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PARADAY ROTATION MEASUREMENTS IN HIGH- β FUSION EXPERIMENTS

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SUMMARY

Faraday rotation is the rotation of the direction of polarization of a plane-polarized electromagnetic wave as it propagates parallel to a magnetic field in a plasma-magnetic field medium. The right and left circularly polarized components of the wave travel at different phase velocities producing a net relative phase shift during propagation which causes the rotation of the plane of polarization. The magnitude of the rotation is proportional to the integral of the product of the magnetic field and plasma electron density over the path length of the propagating wave in the plasma-field medium. A measurement of the rotation can be combined with either a measurement of the plasma density or the magnetic field along the same path to give respectively the internal magnetic field, B_i , or the plasma electron density, n_e .

The application of the rotation measurement to a linear magnetic-plasma system is described in which the $\int n_e(l)dl$ is determined by interferometric measurements and combined with the measured rotation $\int B_i(l)n_e(l)dl$ to give the average value of the internal magnetic field $B_i(r)$ which exists over the plasma length l at the radius r . The measurements of $B_i(r)$ are combined with those of the external magnetic field B_e to determine the average plasma beta as a function of radius, $\beta = 1 - B_i^2(r)/B_e^2$. The Faraday rotation measurement uses a 1.5-mm diameter beam of visible radiation (6328 Å) from a He-Ne gas laser as a plasma-magnetic field probe and a calcite Wollaston prism as the polarization analyzer with silicon diode detectors. The rotation is a small effect at optical wavelengths even for substantial densities and fields. The rotation is proportional to the square of the probing wavelength which suggests the use of infrared laser wavelengths. However, density gradients perpendicular to the beam direction cause beam deflections that also scale with λ^2 . The size and spatial sensitivity of detectors is also an important consideration in the selection of the probing wavelength.

1. INTRODUCTION

A measurement of the Faraday rotation of a plane polarized wave propagating through a plasma-magnetic field medium can be used to determine the integral of the product of the magnetic field and plasma density over the length of the medium. Such Faraday rotation measurements can be combined with plasma density profile measurements to give the average value of the internal magnetic field B_i which exists over the plasma length. The measurements of B_i can be combined with the external magnetic field B_e to determine the average plasma beta along the path. Using a laser beam several investigations have demonstrated the rotation of polarization in a plasma with superimposed magnetic field. The selection of the laser wavelength, λ , depends on several factors including the λ^2 dependence of the Faraday rotation, the λ^2 dependence of the plasma refractive effects, and the sensitivity and uniformity of response of detectors.

The interaction between electromagnetic waves in the optical region and plasma is weak, and is manifested primarily in a slightly altered propagation velocity of the wave. In a plasma, the refractive index, which is the ratio of the phase velocity in vacuum to that in the plasma, is primarily a function of electron density and this is the main plasma parameter determined by various measurements of phase changes induced in a wavefront as a result of its transversal of a plasma. Probing with optical radiation has advantages: the plasma conditions are not perturbed by the measurement and the small size of optical wavelengths permits high spatial resolution. However, the sensitivity is limited and the usefulness of refractivity measurements is generally restricted to plasma densities greater than $\sim 10^{14} \text{ cm}^{-3}$. A laser beam is a particularly useful optical probe since it is bright and monochromatic and competes favorably with the plasma self-luminosity in a narrow spectral band, while simultaneously being well collimated because of its small divergence. "Standard measurements" of refractive effects include interferograms, schlieren photographs, and shadowgrams as well as Faraday rotation measurements. The optical refractivity of plasmas and the associated plasma parameters which can be determined by various measurements of phase changes induced in a wavefront as a result of its transversal of a plasma have been reviewed by Jahoda and Sawyer[1].

2. REVIEW OF THEORY

An electromagnetic wave propagating through a plasma exerts a force on and gives rise to accelerations of the charged plasma particles which in turn modify the time-varying electromagnetic (EM) field components of the wave. Self-consistent solutions for an EM wave in a plasma can be obtained in principle by solving Maxwell's equations with appropriate boundary conditions together with the Lorentz force equation of motion for charged particles in an EM field. Generally the magnetic forces exerted by a wave field are much weaker than the electric forces and can be neglected.

