omega$ and approximating $\sin \frac{n\Delta\omega}{2}$ by $n\Delta\omega/2$ in equation (11) and (12). Doing this, we get

$$\begin{aligned} \Delta T_f(\theta) = & \frac{a}{\pi k_f} \left(\frac{m^2 + 10m + 20}{8m^2 + 48m + 64} \right) \int_0^{2\pi} q''(\omega) d\omega - \frac{a}{\pi k_f} \int_0^{2\pi} q''(\omega) \\ & \ln \left\{ 2 \sin \frac{\theta - \omega}{2} \right\} d\omega + \frac{a^2q_f'''}{k_f} \left(\frac{3m^3 + 26m^2 + 68m + 48}{8m^3 + 48m^2 + 64m} \right). \end{aligned} \quad (13)$$

In equation (13), the series

$$\sum_{n=1}^{\infty} \frac{1}{n} \cos n(\theta - \omega)$$

is replaced by [7]

$$-\ln \left\{ 2 \sin \frac{(\theta - \omega)}{2} \right\}.$$

In equation (13), the first two terms represent the film temperature drop, $\Delta T_s(\theta)$, due to arbitrarily varying surface heat flux. The last term represents the film temperature drop, ΔT_v , due to the uniform volumetric heat generation in the coolant. Although the volumetric heat generation rate is uniform in the coolant, the film temperature drop ΔT_v arises because of the presence of boundary layer. Coolant velocity is zero at the surface of the wall and it increases into the boundary layer away from the wall. Therefore, there is a gradual decrease in axial convection of heat from the center to the wall of the tube. As a result, coolant temperature increases towards the wall and heat conducts to the center of the tube to be convected by the faster moving coolant. A Nusselt number for the surface heat flux can be defined by the following equation.

$$Nu_s(\theta) = \frac{(2a)q''(\theta)}{\Delta T_s(\theta)k_f} \quad (14)$$

An equivalent Nusselt number for the volumetric heat generation can be defined as

$$Nu_v = \frac{(2a)(aq_f''')}{\Delta T_v k_f} = \left\{ \frac{2(8m^3 + 48m^2 + 64m)}{3m^3 + 26m^2 + 68m + 48} \right\}. \quad (15)$$

In equation (15), (aq_f''') is equivalent to heat flux.

Familiar results for Nusselt number can be obtained from the above equations. For uniform surface heat flux, $\beta = \pi$. The film temperature drop corresponding to constant surface heat flux becomes, from equation (13),

$$\Delta T_s = \frac{2aq''}{k_f} \left(\frac{m^2 + 10m + 20}{8m^2 + 48m + 64} \right).$$

The Nusselt number is then given by

$$Nu_s = \frac{8m^2 + 48m + 64}{m^2 + 10m + 20} \quad (16)$$

In the absence of any transverse component of magnetic field, the fully developed laminar velocity profile is parabolic ($m=2$). Equation (16) then gives $Nu_s=48/11=4.36$, which is the Nusselt number for constant heat flux and parabolic velocity profile in a circular tube. When the perpendicular Hartmann number is very large, the velocity profile is nearly flat like that for slug flow. In the limit m approaches infinity, equation (16) gives $Nu_s=8$, which is the Nusselt number for slug flow in a circular tube with constant surface heat flux. Nusselt number defined for the film temperature drop due to the uniform volumetric heat generation, equation (15), has the value of $32/13=2.46$ for parabolic velocity profile ($m=2$) and $16/3=5.33$ for slug flow (large m). Therefore, a strong magnetic field perpendicular to the flow of an electrically conducting coolant will reduce the film temperature drops due to uniform surface heat flux and volumetric heat generation by a factor of about two. When the surface heat flux varies along the circumference, then the film temperature drop and the Nusselt number due to the surface heat flux will be functions of θ . In the next section, we discuss the effects of the flatness of velocity profile and the variation of surface heat flux on Nusselt number by using specific examples from fusion reactor application.

