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**COMPOSITE FLYWHEEL DEVELOPMENT
COMPLETION REPORT**

(October 1, 1976 - September 30, 1977)

R. L. Huddleston
J. J. Kelly
C. E. Knight
R. E. Pollard
D. W. Post

May 1978

**UNION
CARBIDE**

**OAK RIDGE Y-12 PLANT
OAK RIDGE, TENNESSEE**

*prepared for the U.S. DEPARTMENT OF ENERGY under
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Fabrication Systems Department
Y-12 Development Division

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COMPOSITE FLYWHEEL DEVELOPMENT

INTRODUCTION

Union Carbide Corporation-Nuclear Division's (UCC-ND) Composite Flywheel Program objectives and accomplishments for the FY 1977 period are summarized in this report. This work was conducted for and funded by the Advanced Physical Methods Branch, Division of Energy Storage Systems, Office of Conservation, Department of Energy, Washington, DC.

As one aspect of the Department of Energy's (DOE) total energy conservation program, DOE is sponsoring the development of heat engine/flywheel hybrid vehicles. Incorporation of a high-speed flywheel energy storage system into a heat-engine-powered vehicle is integral to this goal. This hybrid system will be designed for peak energy storage and retrieval on demand to promote more efficient heat-engine operation and, also, to provide a system for regenerative braking. The composite flywheel and its containment-suspension mount system are the key elements being developed in the UCC-ND program.

Prior Work

The UCC-ND Composite Flywheel Development program was initiated at the Oak Ridge Y-12 Plant^(a) on May 1, 1976, with the primary objective for the FY 1976 - 1976T period being the design, fabrication, and testing of a prototype, vehicular-sized, composite flywheel. This initial effort led to the successful testing of the UCC-ND bandwrap flywheel design, seen in Figure 1, with a demonstrated energy density performance of 10.1 Wh/lb.¹

FY 1977 Program Objectives

The overall objectives of UCC-ND's program are to develop state-of-the-art, high-speed, composite flywheel and associated containment systems. The FY 1977 program addressed the following specific technical objectives:

1. Design and fabricate an adequate, but not necessarily optimum, vehicular-sized containment housing and a special load-cell-instrumented mounting suitable for installation in a spin test stand.
2. Design and fabricate an improved, prototype composite flywheel which will be available for evaluation and failure testing in the instrumented containment package. Efforts will be keyed on developments to increase the average energy density of the nominal 0.5-kWh flywheel and hub assembly to a level above the 10.1 Wh/lb achieved in FY 1976T.

(a) Operated by the Union Carbide Corporation's Nuclear Division for the Department of Energy.

Figure 1. BANDWRAP KEVLAR-49/EPOXY FLYWHEEL. (Fabricated in FY 1976T)

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The instrumented containment package and improved prototype composite flywheel were to be available at the completion of FY 1977 for subsequent testing and evaluation. Failure testing of the flywheel and future similar flywheels in the instrumented containment/mount assembly can provide a minimum amount of needed data on transient loading and containment damage for design and development of a safe vehicular flywheel-containment system. Test and evaluation of the FY 1977 flywheel and containment assembly will be a portion of UCC-ND's FY 1978 program.

FY 1977 Development Plans

A development plan, consistent with completion of the FY 1977 program objectives, was prepared and followed (Figure 2). To mesh with other on-going activities, the plan was updated and modified periodically as required. The composite flywheel design and fabrication activities were conducted as noted, culminating with the fabrication of an improved "bandwrap"-design flywheel with a prestressed rim. Design and fabrication of the containment-mount assembly was also completed. Two related activities which have been deferred for inclusion in UCC-ND's FY 1978 program are instrumentation and static testing of the containment-mount assembly. The postponement of these activities

● ▲ Target dates achieved.

Figure 2. DEVELOPMENT PLANS FOR FY 1977. (Leading to the Fabrication of an Improved Composite Flywheel and Containment-Mount Assembly by September 30, 1977)

recognizes FY 1978 program objectives which include the design and fabrication of a spin-test facility at the UCC-ND Y-12 Plant. Instrumentation and calibration activities of the containment-mount assembly are more appropriately done in the spin-test chamber.

FLYWHEEL DEVELOPMENT ACTIVITIES

BANDWRAP DESIGN IMPROVEMENTS

Successful tests on the first bandwrap-design flywheel pointed up the need for design improvements to increase the energy-storage efficiency. The flywheel tested in the FY 76-76T program¹ showed a delamination approximately midway through the wall and completely around the circumference. There was also separation of two bands from the rim outer surface on the cylindrical section, indicated by a small amount of fiber pull from the rim. Both of these conditions were precipitated by the poor transverse strength of the composite.

These conditions lead to two needed design improvements: (1) a need for the rim to have a full outer contour to eliminate dependence on the rim-to-band bond; (2) the design must utilize a composite material which has adequate transverse strength to survive the imposed radial stresses, or incorporate design changes which reduce the radial stresses. Developments for the bandwrap design in FY 77²⁻⁴ include: (1) a transverse-strength characterization of several Kevlar-49/epoxy composite systems; (2) derivation of the catenary shape for the rim outer contour; (3) an analysis of a dead-weight-loaded thick rim; (4) the theoretical and experimental development of a prestressed rim; (5) an analytical evaluation of a hybrid Kevlar-29/Kevlar-49 composite thick rim.

Transverse-Strength Characterization

One of the critical factors in the application of thick-rim flywheels is the composite strength or failure strain transverse to the fiber. The first flywheel to be tested in this program (Figure 1) failed in transverse or radial tension in the Kevlar/epoxy rim. The failure occurred at a calculated stress level of approximately 1 ksi. Transverse-tension flexure tests reported in the FY 1976/76T *Composite Flywheel Development Completion Report*¹ on the material system used in the tested flywheel confirmed this failure stress level.

Literature data indicated a transverse strength of ~ 4 ksi, while this flywheel test generated transverse fractures at an ~ 1 -ksi calculated radial stress. A more extensive review of the literature data revealed that a prepreg material (3M Company "Scotchply" SP-306) was tested to establish the ~ 4 -ksi transverse strength. This prepreg is composed of Kevlar-49 fiber and 3M Company PR-286 epoxy resin (a 3M proprietary system). The resin requires a high-temperature cure and, thus, is not suitable for winding thick rims. The PR-286 resin reportedly bonds very well to the Kevlar-49 fiber. Some of the prepreg material was purchased for test and evaluation.

The epoxy system used thus far in the program is Dow Chemical Company DER 332 epoxy with Jefferson Chemical Company Jeffamine T403 hardener. Scanning electron microscope (SEM) views of a transversely fractured composite specimen are presented in Figure 3. Based on the smoothness of the epoxy surfaces where fibers had lain, there is very little bonding of the fiber and resin. There are also appearances of a lack of fiber "wetting"

by this resin. The key to achieving a 4-ksi transverse strength then appears to be promotion of a satisfactory bond to the fiber. The epoxy system that will bond must also be a low or room-temperature-cure system to be useful in thick-rim flywheel development.

(a) At 1000X.

(b) At 5000X.

Figure 3. SCANNING ELECTRON MICROSCOPE VIEWS OF THE KEVLAR-49/EPOXY (DER 332/T403) FRACTURE SURFACE IN TRANSVERSELY FAILED SPECIMENS.

Specimens used in the test results reported previously¹ were machined from wound cylinders; and, due to the difficulty in machining Kevlar/epoxy composites, these specimens were not of the highest quality. This problem may have contributed to the high data variability and degraded the tensile-strength values. New specimens were produced by fabricating rings with an as-wound cross section of a 0.500 by 0.500-inch nominal size. The rings were cut into three segments for flexure tests. Two of the Kevlar/epoxy material systems reported previously¹ were tested again with new rings.

Tests conducted during this fiscal year are summarized in Table 1. The first material-system results are 26% lower than those previously reported, but the variability has improved by 60%. The second material-system results are improved over those previously reported by 72%, and the variability is improved by 250%. An additional Kevlar/epoxy system and two S-glass/epoxy system flexure tests are also reported. The last material system is the 3M Company SP-308 Kevlar/epoxy prepreg. These specimens were straight-sided transverse tensile coupons. This material has a literature-reported strength of 4 ksi, but that value was not achieved in these tests. Scanning electron microscope views of the fracture surface in

this material are presented in Figure 4. In comparison with the SEM views of the composite fracture surface shown in Figure 3, there appears to be some improvement in resin wetting and bonding to the fiber.

Table 1
TRANSVERSE TENSILE STRENGTH OF SELECTED COMPOSITES

Material System	Quantity (vol %)			Tensile Strength (psi)	Maximum Hoop Compressive Stress (ksi)	Modulus of Elasticity (msi)
	Fiber	Resin	Void			
Kevlar-49/ (DER332/T403)	74.5	22.9	2.6	870 (CV = 18.2%) (2)	23	0.89
Kevlar-49/ (XD7818/D230/A398)	76.0	22.8	1.2	1000 (CV = 12.8%)	26	0.94
Kevlar-49/ (DER332/T403/A398)	72.9	24.6	2.5	960 (CV = 10.9%)	25	0.82
S-glass/ (DER332/T403)	74.0	22.2	3.8	2920 (CV = 1.3%)	75	2.95
S-glass/ (ERL2258/MPDA)	74.9	22.1	3.0	3380 (CV = 10.0%)	89	3.18
3M SP308 Kevlar-49 prepreg				1975 (CV = 24.6%)		0.72

(1) DER332 and XD7818 are products of Dow Chemical; T403, D230, and A398 are products of Jefferson Chemical; ERL2258 is a product of Union Carbide; MPDA is metaphenylenediamine, a standard chemical product; 3M SP308 is a product of 3M.

(2) Coefficient of variation, CV = (standard deviation/mean) x 100 percent.

Experience with low-void-content (<0.5%) S-glass/epoxy composites reveals that the results reported in Table 1 are low by a factor of two or three. If special resin-impregnation techniques are employed to lower the void contents of these composites, certainly the S-glass/epoxy data would improve; and, possibly, the Kevlar/epoxy data would also improve.

The maximum hoop compressive flexure stress is monitored to be sure that the composite fails in transverse tension. The stress/strain response was linear to failure in all these tests, so the failure strains may be easily calculated.

Rotational Catenary Contour

When the flywheel is rotating, any portion of the band that is unsupported will tend to displace such that it is in its minimum-potential-energy configuration. If the band were represented as a flexible, inextensible cord that is freely spinning about the axis of rotation (no rim), then it would take a shape similar to that in Figure 5. One approach then is to shape the rim contour to fit in this envelope. However, this design uses too much axial space. Since the band is attached to the rim, it may lie in a normal path from the rotation axis to the rim and still be in a minimum-potential energy configuration.

SM-77-0046-10

(a) At 500X.

SM-77-0046-9

(b) At 1000X.

Figure 4. FRACTURE SURFACE OF THE 3M SP308-KEVLAR/EPOXY PREPREG TRANSVERSE TENSILE SPECIMEN.