Consider the plane wave solutions for an electric field propagating in the z direction of the form

$$\underline{E}(z,t) = \underline{E}_0 \exp[i(kz - \omega t)] \quad , \quad (1)$$

where the real part of $\underline{E}$ represents the physical quantity, the wave number $k = 2\pi/\lambda$ defines

wavelength λ and ω is the wave frequency. A solution requires specifying the "dispersion relation", the functional relation between ω and k . The wave equation for the electric field can be derived from Maxwell's equations and with the assumption of pure transverse waves in the absence of a static transverse magnetic field one obtains

$$\nabla^2 \underline{E} - \frac{4\pi}{c^2} \frac{\partial \underline{j}}{\partial t} = \frac{1}{c^2} \frac{\partial^2 \underline{E}}{\partial t^2} = 0, \quad (2)$$

where the units are Gaussian and $\underline{j}$ refers to current densities. The functional relation between $\underline{j}$ and $\underline{E}$ must be determined in order to solve for eq. (2).

Since an applied magnetic field exerts a force in a perpendicular direction on a charge moving with the velocity induced by the electric field of the waves, the equations of motion of the plasma particles in two perpendicular Cartesian coordinates become coupled. In the absence of collisions, the equations of motion in a right-hand coordinate system for an electron, the ions can be neglected because of the large disparity between the electron and ion masses, subject to the electric field component of a transverse wave and a steady magnetic field, B_0 , applied in the direction of wave propagation become

$$\frac{d\underline{v}_\pm}{dt} = -\frac{e}{m} \underline{E}_\pm \pm i\omega_e \underline{v}_\pm, \quad (3)$$

where ω_e is the electron gyrofrequency, $\underline{v}_\pm = \underline{v}_x \pm i\underline{v}_y$ and $\underline{E}_\pm = \underline{E}_x \pm i\underline{E}_y$. As $\underline{E}_x$ and $\underline{E}_y$ oscillate in time the two-dimensional vectors $\underline{E}_\pm$ will rotate in the x-y plane. For the special case of $|\underline{E}_x| = |\underline{E}_y|$ two oppositely rotating "circular polarizations" result, one of which will correspond to the direction of the force on an electron in the magnetic field and the other of which will oppose the direction of this force.

The solution of eq. (3) is

$$\underline{v}_\pm = i \frac{e}{m_e} \frac{\underline{E}_\pm}{(\omega \pm \omega_e)} \quad (4)$$

and the current due to electrons is

$$\underline{j}_\pm = -en_e \underline{v}_\pm. \quad (5)$$

Substituting into the wave equation (2) and using eq. (1) yields the dispersion relation:

$$k_\pm = \frac{\omega}{c} \left[1 - \frac{\omega_p^2}{\omega^2 (1 \pm \omega_e/\omega)} \right]^{1/2}. \quad (6)$$

B. MAGNITUDE OF ROTATION

As was shown above in the presence of a static axial magnetic field there are two distinct propagation velocities through a plasma associated respectively with the two E -field amplitudes, $E_{\pm} = E_x \pm iE_y$. Solving for E_x and E_y and taking real parts give

$$E_x = \frac{E_+}{2} \cos(k_+ z - \omega t) + \frac{E_-}{2} \cos(k_- z - \omega t) \quad (7)$$

$$E_y = \frac{E_+}{2} \sin(k_+ z - \omega t) - \frac{E_-}{2} \sin(k_- z - \omega t).$$

Consider an electric wave of unit amplitude, linearly polarized in the x direction, incident at the magnetized plasma boundary $z = 0$. These boundary conditions require $E_+ = E_- = 1$. The x and y components at arbitrary distance z into the plasma become:

$$E_x = \cos\left(\frac{k_+ + k_-}{2} z - \omega t\right) \cos\left(\frac{k_+ - k_-}{2} z\right) \quad (8)$$

$$E_y = \cos\left(\frac{k_+ + k_-}{2} z - \omega t\right) \sin\left(\frac{k_+ - k_-}{2} z\right).$$

The two components oscillate in phase, linear polarization, but the last factor in each term indicates that as z increases the maximum amplitudes of the x and y components vary sinusoidally with a 90° phase difference. This means that the plane of the linear polarization at $z = l$ is rotated by

$$\theta = \frac{k_+ - k_-}{2} l \quad (\text{radians}). \quad (9)$$

Since a right-hand coordinate system with B along $+z$ was assumed in the equations of motion (3) and it follows from eq. (6) that $k_+ > k_-$, the rotation is counterclockwise looking toward the source of radiation at negative z and is in the same sense as the electrons gyrate about the magnetic field.