III. RESULTS AND DISCUSSION

Coolant channels with circular cross sections are often used in fusion reactor thermal-hydraulic design because circular tubes have good heat transfer performance, high structural strength, and are easy to manufacture with close tolerance [5]. Figure 2 shows the cross section of a first-wall coolant tube used in the thermal-hydraulic design of the fusion power core of the reversed-field-pinch fusion reactor, TITAN-I [5]. The tube material is the vanadium alloy, V-3Ti-1Si, and the coolant is liquid lithium. The inner radius of the tube is 4 mm and its outer radius is 5.05 mm. The radiation heat flux incident on the surface of the first-wall tube facing the plasma is denoted by q_R'' and the volumetric nuclear heat generation rates in the tube wall and lithium are represented by q_w''' and q_f''' , respectively. Neutron wall loading of TITAN-I is 18.1 MW/m^2 . The nuclear heat generation at the first wall are $q_w''' \approx 100 \text{ MW/m}^3$ and $q_f''' \approx 75 \text{ MW/m}^3$. At the design point where 95% of the alpha and ohmic dissipative power in the plasma is radiated, the radiation heat flux on the first wall is 4.6 MW/m^2 . At different wall loading or different radiation fraction, the heat flux on the first wall will have different values. The effects of nonuniformity of the surface heat flux and the flatness of the velocity profile caused by perpendicular magnetic field, on heat transfer will be presented in this section. The first-wall coolant channel of TITAN-I will be used as example.

The surface heat flux on the inner surface of a first-wall tube can be written as

$$q''(\theta) = \begin{cases} q_{NH}'' + \frac{b}{a} q_R'' \cos \theta & ; -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2} \\ q_{NE}'' & ; \frac{\pi}{2} \leq \theta \leq \frac{3\pi}{2} \end{cases} \quad (17)$$

where

$$q_{NH}'' = \frac{q_w'''(b^2 - a^2)}{2a}.$$

The outer radius of the coolant channel is b . Equation (17) assumes that the radiation heat flux is cosine on the tube surface and that conduction in the tube wall along θ is negligible.

The total film temperature drop due to nonuniform surface heat flux and uniform volumetric heat generation is given by equation (13). Equations (15) and (16) determine the Nusselt numbers for uniform volumetric heat generation in the coolant and constant surface heat flux, respectively, as functions of the power index (m) of the velocity profile. Figure 3 shows this variation. At $m=2$, for parabolic profile, $Nu_s=48/11$ due to constant surface heat flux and $Nu_v=32/13$ due to uniform volumetric heat generation. As m increases, Nu approaches the slug-flow magnitudes of 8 and $16/3$ for the uniform surface heat flux and volumetric heat generation, respectively. This shows that whenever a perpendicular magnetic field flattens the velocity profile, thus reducing the boundary layer thickness, heat transfer will become better and film temperature drop will decrease.

The effect of circumferential variation of surface heat flux is shown in Fig. 4. The value of m is 250 which gives nearly slug flow corresponding to high perpendicular Hartmann number (≥ 200). The surface heat flux is given by equation (17) with $q_R''=4.6 \text{ MW/m}^2$ and $q_w'''=100 \text{ MW/m}^3$. The tube dimensions are the same as those for the first-wall tube for TITAN-I. The Nusselt number is the smallest at $\theta=0^\circ$ where the radiation heat flux is the maximum. The value of Nusselt number at $\theta=0^\circ$ is about 3. It is much less than 8 for constant heat flux. At about 85° , Nu becomes infinite. The reason is that at this point the wall/coolant interface temperature becomes equal to the mean coolant temperature thus making the film temperature drop zero. Beyond this point, Nu is negative. Negative Nusselt number means that the mean coolant temperature is higher than the interface temperature so that the film temperature drop becomes negative. A schematic representation of the temperature profile which explains this is shown in Fig. 5. When the Nusselt number is negative, the magnitude of the film temperature drop has to be subtracted from the mean fluid temperature to obtain the temperature at the inner surface of the wall of the coolant tube.