Figure 5. ROTATIONAL CATENARY SHAPE OF AN INEXTENSIBLE, FLEXIBLE CORD THAT IS SPINNING ABOUT THE AXIS OF ROTATION.

The rotational catenary portion of the band will maintain its shape as the flywheel spins and, thereby, apply no tensile stress to the interface bond from the band to the rim. The fact that the rim expands at a greater rate than the band will induce compression in the interface along the radiused corners and likewise along the catenary-shaped interface. There is a significant shear deformation of the interface layer, however, and this action must be accommodated. Since the interface is in normal compression, the interface layer may be a low-modulus material to accommodate the shear.

It is also desirable to maintain the rim width out to as large a radius as practical. For example, the first bandwrap flywheel was constant in width to a 9.000-inch radius, where the outer-corner radii (1.000 inch) of the rim started. By starting the rotational catenary shape at a tangent to this corner radius, a suitable contour can be derived which maintains a near-rectangular-rim cross section. One such contour is shown in Figure 6. Derivation of the rotational catenary shape is given in Appendix A.

Figure 6. CONTOURED RIM, WITH THE ROTATIONAL CATENARY SHAPE TANGENT TO THE CORNER RADII AT THE 60-DEGREE ANGLE MAKING UP THE OUTER CONTOUR.

Analysis of the Dead-Weight-Loaded Bandwrap Design

For this flywheel design to be successful, the pressure-loading radial stress must reduce the rotational-loading radial stress to an acceptable level. The radial-stress distribution in a rotating ring of Kevlar-49/epoxy composite with a 0.75 radius ratio is plotted in Figure 7. The radial-stress distribution for internal-pressure loading is also plotted, and the sum of these for the specific loadings plotted is illustrated by the dashed line. The corresponding hoop-stress distributions are given in Figure 8. One obvious drawback to the pressure loading is the increased inside-radius hoop stress. The magnitudes of rotational loading and pressure loading were selected to give a maximum hoop tensile stress of 260 ksi and a maximum radial tensile stress of 1 ksi. The maximum radial tensile stress occurs at a radius ratio of 0.94; or, in this specific case, $r = 9.4$ inches. The relative loading magnitudes may also be selected such that the radial stress is compressive throughout, but this condition results in excessive radial compressive stresses and an even higher peak-inside-hoop tensile

stress. The effective internal-pressure loading would be 24 ksi at an angular velocity of 3470 rad/sec (33,155 rpm).

Figure 7. RADIAL-STRESS DISTRIBUTION IN A KEVLAR-49/EPOXY RING. (Ring has a 15-Inch Inside Diameter, a 20-Inch Outside Diameter, and Rotates at 37,800 rpm with a Dead-Weight Layer on the Inside Radius, Yielding an Effective Pressure of 15,900 psi)

The effective internal pressure is provided by a heavy low-modulus material layer of finite thickness placed on the rim inside radius. The layer thickness may be easily determined from centrifugal-force calculations for a given material density and required pressure, assuming that the layer cannot support any of the load. If lead is used with a specific

weight of 0.41 lb/in^3 to give a pressure of 15.9 ksi, the layer thickness is 0.130 inch. To provide a pressure of 24 ksi requires a 0.200-inch-thick layer.

The energy-per-unit-weight storage efficiency is decreased by the use of dead-weight loading. If the rim had adequate radial strength to allow the inside radius hoop stress to reach 260 ksi, the theoretical energy stored per unit weight (E_w) would be 72 watt-hr/lb at a rotational speed of 45,500 rpm, neglecting any support-structure components for the rim. The E_w for the rim with a dead-weight layer of lead, 0.130 inch thick, is 46 watt-hr/lb at a speed of 37,800 rpm. If only the composite weight (wc) in the rim is used, the E_{wc} becomes 63 watt-hr/lb. The E_w for the rim with a dead-weight layer of lead 0.200 inch thick is 35 watt-hr/lb; and, for the composite weight alone, E_{wc} is 54 watt-hr/lb at a speed of 33,250 rpm.

Aside from the control of radial stress by dead-weight loading, there are two other potential benefits: One is the lower operating speed required to store the required energy. This condition simplifies the dynamic behavior with respect to the number of critical frequencies encountered and lowers the probability of fiber sling at high peripheral velocities. A second potential benefit is the use of low-cost material for the dead weight. This achievement allows better use of the higher values of E_{wc} given previously when looking at the economic aspects of flywheels.

A dead-weight-loaded bandwrap design was analyzed by finite-element methods. The model, seen in Figure 9, is symmetrical about the R axis. The dead-weight layer is seen on the rim inside radius and stops short of fully covering the rim inner surface to allow for the band-winding tooling. It has a 45-degree sloped edge in anticipation of metal flow during spin tests. The overall rim width is reduced from that of the design tested earlier. The reduction in width corresponds with the increased energy storage in the dead-weight layer. The rim outer contour has a rotational catenary shape that comes tangent to the 1.000-inch corner radius at an angle of 60 degrees from the side.

Figure 8. HOOP-STRESS DISTRIBUTION IN A KEVLAR-49/EPOXY RING. (Ring has a 15-Inch Inside Diameter, a 20-Inch Outside Diameter, and Rotates at 37,800 rpm with a Dead-Weight Layer on the Inside Radius to Yield an Effective Pressure of 15,900 psi)

Figure 9. FINITE-ELEMENT MESH FOR ANALYSIS OF A DEAD-WEIGHT-LOADED BANDWRAP DESIGN WITH A ROTATIONAL CATENARY OUTER CONTOUR.

The analysis was conducted in the same manner as described in the previous report.⁴ The model was first run in the axisymmetric mode to find the rim stresses and displacements. Displacements of the rim cross section were input to the plane-stress model to find the band and interface stresses.

The stresses on the plane of symmetry in the rim at an angular velocity of 3050 rad/sec are illustrated in Figure 10. At this angular velocity, the flywheel stores 0.56 kWh of energy and has an E_w of 24 watt-hr/lb. The radial stress in this plane is seen to be well controlled. However, the highest radial stresses occur in the vicinity of the rim inside corner. This region does not get full benefit from the dead-weight loading. The maximum stress there is 1120 psi.

Several runs were made with different interface conditions. In every case where the interface or portion of the interface bondline is modeled as an epoxy or similar rigid material, a normal tensile stress on the interface is developed because of the strain differences between the rim and band across the interface. This condition also leads to compressive longitudinal stresses in the band. The best model for the bondline is a low-shear-stiffness material. In this case, the normal stress on the interface is predominantly compressive, and the shear stress is negligible. The interface normal-stress distribution for this case is indicated in Figure 11; longitudinal stresses in the band are illustrated in Figure 12.

Figure 10. STRESSES ALONG A RADIAL LINE IN THE PLANE OF SYMMETRY BY FINITE-ELEMENT ANALYSIS OF THE MODEL IN FIGURE 9. (The Angular Velocity is 29,100 rpm)

The analysis results on this design are satisfactory, but the design should be changed to allow the dead-weight layer to get closer to the rim inside corner. This design would lower the transverse stress in the corner region. In order to use the existing band-winding tooling, however, the rim inside radius would be increased by the dead-weight layer thickness. After analysis of this change, a flywheel may be built and tested.

Kevlar-29/Kevlar-49 Hybrid Rim

One of the concepts for controlling radial stress is the use of graded elastic properties in the rim. If the hoop modulus, in particular, could be continuously variable with radial position from a low value at the inside radius to a high value at the outside, the radial stress could be theoretically eliminated. In the practical sense, however, this material (with a continuously variable modulus) does not exist.

Figure 11. INTERFACE NORMAL PRESSURE BETWEEN THE RIM AND BAND OF THE MODEL IN FIGURE 9. (The Interface Material has Low Shear Stiffness, and the Angular Velocity is 29,100 rpm)

Figure 12. LONGITUDINAL STRESS IN THE BAND PORTION OF THE MODEL IN FIGURE 9. (The Interface Material has Low Shear Stiffness, and the Angular Velocity is 29,100 rpm)

There are a number of high-performance composite materials with different values of longitudinal elastic modulus such as fiberglass, graphite, boron, and two types of Kevlar. The two types of Kevlar available have modulus ratios that look particularly beneficial for combination in a thick rim. The fiber modulus of Kevlar-49 is reported to be 19 Msi, (19×10^6 psi), that of Kevlar-29 is 9 Msi.⁵ The composite hoop moduli reported were 11.0 and 7.25 Msi, respectively, at 60 vol % fiber. (The value on Kevlar-29 does not agree with the rule-of-mixtures calculation.) The flywheel rims have had a fiber volume fraction of approximately 70%. Ratioing the given moduli for a 70% fiber composite gives moduli

of 12.8 and 8.5 Msi, respectively. These values agree well with the experimental values determined from in-house tests.

Assuming that the rim is made of Kevlar-29 in the inner half and Kevlar-49 in the outer half, the peak radial tensile stress in a homogeneous rim occurs very near the rim midpoint. Using the previously defined elastic properties, calculator solutions for the orthotropic rotating ring were obtained for given radius-ratio rims. By trial and error, the overall rim/radius ratio was found which matched displacements at the Kevlar-29 and Kevlar-49 interface. This radius ratio is 0.782 (r_i/r_o). The interface, then, has a zero radial stress. The maximum radial tensile stress is 1.3 ksi in the outer half at a speed where the hoop stress is 250 ksi. The corresponding rim of all Kevlar-49 has a radial stress of 5.9 ksi at that speed. If the overall radius ratio on the hybrid rim is lower than 0.782, interface tension exists and adds to the maximum radial tensile stress; if the overall radius ratio is higher (thinner rim), interface compression exists and decreases the radial tensile stress.

This Kevlar combination looks like it would make a potentially effective hybrid rim. It only needs a more detailed design study to select the proper combination. Obviously, the individual rim portions could be prestressed, and an interference condition could be built in at the Kevlar-29/Kevlar-49 interface.

Prestressed Rims

The prestressed rim will allow lower-radius-ratio rims to be utilized in flywheels without premature delamination failure. From a safety standpoint, the delamination failure mode may be desirable if it does not precipitate catastrophic failure. If this mode is desired, then the design may be adjusted to fail by delamination at a suitable margin prior to hoop failure. This failure mode may be accomplished by building in the prestress just needed for the flywheel operating speed range.

The rotational stresses in an orthotropic ring are illustrated in Figure 13. These curves are for a Kevlar-49/epoxy composite of 0.75 radius ratio. This radius ratio is lower than would be allowable when based on the transverse tensile strength and, thus, the homogeneous ring with no prestressing would fail by delamination at a low speed.

The amount of prestress needed is that which balances the radial tensile stress at the desired rotational speed. If the desired speed is the point of hoop failure, then the prestress magnitude is equal to the radial stress minus the radial tensile strength. At this level there is simultaneous radial and hoop failure. The prestress needed to produce this condition in the ring of Figure 13 is noted in Figure 14. Also notice that the hoop-stress distribution is not leveled but is reversed, with the highest hoop stresses on the outer radius.