A simple physical interpretation of the sense of rotation is as follows. In the magnetized medium one of the oppositely rotating circular polarizations rotates in the same sense as the electrons gyrate. This is the component that shows a resonance effect when the propagation frequency and the gyrofrequency become equal, and is therefore identified with the k_- part of eq. (6). Since $k_- < k_+$, its associated phase propagation velocity, $c_- = \frac{\omega}{k_-}$, is greater than c_+ . At a fixed position in space, the faster wave is leading in phase relative

to the slower wave, therefore the net rotation of the linear polarization will also be in the sense of the faster wave, i.e., in rotation with the electrons.

Using the first order terms in a power expansion of eq. (6) in the small parameters ω_p/ω and ω_e/ω to evaluate $k_+ - k_-$, the rotation, eq. (9), becomes

$$\theta = \frac{\omega_e \omega_p^2}{2c\omega^2} l = \frac{1}{2\pi} \frac{e^3}{m_e^2 c^4} \lambda^2 n_e B l$$

$$= 2.63 \times 10^{-17} \lambda^2 n_e B l \quad (\text{radians}), \quad (10)$$

where the wavelength λ and the path length l are in cms, the electron density n_e is cm^{-3} and the axial magnetic field B is in gauss.

The substitution of numerical values in eq. (10) shows the rotation is small at optical wavelengths even for substantial plasma densities and fields. For example, at the He-Ne laser wavelength of 6328 Å, for a 20-kG field, one-meter path length and 5×10^{16} electrons per cm^3 the rotation is 36 minutes of arc. The λ^2 dependence suggests a significant increase in the rotation angle by using infrared laser wavelengths. However, density gradients perpendicular to the beam direction cause beam deflections that also increase with λ^2 . Consequently the advantage of increased rotation angle may in practice be more than offset by increased refractive beam deviations, particularly in view of the small size of infrared detectors and the variations of spatial sensitivity over their surface.

3. MEASUREMENT OF ROTATION

Rotations of the direction of linear polarization are measured by observing the intensity variation transmitted by a polarization analyzer of fixed orientation. The transmitted intensity is proportional to $\cos^2 \alpha$, where α is the angle between the polarization to be measured and the maximum pass direction of the analyzer. If a transient plasma modulates a steady laser source, thereby permitting measurement of the ac component change in the signal level, the greatest sensitivity is achieved at $\alpha \sim 45^\circ$. If, on the other hand, a transient laser pulse illuminates a plasma that remains constant during the light pulse duration, then the total transmitted laser signal is measured and the greatest fractional change (comparing the case of plasma present versus no plasma) will occur for minimum total transmission, i.e., near $\alpha = 90^\circ$. In both cases the sensitivity can be doubled by using two analyzers set at positive and negative rotations to the unrotated polarization, respectively, and subtracting the two results. Taking the difference signal has the further advantage of cancelling intensity variations due to phenomena other than the rotation of the plane of polarization. In addition the sum of the two signals can serve as the normalization needed to determine the rotation angle θ from the difference signal. The angle θ gives an averaged value over the total path length of the plasma-magnetic field region

$$\theta = 1.51 \times 10^{-15} \lambda^2 \int_0^L B_1 n_e dl \quad (\text{deg}), \quad (11)$$

where B_1 is the magnetic field (gauss), n_e the plasma electron density (cm^{-3}), L the plasma length (cm), and λ the wavelength of the laser beam (cm). Either the average B_1 or average n_e can be deduced if independent knowledge of the other parameter is available.