Figure 6 shows the variation of Nu with m at $\theta=0$ and peak radiation heat flux

of $q_R''=4.6 \text{ MW/m}^2$. The uniform heat flux at the inner surface of the tube due to volumetric heat generation in the tube wall, q_{NH}'' , is 0.145 MW/m^2 . This is about 32 times smaller than the peak radiation heat flux at $\theta=0$. Under this condition of highly peaked surface heat flux, the increase in Nu in going from parabolic to nearly flat velocity profile is very small, only 10% (2.7 to 3.0) compared to about 85% (4.36 to 8) increase for the case of constant heat flux. This means that when the surface heat flux is highly peaked, perpendicular magnetic field, however large, does increase very little the Nu at the critical point of peak heat flux, that is, at $\theta=0^\circ$ under the present condition. This will have significant impact on the thermal-hydraulic design of the first wall, divertor and limiter of a fusion reactor using liquid metal as coolant.

The effects of peakedness of the surface heat flux on the Nusselt number for parabolic ($m=2$) and nearly flat ($m=250$) velocity profiles are shown in Fig. 7. In this figure, $Nu(\theta=0^\circ)$ is plotted against q_R''/q_{NH}'' . The ratio q_R''/q_{NH}'' is a measure of the peakedness of the surface heat flux. The heat flux at the inner surface of the tube wall due to the nuclear heating in the wall, q_{NH}'' , is constant along the circumference. The peak radiation heat flux, q_R'' , is varied to change the ratio. The curve for $m=250$ simulates the flattening of the velocity profile of a conducting fluid by a perpendicular magnetic field. At $q_R''/q_{NH}''=0$, $Nu=8$, the case for constant heat flux and slug flow. The Nusselt number at $\theta=0^\circ$ decreases rapidly with the increase of the ratio q_R''/q_{NH}'' . At $q_R''/q_{NH}''=15$, Nu falls to about 3 and remains almost constant with further increase of peakedness of the surface heat flux. Therefore, the more peaked the surface heat flux is, the less effective becomes the perpendicular magnetic field in increasing the heat transfer.

In Fig. 7, the curve for $m=2$ shows that at $q_R''/q_{NH}''=0$, $Nu=4.36$ which is the Nusselt number for parabolic velocity profile and constant surface heat flux. With the increase of the ratio q_R''/q_{NH}'' , the Nusselt number decreases and reaches a value of 2.7 for highly peaked surface heat flux. It can be noted that for parabolic velocity

profile, the effect of peakedness of the surface heat flux in reducing the Nusselt number is less pronounced compared to that for nearly flat velocity profile ($m=250$). The maximum reduction of Nu for parabolic velocity profile is about 37%, whereas it is about 62% for nearly flat velocity profile. Another point that should be noted is that, when the surface heat flux is highly peaked, the difference between the Nusselt numbers for parabolic velocity profile and nearly slug flow is small, about 10% (2.7 compared to 3). This signifies that the nonuniformity of the surface heat flux is more effective in reducing the Nusselt number or increasing the film temperature drop in the presence of a perpendicular magnetic field than in the absence of it (parabolic profile).

In the thermal-hydraulic and structural design of the first wall, limiter/divertor plates and blanket of a fusion reactor, the effects of nonuniformity of surface heat flux, volumetric heat generation in the coolant, and perpendicular magnetic field can be significant and should be included for a safe design. As an example of these effects, we can look at the various film temperature drops for laminar flow in the first-wall coolant tube of TITAN-I. The surface heat flux and the volumetric heat generation rates in the wall material and coolant are as mentioned above. At the point of maximum surface heat flux, $\theta = 0^\circ$, the film temperature drops are $317^\circ C$ for parabolic profile and $278^\circ C$ for nearly flat profile. If the effect of nonuniformity of the surface heat flux is neglected, these two film temperature drops would become $196^\circ C$ and $104^\circ C$, respectively. The amount of underestimation is quite large. The film temperature drops due to volumetric heat generation are $21^\circ C$ for parabolic profile and $9.7^\circ C$ for nearly flat profile. This temperature drop, which appears not to be included in fusion design, can be quite large for coolants with low thermal conductivity such as FLiBe.

IV. CONCLUSIONS AND RECOMMENDATIONS

The effects of nonuniform surface heat flux, uniform volumetric heat generation and velocity profile on the film temperature drop in fully developed laminar flow in a circular tube are discussed by deriving an analytical solution. The velocity profile of the coolant is represented by a power law which can be used to represent the flattening of the profile by a perpendicular magnetic field if the coolant is electrically conducting. The following conclusions can be drawn from this analysis.