A prestress distribution in a multiregion cylindrical pressure vessel is normally accomplished by interference-fit assembling of a set of thin cylinders, with the magnitude

of interference for each cylinder determining the final prestress distribution. This approach would be difficult to carry out in practice with Kevlar-49/epoxy cylinders.

Figure 13. ROTATIONAL STRESSES IN AN ORTHOTROPIC RING, CONSTRUCTED OF KEVLAR-49/EPOXY COMPOSITE.

The most direct control on prestress in fiber composites is through variation of the winding tension. A theory for residual stress control or elimination by programmed winding tension was presented by Liu and Chamis.⁶ This theory assumed that the residual stresses were directly a result of winding tension alone and neglected other variables. In the normal filament-winding process, the other variables such as resin shrinkage, resin flow, and orthotropic thermal stresses usually dominate the generation of residual stresses. Furthermore, a significant portion of the winding tension is relaxed almost immediately upon application to the winding surface. Based on these facts, the

Figure 14. IDEAL RADIAL PRESTRESS AND THE RESULTING TOTAL RADIAL AND HOOP STRESS DISTRIBUTIONS.

theory could be applied if the ring was built up by winding and curing thin layers and using a winding tension that would result in the correct retained tension in each cured layer. The equations given by Liu and Chamis for the final stress state after cure and removal from the mandrel are:

$$\sigma_r = \frac{2sb^2c^{s-1}f(b)}{c^{2s} - b^{2s}} \left[\left(\frac{c}{r}\right)^{s+1} - \left(\frac{r}{c}\right)^{s-1} \right] - b^{s-1} \left[\alpha \left(\frac{b}{r}\right)^{s+1} + \beta \left(\frac{r}{b}\right)^{s-1} \right] f(r), \text{ and} \quad (1)$$

$$\sigma_\theta = T(r) - \frac{2s^2b^2c^{s-1}f(b)}{c^{2s} - b^{2s}} \left[\left(\frac{c}{r}\right)^{s+1} + \left(\frac{r}{c}\right)^{s-1} \right] + sb^{s-1} \left[\alpha \left(\frac{b}{r}\right)^{s+1} - \beta \left(\frac{r}{b}\right)^{s-1} \right] f(r), \quad (2)$$

where:

σ_r represents the radial stress,

σ_θ the hoop stress,

$T(r)$ the winding tension as a function of radius,

E_r the radial modulus of the composite,

E_θ the hoop elastic modulus of the composite,

E_m the elastic modulus of the mandrel material,

a the mandrel inside radius,

b the mandrel outside radius,

c the composite outside radius,

r the radial coordinate,

$\mu_{\theta r}$ the composite Poisson's ratio ($\epsilon_r/\epsilon_\theta$ for applied σ_ϵ), and

μ_m the mandrel Poisson's ratio.

In addition:

$$s = \sqrt{E_\theta/E_r}, \quad (3)$$

$$B = \frac{E_\theta}{E_m(b^2 - a^2)} \left[(1 - \mu_m)b^2 + (1 + \mu_m)a^2 \right] \quad (4)$$

$$\alpha = s - \mu_{\theta r} - B \quad (5)$$

$$\beta = s + \mu_{\theta r} + B \quad \text{, and} \quad (6)$$

$$f(r) = \int_r^c \frac{\rho^s T(\rho) d\rho}{\beta \rho^{2s} + \alpha b^{2s}} \quad \text{(where } \rho \text{ represents the radial coordinate variable of integration).} \quad (7)$$

These equations are developed by assuming continuous variation of control variables, but they should reasonably approximate the conditions for step-wise changes.

Another approach is to solve the simultaneous equations generated by the interference-fit assembly where the interference fit of each layer is proportional to the retained average hoop tension in the layer. These equations have been used with optimization-search methods to calculate parameters for prestressed multiregion high-pressure cylindrical vessels in the pressure vessel industry.⁷

Although the theory looks well developed and straight forward, it remains to be proven experimentally. As stated before, the prestress produced in a fiber composite ring is a function of several process variables, not just the winding tension. The resin flow and shrinkage during cure, orthotropic thermal stresses (if a heat-curing resin is used), mandrel stiffness, winding speed, and tension relaxation are some of the factors affecting the resultant prestress.

Rather than develop a theory to account for all these variables, an experimental approach is taken here. This approach establishes the prestress that exists in a wound-and-cured layer as a function of the applied winding tension while trying to hold the other variables reasonably constant. An empirical relationship between applied winding tension and average layer prestress is obtained. This relationship can be used with the theory of Liu and Chamis or with the equations developed for multiregion cylindrical vessels⁷ to determine the applied winding tension schedule required for the needed prestress.

A thin steel mandrel was instrumented with strain gages for measuring the tension relaxation during winding. A test winding was initiated, and mandrel strains were recorded at specific thickness increments. The average hoop tension versus wound thickness was plotted (Figure 15). The tension change due to resin cure at the final wound thickness is also shown. A series of 6-inch rings, 0.125 inch thick, were wound at five different tension levels. The materials were 380-denier Kevlar-49 yarn and DER 332/T403 (100/43 pbw) epoxy resin, cured at room temperature. The mandrel was a rigid steel disc. The rings were strain gaged and then removed from the mandrel to measure the retained tension. The resulting relation is presented in Figure 16. By

Figure 15. AVERAGE HOOP TENSION IN A LAYER BEING WOUND WITH KEVLAR-49/EPOXY, USING 380-DENIER YARN AT A 1000-GRAM WINDING TENSION.

extrapolation, winding at 275 grams in this thin ring would leave a zero retained tension.

The curve is very linear, with a higher retained tension fraction at higher winding tensions. Several of the rings were hydro tested and the results showed no evidence of strength degradation due to the high winding tension.

Several test layers were wound and cured on an instrumented mandrel to check the relation in Figure 16. Several important factors in processing were discovered in these tests. The cylindrical mandrel needed to be free from edge restraint due to the side plates and yet stay centered in the assembly. This requirement was accomplished by bonding rubber sheeting between the mandrel and side plates. There was also restraint of the composite layer edges because there was some bonding to the side plates even though

Figure 16. RELATION BETWEEN THE APPLIED WINDING TENSION AND THE RETAINED TENSION IN WOUND AND CURED THIN RINGS OF KEVLAR-49/EPOXY COMPOSITE MATERIAL.

they were coated with a mold release. This problem was solved by applying Teflon sheeting to the side-plate faces. These factors help assure a uniform prestress across the layer width.

Three small-scale test rings were wound and evaluated. Each ring had a six-inch inside diameter, an eight-inch outside diameter, and had ten wound and cured layers. The first two rings failed by inner-layer hoop-compressive-stress failure when the ring was completed or near completion; the third test ring was satisfactory and established the parameters to be used in a full-scale rim fabrication.

The first test ring was wound on a six-inch steel mandrel that was 40 mils thick. The mandrel dimensions and other parameters were used in Equations 1 through 7, along with a winding tension schedule to determine the predicted prestress conditions.

The mandrel was a steel cylinder (6" OD x 0.040" T). The mandrel was strain gaged on the inside, and the strains were monitored as the layers were wound. Ten layers (each 0.100 inch thick) were wound. A layer was wound and allowed to cure at room temperature before winding the next layer. Strains were monitored when the layer was completed and still wet, then when it was cured. There was only a slight relaxation during layer curing. The strain increment when a layer is applied and the properties of the composite-plus-mandrel cylinder to which the layer is applied are used to calculate the actual retained tensile stress in the applied layer. The calculation is performed for each layer and the results superimposed to find the resultant prestress distributions.

Table 2 summarizes the calculated-versus-realized parameters in the first test ring. The actual retained tensile stress in the layers deviated significantly from the desired levels. The reason

Table 2
PRESTRESS RESULTS IN TEST RIM 1
(380 Denier Kevlar-49 Yarn)

Applied Winding Tension (g)	Retained Layer Stress (ksi)		Layer Inside Radius (in)	Radial Stress (ksi)		Hoop Stress (ksi)	
	Theoretical	Experimental		Theoretical	Experimental	Theoretical	Experimental
275	0	2	3.0	0	0	-66.6	-69.0
600	11	11	3.1	-2.12	-2.23	-54.1	-53.7
925	22	20	3.2	-3.66	-3.83	-38.2	-37.0
1250	33	33	3.3	-4.61	-4.84	-21.2	-15.9
1600	44	41	3.4	-5.00	-5.17	-3.5	1.2
1925	55	72	3.5	-4.84	-4.98	15.0	46.8
1925	55	50	3.6	-4.17	-3.54	23.2	32.6
1925	55	50	3.7	-3.33	-2.57	30.7	40.3
1925	55	27	3.8	-2.34	-1.43	37.7	20.6
1925	55	36	3.9	-1.23	-0.89	44.6	34.6
			at r = 4.0	0	0	51.3	34.6

for this deviation is not clear; but it could be due to a variation in the winding tension, variation in the resin viscosity, and/or inaccurate strain-gage readings. The control seems to

be much more erratic after the mandrel has yielded, which occurred as Layer 5 was applied. This result would seem to indicate inaccurate strain-gage readings due to plastic-mandrel strain.

The theoretical-versus-experimental prestress distributions are shown by the graph of Figure 17. The deviations previously mentioned appear in these distributions. If the strain-gage readings are not reliable, the actual distributions may be closer to the theoretical. The inner layers of the ring experienced a hoop-compressive, microbuckling-type failure when the steel mandrel was removed. This action occurred at a calculated stress level between 65 and 69 ksi. Measurement of the ring inside diameter compared well with the total strain readings, confirming this stress level.

Failure of the inner layers prevented a residual stress analysis by incremental material-removal techniques.

By manipulation of parameters in the equations predicting the prestress, it is determined that a lower-stiffness mandrel produces the highest prestress level for a given maximum winding tension. Additionally, the best tension schedule to balance the radial-stress distribution in the 0.75 radius-ratio rim is a linear variation from zero tension in the inner layer to a maximum tension in the outer layer.

A second winding-tension schedule was developed for test and evaluation. Plexiglas was selected for the mandrel material to give a low stiffness as well as provide a linear stress/strain response through the strain range imposed during the test winding. The theoretical schedule limits the inner-layer compressive stress to 50 ksi and produces a maximum radial compressive stress of 4.1 ksi.

Table 3 summarizes the Schedule 2 parameters and gives the results of the test winding. The retained tensile stress in the wound layers, on the average, exceeded the desired levels by 19%. The resultant stress distributions are compared with the theoretical distributions in Figure 18. The agreement is much better on this test ring, but the excess retained tension caused this ring to fail also by an inner-layer-microbuckling compressive failure. The failure occurred while the final layer was curing and the inner layer stress was 65 ksi compression.