Faraday rotation measurements of plasmas have been reported for continuous wave helium-neon lasers operating at 6328 Å by Falconer and Ramsden [2], and by Gribble, Little, Morse, and Quinn [3], and at 3.39 μ by Dougal, Craig, and Gribble [4], and by Bogen and Rnsböldt [5], and for two types of pulsed lasers, the ruby laser at 6943 Å by Grassmann and Wueff [6], and the water vapor laser at 28 μ by Dellis, Earl, Malein, and Ward [7].

A schematic diagram of the Faraday-rotation experimental arrangement, as used in the Scylla IV experiment, is shown in Fig. 1. A 70 mW He-Ne gas laser at 6328 Å sends a plane polarized light beam through the plasma region parallel to the axis of the 1-m long, 11.2-cm diameter compression coil, into a polarization analyzer, and onto a pair of silicon detectors. Two narrow-band interference filters in series, with transmissions peaked at 6328 Å, serve to reject the plasma light as well as the 6943 Å ruby laser light of an interferometer used to measure $\int_0^L n_e dl$. The polarization analyzer consists of a 13° calcite Wollaston prism which splits the plane-polarized laser beam into two perpendicular components of polarization. The prism deviates both components by 13° in opposite directions to yield a 26° separation between the two beam components. The prism is positioned so that in the absence of plasma the plane of polarization of the laser beam is 45° with respect to that of the prism components in order that the two signals will have equal intensity.

The silicon detectors consist of a selected, matched pair of photo-conductive detectors with 20-V/W sensitivity into a 100- Ω load, a measured risetime of 0.1 μ s, and a measured variation in sensitivity over the 0.7-cm diameter active area of less than 2%. Each detector monitors one component of the laser beam. A wide band, high-common-mode rejection ratio, integrated amplifier provides a difference signal from the two detectors. Two additional integrated circuit amplifiers, each with the same gain as the differential amplifier, are each connected to a detector to obtain individual detector signals as well as a sum signal. The Faraday rotation is proportional to the ratio of the difference to the equivalent sum of the detector signals after amplification.

In the arrangement of Fig. 1, lens A serves to focus the laser beam (~ 2 mm diameter) to a minimum diameter of ~ 2 mm at the center of the compression coil. Lens B serves to reduce the divergence of the beam due to refractive effects and to distribute it over a reasonable area on the detectors. A conventional shutter is used to minimize the exposure of the detectors to the laser beam. In addition a polarization rotator is used to rotate the plane of polarization to the desired angle relative to the analyzer and the plane of the mirror surfaces. The rotator can also be used to check that the observed signals are due to actual rotation by observing the reversal of the difference signal caused by a 90° rotation of the incident plane of polarization.

The resolution of the Faraday rotation is determined by the signal-to-noise ratio of the laser-electronic system and by the effects of beam refractivity arising from transverse electron-density gradients in the plasma. Refractive effects do not usually limit the measurements at $\lambda = 6328$ Å. The signal-to-noise ratio limitation was determined by the laser power and the 3-mV peak-to-peak noise at the differential amplifier output. Considering losses in the optics, a power of 20 mW into the analyzer system resulted in a minimum resolvable rotation of 0.01° . With the plasma length and densities in the Scylla IV device,

this corresponds to a magnetic field of ~ 0.8 kG, which is 0.8% of the maximum compression field.

Typical oscillogram traces of the Faraday rotation, signals are shown in the lower traces of Fig. 2. The rotation signal (lower oscillogram) with an initial reversed bias field shows a small rotation at the beginning of the discharge due to the bias field followed by a reversal of the rotation due to the main magnetic field. The purpose of this experiment was to measure magnetic field strength in the plasma region. A Mach-Zehnder interferometer was used simultaneously to provide $\int_0^L n_e dl$ and allow the determination of B_1 from the combined measurement which assumed that

$$\int_0^L B_1 n_e dl = \bar{B}_1(r) \int_0^L n_e dl, \quad (12)$$

where $\bar{B}_1(r)$ is the average magnetic field that exists over the plasma length at the radius r .