1. The flattening of velocity profile increases the heat transfer through a reduction of the thickness of the boundary layer. With constant surface heat flux, the Nusselt number increases by 83% (from $48/11$ to 8) from parabolic to nearly flat velocity profile. The Nusselt number corresponding to uniform volumetric heat generation, defined by equation (15), increases by 117% (from $32/13$ to $16/5$).
2. The nonuniformity of surface heat flux decreases the Nusselt number at the point of maximum heat flux. With the increase of peakedness of the surface heat flux according to equation (17), the Nusselt number decreases from 4.36 to 2.7 for parabolic velocity profile, and from 8 to 3 for nearly slug flow.
3. The point (2) above infers that the effect of nonuniformity of the surface heat flux is more pronounced for flat velocity profile than for parabolic profile. Therefore, the heat transfer in liquid-metal coolant in the presence of a perpendicular magnetic field in a fusion reactor will be significantly affected by the nonuniformity of the surface heat flux.

The results obtained in this paper have significant implications for the design of coolant channels for the removal of thermal energy from the first wall, limiter/divertor plates, and blanket of a fusion reactor. The surface heat flux varies along the circumference of the coolant channels for the first wall and divertor/limiter plates due to the incidence of radiation from the plasma. The maximum wall temperatures of the coolant channels for the first wall and divertor/limiter plates will

be severely underestimated if the effect of nonuniformity of the surface heat flux is not taken into account. This is particularly important when the surface heat flux is high and the coolant is electrically conducting so that the velocity profile becomes nearly flat due to the effect of a large perpendicular magnetic field.

It appears that the component of the film temperature drop due to uniform volumetric heat generation in the coolant is not accounted for in the design of the coolant channels for fusion reactors. In a deuterium/tritium fueled fusion reactor, significant amount of volumetric heat will be generated in the coolant if it is not transparent to the fusion neutrons. In this case, the film temperature drop due to the volumetric heat generation must be added to obtain the total film temperature drop and to determine the correct channel-wall temperature. For liquid-metal coolants, this film temperature drop may not be large because of the high thermal conductivity of the coolant. On the other hand, if the thermal conductivity of a coolant is low and the volumetric heat generation rate is appreciable, this film temperature drop can be quite large.

The analysis done in this paper is applicable to fully developed laminar flow only. Similar analysis should be done for turbulent flow since the coolant flow is expected to be both laminar and turbulent in the coolant channels in different components of a fusion reactor. Although the magnitudes of these film temperature drops for turbulent flow may be different from those for laminar flow, the trend is expected to be the same.

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FIGURE CAPTIONS:

Figure 1: The three steps in the solution of the convection heat transfer equation with arbitrary surface heat flux.

Figure 2: Cross section of a circular coolant channel showing the dimensions, surface heat flux, volumetric heat generation rates, and temperature drops.

Figure 3: Dependence of Nusselt number on velocity profile (values of m). (a) Nusselt number for constant surface heat flux, Nu_s , and (b) Nusselt number for uniform volumetric heat generation, Nu_v .

Figure 4: Circumferential variation of Nusselt number due to circumferentially varying surface heat flux.

Figure 5: A schematic of the temperature profile explaining negative Nusselt number and film temperature drop.

Figure 6: Variation of the $Nu(0^\circ)$ due to nonuniform surface heat flux q''_s with m . The peak radiation heat flux at $\theta=0^\circ$ is 4.6 MW/m^2 .

Figure 7: Variation of $Nu(0^\circ)$ with q''_R/q''_{NH} for nearly flat ($m=250$) and parabolic ($m=2$) velocity profiles. The ratio q''_R/q''_{NH} is a measure of the peakedness of the surface heat flux.

$$(a) k_f \frac{\partial T}{\partial r} \Big|_{r=a} = \begin{cases} q'', & \text{for } -\beta \leq \theta \leq \beta; \\ 0, & \text{for } \beta \leq \theta \leq (2\pi - \beta). \end{cases}$$

$$\frac{\partial T}{\partial \theta} = 0 \begin{cases} \text{at } \theta = 0; \\ \text{and } \theta = \pi. \end{cases}$$

(b) Segmental heating, $\beta = \Delta\omega/2$.

(c) $q''(\theta)$ is arbitrary function of θ .

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