These two test rings demonstrate the capability to achieve prestress levels that can effectively eliminate radial tensile stress on the composite rim. The control as yet is not satisfactory, but should improve with additional development.

Figure 17. THEORETICAL VERSUS EXPERIMENTAL RESIDUAL STRESS DISTRIBUTION IN TEST RIM 1.

Table 3
PRESTRESS RESULTS IN TEST RIM 2
(380 Denier Kevlar-49 Yarn)

Applied Winding Tension (g)	Retained Layer Stress (ksi)		Layer Inside Radius (in)	Radial Stress (ksi)		Hoop Stress (ksi)	
	Theoretical	Experimental		Theoretical	Experimental	Theoretical	Experimental
250	0	0.7	3.0	0	0	-50.0	-66.2
475	6.5	8.3	3.1	-1.60	-2.14	-42.4	-50.2
650	13.0	14.4	3.2	-2.80	-3.64	-31.6	-37.5
850	19.4	21.4	3.3	-3.59	-4.67	-20.6	-24.3
1040	25.9	30.0	3.4	-4.02	-5.25	-9.5	-9.5
1250	32.4	34.8	3.5	-4.11	-5.37	1.7	1.1
1425	38.9	44.0	3.6	-3.87	-5.19	13.2	16.8
1625	45.4	59.5	3.7	-3.34	-4.60	25.2	40.6
1825	51.9	65.4	3.8	-2.51	-3.41	37.6	53.1
2000	58.4	75.0	3.9	-1.40	-1.91	50.7	74.3
			at r = 4.0	0	0	58.1	74.3

A third test ring was fabricated and evaluated. The winding-tension schedule was scaled down to prevent the hoop compressive failure of the inner layers which occurred in the first two test rings. This test ring was successfully fabricated with none of the inner-layer compressive failure experienced on the first two rings. The winding-tension schedule was scaled to produce a maximum inner-layer hoop compressive stress of 3.5 ksi. However, the retained tensions, based on the strain-gage results, again exceeded the expected values. The results calculated from the strain-gage measurements during fabrication indicate an inner-layer hoop compressive stress of 50 ksi and a maximum radial compressive stress of 4 ksi.

Figure 18. THEORETICAL VERSUS EXPERIMENTAL RESIDUAL STRESS DISTRIBUTION IN TEST RIM 2.

The theoretical and experimental prestress results from Test Ring 3 are reported in Figures 19 and 20. The negative of the radial rotation stresses for two different rim speeds is also shown in Figure 19. The lower speed is the point where the theoretical prestress results in a net zero radial stress; the higher speed is the point where the experimental prestress results in a net zero radial stress. The total hoop stresses (rotation plus prestress) are also indicated in Figure 20 for the same two speeds.

The total theoretical energy stored per unit weight, based on the rim weight alone, is 31 Wh/lb at 74,500 rpm and 44 Wh/lb at 87,800 rpm for a six-inch-inside-diameter by eight-inch-outside-diameter rim (29,800 and 35,100 rpm, respectively, for a 15 x 20-inch

rim). Even though the failure mode in this case would still be by delamination, it offers a very significant improvement over the bandwrap design performance.

Figure 19. THEORETICAL AND EXPERIMENTAL RADIAL PRESTRESSES AND THE NEGATIVE OF THE ROTATIONAL RADIAL STRESSES. (These Stresses would Respectively Balance in a Three-Inch-Inside-Radius by Four-Inch-Outside-Radius Kevlar-49/Epoxy Composite Ring)

The successful completion of small-scale Ring 3 led to the fabrication of a full-scale rim. A Plexiglas mandrel was scaled to the full-size rim, with the mandrel outside diameter oversized so that the rim inside diameter would be 15.000 inches when completed. A set of test layers were wound onto the mandrel and cured in succession, with each layer 0.25 inch thick.

The purpose of this study was to check the relation of Figure 16 for the larger mandrel diameter and higher yarn denier. Tension levels were 1900, 3700, 5600, and 7500 grams. The resulting data points are plotted in Figure 21 and compared with the relation from Figure 16. Fiber buckling occurred while the first layer was being wound, due to the low mandrel stiffness. That action probably caused the first data point to be low. Compressive fracture of the inner layer occurred while the fourth layer was being applied and caused the fourth data point to be invalid. Agreement with the relation of Figure 16 is not outstanding, but there was not enough time to generate new data, and the good data provide a reasonable confirmation of the relation.

The winding-tensile-stress schedule used in small-scale Ring 3 was factored down by 15% to be conservative and used for the full-scale rim winding. The mandrel was 15.042

inches in outside diameter by 13.793 inches in inside diameter, with strain gages mounted on the inside. There was rubber sheeting bonded to the side plates and mandrel, and the

Figure 20. THEORETICAL AND EXPERIMENTAL HOOP PRESTRESS DISTRIBUTION AND THE TOTAL HOOP-STRESS DISTRIBUTION OF THE SPEEDS WHERE THE RADIAL PRESTRESSES PRODUCE A NET ZERO RADIAL STRESS.

side-plate faces were covered with Teflon film. The winding progressed smoothly, with three layers wound each week.

Figure 21. RETAINED TENSION DATA AS A FUNCTION OF THE APPLIED WINDING TENSION FROM A TEST CYLINDER, USING 1420-DENIER YARN.

The prestress condition calculated from the measured strains are plotted in Figure 22, and the data are tabulated in Table 4. The calculations are a layer-by-layer determination of the stresses in the existing and applied layers. The equations for the orthotropic ring with internal and external pressure were programmed on a hand calculator, and the data were reduced. The pressure at the mandrel/inner layer interface was calculated from the strain readings and the mandrel material properties. The external pressure on the existing layered ring applied by the new layer is determined from the strains and the average elastic properties of the existing ring. The internal and external pressures on the existing ring yield stresses to superimpose on the stresses already present. The external pressure on the existing ring is used to calculate the tensile stress held in the new layer.

ANALYSIS OF THE DELTAWRAP DESIGN

The "deltawrap" flywheel design incorporates a rim and a hub, and an overwrap connecting the two. The elements of the design are illustrated in Figure 23. The rim is similar to rims used in other flywheel designs,¹ and many of the rim considerations are

Figure 22. PRESTRESS DISTRIBUTIONS AND TOTAL STRESS DISTRIBUTIONS AT TWO SPEEDS IN THE FULL-SCALE FLYWHEEL RIM.

Table 4
PRESTRESS RESULTS IN THE PROTOTYPE RIM
(1420 Denier Kevlar-49 Yarn)

Applied Winding Tension (g)	Retained Layer Stress (ksi)		Layer Inside Radius (in)	Radial Stress (ksi)		Hoop Stress (ksi)	
	Theoretical	Experimental		Theoretical	Experimental	Theoretical	Experimental
850	0	0.9	7.50	0	0	-30.0	-33.8
1400	3.9	5.7	7.75	-0.96	-1.33	-25.4	-24.3
1850	7.8	9.8	8.00	-1.68	-2.06	-18.9	-16.1
2250	11.7	12.6	8.25	-2.16	-2.51	-12.3	-10.0
2700	15.6	15.1	8.50	-2.41	-2.71	- 5.7	- 4.5
3150	19.5	25.4	8.75	-2.46	-2.76	1.0	10.2
3550	23.4	21.3	9.00	-2.32	-2.40	7.9	9.1
4000	27.2	28.0	9.25	-2.00	-2.09	15.1	19.5
4450	31.1	29.6	9.50	-1.51	-1.54	22.6	25.1
4950	35.0	31.7	9.75	-0.84	-0.83	30.4	31.3
			at r = 10.00	0	0	34.8	31.3

independent of this design. The hub simply provides a connection between the overwrap and driving shaft. The hub must be designed in close relationship with the overwrap design.

The key element in the deltawrap design is the overwrap. The overwrap performs several functions: The primary function is to connect the rim and hub and, thereby, the shaft; the overwrap also provides torque-load capability, rigidity in the rim/hub coupling to reduce dynamic problems, and external pressure on the rim to lower the rim hoop stresses.

If sufficient advantage can be taken of the external pressure applied to the rim by the overwrap, the energy storage capacity may be increased. The isotropic material flywheel may be fabricated in a shape derived to maintain a constant stress throughout. It is a more efficient energy-storage flywheel because the higher body-force loadings in the elements near the outside are carried by transfer back to elements near the hub. If the composite flywheel is fabricated by circumferential winding, most composites are too weak in transverse loading to carry the body forces back to the inner elements. The fibers in the overwrap are nearly radial. It is easily shown that the radial rod has its highest stress at the center of rotation and, that for the same speed, has a lower maximum stress than the rotating rim. Therefore, the radial rod can carry some additional load before failure.

Figure 23. DELTAWRAP FLYWHEEL CONCEPTUAL DESIGN.

From a simplistic view, the rod has the same radial displacement capability as the thin rim since it is at the same strain level, but it will not displace the same in the rotational gravity field because the radial rod is not uniformly stressed. Thus, there is automatically an interference condition when the radial fibers are wrapped around the rim and the assembly is spun. This situation applies additional tensile stress in the radial fibers and external loading to the rim. If the overwrap thickness is designed properly, the additional tensile stress in the radial fibers will not exceed failure levels. Furthermore, the external pressure on the rim will reduce its hoop stresses and allow the whole assembly to spin faster before it reaches the failure speed.

The rim element should have a radius ratio low enough to store the necessary amount of energy. If the radial strength is exceeded, then some design alternative such as those presented in earlier sections of this report must be selected. Of course, the rim design used must be analyzed in conjunction with the overwrap and hub to properly evaluate any proposed engineering design.

The hub simply connects the overwrap with the driving shaft. The design needs to be such that the hub is self locking to both the shaft and overwrap during spinup of the flywheel.

Modeling the Overwrap

The overwrap is a variable-thickness, variable-fiber-orientation, structural component. This combination complicates the analysis and requires preparation of a computer program to calculate the thickness profile and fiber-orientation angle as a function of the radial position. The thickness profile is then modeled by the finite-element mesh for analysis. The varying fiber orientation causes the material properties to vary and must be accounted for in the model.

Calculation of thickness and fiber angle is based on the geometrical considerations illustrated in Figure 24. At any given radial coordinate, r , the thickness calculation is based

Figure 24. GEOMETRICAL RELATIONSHIP FOR CALCULATING THE OVERWRAP-THICKNESS PROFILE AND FIBER ANGLE.

on the arc coverage of a band on the circle of radius, r , the number of bands, and the band thickness. The thickness is calculated by the equation:

$$t = r(\phi_2 - \phi_1) \frac{2N}{\pi r} t_b \quad (8)$$

where:

- t represents the thickness buildup,
- ϕ_2 and ϕ_1 the angles shown (Figure 22),
- N the number of bands in the set, and
- t_b the band thickness.