These measurements of B_1 can be combined with that of the external magnetic field B_e to determine the average plasma β as a function of radius,

$$\beta = 1 - B_1^2/B_e^2. \quad (13)$$

The measurements can also be used to obtain the total plasma temperature at a given radius averaged over the length of the plasma from pressure balance with the magnetic field

$$k(T_e + T_1)_1 = \frac{1}{8\pi n} (B_e^2 - B_1^2) = \frac{\beta}{n} \frac{B_e^2}{8\pi}. \quad (14)$$

Here, it is assumed that $n_e = n_1 = n$.

Figure 3 shows radial profiles of B_1 , β , n_e , and T_1 obtained from simultaneous Faraday rotation measurements and density measurements in the 1-m Scylla IV device[3]. The plasma density was obtained from Mach-Zehnder interferometer measurements which provide the integral of the plasma electron density over the length of the plasma. The plasma beta was obtained from the Faraday rotation, density, and external magnetic field measurements through the use of eqs. (12) and (13). The plots of the perpendicular plasma temperature, T_1 , were determined from β , n , and B_e through the scalar pressure balance of eq. (14).

4. FARADAY ROTATION MEASUREMENTS AT 10.6 μ

Faraday rotation measurements have been made on a 40-cm diameter, 1-m long, θ -pinch Implosion Heating Experiment (IHX) by Jarboe[8] using a CO₂ laser at 10.6 μ . The experimental arrangement used a double-pass system with a spherical mirror at one end of the implosion heating coil to reflect the laser beam back through the plasma to the CuGe detector located at the same end of the coil as the laser. The center of curvature of the spherical mirror was located at the midplane of the cylindrical implosion heating coil. The lowest-order refractive effects are corrected, since the beam is reflected to approximately retrace itself through the plasma. This method has the advantage that the beam exits the plasma region with

the same position and direction, independent of first-order refraction. A Ge Brewster plate was used as the polarization analyzer.

The Faraday rotation measurement was combined with a simultaneous measurement of the plasma electron density by a fractional-fringe interferometer to determine the magnetic field during the implosion phase of the plasma discharge. Figure 4 compares the magnetic field as a function of time measured by Faraday rotation and by internal magnetic probes. The plasma density is also shown. Faraday rotation provided a fairly reliable measurement of the magnetic field in these experiments. The important features of the technique are the correction of refraction effects through the use of proper optics and the minimization of such effects by the proper choice of the angle between the plane of polarization and the axis of the polarization analyzer.

The recent availability of several strong CW laser lines between 118 μ and 884 μ by CO₂ laser pumping of passive cavities filled with appropriate molecular gases increases potentially available sensitivity of effects that scale directly with refractivity, proportional to wavelength squared, such as Faraday rotation. Although beam bending also scales as λ^2 , this limitation may be overcome in appropriate circumstances. The first plasma diagnostic experiments using lasers beyond 10 μ were performed by Dellis et al.[7]. They determined the electron density of a plasma in a magnetic field by measuring the Faraday rotation of a polarized beam from a pulsed 28- μ water vapor laser. In this experiment B_z was known from magnetic probe measurements.

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FIGURE CAPTIONS

- Fig. 1. Diagram of the experimental arrangement of a Faraday-rotation apparatus used on the Scylla IV theta pinch experiment. (Reference [3]).
- Fig. 2. Oscillograms of the applied magnetic field B_z (upper traces) and the Faraday-rotation signal θ_R (lower traces) from the 1-m Scylla IV theta pinch experiment. Upper oscillogram: Bias field = 0; Filling pressure = 15 mtorr of deuterium. Lower oscillogram: Bias field = -1.2 kG; Filling pressure = 15 mtorr of deuterium. (Reference [3]).
- Fig. 3. Radial profile graphs of B_z and β (upper plot) and n and T lower plot as a function of plasma radius taken at the time of peak compression field with a deuterium filling pressure of 15 mtorr and a bias field of -1.2 kG. (Reference [3]).
- Fig. 4. Magnetic field as a function of time measured by Faraday rotation and by magnetic probes in a θ -pinch implosion heating experiment at a radius of 12 cm. The plasma electron density is also shown. (Courtesy of Dr. T.R. Jarboe, Los Alamos Scientific Laboratory.)