Along the inner band edge, the angle is:

$$\phi_1 = \sin^{-1} \frac{R_i}{r} \quad (9)$$

along the outer edge, the angle is:

$$\phi_2 = \frac{\pi}{2}, \text{ for } R_i \leq r \leq R_i + W, \text{ and} \quad (10)$$

$$\phi_2 = \sin^{-1} \frac{R_i + W}{r}, \text{ for } R_i + W \leq r \leq R_o \quad (11)$$

where:

- R_i represents the inner radius of the band set,
- W the band width, and
- R_o the outer radius of the band set.

The fiber-orientation angle with respect to the radial direction varies between ϕ_1 and ϕ_2 at the radial position, r . The average of ϕ_1 and ϕ_2 is specified as the fiber angle at radius, r . The thickness profile and fiber angle are plotted in Figure 25 for the case of $R_i = 0.750$ inch, $R_o = 10.0$ inches, $W = 0.750$ inch, $t_b = 0.003$ inch, and $N = 40$ bands.

The fiber-angle variation with the radial coordinate, of course, causes the material properties, with respect to the radial-hoop coordinates, to vary. The fiber-angle variation between ϕ_1 and ϕ_2 at a given radial coordinate also causes the properties to vary. There are a number of approaches to find the material-stiffness matrix which will represent the oriented material at the given radial position. The composite is modeled as a symmetric

angle-ply laminate. The first option is between bonded plies and unbonded (shear-failure) plies. On initial loading, the plies are bonded; and, prior experience with angle-ply wound

structures indicates that they remain bonded up to the ultimate laminate strength.

Figure 25. THICKNESS AND FILM-ANGLE PROFILE OF THE OVERWRAP ON THE DELTAWRAP FLYWHEEL.

The next option is whether to calculate the properties for the average fiber angle at a given radial position or calculate properties for the range of fiber angles at that position and average the material-stiffness matrices. Since the material-stiffness matrix transforms with the angle as a fourth-rank tensor, the resulting stiffness matrix for the two approaches will not be the same. The latter approach is more involved, but should be more representative of the average material at a given position and is the approach used. This approach demands that the calculations be per-

formed in the thickness and fiber-angle program rather than in the finite-element code. The average material properties are derived from this stiffness matrix and output with the thickness and average fiber angle for each specified radial coordinate. The computer program listing and the output corresponding to that in Figure 25 are given in Appendix B.

Finite-Element Analysis

The selected radii to output data from the thickness calculation program should correspond to the finite-element mesh used to analyze the structure. Since the material properties vary with the radial position, a different material type will have to be assigned at each radial position unless the property values are reasonably close for two or more positions.

A mesh-generation program was utilized to prepare the finite-element mesh shown in Figure 26. The upper illustration shows the boundaries of the hub, overwrap, and rim portions of the deltawrap design. Thin interface layers are between the hub and overwrap and the overwrap and rim to represent the bondlines. The lower illustration draws the actual quadrilateral mesh used in the analysis. The Z axis is the center of rotation for the axisymmetric model, and the model is symmetric about the R axis.

The hub is made of an isotropic metal, such as aluminum. The overwrap is made of incrementally positioned parallel fiber bands, resulting in full coverage of the rim circumference. The rim is made of a circumferentially wound unidirectional composite. The interfaces may be modeled with epoxy or rubber bonds.

Figure 26. FINITE-ELEMENT MODEL FOR THE DELTAWRAP FLYWHEEL DESIGN.

The finite-element code needs some special capabilities to properly analyze composite materials. The composite material is usually elastic and orthotropic. Most general finite-element codes have the ability to input orthotropic material properties. There is no generally accepted failure criterion for composites, so most codes have not put any in their program. The failure criteria that can be used usually require either the strain or stress components in the local fiber coordinate system. Unfortunately, these components are not usually output from any of the codes unless they are aligned with the overall coordinate axes or the element local axes. The components needed may be gained by altering the code or post processing.

A constant-strain, triangle, finite-element code in use for many years was modified to produce the fiber-coordinate stress components. This code was used to analyze the model in Figure 26. A new code, called ADINA,⁸ was also used to analyze the model. This code has orthotropic material capability and uses the more refined isoparametric element. A post processor is necessary to determine the local fiber-coordinate stress components. The analyses done with ADINA compare very closely with those done with the older code. This consistency lends confidence to the results of both, but the ADINA code will be used in the future because it has extensive dynamic and nonlinear-analysis capability.

Results from the analysis of the model in Figure 26 are presented in the next few drawings. The amount of data generated by a finite-element analysis can be enormous, so the presentation of data becomes a significant problem. A post-processing graphics program, called MOVIE,⁹ is used to develop plots of the data. The model has been magnified ten times in the axial direction for presentation purposes. A three-dimensional perspective view of the magnified Figure 26 mesh is shown in Figure 27. The hub, overwrap, and rim parts are exploded in this view.

The hoop or fiber-direction stress component in the rim is indicated in Figure 28, plotted as a surface above the element-mesh cross section. The maximum hoop stress is 215 ksi at the

Figure 27. MAGNIFIED AND EXPLODED DELTAWRAP FINITE-ELEMENT MESH.

inner surface. The more critical radial stress is plotted in Figure 29 above the mesh cross section. A prestressed rim would be necessary to prevent radial delamination at a low speed.

Figure 28. RIM HOOP STRESS DISTRIBUTION.

The overwrap stresses are plotted in Figures 30 and 31. The component transverse to the fiber direction in the lamination plane is displayed in Figure 30. This result indicates that significant transverse cracking will occur at a relatively low speed. Transverse microcracking may not be a problem since it has been a common occurrence in the filament-wound

Figure 29. RIM RADIAL STRESS DISTRIBUTION.

pressure vessel industry for years. If the composite had high enough transverse strain capability, there would be no cracking. The fiber-direction stress component, displayed in Figure 31, indicates a peak stress on the outside surface at the position where the overwrap loses contact with the hub. This component is relatively low at other locations, so the peak significantly reduces the overwrap performance.

Based on these results, some design changes should offer some improvement. The hub design could be changed to lower its stiffness where the overwrap and hub separate, thereby reducing the overwrap peak stress. It may be better to allow the overwrap thickness variation to be symmetric about a mid plane rather than forcing all the variation to the inside. These changes need to be evaluated by finite-element analysis before any firm conclusions can be drawn.

FABRICATION OF THE FY 1977 FLYWHEEL

The studies described in the preceding sections led to the selection of an improved bandwrap design with a prestressed rim for the FY 1977 flywheel. The full-scale flywheel

Figure 30. COMPOSITE STRESS PLOT OF AN OVERWRAP IN THE TRANSVERSE DIRECTION.

Figure 31. COMPOSITE STRESS PLOT OF AN OVERWRAP IN THE FIBER DIRECTION.

rim described in an earlier section was used. The rim was diamond-knife machined with a radiused outer corner and a rotational catenary-shaped outer contour, as illustrated in Figure 6. Coordinates of the contour are listed in Appendix C. The rim was assembled with the band-winding fixtures, and the bands were wound. The flywheel shown in Figure 32 will be tested late in FY 1978.

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Figure 32. FLYWHEEL WITH ROTATIONAL, CATENARY-SHAPED OUTER CONTOUR AND A PRESTRESSED RIM.

A performance estimate can be made that is based on the prestress results in Figure 22 and the rim rotational stresses. The estimate is that the rim's radial stress will remain

compressive up to a speed of 29,800 rpm and that the delamination failure will occur at a speed of 35,100 rpm. This estimate assumes that there is no failure of the bands or hub. The finite-element stress analysis indicates that this assumption is valid. The overall flywheel energy density will be 31 watt-hr/lb at 29,800 rpm and 44 watt-hr/lb at 35,100 rpm.

This performance estimate is based on the current prestress condition. Since the flywheel will not be tested for approximately nine months, there may be some creep-and-stress relaxation. A sample ring was parted from the as-wound rim for the purpose of doing a residual stress analysis by the incremental-material-removal method. This ring will be stored with the flywheel until the time of testing. The residual stress analysis done at that time will determine if there has been any significant creep problem. Of course, this problem would need a more thorough investigation for deciding the lifetime suitability of prestressed rims in flywheels.

CONTAINMENT ASSEMBLY

Quite obviously the immediate problems of developing a weight-efficient flywheel are associated with providing a composite flywheel which can operate reliably in the 20 - 40 watt-hr/lb range. However, if the intended vehicular application is noted, next in importance might be assuring the safety of personnel in the event of catastrophic failure of the flywheel.

In the case of isotropic flywheels, this problem is obviated by carefully designing to stress levels which are so low that reliability is assured. This approach has been quite successful since the material properties are well characterized and their implementation into full-size applications is well understood.

As evidenced by current performance levels of the composite flywheels, the behavior of these materials in the needed sizes and configurations is still being developed and investigated; hence, a more reliable way of providing safety of personnel may be to provide containment housings to contain fragments with mount designs capable of reducing the failure loads to acceptable levels. The intent of the test assembly described here is to provide the design data necessary to accomplish these tasks.

Design Considerations

The basic objective was to design a containment test assembly which would contain the fragments and measure the failure loads resulting from maximum speed bursts of full-scale (1.12 kWh) flywheels. Obtaining data from this "worst-case" situation was felt to be the most efficient and desirable means of accomplishing the following goals:

1. Evaluate the magnitudes of the hazards afforded by composite flywheel failures.
2. Evaluate the feasibility of providing "fail-safe" containment and mounts for even this overspeed condition.
3. Optimize the containment and mount system for either the overspeed or operating-speed failures.

Noting that the intent of the assembly is to measure variables which are normally considered as design criteria for the assembly, a basic premise of the design was to provide versatility which would allow slowly increasing risks to the assembly itself. Conservative design of a containment ring does not provide difficult design problems; however, application of the same conservatism, coupled with uncertainties concerning flywheel failure modes and times to try to predict upper-bound failure loads, can quickly provide seemingly unrealistic design criteria.

Consider the curve of Figure 33, which is a plot of the average torque required to stop the UCC-ND bandwrap flywheel as a function of the time over which the load is applied (wreck

Figure 33. CALCULATED AVERAGE TORQUE REQUIRED TO STOP THE FLYWHEEL AS A FUNCTION OF FAILURE TIME.

time). From these simplistic considerations of angular impulse, it is seen that if the wreck time corresponds to the time required for an elastic wave to travel across the rim cross section (about $20 \mu\text{sec}$), then the torque would be on the order of $74 \times 10^6 \text{ ft-lb}$. This would correspond to an axial and radial "unzip" mode. If, instead, the flywheel failure mode consists of first radially delaminating into rings and then hoop failure of the individual rings, the wreck time grows by orders of magnitude and the expected torque

correspondingly decreases to the order of thousands of ft lb. Faced with these considerations, it was felt that the basic objective could best be accomplished with a test assembly having the basic concepts illustrated in Figure 34.

Figure 34. CONTAINMENT TEST ASSEMBLY CONCEPT.

Consider a containment assembly mounted on a torque tube which would serve as the load transducer. The shaking and torsional loads occurring during flywheel failures would be determined by measuring the transient torsional and bending deformations in the tube with strain gages. The containment assembly would consist of a housing with top and bottom plates to provide axial containment and a removable ring to provide radial containment. The removable feature of the ring serves dual purposes: First, it allows the investigation of various materials, configurations, and sizes of containment; second, it provides versatility in mount conditions by permitting the ring to either be freely rotatable and serve as a momentum-decoupling inertia ring, or to be tied down with various degrees of restraint. This versatility makes it possible to probe the magnitudes of the failure loads with minimum risk to the test assembly. There is no suspension system, for it is intended that the assembly be installed in a turbine-driven test stand for companion data gathering during flywheel development testing.

Establishing the Design Loads

Radial Containment - As in the Lockheed¹⁰ containment designs, it was assumed that a bihub rupture of the flywheel occurs and produces a hydrostatic pressure on the inside of the containment ring (Figure 35). The magnitude of the pressure is established by requiring

Figure 35. BIHUB FAILURE MODE, PRODUCING HYDROSTATIC PRESSURE ON THE INSIDE RADIUS OF THE CONTAINMENT RING.

that the centrifugal forces present in the flywheel at rupture be in equilibrium with the resultant force of the pressure distribution. For the assumed UCC-ND-flywheel, 1.12-kWh burst (Appendix C), these relationships predicted that a containment ring having the same axial length as the rim of the flywheel (3.4 inch) would have to sustain an equivalent uniform internal pressure of 40,330 psi. This pressure was used in the stress solutions for a thick-walled cylinder to size the radial thickness of the ring.

Axial Containment - The low-strength properties of composite flywheels in directions normal to the fiber orientations provides the possibility of large axial loads occurring during flywheel failures. Since the composites have transverse strengths which are typically only 2 or 3% of the fiber-oriented strength, it could be expected that if axial restraints are present, the radial "packing" occurring during the flywheel failure could provide significant axial thrust.

The transverse loads which are developed are analogous to the side-thrust earth loads developed on retaining walls from cohesive soils. Indeed, this analogy is applied to obtain an estimate of the axial loads developed during failure.

Just as in soil analysis¹¹ (where the transverse loads are found as a function of the normal load, the cohesion, and the internal friction), the axial load during flywheel failure is postulated to result from the radial compression of the flywheel rim and be a function of the shear strength and internal friction of the composite.

Using the radial pressures determined in the previous section and allowing for some axial expansion during failures of the flywheel, the predicted axial load (Appendix C) is 336,170 lb.

Torsional Load - The torsional load criterion is determined by assuming that the radial containment ring is free to rotate and hence serves as a momentum-decoupling device. This

assumption obviates the problem of having to design something for the tremendously large loads which were previously considered.

This arrangement leaves the axial loads as being the only loads directly applied to the containment structure. Calculation of the friction torque (Appendix C) developed from these axial loads on the containment lid and base plate, resulted in an anticipated torque of 206,613 ft lb.

Shaking Loads - The shaking loads are considered to be developed by the flywheel orbiting on the inner diameter of the containment ring. If the containment ring were laterally unrestrained, then the containment ring/flywheel mass system would move about the mass center. This provision would reduce the shaking loads to that provided by the radial friction forces between the containment ring base and the containment base plate during the radial motion of the ring.

However, an evaluation of the shaking loads was made by assuming that the containment ring is fixed. Classical solutions of an eccentric mass rotating inside another spring-mass system¹² show that, at high speeds, the deflection of the system and hence the loads is a function of the eccentricity, the masses involved, and the spring stiffness.

Although the previous criteria predetermined the majority of these variables, a check calculation (Appendix C) showed that, for the containment assembly as designed, the expected loads could be 9236 lb and that the bending stresses in the tube could permit a reduction of the overall containment assembly weight by a factor of 16 and still permit testing without damaging the torque tube.

Description of the Assembly

Seen in Figure 36 is a cutaway view of the test assembly with a flywheel indicated in position. Figure 37 shows the containment assembly with and without the top containment lid. All structural components were made from Type E-4340 steel and heat treated to R_C 38 - 42 to provide the desired strength and ductility characteristics.

The containment ring is 20 11/16 inches in ID, 26 11/16 inches in OD, and 4 3/4 inches long. The containment housing is a bolted assembly consisting of the containment baseplate, housing ring, and lid with a nominal thickness of 1 inch. Note that the lid and baseplate have reduced thicknesses (7/8 inch) at the radial position of the containment ring to remove axial joints from the radial "line of sight" and assure the full radial containment thickness for all debris.

This containment assembly can either be bolted directly to the mounting plate (as shown in the figure), or it can be clamped between flanges to permit evaluations of friction-braked mounting situations.

Figure 36. PRINCIPAL COMPONENTS OF THE CONTAINMENT ASSEMBLY.

(a) With Lid.

(b) With Lid Removed.

K/PH 77-4526

Figure 37. CONTAINMENT ASSEMBLY.

The torque tube is 16 inches long and has 12-inch-diameter flanges. The tube section is 4.22 inches in ID and 6.0 inches in OD. Attachment to the baseplate is accomplished with eight 1¼-7NC socket-head cap screws on a 9-inch bolt circle. Identical attachment is accomplished at the torque-tube-base/test-stand interface.

A compilation of the weights of the containment assembly is provided in Table 5. The total weight of the package is prohibitive at 776.1 lb; however, it is noted that sizings were accomplished with conservative failure modes for maximum-speed-burst situations, and it is expected that data obtained from the test program will allow a considerable reduction in weight.

Table 5
CALCULATED WEIGHTS OF THE
CONTAINMENT ASSEMBLY

Component	Weight (lb)
Containment Ring	299.9
Housing Lid	164.5
Housing Ring	117.9
Housing Baseplate	193.8
Total	776.1

Flywheel/Containment Assembly Testing Plans

A preliminary outline has been prepared for testing the FY 1977 composite flywheel-containment assembly. FY 1978 program objectives, which were defined in late FY 1977, include fabrication of a spin-test facility at the Y-12 Plant and testing the FY 1977 products in this facility. Additional detail or modification of the plan may be required to reflect specific facility characteristics. An outline of the plan is:

1. Instrument the containment-mount assembly with torsional/bending strain gages for acquisition of failure data.
2. Mount the instrumented containment assembly within the test stand in a relatively rigid mode.

3. Statically calibrate the instrumented containment assembly.
4. Low and/or high-speed force and couple balance the flywheel.
5. Mount the flywheel to the turbine shaft and position the subassembly within the containment assembly.
6. Attach eddy-current and photosensitive displacement probes as required to monitor axial and radial displacements.
7. Attach instrumentation leads as required for test data acquisition.
8. Attach appropriate tape-recording equipment for recording test data for post failure evaluations.
9. Pump the test stand down to a test vacuum level of < 1 torr.
10. Drive the flywheel to prefailure speeds for one or more cycles, assessing the dynamic behavior and, subsequently, drive the flywheel to its failure speed.
11. Evaluate flywheel-stability, energy-density-performance, and failure modes.
12. Evaluate containment damage and transient loading data.
13. Assess the overall test results and recommend additional testing, using the instrumented containment assembly for data acquisition.

Safety Considerations

Safety awareness is an integral element of the UCC-ND Composite Flywheel Program. As was indicated in the preceding discussion of vehicular containment design considerations, development of the composite flywheel containment system which will assure safe vehicular operation is vitally important to the development of hybrid vehicles. The transient loading data which will be generated in the UCC-ND Composite Flywheel/Containment System Development Program will be used in the design of a vehicular inertial energy storage system which is weight efficient, but possesses the structural integrity required for safe operation. Equally important is the design and operation of test facilities where prototype flywheels are to be evaluated. The large inertial energies which are to be stored, uncertainties relating to the dynamic characteristics and failure modes of untested design concepts, and the manner in which the stored energy is dissipated to the containment structure are factors to be considered in such testing programs. Design conservatism is no detriment to such facilities where the safety of personnel is a critical concern.

SUMMARY OF RESULTS

During the FY 1977 period, various design deficiencies of the FY 76-76T bandwrap flywheel were identified, and concepts to improve the performance of this flywheel

design were evaluated. Specific areas addressed include: (1) transverse-strength characterization of several Kevlar-49/epoxy composite systems; (2) derivation of the catenary shape for the rim outer contour; (3) analysis of a dead-weight-loaded thick rim; (4) theoretical/experimental development of the prestressed rim; (5) analytical evaluation of a Kevlar-29/Kevlar-49 hybrid composite thick rim.

An improved bandwrap-design flywheel was fabricated which included design modifications noted in Concepts 2 and 4. The estimated performance of this flywheel translates to an overall flywheel energy density of 44 watt-hr/lb at a rotational speed of 35,100 rpm. As was experienced with the FY 76-76T bandwrap flywheel, the anticipated failure mode is rim delamination, assuming that no failure of the bands or hub occurs. However, the projected performance of the FY 1977 product will be a significant improvement over the energy density of 10.1 watt-hr/lb achieved previously.

A preliminary analysis of the deltawrap flywheel design concept was completed. Potential benefits of the design include: (1) improved torsional loading capability; (2) reduction in dynamic problems due to improved rim/hub coupling rigidity; (3) reduction in rim-hoop stresses due to external rim pressure. This analytical evaluation included the development of codes to model the overwrap composite thickness profile and calculate material properties required as input for the finite-element analysis.

A prototype vehicular-sized flywheel-containment assembly was designed and fabricated. The conservatively designed assembly includes provisions for instrumentation which will be used to measure flywheel failure loads. These loads, when quantified, will be used as design input for future flywheel containment-suspension-mount assemblies which will be weight efficient, but will permit safe vehicular operations. Both the containment assembly and the improved bandwrap flywheel will be tested and evaluated as a portion of the UCC-ND FY 1978 Composite Flywheel Program.

APPENDIX A

ROTATIONAL CATENARY BAND SHAPE

The shape of a flexible cord in a rotational potential field is the shape naturally assumed by the cord because of the body forces that act and its negligible resistance to bending. This is a problem comparable to the hanging cable (called a "catenary") suspended at two points in a constant gravitational field.

Consider an inextensible, flexible cord of a length, L , attached at two points (x_0, y_0) and (x_1, y_1) . The potential energy of the cord in the rotational potential field is proportional to the integral:

$$V = \int \frac{1}{2} \mu \omega^2 x^2 \sqrt{1 + (y')^2} dx \quad , \quad (A-1)$$

where:

V represents the potential energy,

μ the cord mass per unit length,

ω the angular velocity,

x the radial coordinate from the axis of rotation, and

y the coordinate along the axis of rotation.

The problem is to minimize the potential energy, subject to the constraint of the fixed length, L . This problem belongs to the isoperimetric class of problems from calculus of variations. The modified-potential-energy integral is formed, using the Lagrangian multiplier:

$$\bar{V} = V + \lambda L = \int \left(\frac{1}{2} \mu \omega^2 x^2 + \lambda \right) \sqrt{1 + (y')^2} dx \quad , \quad (A-2)$$

where:

$\bar{V}$ represents the modified potential energy, and

λ the Lagrangian multiplier.

The Euler equation that minimizes this integral becomes:

$$\frac{\partial F}{\partial y'} = \left(\frac{1}{2} \mu \omega^2 x^2 + \lambda \right) \frac{y'}{\sqrt{1 + (y')^2}} = C \quad , \quad (A-3)$$

where

F represents the function inside the integral of the previous equation.

C is a constant of integration.

Simplifying and integrating this equation yields:

$$y = y_0 - \int_{x_0}^x \frac{dx}{\sqrt{\frac{1}{C^2} \left(\frac{1}{2} \mu \omega^2 x^2 + \lambda \right)^2 - 1}} \quad (\text{A-4})$$

for the equation of the shape, starting from the point (x_0, y_0) . This equation can be rearranged into the elliptic integral of the first kind; however, it can also be programmed on a calculator to be solved numerically. The shapes useful for band winding will be symmetrical about the x axis and have a vertical slope at the maximum value of x . The constants, C and λ , are chosen to meet these conditions.

APPENDIX B

METHOD FOR TRANSFORMING COMPOSITE MATERIAL PROPERTIES AND
CALCULATING THICKNESS BUILDUP IN THE OVERWRAP BANDS FOR THE
DELTAWRAP FLYWHEEL

```

C. *****
C. THIS PROGRAM CALCULATES TRANSFORMED MATERIAL PROPERTIES OF
C. UNIDIRECTIONAL COMPOSITE MATERIAL ANGLE-PLY LAMINATES AND
C. THICKNESS BUILDUP AT SPECIFIED RADIAL COORDINATE POSITIONS
C. FOR THE OVERWRAP PORTION OF THE "DELTAWRAP" FLYWHEEL.
C. THE RADIUS, THICKNESS, AVERAGE FIBER ORIENTATION ANGLE, AND
C. THE AVERAGE TRANSFORMED MATERIAL PROPERTIES ARE OUTPUT.
C. *****
C. *****
C. THE FIRST LINE OF INPUT IS THE UNIDIRECTIONAL ELASTIC PROPERTIES
C. WITH THE 1 AXIS ALIGNED WITH THE FIBER DIRECTION.
C. RO IS THE HUB RADIUS OR INNER RADIUS OF COMPOSITE,
C. RM IS THE OUTSIDE RADIUS OF COMPOSITE,
C. W IS THE BAND WIDTH,
C. NB IS THE NUMBER OF BANDS IN THE SET,
C. TB IS THE BAND THICKNESS,
C. NEL IS THE TOTAL NUMBER OF ELEMENT ROWS IN THE MODEL,
C. REL(I) IS THE RADIAL COORDINATE CORRESPONDING TO NE(I),
C. NE(I) IS THE NUMBER OF ELEMENT ROWS TO COORDINATE REL(I),
C. NOTE: REL(I) AND NE(I) ARE INPUT IN PAIRS UNTIL NE(I) EQUALS
C. NEL (MAX OF 10 PAIRS) AND REL(1) = RO, NE(1) = 0.
C. *****
      REAL NU12,NU13,NU23,M,N,M2,N2,M4,N4,MN,NUNS,NUNT,NUST
      DIMENSION REL(10), NE(10), RTHK(100), THK(100)
      ACCEPT [E11,E22,E33,NU12,NU13,NU23,G12,G13,G23]
      ACCEPT [RO,RM,W,NB,TB,NEL]
      DO 1 I = 1,10
      ACCEPT [REL(I), NE(I)]
      IF(NE(I) .EQ. NEL) GO TO 2
1    CONTINUE
2    CONTINUE
      OPEN(3,OUTPUT,/TEMP/)
10  V = 1./(1. - (NU12**2)*E22/E11 - (NU23**2)*E33/E22 - (NU13**2)
      *E33/E11 - 2.*NU12*NU23*NU13*E33/E11)
      C11 = (1. - (NU23**2)*E33/E22)*V*E11
      C22 = (1. - (NU13**2)*E33/E11)*V*E22
      C33 = (1. - (NU12**2)*E22/E11)*V*E33
      C12 = (NU12*E22/E11 + NU23*NU13*E33/E11)*V*E11
      C13 = (NU13*E33/E11 + NU12*NU23*E33/E11)*V*E11
      C23 = (NU23*E33/E22 + NU12*NU13*E33/E11)*V*E22
      C44 = G23
      C55 = G13
      C66 = G12
      RAD = 57.29577951
      PI = 3.141592654
      WRITE [1,101] E11,E22,E33,NU12,NU13,NU23,G12,G13,G23
      WRITE [1,104] RO,RM,W,NB,TB,NEL
      WRITE [1,103]
      II = 2

```

```

DO 100 I = 1,NEL
  IF(I .GT. NE(II)) II = II+1
  R1 = REL(II-1)
  R2 = REL(II)
  DR = (R2-R1)/(NE(II) - NE(II-1))
  RANG = R1 + (I-NE(II-1))*DR - DR/2.
  RTHK(I) = R1 + (I-NE(II-1))*DR
  PHI2 = PI/2.
  IF(RTHK(I) .GT. RO+W) RTAN2 = SQRT(RTHK(I)**2 - (RO+W)**2)
  RTAN1 = SQRT(RTHK(I)**2 - RO**2)
  IF(RTHK(I) .GT. RO+W) PHI2 = ATAN((RO+W)/RTAN2)
  PHI1 = ATAN(RO/RTAN1)
  THK(I) = 2.*TB*NB*(PHI2 - PHI1)/PI
  PHI2 = 0.0
DO 100 I = 1,NEL
  IF(I .GT. NE(II)) II = II+1
  R1 = REL(II-1)
  R2 = REL(II)
  DR = (R2-R1)/(NE(II) - NE(II-1))
  RANG = R1 + (I-NE(II-1))*DR - DR/2.
  RTHK(I) = R1 + (I-NE(II-1))*DR
  PHI2 = PI/2.
  IF(RTHK(I) .GT. RO+W) RTAN2 = SQRT(RTHK(I)**2 - (RO+W)**2)
  RTAN1 = SQRT(RTHK(I)**2 - RO**2)
  IF(RTHK(I) .GT. RO+W) PHI2 = ATAN((RO+W)/RTAN2)
  PHI1 = ATAN(RO/RTAN1)
  THK(I) = 2.*TB*NB*(PHI2 - PHI1)/PI
  PHI2 = 0.0
  IF(RANG .GT. RO+W) RTAN2 = SQRT(RANG**2 - (RO+W)**2)
  RTAN1 = SQRT(RANG**2 - RO**2)
  IF(RANG .GT. RO+W) PHI2 = ATAN(RTAN2/(RO+W))
  PHI1 = ATAN(RTAN1/RO)
  DPHI = (PHI1 - PHI2)/10.
  C11P = 0.0
  C22P = 0.0
  C12P = 0.0
  C33P = 0.0
  C13P = 0.0
  C23P = 0.0
  C55P = 0.0
DO 50 J = 1,10
  BETA = PHI1 - J*DPHI + DPHI/2.
  M = SIN[BETA]
  N = COS[BETA]
  M2 = M**2
  N2 = N**2
  M4 = M**4
  N4 = N**4
  MN = M2*N2
30 C11P = (M4*C11 + MN*(2.*C12 + 4.*C66) + N4*C22)/10. + C11P
  C22P = (N4*C11 + MN*(2.*C12 + 4.*C66) + M4*C22)/10. + C22P
  C12P = (MN*(C11 + C22 - 4.*C66) + (M4+N4)*C12)/10. + C12P
  C33P = C33/10. + C33P
  C13P = (M2*C13 + N2*C23)/10. + C13P
  C23P = (N2*C13 + M2*C23)/10. + C23P
  C55P = (N2*C44 + M2*C55)/10. + C55P
50 CONTINUE
  BETA = (PHI1 + PHI2)/2.
  X = C11P*C22P*C33P + 2.*C12P*C23P*C13P - C22P*C13P**2

```



```

      - C11P*C23P**2 - C33P*C12P**2
S11P = (C22P*C33P - C23P**2)/X
S22P = (C11P*C33P - C13P**2)/X
S12P = (C13P*C23P - C12P*C33P)/X
S13P = (C12P*C23P - C13P*C22P)/X
S23P = (C12P*C13P - C23P*C11P)/X
S55P = 1./C55P
31 EN = E33
   ES = 1./S11P
   ET = 1./S22P
   NUNS = -EN*S13P
   NUNT = -EN*S23P
   NUST = -ES*S12P
   GNS = 1./S55P
   BETA = BETA*RAD
   WRITE(1,102) RTHK(I),THK(I),BETA,EN,ES,ET,NUNS,NUNT,NUST,GNS
   WRITE(3,(8F7.3)) BETA,EN,ES,ET,NUNS,NUNT,NUST,GNS
100 CONTINUE
101 FORMAT(////6HMODULI /6HE11 = F6.3/6HE22 = , F6.3/6HE33 = ,F6.3//
        16HPOISSON'S RATIOS /7HNU12 = F6.3/7HNU13 = F6.3/7HNU23 = F6.3//
        12HSHEAR MODULI /6HG12 = F6.3/6HG13 = F6.3/6HG23 = F6.3//)
102 FORMAT (F7.3,F8.4,F6.1,3F7.2,4F7.3)
103 FORMAT (//22X,42H-----MODULI----- --POISSON'S RATIOS- /
        7H RADIUS, 8H THICK, 6H BETA, 3X, 2HEN, 5X, 2HES, 5X,
        2HET, 4X, 4HNUNS, 3X, 4HNUNT, 3X, 4HNUST, 4X, 3HGNS //)
104 FORMAT (4HRO =,F6.3/4HRM =,F6.3/3HW =,F6.3/4HNB =,I4/4HTB =,
        F6.4/5HNEL =,I4)
200 CONTINUE
   CLOSE
   END

```

MODULI

E11 = 14.000

E22 = 0.800

E33 = 0.800

POISSON'S RATIOS

NU12 = 0.340

NU13 = 0.340

NU23 = 0.340

SHEAR MODULI

G12 = 0.300

G13 = 0.300

G23 = 0.300

RO = 0.750

RM = 10.000

W = 0.750

NB = 126

TB = 0.0030

NEL = 50

RADIUS	THICK	BETA	-----MODULI-----			--POISSON'S RATIOS--			GNS
			EN	ES	ET	NUNS	NUNT	NUST	
0.900	0.1409	12.3	0.80	0.82	11.56	0.313	0.005	0.077	0.300
1.050	0.1865	19.9	0.80	1.03	8.51	0.228	-0.003	0.159	0.300
1.200	0.2155	24.1	0.80	1.34	7.31	0.167	0.001	0.210	0.300
1.350	0.2363	27.0	0.80	1.66	6.75	0.131	0.007	0.244	0.300
1.500	0.2520	29.1	0.80	1.95	6.44	0.109	0.012	0.267	0.300
1.650	0.1610	39.7	0.80	2.24	3.60	0.068	0.022	0.494	0.300
1.800	0.1336	46.9	0.80	2.51	2.09	0.031	0.053	0.771	0.300
1.950	0.1162	51.6	0.80	2.88	1.48	0.006	0.095	1.007	0.300
2.100	0.1036	55.2	0.80	3.35	1.19	-0.010	0.137	1.207	0.300
2.250	0.0938	58.1	0.80	3.92	1.04	-0.019	0.174	1.368	0.300
2.600	0.0776	61.9	0.80	4.98	0.91	-0.023	0.222	1.550	0.300
2.950	0.0665	65.8	0.80	6.51	0.83	-0.020	0.265	1.648	0.300
3.300	0.0584	68.7	0.80	7.88	0.80	-0.015	0.289	1.623	0.300
3.650	0.0521	71.0	0.80	9.02	0.79	-0.010	0.304	1.536	0.300
4.000	0.0471	72.8	0.80	9.92	0.79	-0.005	0.313	1.426	0.300
4.350	0.0430	74.3	0.80	10.62	0.79	-0.001	0.319	1.313	0.300
4.700	0.0396	75.6	0.80	11.17	0.79	0.002	0.323	1.208	0.300
5.050	0.0367	76.6	0.80	11.61	0.79	0.004	0.326	1.114	0.300
5.400	0.0342	77.5	0.80	11.95	0.79	0.006	0.329	1.031	0.300
5.750	0.0320	78.3	0.80	12.23	0.79	0.008	0.330	0.959	0.300
6.100	0.0301	79.0	0.80	12.45	0.79	0.009	0.332	0.897	0.300
6.450	0.0284	79.7	0.80	12.64	0.79	0.010	0.333	0.842	0.300
6.800	0.0269	80.2	0.80	12.79	0.79	0.011	0.334	0.795	0.300
7.150	0.0256	80.7	0.80	12.92	0.79	0.012	0.334	0.754	0.300
7.500	0.0244	81.2	0.80	13.03	0.79	0.012	0.335	0.718	0.300
7.611	0.0240	81.4	0.80	13.10	0.79	0.013	0.335	0.697	0.300
7.722	0.0236	81.6	0.80	13.12	0.79	0.013	0.336	0.687	0.300
7.833	0.0233	81.7	0.80	13.15	0.79	0.013	0.336	0.678	0.300
7.944	0.0230	81.8	0.80	13.18	0.79	0.013	0.336	0.669	0.300
8.056	0.0226	81.9	0.80	13.20	0.79	0.014	0.336	0.660	0.300
8.167	0.0223	82.0	0.80	13.23	0.79	0.014	0.336	0.652	0.300

8.278	0.0220	82.1	0.80	13.25	0.79	0.014	0.336	0.644	0.300
8.389	0.0217	82.2	0.80	13.27	0.79	0.014	0.336	0.636	0.300
8.500	0.0214	82.3	0.80	13.29	0.79	0.014	0.336	0.629	0.300
8.611	0.0211	82.4	0.80	13.31	0.79	0.014	0.337	0.621	0.300
8.722	0.0209	82.5	0.80	13.33	0.79	0.014	0.337	0.615	0.300
8.833	0.0206	82.6	0.80	13.35	0.79	0.015	0.337	0.608	0.300
8.944	0.0203	82.7	0.80	13.36	0.79	0.015	0.337	0.602	0.300
9.056	0.0201	82.8	0.80	13.38	0.79	0.015	0.337	0.595	0.300
9.167	0.0198	82.9	0.80	13.40	0.79	0.015	0.337	0.589	0.300
9.278	0.0196	83.0	0.80	13.41	0.79	0.015	0.337	0.584	0.300
9.389	0.0194	83.1	0.80	13.43	0.79	0.015	0.337	0.578	0.300
9.500	0.0191	83.2	0.80	13.44	0.79	0.015	0.337	0.573	0.300
9.571	0.0190	83.2	0.80	13.45	0.79	0.015	0.337	0.568	0.300
9.643	0.0189	83.3	0.80	13.46	0.80	0.015	0.337	0.565	0.300
9.714	0.0187	83.3	0.80	13.47	0.80	0.015	0.337	0.562	0.300
9.786	0.0186	83.4	0.80	13.48	0.80	0.016	0.337	0.559	0.300
9.857	0.0184	83.4	0.80	13.49	0.80	0.016	0.337	0.556	0.300
9.929	0.0183	83.5	0.80	13.49	0.80	0.016	0.337	0.553	0.300
10.000	0.0182	83.5	0.80	13.50	0.80	0.016	0.338	0.550	0.300

STOP

(\$MAIN\$)200+2

APPENDIX C

CALCULATION OF CONTAINMENT ASSEMBLY DESIGN LOADS

Radial Containment Design Load

Assuming a bihub burst of the flywheel, as shown in Figure 5, the centrifugal force, F , over a radial section of the flywheel is:

$$F = \rho \frac{t\omega^2}{3g} (R_o^3 - R_i^3) \quad , \quad (C-1)$$

where:

- ρ represents the flywheel density (0.05 lb/in³),
- t the flywheel axial thickness (3.4 inches),
- ω the flywheel angular velocity (4019.5 rad/sec),
- R_o the flywheel outer radius (10.0 inches), and
- R_i the flywheel inner radius (7.5 inches).

Assuming intimate contact of the flywheel over the entire semicircular periphery, the equivalent pressure load on the inner surfaces of a containment ring having equal axial length is found from:

$$2F = \int_{-\pi/2}^{\pi/2} P_i t \cos \theta R_o d\theta \quad , \text{ where:} \quad (C-2)$$

$$P_i = \frac{F}{t R_o} \quad (C-3)$$

Then:

$$P_i = \rho \frac{\omega^2}{3g} \left(\frac{R_o^2 - R_i^3}{R_o} \right) = 40,330 \text{ psi.} \quad (C-4)$$

Axial Containment Design Load

Axial Load Distribution During Flywheel Failure - Consider that a flywheel bihub burst is restrained by the containment ring, as in the previous section. Then, considering the weak

transverse strength of the composite and noting the magnitude of the equivalent pressure of the previous section, the flywheel section should deform and assume the axial thickness of the inside of the containment housing. Axial and radial equilibria of a cross section could then be represented, as shown in Figure C-1: (1) the centrifugal force of the element

Figure C-1. ASSUMED STRESS DISTRIBUTION OF THE FLYWHEEL RIM SECTION TO RADIALLY AND AXIALLY CONTAIN IT.

balanced by an equivalent pressure from the containment ring, and (2) an axial pressure distribution varying with radial position which is dependent on the radial pressure, p_c , the shear strength, and internal friction of the composite. Analyses for these types of conditions exist in soil mechanics¹¹ which show the functional relationship for an element, as shown in Figure C-1, to be:

$$p_o = p_c \tan^2 \left(45^\circ - \frac{\phi}{2} \right) - 2C \tan \left(45^\circ - \frac{\phi}{2} \right) , \quad (C-5)$$

where:

C represents the unit cohesion, and

ϕ the angle of internal friction.

If it is assumed that the equivalent pressure, p_c , is that pressure required to balance the centrifugal force of the flywheel, then:

$$p_c = \frac{F}{t_c R_o} = \frac{t_f}{t_c} p_i = \frac{3.4}{4.5} (40,330) = 30,471 \text{ psi.} \quad (C-6)$$

Further, assume that (for Equation C-5):

C represents the shear strength of the composite, and

$\phi = \tan^{-1}$ (friction coefficient for Kevlar on Kevlar).

If it is assumed that C is equal to 6000 psi and ϕ equals 0.47; then, from Equation C-5:

$$p_o = 4669.7 \text{ psi.}$$

The axial load is determined by integrating the axial pressure distribution over the area of contact of the flywheel and containment lid:

$$A = \int_0^{2\pi} \int_{R_i}^{R_o} \left(\frac{p_o r}{R_o - R_i} - \frac{p_o R_i}{R_o - R_i} \right) r d\theta dr, \text{ or} \quad (C-7)$$

$$= 2\pi \left[\frac{p_o}{3} \left(\frac{R_o^3 - R_i^3}{R_o - R_i} \right) - \frac{p_o R_i}{2} \frac{(R_o^2 - R_i^2)}{R_o - R_i} \right] \quad (C-8)$$

Using the values of R_o , R_i , and p_o given previously, the axial load is calculated to be 336,170 lb.

Torsional Load

It is assumed that the containment ring is freely rotatable and the torsional load results from the frictional forces developed by the axial pressure distributions. Then, the total torque is equal to the sum of the torques from the lid and baseplate or:

$$T = 2 \int_0^{2\pi} \int_{R_i}^{R_o} \mu p(r) r da, \text{ or} \quad (C-9)$$

$$= 2 \int_0^{2\pi} \int_{R_i}^{R_o} \mu \left(\frac{p_o r}{R_o - R_i} - \frac{p_o R_i}{R_o - R_i} \right) r d\theta r dr, \text{ or} \quad (C-10)$$

$$= 4 \pi \mu \left[\frac{p_o}{4} \left(\frac{R_o^4 - R_i^4}{R_o - R_i} \right) - \frac{p_o R_i}{3} \left(\frac{R_o^3 - R_i^3}{R_o - R_i} \right) \right] \quad (C-11)$$

Assuming that the coefficient of friction of the composite on steel, μ , is equal to 0.4; and, using the values of R_o , R_i , and p_o as before, the total torque is 206,613 ft lb.

Shaking Load

Assume that the shaking load results from the flywheel orbiting on the inner surface of the containment ring; then, from the classical solution of an eccentric mass rotating within another spring mass system, the maximum deflection developed at speeds much greater than the critical of the system is:¹²

$$\delta = \frac{m}{M} e \quad , \quad (C-12)$$

where:

m represents the mass of the rotating mass (ie, the flywheel mass; 25.54 lb),

M the system mass (ie, the test assembly mass; 963.64 lb), and

e the eccentricity (ie, the radial clearance of the flywheel and the containment ring; 11/32 inch).

Since, for a cantilever beam loaded by an end load, P :

$$\delta = \frac{PL^3}{3EI} \quad , \text{ or:} \quad (C-13)$$

$$P = \frac{3EI\delta}{L^3} \quad , \quad (C-14)$$

where:

E represents the modulus of the torque tube (30×10^6 psi),

I the section modulus of the torque tube (48 in^4), and

L the length of the torque tube (16 in),

then, the shaking load is:

$$P = \frac{3 (30 \times 10^6) (48) \left(\frac{24.55}{963.64} \right) \left(\frac{11}{32} \right)}{(16)^3} = 9236 \text{ lb.} \quad (C-15)$$

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